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Abstract

We present and study the properties of a sticky information exchange rate model where consumers and producers update their information sets infrequently. We find that introducing inattentive consumers has important implications. Through a mechanism resembling the limited participation models, we can address the exchange rate volatility for reasonable values of risk aversion. We observe more persistence in output, consumption and employment which brings us closer to the data. Impulse responses to monetary shocks are hump shaped consistent with the empirical evidence. Forecast errors of inattentive consumers provide a channel to reduce the correlation of relative consumption and real exchange rate. However, we find that decline in the correlation is quantitatively small.

1　Introduction

Empirical evidence indicates that nominal and real exchange rates have been excessively volatile relative to major economic aggregates during the post-Bretton Woods period[1]. This paper presents a two country model with the assumption of infrequent information updating for consumers and producers. We show that sticky information on the consumer side provides a new mechanism to generate volatile exchange rates. The literature suggests at least two other approaches to modelling endogenous exchange rate volatility in a rational expectations framework: the first is pursued by Backus, Kehoe, and Kydland (1995), the other by Chari, Kehoe and McGrattan (2002)[2].

In the framework of Backus, Kehoe, and Kydland (1995), the impact of productivity shocks on international prices is magnified by a relatively low price elasticity of imports, choosing parameter values on the low end of the range commonly adopted by the literature. This strategy is labeled the "Elasticity Approach" by Corsetti, Dedola and Leduc (2009). The main problem with this approach is the trade-off between the volatility of relative prices and trade flows. When the trade structure is defined by a constant-elasticity-of-substitution aggregator over domestic and foreign goods[3], the model inherits an inverse relationship between the volatility of trade flows and international prices. The lower (higher) the elasticity of substitution between traded goods, the larger (smaller) the response of prices to shocks, whereas the opposite is true for quantities. As a result, a low import elasticity can generate the exchange rate volatility observed in the data, but this leaves the volatility of net exports counterfactually low.

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[1]We use data for the U.S. Dollar and a synthetic aggregate of the Euro-zone to quantify exchange rate volatility. Similar patterns have been consistently uncovered between the U.S. and other major OECD countries. See Chari, Kehoe and McGrattan (2002).

[2]See Corsetti, Dedola and Leduc (2009) for a comprehensive discussion.

[3]Composite good is aggregated as in Armington (1969).

Chari, Kehoe and McGrattan (2002) exploit the positive and strict link between the ratio of marginal utilities of consumption and the real exchange rate that characterizes economies with complete markets. We label this strategy the "Risk Aversion Approach". If risk aversion is sufficiently high, the variability of the ratio of home to foreign consumption observed in the data can correspond to large equilibrium movements in the real exchange rate. However, the necessary amount of risk aversion required to address real exchange rate volatility is on the high end of business cycle calibrations[4].

This paper proposes a new approach to address exchange rate volatility. We present and study the properties of a sticky information exchange rate model where consumers and producers update their information sets infrequently[5]. Similar to an environment with limited participation models, exchange rates are linked to the marginal utilities of attentive consumers who updated their information set in the current period. When a shock alters the supply side in this economy, consumption plans of inattentive consumers remain unchanged as they remain unaware of this information. The goods market is cleared by the demand response of attentive consumers who are able to update their consumption plans. As the fraction of attentive consumers decreases, their response needs to increase to clear the market. As a result, the consumption of attentive consumers is more volatile than aggregate consumption, and gets more volatile as we decrease the frequency of information updating for

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[4]Chari, Kehoe and McGrattan (2002) set the degree of risk aversion as 5, which corresponds to an elasticity of intertemporal substitution (EIS) of 0.2. Guvenen (2006) provides a comprehensive discussion on estimates of EIS, and the implications of EIS for real interest rates and consumption. Furthermore, an upper bound for risk aversion is critical for calculations regarding the welfare costs of business cycles. By using consumption data, Lucas (2003) calculates an upper bound of 2.5 for risk aversion.

[5]Microfoundations of sticky information models rely on the *inattentiveness* framework proposed by Reis (2006a) and Reis (2006b). Agents are subject to an information processing and updating cost, therefore they optimally choose the duration between the updates in this setup. Once they update their information set, they learn all shocks and all variables up to that date. Sticky information models assume that information updating is exogenous. Micro evidence of inattentiveness is based on the updating frequency of expectations, reported in public and professional forecaster surveys. See Carroll (2003), Mankiw, Reis and Wolfers (2004).

consumers. Since the real exchange rate is determined by the marginal utilities of attentive consumers, we observe higher volatility in real exchange rates. With an average information updating duration of 4 quarters, real exchange rates generated by the model are as volatile as in the data for a risk aversion[6] of 2.

When we look at frictions on the producer side assuming attentive consumers, we observe that the sticky information model is virtually identical to the sticky price model[7]. Introducing inattentiveness to the consumer side brings the model in line with the data by (i) increasing the volatility of exchange rates, (iii) generating hump-shaped impulse responses[8] for quantities to a monetary shock, therefore increasing persistence and (iii) reducing the correlation between relative consumption and real exchange rates[9]. Forecast errors of inattentive consumers provide a channel to reduce the Backus-Smith correlation[10].

Organization of the paper is as follows. First, we introduce our model in a nested framework, where we distinguish a standard sticky price model and the proposed sticky information model. We also introduce an alternative wage-posting model within the sticky information framework. Next, we present results regarding the "Risk Aversion Approach"[11] by using

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[6]Trabant and Uhlig (2010) refer to a value of 2 as a consensus in macro literature.

[7]Differences regarding the correlations of output and inflation, the speed of price response to monetary shocks are out of this study's scope. We concentrate on the moments which describe the international business cycles.

[8]See Kim (2001) and Landry (2009) for VAR evidence regarding the impulse responses to a monetary shock.

[9]Notice that the real exchange rate is related to the consumption of *attentive* consumers, not the aggregate consumption in this framework. However, we observe that size of the decline in the correlation is fairly small. Considering the simple structure of the model, this channel needs to be further investigated.

[10]Theoretical models produce large and positive correlations between the real exchange rate and relative consumption, as the real exchange rate is tightly linked to the ratio of marginal utilities of consumption. Standard theory implies that consumption is higher wherever it is cheaper, in stark contrast with the data. Real exchange rates in the data appreciate when domestic consumption is higher than foreign consumption, leading to a low and often negative correlation between real exchange rates and relative consumption. Therefore, consumption is higher where it is more expensive. See Backus-Smith (1993) and Chari, Kehoe and McGrattan (2002).

[11]Regarding the "Elasticity Approach", we observe the price-quantity volatility trade-off with our no-

a model with attentive consumers. We proceed by giving the results with inattentive consumers, discussing the mechanism that generates more exchange rate volatility and checking the robustness of our volatility amplification result. Then, we compare alternative models by reporting a set of business cycle moments. Final section concludes.

2 Model

We start by describing the economy where consumers update their information set every period. That is, consumers are assumed to be *attentive*. Then, we describe the economy with inattentive consumers. For producers, we summarize the price-setting problem[12] under two alternative assumptions: the first setup features sticky prices (infrequent *price* updating), while the second assumes sticky information (infrequent *information* updating). Our benchmark model features inattentive consumers and inattentive producers (IC-IP model), and we assume flexible labor response. Alternative models are also introduced for comparison. We can summarize underlying assumptions as follows: (i) Attentive consumers and sticky prices (AC-SP model), (ii) Attentive consumers and inattentive producers (AC-IP model), (iii) inattentive consumers and inattentive producers with wage posting assumption (IC-IP-WP model).

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frictions model, e.g. attentive consumers, attentive producers, and a flexible price setting environment. Since we develop a framework with nominal rigidities, we compare our mechanism with the "Risk Aversion Approach" of Chari, Kehoe and McGrattan (2002). See Backus, Kehoe, and Kydland (1995) and Corsetti, Dedola and Leduc (2009) for further discussion.

[12]We assume time-dependent price/information updating.

2.1   Households

2.1.1   Environment

The world economy consists of two countries, home and foreign[13], each specialized in the production of a composite traded good. In this economy, we have complete markets with one-period contingent bonds. Households maximize lifetime utility,

$$\max E_0 \sum_{t=0}^{\infty} \beta^t U\left(C_t, N_t\right)$$

subject to a sequence of budget constraints, which can be expressed in domestic currency units as:

$$W_t N_t + B_t + \Pi_t + \tau_t \geq P_t C_t + \sum_{s^{t+1}} v_{t,t+1}(s^{t+1}) B_{t+1}(s^{t+1})$$

where C_t is the composite consumption good[14]. N_t is the labor supply and W_t is the nominal wage rate. Π_t is profits of intermediate goods producers and τ_t is transfers of home currency. P_t represents the price index for home country. B_t is the amount of contingent securities owned when history s^t is realized[15], and $v_{t,t+1}(s^{t+1})$ are the prices for one period Arrow securities. We assume that securities pay off in domestic currency. Decision variables for the household are asset holdings and labor supply.

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[13]Countries are assumed to be of equal size, and foreign country variables are denoted with an asterisk.

[14]Home and foreign goods are aggregated by a CES index. Details are given in the next section.

[15]History consists of monetary and productivity shocks in the home and foreign country.

2.1.2 Composite Consumption Index

Consumption preferences are described by the following composite index of domestic and imported bundles of goods:

$$C_t \equiv \left[(1-\gamma)^{\frac{1}{\eta}} C_{H,t}^{\frac{\eta-1}{\eta}} + \gamma^{\frac{1}{\eta}} C_{F,t}^{\frac{\eta-1}{\eta}} \right]^{\frac{\eta}{\eta-1}}$$

where $\eta > 0$ is the elasticity of substitution between domestic and foreign goods and γ is the weight of imported goods in the consumption basket[16]. Each consumption bundle $C_{H,t}$ and $C_{F,t}$ is composed of imperfectly substitutable varieties, with elasticity of substitution $\nu > 1$. Optimal allocation of expenditure between each variety of goods yields,

$$C_{H,t}(i) = \left(\frac{P_{H,t}(i)}{P_{H,t}} \right)^{-\nu} C_{H,t}; \quad C_{F,t}(i) = \left(\frac{P_{F,t}(i)}{P_{F,t}} \right)^{-\nu} C_{F,t}$$

where each variety is indexed by i, $C_{H,t} \equiv \left[\int_0^1 C_{H,t}(i)^{\frac{\nu-1}{\nu}} di \right]^{\frac{\nu}{\nu-1}}$ and $C_{F,t} \equiv \left[\int_0^1 C_{F,t}(i)^{\frac{\nu-1}{\nu}} di \right]^{\frac{\nu}{\nu-1}}$. Optimal expenditure on home and foreign goods gives,

$$C_{H,t} = (1-\gamma) \left(\frac{P_{H,t}}{P_t} \right)^{-\eta} C_t; \quad C_{F,t} = \gamma \left(\frac{P_{F,t}}{P_t} \right)^{-\eta} C_t$$

where $P_t \equiv \left[(1-\gamma) P_{H,t}^{1-\eta} + \gamma P_{F,t}^{1-\eta} \right]^{\frac{1}{1-\eta}}$ is the CPI index. We can express the log-linearized inflation dynamics as follows[17],

$$\hat{\pi}_t = (1-\gamma)\hat{\pi}_{H,t} + \gamma\hat{\pi}_{F,t}$$

where hat notation represents the log-deviations from steady state.

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[16]For the foreign country, goods produced at home country are the import goods. Therefore, γ is the share of home goods in the foreign consumption basket.

[17]Log-linearization is around the zero-inflation steady state, assuming symmetry across home and foreign countries.

2.1.3 Optimality Conditions

We denote the marginal utility of consumption by λ^c and the marginal disutility of labor as λ^n. The first order condition with respect to security holdings gives the price of each security that pays off conditional on a particular history

$$v_{t,t+1}(s^{t+1}) = \beta \left[\frac{\lambda_{t+1}^c(s^{t+1})}{\lambda_t^c(s^t)} \frac{P_t(s^t)}{P_{t+1}(s^{t+1})} \right]$$

This equation holds for every possible history. To observe the relation of exchange rates to domestic and foreign marginal utilities, prices of securities[18] can be expressed as follows,

$$v_{t,t+1}(s^{t+1}) = \beta \frac{\lambda_{t+1}^c(s^{t+1})}{\lambda_t^c(s^t)} \frac{P_t(s^t)}{P_{t+1}(s^{t+1})} = \beta \frac{\lambda_{t+1}^{c*}(s^{t+1})}{\lambda_t^{c*}(s^t)} \frac{P_t^*(s^t)}{P_{t+1}^*(s^{t+1})} \frac{e_t(s^t)}{e_{t+1}(s^{t+1})}$$

where e_t is the nominal exchange rate. Iterating backwards, we obtain

$$\frac{\lambda^c(s^t)}{P(s^t)} \frac{P(s^0)}{\lambda^c(s^0)} = \frac{\lambda^{c*}(s^t)}{e(s^t)P^*(s^t)} \frac{P^*(s^0)e(s^0)}{\lambda^{c*}(s^0)}$$

We find the risk sharing condition which relates the real exchange rate to the marginal utilities,

$$e_t = \left(\frac{\lambda^c(s^0)}{\lambda^{c*}(s^0)} \frac{P^*(s^0)}{P(s^0)} e(s^0) \right) \frac{\lambda_t^{c*}}{\lambda_t^c} \frac{P_t}{P_t^*}$$

Linearization of this expression and defining the real exchange rate as $rer_t \equiv e_t \frac{P_t^*}{P_t}$ gives

$$\hat{rer}_t = \hat{\lambda}_t^{c*} - \hat{\lambda}_t^c$$

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[18]Securities pay off in domestic currency.

By taking period t conditional[19] expectations, we find that the gross nominal interest rate is

$$R_t^{-1} \equiv E_t \left[v_{t,t+1} \right] = \beta E_t \left[\frac{\lambda_{t+1}^c}{\lambda_t^c} \frac{P_t}{P_{t+1}} \right]$$

Labor supply is determined by the static first order condition, which sets the real wages equal to the marginal rate of substitution between consumption and leisure,

$$\frac{W_t}{P_t} = -\frac{\lambda_t^n}{\lambda_t^c}$$

2.2 Consumers with Sticky Information

In this section we describe the decision making process of the household under inattentiveness assumption. A household is composed of a shopper and a planner. The shopper makes the intra-temporal decision. She allocates the best bundle of varieties without information frictions and does not share information about prices with the planner[20]. The planner solves an inter-temporal problem to allocate total expenditure. She faces a stochastic income process, defined as $Y_{t,j} \equiv \frac{W_{t,j} N_{t,j} + \pi_t + T_t}{P_t}$, taking labor supply as given. Here, the second index is the number of periods by which the information set is outdated. If she knows all variables up to date t, the probability of updating her information set at date t+1 is $1 - \delta$.

If we assume a complete markets structure in this economy, that would equate the marginal utilities among the consumers. Therefore there would be no heterogeneity from the consumption responses, which is crucial for our results. Instead, if we impose incomplete markets, this would require keeping track of the asset levels of all information cohorts. This strategy is

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[19]We drop the s^t notation for the ease of exposition.

[20]This assumption is necessary to have a tractable demand condition for varieties. If we relax this assumption, producers need to account for the demand from different information cohorts.

feasible for staggered information updating[21], and it is possible to solve for the level of assets for each information cohort. To keep the problem more tractable, we assume that all assets are divided among all agents at the end of every period following Mankiw and Reis (2006). With this structure, there is no heterogeneity in the asset levels. This setup gives us similar optimality conditions to the staggered information updating, but with different weights for the aggregation. Under this assumption, we can state the problem of the attentive consumer as follows

$$V(\frac{B_t}{P_t}; \Im = t) = \max_{\{C_{t,0}\}} U(C_{t,0}, N_{t,0}) + \beta\delta V(\frac{B_{t+1}}{P_{t+1}}, ; \Im = t) + \beta(1-\delta)V(\frac{B_{t+1}}{P_{t+1}}, ; \Im = t+1)$$

subject to

$$\frac{B_{t+1}}{P_{t+1}} = R_{t,t+1}\frac{P_t}{P_{t+1}}\left[\frac{B_t}{P_t} - C_{t,j} + Y_{t,j}\right]$$

where the second state variable $\Im$ is the date of the last information set update. The Euler equation for the attentive consumer[22] is

$$\frac{\lambda_{t,0}^c}{P_t} = \beta E_t\left[R_t\frac{\lambda_{t+1,0}^c}{P_{t+1}}\right]$$

Defining $rr_t \equiv E_t\left[R_t\frac{P_t}{P_{t+1}}\right]$ as the real interest rate, log-linearization around the deterministic steady state gives the following optimality conditions

$$\hat{\lambda}_{t,0}^c = E_t\left[\hat{\lambda}_{t+1,0}^c + \hat{rr}_t\right]$$

$$\hat{\lambda}_{t,j}^c = E_{t-j}\hat{\lambda}_{t,0}^c$$

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[21]Assuming that consumers update their information set every N periods, the fraction of each information cohort is $\frac{1}{N}$.

[22]Details are provided in the appendix.

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Aggregate consumption is given by $\hat{c}_t = \sum_{j=0}^{\infty}(1-\delta)\delta^j \hat{c}_{t,j}$. In this economy, the real exchange rate is determined by assuming the implicit insurance contract pays out in domestic currency, and the marginal utilities of attentive consumers satisfy

$$E_t \frac{U_{c,t+1,0}}{U_{c,t,0}} \frac{P_t}{P_{t+1}} = E_t \frac{U^*_{c,t+1,0}}{U^*_{c,t,0}} \frac{P^*_t}{P^*_{t+1}} \frac{e_t}{e_{t+1}}$$

Log-linearizing this equation and using the definition of the real exchange rate, we find the real exchange rate depends on the marginal utilities of attentive consumers,

$$\hat{rer}_t = \hat{\lambda}^*_{t,0} - \hat{\lambda}_{t,0}$$

2.3 Labor Market with Sticky Information

We consider two different structures for the labor market. Our benchmark case is "flexible labor response", where the shopper sets the labor response by observing the real wage and taking the consumption decision of the planner as given. Labor responses for each information cohort satisfy the following equilibrium condition,

$$\frac{W_t}{P_t} = -\frac{\lambda^n_{t,j}}{\lambda^c_{t,j}}$$

Aggregate labor can be calculated by using this condition. Alternatively, following Mankiw and Reis(2006) closely, we consider a "wage posting" model. In this case, each household is a monopolistic supplier of a specific labor variety. The demand condition for the labor variety is given by $N_{t,j} = \left(\frac{W_{t,j}}{W_t}\right)^{-\chi} N_t$. The planner posts a nominal wage rate using the available information. Using results from the consumption decision and plugging in the demand for labor variety, we obtain the following condition for wage posting in the case of attentive

consumers,

$$\frac{W_{t,0}}{P_t} = -\mu_\chi \frac{\lambda_{t,0}^n}{\lambda_{t,0}^c}$$

where $\mu_\chi = \frac{\chi}{\chi-1}$ is the markup over the marginal rate of substitution between consumption and leisure. Agents who have outdated information post wages by forecasting the decision of attentive agents

$$\hat{w}_{t,j} = E_{t-j}\hat{w}_{t,0}$$

The aggregate nominal wage rate is given by $\hat{w}_t = \sum_{j=0}^{\infty}(1-\delta)\delta^j\hat{w}_{t,j}$.

2.4 Producers

Intermediate goods are produced by labor: $Y_H(i) = A_t N_t(i)$. We assume that firms set prices in buyers' currencies to maximize their expected profits. Demand from the domestic country for the variety produced by firm i is given by $Y_H(i) = \left\{\frac{P_H(i)}{P_H}\right\}^{-\nu} Y_H$. We define nominal marginal cost as $MC_t = \frac{W_t}{A_t}$.

Sticky Prices : Producers are attentive, they update their information set every period. They update their *prices* when they receive a Calvo signal. The probability of updating their prices is $1 - \theta$, while price stays constant with probability θ. They set prices in the local currencies for domestic and foreign country to maximize their expected profits

$$\max_{P_H(i),P_H^*(i)} \sum_{k=0}^{\infty} \theta^k E_t \left[v_{t,t+k}\left\{P_{H,t}(i)Y_{H,t+k}(i) - MC_{t+k}Y_{H,t+k}(i)\right\}\right]$$
$$+ \sum_{k=0}^{\infty} \theta^k E_t \left[v_{t,t+k}\left\{e_{t+k}P_{H,t}^*(i)Y_{H,t+k}^*(i) - MC_{t+k}Y_{H,t+k}^*(i)\right\}\right]$$

Using the demand for the variety, the first order condition for home prices for locally produced goods is

$$\sum_{k=0}^{\infty} \theta^k E_t \left[v_{t,t+k} Y_{H,t+k}(i) \right] = \frac{\nu}{\nu - 1} \sum_{k=0}^{\infty} \theta^k E_t \left[v_{t,t+k} \frac{MC_{t+k}}{P_{H,t}(i)} Y_{H,t+k}(i) \right]$$

After log-linearization, we can express this result as a sticky price Philips curve relation between the real marginal cost and inflation,

$$\hat{\pi}_{H,t} = \kappa \hat{mc}_t + \beta E_t \left[\hat{\pi}_{H,t+1} \right]$$

where $\kappa \equiv \frac{(1-\theta\beta)(1-\theta)}{\theta}$ and real marginal cost is $\hat{mc}_t \equiv \hat{MC}_t - \hat{P}_{H,t} = \hat{W}_t - \hat{P}_{H,t} - \hat{A}_t$. Import inflation for the foreign country is given by

$$\hat{\pi}_{H,t}^* = \frac{(1 - \theta\beta)(1 - \theta)}{\theta} (\hat{mc}_t + \hat{\psi}_{H,t}^*) + \beta E_t \left[\hat{\pi}_{H,t+1}^* \right]$$

where the law of one price gap is defined as $\hat{\psi}_{H,t}^* \equiv \hat{P}_{H,t} - \hat{P}_{H,t}^* - \hat{e}_t$.

Sticky Information : Firms update their *information set* with probability $(1 - \theta)$ each period. They proceed using their outdated information with probability θ. The firm which sets the price at time t according to the information received j periods ago solves the following static problem

$$\max_{P_H(j), P_H^*(j)} E_{t-j} \left[P_{H,t}(j) Y_{H,t}(j) - \frac{W_t}{A_t} Y_{H,t}(j) \right] + E_{t-j} \left[e_t P_{H,t}^*(j) Y_{H,t}^*(j) - \frac{W_t}{A_t} Y_{H,t}^*(j) \right]$$

The first order condition for home prices of locally produced goods is

$$E_{t-j} \left[Y_{H,t}(j) \right] = \frac{\nu}{\nu - 1} E_{t-j} \left[\frac{MC_t}{P_{H,t}(j)} Y_{H,t}(j) \right]$$

In this case home country inflation for domestic goods is a function of lagged expectations

$$\hat{\pi}_{H,t} = \frac{1-\theta}{\theta} \hat{mc}_t + \frac{1-\theta}{\theta} \left(\sum_{j=1}^{\infty} \theta^j E_{t-j} \left[\Delta \hat{mc}_t + \hat{\pi}_{H,t} \right] \right)$$

Import inflation in the foreign country is

$$\hat{\pi}_{H,t}^* = \frac{1-\theta}{\theta} \left(\hat{mc}_t + \hat{\psi}_{H,t}^* \right) + \frac{1-\theta}{\theta} \left(\sum_{j=1}^{\infty} \theta^j E_{t-j} \left[\Delta \hat{mc}_t + \Delta \hat{\psi}_{H,t}^* + \hat{\pi}_{H,t}^* \right] \right)$$

Regarding the inflation dynamics, we observe a forward looking relation with sticky prices. Current inflation is a function of the expectation of future inflation. On the other hand, we observe that inflation is a function of lagged expectations of current inflation with the sticky information assumption. We discuss the implications of the different price setting mechanisms[23] by assuming attentive consumers in our results section.

2.5 Monetary Policy and Market Clearing

We close the model by defining the monetary policy rule and imposing the market clearing condition. Interest rates follow a Taylor-type policy rule with a stochastic component

$$\hat{R}_t = \rho_R \hat{R}_{t-1} + (1 - \rho_R) \left[\psi_\pi \pi_t + \psi_y \hat{y}_t \right] + \epsilon_{R,t}$$

Market clearing condition for domestic goods is given by

$$Y_t = C_{H,t} + C_{H,t}^*$$

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[23]See Mankiw and Reis (2002) for a closed economy model.

We define the relative price of imports[24] as $q_t \equiv \frac{P_{F,t}}{P_{H,t}}$. Using the optimal allocation from the demand functions, the market clearing condition for domestic goods can be expressed as follows

$$\hat{y}_t = (1 - \gamma)\hat{c}_t + \gamma\hat{c}_t^* + \gamma(1 - \gamma)\eta\hat{q}_t - \gamma(1 - \gamma)\eta\hat{q}_t^*$$

2.6 Parametrization and Calibration Strategy

We log-linearize the system around the zero-inflation steady state, which yields a system of second order difference equations in the case of frictionless and sticky price models[25]. These systems can be solved by standard methods outlined in Klein(2000). Sticky information models include the lagged expectations of variables. We can write our models in the following form:

$$AE_tY_{t+1} + B_0Y_t + \sum_{i=1}^{I} B_iE_{t-i}Y_t + CY_{t-1} + GW_t = 0$$

where Y_t is vector of endogenous variables and W_t is vector of exogenous variables with a law of motion $W_t = NW_{t-1} + \epsilon_t$. The solution is in the form of $Y_t = \sum_{j=0}^{\infty} \Theta_j\epsilon_{t-j}$. We can manipulate this structure by plugging the solution into the system and truncating at a large number of lags. This reduces the model to a block tridiagonal structure which can be easily solved[26].

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[24]The relative price of imports is equivalent to terms of trade when producers update their prices and information set every period. Terms of trade is the price of imports in terms of exports, which we can express as $tot_t = \frac{P_{F,t}}{e_t P_{H,t}^*} = q_t\psi_{H,t}^*$.

[25]A summary of log-linearized models, including the frictionless (flexible prices, attentive consumers and producers), with sticky prices (assuming attentive consumers, AC-SP model) and with sticky information (featuring inattentive consumers and producers, IC-IP model), are provided in the Appendix.

[26]Earlier literature introduced lagged expectations as new variables to the endogenous state vector. This approach increases the computational burden, and accuracy depends on the number of lags included. Meyer-Gohde (2010) provides a new solution method for this class of models. A summary of the method is provided in the Appendix.

Our choice of the parameter values is summarized in Table 1. We assume a utility function of the form

$$U(C_t, N_t) = \frac{C_t^{1-\sigma}}{1-\sigma} - \xi \frac{N_t^{1+1/\phi}}{1+1/\phi}$$

Notice that utility is separable between consumption and leisure.

For the preference parameters, we assume a discount factor $\beta = 0.995$, which implies an annual real return of 2 percent at steady state[27]. The curvature parameter of the utility function (σ) determines the degree of risk aversion. We set this parameter as 2 unless otherwise stated. Regarding the home bias in the consumption basket, γ is set to 0.06 following Chari, Kehoe and McGrattan (2002). They target the import share of U.S. output for this parameter.

The Frisch elasticity of labor supply is determined by ϕ. Many macro studies[28] set this elasticity to 3. Micro-econometric studies suggest lower values. Kimball and Shapiro (2008) report estimates around unit elasticity. We set $\phi = 2$ for our exercises. Following Backus, Kehoe and Kydland (1994) and Chari, Kehoe and McGrattan (2002), we set the elasticity of substitution between home and foreign goods as $\eta = 1.5$.

The elasticity of substitution across varieties of goods, ν, is set to 10. This is consistent with a price markup of 11 percent as documented in the U.S. data by Basu (1996). The elasticity of substitution among labor varieties is set to 10 for the wage posting model, following Mankiw and Reis (2006). We set the degree of price/information stickiness for the producers

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[27]Steady state labor supply is determined by ξ, log-linearized solution does not depend on this parameter.
[28]Such as Kydland and Prescott (1982), Cooley and Prescott (1995) and Prescott (2002, 2004).

to $\theta = 0.75$. This implies an average duration of 4 quarters for price/information updating.

We follow Chari, Kehoe and McGrattan (2002) to describe our exogenous productivity processes. Assuming symmetry across countries, we set the persistence and standard deviation of the productivity shocks as $\rho_A = 0.95$, and $\sigma_A = 0.7$ percent respectively. Cross-country correlation of these shocks is set to 0.25.

For the monetary policy rule, we use $\rho_R = 0.9$, $\psi_\pi = 1.8$ and $\psi_y = 0.07$ following the estimates of Clarida, Gali and Gertler (1998). We check the sensitivity of real exchange rate volatility using other estimates of the Taylor rule from the literature.

We choose the standard deviation of the monetary shocks so that the volatility of output is the same in the model as in the U.S. data[29] for each specification. We set the cross-country correlation of monetary shocks as 0.5 and assume the shock is symmetric for the rest of the world, i.e. the standard deviation of the foreign country monetary shock is the same.

To pin down the degree of information stickiness on the consumer side, we carry out an exercise with consumption growth following Mankiw and Reis (2006). If consumption[30] follows a random walk, then the variance of growth rate from t to t+2 should be twice as the variance of the growth rate from t to t+1. However, in the US data, we observe that $\left(2 \times \frac{Var(c_t - c_{t-1})}{Var(c_t - c_{t-2})}\right)$ is equal to 0.79, which means consumption adjusts gradually to the shocks governing the economy[31]. Furthermore, if consumption follows a random walk, the autocorrelation of consumption growth should be 0. We calibrate our sticky price and sticky information models to match output volatility as described above. Results are reported in Table 2. We find that

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[29]Details of data sources are described in the Appendix.

[30]Transformed by taking the logarithm of the data.

[31]Mankiw and Reis (2006) use $\frac{std(c_t - c_{t-1})}{std(c_t - c_{t-4})}$ as a calibration target.

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the variance ratio is greater than 1 for our no frictions model, sticky price model and sticky information model with attentive consumers. It is evident that introducing rigidities on the producer side quickens the consumption response, contradicting the data. Information stickiness on the consumer side helps us to bring the model closer to the data for these two moments. Mankiw and Reis (2006) and Reis (2009) report estimation results for US and Europe, for closed economy models. The range for δ in these studies is between 0.64 and 0.92. We report results for $\delta = 0.5$ and $\delta = 0.75$ for our exercises, and we set $\delta = 0.75$ for our benchmark calibration[32].

Looking at our alternative models, setting $\delta = 0$ is equivalent to assuming attentive consumers. For the sticky price model (AC-SP), we always assume attentive consumers. Benchmark model with inattentive consumers and producers (IC-IP) and wage posting extension (IC-IP-WP model) collapses to the model with attentive consumers and inattentive producers (AC-IP) when δ is set to 0.

3   Results

We start with numerical results of "Risk Aversion Approach" to address real exchange rate volatility and explain the underlying mechanism. We assume attentive consumers for this exercise. Then, we present the new approach proposed in this paper by introducing inattentive consumers. We show that our results are robust to alternative specifications regarding monetary policy rules, elasticity of substitution between home and foreign goods, degree of nominal rigidities, preferences and the labor market mechanism. Results suggest that exchange rate volatility becomes closer to the data under all alternative specifications. We also present the results with habit formation and attentive consumers to emphasize the dis-

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[32] Average duration of information updating is given by $\frac{1}{1-\delta}$.

tinction from assuming inattentiveness on the consumer side. We show that real exchange rate volatility declines as we increase the level of habit formation.

Next, we report business cycle statistics for alternative models. We show that different forms of rigidity in price setting behaviour produce similar results regarding international business cycles. We discuss the business cycle statistics of our sticky information model under three alternative specifications: first one assuming attentive consumers, then introducing inattentive consumers with flexible labor response, and finally under wage posting assumption.

3.1   Exchange Rate Volatility with Attentive Consumers

First, we derive the relationship between real exchange rate volatility and the level of risk aversion for models with attentive consumers. This exercise helps to understand the dynamics of the "Risk Aversion Approach" á la Chari, Kehoe and Mcgrattan (2002). In this class of models, the real exchange rate is determined by the risk sharing condition:

$$r\hat{e}r = \hat{\lambda}_t^{c*} - \hat{\lambda}_t^c$$

With separable utility, $\hat{\lambda}_t = -\sigma \hat{c}_t$, we can express the real exchange rate in terms of relative consumption, $r\hat{e}r_t = \sigma(\hat{c}_t - \hat{c}_t^*)$. Dividing by the variance of output, expanding the relative consumption variance and imposing symmetry gives

$$\frac{std(r\hat{e}r)}{std(y)} = RISK\ AVERSION \times \sqrt{2\left(1 - corr(\hat{c}, \hat{c}^*)\right)}\frac{std(\hat{c})}{std(y)}$$

This relation shows a direct link between the level of risk aversion (parametrized as σ) and real exchange rate volatility. Results are reported in Table 3. We observe that cross country consumption correlation and volatility of consumption are not the main driving forces when

we change the level of risk aversion. We also observe that we need to set risk aversion as 5 to match the real exchange rate volatility. This result does not change whether we impose a sticky price or sticky information structure for the producer side.

3.2 Real Exchange Rate Volatility with Inattentive Consumers

When we have sticky information on the consumer side, the real exchange rate is determined by an asset pricing condition based on the marginal utilities of attentive consumers,

$$E_t \frac{\lambda^c_{t+1,0}}{\lambda^c_{t,0}} \frac{P_t}{P_{t+1}} = E_t \frac{\lambda^{c*}_{t+1,0}}{\lambda^{c*}_{t,0}} \frac{P^*_t}{P^*_{t+1}} \frac{e_t}{e_{t+1}}$$

using the real exchange rate definition and log-linearizing gives

$$\hat{rer} = \hat{\lambda}^{c*}_{t,0} - \hat{\lambda}^c_{t,0}$$

Following similar steps to the case with attentive consumers yields

$$\frac{std(\hat{rer})}{std(y)} = RISK\ AVERSION \times \sqrt{2\left(1 - corr(\hat{c}_0, \hat{c}^*_0)\right)} \frac{std(\hat{c}_0)}{std(y)}$$

This equation links the volatility of real exchange rate with the attentive consumer's consumption[33]. Aggregate consumption is a weighted sum of the responses from all information cohorts[34], $\hat{c}^{agg}_t = (1 - \delta) \sum_{i=0}^{\infty} \hat{c}_{t,i}$. We can express the consumption response of an agent who updated her information set i periods ago as her expectation of the long rate conditioning on the available information, that is $\hat{c}_{t,i} = -\frac{1}{\sigma} E_{t-i} \hat{lr}_t$. The long rate is defined as $\hat{lr}_t = \sum_{j=0}^{\infty} \hat{rr}_{t+j}$.

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[33] Denoted with the subscript 0, as her information set is updated in the current period.
[34] See Appendix for the details.

We start by establishing that the volatility of attentive consumers' consumption is at least as high as aggregate consumption. For any moving average process x_t, $\text{var}(x_t) > \text{var}(E_{t-j}x_t)$ when $j > 0$. Since we can express our solution as a moving average process, then $\text{var}(\hat{lr}_t) > \text{var}(E_{t-j}\hat{lr}_t)$ for $j > 0$. It is easy to show that $\text{var}(\hat{c}_0) > \text{var}(\hat{c}_j)$ for $j > 0$. It follows that aggregate consumption is less volatile than the consumption of attentive consumers, $\text{var}(\hat{c}_0) > \text{var}(\hat{c}^{agg})$ for $\delta > 0$. We can also analytically show that attentive consumers' volatility increases as we increase the degree of information stickiness on the consumer side. Numerical results for varying degrees of information stickiness on the consumer side are reported in Table 4.

To understand the intuition, we plot the impulse response to a one unit negative innovation[35] which decreases home interest rate in Figure 1. Output and aggregate consumption move very closely. On impact, only the consumers who updated their information set in the current period have this shock in their information set. Therefore, aggregate consumption response is a fraction of the attentive consumer's response. The consumption plans of inattentive consumers remain the same since they do not have information on that. The goods market is cleared by the demand response of attentive consumers who are able to update their consumption plans. As the fraction of attentive consumers goes down, their response needs to increase to clear the market. Therefore, attentive consumers have more volatile consumption compared to the other cohorts of information. Consequently, their consumption is more volatile than the aggregate consumption and output. Since the real exchange rate is determined by the marginal utilities of attentive consumers, we observe higher volatility in real exchange rates.

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[35]1 unit negative shock to Taylor rule, ϵ_R.

3.3 Sensitivity Analysis

We report the volatility of real exchange rates under alternative specificiations for varying degrees of information stickiness on the consumer side in Table 5. Using two alternative monetary policy rules[36], we observe that our volatility amplification result is robust to different monetary policy rules.

A lower degree of information stickiness on the producer side generates excessive volatility in nominal exchange rates and decreases the persistence of real exchange rates. A higher (lower) level of import share (home bias) in preferences creates more volatility in net exports, while the low elasticity experiment decreases this moment. In comparison with the benchmark case, lower elasticity with inattentive consumers keeps the cross-country output correlation higher than that of consumption. We observe that our volatility amplification result survives under all specifications, which brings us closer to the data.

When we look at the results with Cobb-Douglas preferences[37], we observe that real exchange rates are less volatile than our benchmark model. The strong comovement between consumption and labor makes the marginal utility of consumption less volatile. This causes a decline in the volatility of the real exchange rate for all levels of inattentiveness, but the real exchange rate becomes more volatile when we increase the degree of inattentiveness.

The wage posting model also has some success about addressing real exchange rate volatility. Other features of this model will be discussed further when we report all business cycle moments.

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[36]First from the estimates of Clarida, Gali and Gertler (2000), and second from Rudebusch (2002).

[37]Utility function in this case is given by $U(C_t, N_t) = \frac{\left(C_t^{\varphi}(1-N_t)^{1-\varphi}\right)^{1-\sigma}}{1-\sigma}$.

Finally, we introduce external habit formation into our utility function to emphasize the difference from sticky information on the consumer side. Marginal utility of consumption in this case is given by $\hat{\lambda}^c = -\sigma(\hat{c}_t - h\hat{c}_{t-1})$. We report the results for varying degrees of habit. As the degree of habit increases, we observe that marginal utility becomes less volatile. Numerical results[38] are reported in Table 6, showing that habit formation reduces the volatility of real exchange rates.

3.4 Calibration Results and Impulse Responses

We focus on the business cycle moments and transmission of monetary shocks in this section. To understand the effect of imposing different frictions on the producer side, we compare the sticky price model and the sticky information model with attentive consumers. Next, we discuss the business cycle properties of the benchmark model with inattentive consumers and present the results under two alternative specifications.

3.4.1 Attentive Consumers

Table 7 reports business cycle moments for alternative models. Comparing models with attentive consumers, we observe that the form of the friction on the producer side has a negligible affect on the moments generated by the model[39].

For models with attentive consumers, we observe that consumption and employment are more volatile[40] in the model compared to the data. Net exports are less volatile than the data, but we should note that the volatility of net exports is sensitive to the degree of home

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[38]We can also show that analytically.

[39]We focus on the moments which describe the properties of international business cycles. Assumptions on the producer's price setting behaviour is crucial in terms of addressing issues on the dynamics of inflation. Key issues are comovement between the output and changes in inflation, and the delayed response of inflation to monetary shocks. See Mankiw and Reis (2002) for details.

[40]For simplicity, we abstract from capital accumulation. Chari, Kehoe and McGrattan (2002) target consumption volatility by changing an investment adjustment cost parameter.

bias and import elasticity. Models with attentive consumers generate less persistence in quantities and prices compared to the data. Our model captures the fact that cross country consumption correlation is lower than output correlation, but it generates a higher employment correlation with respect to the data. The real exchange rate and relative consumption exhibit perfect correlation, widely referred as the Backus-Smith puzzle.

Since monetary shocks play the dominant role in determining the dynamics of our model, we focus on the impulse responses to a home monetary shock[41] to understand the effect of introducing inattentive consumers. Figure 2 plots the impulse response functions for the sticky information model with attentive consumers. We observe that home consumption increases following a decline in the interest rate. Due to increased demand from home consumers, domestic and foreign output increases, and inflation rises in both countries. The foreign interest rate increases via feedback from the monetary authority to increased output and inflation. Foreign consumption decreases as a result of the increase in the interest rate. Transmission of a monetary shock is negative in consumption and positive in output. This helps to explain the fact that cross country output correlation is higher than that of consumption in the data. As the shock dissipates, quantities and real exchange rates return to their steady state values monotonically. Therefore, our model with attentive consumers generates low persistence in quantities. Real exchange rate persistence is also low since it is tightly linked to relative consumption in this model.

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[41]Direction of the impulse responses to a productivity shock remains same across the models for key variables. When home productivity increases, prices of home goods decrease. This leads to a rise in demand for home goods, which raises home and foreign consumption. Home consumption increases less than home output. By the decline in home inflation, the home interest rate decreases. Since demand shifts away from foreign goods, foreign output and inflation decrease. By the monetary policy rule, foreign interest rate goes down. The increase in home (attentive) consumption is greater than foreign (attentive) consumption. We observe hump shaped impulse responses, due to the negative comovement between output and inflation combined with the feedback from the interest rate rule. See Steinsson (2008) for a more comprehensive discussion of real shocks.

3.4.2   Inattentive Consumers

We observe that nominal and real exchange rate volatility is magnified with inattentive consumers and the persistence of quantities and prices becomes closer to the data[42]. Since real exchange rates are determined by the attentive consumer's consumption instead of aggregate consumption, inattentiveness on the consumer side provides a channel for a lower Backus-Smith correlation. The real exchange rate with inattentive consumers is given by

$$\hat{rer}_t = RISK \ \ AVERSION \times (\hat{c}_{t,0} - \hat{c}^*_{t,0})$$

Aggregating consumption from information cohorts and defining forecast errors on the real exchange rate movements as $\hat{f}_{t,j} = \hat{rer}_t - E_{t-j}\hat{rer}_t$

$$\hat{rer}_t = RISK \ \ AVERSION \times (\hat{c}_t - \hat{c}^*_t) + (1-\delta)\sum_{j=1}^{\infty}\delta^j \hat{f}_{j,t}$$

therefore the correlation of real exchange rates and relative consumption depends on the size of forecast errors made by the agents who have outdated information. However, calibration results show the size of the decline is quantitatively small. This channel needs to be further investigated. Inattentive consumer models perform less well on some issues compared to the models with attentive consumers. The cross country consumption correlation is higher than that of output, and we obtain procyclical net exports.

We plot the impulse responses from the benchmark sticky information model[43] in Figure 3 to compare with the sticky information model with attentive consumers. We previously investigated the results on exchange rate volatility, therefore we skip the distinction between

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[42]Results are reported in Table 7.

[43]which features inattentive consumers and producers. The labor market is characterized by flexible labor response assumption.

aggregate consumption and consumption of the attentive consumer here. Demand from home consumers increases gradually in this case. Consumers react to the monetary shock as they update their information set. Therefore, the decline in home output and consumption is not as fast as in the full information case. These dynamics help us to get more persistence in quantities, moving the model closer to the data. We also observe that the reduced increase in home demand changes the nature of the transmission dynamics for a monetary shock. The direction of inflation response in foreign country also changes with inattentive consumers. A larger exchange rate depreciation[44] creates a decline in import good inflation in the foreign country. The decline in the inflation is reflected in interest rates, which leads to a positive consumption response as opposed to the negative one for the case with attentive consumers. Weak demand response also leads to a decline in the consumption of import goods in the home country since foreign goods became more expensive for home consumers due to the depreciation. This leads to a positive net exports response with inattentive consumers.

Introducing inattentive consumers generates a positive transmission in consumption and a negative transmission in output in response to a monetary shock, therefore cross-country consumption correlation is higher than that of output. This result is sensitive to the elasticity of substitution between home and foreign goods. When we calibrate our benchmark model for a lower import elasticity(by setting $\eta = 0.5$), we obtain slightly counter-cyclical net exports, and cross country correlation of output is higher than that of consumption. Results from using a lower elasticity in the benchmark model are reported in Table 8. Aside from parametrization, abstracting from capital is also an important influence on our results. Countercyclical trade fluctutations reflect in large part on the dynamics of capital formation: expansions are associated with investment booms financed by borrowing from international capital markets. Since we assume labor is the only production input, moments of net exports

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[44]Relative to the case with attentive consumers.

are hard to capture with our model.

For the same information updating frequency, wage posting model seems to perform better in terms of explaining persistence in the data, but it generates less volatility in the exchange rates compared to the benchmark model with flexible labor response. Table 8 reports the business cycle moments for the wage posting model. To understand the effect of the labor market specification, we plot impulse responses for selected variables for our benchmark model with flexible labour response and the wage posting model in Figure 4. Real wages almost stay constant for wage posting model, compared to the quick adjustment for other models. We observe that increased demand raises inflation much less than in the other models, because the response of marginal cost is smaller. This generates a hump shaped[45] impulse response in output and consumption which increases the persistence in quantities. A larger response in inflation makes the decline in interest rates quicker, therefore the attentive consumer's consumption drops quickly in the models with flexible labor response. Since real exchange rates are linked to the attentive consumer's consumption, we observe more persistence in the wage posting model.

4 Conclusion

We present and study the properties of a model which imposes infrequent information updating for consumers and producers. Comparing a sticky price and sticky information model with attentive consumers, we find that the form of frictions on the producer side has a negligible affect for the international business cycles. On the other hand, imposing sticky information on the consumer side provides a new mechanism to address the exchange rate volatility without setting the degree of risk aversion too high.

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[45]The peak in impulse responses for flexible labor response model is in the second period, whereas wage posting model postpones the peak point further.

Introducing inattentive consumers exhibit a similar mechanism to the limited participation models of asset pricing literature. In this framework, exchange rates are linked to the relative consumption of attentive consumers who updated their information set in the current period. Their consumption is more volatile than aggregate consumption because inattentive consumers cannot adjust their consumption plans to the current shocks. As the fraction of attentive consumers falls, we observe more volatility in their consumption. This increases the volatility of marginal utilities, resulting in more volatile exchange rates. Setting the degree of risk aversion at a consensus value, where the intertemporal elasticity of substitution is 0.5, an average duration of four quarters between information updates can account for the exchange rate volatility observed in the data.

Sticky information on the consumer side brings the model closer to the data in other dimensions as well. We observe hump shaped impulse responses to monetary shocks, which increases the persistence of output, consumption and employment. We also see a small decline in the correlation of relative consumption and real exchange rates due to the forecast errors of inattentive consumers.

Possible extensions to improve the fit of the model are introducing capital into the production function and having non-tradable goods in the consumption basket. Furthermore, imposing staggered information updating and incomplete markets can allow us to examine the implications for current account dynamics.

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Table 1: **Parameter Values**

Description	Parameter	Value
Risk Aversion	σ	2
Frisch Elasticity	ϕ	2
Discount Factor	β	0.995
Elasticity of Substitution		
goods	ν	10
labor(WP model)	χ	10
home and foreign	η	1.5
Import Share	γ	0.94
Price/Information Stickiness		
producers	θ	0.75
consumers	δ	0.75
Monetary Policy Rule		
inertia	ρ_R	0.9
inflation	ψ_π	1.8
output	ψ_y	0.9
$\text{corr}(\epsilon_R, \epsilon_{R^*})$		0.5
Productivity Process		
persistence	ρ_A	0.95
st.dev.	σ_A	0.7
$\text{corr}(\epsilon_A, \epsilon_{A^*})$		0.25

Notes: Countries are assumed to be symmetric in terms of parameters and exogenous processes. The standard deviation of monetary shock is set to target output volatility.

Table 2: **Sticky Information: Consumers**

	Data	NoF	AC-SP	AC-IP	IC-IP	IC-IP	IC-IP-WP
Cons. Info. Stickiness(δ)	–	0	0	0	0.5	0.75	0.75
$2\frac{Var(c_t-c_{t-1})}{Var(c_t-c_{t-2})}$	0.79	1.03	1.20	1.22	0.95	0.79	0.67
$\rho(\Delta c_t)$	0.26	-0.03	-0.16	-0.18	0.05	0.26	0.49

Notes: We report the unfiltered ratio of variances for consumption growth and the autocorrelation of consumption growth for different models. Second column is the model with no frictions (NoF), third one is the sticky price model with attentive consumers (AC-SP), and others are results from the benchmark sticky information (featuring inattentive consumers and producers, IC-IP) model for varying degrees of stickiness on the consumer side. We assume flexible labor response for the benchmark model. The last column reports results from IC-IP model with wage posting assumption. Average duration of information updating is $\frac{1}{1-\delta}$. All models except the no frictions model are calibrated to match HP-filtered US output volatility by changing the standard deviation of the monetary shock.

Table 3: **Risk Aversion Approach**

	Data	AC-SP Model			AC-IP Model		
Risk Aversion	–	1	3	5	1	3	5
std($r\hat{e}r$)	4.81	1.06	3.12	5.19	1.08	3.14	5.21
std($\hat{c}$)	0.82	1.02	1.03	1.03	1.03	1.03	1.03
std($\hat{c} - \hat{c}^*$)	0.84	1.06	1.04	1.04	1.08	1.05	1.04
corr($\hat{c}, \hat{c}^*$)	0.30	0.46	0.49	0.49	0.45	0.49	0.49

Notes: Sticky price (AC-SP) and sticky information (AC-IP) models with attentive consumers are calibrated to match the standard deviation of US output. All series are HP-filtered. Standard deviations are normalized by dividing the output volatility. We report volatility of real exchange rates, consumption, relative consumption and cross country consumption correlation for varying degrees of risk aversion.

Table 4: **Real Exchange Rate Volatility with Inattentive Consumers**

	Data	AC-IP	IC-IP	IC-IP
Cons. Info. Stickiness(δ)	–	0	0.5	0.75
std($r\hat{e}r$)	4.81	2.11	3.28	5.08
std($\hat{c}^{agg}$)	0.82	1.03	1.00	0.95
std($\hat{c}_0$)	–	1.03	1.64	2.61
corr($\hat{c}_0$,$\hat{c}_0^*$)	–	0.48	0.49	0.52

Notes: Benchmark sticky information model (IC-IP, with inattentive consumers and producers) is calibrated to match the standard deviation of US output for varying degrees of information stickiness on the consumer side(δ). Average duration of information updating is $\frac{1}{1-\delta}$. The degree of information stickiness on the producer side(θ) is set to 0.75 and the level of risk aversion is 2. All volatilities are normalized by dividing the output volatility. All series are HP-filtered. Standard deviations of real exchange rates, aggregate consumption, consumption of attentive consumers and cross country consumption correlation(attentive consumers) are reported.

Table 5: **Real Exchange Rate Volatility Under Alternative Specifications**

	$\delta = 0$	$\delta = 0.5$	$\delta = 0.75$
Benchmark Model	2.11	3.28	5.08
Monetary Policy			
$\quad\rho_R = 0.92, \psi_\pi = 1.24, \psi_y = 0.33$	2.10	3.22	4.92
$\quad\rho_R = 0.79, \psi_\pi = 2.15, \psi_y = 0.23$	2.11	3.41	5.50
Lower Elasticity($\eta = 0.5$)	2.11	3.34	5.42
Lower Rigidity on Producers($\theta = 0.5$)	2.12	3.45	5.31
Higher Import Share($\gamma = 0.24$)	2.28	3.25	4.32
Cobb-Douglas Preferences	1.20	1.80	2.80
Wage Posting Model	2.11	2.83	3.63

Notes: Standard deviation of real exchange rates (relative to output) under alternative specifications are reported. Benchmark sticky information model (IC-IP, with inattentive consumers and producers) is calibrated to match the standard deviation of US output for varying degrees of information stickiness on the consumer side(δ). Average duration of information updating is $\frac{1}{1-\delta}$. For the benchmark model; (i) monetary policy parameters are $\rho_R = 0.9, \psi_\pi = 1.8, \psi_y = 0.07$, (ii) import elasticity(η) is 1.5, (iii) degree of information stickiness on the producer side(θ) is 0.75, (iv) import share(γ) is 0.06 and (v) we assume flexible labor response. Consumption exponent of Cobb-Douglas utility is set to 0.36.

Table 6: **Real Exchange Rate Volatility with Habit Formation**

	h=0	h=0.5	h=0.75
AC-SP model	2.09	1.46	1.12
AC-IP model	2.11	1.49	1.16

Notes: Standard deviation of real exchange rates (relative to output) under varying degrees of habit formation are reported. Models are calibrated to match the standard deviation of US output. The degree of information/price stickiness on the producer side(θ) is set to 0.75. Consumers are assumed to be attentive. AC-SP model introduces sticky prices and the AC-IP model features inattentive producers. Risk aversion is set to 2.

Table 7: **Selected Business Cycle Moments: Inattentive Consumers**

	Data	Benchmark	Attentive Consumers	
		IC-IP	AC-SP	AC-IP
Consumer Info. Stickiness(δ)	–	0.75	–	0
Output Volatility	1.54	1.54	1.54	1.54
Volatilities (Relative to GDP)				
Consumption	0.82	0.95	1.03	1.03
Employment	0.67	1.05	1.03	1.01
Real Exchange Rate	4.81	5.08	2.09	2.11
Nominal Exchange Rate	5.05	5.83	2.64	3.01
Net Exports	0.24	0.14	0.09	0.10
Autocorrelations				
Output	0.88	0.76	0.53	0.52
Consumption	0.89	0.78	0.52	0.52
Employment	0.91	0.72	0.52	0.51
Real Exchange Rate	0.84	0.52	0.53	0.53
Nominal Exchange Rate	0.85	0.61	0.60	0.63
Net Exports	0.86	0.64	0.71	0.70
Correlations				
cross-country				
Output	0.44	0.36	0.56	0.57
Consumption	0.30	0.52	0.48	0.48
Employment	0.19	0.40	0.55	0.58
Real Exchange Rate and				
Nominal Exchange Rate	0.99	0.87	0.84	0.86
Relative Consumption	-0.22	0.88	1.00	1.00
Output	0.04	0.52	0.46	0.46
Output and Net Exports	-0.49	0.44	-0.25	-0.24

Notes: All series are logged and HP-filtered. IC-IP model is the benchmark sticky information model with inattentive consumers and producers. AC-IP model features attentive consumers, and inattentive producers, AC-SP model is the sticky price model with attentive consumers.

Table 8: **Selected Business Cycle Moments: Extensions**

	Data	Benchmark	Wage Posting	Low Elasticity
Consumer Info. Stickiness(δ)	–	0.75	0.75	0.75
Import Elasticity(η)	–	1.5	1.5	0.5
Output Volatility	1.54	1.54	1.54	1.54
Volatilities (Relative to GDP)				
Consumption	0.82	0.95	0.88	1.00
Employment	0.67	1.05	1.09	1.07
Real Exchange Rate	4.81	5.08	3.65	5.42
Nominal Exchange Rate	5.05	5.83	3.57	6.41
Net Exports	0.24	0.14	0.22	0.04
Autocorrelations				
Output	0.88	0.76	0.87	0.77
Consumption	0.89	0.78	0.87	0.78
Employment	0.91	0.72	0.82	0.73
Real Exchange Rate	0.84	0.52	0.64	0.52
Nominal Exchange Rate	0.85	0.61	0.61	0.64
Net Exports	0.86	0.64	0.86	0.77
Correlations				
cross-country				
Output	0.44	0.36	0.22	0.52
Consumption	0.30	0.52	0.58	0.51
Employment	0.19	0.40	0.25	0.49
Real Exchange Rate and				
Nominal Exchange Rate	0.99	0.87	0.97	0.86
Relative Consumption	-0.22	0.88	0.86	0.88
Output	0.04	0.52	0.54	0.45
Output and Net Exports	-0.49	0.44	0.62	-0.03

Notes: All series are logged and HP-filtered. Benchmark model is the sticky information model with inattentive consumers and producers. Labor market is characterized by flexible labor response assumption. Third column reports the results with wage posting assumption in the labor market. Fourth column reports results with a lower import elasticity for the benchmark model.

Figure 1: Inattentive Consumers: Impulse Response to Home Monetary Shock
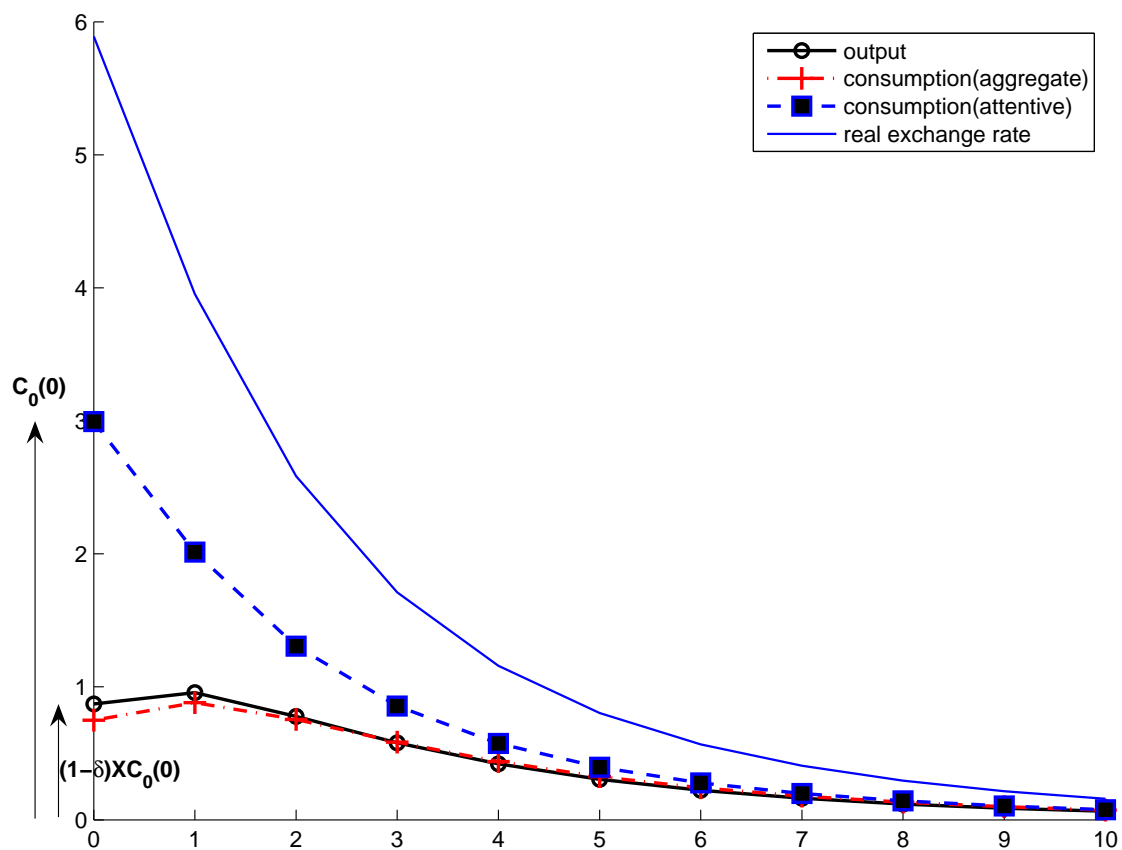
Figure 2: Sticky Information Model with Attentive Consumers
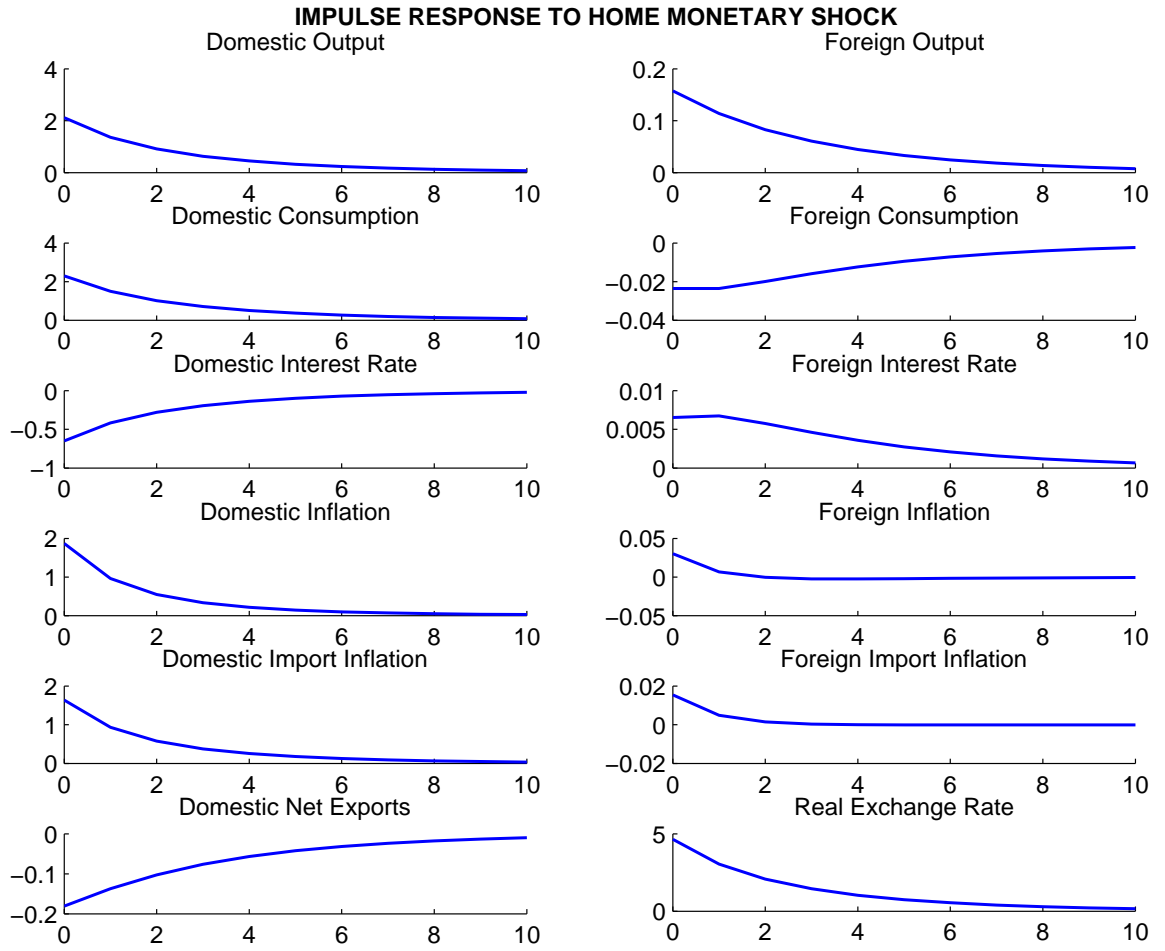
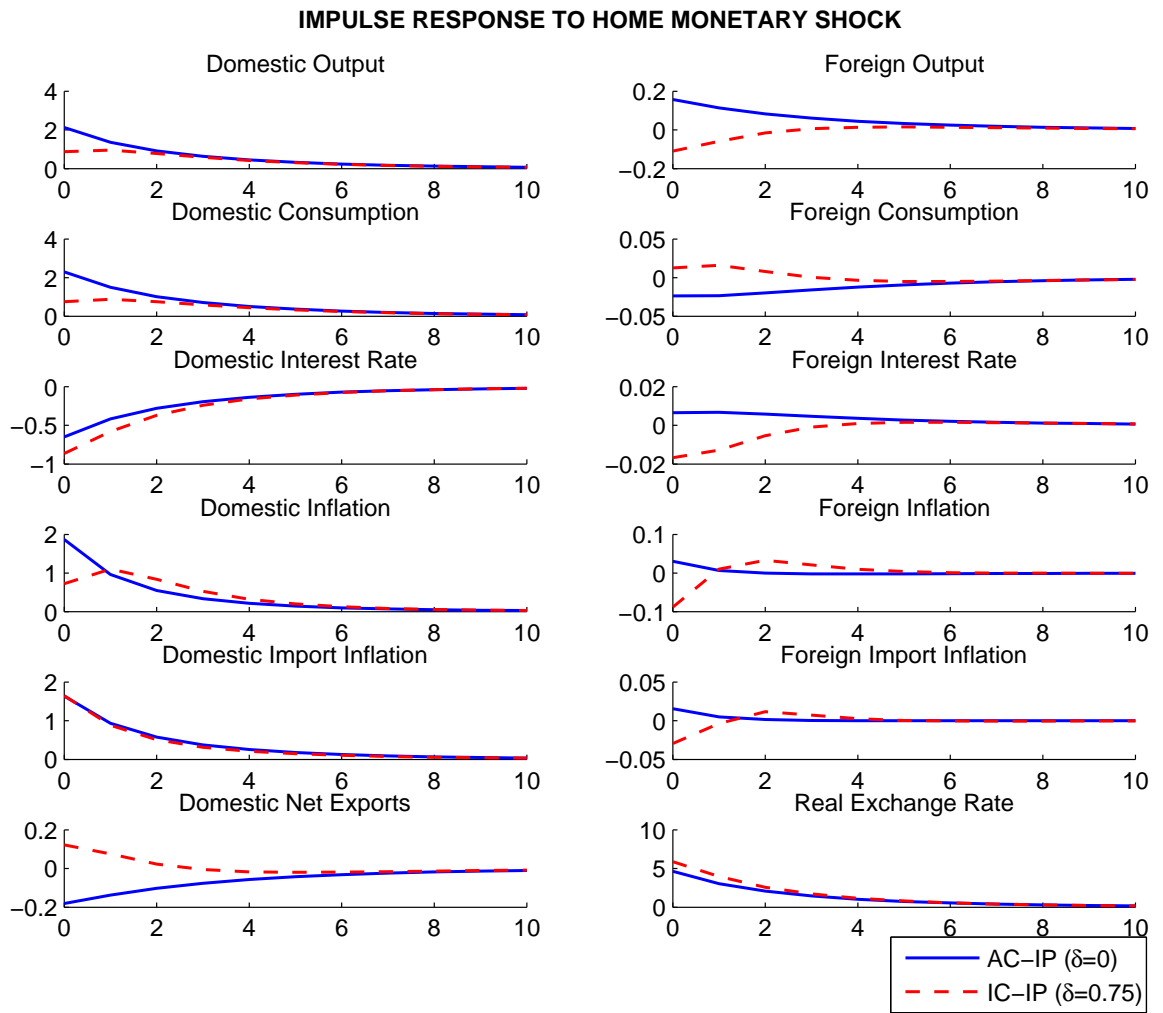Figure 3: Sticky Information Model with Flexible Labor Response

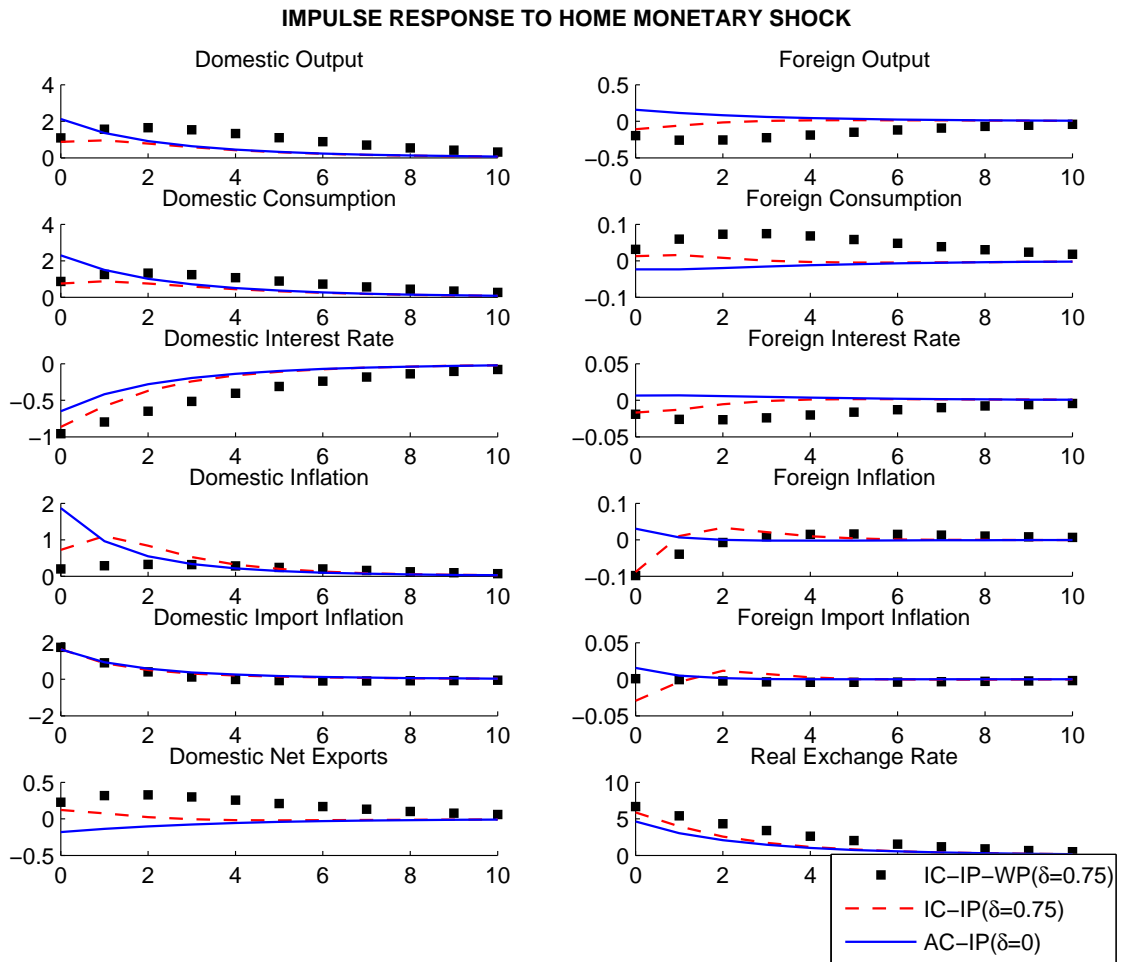**IMPULSE RESPONSE TO HOME MONETARY SHOCK**

Figure 4: Comparing Flexible Labor Response and Wage Posting Specifications

IMPULSE RESPONSE TO HOME MONETARY SHOCK

Appendix A : Data

Data is quarterly. Our sample period is between 1973Q1 and 2005Q4. Data sources are the FRED2 database from the Federal Reserve Bank of St. Louis, Area Wide Model (AWM) of the European Central Bank, OECD Economic Outlook and International Financial Statistics (IFS) by the IMF. All series are logged and HP-filtered. The ratio of net exports to GDP is filtered without using a log transformation.

US Output Real GDP series is obtained from GDPC96-Fred2.

Euro Area Output YER-AWM series is used for real output.

US Price Index Quarterly series based on Consumer Price Index for All Urban Consumers CPIAUCSL-Fred2. Monthly series are converted to quarterly by arithmetic averaging.

Euro Area Price Index Based on harmonized index, HICP-AWM.

US Consumption Real Consumption series is obtained from PCECC96-Fred2.

EU Consumption PCR-AWM series is used for real consumption.

US Employment . CE16OV-Fred2 series for civilian employment.

EU Employment LNN-AWM series for employment.

Exchange Rates Fixed conversion rates between the national currency units and the Euro and real GDP weights from the AWM database[46] are used to obtain an artificial bilateral exchange rate series prior to 1999. The Euro-Dollar exchange rate from IFS is used after 1999. Prior to 1999, we define the nominal exchange rate as $E_t \equiv \Pi_{i=1}^n (f_i E_{i,t})^{w_i}$. We calculate the real exchange rate as $RER_t = \frac{E_t P_{EU}}{P_{US}}$.

Net Exports Ratio of difference between Exports (EXPGSC96) and Imports (IMPGSC96) to Real GDP.

---

[46]The weights are Austria=0.03, Belgium=0.036, Finland=0.017, France=0.201, Germany=0.283, Greece=0.025, Ireland=0.015, Italy=0.195, Luxembourg=0.003, Netherland=0.06, Portugal=0.024, Spain=0.111.

Appendix B : Sticky Information Derivations

Consumer's Problem

A household is composed of a shopper-planner pair. The shopper chooses the optimal bundle of varieties and does not share the information about relative prices with the planner. The planner solves an intertemporal problem to allocate total expenditure. She faces a stochastic income process, defined as $Y_{t,j} \equiv \frac{W_{t,j}N_{t,j}+\pi_t+T_t}{P_t}$. where the second index is the number of periods by which her information set is outdated. If she knows all variables up to date t, and probability of updating information set for date t+1 variables is $1 - \delta$.

In the case of an attentive consumer, i.e. whose information set is updated at period t, the planner's dynamic program is

$$V(\frac{B_t}{P_t}; \Im = t) = \max_{\{C_{t,0}\}} U(C_{t,0}, N_{t,0}) + \beta\delta V(\frac{B_{t+1}}{P_{t+1}},; \Im = t) + \beta(1-\delta)V(\frac{B_{t+1}}{P_{t+1}},; \Im = t+1)$$

Assets are divided among all agents at the end of every period. By this transfer mechanism, there is no heterogeneity in the asset levels. The sequence of budget constraints is given by

$$\frac{B_{t+1}}{P_{t+1}} = R_{t,t+1}\frac{P_t}{P_{t+1}}\left[\frac{B_t}{P_t} - C_{t,j} + Y_{t,j}\right]$$

We define the real interest rate as $rr_t \equiv R_{t,t+1}\frac{P_t}{P_{t+1}}$. Envelope condition gives

$$V'(\frac{B_t}{P_t}; \Im = t) = \beta\delta rr_t V'(\frac{B_{t+1}}{P_{t+1}}; \Im = t) + \beta(1-\delta)rr_t V'(\frac{B_{t+1}}{P_{t+1}}; \Im = t+1)$$

Denoting $\lambda^c_{t,i} = U_{c,t,i}$, optimality conditions for the attentive consumers are

$$\lambda^c_{t,0} = \beta\delta rr_t V'(\frac{B_{t+1}}{P_{t+1}}; \Im = t) + \beta(1-\delta)rr_t V'(\frac{B_{t+1}}{P_{t+1}}; \Im = t+1)$$

Combining these two conditions, we obtain

$$\lambda_{t,0}^c = V'(\frac{B_t}{P_t}; \Im = t)$$

Next, we repeat the same steps for the consumers who updated their information set at period (t-1). The value function for the planner is given by

$$V(\frac{B_t}{P_t}; \Im = t - 1) = \max_{\{C_{t,1}\}} U(C_{t,1}, N_{t,1}) + \beta\delta V(\frac{B_{t+1}}{P_{t+1}}; \Im = t - 1) + \beta(1 - \delta)V(\frac{B_{t+1}}{P_{t+1}}; \Im = t + 1)$$

Optimality conditions are given by

$$V'(\frac{B_t}{P_t}; \Im = t - 1) = \beta\delta rr_t V'(\frac{B_{t+1}}{P_{t+1}}; \Im = t - 1) + \beta(1 - \delta)rr_t V'(\frac{B_{t+1}}{P_{t+1}}; \Im = t + 1)$$

$$\lambda_{t,1}^c = \beta\delta rr_t V'(\frac{B_{t+1}}{P_{t+1}}; \Im = t - 1) + \beta(1 - \delta)rr_t V'(\frac{B_{t+1}}{P_{t+1}}; \Im = t + 1)$$

Combining these results we obtain

$$\lambda_{t,1}^c = V'(\frac{B_t}{P_t}; \Im = t - 1)$$

In general, we have the following relation in log-linear form

$$\hat{\lambda}_{t,j}^c = E_{t-j}\hat{\lambda}_{t,0}^c$$

Iterating one period ahead, $\lambda_{t+1,1}^c = E_t\lambda_{t+1,0}^c$, and combining this with the optimality condition for attentive consumers gives us,

$$\lambda_{t,0}^c = \beta\delta rr_t V'(\frac{B_{t+1}}{P_{t+1}}; \Im = t) + \beta(1 - \delta)rr_t V'(\frac{B_{t+1}}{P_{t+1}}; \Im = t + 1)$$

$$\lambda_{t,0}^c = \beta\delta rr_t \lambda_{t+1,1}^c + \beta(1 - \delta)rr_t \lambda_{t+1,0}^c$$

these steps yield the Euler equation

$$\hat{\lambda}^c_{t,0} = E_t\left(\hat{\lambda}^c_{t+1,0} + \hat{rr}_t\right)$$

where the real interest rate is

$$\hat{rr}_t = \hat{R}_t - E_t\hat{\pi}_{t+1}$$

Writing the equations for attentive and inattentive consumers and iterating forward (for separable utility $\hat{\lambda}^c_{t,0} = -\sigma\hat{c}_{t,0}$)

$$\hat{c}_{t,0} = E_t\left[\hat{c}_{t+1,0} - \frac{1}{\sigma}\hat{rr}_t\right]; \quad \hat{c}_{t+1,0} = E_{t+1}\left[\hat{c}_{t+2,0} - \frac{1}{\sigma}\hat{rr}_{t+1}\right]$$

$$\hat{c}_{t,j} = E_{t-j}\hat{c}_{t,0} = E_{t-j}\left[\hat{c}_{t+2,0} - \frac{1}{\sigma}(\hat{rr}_t + \hat{rr}_{t+1})\right]$$

$$\hat{c}_{t,j} = E_{t-j}\left[\hat{c}_{t+T,0} - \frac{1}{\sigma}\sum_{i=0}^{T}\hat{rr}_{t+i}\right]$$

Next, we take the limit as $T \to \infty$, and define the long interest rate $\hat{lr}_t = \sum_{i=0}^{T}\hat{rr}_{t+i}$. As time elapses to infinity all become aware of past news so $lim_{i\to\infty}E_t\hat{rr}_{t+i} = 0$. Moreover, since the probability of remaining inattentive falls exponentially with the length of the horizon, we approach this limit fast enough to ensure that the sum in the second term converges. As for the first term, $lim_{i\to\infty}E_t(\hat{c}_{t+i,0}) = 0$. The shocks in the economy die out in the long run, so consumption is expected to be at the steady state level in the limit. The long interest rate can be defined recursively as follows

$$\hat{lr}_t = E_t\sum_{i=0}^{\infty}\hat{rr}_{t+i}$$

$$E_t\hat{lr}_{t+1} = E_t\sum_{i=0}^{\infty}\hat{rr}_{t+1+i}$$

$$\hat{lr}_t = \hat{rr}_t + E_t\hat{lr}_{t+1}$$

and the consumption Euler equation can be written as

$$\hat{c}_{t,j} = -\frac{1}{\sigma} E_{t-j} \left[\hat{lr}_t\right]$$

We can write aggregate consumption as $\hat{c}_t^{agg} = \sum_{j=0}^{\infty}(1-\delta)\delta^j \hat{c}_{t,j}$,

$$\hat{c}_t^{agg} = -\frac{1}{\sigma} \sum_{j=0}^{\infty}(1-\delta)\delta^j E_{t-j} \left[\hat{lr}_t\right]$$

In the benchmark case of flexible labor response, the shopper makes the decision by observing real wages and taking the consumption decision of the planner as given. The marginal rate of substitution between consumption and leisure is equal to real wage.

$$(\hat{W}_t - \hat{P}_t) = \hat{\lambda}_{t,j}^n - \hat{\lambda}_{t,j}^c$$

Aggregate labor response with separable utility satisfies

$$(\hat{W}_t - \hat{P}_t) = \frac{1}{\phi}\hat{n}_t^{agg} + \sigma\hat{c}_t^{agg}$$

We describe the economy with wage posting in the next section.

Wage Posting

Here, the labor market features workers as the supplier of a specific variety of labor and firms, indexed by i, have a hiring department purchasing a continuum of varieties of workers, indexed by k, in the amount $N_{t,i}(k)$ at the price $W_{t,k}$. Firms combine these varieties into the labor input $N_{t,i}$ according to a Dixit-Stiglitz aggregator. The hiring department of the firm

solves the following problem

$$\min_{\{N_{t,i}(j)\}_{j\in[0,1]}} \int_0^1 W_{t,k} N_{t,i}(k) dk$$

$$s.t. \quad N_{t,i} = \left[\int_0^1 N_{t,i}(k)^{\frac{\chi-1}{\chi}} dk \right]^{\frac{\chi}{\chi-1}}$$

this problem has solution $N_{t,i}(k) = N_{t,i}(\frac{W_{t,k}}{W_t})^{-\chi}$ where W_t is the static wage index $W_t = \left[\int_0^1 W_{t,k}^{1-\chi} dk\right]^{\frac{1}{1-\chi}}$. Aggregation over demand from firm i gives the demand for labor variety k

$$\int_0^1 N_{t,i}(k)dk = \left(\frac{W_{t,k}}{W_t}\right)^{-\chi} \int_0^1 N_{t,i} di$$

Plugging in the labor demand $N_{t,0} = \left(\frac{W_{t,0}}{W_t}\right)^{-\chi} N_t$, the problem for wage posting becomes

$$V(\frac{B_t}{P_t}; \Im = t) = \max_{\{W_{t,0}\}} U\left(C_{t,0}, \left(\frac{W_{t,0}}{W_t}\right)^{-\chi} N_t\right)$$

$$+\beta\delta V(\frac{B_{t+1}}{P_{t+1}}, ; \Im = t) + \beta(1-\delta)V(\frac{B_{t+1}}{P_{t+1}}, ; \Im = t+1)$$

subject to

$$\frac{B_{t+1}}{P_{t+1}} = R_{t,t+1}\frac{P_t}{P_{t+1}}\left[\frac{B_t}{P_t} - C_{t,0} + Y_{t,0}\right]$$

where the income process is $Y_{t,0} = \frac{W_{t,0}}{P_t}\left(\frac{W_{t,0}}{W_t}\right)^{-\chi} N_t + \frac{\Pi_t}{P_t} + \frac{\tau_t}{P_t}$. First order condition is

$$\frac{W_{t,0}}{P_t} = \frac{\chi}{\chi-1}\frac{-\lambda_{t,0}^n}{\lambda_{t,0}^c} = \xi\frac{\chi}{\chi-1}\frac{N_{t,0}^{1/\phi}}{\lambda_{t,0}^c}$$

Euler equation gives

$$\frac{P_t N_{t,0}^{1/\phi}}{W_{t,0}} = \beta E_t\left[\frac{P_{t+1}N_{t+1,0}^{1/\phi}}{W_{t+1,0}}rr_t\right]$$

linearization gives,

$$\frac{1}{\phi}\hat{N}_{t,0} + \hat{p}_t - \hat{w}_{t,0} = E_t\left[\frac{1}{\phi}\hat{N}_{t+1,0} + \hat{p}_{t+1} - \hat{w}_{t+1,0} + \hat{rr}_t\right]$$

The workers who have outdated information post the wages by forecasting the decision of attentive workers,

$$\hat{w}_{t,j} = E_{t-j}\hat{w}_{t,0}$$

Combining these equations, and iterating forward yields

$$\hat{w}_{t,j} = E_{t-j}\hat{w}_{t,0} = E_{t-j}\left[\frac{1}{\phi}\hat{N}_{t,0} + \hat{p}_t - \frac{1}{\phi}\hat{N}_{t+1,0} - \hat{p}_{t+1} + \hat{w}_{t+1,0} - \hat{rr}_t\right]$$

$$E_{t-j}\hat{w}_{t+1,j} = E_{t-j}\left[\frac{1}{\phi}\hat{N}_{t+1,0} + \hat{p}_{t+1} - \frac{1}{\phi}\hat{N}_{t+2,0} - \hat{p}_{t+2} + \hat{w}_{t+2,0} - \hat{rr}_{t+1}\right]$$

$$\hat{w}_{t,j} = E_{t-j}\left[\hat{p}_t + \frac{1}{\phi}\hat{N}_{t,0} - \sum_{i=0}^{T}\hat{rr}_{t+i} + \left(\hat{w}_{t+T,0} - \hat{p}_{t+T} - \frac{1}{\phi}\hat{N}_{t+T,0}\right)\right]$$

Now, using the definition of the long rate and taking the limit

$$\hat{w}_{t,j} = E_{t-j}\left[\hat{p}_t + \frac{1}{\phi}\hat{N}_{t,0} - \hat{lr}_t\right]$$

Using labor demand from firms $\hat{N}_{t,0} = \chi\left(\hat{w}_t - \hat{w}_{t,0}\right) + \hat{N}_t$, and $E_{t-j}\hat{w}_{t,0} = w_{t,j}$,

$$\hat{w}_{t,j} = E_{t-j}\left[\hat{p}_t + \frac{\chi}{\phi}\left(\hat{w}_t - \hat{w}_{t,0}\right) + \frac{1}{\phi}\hat{N}_t - \hat{lr}_t\right]$$

$$(\phi + \chi)\hat{w}_{t,j} = E_{t-j}\left[(\phi + \chi)\hat{p}_t + \chi(w_t - p_t) + \hat{N}_t - \phi\hat{lr}_t\right]$$

Aggregating to find the wage rate, $\hat{w}_t = \sum_{j=0}^{\infty}(1-\delta)\delta^j \hat{w}_{t,j}$,

$$\hat{w}_t = \sum_{j=0}^{\infty}(1-\delta)\delta^j E_{t-j}\left[\hat{p}_t + \frac{\chi}{\phi+\chi}(w_t - p_t) + \frac{1}{\phi+\chi}\hat{N}_t - \frac{\phi}{\phi+\chi}\hat{lr}_t\right]$$

which yields our final result for real wages

$$(\hat{w}_t - \hat{p}_t) = \frac{\delta(\phi+\chi)}{\phi+\delta\chi}(\hat{w}_{t-1} - \hat{p}_{t-1}) + \frac{(1-\delta)(\phi+\chi)}{\phi+\delta\chi}\left[\frac{1}{\phi+\chi}\hat{N}_t - \frac{\phi}{\phi+\chi}\hat{lr}_t\right] - \frac{\delta(\phi+\chi)}{\phi+\delta\chi}\hat{\pi}_t$$
$$+\frac{\delta(\phi+\chi)}{\phi+\delta\chi}\sum_{j=0}^{\infty}(1-\delta)\delta^j E_{t-1-j}\left[\hat{\pi}_t + \frac{\chi}{\phi+\chi}\Delta(w_t - p_t) + \frac{1}{\phi+\chi}\Delta\hat{N}_t - \frac{\phi}{\phi+\chi}\Delta\hat{lr}_t\right]$$

Firms

Firms are committed to producing as much as necessary to clear the market. Intermediate goods are produced solely by labor, $Y_H(i) = A_t N_t(i)$. Firms update their expectations with probability $(1-\theta)$ probability each period. They proceed with the outdated information with probability θ. The firm which sets the price at time t according to the information received j periods ago solves the following problem

$$\max_{P_H(j),P_H^*(j)} E_{t-j}\left[P_{H,t}(j)Y_{H,t}(j) - \frac{W_t}{A_t}Y_{H,t}(j)\right] + E_{t-j}\left[e_t P_{H,t}^*(j)Y_{H,t}^*(j) - \frac{W_t}{A_t}Y_{H,t}^*(j)\right]$$

Plugging in the demand functions,

$$\max_{P_H(j),P_H^*(j)} E_{t-j}\left[P_{H,t}(j)\left(\frac{P_{H,t}(j)}{P_{H,t}}\right)^{-\nu}Y_{H,t} - \frac{W_t}{A_t}\left(\frac{P_{H,t}(j)}{P_{H,t}}\right)^{-\nu}Y_{H,t}\right]$$
$$+E_{t-j}\left[e_t P_{H,t}^*(j)\left(\frac{P_{H,t}^*(j)}{P_{H,t}^*}\right)^{-\nu}Y_{H,t}^* - \frac{W_t}{A_t}\left(\frac{P_{H,t}^*(j)}{P_{H,t}^*}\right)^{-\nu}Y_{H,t}^*\right]$$

where $\frac{W_t}{A_t}$ is the nominal marginal cost, MC_t. The first order condition for home prices of locally produced goods is

$$E_{t-j}\left[Y_{H,t}(j)\right] = \frac{\nu}{\nu-1}E_{t-j}\left[\frac{MC_t}{P_{H,t}(j)}Y_{H,t}(j)\right]$$

Log-linearization and defining real marginal cost as $\hat{mc}_t = \hat{MC}_t - \hat{P}_{H,t}$ gives

$$\hat{P}_{H,t}(j) = E_{t-j}\left[\hat{mc}_t + \hat{P}_{H,t}\right]$$

We have a continuum of firms, the fraction which updates information in any given period is $1-\theta$, and consequently the fraction of firms which updated their information j periods ago is $(1-\theta)\theta^j$, therefore we can write the price index as follows

$$P_{H,t} = (1-\theta)\left(\sum_{j=0}^{\infty}\theta^j P_H(j)^{1-\nu}\right)^{\frac{1}{1-\nu}}$$

Log-linearization gives

$$\hat{P}_{H,t} = (1-\theta)\left(\sum_{j=0}^{\infty}\theta^j \hat{P}_H(j)\right) = (1-\theta)\left(\sum_{j=0}^{\infty}\theta^j E_{t-j}\left[\hat{mc}_t + \hat{P}_{H,t}\right]\right)$$

Collecting terms, taking the lag and rearranging gives the sticky information Philips curve for inflation

$$\hat{\pi}_{H,t} = \frac{1-\theta}{\theta}\hat{mc}_t + \frac{1-\theta}{\theta}\left(\sum_{j=1}^{\infty}\theta^j E_{t-j}\left[\Delta\hat{mc}_t + \hat{\pi}_{H,t}\right]\right)$$

Import inflation is derived in a similar fashion. In the wage posting model, we assume that within the firm there are two departments making decisions. The hiring department takes as given the choice of how much to produce and hires the combination of labor inputs that

minimizes costs using full information. The labor demand equation in the worker's problem characterizes the solution to this problem. The sales department sets a price that takes into account its monopoly power and the demand for its product.

Appendix C : Summary of Linearized Models

I summarize the log-linearized system of equations for three models in this appendix. The first one is labeled as "no frictions" model, where consumers and producers have full information and producers can update their prices each period, i.e. $\delta = 0$ and $\theta = 0$. Sticky price model refers to the model where agents have full information, but producers can update their *prices* when they receive a Calvo signal. The last model is the benchmark sticky information model, where consumers and producers update their *information set* with a Calvo signal.

No Frictions Model

Consumption Euler equations are

$$\hat{R}_t - E_t\hat{\pi}_{t+1} = -E_t\hat{\lambda}^c_{t+1} + \hat{\lambda}^c_t \tag{A.1}$$

$$\hat{R}^*_t - E_t\hat{\pi}^*_{t+1} = -E_t\hat{\lambda}^{c*}_{t+1} + \hat{\lambda}^{c*}_t \tag{A.2}$$

Risk sharing condition

$$\hat{rer}_t = \hat{\lambda}^{c*}_t - \hat{\lambda}^c_t \tag{A.3}$$

Real wages are given by

$$\hat{W}_t - \hat{P}_t = \hat{\lambda}^n_t - \hat{\lambda}^c_t \tag{A.4}$$

$$\hat{W}^*_t - \hat{P}^*_t = \hat{\lambda}^{n*}_t - \hat{\lambda}^{c*}_t \tag{A.5}$$

Relative PPP condition,

$$\Delta\hat{e}_t = \hat{\pi}_t - \hat{\pi}^*_t + \hat{rer}_t - \hat{rer}_{t-1} \tag{A.6}$$

Monetary policy

$$\hat{R}_t = \rho_R \hat{R}_{t-1} + (1 - \rho_R) \left[\psi_\pi \hat{\pi}_t + \psi_y \hat{y}_t \right] + \epsilon_{R,t} \tag{A.7}$$

$$\hat{R}_t^* = \rho_R \hat{R}_{t-1}^* + (1 - \rho_R) \left[\psi_\pi \hat{\pi}_t^* + \psi_y \hat{y}_t^* \right] + \epsilon_{R^*,t} \tag{A.8}$$

Production functions;

$$\hat{y}_t = \hat{A}_t + \hat{N}_t \tag{A.9}$$

$$\hat{y}_t^* = \hat{A}_t^* + \hat{N}_t^* \tag{A.10}$$

Exogenous shocks to productivity

$$\hat{A}_t = \rho_A \hat{A}_{t-1} + \epsilon_{A,t} \tag{A.11}$$

$$\hat{A}_t^* = \rho_A^* \hat{A}_{t-1}^* + \epsilon_{A^*,t} \tag{A.12}$$

Goods market clearing (with no frictions, relative price of imports is equal to terms of trade)

$$\hat{y}_t = (1 - \gamma)\hat{c}_t + \gamma \hat{c}_t^* + \gamma(1 - \gamma)\eta \hat{tot}_t - \gamma(1 - \gamma)\eta \hat{tot}_t^* \tag{A.13}$$

$$\hat{y}_t^* = \gamma \hat{c}_t + (1 - \gamma)\hat{c}_t^* - \gamma(1 - \gamma)\eta \hat{tot}_t + \gamma(1 - \gamma)\eta \hat{tot}_t^* \tag{A.14}$$

Price setting equations give

$$\hat{W}_t - \hat{P}_t + \gamma \hat{tot}_t = \hat{A}_t \tag{A.15}$$

$$\hat{W}_t^* - \hat{P}_t^* + \gamma \hat{tot}_t^* = \hat{A}_t^* \tag{A.16}$$

$$(1 - \gamma)\hat{tot}_t = \hat{rer}_t - \gamma \hat{tot}_t^* \tag{A.17}$$

$$(1 - \gamma)\hat{tot}_t^* + \hat{rer}_t = -\gamma \hat{tot}_t \tag{A.18}$$

The linearized net exports to output ratio is given by

$$\hat{nx}_t = \hat{y}_t - \hat{c}_t \tag{A.19}$$

$$\hat{nx}_t^* = \hat{y}_t^* - \hat{c}_t^* \tag{A.20}$$

Marginal utilities are given by

$$\hat{\lambda}_t^c = -\sigma \hat{C}_t \tag{A.21}$$

$$\hat{\lambda}_t^n - \hat{\lambda}_t^c = \frac{1}{\phi}\hat{N}_t + \sigma \hat{C}_t \tag{A.22}$$

$$\hat{\lambda}_t^{c*} = -\sigma \hat{C}_t^* \tag{A.23}$$

$$\hat{\lambda}_t^{n*} - \hat{\lambda}_t^{c*} = \frac{1}{\phi}\hat{N}_t^* + \sigma \hat{C}_t^* \tag{A.24}$$

Vector of state variables is

$$x_{24\times1} \equiv (\hat{\lambda}^c, \hat{\lambda}^{c*}, \hat{\lambda}^n - \hat{\lambda}^c, \hat{\lambda}^{n*} - \hat{\lambda}^{c*}, \hat{c}, \hat{c}^*, \Delta\hat{e}, \hat{rer}, \hat{tot}, \hat{tot}^*, \hat{y}, \hat{y}^*, \hat{\pi}, \hat{\pi}^*,$$

$$(\hat{W} - \hat{P}), (\hat{W}^* - \hat{P}^*), \hat{R}, \hat{R}^*, \hat{N}, \hat{N}^*, \hat{A}, \hat{A}^*, \hat{nx}_t, \hat{nx}_t^*)'$$

Vector of exogenous variables is

$$\epsilon_{4\times1} \equiv (\epsilon_R, \epsilon_{R^*}, \epsilon_A, \epsilon_{A^*})'$$

Sticky Price Model

Some equations remain the same as in the frictionless model. Consumption Euler equations(A.1,A.2), risk sharing condition(equation A.3), real wage equations(A.4,A.5), PPP condition(equation A.6), monetary policy rules (equations A.7,A.8), production functions (equations A.9,A.10), productivity processes(equations A.11,A.12)), net exports equations(A.19,A.20) and marginal

utility equations(A.21,A.22,A.23,A.24) are given in the previous section. Other equations of the model are described as follows.

Relative price of import goods is given by

$$\hat{q}_t = \hat{q}_{t-1} + \hat{\pi}_{F,t} - \hat{\pi}_{H,t} \tag{A.25}$$

$$\hat{q}_t^* = \hat{q}_{t-1}^* + \hat{\pi}_{H,t}^* - \hat{\pi}_{F,t}^* \tag{A.26}$$

Goods market clearing conditions are(relative prices of imports defined above are not necessarily equal to terms of trade),

$$\hat{y}_t = (1 - \gamma)\hat{c}_t + \gamma\hat{c}_t^* + \gamma(1 - \gamma)\eta\hat{q}_t - \gamma(1 - \gamma)\eta\hat{q}_t^* \tag{A.27}$$

$$\hat{y}_t^* = \gamma\hat{c}_t + (1 - \gamma)\hat{c}_t^* - \gamma(1 - \gamma)\eta\hat{q}_t + \gamma(1 - \gamma)\eta\hat{q}_t^* \tag{A.28}$$

Inflation indices are

$$\hat{\pi}_t = (1 - \gamma)\hat{\pi}_{H,t} + \gamma\hat{\pi}_{F,t} \tag{A.29}$$

$$\hat{\pi}_t^* = \gamma\hat{\pi}_{H,t}^* + (1 - \gamma)\hat{\pi}_{F,t}^* \tag{A.30}$$

Home marginal cost is

$$\hat{mc}_t = \hat{W}_t - \hat{P}_t + \gamma\hat{q}_t - \hat{A}_t \tag{A.31}$$

Foreign marginal cost is

$$\hat{mc}_t^* = \hat{W}_t^* - \hat{P}_t^* + \gamma\hat{q}_t^* - \hat{A}_t^* \tag{A.32}$$

The definition of law of one price gap is given by

$$\hat{\psi}_{F,t} = r\hat{e}r_t - (1-\gamma)\hat{q}_t - \gamma\hat{q}_t^* \tag{A.33}$$

$$\hat{\psi}_{H,t}^* = -r\hat{e}r_t - \gamma\hat{q}_t - (1-\gamma)\hat{q}_t^* \tag{A.34}$$

Home inflation on locally produced goods is

$$\hat{\pi}_{H,t} = \kappa\hat{mc}_t + \beta E_t\hat{\pi}_{H,t+1} \tag{A.35}$$

where $\kappa \equiv \frac{(1-\theta\beta)(1-\theta)}{\theta}$, and $\hat{mc}_t = \hat{W}_t - \hat{P}_{H,t} - \hat{A}_t$. Foreign inflation on locally produced goods is

$$\hat{\pi}_{F,t}^* = \kappa\hat{mc}_t^* + \beta E_t\hat{\pi}_{F,t+1}^* \tag{A.36}$$

where $\kappa \equiv \frac{(1-\theta\beta)(1-\theta)}{\theta}$ and $\hat{mc}_t^* = \hat{W}_t^* - \hat{P}_{F,t}^* - \hat{A}_t^*$. Price setting equations for import goods are

$$\hat{\pi}_{H,t}^* = \kappa\hat{mc}_t + \kappa\hat{\psi}_{H,t}^* + \beta E_t\hat{\pi}_{H,t+1}^* \tag{A.37}$$

$$\hat{\pi}_{F,t} = \kappa\hat{mc}_t^* + \kappa\hat{\psi}_{F,t} + +\beta E_t\hat{\pi}_{F,t+1} \tag{A.38}$$

Terms of trade definition is

$$\hat{tot}_t = \hat{q}_t + \hat{\psi}_{H,t}^* \tag{A.39}$$

$$\hat{tot}_t^* = \hat{q}_t^* + \hat{\psi}_{F,t} \tag{A.40}$$

Vector of state variables is

$$x_{34 \times 1} \equiv (\hat{\lambda}^c, \hat{\lambda}^{c*}, \hat{\lambda}^n - \hat{\lambda}^c, \hat{\lambda}^{n*} - \hat{\lambda}^{c*}, \hat{c}, \hat{c}^*, \Delta \hat{e}, r\hat{e}r, \hat{q}, \hat{q}^*, t\hat{o}t, t\hat{o}t^*, \hat{y}, \hat{y}^*, \hat{\pi}, , \hat{\pi}^*,$$

$$\hat{\pi}_H, \hat{\pi}_F, \hat{\pi}_H^*, \hat{\pi}_F^*, \hat{mc}, \hat{mc}^*, \hat{\psi}_F, \hat{\psi}_H^*, (\hat{W} - \hat{P}), (\hat{W}^* - \hat{P}^*), \hat{R}, \hat{R}^*, \hat{N}, \hat{N}^*, \hat{A}, \hat{A}^*, \hat{nx}, \hat{nx}^*)'$$

Vector of exogenous variables is

$$\epsilon_{4 \times 1} \equiv \left(\epsilon_R, \epsilon_{R^*}, \epsilon_A, \epsilon_{A^*} \right)'$$

Sticky Information Model

Common equations of this model are the PPP condition (equation A.6), monetary policy rules (equations A.7,A.8), production functions (equations A.9,A.10), goods market clearing conditions (equations A.27,A.28), net exports equations(equations A.19,A.20), terms of trade definitions (equations A.39,A.40), relative prices of import goods(equations A.25,A.26), CPI definitions (equations A.29,A.30), marginal cost equations(A.31,A.32) and law of one price gaps(equations A.33,A.34) . The remaining equations of the model are described as follows. Definitions of real interest rates and long interest rates

$$\hat{rr}_t = \hat{R}_t - E_t \hat{\pi}_{t+1} \tag{A.41}$$

$$\hat{rr}_t^* = \hat{R}_t^* - E_t \hat{\pi}_{t+1}^* \tag{A.42}$$

$$\hat{lr}_t = \hat{rr}_t + E_t \hat{lr}_{t+1} \tag{A.43}$$

$$\hat{lr}_t^* = \hat{rr}_t^* + E_t \hat{lr}_{t+1}^* \tag{A.44}$$

The aggregated sum of expected long interest rates

$$\hat{LR}_t = (1 - \delta) \sum_{j=0}^{\infty} \delta^j E_{t-j} \left[\hat{lr}_t \right] \tag{A.45}$$

$$\hat{LR}_t^* = (1 - \delta) \sum_{j=0}^{\infty} \delta^j E_{t-j} \left[\hat{lr}_t^* \right] \tag{A.46}$$

Real exchange rate is determined by

$$\hat{rer}_t = \hat{lr}_t^* - \hat{lr}_t \tag{A.47}$$

Defining auxiliary variables for price setting equations

$$a\hat{ux}_{1,t} = \Delta \hat{mc}_t + \hat{\pi}_{H,t} \tag{A.48}$$

$$a\hat{ux}_{2,t} = \Delta \hat{mc}_t^* + \hat{\pi}_{F,t}^* \tag{A.49}$$

$$a\hat{ux}_{3,t} = \Delta \hat{mc}_t + \Delta \hat{\psi}_{H,t}^* + \hat{\pi}_{H,t}^* \tag{A.50}$$

$$a\hat{ux}_{4,t} = \Delta \hat{mc}_t^* + \Delta \hat{\psi}_{F,t} + \hat{\pi}_{F,t} \tag{A.51}$$

Home inflation on locally produced goods is

$$\hat{\pi}_{H,t} = \frac{1 - \theta}{\theta} \hat{mc}_t + \frac{1 - \theta}{\theta} \left(\sum_{j=1}^{\infty} \theta^j E_{t-j} a\hat{ux}_{1,t} \right) \tag{A.52}$$

Foreign inflation on locally produced goods is

$$\hat{\pi}_{F,t}^* = \frac{1 - \theta}{\theta} \hat{mc}_t^* + \frac{1 - \theta}{\theta} \left(\sum_{j=1}^{\infty} \theta^j E_{t-j} a\hat{ux}_{2,t} \right) \tag{A.53}$$

Price setting equations for import goods are

$$\hat{\pi}^*_{H,t} = \frac{1-\theta}{\theta}\left(\hat{mc}_t + \hat{\psi}^*_{H,t}\right) + \frac{1-\theta}{\theta}\left(\sum_{j=1}^{\infty}\theta^j E_{t-j}\hat{aux}_{3,t}\right) \tag{A.54}$$

$$\hat{\pi}_{F,t} = \frac{1-\theta}{\theta}\left(\hat{mc}^*_t + \hat{\psi}_{F,t}\right) + \frac{1-\theta}{\theta}\left(\sum_{j=1}^{\infty}\theta^j E_{t-j}\hat{aux}_{4,t}\right) \tag{A.55}$$

Aggregate consumption equations are

$$\hat{c}^{agg}_t = -\frac{1}{\sigma}\hat{LR}_t \tag{A.56}$$

$$\hat{c}^{agg*}_t = -\frac{1}{\sigma}\hat{LR}^*_t \tag{A.57}$$

Aggregate labor equations are

$$\frac{1}{\phi}\hat{n}^{agg}_t + \sigma\hat{c}^{agg}_t = (\hat{W}_t - \hat{P}_t) \tag{A.58}$$

$$\frac{1}{\phi}\hat{n}^{*agg}_t + \sigma\hat{c}^{*agg}_t = (\hat{W}^*_t - \hat{P}^*_t) \tag{A.59}$$

Vector of state variables is

$$x_{38\times1} \equiv (\hat{rr}, \hat{rr}^*, \hat{lr}, \hat{lr}^*, \hat{LR}, \hat{LR}^*, \Delta\hat{e}, \hat{rer}, \hat{tot}, \hat{tot}^*, \hat{q}, \hat{q}^*, \hat{y}, \hat{y}^*, \hat{\pi}, , \hat{\pi}^*,$$

$$\hat{\pi}_H, , \hat{\pi}_F, \hat{\pi}^*_H, \hat{\pi}^*_F, \hat{mc}, \hat{mc}^*, \hat{\psi}_F, \hat{\psi}^*_H, (\hat{W} - \hat{P}), (\hat{W}^* - \hat{P}^*)$$

$$\hat{R}, \hat{R}^*, \hat{nx}, \hat{nx}^*, \hat{aux}_1, \hat{aux}_2, \hat{aux}_3, \hat{aux}_4, \hat{c}^{agg}, \hat{c}^{agg*}, \hat{n}^{agg}, \hat{n}^{agg*})'$$

Vector of exogenous variables is

$$\epsilon_{4\times1} \equiv \left(\epsilon_R, \epsilon_{R^*}, \epsilon_A, \epsilon_{A^*}\right)'$$

For the wage posting model, aggregate labor response is described by following the equations(equations A.58 and A.58 are replaced),

$$(\hat{w}_t - \hat{p}_t) = \frac{\delta(\phi + \chi)}{\phi + \delta\chi}(\hat{w}_{t-1} - \hat{p}_{t-1}) + \frac{1-\delta}{\phi + \delta\chi}\hat{n}_t^{agg} - \frac{\phi(1-\delta)}{\phi + \delta\chi}\hat{lr}_t$$
$$- \frac{\delta(\phi + \chi)}{\phi + \delta\chi}\hat{\pi}_t + \frac{\delta(\phi + \chi)}{\phi + \delta\chi}\sum_{j=0}^{\infty}(1-\delta)\delta^j E_{t-1-j}a\hat{u}x_{w1,t}$$
$$(\hat{w}_t^* - \hat{p}_t^*) = \frac{\delta(\phi + \chi)}{\phi + \delta\chi}(\hat{w}_{t-1}^* - \hat{p}_{t-1}^*) + \frac{1-\delta}{\phi + \delta\chi}\hat{n}_t^{agg*} - \frac{\phi(1-\delta)}{\phi + \delta\chi}\hat{lr}_t^*$$
$$- \frac{\delta(\phi + \chi)}{\phi + \delta\chi}\hat{\pi}_t^* + \frac{\delta(\phi + \chi)}{\phi + \delta\chi}\sum_{j=0}^{\infty}(1-\delta)\delta^j E_{t-1-j}a\hat{u}x_{w2,t}$$

Auxiliary variables for wage equations

$$a\hat{u}x_{w1,t} = \left[\hat{\pi}_t + \frac{\chi}{\phi + \chi}\Delta(w_t - p_t) + \frac{1}{\phi + \chi}\Delta\hat{n}_t^{agg} - \frac{\phi}{\phi + \chi}\Delta\hat{lr}_t\right]$$
$$a\hat{u}x_{w2,t} = \left[\hat{\pi}_t^* + \frac{\chi}{\phi + \chi}\Delta(w_t^* - p_t^*) + \frac{1}{\phi + \chi}\Delta\hat{n}_t^{agg*} - \frac{\phi}{\phi + \chi}\Delta\hat{lr}_t^*\right]$$

Steady State

We normalize the level of prices to 1 and impose symmetry. Therefore, $\bar{P} = \bar{P}_H = \bar{P}_F = \bar{P}^* = \bar{P}_H^* = \bar{P}_F^* = \bar{e} = \bar{rer} = \bar{tot} = \bar{tot}^* = 1$. Productivity levels are $\bar{A} = \bar{A}^* = 1$. Inflation is zero at steady state, and interest rates are $\bar{R} = \bar{R}^* = \bar{rr} = \bar{rr}^* = \frac{1}{\beta}$. Quantities are given by $\bar{Y} = \bar{C} = \bar{C}_H = \bar{C}_F = \bar{Y}^* = \bar{C}^* = \bar{C}_H^* = \bar{C}_F^* = \bar{A}\bar{N} = \bar{A}^*\bar{N}^*$. Price setting equations give,

$$\bar{P}(j) = \frac{\nu}{\nu - 1}\frac{\bar{W}_t}{\bar{A}_t}$$

which implies the steady state value of nominal and real wage is $\frac{\nu - 1}{\nu}$. Using the labor supply condition and separable utility

$$-\frac{\bar{\lambda}^n}{\bar{\lambda}^c} = \frac{\bar{W}_t}{\bar{P}_t}; \qquad \frac{\bar{N}^{1/\phi}}{\bar{C}^{-\sigma}} = \frac{\nu - 1}{\nu}$$

Using $\bar{Y} = \bar{C} = \bar{N}$,

$$\bar{Y} = \bar{C} = \bar{N} = \left(\frac{\nu - 1}{\nu}\right)^{\frac{1}{1/\phi + \sigma}}$$

Calculating Theoretical Moments

The covariance matrix of vector of innovations (ϵ_t) is denoted by Σ. The solution for models without lagged expectations is given by $x_t = Ax_{t-1} + \epsilon_t$. We can calculate the unconditional covariance matrix of the state vector x_t(Γ_0) as follows

$$Var(x_t) = AVar(x_{t-1})A' + B\Sigma B' \Gamma_0 = A\Gamma_0 A' + B\Sigma B'$$

$$vec(\Gamma_0) = vec(A\Gamma_0 A') + vec(B\Sigma B')$$

Using $vec(X_1 Y X_2) = X_2^T \otimes X_1 vec(Y)$

$$vec(\Gamma_0) = (I - A \otimes A)^{-1} vec(B\Sigma B')$$

Autocovariances are given by,

$$\Gamma_1 = Cov(x_t, x_{t-1}) = Cov(Ax_{t-1} + B\epsilon_t, x_{t-1}) = A\Gamma_0$$

$$\Gamma_k = Cov(x_t, x_{t-k}) = Cov(A^k x_{t-k} + ..., x_{t-k}) = A^k \Gamma_0$$

When we solve the models with lagged expectations, the solution is of the following form :
$x_t = \sum_{j=0}^{\infty} \Theta_j \epsilon_{t-j}$, and autocovariances are calculated accordingly.

For HP filtered moments, we use a two-sided filter following King and Rebelo(1993). For any series F, our filter is defined as $F^{HP} = B(L)F$, where $B(L) = \sum_{j=-\infty}^{\infty} b_j L^j$. At quarterly

frequency, setting HP parameter λ to 1600, filter coefficients are given by

$$b_j = b_{-j} = -(.894^j)\left[(0.0561cos(.112j)) + (0.0558sin(.112j))\right]$$

For $j = 0$, $b_0 = 1 - 0.0561 = 0.9439$. Proceeding with derivations we can show that,

$$Var_iF^{HP} = E\left(F_t^{HP}F_{t-i}^{HP}\right) = E\left([B(L)F]\left[B(L)L^iF'\right]\right) = \sum_{j=-\infty}^{\infty}\sum_{k=-\infty}^{\infty}b_jb_{j-k-i}Var_kF$$

The final result for i th covariance is given by,

$$Var_iF^{HP} = Var_0F\sum_{j=-\infty}^{\infty}b_jb_{j-i} + \sum_{k=1}^{\infty}\sum_{j=-\infty}^{\infty}(b_jb_{j+k-i}Var_kF + b_jb_{j-k-i}Var_kF)$$

Appendix D : Solution of the Models with Lagged Expectations

Sticky price models can be written as second order difference equations and solved by standard methods outlined in Klein(2000). This appendix closely follows Meyer-Gohde(2010). Consider the following model with lagged expectations

$$AE_tY_{t+1} + B_0Y_t + \sum_{i=1}^{I} B_iE_{t-i}Y_t + CY_{t-1} + GW_t = 0$$

where Y_t is $n \times 1$ vector of endogenous variables, and W_t is $k \times 1$ vector of exogenous variables with a law of motion $W_t = NW_{t-1} + \epsilon_t$, or alternatively, with the moving average representation $W_t = \sum_{j=0}^{\infty} N^j\epsilon_{t-j}$. The solution is of the form of $Y_t = \sum_{j=0}^{\infty} \Theta_j\epsilon_{t-j}$ with coefficients Θ_j $(n \times k)$. The one period ahead realization is $Y_{t+1} = \sum_{j=0}^{\infty} \Theta_j\epsilon_{t+1-j}$; taking expectations yields

$$E_tY_{t+1} = E_t\sum_{j=0}^{\infty} \Theta_j\epsilon_{t+1-j} = \sum_{j=1}^{\infty} \Theta_j\epsilon_{t+1-j} = \sum_{j=0}^{\infty} \Theta_{j+1}\epsilon_{t-j}$$

Similarly, $Y_{t-1} = \sum_{j=0}^{\infty} \Theta_j\epsilon_{t-1-j}$. For the past expectations, when $i = 0 : Y_t = \sum_{j=0}^{\infty} \Theta_j\epsilon_{t-j}$, $i = r : E_{t-r}Y_t = \sum_{j=r}^{\infty} \Theta_j\epsilon_{t-j}$. Expanding the expression,

$$\sum_{i=0}^{I} B_iE_{t-i}Y_t = B_0Y_t + B_1E_{t-1}Y_t + B_2E_{t-2}Y_t + B_3E_{t-3}Y_t + ... + B_IE_{t-I}Y_t$$

$$= B_0\sum_{j=0}^{\infty} \Theta_j\epsilon_{t-j} + B_1\sum_{j=1}^{\infty} \Theta_j\epsilon_{t-j} + B_2\sum_{j=2}^{\infty} \Theta_j\epsilon_{t-j} + B_3\sum_{j=3}^{\infty} \Theta_j\epsilon_{t-j} + ... + B_I\sum_{j=I}^{\infty} \Theta_j\epsilon_{t-j}$$

$$= B_0\Theta_0\epsilon_t + (B_0 + B_1)\Theta_1\epsilon_{t-1} + (B_0 + B_1 + B_2)\Theta_2\epsilon_{t-2} + ... + \left(\sum_{j=0}^{I} B_j\right) \sum_{k=j+1}^{\infty} \Theta_k\epsilon_{t-k}$$

Defining $\tilde{B}_j \equiv \left(\sum_{i=0}^{min(I,j)} B_i\right)$, we can write this expression as follows

$$\sum_{i=0}^{I} B_iE_{t-i}Y_t = \sum_{j=0}^{\infty} \tilde{B}_j\Theta_j\epsilon_{t-j}$$

Plugging the MA representation, the system in terms of the MA coefficients is

$$A \sum_{j=0}^{\infty} \Theta_{j+1} \epsilon_{t-j} + \sum_{j=0}^{\infty} \tilde{B}_j \Theta_j \epsilon_{t-j} + C \sum_{j=0}^{\infty} \Theta_j \epsilon_{t-1-j} + G \sum_{j=0}^{\infty} N^j \epsilon_{t-j} = 0$$

We need to solve for the MA coefficient matrices, Θ_0 ... Θ_I, for a large I. These coefficients solve the following system of equations,

$$\left[A\Theta_1 + \tilde{B}_0 \Theta_0 + G \right] \epsilon_t = 0, \forall \epsilon_t$$

$$\left[A\Theta_2 + \tilde{B}_1 \Theta_1 + C\Theta_0 + GN \right] \epsilon_{t-1} = 0$$

$$\left[A\Theta_3 + \tilde{B}_2 \Theta_2 + C\Theta_1 + GN^2 \right] \epsilon_{t-2} = 0$$

$$....$$

$$\left[A\Theta_{j+1} + \tilde{B}_j \Theta_j + C\Theta_{j-1} + GN^j \right] \epsilon_{t-j} = 0$$

We have I matrix equations with $I+1$ unknowns, Θ_0 .. Θ_I. The coefficients of the recursion are non-varying when $j \geq I$. Therefore, the last equation is obtained by solving a second order difference equation

$$A\Theta_{j+1} + \tilde{B}_I \Theta_j + C\Theta_{j-1} + GN^I x_j = 0 \quad j \geq I$$

$$x_I = I_k \quad and \quad x_{j+1} = x_j$$

The dimensions of the matrices are A , $\tilde{B}_I$ and C: $n \times n$, $G : n \times k$, $N : k \times k$ and x: $k \times 1$. The solution is $\Theta_j = \alpha_\theta \Theta_{j-1} + \alpha_N x_j$ and $\Theta_I = \alpha_\theta \Theta_{I-1} + \alpha_N$. In our system, $I \to \infty$, therefore we need to plug in the limiting matrix $\tilde{B}_I$ and take a large enough number of lags. We can write the resulting system of equations in a tridiagonal structure by setting the initial

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condition as $\theta_{-1} = 0$.

$$
\begin{pmatrix}
\tilde{B}_0 & A & 0 & 0 & 0 & 0 & 0 & 0 \\
C & \tilde{B}_1 & A & 0 & 0 & 0 & 0 & 0 \\
0 & C & \tilde{B}_2 & A & 0 & 0 & 0 & 0 \\
.. & .. & .. & .. & .. & .. & .. & .. \\
0 & 0 & 0 & 0 & 0 & C & \tilde{B}_{I-1} & A \\
0 & 0 & 0 & 0 & 0 & 0 & -\alpha_\theta & I_n
\end{pmatrix}_{(I+1)n \times (I+1)n}
\begin{pmatrix}
\theta_0 \\
\theta_1 \\
\theta_2 \\
.. \\
\theta_{I-1} \\
\theta_I
\end{pmatrix}_{(I+1)n \times k}
=
\begin{pmatrix}
-G \\
-GN \\
-GN^2 \\
.. \\
-GN^{I-1} \\
\alpha_N
\end{pmatrix}_{(I+1)n \times k}
$$