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# **Can carpooling clean the air? The economics of HOV lanes, hybrid cars and the Clean Air Act.**

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# Can Carpooling Clean the Air?

The Economics of HOV Lanes, Hybrid Cars and the Clean Air Act

By

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## **Can Carpooling Clean the Air?**

### **Abstract**

Private vehicles are a significant source of air pollution in many areas of the United States. Areas with already high levels of air pollution are required by the Clean Air Act to take steps to reduce automobile use and the associated emissions. The behavioral implications of many travel demand management techniques are poorly understood. In this dissertation I focus on carpooling. Policy makers encourage commuters to carpool through High Occupancy Vehicle (HOV) Lanes, free parking for carpoolers, attempts to connect carpoolers, and casual carpoolers (often called slugging). Despite these efforts, carpooling rates have been falling over time.

One reason for the decrease in carpooling rates, is that carpooling comes with an additional set of personal costs. These costs include reduced route flexibility, assembly costs, and a loss of privacy when another person shares the car. Encouraging carpooling may not improve traffic conditions as much as advocates claim since new carpoolers may be people who would otherwise not have driven. Encouraging carpooling does not eliminate the root of all traffic problems: under- or un-priced road space.

Traditional travel demand models take carpool mode share as exogenous. In this dissertation, I make the decision to carpool endogenous, and build a traffic equilibrium model based on the micro-economic foundations of individual route choices. I then use my model to evaluate High Occupancy Vehicle (HOV) lanes. I apply insights from these studies to a particular policy in California that sought to allocate space on HOV lanes to buyers of hybrid cars as an incentive to adopt this new technology.

My dissertation is divided into four chapters. In Chapter 1, I review current models of carpooling behavior and route choice. In Chapter 2, I develop my micro-foundation model of carpooling behavior. In the Chapter 3, I apply and extend my model to the study of HOV lanes. In Chapter 4, I use data from the used car market to understand what happened when California allocated space in HOV lanes to hybrid car owners.

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## Chapter 1

# Transportation Demand Models

In this chapter I discuss structural transportation models applied to carpooling and congestion. In the next chapter I will present my own model of carpooling and congestion, but before doing so I review transportation models and put my work into context with the existing literature. In the first section of this chapter, I discuss engineering models that model trip generation as an exogenous process. The next three sections discuss models where carpooling and trips taken are endogenous to the model. I call these ‘economic traffic models’. There are three types of economic traffic models: discrete choice models, deterministic models of utility maximization, and cost minimization models.

### 1.1 Engineering Models of Transportation Demand

The workhorse of transportation demand modeling is the four step model (FSM). Planners and transportation engineers use the FSM to examine questions such as how many lanes should a bridge have, what is the financial viability of a project, and what are the potential environmental impacts of a project. The FSM dominates transportation planning despite well known inadequacies in terms of realism and its inability to answer many relevant policy questions (McNally, 2007; McNally and Recker, 1986).

The first step in the FSM is trip generation to determine the number of trips taken from each trip origin and the number of trips attracted to each destination. The models use demographic and land use information to generate origin-destination (O-D) matrices which forecast the number and type of trips coming from and going to each area. Trip generation is not modeled using economic fundamentals, and instead is based off of historical associations between demographic variables and historical trips levels. Some models include friction factors which express the reduction in trips taken with greater travel times, but this is an aggregate measure. The FSM's trip generation step might be adequate for predicting future demand, but presents a problem in analyzing structural changes such as an upgraded bus system, new transportation technologies, incentives for carpooling or congestion pricing.

Step two in the FSM is trip distribution. This step uses a gravity model or similar method to connect origins with destinations and hence load the demand generated through the first step onto the transportation network. The next two steps are mode choice and route choice. Mode choice determines the proportion of trips that travel by each mode, where modes usually include transit, HOV2, HOV3, and driving alone. Potential mode choices may include cycling, walking or not taking a trip at all. Route choice allocates origin-destination pairs by a particular mode to a route. This step relies on Wardrop's principle of user equilibrium (equivalent to a Nash Equilibrium in economics) that states each traveler chooses the path with the lowest travel time subject to the decisions of all the other travelers. The FSM has a significant advantage over other models in its ability to model large metropolitan areas and account for complicated geography. Even with simplistic assumptions of trip generation and route choice, an FSM model may take millions of dollars to calibrate and weeks to converge.

The alternative to FSMs is activity based modeling which shifts the unit of analysis from trips to activities. Activity based modeling recognizes that the demand for travel is derived from the demand to pursue activities that vary in both time and space. Activ-

ity based models can better incorporate observed travel behavior such as trip-chaining and induced demand but require that the researcher collect travel diaries detailing all activities pursued by an individual over the course of the day for a more holistic analysis of travel behavior. These models are more difficult to categorize than FSMs. They allow for a richer description of travel behavior, but do not necessarily have micro-economic foundations. Activities may be chosen as a function of utility maximization (Ben-Akiva and Bowman, 1998; Wen and Koppelman, 2000), but often trips are generated in activity models through rule based decision making (Vause, 1997) or using a statistical approach (Vaughn and Pas., 1997; Speckman, Vaughn, and Pas, 1997). Again, while potentially useful for understanding travel behavior in the aggregate and for modeling large metropolitan areas, approaches without micro-economic foundations may not capture the important aspects of travel demand and system behavior. In the next subsection, I summarize a few transportation models that do not have micro-economic foundations.

### **1.1.1 Commuter Welfare Approach to High Occupancy Vehicle Lane Evaluation: An Exploratory Analysis**

By Fred Mannering and Mohammad Hamed, *Transportation Research Part A*, 1990.

Previous studies have evaluated HOV lane policies by comparing passengers per mile, travel time, fuel consumption, distance traveled, pollution costs and parking costs. Mannering and Hamed make the valid point that analysts should instead use welfare criteria to evaluate HOV lane performance. While using a welfare metric should put this work into the economic model section, the behavioral model is more similar to the FSM. The authors examine a single origin/destination pair near Penn State University but simply assume three levels of HOV percentages, 17%, 30%, and 40% for the traffic simulation model.

### **1.1.2 The Effects of New High-Occupancy Vehicle Lanes on Travel and Emissions**

By Robert Johnston and Raju Ceerla, *Transportation Research Part A*, 1996.

The authors run a FSM with multiple feedback loops to understand the impact of adding 206 new freeway lane-miles of HOV-only lanes to the Sacramento region. Their model includes friction factors from a survey done in Seattle, Washington. These friction factors attempt to model the reduction in commute trips that result from high travel times. The friction factors are applied individually to each mode considered and thus do not represent decisions to switch modes depending on travel time differentials between HOV and general purpose lanes. The authors test many scenarios: not building anything new, building an HOV lane, peak period tolls of \$0.50/mile on freeways and \$0.25 on arterial roads, replacing a general purpose lane with an HOV lane, \$0.30/mile citywide tolls, light rail, transit oriented development and combinations of the above. The results were sensitive to initial assumptions but the authors found that building a new HOV lane increased vehicle miles travelled (VMT) but decreased delays relative to the no-build scenario. Their results suggest that converting a general purpose lane to HOV also increased VMT and increased vehicle delay substantially. While likely one of the more realistic papers on the impacts of HOV lanes, this paper's results cannot be generalized outside of the Sacramento metropolitan area.

### **1.1.3 High Occupancy Vehicle Lanes: Not Always More Effective than General Purpose Lanes**

By Joy Dahlgren. *Transportation Research Part A*, 1998.

Dahlgren uses a bottleneck model to examine congestion, making the share of HOV vehicles on the road an exogenous function of the time differentials between HOV and general purpose lanes. I am classifying this as an engineering model of traffic conges-

tion because Dahlgren takes as exogenous the relationship between percentage of people who carpool and the time savings to carpool.

Dahlgren does discuss the assumptions behind a behavioral model of HOV lane performance, and this is one of the better papers on HOV lanes. However, Dahlgren's analysis falls short of modeling economic decision making on the part of commuters.

## 1.2 Discrete Choice Models

Discrete Choice models have a long history with the transportation literature, starting with the additive random-utility model of McFadden (McFadden, 1974). In discrete choice models, user  $n$  decides between alternatives  $j = 1, \dots, J$  by choosing the alternative with the highest utility given by:

$$U_{j,n} = V(z_{j,n}, s_n; \beta) + \epsilon_{j,n} \quad (1.1)$$

Here  $V(\cdot)$  is known as the systematic utility,  $z_{j,n}$  is a vector of alternative specific attributes and  $s_n$  is a vector of characteristics specific to the decision maker. The unobservable part of the model captures idiosyncratic preferences and is represented by  $\epsilon_{j,n}$ .

An individual is said to choose alternative  $j$  if  $U_{j,n} > U_{i,n} \forall j \neq i$ . This can be rewritten as the probability that decision maker  $n$  chooses alternative  $j$ :

$$\begin{aligned} P_{j,n} &= Pr(U_{j,n} > U_{i,n} \forall j \neq i) \\ &= Pr(V_{j,n} + \epsilon_{j,n} > V_{i,n} + \epsilon_{i,n} \forall j \neq i) \\ &= Pr(\epsilon_{j,n} - \epsilon_{i,n} > V_{j,n} - V_{i,n} \forall j \neq i) \end{aligned} \quad (1.2)$$

The researcher can specify a functional form for  $V(\cdot)$ , an error structure for  $\epsilon_{j,n}$  and estimate a model with direct applications to welfare analysis. This technique can be

used to model transportation demand and mode choice, with an example discussed in the next subsection.

### 1.2.1 The Models and Economics of Carpools

By Hai-Jun Huang, Hai Yang and Michael Bell *Annals of Regional Science*, 2000.

This paper presents both a deterministic model of carpooling behavior and a discrete choice model. The deterministic model has commuters deciding between carpooling or driving alone by picking the mode with the lowest cost, where costs are:

$$\begin{aligned} c_x &= \beta t(v) + (f + a)/2 \\ c_y &= \beta t(v) + f \end{aligned} \tag{1.3}$$

The cost of carpooling,  $c_x$ , is the value of time,  $\beta$ , times the amount of time spent on the line haul portion of the trip, plus fuel costs,  $f$ , divided by two, and assembly costs,  $a$ , divided by two. The costs of driving alone,  $c_y$ , is simply the time costs of driving  $\beta v(t)$  plus fuel costs,  $f$ . The problem with this model is that everyone either drives or everyone carpools. The authors add realism by turning it into a simple discrete choice model. Agents have two options, to carpool or drive alone. They receive a generalized utility from carpooling and driving alone as such:

$$\begin{aligned} U_x &= U - c_x + \xi_x \\ U_y &= U - c_y + \varphi + \xi_y \end{aligned} \tag{1.4}$$

where  $U$  is a constant representing the utility receiving through a trip,  $c_x$  is the monetary cost of carpooling and  $c_y$  is the generalized cost of driving, while  $\xi_x$  and  $\xi_y$  both represent the random utility components from carpooling and driving alone. The variable  $\varphi$  represents the summation of attitudinal or psychological factors that make commuters have a subjective preference for driving alone. The authors derive optimality conditions but do not calibrate their model to data or explore HOV lanes.



### 1.2.2 Differentiated Road Pricing, Express Lanes and Carpools: Exploiting Heterogeneous Preferences in Policy Design

By Small, Winston and Yan, 2006, Working Paper.

This paper uses a survey of travelers along State Route 91 in Southern California to estimate an empirical model of route choice where travelers have a choice between driving alone on general lanes or paying a toll to use the HOT lane, and secondly whether or not to carpool, where carpoolers receive a 50% discount on tolls if they have three or more people in their car. After exploring the value of time and commuters' value for reliability, the authors then run a simulation to understand changes in consumer surplus that result from changes in route and toll structure.

The authors estimate a nested logit. Since a transponder is required to drive on the express lane the decision to acquire a transponder is estimated separately. Car-occupancy rates are modeled in the second stage along with the choice on whether or not to drive on the express lane conditional on having obtained a transponder. To estimate travel demand, the authors integrate the nested-logit probability formula over the distribution of the random parameters and obtain the demand for each alternative as:

$$D_j = \sum_n w_n S_{j,n} \quad (1.5)$$

Here  $D_j$  is demand for alternative  $j$ ,  $w_n$  is the number of people represented by motorist  $n$ , and  $S_{j,n}$  is the share of type  $n$  commuter in transportation mode  $j$ . Traffic volume on route  $j$  is thus  $V_j \equiv D_j/O_j$  where  $O_j$  is the occupancy of route  $j$ . Travel delays are thus proportional to the fourth power of the volume-capacity ration with capacity set at 2,000 vehicles per hour per lane.

Small, Winston and Yan find that HOV lanes provide improvements for both carpoolers and non-carpoolers by doubling the share of people who choose to carpool (travel times go from 20 minutes in the base case scenario to 18.8 minutes on the general

lane and 11.8 minutes on the HOV lane). They also find that HOV lanes induce people who were not traveling on the corridor to travel on the corridor. They demonstrate that discrete choice models can be used to explore the efficiency considerations and create a behavioral explanation of carpooling behavior. Additional papers explore carpooling and HOV lanes using discrete choice models, they including:

- “Choice of Route, Occupancy, and Time-of-Day with Value Priced Tolls”, by Yan, Small and Sullivan, 2001, Working Paper.
- “When Should Carpool Lanes Be Introduced in a Multi-Lane Highway?”, by Hai Yang, 2010, *Journal of Advanced Transportation*.

The limitation of this literature is that the discrete choice models uncover reduced form relationships between route choice and hence do not model efficiency changes as a function of the structural aspects of the route. Arnott, De Palma, and Lindsey (1993a,b) call for a structural model to explain congestion, which is what I present in the next chapter. In Small, Winston and Yan, it is unclear which drivers are choosing to drive on the newly added HOV lane, and how passenger choices get transmitted into travel time which again feeds back into passenger choices. Another limitation is that discrete choice models have the particular feature of increasing in total welfare as a result of adding an alternative (in these cases an HOV lane) because of the idiosyncratic structure of the errors. In the case of Small, Winston and Yan, the model is estimated only for parameters representing SR-91, and it is not clear whether or not the results apply to all HOV lanes.

### **1.3 Deterministic Models of Route Choice: Utility Maximization**

Deterministic models of route choice typically model travel demand for trips, as a constrained utility maximization, a demand system or a cost minimization problem. The

first theoretical framework we examine comes from Becker's (Becker, 1965) model of utility maximization subject to budget and time constraints:

$$\begin{aligned}
 & \max_{G, T_w, \{T_k\}} && U(G, T_w, \{T_k\}) \\
 & \text{s.t.} && G + \sum_k P_k \leq wT_w + Y \\
 & && \bar{T} \leq T_w + \sum_k T_k
 \end{aligned} \tag{1.6}$$

In this model, utility  $U$  depends on consumption of goods,  $G$ , time spent at work,  $T_w$ , and times spend in  $k$  other activities  $T_k$ . The budget constraint ensures that expenditures remain under exogenous income,  $Y$  and wage income,  $wT_w$  where  $w$  is the wage rate and the price of goods is normalized to one. In addition to the budget constraint there is a time constraint where the total amount of time available,  $\bar{T}$  must be greater than time spent working,  $T_w$ , and the sum of time spend on activities  $\sum_k T_k$ . The model can be extended to represent variable commuting times, constraints on work hours, home production, psychological biases and general equilibrium effects as described in (Small and Verhoef, 2007).

## 1.4 Deterministic Models of Route Choice: Cost Minimization

Additionally, many researchers predict travel behavior based on cost minimization behavior (Konishi and Mun, 2010; de Palma, Kilani, and Lindsey, 2008; Arnott, De Palma, and Lindsey, 1993a,b; Vickrey, 1969). The most prominent of these models is the bottleneck model first conceptualized by Vickery (1969) and formalized by Arnott, De Palma, and Lindsey (1993a,b). The Arnott et al. framework is reviewed below; additionally a number of papers have been written using cost minimization to explain route choice.

### 1.4.1 A Structural Model of Peak-Period Congestion: A Traffic Bottleneck with Elastic Demand

By Richard Arnott, Andre de Palma and Robin Lindsey, 1993, *American Economic Review*.

This paper models the trade-off between getting to work at an inconvenient time (earlier or later than the starting date) versus waiting in traffic. In the model,  $N$  identical agents travel from home to work along a road where traffic is uncongested except at a bottleneck where only  $s$  cars can pass through at a time. If the arrival rate at the bottleneck exceeds  $s$ , then a queue forms. Travel time from home to work is composed of a fixed time component and variable travel time which is a function of  $t$ , the departure time from home:

$$T(t) = T^f + T^v(t) \quad (1.7)$$

Without impacting the results, the authors set  $T^f = 0$ . The number of cars in the queue is denoted  $D(t)$  and thus the variable amount of time spend waiting in the queue is:

$$T^v(t) = \frac{D(t)}{s} \quad (1.8)$$

Let  $\hat{t}$  be the most recent time without a queue, and let  $r(t)$  be the departure rate from home. Thus the queue length is:

$$D(t) = \int_{\hat{t}}^t r(u)du - s(t - \hat{t}). \quad (1.9)$$

Agents perceive early and late arrival costs as costly, so the authors model private costs as linear in travel time and schedule delay:

$$C(t) = \alpha T^v(t) + \beta(\text{time early}) + \gamma(\text{time late}) \quad (1.10)$$

where  $\alpha$  is the cost of travel time,  $\beta$  is the unit cost of arriving early at work and  $\gamma$  is the unit cost of arriving late. Commuters cross the bottleneck in the same order as they left home. The authors discuss various toll and no-toll equilibria as well as situations with heterogeneous agents. For brevity I discuss the no-toll equilibrium only, but I use the same notation as the paper and the reader should look at the original paper to see the extensions.

The authors use a Nash equilibrium solution whereby no commuter can reduce their time costs by changing their arrival time. The first commuter and the last commuter must be equally well off in equilibrium. Thus the equal trip price condition for a commuter who arrives early is:

$$\bar{p} = \alpha T^v(t) + \beta[t^* - t - T^v(t)] \quad (1.11)$$

and for the commuter who arrives late, the equal price condition is:

$$\bar{p} = \alpha T^v(t) + \gamma[t + T^v(t) - t^*]. \quad (1.12)$$

Differentiating Equation 1.11, we find:

$$\frac{dT^v(t)}{dt} = \frac{\beta}{\alpha - \beta}. \quad (1.13)$$

Solving and differentiating Equation 1.9, yields:

$$\frac{dD(t)}{dt} = r(t) - s. \quad (1.14)$$

Let  $t_q$  be the beginning of the rush hour, and  $t_{q'}$  be the end of the rush hour, and  $\tilde{t}$  be the departure time for on-time arrival [ $\tilde{t} = t^* - T^v(t)$ ]. Combining Equations 1.13 and 1.14

with Equation 1.8, we can write that:

$$r(t) = \frac{\alpha s}{\alpha - \beta} \quad \text{for } t \in [t_q, \tilde{t}]. \quad (1.15)$$

Similarly manipulating Equations 1.12, 1.9 and 1.8 can be shown to yield:

$$r(t) = \frac{\alpha s}{\alpha - \gamma} \quad \text{for } t \in [\tilde{t}, t_{q'}]. \quad (1.16)$$

The time between the first and the last commuters departures must be  $N/s$ , thus:

$$\frac{N}{s} = t_{q'} - t_q. \quad (1.17)$$

The last commuter and the first commuter do not wait in the queue, but their equilibrium costs remain equal:

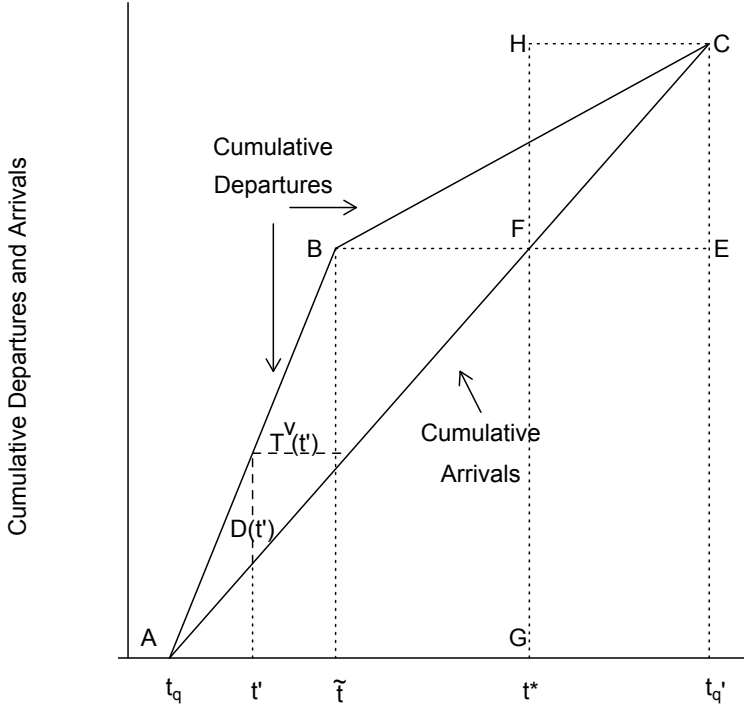
$$\beta(t^* - t_q) = \gamma(t_{q'} - t^*). \quad (1.18)$$

Combining these two equations we can write the beginning and the end of the rush hour as:

$$\begin{aligned} t_q &= t^* - \left( \frac{\gamma}{\beta + \gamma} \right) \frac{N}{s} \\ t_{q'} &= t^* - \left( \frac{\beta}{\beta + \gamma} \right) \frac{N}{s}. \end{aligned} \quad (1.19)$$

The cost of time spent in traffic for those arriving at  $t^*$  is equivalent to the time cost for those arriving early or late. The commuter who arrives exactly on time only faces travel time costs:

$$\alpha T^v(\tilde{t}) = \alpha(t^* - \tilde{t}). \quad (1.20)$$



**Figure 1.1:** The No Toll Equilibrium, Arnott et al 1993

Using the equilibrium condition we can set the costs of the traveller who arrives at  $t^*$  equal to the costs of the first commuter and solve for  $\tilde{t}$ :

$$\begin{aligned} \alpha(t^* - \tilde{t}) &= \beta t^* - \beta \left( t^* - \left( \frac{\gamma}{\beta + \gamma} \right) \frac{N}{s} \right) \\ \tilde{t} &= t^* - \left( \frac{\beta}{\alpha} \right) \left( \frac{\gamma}{\beta + \gamma} \right) \frac{N}{s}. \end{aligned} \quad (1.21)$$

Using these expressions the authors trace out the solution which is reproduced in Figure 1.1. The length of the queue is the vertical distance between cumulative departures and cumulative arrivals, while the travel time is the horizontal difference. The queue builds up starting at  $t_q$  until  $\tilde{t}$  until it ends at time  $t_q'$ . The total travel costs can be seen as  $\alpha$  times area ABCA and the schedule delay cost is  $\beta$  times AFGA plus  $\gamma$  times area CFHC.

The authors develop many extensions to this model, including elastic demand for trips, tolling regimes, heterogeneous users, optimal capacity and the potential for self-financing roads. In the 1990 paper they compare total costs of commuting between tolling regimes, while in the 1993 paper the authors present the case of elastic demand by specifying demand as a function of generalized price and then comparing consumer surplus across toll regimes. Additional work has been done with bottleneck models, including (Yang and Huang, 1997).

#### 1.4.2 Carpooling and Congestion Pricing: HOV and HOT Lanes

By Hideo Konishi and Se-il Mun, 2010, *Regional Science and Urban Economics*.

The authors develop a model similar to what I will present in Chapter 2, but with some important distinctions. They use a cost minimization framework, and allow assembly costs, but not time costs, to vary over individuals. They also assume inelastic transportation demand, thus leaving out induced demand. Consumer cost is modeled as a function of the commute cost which is a function of congestion  $C(q_i)$ , plus an assembly cost,  $t$  that varies across commuters according to the distribution function  $F : \mathbb{R}_+ \rightarrow [0, 1]$ , and finally a toll  $\tau$  that varies by lane and carpooling decision.

$$C(q_i) + et + \tau_i^e \tag{1.22}$$

The variable  $e$  is an indicator variable that denotes the commuter's carpooling decision,  $e = 0$  if not carpooling and  $e = 1$  if the commuter does carpool. Tolls and congestion have a subscript to denote which lane the commuter drives in, general purpose, HOV or HOT.

The authors make minimal assumptions about the shape of  $C(q_i)$  and solve for a social cost function as the integral of costs over users and lanes with congestion determined endogenously. They allow for commuters to choose 2, 3, 4 or 5 person carpools



with the caveat that all carpoolers much choose the same occupancy level, and thus there cannot be a mix of 2 and 3 person carpools. Another way this model differs from the model presented in this dissertation is by the focus on time costs and a total neglect of operating costs. While the operating costs of driving relative to time may be small in areas such as New York City or congested Los Angeles, I believe this omission fails to capture an important aspect of carpooling. Empirically, people carpool even in areas without differential tolls for carpoolers, HOV lanes or HOT lanes. One of the reasons why is that carpoolers save on the monetary costs of driving and this is likely one of the drivers of the decline in carpool rates, as well as an explanation for higher carpooling rates in newly arrived immigrant communities where incomes are lower (Blumenberg and Smart, 2010).

Konishi and Mun find HOV lanes can be an improvement over general purpose lanes only under certain sets of parameters, but can aggravate congestion in other cases. HOV lanes encourage car-pooling and reduce total traffic, but cause distortions by creating different levels of congestion between general purpose and HOV lanes. HOT lanes can mitigate this by allowing solo-drivers on HOV lanes, but they also discourage car-pooling thus a conversion of HOV to HOT lanes may decrease congestion in this particular model set up. This conclusion highlights the failing of this model to include different values of time. As Verhoef and Small (Verhoef and Small, 2004) argue, ignoring heterogeneity can underestimate the benefits of congestion policy.

Additional papers have been used to explore traffic demand using cost minimization, they include:

- “Peak-Load Pricing of a Transportation Route with an Unpriced Substitute”, by Ralph Braid, 1996, *Journal of Urban Economics*.
- “The Merits of Separating Cars and Trucks”, by Andre De Palma, Robin Lindsey, and Moez Kilani, 2008, *Journal of Urban Economics*.

- “The Car Pooling Problem: Heuristic Algorithms Based on Savings Functions”, by Emilio Ferrari, Riccardo Manzini, Arrigo Pareschi, Alessandro Persona, Alberto Regattieri, 2003, *Journal of Advanced Transportation*.

## 1.5 Deterministic Models of Route Choice: Demand as a Function of Generalized Price

These models are a bridge between utility maximization models and cost minimization models, but are generally used as a way to model induced demand. Small and Verhoef (2007) model transportation demand on a single road with an inverse demand function  $d(V)$ , an average variable cost function,  $c(V)$ , and a toll  $\tau$

$$d(V) = p \equiv c(V) + \tau \quad (1.23)$$

Thus the average benefit from road use is the value of travel to users:

$$B = \int_0^V d(v)dv. \quad (1.24)$$

Holding capacity fixed, total cost in the short run is:

$$C = Vc(V) + \rho K \quad (1.25)$$

where  $\rho K$  is the annualized cost of capital expenditures  $K$ . Social surplus is defined as  $W$  and is defined as total benefit minus total cost,  $W \equiv B - C$ . The model is then developed to explore the alternatives to various tolling regimes, multiple bottlenecks, the value of information in route choice and the value of capacity. Papers that describe an inverse demand function to describe travel behavior include:

- “Product Differentiation on Roads”, by Erik Verhoef and Kenneth Small, 2004, *Journal of Transport Economics and Policy*.
- “The Value of ‘Value Pricing’ of Roads: Second-Best Pricing and Product Differentiation”, by Ken Small and Jia Yang, 2008, *RFF Discussion Paper*.

## 1.6 Equivalence Between Cost Minimization and Utility Maximization

Under some circumstances constrained maximization of utility under a budget constraint and cost minimizing behavior can be shown as equivalent. To show the equivalence between a cost minimization approach and a utility maximization approach I return to the problem stated in 1.6 with a simplified choice set that allows commuters to choose between consumption,  $G$ , and leisure,  $l$ , where leisure is denoted as the total amount of time available minus time spent at work and in travel,  $l \equiv \bar{T} - T_w - \tau$ , and  $\tau$  is the time cost of commuting. Since consumers are choosing between modes, users choose a mode with a time costs and an associated monetary cost  $\psi$ . subject to a time constraint and a full income constraint. Mode choice is discrete, users choose whether to not drive, carpool or drive alone.

$$\begin{aligned}
 & \max_{G,l,\psi,\tau} U(G, T_w) \\
 \text{s.t.} \quad & G + \psi \leq wT_w + Y \\
 & \bar{T} \leq T_w + l + \tau \\
 & (\tau, \psi) \in \{(\kappa, 0), (t + a, M/2), (t, M)\}
 \end{aligned} \tag{1.26}$$

In the most general case, I solve 1.26 for the utility maximizing quantity of other goods,  $G^*$ , and the optimal amount of time spend working,  $T_w^*$ , as a function of the exogenous parameters as well as  $\tau$ , and  $\psi$ . This results in  $G^*(\tau, \psi : w, Y, \bar{T})$ , and

$T_w^*(\tau, \psi : w, Y, \bar{T})$ , which can be plugged back into  $U$ , to arrive a utility function that relied on  $\tau$  and  $\psi$ :  $V(\tau, \psi : w, Y, \bar{T})$ . Commuters choose the set of  $(\tau, \psi)$  that result in the highest value of  $V$ . A commuter will take transit if:

$$\begin{aligned} V(\kappa, 0 : w, Y, \bar{T}) &> V(t + a, M/2 : w, Y, \bar{T}) \\ V(\kappa, 0 : w, Y, \bar{T}) &> V(t, M : w, Y, \bar{T}) \end{aligned} \tag{1.27}$$

A commuter will carpool if:

$$\begin{aligned} V(t + a, M/2 : w, Y, \bar{T}) &> V(\kappa, 0 : w, Y, \bar{T}) \\ V(t + a, M/2 : w, Y, \bar{T}) &> V(t, M : w, Y, \bar{T}) \end{aligned} \tag{1.28}$$

A commuter will drive alone if:

$$\begin{aligned} V(t, M : w, Y, \bar{T}) &> V(\kappa, 0 : w, Y, \bar{T}) \\ V(t, M : w, Y, \bar{T}) &> V(t + a, M/2 : w, Y, \bar{T}) \end{aligned} \tag{1.29}$$

In the case of CES, Leontief, Linear, Cobb-Douglas, Log Cobb-Douglas and Stone Geary functions, the value function is monotonically increasing in full income. The way the full income constraints have been written in 1.26, full income is  $\bar{T}w - \tau w + Y - \psi$ . Since  $\bar{T}, w$  and  $Y$  are exogenous variables, maximizing full income is equivalent to minimizing  $w\tau + \psi$ , or choosing from the set of  $(\tau, \psi) \in \{(\kappa, 0), (t + a, M/2), (t, M)\}$  that results in the smallest generalized costs.

### 1.6.1 Cost Minimization and Utility Maximization with Cobb-Douglas Utility

In the Cobb-Douglas case where utility is  $U(G, T_w) = G^{.5}l^{.5}$ , we can solve for the actual value function:

$$V(\tau, \psi : w, Y, \bar{T}) = \frac{w(T - \tau) + Y - \psi}{2\sqrt{w}} = \frac{wT + Y}{2\sqrt{w}} - \frac{\tau\sqrt{w}}{2} - \frac{\psi}{2\sqrt{w}} \quad (1.30)$$

Agents choose the  $\tau$  and  $\psi$  that maximizes their value function. Thus for commuters who choose transit, their value function is  $V(\kappa, 0 : w, Y, \bar{T})$ , commuters who choose carpooling have the value function  $V(t + a, M/2 : w, Y, \bar{T})$ , and commuters who choose to drive alone have the value function  $V(t, M : w, Y, \bar{T})$ . If we allow the wage rate to vary across individuals,  $w_i$ , then we can solve for the number of transit riders, carpoolers and commuters by finding the critical values of  $w_i$  where commuters are indifferent between modes:

$$\frac{w_1 T + Y}{2\sqrt{w_1}} - \frac{V\sqrt{w_1}}{2} = \frac{w_1 T + Y}{2\sqrt{w_1}} - \frac{(t+a)\sqrt{w_1}}{2} - \frac{M}{4\sqrt{w_1}} \quad (1.31)$$

$$\frac{w_2 T + Y}{2\sqrt{w_2}} - \frac{(t+a)\sqrt{w_2}}{2} - \frac{M}{4\sqrt{w_2}} = \frac{w_2 T + Y}{2\sqrt{w_2}} - \frac{t\sqrt{w_2}}{2} - \frac{M}{2\sqrt{w_2}} \quad (1.32)$$

These two conditions can be rearranged to:

$$V w_1 = (t(v) + a)w_1 + \frac{M}{2} \quad (1.33)$$

$$(t(v) + a)w_2 + \frac{M}{2} = t(v)w_2 + M \quad (1.34)$$

These are identical to the equations found from the cost minimization problem we will see in Chapter 2, with the one exception being that  $\beta_i$  is replaced with  $w_i$ .

## 1.7 Conclusion

In this chapter I discussed various methods of modeling transportation demand. In the next chapter I will develop a cost-minimization approach to look at the the efficiency impacts of HOV lanes using a simple theoretical model.

## Chapter 2

# A Structural Model of Carpooling Behavior

### 2.1 Introduction

The United States has built over 2,300 lane-miles<sup>1</sup> of HOV lanes. While the purpose of these lanes is to reduce vehicle-trips by encouraging more people to carpool, the effectiveness of HOV lanes is questionable (Kwon and Varaiya, 2008; Dahlgren, 1998; Johnston and Ceerla, 1996). Even with the 2,300 lane-miles of HOV lanes<sup>2</sup>, the percentage of commuters who carpool has been dropping over the years from 14.1% in 1985 to 8.7% in 2003.<sup>3</sup> Plausible explanations for the decline in carpooling include higher rates of automobile ownership, higher wages, changes in the real price of gasoline, a possible decline in social capital, and the growth of suburbs. Understanding these connections and how they impact congestion and energy policy is not straight-forward.<sup>4</sup> Despite the billions

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<sup>1</sup>[http://www.metro.net/projects\\_studies/hov/faqs.htm](http://www.metro.net/projects_studies/hov/faqs.htm)

<sup>2</sup>In addition there are matching programs for carpools and vanpools, preferential (or free) parking spaces for carpools, direct subsidies for carpooling and other carpooling incentive programs

<sup>3</sup>U.S. Department of Housing and Urban Development, American Housing Survey for the United States: 2003, <http://www.census.gov/hhes/www/ahs.html> and summarized [http://www.bts.gov/publications/national\\_transportation\\_statistics/2005/html/table\\_01\\_38.html](http://www.bts.gov/publications/national_transportation_statistics/2005/html/table_01_38.html)

<sup>4</sup>For a review of carpool trends from 1970-1990 see (Ferguson, 1997).

invested in carpooling incentives, and the millions of Americans who carpool to work, we know relatively little about carpooling behavior and HOV lanes. This chapter of my dissertation models carpooling behavior as a cost minimization problem and applies that model to the analysis of high occupancy vehicle lanes.

Carpooling is one of many mode choices. It allows commuters to split the monetary costs of driving by sharing rides to and from work. Carpoolers sacrifice time, route and schedule flexibility to save on the monetary costs of driving. Sharing a car with another commuter may increase or decrease utility compared to driving alone or taking transit, and some consumers may or may not prefer to share driving responsibilities (Lee, 1984). Transportation policy experts have typically grouped carpools into family members driving to work together ‘fam-pools’, co-workers, friends or strangers meeting through word-of-mouth and formal matching programs, van-pools, and casual carpools<sup>5</sup>(??). Carpooling generally involves spending more time on each end of the journey picking up and dropping off other members. I model this as an assembly cost, and to keep the model simple, I include in this assembly cost the time costs of scheduling carpools, finding carpool partners, and the utility/disutility of companionship and the sharing of driving duties. In the absence of preferential treatment for carpools, these assembly costs will make carpooling take longer than driving alone. While carpooling generally requires an increased time cost (in the absence of HOV lanes), carpoolers enjoy a decreased monetary cost because they share vehicle, toll and fuel costs.

Public transportation is both a substitute and a complement for carpooling. Price-rationed commuters who may not drive a single-occupant vehicle to work, may choose to split the monetary costs with another commuter and carpool. If a commuter has a variable schedule, he or she may choose to carpool to work with the knowledge that he or she can take transit home if conditions change. Carpooling has other impacts

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<sup>5</sup>A casual carpool is when drivers and passengers meet at a designated place (for instance the Berkeley BART station) to drive to a central business district (in this case downtown San Francisco) without making prior arrangements.

on the transportation system, fewer people will trip chain when carpooling, and if two carpoolers leave one of their cars behind, other members of the household may still use that car (Johnston and Ceerla, 1996).

As discussed in the previous chapter, currently carpooling is modeled in four-step transportation models by exogenously assuming a rate of carpooling. Academics in engineering and policy schools typically assume an exogenous rate of carpooling and run traffic simulations to understand the impact of carpooling (Dahlgren, 1998; Johnston and Ceerla, 1996) although a few economic models of carpooling allow carpooling to be determined endogenously as a function of monetary costs, time costs and the utility or disutility of companionship (Lee, 1984; Yang and Huang, 1999; Huang, Yang, and Bell, 2000; Ben-Akiva and Atheron, 1977). Lee (1984) discusses the economics of carpooling includes a cost of companionship which could be positive or negative depending on how a passenger feels about the other passengers, but does not incorporate congestion or the idea that the time and gasoline costs of the long haul depend on congestion created by other agents. Yang and Huang (1999) build a simple model of carpool formation as a cost-minimization program, making congestion an endogenous part of the model. Endogenizing congestion is an essential component to examining the impact of carpool lanes and the impact of carpooling on traffic, but Huang and Yang's model does not allow for the impact of induced demand or heterogeneous agents. The only way that traffic increases in Yang and Huang's model is if fewer people carpool, and the only way traffic decreases is if more people carpool. Users do not make more or fewer trips and they cannot switch to transit or non-motorized trips. This severely limits the usefulness of Yang and Huang's model.

The impact of induced demand is an important mechanism in Dahlgren's analysis of HOV lanes (Dahlgren, 1998). Other studies have found that when capacity expands, the number of trips increases in response (Duranton and Turner, 2009). If HOV lanes really increase the capacity of a road, it is important to incorporate induced demand



responses into models of HOV effectiveness. I build upon Huang and Yang's initial analysis of modeling carpooling as a cost-minimization problem, but I add a third option corresponding to transit, off-peak or no trip to capture induced demand. I also allow for some heterogeneity in agents' values of time. Omitting heterogeneity results in a razor's edge situation where either everyone carpools or nobody carpools as in Huang and Yang's model. If time savings are a way to induce carpools to form, users with a higher value of time may choose to carpool; accounting for this heterogeneity is an important part of Small and Yan's welfare analysis of HOV lanes (Small and Yan, 2008). It is my hypothesis that induced demand and heterogeneity can drastically change the social welfare benefits of a project.

## 2.2 Theoretical Model

This section introduces the theoretical model underlying carpooling decisions. I assume cost minimizing agents following work by Vickrey (1969) and Arnott, De Palma, and Lindsey (1993b) to model congestion along two lanes on a line haul where congestion is determined endogenously. In this section the two lanes are general purpose lane. Traffic is assumed to be assigned evenly between the lanes. Further in the paper, one lane will be a general purpose lane while the other lane will be an HOV lane. In this section, I solve for the decentralized solution, in the next section section I look at the traffic planner's equilibrium, I then examine ride-sharing incentives and HOV lanes.

Commuters will carpool if the cost of carpooling,  $C_y$ , is less than the cost of driving alone,  $C_z$ , where the cost of commuting is a function of monetary costs such as fuel, insurance, depreciation of the car, parking and tolls, and time costs in the line haul and assembly portions of the trip. To incorporate induced demand, we add a third group of people, those that choose transit, non-motorized transportation, off-peak travel or work from home/refrain from travel. I refer to this group as non-drivers as its

interpretation includes users who choose transit, bicycle, off-peak travel or anyone that has been priced off the road by money or time. This group has reservation time cost,  $V$ , which is a constant that does not vary by agent. The cost of choosing option  $i$ , is  $C_i$  where  $i = x, y, z$ . We can write the costs of carpooling, driving alone and not driving as:

$$\begin{aligned} C_y &= \beta_i t(v) + \beta_i a_i + M/2 \\ C_z &= \beta_i t(v) + M \\ C_x &= \beta_i V \end{aligned} \tag{2.1}$$

where  $t(v)$  is the amount of time it takes on the line haul portion of the commute as a function of traffic volume and  $M$  is the monetary cost of commuting. The assembly costs should be decomposed into time assembly costs, the additional fuel costs of driving and picking up another person and the utility/disutility of driving. For now all but time assembly costs are assumed to be negligible or at least correlated with the value of time. Commuter  $i$ 's value of time is  $\beta_i$ , and  $a_i$  is individual  $i$ 's additional time cost to assemble the carpool. The constant  $V$  represents the reservation cost of alternative transportation. Unlike Huang and Yan's model,  $\beta$  and  $a$  are allowed to vary over individuals thus introducing heterogeneity into a model of carpool behavior. By allowing a portion of the population to take transit or other, it is possible to take into account demand responses to changing congestion levels.

In this paper, I solve the model by ignoring heterogeneity in assembly costs,  $a_i = a \forall i$ , and assuming a uniform distribution for  $\beta_i$ ,  $\beta_i \sim \text{uniform}(\underline{\beta}, \bar{\beta})$ . Without loss of generality, I can normalize the population to 1,  $x + y + z = 1$ . Using these simplifying assumptions I can solve for the critical values of time,  $\beta_1$  and  $\beta_2$ , that separate transit riders from carpoolers and carpoolers from single passengers by setting  $C_x(\beta_1) = C_y(\beta_1)$  and  $C_y(\beta_2) = C_z(\beta_2)$ :

$$\beta_1 V = \beta_1 t + \beta_1 a + \frac{M}{2} \tag{2.2}$$

$$\beta_2 t + \beta_2 a + \frac{M}{2} = \beta_2 t + M. \quad (2.3)$$

It is possible to extend this model to carpools of 3 or more people by allowing commuters to choose between these additional modes. A model where commuters choose between not driving, carpooling with three people, carpooling with two people or driving alone would require the calculation of three critical  $\beta$ 's. For tractability we restrict carpools to two people. Solving Equations 2.2 and 2.3 gives expressions for the critical  $\beta$ 's:

$$\beta_1^* = \frac{M}{2(V - t(v) - a)} \quad (2.4)$$

$$\beta_2^* = \frac{M}{2a} \quad (2.5)$$

which hold for an interior solution,  $\underline{\beta} < \beta_1^* < \beta_2^* < \bar{\beta}$ . When this is not the case, the expressions in Equations 2.4 and 2.5 may lead to solutions that are negative or greater than  $\bar{\beta}$ . I can divide these up into a few subcases. When  $\frac{M}{2a} > \bar{\beta}$ , then nobody drives alone and  $\beta_2^* = \bar{\beta}$ . When  $\frac{M}{2(V - t_{min} - a)} > \bar{\beta}$  then nobody drives period leaving everyone to take transit and  $\beta_1^* = \beta_2^* = \bar{\beta}$ . If  $V > t_{max} + a$  and  $\underline{\beta} = 0$ , then there will always be some transit riders. Using  $\beta_1^*$  and  $\beta_2^*$ , I integrate over the distribution of  $\beta$  to solve for the proportion of the population that chooses transit, carpooling and single passenger vehicle:

$$x = \int_{\underline{\beta}}^{\beta_1^*} f(\beta) d\beta \quad (2.6)$$

$$y = \int_{\beta_1^*}^{\beta_2^*} f(\beta) d\beta \quad (2.7)$$

$$z = \int_{\beta_2^*}^{\bar{\beta}} f(\beta) d\beta \quad (2.8)$$

**Table 2.1:** Model Parameters and Policy Interpretations

Variable	Interpretation	Relevant Policy Interventions
M	Monetary Costs of Driving	Taxes/Subsidies on Automobiles and fuel, Tolls and Congestion Pricing
a	Assembly Costs of Carpooling	Dense Development, Matching Programs for Carpoolers, Casual Carpool Support, Guaranteed Ride Home
V	Time Costs of Transit/Not Driving	Taxes/Subsidies on Transit and Non-motorized Travel, Support for Telecommuting, Bike Lanes
$\delta$	Line Haul Travel Time with Zero Congestion	New Roads, Speed Limits
$\alpha$	Impact of Congestion on Travel Time	New Lanes on Existing Roads, Micro-Traffic Controls that Reduce Congestion
$\bar{\beta}, \underline{\beta}$	High and Low Values of Time	Labor Policies that Impact Wages and the Value of Time

Recall that the number of cars on the road,  $v$ , is equivalent to all the single-passenger vehicles and half of the individuals that decide to carpool:

$$v = \frac{y}{2} + z \quad (2.9)$$

If I assume that the external time costs of each car is some constant  $\alpha$ , then I can write the time it takes to travel the corridor as:

$$t(v) = \delta + \alpha v \quad (2.10)$$

where  $\delta$  is the amount of time that it takes to drive the corridor without any traffic on the road. In this model, policy interventions are modelled as changes in the parameters  $M$ ,  $a$ ,  $V$ ,  $\delta$ ,  $\alpha$ ,  $\bar{\beta}$  and  $\underline{\beta}$ . These parameters, and how policies may shift the parameters, are described in Table 2.1.

Using Equations 2.4 through 2.10 I solve for  $t(v)$ ,  $x$ ,  $y$ ,  $z$ .<sup>6</sup> To solve for the amount of time it takes on the long-haul portion of the trip in equilibrium, I use a Nash Equilibrium (in transportation it is called Waldrop's Principle) and take the positive root, to find an expression for time as a function of the problem's parameters,  $t^*(M, a, V, \bar{\beta}, \underline{\beta}, \alpha, \delta)$ :

$$t^* = -\frac{1}{8a} \left( 4a(a - V - \delta - \frac{\alpha \bar{\beta}}{\bar{\beta} - \underline{\beta}}) + \frac{M\alpha}{\bar{\beta} - \underline{\beta}} + \sqrt{16a \left( \frac{\alpha(MV + 4a(a - V)\bar{\beta})}{\bar{\beta} - \underline{\beta}} + 4a\delta(a - V) \right) + \left( 4a(a - V - \delta - \frac{\alpha \bar{\beta}}{\bar{\beta} - \underline{\beta}}) + \frac{M\alpha}{\bar{\beta} - \underline{\beta}} \right)^2} \right) \quad (2.11)$$

Since  $t^*(M, a, V, \bar{\beta}, \underline{\beta}, \alpha, \delta)$  is such an unwieldy expression, I write  $x^*(\theta)$ ,  $y^*(\theta)$  and  $z^*(\theta)$  as functions of  $t^*(M, a, V, \bar{\beta}, \underline{\beta}, \alpha, \delta)$ :

$$x^*(M, a, V, \bar{\beta}, \underline{\beta}, t^*) = \frac{\frac{M}{2(V - t^* - a)} - \underline{\beta}}{\bar{\beta} - \underline{\beta}} \quad (2.12)$$

$$y^*(M, a, V, \bar{\beta}, \underline{\beta}, t^*) = \frac{\frac{M}{2a} - \frac{M}{2(V - t^* - a)}}{\bar{\beta} - \underline{\beta}} \quad (2.13)$$

$$z^*(M, a, \bar{\beta}, \underline{\beta}) = \frac{\bar{\beta} - \frac{M}{2a}}{\bar{\beta} - \underline{\beta}} \quad (2.14)$$

An interesting result is that the number of solo drivers on the road is not a function of  $t^*$ , but instead a function of assembly costs, monetary costs of driving and values of time since solo drivers face the same amount of congestion on the line haul. While Equations 2.11 through 2.14 are cumbersome, they can easily be solved for a

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<sup>6</sup>Specifically, this is done by writing  $t = \delta + \alpha \left[ \int_{\beta_2^*}^{\bar{\beta}} f(\beta) d\beta + \frac{1}{2} \int_{\beta_1^*}^{\beta_2^*} f(\beta) d\beta \right]$ . Using the assumption that  $\beta$  has a uniform distribution, we can rewrite  $t = \delta + \alpha \left[ \frac{\bar{\beta} - \beta_2^*}{\bar{\beta} - \underline{\beta}} + \frac{1}{2} \frac{\beta_2^* - \beta_1^*}{\bar{\beta} - \underline{\beta}} \right]$ . Next we substitute Equations 2.4 and 2.5, and write  $t$  as a function of  $t$  and the model parameters:  $t = \delta + \alpha \left[ \frac{\bar{\beta} - \frac{M}{4a} - \frac{M}{4(V - t - a)}}{\bar{\beta} - \underline{\beta}} \right]$ , then solve for  $t^*$ .

particular set of parameters. Using the parameters,  $M = 2,000, \bar{\beta} = 4,000, \underline{\beta} = 0, a = 2, \alpha = 0.99, \delta = 5, V = 60$ , I find that  $t^* = 5.2926, x^* = 0.48\%, y^* = 12.0\%, z^* = 87.5\%$ .

The function  $t^*$  is a measure of congestion, and its inverse  $t^{-1}(t^*)$  returns the volume of traffic and can be related to local air pollution and greenhouse gas emissions.

### 2.3 Traffic Planner Solution

The previous section discussed the decentralized equilibrium where each driver minimized commuting costs. In this section I examine a centralized solution where total commuting costs are minimized and how it differs from the decentralized solution. Total social cost is the integral of costs for each commute mode over the relevant values of time:

$$TSC = \int_{\underline{\beta}}^{\beta_1} C_x f(\beta) d\beta + \int_{\beta_1}^{\beta_2} C_y f(\beta) d\beta + \int_{\beta_2}^{\bar{\beta}} C_z f(\beta) d\beta \quad (2.15)$$

Replacing  $C_x$ ,  $C_y$ , and  $C_z$  with the Equations in 2.1, we can rewrite the total social costs as:

$$TSC = \int_{\underline{\beta}}^{\beta_1} \beta_i V f(\beta) d\beta + \int_{\beta_1}^{\beta_2} (\beta_i t(v) + \beta_i a + M/2) f(\beta) d\beta + \int_{\beta_2}^{\bar{\beta}} (\beta_i t(v) + M) f(\beta) d\beta \quad (2.16)$$

As before,  $t(v)$  is a function of the number of carpoolers and single drivers, which is in turn a function of  $\beta_1$  and  $\beta_2$ , I need to write  $t$  as a function  $t(\beta_1, \beta_2)$ . Assuming a uniform distribution for  $\beta_i \sim \text{uniform}(\underline{\beta}, \bar{\beta})$ , we can re-write the portion of commuters who carpool,  $y$  and drive alone  $z$  as:

$$y = \frac{\beta_2 - \beta_1}{\bar{\beta} - \underline{\beta}} \quad (2.17)$$

$$z = \frac{\bar{\beta} - \beta_2}{\bar{\beta} - \underline{\beta}} \quad (2.18)$$

Substituting Equations 2.17 and 2.18 into Equation 2.16 and rearranging we obtain an intuitive expression for total social costs:

$$TSC = \underbrace{\frac{\beta_1^2 - \underline{\beta}^2}{2(\bar{\beta} - \underline{\beta})}V}_{\text{Time Costs to Non-Drivers}} + \underbrace{\frac{\bar{\beta}^2 - \beta_1^2}{2(\bar{\beta} - \underline{\beta})}t}_{\text{Line Haul Costs for All Drivers}} + \underbrace{\frac{\beta_2^2 - \beta_1^2}{2(\bar{\beta} - \underline{\beta})}a}_{\text{Assembly Time Costs for Carpoolers}} + \underbrace{\frac{\bar{\beta} - \frac{\beta_1}{2} - \frac{\beta_2}{2}}{\bar{\beta} - \underline{\beta}}M}_{\text{Monetary Costs for All Drivers}} \quad (2.19)$$

Where the time required to travel the corridor in equilibrium is:

$$t(\beta_1, \beta_2) = \delta + \alpha \left( \frac{\beta_2 - \beta_1}{2(\bar{\beta} - \underline{\beta})} + \frac{\bar{\beta} - \beta_2}{\bar{\beta} - \underline{\beta}} \right) = \delta + \alpha \left( \frac{\bar{\beta} - \frac{\beta_1}{2} - \frac{\beta_2}{2}}{\bar{\beta} - \underline{\beta}} \right) \quad (2.20)$$

Thus we can rewrite the entire social cost function, integrating over  $\beta_i$  and plugging Equation 2.20 into 2.16. This defines the total social cost as a function of the parameters and the choice variables  $\beta_1$  and  $\beta_2$ :

$$TSC = \frac{1}{\bar{\beta} - \underline{\beta}} \left[ \frac{\beta_1^2 - \underline{\beta}^2}{2}V + \frac{\bar{\beta}^2 - \beta_1^2}{2} \left( \delta + \alpha \frac{\bar{\beta} - \frac{\beta_1}{2} - \frac{\beta_2}{2}}{\bar{\beta} - \underline{\beta}} \right) + \frac{\beta_2^2 - \beta_1^2}{2}a + \left( \bar{\beta} - \frac{\beta_1}{2} - \frac{\beta_2}{2} \right)M \right] \quad (2.21)$$

The traffic planner minimizes the social costs of commuting, Equation 2.21 using  $\beta_1$  and  $\beta_2$ . The first order conditions to this problem are:

$$\frac{\partial TSC}{\partial \beta_1} = \frac{1}{\bar{\beta} - \underline{\beta}} \left[ \beta_1 V - \beta_1 \left( \delta + \alpha \frac{\bar{\beta} - \frac{\beta_1}{2} - \frac{\beta_2}{2}}{\bar{\beta} - \underline{\beta}} \right) - \beta_1 a - \frac{M}{2} - \frac{\alpha(\bar{\beta}^2 - \beta_1^2)}{4(\bar{\beta} - \underline{\beta})} \right] = 0 \quad (2.22)$$

$$\frac{\partial TSC}{\partial \beta_2} = \frac{1}{\bar{\beta} - \underline{\beta}} \left[ \beta_2 a - \frac{M}{2} - \frac{\alpha(\bar{\beta}^2 - \beta_1^2)}{4(\bar{\beta} - \underline{\beta})} \right] = 0 \quad (2.23)$$

In expressions 2.22 and 2.23, the components have economic interpretations. An increase in  $\beta_1$  increases the number of transit/non-travellers while decreasing the number of motorists. The first term inside the parentheses in Equation 2.22 is  $2\beta_1 V$ , which is the additional cost of adding non-motorists. The second term,  $-2\beta_1 t$  is the time saving from taking an additional carpooler off the line haul. The third term  $2\beta_1 a$  is the time saving from assembly costs of moving an additional traveller from carpooling to transit. The fourth term,  $-M/2$ , is the monetary costs saved by increasing  $\beta_1$  while the last term is a general equilibrium effect.

The FOC with regards to  $\beta_2$ , Equation 2.23, is simpler than 2.22. Since the line haul time between carpoolers and solo travelers is the same, increasing  $\beta_2$  decreases the total assembly costs,  $2\beta_2 a$  while increasing total monetary costs  $M/2$ . The last term in Equation 2.23 is the general equilibrium costs of decreasing the traffic on the road by one half a car.

These FOCs in the traffic planner problem are equivalent to solving Equations 2.2 and 2.3 except they take into account the general equilibrium impacts of congestion. This is obvious once we simplify and rearrange the traffic planner's FOCs:

$$\underbrace{\beta_1 V}_{\substack{\text{Cost of} \\ \text{Marginal} \\ \text{Transit} \\ \text{Rider}}} = \underbrace{\beta_1 t + \beta_1 a + \frac{M}{2}}_{\substack{\text{Time and} \\ \text{Monetary Costs} \\ \text{of Marginal} \\ \text{Carpooler}}} + \underbrace{\frac{\alpha(\bar{\beta}^2 - \beta_1^2)}{4(\bar{\beta} - \beta)}}_{\substack{\text{General Equilibrium} \\ \text{Effect}}} \quad (2.24)$$

Here the cost of the marginal transit rider is equated to the time and monetary costs of the marginal carpooler, plus the general equilibrium effects, just as in Equation 2.2. The general equilibrium effects describe the benefit to all motorists of shorter travel time, a second order effect which describes the induced demand response to shorter travel times,



and a corresponding monetary cost from that induced demand. Similarly, Equation 2.25 replicates the condition found in Equation 2.3 when we set  $C_y = C_z$  except with these additional general equilibrium effects.

$$\underbrace{\beta_2 t + \beta_2 a + \frac{M}{2}}_{\substack{\text{Time and} \\ \text{Monetary Costs} \\ \text{of Marginal} \\ \text{Carpooler}}} = \underbrace{\beta_2 t + M}_{\substack{\text{Time and} \\ \text{Monetary Costs} \\ \text{of Marginal} \\ \text{Solo Driver}}} + \underbrace{\frac{\alpha(\bar{\beta}^2 - \beta_1^2)}{4(\bar{\beta} - \beta)}}_{\substack{\text{General Equilibrium} \\ \text{Effect}}} \quad (2.25)$$

The critical values of time,  $\beta_1^*$  and  $\beta_2^*$  can be solved for using Equations 2.22 and 2.23. First we use the FOC that corresponds to  $\beta_2$  to solve for  $\beta_2^*(\beta_1)$ :

$$\beta_2^* = \frac{M}{2a} + \frac{\alpha(\bar{\beta}^2 - \beta_1^2)}{4a(\bar{\beta} - \beta)} \quad (2.26)$$

The solutions for  $\beta_1^*(M, a, V, \bar{\beta}, \beta, \alpha, \delta)$  and  $\beta_2^*(M, a, V, \bar{\beta}, \beta, \alpha, \delta)$  involves plugging Equation 2.26 into Equation 2.24 and rearranging to find a cubic function of  $\beta_1$ :

$$\underbrace{\frac{-\alpha^2}{8a(\bar{\beta} - \beta)^2}}_A \beta_1^3 + \underbrace{\frac{3\alpha}{4(\bar{\beta} - \beta)}}_B \beta_1^2 + \underbrace{\left[ V - \delta - \frac{\alpha\bar{\beta}}{\bar{\beta} - \beta} - a - \frac{\alpha^2\bar{\beta}^2}{8a(\bar{\beta} - \beta)^2} + \frac{\alpha M}{4a(\bar{\beta} - \beta)} \right]}_C \beta_1 - \underbrace{\left[ \frac{\alpha\bar{\beta}^2}{4(\bar{\beta} - \beta)} + \frac{M}{2} \right]}_D = 0 \quad (2.27)$$

Rewriting this equation using A, B, C and D as the coefficients on  $\beta_1^3$ ,  $\beta_1^2$ ,  $\beta_1$  and  $\beta_1^0$ , we can use the general form for the solution to a cubic function:

$$\beta_1^* = -\frac{B}{3A} - \frac{1}{3A} \sqrt[3]{\frac{1}{2} \left[ 2B^3 - 9ABC + 27A^2D + \sqrt{(2B^3 - 9ABC + 27A^2D)^2 - 4(B^2 - 3AC)^3} \right]} - \frac{1}{3A} \sqrt[3]{\frac{1}{2} \left[ 2B^3 - 9ABC + 27A^2D - \sqrt{(2B^3 - 9ABC + 27A^2D)^2 - 4(B^2 - 3AC)^3} \right]} \quad (2.28)$$

This result for  $\beta_1^*$  is complicated, but can be used to solve for  $\beta_2^*$  and  $t^*(M, a, V, \bar{\beta}, \underline{\beta}, \alpha, \delta)$ .

$$\beta_1^* = \frac{-2a(\bar{\beta} - \underline{\beta})}{\alpha} + \frac{8a(\bar{\beta} - \underline{\beta})^2}{3\alpha^2} \sqrt[3]{\frac{1}{2} [R + \sqrt{Z}]} + \frac{8a(\bar{\beta} - \underline{\beta})^2}{3\alpha^2} \sqrt[3]{\frac{1}{2} [R - \sqrt{Z}]} \quad (2.29)$$

where

$$R = \frac{-3\alpha^3}{32a^2(\bar{\beta} - \underline{\beta})^5} \left[ 2M\alpha(\bar{\beta} - \underline{\beta}) + (a\delta - aV - 8a^2)(\bar{\beta} - \underline{\beta})^2 + \alpha\bar{\beta}(\alpha\bar{\beta} + a(\bar{\beta} - \underline{\beta})) \right] \text{ and}$$

$$Z = R^2 - \frac{27\alpha^6}{128a^3(\bar{\beta} - \underline{\beta})^6} \left[ \frac{a}{2} + V - \delta + \frac{\alpha^2\bar{\beta}^2}{8a(\bar{\beta} - \underline{\beta})^2} + \frac{M\alpha}{4a(\bar{\beta} - \underline{\beta})} - \frac{a\bar{\beta}}{\bar{\beta} - \underline{\beta}} \right]^3$$

This expression for  $\beta_1^*$  can theoretically be plugged into  $\beta_2^*$  and thus  $t(M, a, V, \bar{\beta}, \underline{\beta}, \alpha, \delta)$ ,  $x(M, a, V, \bar{\beta}, \underline{\beta}, \alpha, \delta)$ ,  $y(M, a, V, \bar{\beta}, \underline{\beta}, \alpha, \delta)$  and  $z(M, a, V, \bar{\beta}, \underline{\beta}, \alpha, \delta)$  can be calculated but the expressions are unwieldy. Using the parameters,  $M = 2,000$ ,  $\bar{\beta} = 4,000$ ,  $\underline{\beta} = 0$ ,  $a = 2$ ,  $\alpha = 0.99$ ,  $\delta = 5$ ,  $V = 60$ , I find values of  $\beta_1^* = 38.2$  and  $\beta_2^* = 995.0$  that are higher than the decentralized solution indicating fewer solo drivers and more transit riders. The traffic planner equilibrium is compared to the decentralized equilibrium in Table 2.2. The amount of time spent on the line-haul portion of the trip is smaller in the traffic planner equilibrium because more people are taking transit and more people are carpooling.

## 2.4 Ridesharing Incentives

A number of programs have been used to encourage ridesharing, they are summarized in (Brownstone and Golob, 1992; Victoria Transport Policy Institute, 2010). Programs that

**Table 2.2:** Decentralized Equilibrium Versus Traffic Planner Equilibrium

Variable	Decentralized Equilibrium	Traffic Planner Equilibrium
$\beta_1^*$	19.2	38.2
$\beta_2^*$	500	995.0
t	5.93	5.86
x	0.48%	0.95%
y	12.0%	23.9%
z	87.5%	75.1%
Total Social Costs	13,787	13,723

influence the monetary costs of driving include gas taxes, automobile fees, technology requirements, or congestion charges. To understand how monetary costs influence the decentralized equilibrium, I take the derivative of an outcome variable with respect to  $M$ . The impact on the number of solitary drivers can be shown to unambiguously decreasing,

$$\frac{\partial z^*}{\partial M} = -\frac{1}{2a(\bar{\beta} - \beta)} < 0.$$

Finding  $\frac{\partial y^*}{\partial M}$  is more cumbersome, but we can show that

$$\frac{\partial t^*}{\partial M} < 0$$

which is derived in Appendix B. Since  $v^* = t^{-1}(t^*)$ , I can say that

$$\frac{\partial v^*}{\partial M} < 0$$

indicating that either the impact of monetary costs of carpooling is negative or that the growth in carpoolers does not outweigh the decline in single passenger vehicles. If traffic managers are seeking to reduce traffic volumes, one thing they can do is increase the monetary costs of driving.

**Table 2.3:** Decentralized Equilibrium Versus Traffic Planner Equilibrium

Variable	Decentralized Equilibrium	Traffic Planner Equilibrium	Decentralized Equilibrium with Tax
$\beta_1^*$	19.2	38.2	38.2
$\beta_2^*$	500	995.0	995.0
$t$	5.93	5.86	5.86
$x$	0.48%	0.95%	0.95%
$y$	12.0%	23.9%	23.9%
$z$	87.5%	75.1%	75.1%
TSC	13,787	13,723	13,723

To obtain the traffic planner equilibrium from the decentralized equilibrium, I use Equation 2.11 and set the Nash Equilibrium value of  $t^*$  to the traffic planner's solution for  $t$  and solve for the monetary cost,  $M$ , that would equate the two values. Using the parameters  $\bar{\beta} = 4,000$ ,  $\underline{\beta} = 0$ ,  $a = 2$ ,  $\alpha = 0.99$ ,  $\delta = 5$ ,  $V = 60$ , I find that the a monetary cost of  $M = 3,980$  or a tax of  $tax = 1,980$  on top of a monetary cost of  $M = 2,000$  would turn the Nash Equilibrium into the socially optimal solution.

Priority parking spots for carpools, matching programs that try to find neighbors going in similar directions, and programs whose value is correlated with a value of time such as a guaranteed ride home, can be thought of as programs that reduce the assembly costs of carpools. An increase in assembly costs increases the number of solo drivers as it increases the price of carpooling,

$$\frac{\partial z^*}{\partial a} = \frac{M/2a}{2a(\bar{\beta} - \underline{\beta})} > 0.$$

An increase in the highest values of time,  $\bar{\beta}$ , will increase the number of solo occupant vehicles:

$$\frac{\partial z^*}{\partial \bar{\beta}} = \frac{(\bar{\beta} - \underline{\beta}) + (\bar{\beta} - M/2a)}{(\bar{\beta} - \underline{\beta})^2} > 0.$$

Since I modeled group  $x$  as people who take public transportation and people who choose to work from home, I can discuss programs that lower  $V$  as upgrades to the public transportation system or programs that increase the productivity of at-home workers (by decreasing the time cost of working from home). Maintaining the interior solution assumption that,  $V > t(v^{max}) + a > t(v^{max})$ , implies changes in  $V$  have no impact on the number of people that drive alone, it only influences the number of carpoolers and transit riders. In Appendix B I derive the result that  $\frac{\partial t^*}{\partial V} > 0$  which implies that  $\frac{\partial y^*}{\partial V} > 0$  and  $\frac{\partial x^*}{\partial V} < 0$  since  $\frac{\partial z^*}{\partial V} = 0$ .

Simple forms of ridesharing incentives can be modelled. Changes to the assembly costs of carpools can be modeled, time costs of transit or the monetary costs of driving were explored in this section, but in the next section I will explicitly model HOV lanes. HOV lanes change the line haul time cost of carpooling but not driving alone.

## 2.5 Incorporating High Occupancy Vehicle Lanes

HOV lanes decrease the amount of time spend on the line-haul portion of the commute for carpoolers by providing a special lane that only carpoolers can use. This does not directly change the cost equations for solo drivers or those who take transit from equations, but it will influence them indirectly. The presence of an HOV lane will impact the time it takes to travel the general purpose lane, and time savings that come from carpooling could induce riders who would have otherwise taken transit. An HOV lane offers carpoolers the choice between taking the conventional lane which takes time  $t(v - hov)$  or the HOV lane which takes time  $t(hov)$  where  $hov$  is the number of cars that takes the HOV lane. This changes the cost equation for carpoolers to incorporate the choice of taking an HOV lane:

$$C_y = \beta_i \min[t(gp), t(hov)] + \beta_i a_i + M/2 \quad (2.30)$$

Allocation of traffic between the carpool and the general purpose lanes will proceed as follows. All single passenger cars will be assigned to the general purpose lane, and carpoolers will be assigned to the HOV lane until the density of cars on the HOV lane equals the density of cars on the general purpose lane. In this special case where we have one carpool lane and one general purpose lane, modeling the density of cars is equivalent to modeling the number of cars. When the number of carpooling vehicles is greater than the number of single passenger vehicles, then the two lanes will equalize until the time it takes to travel the line-haul is the same for the HOV and the general purpose lanes,  $t(gp) = t(hov)$ .<sup>7</sup> Since  $t$  is a monotonic function of traffic volume, this is equivalent to equating the number of cars between the two lanes. Thus I can write  $t(hov)$  and  $t(gp)$  as functions of  $hov$  and  $gp$ :

$$gp = \begin{cases} z & \text{if } z > y/2 \\ \frac{z}{2} + \frac{y}{4} & \text{if } z \leq y/2 \end{cases} \quad (2.31)$$

$$hov = \begin{cases} \frac{y}{2} & \text{if } z > y/2 \\ \frac{z}{2} + \frac{y}{4} & \text{if } z \leq y/2 \end{cases}$$

Upon inspection it is clear that  $gp \geq hov$  for all values of  $x$ ,  $y$  and  $z$ . Thus  $\min[t(gp), t(hov)] = t(hov)$  and Equation 2.30 can be rewritten as:

$$C_y = \beta_t t(hov) + \beta_i a_i + M/2 \quad (2.32)$$

---

<sup>7</sup>In this case we will model one HOV lane and one general purpose lane, but it is possible to make a more general model with one HOV lane and multiple general purpose lanes.

Costs for solo drivers and transit riders (Equation 2.1) remain the same, but new values of  $\beta_1^*$  and  $\beta_2^*$  result from Equation 2.32:

$$\beta_1^* = \frac{M/2}{(V - t(hov) - a)} \quad (2.33)$$

$$\beta_2^* = \frac{M/2}{(t(hov) + a - t(gp))} \quad (2.34)$$

This is similar to the conditions we derived in Section 2.2, where both  $\beta$ 's are functions of the monetary savings divided by the time savings. If  $hov = gp$  then  $t(hov) - t(gp) = 0$  and Equation 2.34 reverts to Equation 2.5. The equilibrium same as if there were no HOV lane. The conditions for this are when  $z > \frac{y}{2}$  or using Equations 2.14 and 2.13:

$$\bar{\beta} > \frac{3M}{4a} - \frac{M}{4(V - t^* - a)} \quad (2.35)$$

The reader is reminded that  $t^*$  is a function of the parameters and described in Equation 2.11. This describes all cases where the HOV lane does not provide time savings. When the inequality in Equation 2.35 is reversed, then  $hov = y/2$  and  $gp = z$ . Recall that in Section 2.2, we expressed travel time as a function of how long it took to drive the line haul portion of the trip when there were no other cars,  $\delta$ , and a function expressing how each car slowed down the other drivers,  $\alpha v$ . In Section 2.2, we had two lanes for cars to drive on; now that we are modeling each lane separately, we need to adjust the impact of congestion on travel time by using  $2\alpha$  instead of  $\alpha$ :

$$t(hov) = \delta + 2\alpha hov \quad (2.36)$$

$$t(gp) = \delta + 2\alpha gp \quad (2.37)$$

Using the same methodology to solve for the equilibrium, we use expressions for  $y$  and  $z$  to write two equalities for  $t(hov)$  and  $t(gp)$ :

$$t(hov) = \delta + \alpha y = \delta + \alpha \frac{\beta_2^* - \beta_1^*}{\bar{\beta} - \underline{\beta}} \quad (2.38)$$

$$t(gp) = \delta + 2\alpha z = \delta + 2\alpha \frac{\bar{\beta} - \beta_2^*}{\bar{\beta} - \underline{\beta}} \quad (2.39)$$

Substituting in Equations 2.33, 2.34, we have a system of two equations, and two unknowns,  $t(hov)$  and  $t(gp)$ :

$$t(hov) = \delta + \alpha \frac{\frac{M/2}{(t(hov) + a - t(gp))} - \frac{M/2}{(V - t(hov) - a)}}{\bar{\beta} - \underline{\beta}} \quad (2.40)$$

$$t(gp) = \delta + 2\alpha \frac{\bar{\beta} - \frac{M/2}{(t(hov) + a - t(gp))}}{\bar{\beta} - \underline{\beta}} \quad (2.41)$$

We can first solve for  $t(gp)$  in terms of  $t(hov)$  using Equation 2.41. We first solve Equation 2.41 for  $t(gp)[t(hov)]$ , and then plug this into Equation 2.40 which results in an analytical expression for time spend on the HOV and general purpose lanes that is too large to place here. Instead I use the same parameters from previous sections and find the time needed to drive the HOV lane is  $t(hov) = 5.28$  and time spent driving the general purpose lane is  $t(gp) = 6.41$ . This results in  $\beta_1^* = 19.0$  and  $\beta_2^* = 1,150$ . Compared to Section 2.3,  $\beta_1^*$  is lower than the Nash Equilibrium and the Traffic Planner Equilibrium showing that HOV lanes draw users from transit and non-motorized modes. Additionally,  $\beta_2^*$  is much higher suggesting HOV lanes are a strong incentive to carpool under this set of parameters. The percentage of the population that chooses non-motorized travel is 0.47%, carpoolers account for 28.3% and solo drivers account for 71.2%. While HOV lanes can, in some circumstances increase the number of carpoolers and decrease the number of solo drivers, this is not the socially optimal allocation of vehicles. The total



social costs of transportation under HOV lanes are 14,675.1, considerably higher than under the Nash equilibrium general purpose lanes or the traffic planner equilibrium.

## 2.6 Conclusion

This chapter uses previous work to build a model of carpooling behavior and HOV lanes that incorporates heterogeneous values of time and induced demand. Billions of dollars are spent building HOV lanes and promoting carpooling, but with this set of parameters HOVs result in higher travel costs compared to a system with just two general purpose lanes. HOV lanes do seem to reduce traffic volume, but at a large cost to commuters. Table ?? shows that the time to drive on the general purpose lane increases for those commuters with the highest values of time.

Local and state governments are encouraged to build HOV lanes to reduce vehicle-trips, and will sometimes argue that HOV lanes are a win-win for all commuters. This study suggests that HOV lanes may benefit carpoolers and transit riders who switch to carpooling, but at a cost to solo passenger vehicles. With HOV lanes, the increase in carpooling from 12% to 28.3% in this simulation results in more people taking cars to work with lower volumes, but lower volumes than is socially optimal. A price instrument is a more reliable method of travel demand management.

Understanding the relationship between carpooling behavior and traffic outcomes (both volume and total commute costs) requires a structural model with endogenous carpool formation and induced demand. This model is not meant to be a replacement to four-step planning models, but a tool for understanding when and why HOV lanes results volume and cost reductions. The next chapter examines the ability of HOV lanes to both mitigate traffic and reduce total travel costs under a wide range of scenarios.

## Chapter 3

# The Impact of HOV Lanes on Traffic Volume and Commuting Costs

### 3.1 Introduction

Under the 1990 Clean Air Act Amendments, air quality non-attainment<sup>1</sup> areas are required to use transportation control measures to reduce ozone, nitrous oxides and carbon monoxide emissions. The U.S. EPA may sanction non-attainment areas by making it difficult or impossible to expand highway capacity, but this prohibition does not apply to additions or expansions of High Occupancy Vehicle (HOV) lanes<sup>2</sup>. HOV lanes are considered a transportation control strategy. However the question “do HOV lanes reduce the number of trips taken” remains unanswered. Are HOV lanes truly a transportation control measure, or are they a way of expanding capacity without violating the Clean Air Act Amendments? The second question to ask is “what is the impact of HOV lanes

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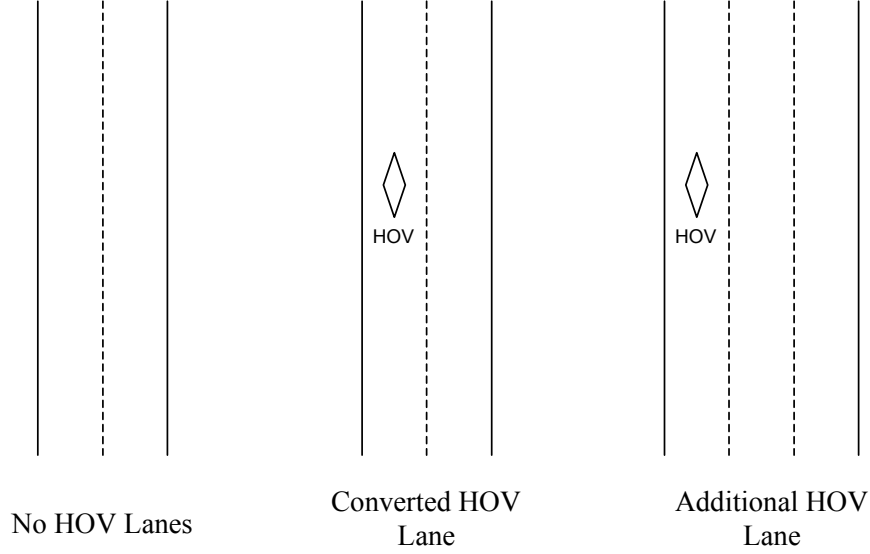
<sup>1</sup>Non-attainment is a technical term by the EPA to denote areas where air pollution levels persistently exceed the national ambient air quality standards.

<sup>2</sup>Clean Air Act Amendments, section 179(b)(1)(B)

on travel costs?” Proponents of HOV lanes say they reduce traffic for carpoolers and for solo drivers by inducing additional people to carpool, but if these carpoolers are being drawn from people who would otherwise take transit, HOV lanes may actually increase traffic volumes. This may not necessarily be a welfare loss, but it contradicts the traffic demand management purposes of HOV lanes.

In this chapter, I use the model developed in Chapter 2 to compare the mode shares, lane travel times, traffic volumes and total travel costs across HOV configurations and a range of parameters. I use traffic volume to proxy for emissions but acknowledge it is an imperfect measure of emissions. Emissions of CO<sub>2</sub> are directly related to miles driven and hence trips taken. Thus using trips taken is an acceptable proxy for CO<sub>2</sub> emissions. However NO<sub>x</sub> emissions, an important ozone precursor, have a more complicated relationship with how the car is driven and under what conditions. A car sitting in traffic generally has higher NO<sub>x</sub> emissions per mile than a car in free-flowing conditions. I cannot answer questions such as the time spent in free-flowing versus congestion conditions with the current version of my model. Instead I answer a more basic question: does carpooling can reduce the number of trips taken?

There are three situations I consider. A highway with  $n$  general purpose lanes, that same highway where one of the general purpose lanes has been converted to a HOV lane and finally a highway where a HOV lane has been added to  $n$  general purpose lanes. The case where  $n = 2$  is presented graphically in Figure 3.1.



**Figure 3.1:** A Two Lane Highway with no HOV Lanes, a Converted HOV Lane and an Additional HOV Lane.

### 3.2 A Highway with $n$ General Purpose lanes and one HOV lane

Just as in Chapter 2, commuters choose the travel mode with the lowest generalized costs. The costs of carpooling ( $C_y$ ), driving alone ( $C_z$ ) and not driving ( $C_x$ ) are:

$$\begin{aligned}
 C_y &= \beta_i \min[t(gp), t(hov)] + \beta_i a_i + M/2 \\
 C_z &= \beta_i t(gp) + M \\
 C_x &= \beta_i V
 \end{aligned}
 \tag{3.1}$$

where  $t(gp)$  and  $t(hov)$  are the lane specific amounts of time it takes on the line haul portion of the commute as a function of traffic volume  $gp$  for general purpose and  $hov$  for the HOV lanes, and  $M$  is the monetary cost of commuting. The assembly costs could

theoretically be decomposed into time assembly costs, the additional fuel costs of driving and picking up another person and the utility/disutility of driving, for now I leave them aggregated as something highly correlated with the value of time. Commuter  $i$ 's value of time is  $\beta_i$ , and  $a_i$  is individual  $i$ 's additional time cost to assemble the carpool. The constant  $V$  represents the reservation cost of alternative transportation. As before, travel times are a function of commuters' cost minimization behavior which also depends on travel times and congestion. The main difference between the HOV lane situation and Chapter 2 is the allocation of traffic between lanes. Carpoolers can drive on both HOV and general purpose lanes, but single passenger vehicles are restricted to general purpose lanes. Thus all single passenger cars will be assigned to the general purpose lane, and carpoolers will be assigned to the HOV lane until the density of cars on the HOV lane equals the density of cars on the general purpose lane. Unlike the previous section where there was one carpool lane and one general purpose lane, now there is one carpool lane and  $n$  general purpose lanes so a per-lane measure of traffic in the general purpose lanes is needed. When the number of carpooling vehicles per lane is greater than the number of single passenger vehicles per lane, then the two lanes will equalize until the time it takes to travel the line-haul is the same for the HOV and the general purpose lanes,  $t(gp/n) = t(hov)$ . Since  $t$  is a monotonic function of traffic volume, this is equivalent to equating the number of cars between the two lanes. Thus, as before, I can write  $t(hov)$  and  $t(gp)$  as functions of  $hov$  and  $gp$ :

$$gp/n = \begin{cases} z/n & \text{if } z/n > y/2 \\ \frac{z+y/2}{n+1} & \text{if } z/n \leq y/2 \end{cases} \quad (3.2)$$

$$hov = \begin{cases} \frac{y}{2} & \text{if } z/n > y/2 \\ \frac{z+y/2}{n+1} & \text{if } z/n \leq y/2 \end{cases}$$

Upon inspection it is clear that there are always at least as many people per lane on the general purpose lanes as the HOV lanes,  $gp/n \geq hov$ , for all values of  $x$ ,  $y$  and  $z$ . Costs for solo, carpoolers, and transit remain the same. Now the solution for  $t(hov)$  and  $t(gp)$  are functions of  $n$  to represent the more general formulation of multiple lanes.

Again we adjust the impact of congestion on travel time by using  $gp/n$  instead of  $gp$  since all drivers are equally distributed across general purpose lanes:

$$t(hov) = \delta + \alpha hov \quad (3.3)$$

$$t(gp) = \delta + \alpha \frac{gp}{n} \quad (3.4)$$

Using the same methodology to solve for the equilibrium, I use expressions for  $y$  and  $z$  to write two equalities for  $t(hov)$  and  $t(gp)$ :

$$t(hov) = \delta + \alpha \frac{\beta_2^* - \beta_1^*}{2(\bar{\beta} - \underline{\beta})} \quad (3.5)$$

$$t(gp) = \delta + \alpha \frac{\bar{\beta} - \beta_2^*}{n(\bar{\beta} - \underline{\beta})} \quad (3.6)$$

Substituting in expressions for  $\beta_2^*$  and  $\beta_2^*$ , results in a system of two equations, and two unknowns,  $t(hov)$  and  $t(gp)$ :

$$t(hov) = \delta + \alpha \frac{\frac{M/2}{(t(hov) + a - t(gp))} - \frac{M/2}{(V - t(hov) - a)}}{2(\bar{\beta} - \underline{\beta})} \quad (3.7)$$

$$t(gp) = \delta + 2\alpha \frac{\bar{\beta} - \frac{M/2}{(t(hov) + a - t(gp))}}{n(\bar{\beta} - \underline{\beta})} \quad (3.8)$$

I can first solve for  $t(gp)$  in terms of  $t(hov)$  using equation 3.8. I first solve equation 3.8 for  $t(gp)[t(hov)]$ , and then plug this into equation 3.7 which results in an analytical expression for time spend on the HOV and general purpose lanes that is too large to place here. I use this time on the general purpose and HOV lanes to solve for the total number of cars on the road or  $z + y/2$ .

### 3.3 A Highway with n General Purpose Lanes

Without an HOV lane, carpoolers and solo drivers are grouped into the same  $n$  general purpose lanes and hence have time costs that reflect the total number of cars,  $v = z + y/2$ :

$$\begin{aligned} C_y &= \beta_i t(v) + \beta_i a_i + M/2 \\ C_z &= \beta_i t(v) + M \\ C_x &= \beta_i V \end{aligned} \quad (3.9)$$

The equations in 3.9 are identical to what we found in Chapter 2 without HOV lanes, however the amount of time that it takes to drive in each lane needs to be generalized to reflect the  $n$  lanes:

$$t(v) = \delta + \alpha v/n. \quad (3.10)$$

Thus  $t(v)$  can be solved by the following equation:

$$t(v) = \delta + \frac{\alpha}{n} \frac{\bar{\beta} - \frac{M}{4a} - \frac{M}{4(V - t(v) - a)}}{\bar{\beta} - \underline{\beta}}. \quad (3.11)$$

This is roughly equivalent to the results found in Chapter 2 with the addition of  $n$ . Thus the increase/decrease in trips from adding an HOV lane is the difference:

$$\delta_{trips} = \left[ z_{N+HOV} + \frac{y_{N+HOV}}{2} \right] - \left[ z_N + \frac{y_N}{2} \right]. \quad (3.12)$$

The increase/decrease in trips from converting a general purpose lane into an HOV lane is:

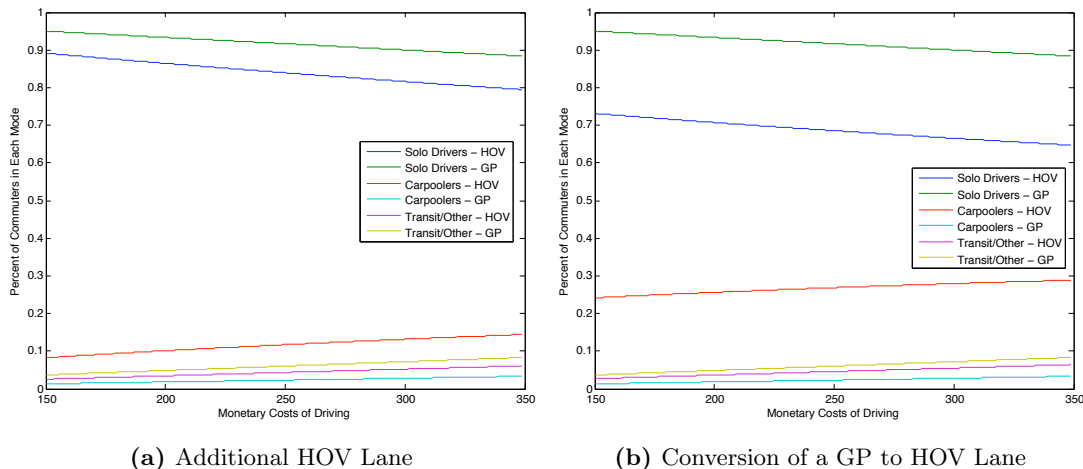
$$\delta_{trips} = \left[ z_N + \frac{y_N}{2} \right] - \left[ z_{N-1+HOV} + \frac{y_{N-1+HOV}}{2} \right]. \quad (3.13)$$

In the next section I estimate Equations 3.12 and 3.13 over a range of parameters to understand when an HOV lane can reduce traffic volume.

### 3.4 Numerical Results

The relevant parameters are  $n$ ,  $M$ ,  $\alpha$ ,  $V$ ,  $a$ ,  $\bar{\beta}$ ,  $\underline{\beta}$  and  $\delta$ . Since the objective of this study is to understand the impact of HOV lanes on carpooling, I focus on the parameter space where each mode is used by some commuters. For all cases we begin with the assumption of two general purpose lanes ( $n=2$ ) and then either convert a general purpose lane to an HOV lane or add an HOV lane to the two general purpose lanes. The default parameters are  $M=150$ ,  $\alpha=20$ ,  $V=75$ ,  $a=15$ ,  $\bar{\beta}=100$ ,  $\underline{\beta}=0$  and  $\delta=30$ . If the time units are minutes, this represents a line haul commute that takes between 30 to 50 minutes depending on congestion and lane configuration, or 75 minutes by transit. Picking up another passenger adds 15 minutes to the commute in assembly costs, being able to drive on less congested HOV lane may mitigate this. The monetary costs are important in their



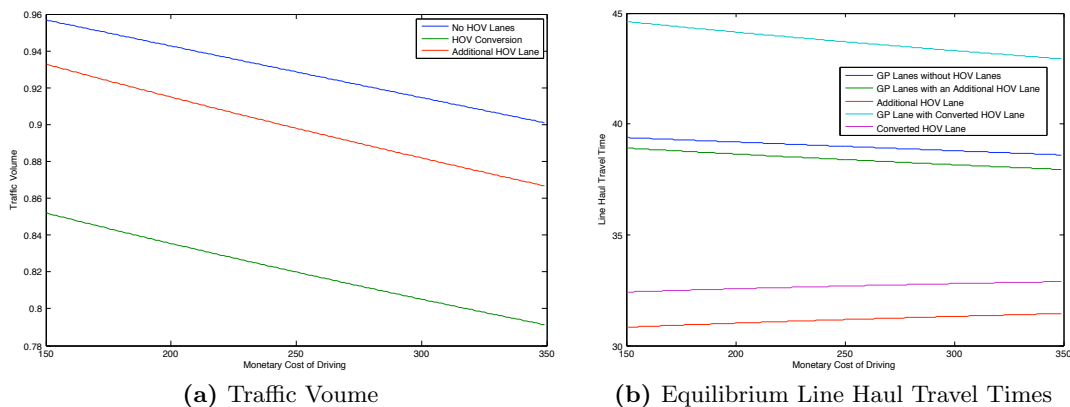


**Figure 3.2:** Mode Share Comparisons With Changing M.

relevance to each other. Some passengers value their time at zero dollars per hour while the highest value of time,  $\bar{\beta}=100$  is equivalent to approximately \$60 per hour which is the upper limit in surveys. The default value of  $\bar{\beta}$  is also twice as much as the default monetary cost of driving  $M=300$ . Translating these into real world costs is difficult. A car costs approximately \$0.40 per mile to drive, if passengers travel at 50 to 75 mph, then the cost per hour of driving is approximately \$20 - 30 per hour. This is a little bit lower than the default monetary cost of driving but using a lower M would result in very few carpoolers and transit riders and investigating these members is the point of this study. In the few subsections I examine a range of monetary costs of driving, ranges of congestion externalities, time costs of transit, assembly costs, maximum value of commute time and the time it takes to travel the highway without traffic.

### 3.4.1 Changing M

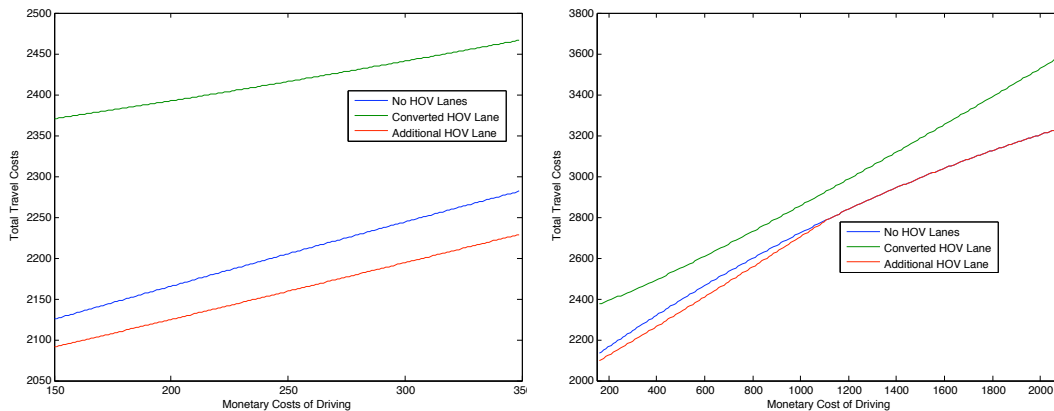
Figure 3.3 shows the results of the simulation when we change the monetary costs of driving, M. When  $M=150$ , 95% of commuters are solo drivers where there are two general



**Figure 3.3:** Traffic Volume and Lane Travel Times that Result from Varying Monetary Costs.

purpose lanes. Adding an HOV lane reduces this to 89%, but as we can see in Figure 3.2b conversion of a general purpose lane to an HOV lane has a much more dramatic impact on carpooling and solo driving. Increasing the monetary price of driving decreases solo drivers, increases carpoolers and increases transit riders in all configurations. Under all values of  $M$ , either adding an HOV lane or converting a general purpose lane to an HOV lane decreases non-drivers because the low travel time on the HOV lane draws these commuters onto the highway. This impact is not enough to increase volume in either scenario.

Figures 3.3a and 3.3b show how the time and volume of cars respectively, vary over  $M$ . The volume is calculated by adding the percentage of solo drivers and half of the percentage of carpoolers. Without HOV lanes, the volume on the road is 0.9566; almost everyone is driving alone. When an HOV lane is added, volume falls to 0.9329, but when an HOV lane is converted from a general purpose lane, volume is reduced to 0.8520. One of the reasons for the dramatic reduction in volume from the conversion of an HOV lane is that travel times on the general purpose lane drastically increase relative to the HOV lanes. This can be seen in Figure 3.3b. In Figure 3.3b, the travel time on the

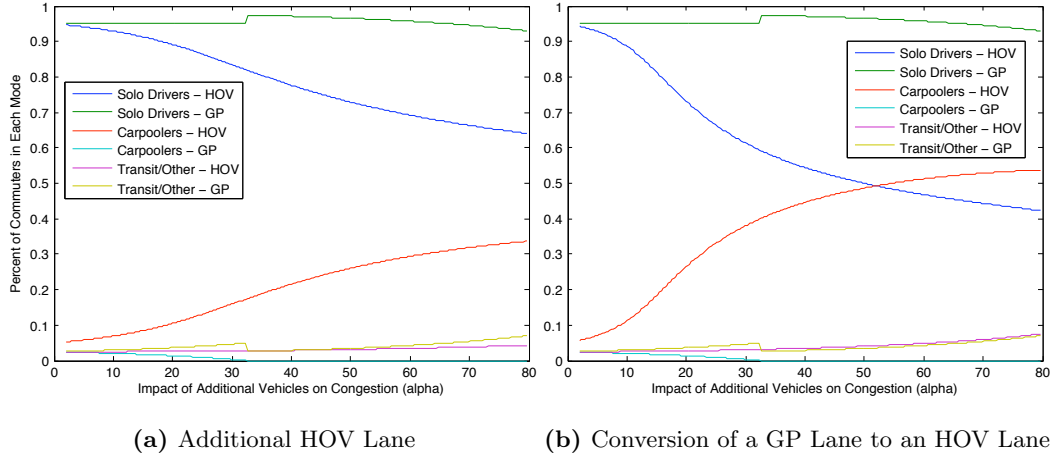


(a) Monetary Costs From \$1.50 - \$3.50 per minute (b) Monetary Costs From \$1.50 - \$20.00 per minute

**Figure 3.4:** Total Generalized Costs to Commuters versus the Monetary Cost of Driving.

general purpose lane when an HOV lane is converted is 44.6250 minutes when  $M=150$ . Without the HOV lane, the travel time would be 39.5664 minutes. Adding an HOV lane reduces the amount of time spent commuting for both solo and carpool drivers to 38.9151 when  $M=150$ , which is unsurprising since an HOV lane addition results in an increase in capacity.

Figure 3.4 is the impact of HOV lanes on total travel costs. Note that in Figure 3.4, travel costs do not include externalities related to accidents, noise or congestion, only the externalities due to congestion through increasing travel times and monetary costs. Nor do travel costs include the costs of actually converting or building an HOV lane. With these caveats in mind, adding an HOV lane can reduce total travel costs by 1.6% when  $M=150$ , but converting a general purpose to an HOV lane increases total travel costs by 11.55%. Increasing monetary costs to individual drivers ( $M$ ) increase total travel costs to society in all lane configurations although if travel costs are very high, the additional HOV lane scenario converges to the no HOV lane scenario. This is because at very high costs of driving many commuters choose to go from driving solo



**Figure 3.5:** Mode Share Comparisons With Changing  $\alpha$ .

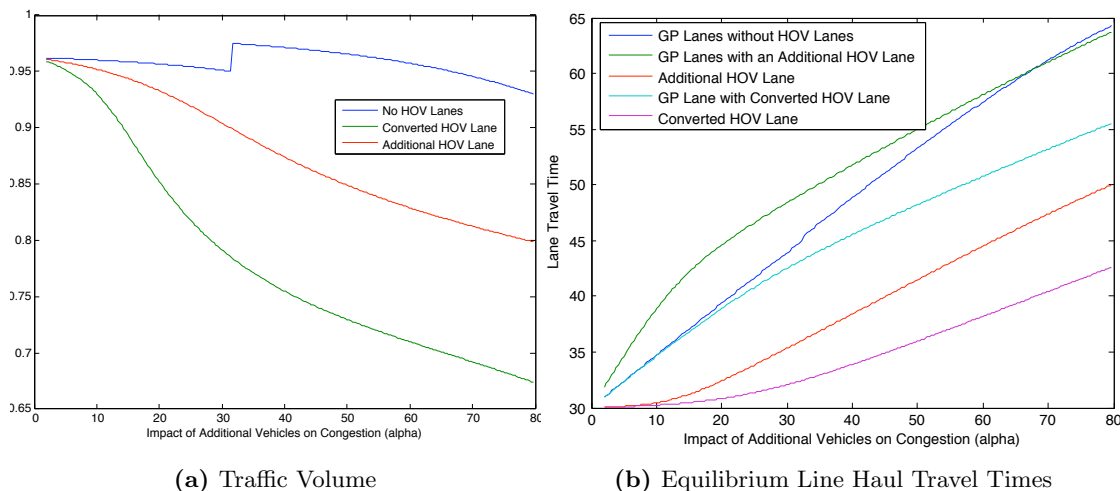
to carpooling regardless of HOV lanes, and the time travel savings from driving on an HOV lane disappears. As  $M$  increases even more, the monetary cost of driving flattens out as more and more commuters choose transit and carpooling over solo driving.

### 3.4.2 Changing $\alpha$

In this section I vary the externality impact of cars which is captured by the parameter  $\alpha$ . Recall Equation 3.3 and 3.4 that relate traffic volume per lane to travel time on that lane: equations to  $t(hov)$  and  $t(gp)$

$$\begin{aligned} t(hov) &= \delta + \alpha hov \\ t(gp) &= \delta + \alpha \frac{gp}{n} \end{aligned} \tag{3.14}$$

The parameter  $\alpha$  related how an additional vehicle contributes to overall travel time. An increase in  $\alpha$  means that the impact each additional car has on other cars increases. Future work will use a more realistic function for translating traffic volume to

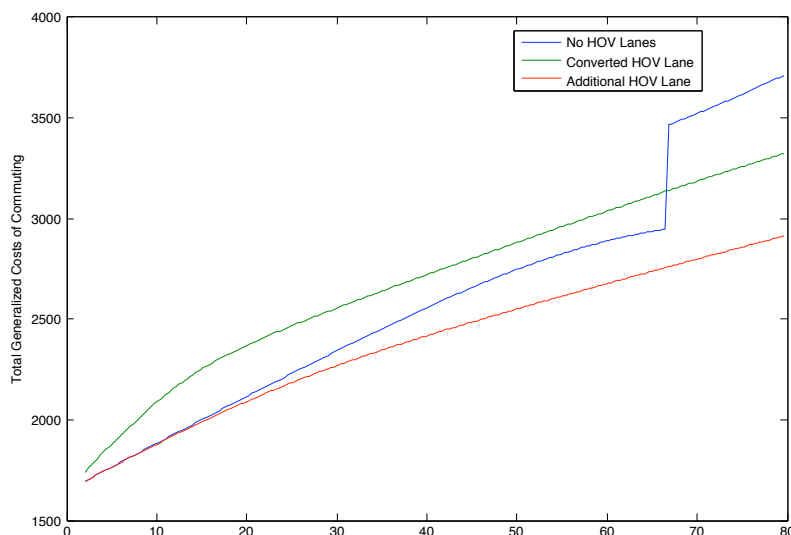


**Figure 3.6:** Traffic Volume and Lane Travel Times that Result from Varying  $\alpha$ .

travel delay. For now I hope to gain insight into the impact of traffic through varying  $\alpha$ . This is a rough approximation to changes I expect when I incorporate a more complicated traffic function.

Another interpretation of  $\alpha$  is that it describes the amount of space on the road. Because  $\alpha$  is an overall measure of the road space and  $\frac{\alpha}{n}$  is a measure of the per lane congestion, a lower  $\alpha$  could also be thought of as a change in overall road capacity. In this case, a change in  $\alpha$  also allows for a change in the relative size of the road.

Figure 3.5 shows the impact of  $\alpha$  on mode share. There is a kink in non-HOV road configurations around  $\alpha = 66$  that corresponds to the point at which people stop carpooling because the travel time on the road is high enough such that the time gains from going from transit to carpooling (V-t-a) are lower than the time savings from going from carpooling to driving solo (the time cost between these two modes is the assembly cost, a). Because the cost of going between these modes is the same  $M/2$  (the carpooler pays  $M/2$  to share a car while the solo drive pays an additional  $M/2$  to avoid



**Figure 3.7:** Total Generalized Costs to Commuters versus  $\alpha$

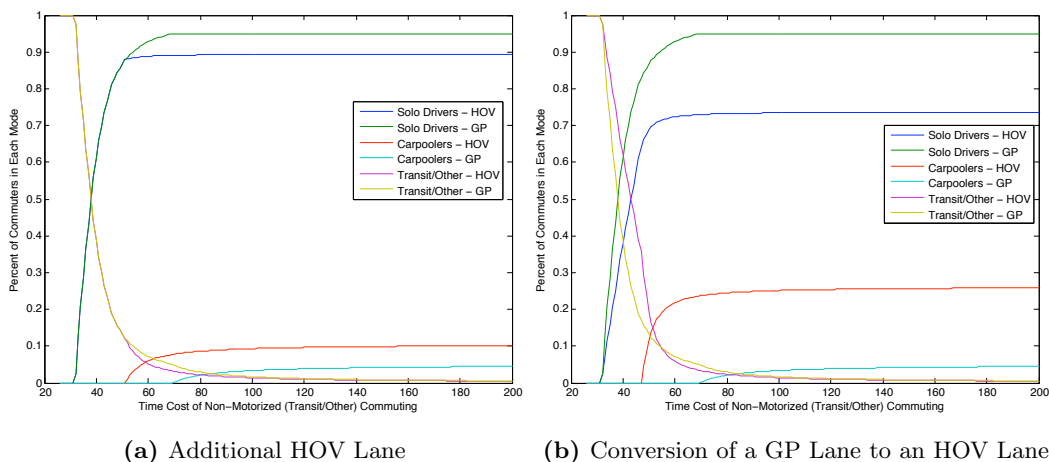
the assembly costs), no driver would choose to carpool. This is not the case when HOV lanes are present because the HOV lanes take less time than the general purpose lanes do.

This jump can be seen more clearly in Figure 3.6a where the volume on the configuration without HOV lanes increases sharply. Other things to note in Figure 3.6a are that for low levels of  $\alpha$ , the volume of cars on the additional HOV lane configuration and the no HOV lane configuration are close. It is not until there is a high impact of congestion that the volumes differ. In Figure 3.6b the equilibrium lane travel times change over  $\alpha$ . In all cases the travel times on the general purpose lanes are longer than the travel times in the HOV lanes.

The relative total travel costs are increasing in  $\alpha$ , with converted HOV lanes having the highest total travel costs, followed by no HOV lanes and finally additional HOV lanes, except in the cases where there is a very high  $\alpha$ . Recall that the externality impact of other vehicles is worst when there is not an HOV lane because there are no extra incentives to carpool. In most cases, the benefit of increasing the incentives to

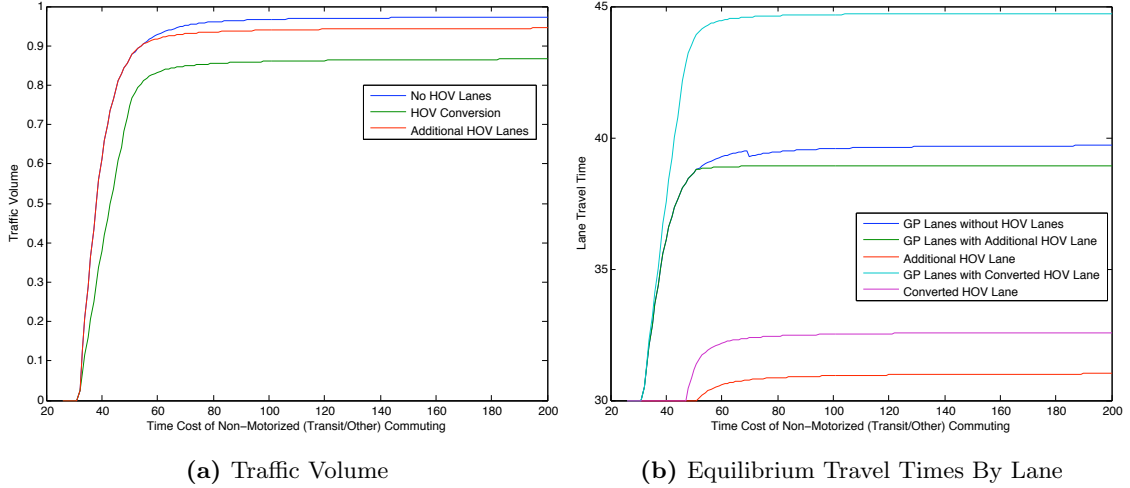
carpool is decreased by either reducing the capacity through conversion or adding an HOV lane. However, with  $\alpha > 66$ , the externality impact of additional vehicles is much higher and the no HOV lane scenario becomes even more costly than the conversion scenario. This suggests that high externality impacts of cars are needed before HOV lanes can bring down total travel costs.

### 3.4.3 Changing V



**Figure 3.8:** Mode Share Comparisons With Changing V.

The parameter  $V$  represents the reservation time cost of either not taking a trip or using a non-motorized type of transportation. Unlike some of the other parameters, the parameter  $V$  can be reduced by public policy such as investments in public transportation or encouraging businesses to allow workers flex-schedules. For very low levels of  $V$ , nobody drives at all since there are no time savings to driving. For higher levels of  $V$ , solo drivers start driving, and an additional lane has no influence on their decision. A converted lane does influence these early solo drivers because the conversion of a general purpose lane effectively reduces the capacity for solo drivers by taking away a

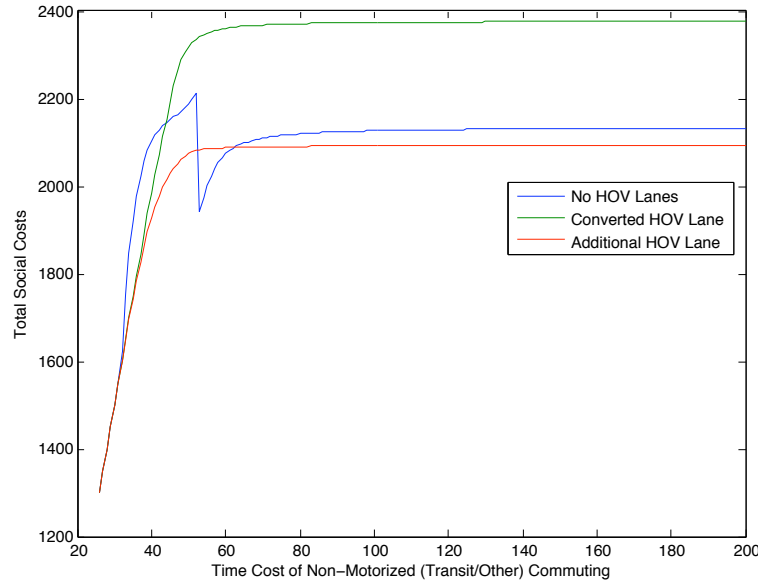


**Figure 3.9:** Traffic Volume and Lane Travel Times that Result from Varying  $V$ .

general purpose lane. For even higher levels of  $V$ , carpooling becomes a low cost mode for those with intermediate values of time, and transit riders turn to carpooling as the time cost of transit,  $V$ , increases. For relatively high values of  $V$ , both conversion of an HOV lane and an additional HOV lane can reduce volume, much as was seen when the monetary cost of driving,  $M$ , was changed. The patterns we saw earlier where a converted lane was more effective at reducing solo driving and increasing carpools is observed once again as we change  $V$ .

While the situations with additional and converted HOV lanes behave as expected, once  $V$  reaches a certain level the total social costs actually decrease. This happens around  $V=70$  and the travel time savings from carpooling become a viable alternative to driving. As we saw with a varying  $\alpha$ , when the travel time savings from carpooling ( $V-t-a$ ) are less than the travel time savings from driving alone,  $a$ , then nobody carpools since the price of switching between each mode is  $M/2$ . When nobody is carpooling on the general purpose lane, the total social costs are actually higher than in the converted HOV and the additional HOV scenarios. Once carpooling becomes an





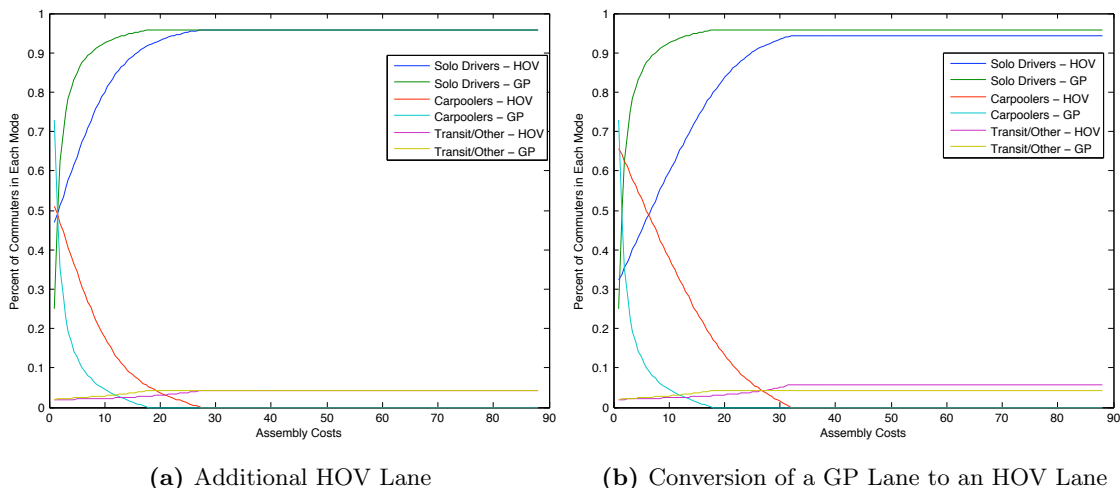
**Figure 3.10:** Total Generalized Costs to Commuters versus  $V$

attractive mode, some solo drivers choose to carpool decreasing the externality cost and hence the total social costs.

### 3.4.4 Changing $a$

In this version of the model, assembly costs are the same for all commuters. In future work, I hope to allow assembly costs similar to vary across individuals, much in the same way as  $\beta_i$  is modeled, through a distribution of  $a_i$ . Changing the assembly costs of carpooling could be construed as programs that give priority parking to carpoolers, upgrade arterial street networks or match commuters through slugging, websites, etc.

In Figure 3.12, we can see that for very high assembly costs, an additional HOV lane has no impact on mode share. This is the case where nobody is carpooling. However, with a converted HOV lane, the conversion results in a reduction in capacity which does decrease the number of solo drivers even at higher values of  $a$ . At these high assembly

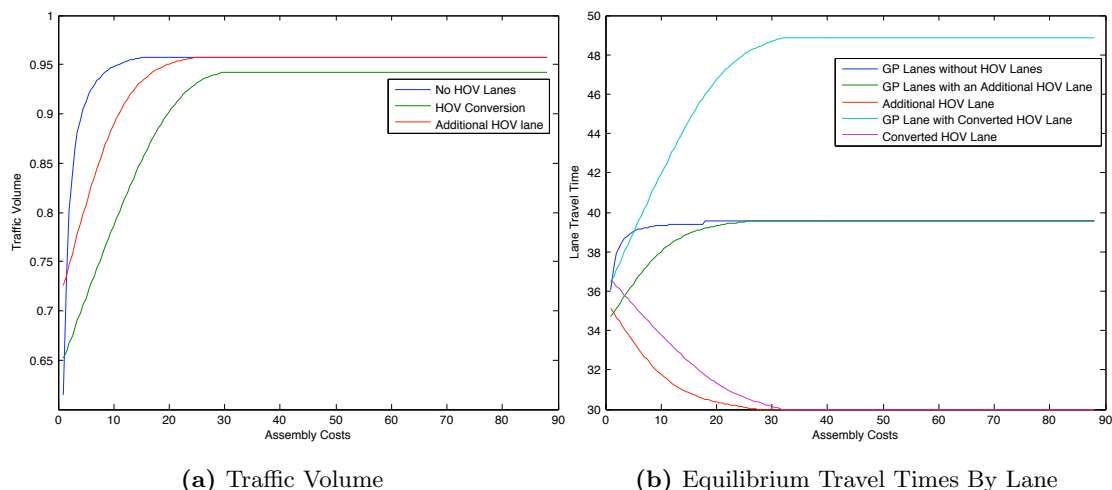


**Figure 3.11:** Mode Share Comparisons With Changing a.

costs, there are still not any carpoolers, but traffic on the general purpose lane decreases because there is one fewer lane. At high assembly costs, the conversion of a general purpose lane results in an extra ten minutes per trip on the general purpose lane while the HOV lane remains empty.

Looking at traffic volume and travel times change as assembly costs change reinforces the story in Figure 3.12. For low assembly costs, traffic volume is lower in situations with HOV lanes. However, when assembly costs reaches a threshold point, transit becomes a more attractive choice for commuters with low values of time and driving alone is a more attractive choice for everyone else. Nobody carools, leaving the HOV lanes empty. Travel times tell a similar story where at some point assembly costs get high enough such that nobody carools and the marginal impact of a is zero.

Looking at generalized travel costs, we can see that for very low assembly costs the situation without HOV lanes has the highest cost, but that costs in the converted HOV lane scenario quickly rise with assembly costs. This is because in the converted HOV lane scenario the travel time costs on the general purpose lane are very high,

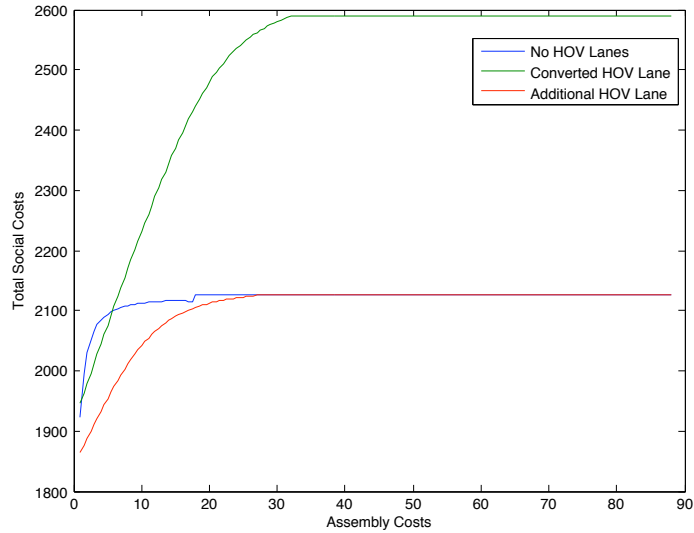


**Figure 3.12:** Traffic Volume and Lane Travel Times that Result from Varying  $a$ .

while the incentive to carpool becomes smaller and smaller until carpooling as a mode actually dies out. This may not matter to the additional HOV lane scenario (although the additional lane in this case is empty), but with the converted HOV scenario the total travel costs are 33% higher than in the other two scenarios when the HOV lane goes unused.

### 3.4.5 Changing $\bar{\beta}$

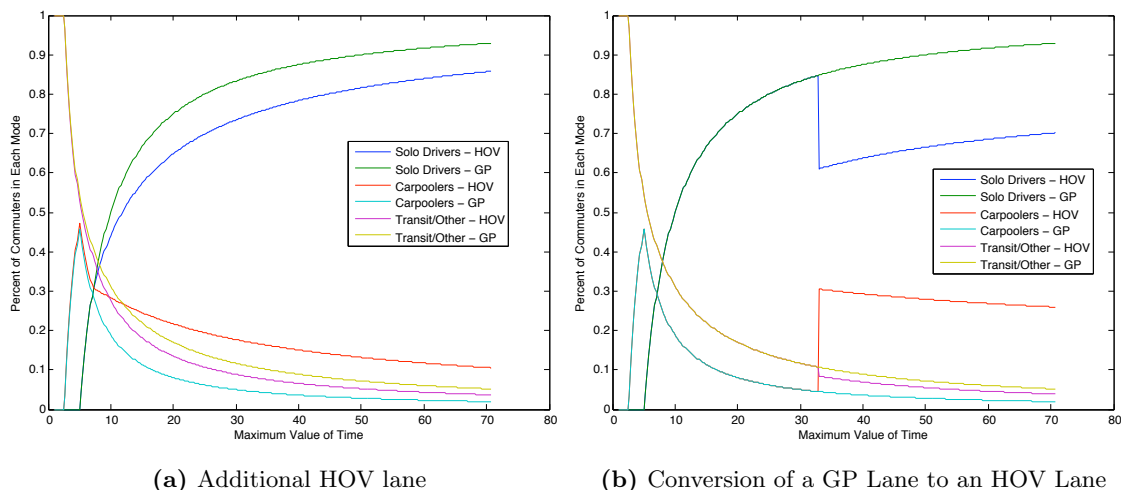
Changing  $\bar{\beta}$  changes the maximum value of time. This has two impacts, it stretches out the income distribution leaving a smaller percentage of commuters with a low value of time, and increases the willingness to pay of the top commuters to save time by driving alone. For low values of  $\bar{\beta}$ , nobody drives and the entire population uses transit. Once  $\bar{\beta}$  reaches a threshold, some commuters begin to carpool. At this point it is only carpoolers on the road, so they drive equally on the HOV and the general purpose lanes. Once  $\bar{\beta}$  becomes high enough to make it worthwhile for drivers to save time by driving alone, solo drivers enter the road and there is divergence in mode share between two



**Figure 3.13:** Total Generalized Costs to Commuters versus a

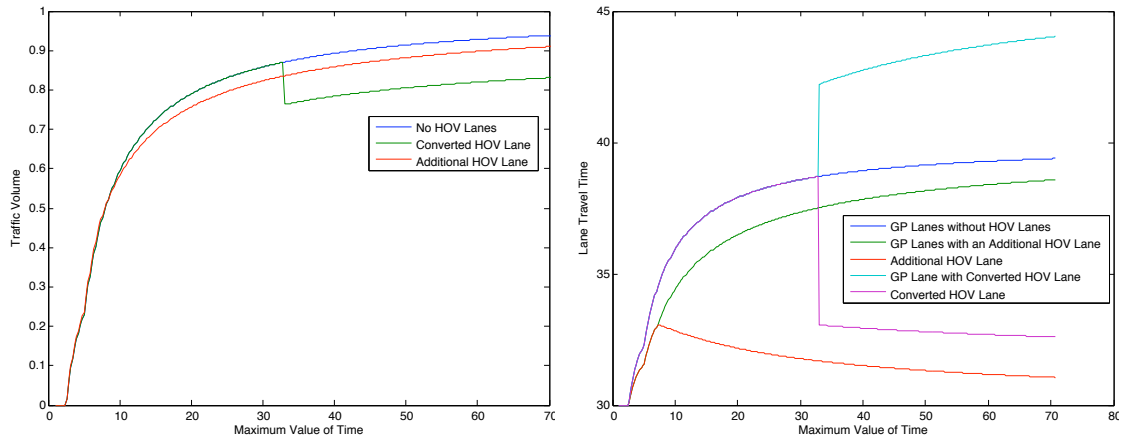
general purpose lanes and general purpose/HOV lane mixes. With the additional lane this occurs almost immediately since commuters are responding to higher values of time and different congestion levels that result from the additional lane. However in the conversion scenario, the mode shares between the general purpose and HOV lanes do not change until the number of solo drivers per lane exceeds the number of carpoolers per lane. Then the change is drastic as the carpoolers all re-sort to the HOV lane and the solo drivers are left on the general purpose lanes.

Low values  $\bar{\beta}$  do not result in large differences in volume between HOV and non-HOV scenarios until  $\bar{\beta}$  reaches 30. Then the conversion of a general purpose lane has a bigger impact on volume than an additional lane. In Figure 3.15a, this can be seen when the traffic volume on the converted HOV scenario diverges sharply from the other two scenarios. This is interesting because it suggests HOV lanes may not be an effective travel demand tool when income distributions are compressed.



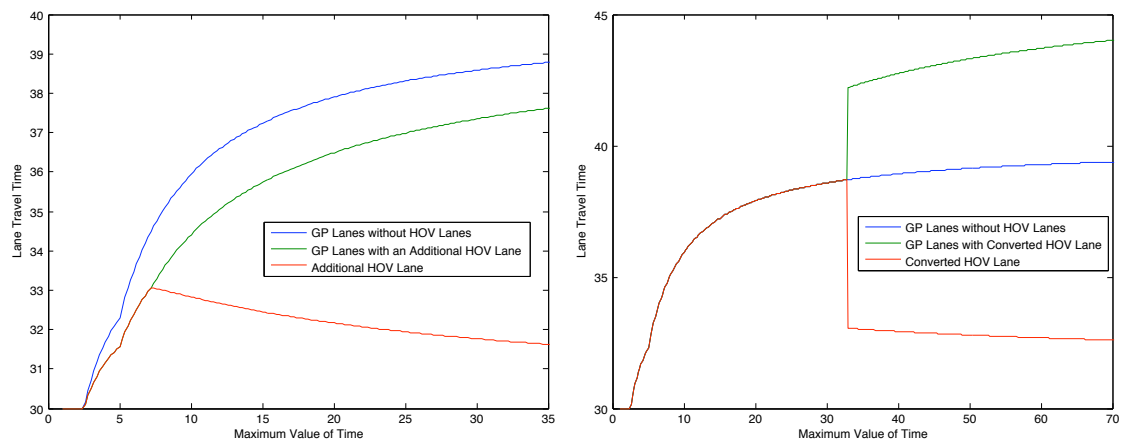
**Figure 3.14:** Mode Share Comparisons With Changing  $\bar{\beta}$ .

To help the reader understand the changes in travel time that result from a higher  $\bar{\beta}$ , Figure 3.15c and 3.15d are zoomed in versions of Figure 3.15b. For low levels of  $\bar{\beta}$ , nobody drives and the travel time is simply  $\delta$  or 30 minutes. For  $\bar{\beta} = M/(V - t - a)$ , carpoolers start driving and at  $\bar{\beta} = 5$ , commuters start to drive alone. The travel times are lower in the additional lane scenario since the extra lane means there is less congestion. Travel times between the HOV and the general purpose lanes do not diverge until the number of solo vehicles per lane,  $\frac{z}{n}$  is greater than the number of carpooling vehicles,  $\frac{y}{2}$ , where  $n=2$  for this configuration. In the conversion scenario, the divergence in land travel time occurs when  $z = \frac{y}{2}$  since the number of lanes is equivalent. This occurs much later and results in an immediate change in travel times between lanes. The change in total travel costs, Figure 3.16 shows a similar story where the additional lane has lower travel costs for all values of  $\bar{\beta}$  but the converted lane diverges sharply at the value of  $\bar{\beta}$  where the lanes separate into HOV and general purpose lanes.



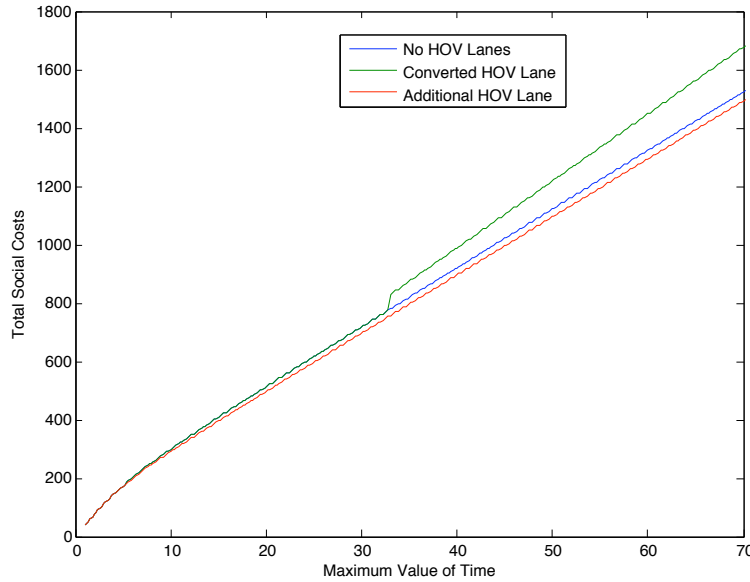
(a) Traffic Volume

(b) Equilibrium Travel Times By Lane, All Lanes



(c) Equilibrium Travel Times in Additional Lane Scenario  
 (d) Equilibrium Travel Times in Conversion Scenario

**Figure 3.15:** Traffic Volumes and Travel Times with Changes in  $\bar{\beta}$ .

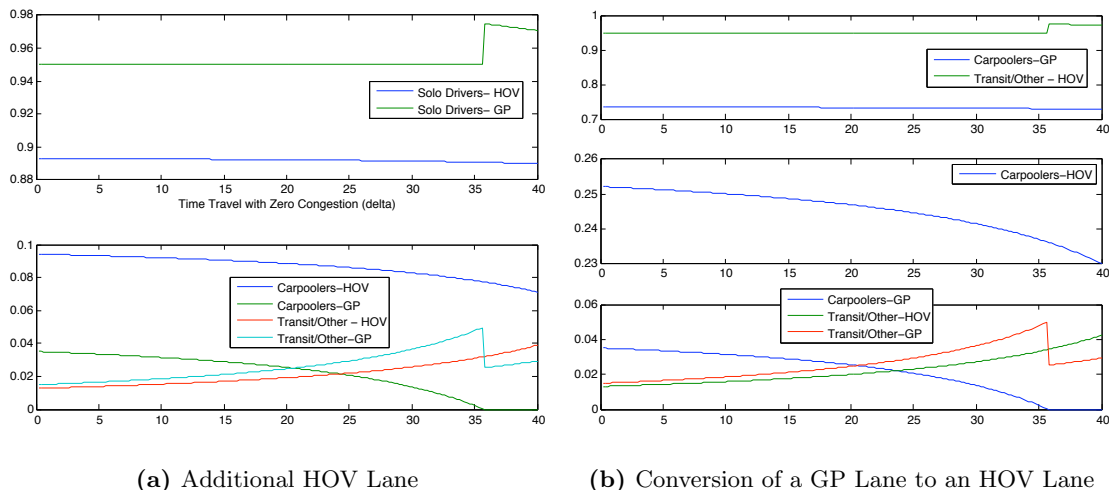


**Figure 3.16:** Total Generalized Costs to Commuters versus  $\bar{\beta}$

### 3.4.6 Changing $\delta$

The parameter  $\delta$  represents the time it takes to go down the line haul portion of the trip when there is no traffic on the road. To understand the influence of  $\delta$  on road share, Figures 3.17a and 3.17b are split because changes in mode share are relatively small. One thing to note is that increasing  $\delta$  decreases the number of carpoolers, not because it makes solo driving more attractive but because a higher  $\delta$  makes taking transit relatively more attractive. In the HOV situations, the number of people being diverted from carpooling to transit increases the time differential on the HOV lane, leading some solo drives to be converted to carpoolers. As before we have the kink that occurs when  $a > V - t - a$  and the cost per minute of driving alone is lower than the cost per minute or carpooling.

Volume is highest when there are no HOV lanes, again suggesting that HOV lanes can reduce volume. However, travel times on the general purpose lanes are also highest



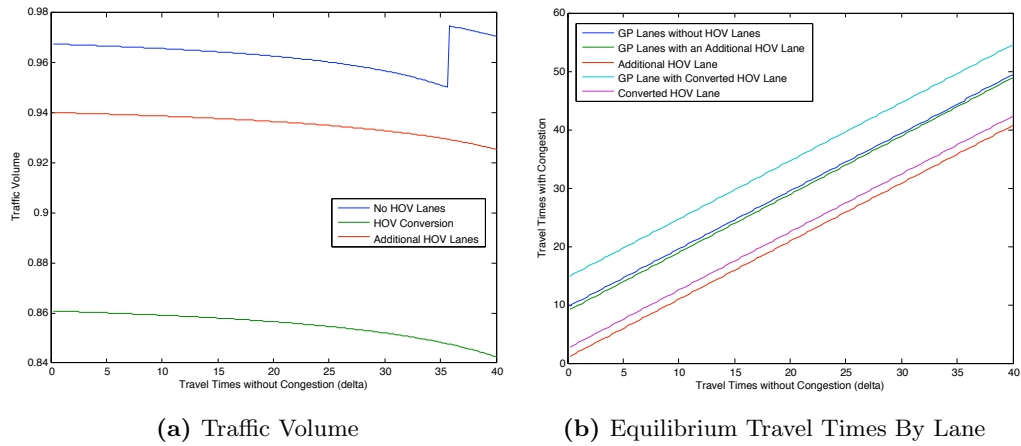
**Figure 3.17:** Mode Share Comparisons With Changing  $\delta$ .

in the HOV conversion scenario suggesting that this volume reduction is coming at a high cost for commuters with a high value of  $\bar{\beta}$ . The total travel costs of the additional HOV lane and the situation without HOV lanes converge as delta increases since the time costs are largely driving the total social costs.

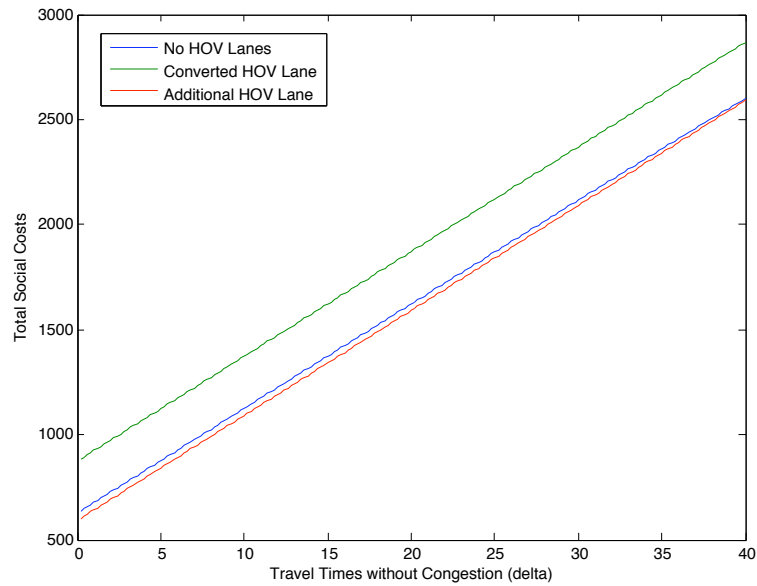
### 3.5 Conclusion

This research was motivated by a desire to understand when and whether HOV lanes can reduce traffic volume and total travel costs to commuters. Both the conversion and the addition of an HOV lane generally decrease traffic volumes, although a converted HOV lane works much better than an additional HOV lane. There are a few exceptions where two or all three of the configurations are the same such as very low values of  $V$  or very high values of  $M$ . Low values of  $\bar{\beta}$  result the conversion and no HOV lane scenarios having the same volumes of traffic, which is higher than the additional HOV lane scenario.





**Figure 3.18:** Traffic Volumes and Travel Times with Changes in  $\delta$ .



**Figure 3.19:** Total Generalized Costs to Commuters versus  $\delta$

These results are tentative as two main extensions are needed for this paper. First, the assembly cost should be varying across commuters. Second, a more realistic traffic congestion function needs to be used instead of  $t = \delta + \alpha * \text{volume}$ .

While both additional and conversions of HOV lanes may work to reduce volume, conversion of an HOV lane to a general purpose lane has significantly higher total commuter costs. The exceptions to this are high values of  $\alpha$ , very low values of  $V$ , and low values of  $a$ . These exceptions arise from behavior where either no solo driving or no carpooling occurs in the various regimes and is likely not representative of reality. They may also be an artifact of the model's restriction that  $a_i = a_j \forall i, j$ . Meanwhile, an additional HOV lane generally lowers total commuter costs and volume, but this does not include the cost of building the additional lane. Building an additional HOV decreases commuter costs relative to the statue quo, but increases costs relative to building an additional general purpose lane. However, since it also reduces traffic volume, traffic managers may be justified in thinking of additional HOV lanes as a win-win traffic management strategy.

## Chapter 4

# Hybrid Cars and HOV Lanes

### 4.1 Introduction

While the previous chapters used theoretical models to understand the impacts of car-pooling and HOV lanes generally, this chapter uses data to understand the impact of a particular program that encourages the purchase of energy efficient vehicles by offering them access to HOV lanes. Road traffic is projected to cost Californians over \$42 billion per year in lost time and higher fuel costs and is a significant contributor to California's air pollution problems (California Air Resources Board (CARB), 2009). California has been on the cutting edge of creative solutions to address congestion and air pollution externalities through the promotion of low or zero emission vehicles, reformulated cleaner-burning fuels, and demand oriented policies to reduce vehicle miles traveled (Sperling and Gordon, 2009). This paper looks at one of these programs. The California Clean Air program was aimed at encouraging the adoption of energy efficient, low emission hybrid vehicles. The program offered special stickers to hybrid car owners that allow them to allow bypass congestion by driving in High Occupancy Vehicle (HOV)<sup>1</sup> lanes without meeting minimum capacity constraints.

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<sup>1</sup>HOV lanes are also known as carpool lanes, express lanes, diamond lanes, commuter lanes or transit lanes.

HOV lanes were built to induce drivers to carpool by providing a free flowing lane with shorter travel times and greater travel time reliability. It was assumed that all drivers would benefit from higher carpooling rates and fewer cars on the road. Society would benefit from lower air pollution and lower fuel consumption. There is growing debate as to whether or not an HOV lane is a strong enough incentive to carpool and even if it were, whether or not more carpooling can really mitigate congestion (Kwon and Varaiya, 2008; Legislative Analyst's Office, 2000; Li et al., 2007; Dahlgren, 1998). By 2004, it was clear that California's HOV lanes suffered from 'empty lane syndrome', when HOV lanes are under-utilized and government officials feel pressure to convert them to general purpose lanes (Schofer and Czepiel, 2000). Moving a small fraction of cars from the general purpose lane to the HOV lanes could relieve some congestion on the general purpose lanes without worsening traffic on the HOV lanes. The question became how to allocate this space on the HOV lanes.

One option was to convert HOV lanes into high occupancy/toll (HOT) lanes. In HOT lanes, carpoolers can use the lane for free or a reduced toll and non-carpoolers pay the full toll to use the lane. This policy was viewed as an entry into congestion pricing, and a way to raise revenue for transportation projects. Alternatively clean air vehicles could be allowed to drive on HOV lanes without meeting minimum-capacity requirement. California chose the latter and instructed the Department of Motor Vehicles to issue 85,000 stickers to owners of qualifying hybrid vehicles. In this paper I show that the 85,000 stickers were worth approximately \$442 million. Accordingly, the excess capacity has a net present value of \$1.77 billion.<sup>2</sup> Converting 85,000 standard cars into hybrid cars would be worth at most \$197 million in air pollution benefits. Clean Air stickers did not even result in 85,000 more hybrid cars; previous studies (Gallagher and Muehlegger, 2008; Diamond, 2009; Chandra, Gulati, and Kandlikar, 2009) have not found evidence that allowing hybrids into HOV lanes encourages the adoption of hybrid cars.

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<sup>2</sup>This estimate is calculated using a 7 percent discount rate.



**Figure 4.1:** Yellow Clean Air Stickers issued by the DMV to AT PZEV hybrid vehicles

#### 4.1.1 The California Clean Air Sticker Program

In September of 2004, Governor Schwarzenegger signed Assembly Bill 2628 (AB 2628) to allow hybrids meeting the state's advanced technology partial zero emission vehicle (AT PZEV) standard and having a 45 mpg or greater fuel efficiency rating<sup>3</sup> to use the HOV lanes without having to carpool. Three hybrid vehicles met the requirements: the Honda Civic hybrid, the Honda Insight and the Toyota Prius.<sup>4</sup>

From August 2005 to February 2007, any California driver with a Prius, Civic Hybrid or Insight could write to the Department of Motor Vehicles (DMV) and obtain a set of stickers for \$8. If the owner sold his or her car, the sticker and the privileges it conferred were transferred to the new owner of the vehicle. The 85,000 stickers were given out in three installments. The first installment of 50,000 stickers was issued starting August 2005. Once those 50,000 stickers were issued, the DMV commissioned a study of the impact of hybrids on HOV lanes. The study found hybrids had not degraded the HOV lanes, so the remaining 25,000 stickers were issued under AB 2628. In September 2006, another bill, AB 2600, expanded the number of stickers by 10,000 and extended

<sup>3</sup>Additionally, 2004-model year or older hybrids with a 45 mile or greater fuel economy rating and meeting either the SULEV, ULEV (ultra low emission vehicles) or PZEV standards.

<sup>4</sup>Effective January 1, 2009, DMV was allowed to issue Clean Air Stickers to the original owners of qualifying hybrids to replace hybrids declared nonrepairable or total loss salvage (AB 1209). Thus, it was possible for a 2008 or 2009 hybrid car to have a sticker on it. As only one case was found in our data, it was not included.

September 2004	AB 2628 signed authorizing Hybrids to use HOV lanes in California pending approval by federal government
August 2005	SAFETEA-LU, federal transportation bill, authorizes states to allow fuel-efficient hybrid cars into HOV lanes
August 2005	State begins issuing California Clean Air stickers to qualifying hybrids
September 2006	AB 2600 authorizes an additional 10,000 stickers and extends program life till January 1, 2011
February 2007	DMV completes distribution of California Clean Air stickers
September 2008	AB 1209 allows hybrid owner with stickers whose cars are declared total loss/salvage to obtain stickers for a new hybrid
January 2009	DMV allowed to issue new stickers to owners of cars with stickers that have been declared total loss/savage
January 2011	California Clean Air sticker is scheduled to end.

**Table 4.1:** Timing of California Clean Air Sticker Program

the program end date to January 1, 2011. Stickers were available for issue until February 2007, when the 85,000 limit was reached. After February 2007, a buyer could obtain a California Clean Air sticker only by buying a used car that had stickers on it. Data from the used car market provides observations on cars with and without stickers from which one can estimate the value of a sticker. I hypothesize that this value is a valid indicator of purchasers' willingness to pay for access to California's HOV lanes, conditional on also driving a used qualifying hybrid.

#### 4.1.2 Theoretical Value of a California Clean Air Sticker

The theoretical value drivers place on using HOV lanes depends on the level of congestion on the general purpose lane versus the HOV lane, travel time savings from using an HOV lane, the increased reliability in travel time from an HOV lane, drivers' valuations of time, and whether or not the user feels safer in an HOV lane. In the Bay Area, commuters who carpool report travel time savings of approximately 17 minutes each way in 2005 (RIDES Associates, 2005). Engineering estimates of travel time savings vary depending on the network and time of day. Brownstone et al. (2007) find average speed differences

between HOV lanes and general-purpose lanes to be approximately 10-30 percent across highways in Orange County, California. In the Bay Area, Kwon and Varaiya (2008) conclude that HOV lanes provide only minimal travel time savings but do provide better reliability.

Assuming that a sticker provides a service in each time period that is valued at  $c_s$ , the net present value of the sticker and the value of present and future services at time  $t$  is:

$$NPV(t) = \int_t^T c_s e^{-r(s-t)} ds \quad (4.1)$$

where  $T$  is the date at which the program will end and  $r$  is the rate of time preference.

Stickers may have had nominal value prior to February 2007.<sup>5</sup> Thus I model the value of a sticker prior to February 2007 as  $V_1$ . Assuming the sticker service flow,  $c_t$ , is constant after February 2007 (i.e.,  $c_t = c$  for all  $t$  between February 2007 and January 1, 2011), the price of a sticker can be written as:

$$P(t) = \begin{cases} V_1 & \text{if } t < \text{February 2007} \\ \frac{c}{r}(1 - e^{-r(T-t)}) & \text{if } t > \text{February 2007} \end{cases} \quad (4.2)$$

In the empirical section of this paper, I model the willingness-to-pay for a Clean Air sticker as decreasing over time, both as a linear time trend and non-parametrically.

### 4.1.3 Hedonic Pricing Model

Hedonic pricing provides a method for decomposing a good into characteristics and estimating the contributory value of each characteristic. Hedonic models have been applied to a wide variety of goods. Griliches (1971) is an example of early hedonic models applied to automobiles. Recent papers using hedonic analysis to understand

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<sup>5</sup>If stickers are available at the DMV for \$8, they should not command a premium of more than \$8 plus some transaction cost in the overall vehicle price.

automotive markets include Kooreman and Haan (2006), Ramachandran, Viswanathan, and Gosain (2007) and Kahn (1986).

The value of a Clean Air sticker is first assumed to enter the automobile price linearly and not as a function of other car characteristics. Thus, the price of a used car can be written as:

$$P(z) = P_{sticker}(sticker) + P(z_1, z_2, \dots, z_n) \quad (4.3)$$

The assumption of linearity is tested using a log transformation of price, common in the hedonic literature. This specification implies the sticker's value is multiplicative to the value of the car:

$$P(z) = e^{(z_1, z_2, \dots, z_n)} e^{P_{sticker}(sticker)} \quad (4.4)$$

Robustness checks on the price of the sticker are estimated by interacting the sticker price with geographic regions, car characteristics, and gas prices. All of these relationships are estimated in Appendix D.

## 4.2 Empirical Model

The price of used car  $i$  at time  $t$  is assumed to be a function of whether it has a Clean Air sticker, car type (make, model, year, etc.), condition (captured by mileage), accessories, location of seller and whether financing is available. Price and the natural log of price are both estimated, while car type is captured by model year and model type (Prius, Civic or Insight). While many used cars were listed as being in “excellent” or “perfect” condition, there was no objective way to grade the condition of the cars other than mileage and whether the car had a salvage title.<sup>6</sup>

Theoretically, sticker value should be decreasing in time. We first allow the sticker price to vary over time non-parametrically using a partially linear regression model. We

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<sup>6</sup>The impact of descriptive words such as “excellent”, “perfect” or “mint” on price was examined for a subset of the data to no effect.



then fit the evolution of sticker price to a linear time trend to see how much the price of a sticker changes on average.

The partially linear model allows the price of the sticker to be a non-parametric function over time while controlling for other covariates with a simple regression model. We write the price of the car as a function of the changing value of the sticker, car characteristics and a normally distributed error term:

$$Price_{it} = F(z_{it}) + x'_i\beta + \varepsilon_i \quad (4.5)$$

where  $z_{it}$  is zero if the car does not have a sticker and takes on the value time  $t$  if the car does have a sticker.  $F$  is a non-linear function tracing sticker price over time and  $x_i$  refers to other car specific characteristics such as make, model, mileage, etc. We use a variation of Yatchew's strategy (Yatchew, 2003) to remove the non-parametric part of the regression so as to consistently estimate  $\hat{\beta}$ , then form residuals  $Price_{it} - x'_i\hat{\beta}$  and run a non-parametric kernel regression on the residuals:

$$Price_{it} = x'_i\hat{\beta} + \varepsilon_i. \quad (4.6)$$

Yatchew's method requires assuming that  $F$  is smooth with  $z_i$  dense in the domain. All other variables are assumed to be scalars, with the normal assumptions  $E(\varepsilon_i|z, x) = 0$  and  $Var(\varepsilon_i|z, x) = \sigma^2$ . Yatchew assumes that the conditional mean of  $x$  is a smooth function of  $z$ ,  $E(x|z) = g(z)$ , where  $g$  is bounded and  $Var(x|z) = \sigma_u^2$ . Thus I can rewrite  $z = g(z) + u$  and difference to obtain:

$$\begin{aligned} y_i - y_{i-1} &= F(z_i) - F(z_{i-1} + (x_i - x_{i-1})')\beta + \varepsilon_i - \varepsilon_{i-1} \\ &= (g(z_i) - g(z_{i-1}))\beta + (u_i - u_{i-1})\beta + (F(z_i) - F(z_{i-1})) + \varepsilon_i - \varepsilon_{i-1} \\ &\approx (u_i - u_{i-1})\beta + \varepsilon_i - \varepsilon_{i-1} \end{aligned} \quad (4.7)$$

since I assume that small changes in  $z_i$  produce small changes in  $g$  and  $F$ . Thus the direct effect of the nonparametric variable is removed and I can estimate  $\hat{\beta}$  on the transformed model using OLS.

The data from the used car market satisfies most of these assumptions in all but one case where  $F$  is not smooth. The sticker is represented as a 0 if there is no sticker, and a 1 through 745 depending on the day during the two year period of May 19, 2008 and June 2, 2010 in which the car appears. While it is reasonable to assume that changes in sticker value from week  $i$  to week  $i + 1$  are small, the change in automobile price from not having a sticker to having a sticker on May 19, 2008 (week 1) is not small. Using Yatchew's strategy I can identify the weekly change in sticker value, but not the initial value of having a sticker.

Instead I remove the non-parametric component of the model with a dummy variable for each value that  $z_i$  can take on in the data. This means I estimate almost 87 dummy variables (the data spans 106 weeks, but not all weeks have data on cars with stickers) and obtain imprecise estimates of sticker value. While the estimates of sticker value are imprecise, I am able to estimate  $\beta$  using OLS without the bias from correlation between  $x_i$  and  $z_i$ . We can then use  $\hat{\beta}$  to form residuals  $y_i - x_i' \hat{\beta}$ , and run a non-parametric kernel regression on:

$$y_i - x_i' \hat{\beta} = F(z_i) + \varepsilon_i. \quad (4.8)$$

We now turn to a description of the data and the estimation of the non-parametric and linear models.

## 4.3 Estimated Value of a Clean Air Sticker

### 4.3.1 Description of the Data

Data were gathered from completed Ebay auctions and list prices from Autotrader.com. Additional data was gathered from the classified sections of four major metropolitan

Data Source	Number of Observations	Percentage of Data	Data Covered in Data Segment
Ebay	231	3.17	May 2008, July 2008, Oct 2008 through June 2010
Autotrader.com	7,067	98.83	May 2008, July 2008, Oct 2008 through June 2010
Total Complete Observations	7,298		May 2008 - June 2010

**Table 4.2:** Sources of Data and Date

newspapers but there were not enough observations from newspapers to precisely estimate a model. The breakdown of these data is presented in Table 4.2. Data were gathered from Autotrader.com and Ebay manually for the month of May and July 2008, and using a program from October 2008 to June 2010. Cars with a Clean Air sticker accounted for 14.0 percent of the cars in the sample. A striking difference between cars with and without a Clean Air sticker is the difference in mileage, presented in Table 4.2. Cars with a sticker are actually worth less on average, but this is before taking mileage into account. The mileage of cars with a sticker is almost 50 percent higher than cars without. Cars with a sticker tended to be older than cars without a sticker, and they had more miles per year driven. This could be because being able to drive on HOV lanes makes driving less costly and more enjoyable. More likely, people who expect to heavily use their cars applied for a sticker. Either explanation points to the need to include mileage in any estimate of sticker value since it is correlated with the Clean Air Sticker and an important component of price.

### 4.3.2 Results

The results for the regression of car characteristics with robustness checks are summarized in Appendix C. Figure 4.2 shows the evolution of the sticker value over time from the non-parametric regression with bootstrapped 95% confidence intervals. The price of

	Price	Average Age of Car	Average Mileage	<u>Average Mileage</u> Age
Cars with a Clean Air Sticker	\$15,300	4.2	72,500	18,200
Cars without a Clean Air Sticker	\$16,800	3.2	50,300	16,200
All Cars	\$16,600	3.4	53,400	16,500

**Table 4.3:** Average Price and Mileage for Cars With and Without Clean Air Stickers

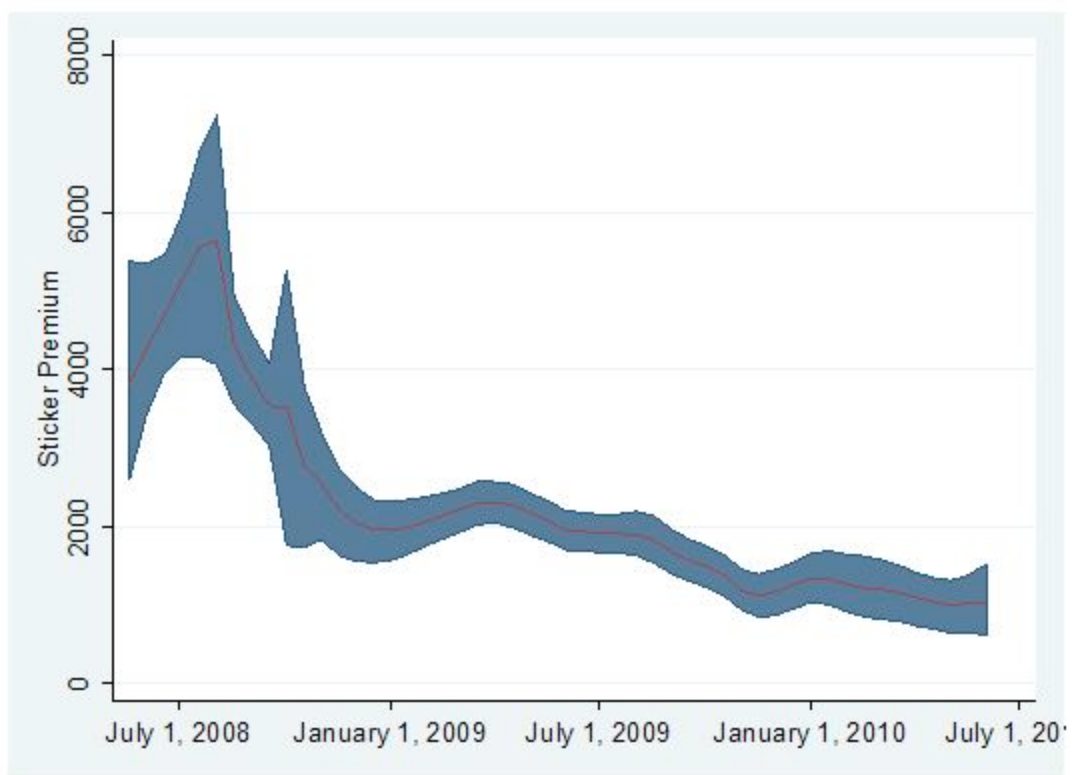
Variable	Estimate
Sticker Intercept	3,130*** (267)
Sticker Slope	-3.96*** (0.528)
Number of Observations	7,292
R-Squared	0.7589

**Table 4.4:** Linear Estimate of Sticker Price Over Time

the sticker appears to be approximately \$4,000 in May 2008, but falls to approximately \$1,000 by June 2010.

While Figure 4.2 provides compelling evidence that the sticker value is significant and decreasing over time, looking at the entire sample I can see that there is more noise in the beginning of the sample because the program to automatically collect the data was not running until late October 2009. Another way to view the price of the sticker over time is to run a regression on a linear time-trend model. In this section examine a model with an intercept for sticker price,  $\alpha$ , and a slope variable with coefficient  $\gamma$ :

I run this regression with the same set of controls and present the results in Table 4.4. The slope and intercept for the impact of sticker on vehicle price are significant and as expected. The intercept indicates that a sticker was worth approximately \$3,130 on May 19, 2008, and has since depreciated at approximately \$4/day. This translates into yearly values of \$1,460.



**Figure 4.2:** Non-Parametric Estimation of Sticker Value Over Time

Using a 7% discount rate and the lower weekly value, this means the stickers were worth \$5,200 each in January 2007, or \$442 million for, all 85,000. If the state were to sell yearly stickers, assuming symmetric demand, they could obtain \$124 million per year or a net present value of \$1.77 billion. While these values seem high, in the next section we see they fit in with previous value of time estimates and are likely a lower bound of the value of access to HOV lanes.

### **4.3.3 Value of Time Estimates**

The value of driving in the HOV lane has many components such as, travel time savings, greater travel time reliability and a greater perception of safety by being able to travel in a less congested lane. None of these effects can be separated using the data collected, but as an empirical check a rough value of travel time savings can be estimated and compared with previous results.

Assuming the Bay Area time savings are 17 minutes each way (RIDES Associates, 2005), similar to the 17 minutes of time savings found by Caltrans along the HOV corridor on I-210 (California Department of Transportation(Caltrans), District 7, 2006) and assuming commuters make two trips a day, five days a week, the Clean Air sticker values time at \$10 per hour. This is below the range of \$20-40 per hour found by Brownstone and Small (2005), but within the \$7-25 per hour found by Barrett (2010), \$30 median value of time in Steinmetz and Brownstone (2005) and 50 percent of the gross wage rate found in Small (1992). The estimate of \$10 per hour is likely an underestimate of HOV driving privileges since it is conditional on having to drive one of three used hybrid cars and would likely be higher without that constraint.

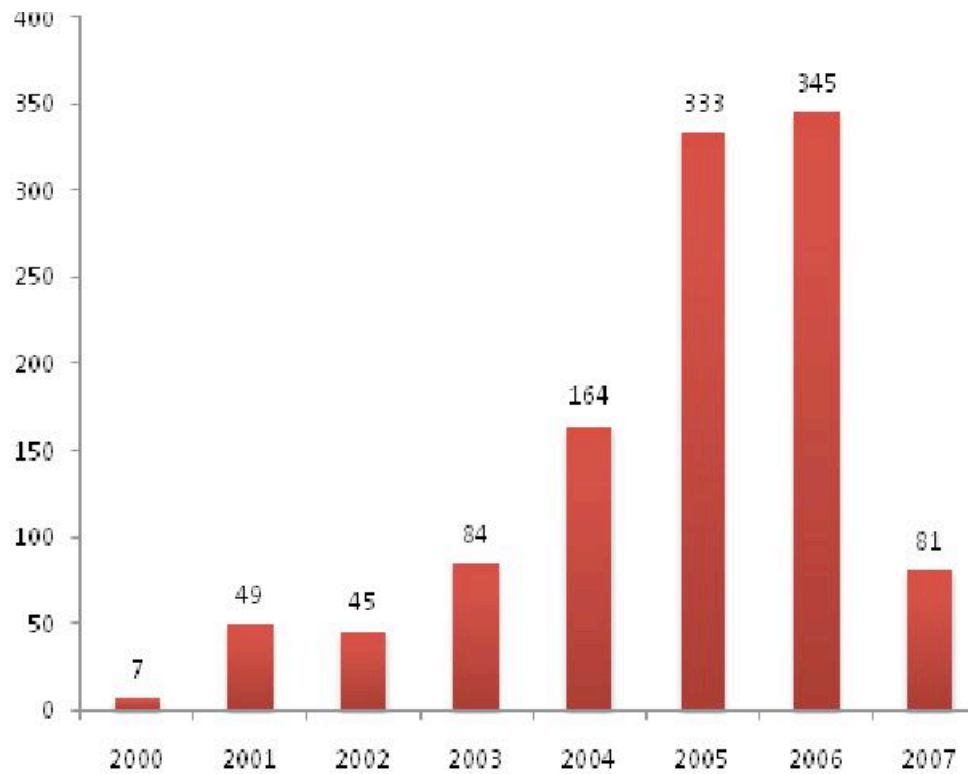
## 4.4 Discussion

Excess capacity on HOV lanes did not have an obvious dollar value before the California Clean Air program allowed motorists to capture significant economic rents. Allowing stickers to be traded enables those rents to be observed. If the excess capacity calculations were correct (Boriboonsomsin and Barth, 2008; Brownstone et al., 2007; Breiland, Chu, and Benouar, 2006), and hybrids did not slow down carpoolers, then the state created up to \$612 million of economic rent through the California Clean Air program.

The Clean Air Program is set to expire January 1, 2011, and the California State Assembly is debating what to do with the excess capacity once the current Clean Air stickers expire. Senate Bill 535, which is sponsored by General Motors, would allow the next generation of hybrids, the plug-in electric hybrids such as the Chevy Volt, to use HOV lanes. Our results suggest that a repeat of the Clean Air Program would not be the best use of the excess capacity. This section discusses whether or not the original California Clean Air program achieved its goal of stimulating the market for hybrid cars, reducing air pollution and alleviating congestion. I also discuss implications of forgone revenue, and the potential for alternative uses of excess capacity, particularly HOT lanes.

California Clean Air stickers were one of many incentives created to encourage the purchase of hybrid cars at the state, federal and local levels. A natural question to ask is whether granting access to hybrids stimulated the demand for hybrids. Three studies have addressed this question. Gallagher and Muehlegger (2008) and Chandra, Gulati, and Kandlikar (2009) found that HOV privileges were not strongly correlated with hybrid market share in the US or in Canada. Diamond (2009) found HOV privileges encouraged hybrid ownership in northern Virginia, but he did not find an impact in other states or in other parts of Virginia with HOV lanes.

My research indicates that HOV privileges have substantial value to motorists, and should stimulate demand for hybrid cars. One reason they may not have stimulated

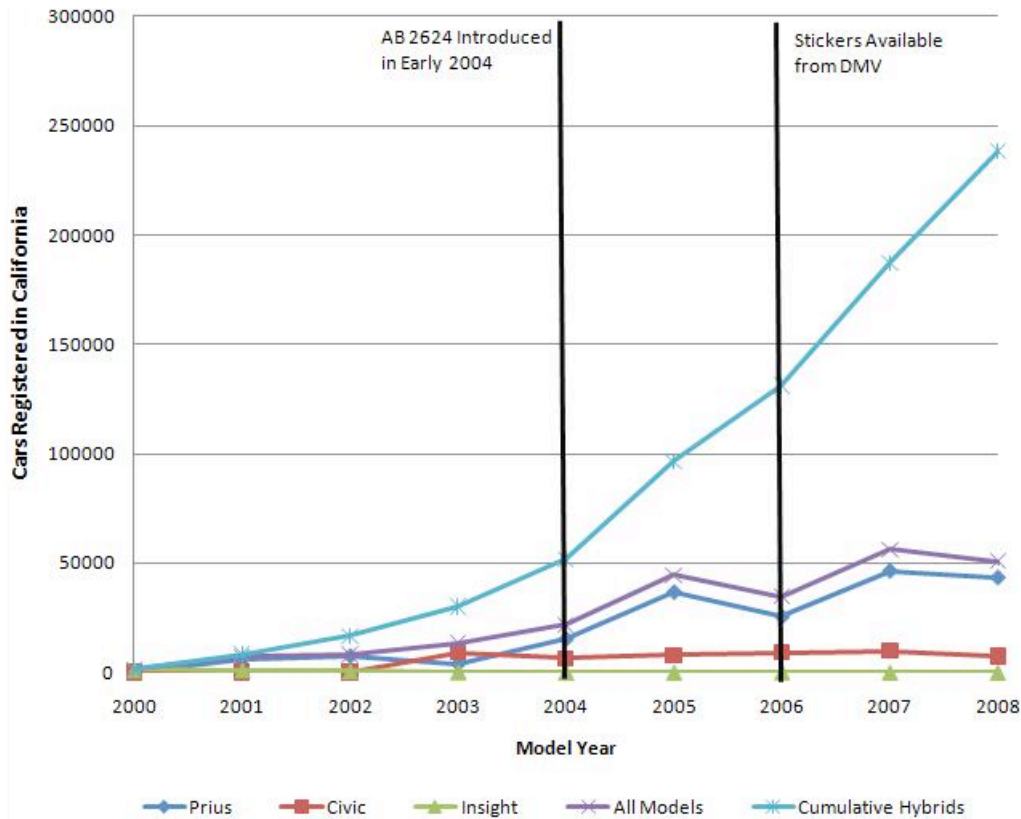


**Figure 4.3:** Number of Cars with HOV Stickers by Model Year

demand in California is that that stickers were given away to vehicles that had already been purchased.

Figure 4.3 displays the number of cars with a sticker in my sample by model year. The stickers were given out starting in August 2005, by then 2006 model year cars were being sold as new models. According to the data I gathered on Autotrader, Ebay and from classified ads, 32% of cars were 2004 or earlier model years while 62% were 2005 or earlier model years. California Clean Air Stickers were not available until the 2006 models were being sold. Almost two-thirds of the stickers were given to cars that were already on the road. It is of dubious value to give a car purchased in 2000 a sticker to encourage its purchase in 2006.





**Figure 4.4:** Hybrids Registered in California

Gallagher and Muehlegger (2008), Chandra, Gulati, and Kandlikar (2009) and Diamond (2009) look at cross-state comparisons to explore the impact of HOV privileges on hybrid purchases. Hybrids were already popular by the time AB 2628 was passed to encourage their purchase. AB 2628 was introduced in early 2004, signed into law in September 2004, and initiated in August 2005. Figure 4.4 shows when AB 2628 was introduced into the State Senate as well as when stickers were actually available. These data are overlaid with Department of Motor Vehicle data for the total number of hybrid cars registered in California by model year.

Model Year	Models Prius	Civic	Insight	All Models	Cumulative
2000	0	0	1,275	1,275	1,275
2001	6,042	0	1,114	7,156	8,431
2002	7,593	0	573	8,166	16,597
2003	4,027	9,077	291	13,395	29,992
2004	15,386	6,385	121	21,892	51,884
2005	36,69	17,790	131	44,612	96,496
2006	25,560	9,007	190	34,757	131,253
2007	46,382	9,760	0	56,142	187,395
2008	43,391	7,534	0	50,925	238,320
2009	17,911	3,834	0	21,745	260,065

**Table 4.5:** Car Registrations in California By Model and Model Year

The data in Figure 4.4 is also presented in Table 4.5. Using DMV data made available by Jeffrey Williams<sup>7</sup>, I looked at the total number of cars by model year in California that were ever registered in California. While a more in-depth study needs to be done, a cursory look at the data in Figure 4.4 does not suggest a massive demand shift for hybrid cars. This is not surprising given the Prius was already limited in supply with long waiting lines for new vehicles.

The name California Clean Air Sticker signifies that the program was intended to lessen pollution from automobiles. Using the most conservative assumptions possible, namely ignoring evidence that the Clean Air stickers failed to stimulate hybrid sales and thus assuming that each of the 85,000 stickers caused a conventional car to be replaced with a hybrid, and making conservative assumptions about the value of air pollution, the upper limit to the value of air pollution reductions is \$197 million (for calculations see Appendix E). If all 85,000 permits had been sold for \$3,400 each, the state could have obtained \$306 million in revenue to reduce air pollution from more cost effective sources.

Another aim of the program was to reduce congestion. An optimal taxation scheme would favor workers over non-workers during the peak period (De Borger, 2009).

<sup>7</sup>Jeffrey Williams is a faculty member in the Department of Agricultural and Resource Economics at the University of California, Davis

To the extent that commuters obtained stickers, the Clean Air Sticker program might have lessened the deadweight loss of congestion by allowing those with a high value of time to “buy out” of congestion by purchasing hybrid cars. As Parry (2002) finds, the biggest efficiency gains from congestion pricing come from separating high value and low value users into fast and slow lanes, not necessarily from encouraging low value users to use public transit or travel during off-peak times. While this market for Clean Air stickers may have been unanticipated by policy makers, it likely improved welfare by allowing drivers with a high value of time to bypass congestion through purchase of a hybrid car with a sticker. This does not mean the entire California Clean Air policy was justified on welfare grounds, just that the market for stickers likely improved welfare in comparison to programs where stickers could not be transferred between owners.

Assuming that the excess capacity on the HOV lanes really can be used without impacting carpoolers, using the excess capacity is worth at least \$85-170 million/year. The California Clean Air sticker program is not the best way to use that capacity. It is a marginal improvement on a system that is already far from optimal and better marginal improvements can be made. Auctioning stickers for hybrids would have raised government revenue for a direct hybrid subsidy with money leftover. Auctioning stickers for any type of car would have raised even more government revenue and would have allocated stickers to those with the highest value of time. If the government still wanted to subsidize hybrids, they could have issued a revenue equivalent subsidy which has been shown to be more effective. Instead of handing out access privileges as a lump sum, transportation officials could have created HOT lanes that allow anybody willing to pay to bypass congestion. This would have been more equitable since it would have allowed all motorists, regardless of vehicle choice, to escape traffic regularly or occasionally instead of just a lucky 85,000.

## 4.5 Conclusion

An un-priced highway will suffer from overuse if travelers do not take into account the external costs of pollution, congestion, accidents and road maintenance. An optimal Pigouvian tax would result in consumers choosing the socially efficient combination of energy efficient cars, number of trips and mode choice, but political concerns have blocked congestion and emission charges. This leaves policy makers with technical fixes for pollution and ad hoc methods to discourage driving such as driving restrictions (Davis, 2008), subsidies for transit, or occupancy restrictions (HOV lanes). It is unclear whether HOV lanes result in more carpools, higher social surplus or less pollution, and they are not necessarily more effective than a general purpose lane (Dahlgren, 1998). Giving away excess capacity on HOV lanes to hybrid cars is an ad hoc measure on top of an already wasteful policy.

California considered using the excess capacity of HOV lanes to experiment with a form of congestion pricing, but instead chose to grant access to its HOV lanes to owners of hybrid cars. These cars achieve higher gas mileage, reducing smog forming pollutants and greenhouse gases. Always forward looking, California was hoping that if motorists could be convinced to switch from conventional cars to hybrids, it could speed a transition to even more advanced technologies such as plug-in hybrids, natural gas vehicles or electric vehicles. This research suggests that granting stickers to hybrids did not achieve either of these goals. The air pollution benefits are worth far less than the value of the stickers, and research by others shows that stickers are less effective than direct subsidies. From a welfare perspective, allowing users with the highest values of time to bypass traffic can contribute to the overall efficiency of the system and so auctioning permits would have been more effective in terms of efficiency and raising revenue for a cash-strapped state.

The California Clean Air sticker program failed to achieve its goals and came at a high opportunity cost to the state. Similar programs in Arizona, Colorado, Florida,

Georgia, Hawaii, Maryland, Texas, Utah and Virginia allow clean air vehicles access to HOV lanes. The value of access to HOV lanes varies within California and is likely to vary across states, but traffic managers in all these areas need to carefully analyze programs that give out access without regard as to whether this is the best way to use this capacity. Arizona considered selling its capitol building to raise funds, California is facing massive shortfalls. Instead of using the valuable space in HOV lanes to fund education, health, transportation or a variety of public work projects, these states are giving away space to support an ineffective project.

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## Appendix A

# Second Order Conditions

To ensure that the solutions found in Section 2.3 are indeed a minimum, we check the second order conditions for Equation 2.21. We calculate  $\frac{\partial^2 TSC}{\partial \beta_1^{*2}}$ ,  $\frac{\partial^2 TSC}{\partial \beta_2^{*2}}$ ,  $\frac{\partial^2 TSC}{\partial \beta_1^* \beta_2^*}$ , and  $\frac{\partial^2 TSC}{\partial \beta_2^* \beta_1^*}$  and see whether or not they guarantee a minimum. The first second derivative we examine is  $\frac{\partial^2 TSC}{\partial \beta_1^{*2}}$ :

$$\frac{\partial^2 TSC}{\partial \beta_1^{*2}} = \frac{2}{\underline{\beta} - \underline{\beta}} \left[ V - t - a + \beta \left( \frac{\alpha}{2(\underline{\beta} - \underline{\beta})} \right) \right] \quad (\text{A.1})$$

One of the assumptions in the beginning was that  $V - t - a > 0$ ,  $\forall x, y, z$ . Using this assumption we know that  $\frac{\partial^2 TSC}{\partial \beta_1^{*2}}$  is positive. The next second derivative is  $\frac{\partial^2 TSC}{\partial \beta_2^{*2}}$  and is positive for all relevant parameters:

$$\frac{\partial^2 TSC}{\partial \beta_2^{*2}} = \frac{2a}{\underline{\beta} - \underline{\beta}} \quad (\text{A.2})$$

The cross-partial derivatives  $\frac{\partial^2 TSC}{\partial \beta_1^* \beta_2^*}$ , and  $\frac{\partial^2 TSC}{\partial \beta_2^* \beta_1^*}$  are equal to each other and also positive:

$$\frac{\partial^2 TSC}{\partial \beta_1^* \beta_2^*} = \frac{\partial^2 TSC}{\partial \beta_2^* \beta_1^*} = \frac{\alpha \beta_1^*}{(\underline{\beta} - \underline{\beta})^2} \quad (\text{A.3})$$

## Appendix B

# Comparative Statics

**B.0.1** Signing  $\frac{\partial t^*}{\partial M}$

$$\frac{\partial t^*}{\partial M} = \frac{-\alpha}{8a(\bar{\beta} - \underline{\beta})} \left( 1 + \frac{M\alpha - 4ar}{\sqrt{(M\alpha - 4ar)^2 + 64a^3(r+a)}} \right)$$

Where  $r = \alpha\bar{\beta} + (\bar{\beta} - \underline{\beta})(\delta - V - a)$ .

The first term is unambiguously negative for all relevant parameters. Whether or not the entire partial derivative is negative(positive) depends on whether or not the last term is greater(less) than -1. The term  $\delta - V - a$  is negative from the initial assumption that transit takes more time than carpooling even at the maximum amount of traffic,  $V > \delta + \alpha * v^{max} + a$  where  $v^{max}$  corresponds to the situation in which every commuter chooses to drive alone,  $v^{max} = 1$ , which implies  $0 > \delta - V - a + (\alpha + 2a)$ . Thus it is possible for all of  $r$  to be negative. If  $r$  is negative, then  $1 + \frac{M\alpha - 4ar}{\sqrt{(M\alpha - 4ar)^2 + 64a^3(r+a)}}$  is unambiguously positive. However, if  $r$  is positive, then it could be that  $M\alpha - 4ar < 0$ , but it will not be true that  $\frac{M\alpha - 4ar}{\sqrt{(M\alpha - 4ar)^2 + 64a^3(r+a)}} < -1$  since this term can be written as  $\frac{q}{\sqrt{q^2 + p}}$  which is less than 1 if  $p > 0$  which it is if  $r > 0$ . Thus we can say that for all positive parameter values:

$$\frac{\partial t^*}{\partial M} < 0.$$

### B.0.2 Signing $\frac{\partial r^*}{\partial V}$

$$\frac{\partial r^*}{\partial V} = \frac{1}{2}(1 + c^{-3/2} \left( \left( \frac{16a\alpha}{\bar{\beta} - \underline{\beta}}(M - 4a) - 64a^2 \right) - 8a(a - V - \delta - \frac{\alpha\bar{\beta}}{\bar{\beta} - \underline{\beta}}) \right))$$

Where  $c = \left( 16a \left( \frac{\alpha(MV + 4a(a - V)\bar{\beta})}{\bar{\beta} - \underline{\beta}} + 4a\delta(a - V) \right) + \left( 4a(a - V - \delta - \frac{\alpha\bar{\beta}}{\bar{\beta} - \underline{\beta}}) + \frac{M\alpha}{\bar{\beta} - \underline{\beta}} \right)^2 \right)$ .

For any real solution  $c > 0$ , and rewrite:

$$\frac{\partial r^*}{\partial V} = \frac{1}{2} \left( 1 + c^{-3/2} 8a \left( \frac{2\alpha}{\bar{\beta} - \underline{\beta}}(M - 4a) - 9a + V + \delta + \frac{\alpha\bar{\beta}}{\bar{\beta} - \underline{\beta}} \right) \right)$$

It can be shown that

$$8a \left( \frac{2\alpha}{\bar{\beta} - \underline{\beta}}(M - 4a) - 9a + V + \delta + \frac{\alpha\bar{\beta}}{\bar{\beta} - \underline{\beta}} \right) < c.$$

This implies that  $\frac{\partial r^*}{\partial V} > 0$ .

## Appendix C

# Regression Tables

The regressions from the body of the paper are presented here. Model 1 is the partially linear model where weekly dummies are used to remove the non-parametric effect. Models 2 is the linear model. Many of the coefficients of car characteristics are positive and go in the direction economic theory would predict. The coefficients for mileage range from -0.054 to -0.056. This translates into a penalty of \$54-\$56 for every thousand miles on the vehicle. Ramachandran, Viswanathan, and Gosain (2007) find values of \$31.08 and \$36.65 per thousand miles on the vehicle, without controlling for mileage squared, miles driven per year or a dummy variable for crossing the 100,000-mile mark. Mileage squared is not significant in any of the regressions, but miles per year is negative and significant across regressions, as is the dummy variable for having mileage over 100,000 miles. Having a salvage title is significant across all regressions and results in an approximately \$4,000 penalty on the car price.

Drivers that expect to use their vehicle heavily will be more likely to apply for a sticker and may want a car with more extras such as a premium sound or a navigation system. A built-in navigation system is worth approximately \$260, significantly less than the dealership price differential of \$2,000 for cars with and without a navigation system. A premium (JBL) sound system is worth somewhere approximately \$800, while mp3 playing capabilities are worth a little over \$600. Leather interiors increase the price of

a car by about \$700 across the regressions, while a 'loaded' package has no statistical significance and Bluetooth capabilities is only significant at the 10% level in one of the regressions.

Where the car was sold was an important determinant of price. Cars sold on Ebay (the omitted category) commanded a \$3,800 lower price than asking prices were on Autotrader and were lower than asking prices in the four newspapers. Using asking prices instead of actual prices should not be a problem in the identification of the sticker because the difference between the asking price and the actual price should not vary depending on whether or not the car has a sticker. Prices from dealers were about \$800 more than prices from private sellers. Monthly dummies on the price of all cars and dummy variables for make/model combinations are not presented, but they are as expected. While not all of these results are relevant to the price of the sticker, they do lend credibility to the model.

Variables	Model 1 Partially Linear	Model 2 Linear Estimate
Mileage	-0.0846*** (0.003)	-0.0849*** (0.003)
Mileage Squared	1.74E-07*** (1.54E-08)	1.76E-07*** (1.51E-08)
Mileage over 100,000 Miles	-473.4*** (131.8)	-493.9*** (130.7)
Salvage Dummy	-4379*** (232.5)	-4388*** (229.4)
Premium Sound System	862.6*** (104.8)	862.2*** (105.3)
MP3 Player	335.3*** (90.5)	334.6*** (90.6)
Leather Interior	598.0*** (207.0)	601.4*** (113.9)
Listed as "loaded"	-696.0*** (121.7)	-696.3*** (120.5)
Bluetooth Capabilities	-167.2 (153.4)	-168.6 (153.5)
Navigation System	253.5*** (83.4)	248.1*** (83.1)
Ebay Listing (Autotrader omitted)	-3672*** (193.4)	-3628*** (195.0)
San Diego Region (LA Omitted)	-4694*** (98.9)	-480.9*** (109.6)
San Francisco Region	-43.8 (59.8)	-54.3 (59.3)
Sacramento Region	-354.9*** (94.7)	-360.9*** (94.3)
Central Valley Region	247.1** (117.4)	246.4** (116.4)
Northern California Region	225.9 (336.3)	192.7 (332.8)
Sold by a Private Seller	-1076.8*** (142.1)	-1025.4*** (134.9)
Gas Price (California Regular, Statewide in cents)	20.31*** (3.76)	19.0*** (3.10)
Make/Model Dummy Variables	Yes	Yes
Date Dummies	Yes	Yes
Constant	14014*** (1342)	14721*** (1109)
Observations	7292	7292
R-squared	0.762	0.759

NOTE: Robust standard errors are in parentheses, Asterisks denote significant at the \*10%, \*\*5% and \*\*\*1% level.

**Table C.1:** Regression Coefficients for the Partially Linear and Linear Models

## Appendix D

# Alternative Specifications

In this section I examine how a Clean Air Sticker changes with alternative specifications. First I look at whether or not the sticker's value changes depending on using a multiplicative model versus an additive model. Next I explore how gas prices interact with sticker and vehicle price. Finally I look at whether or not the sticker is more valuable in various metropolitan areas and the impact on the sticker price of various car characteristics such as the make, year, mileage, and options on a vehicle.

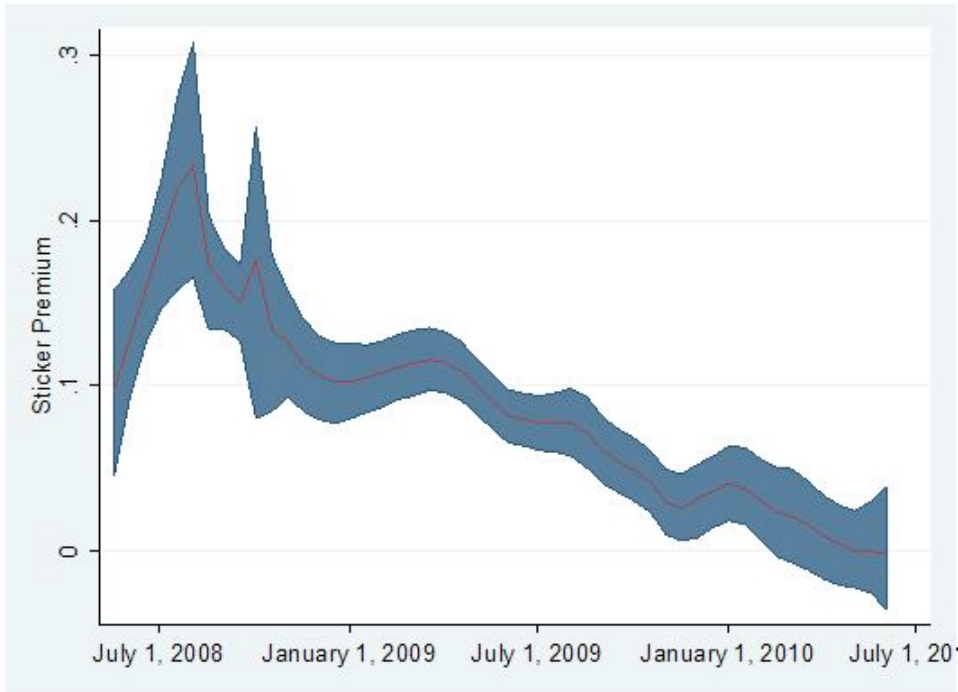
### D.1 Natural Log Specification

The first specification I examine is transforming the price with natural logs. This implies that the sticker and other car attributes enter the price multiplicatively, where  $\text{Price}_{it}$  refers to the price of the automobile,  $x_i$  is a vector of vehicle characteristics and  $F$  is a non-parametric function meeting the assumptions described in Section 4.2

Taking the natural log of both sides, this results in the transformed model:

$$\text{Price}_{it} = e^{x_i' \beta + F(t) - \varepsilon_i} \quad (\text{D.1})$$





**Figure D.1:** Non-Parametric Estimation of Sticker Value Over Time Using Ln(Price)

Using the same technique as in the body of the paper, we generate a graph that traces the price of the sticker over time. This is presented in Figure D.1. Figure D.1 has a similar shape when compared with Figure 4.2, which was made using a regression on simply the price, as opposed to the natural log of price, of a vehicle. The main difference is that the log model appears to have a negative premium in late May/early June.

In addition to the partially linear model, we examine a model with an intercept for sticker price,  $\alpha$ , and a slope variable with coefficient  $\gamma$ :

$$\ln(\text{Price}_{it}) = x'_{it}\beta + F(t) + \varepsilon_i \quad (\text{D.2})$$

We present the results in Table D.1.

Again, many of the results in Table D.1 are similar to the analogous untransformed model results in Table 4.4. The magnitude of the intercept and the slope show the

Variable	Estimate
Sticker Intercept	0.155*** (0.15)
Sticker Slope	-0.0002315*** (0.0000302)
Number of Observations	7,292
R-Squared	0.8070

NOTE: Robust Standard Errors are in parentheses, Asterisks denote significance at the \*10%, \*\*5% and \*\*\*1% level.

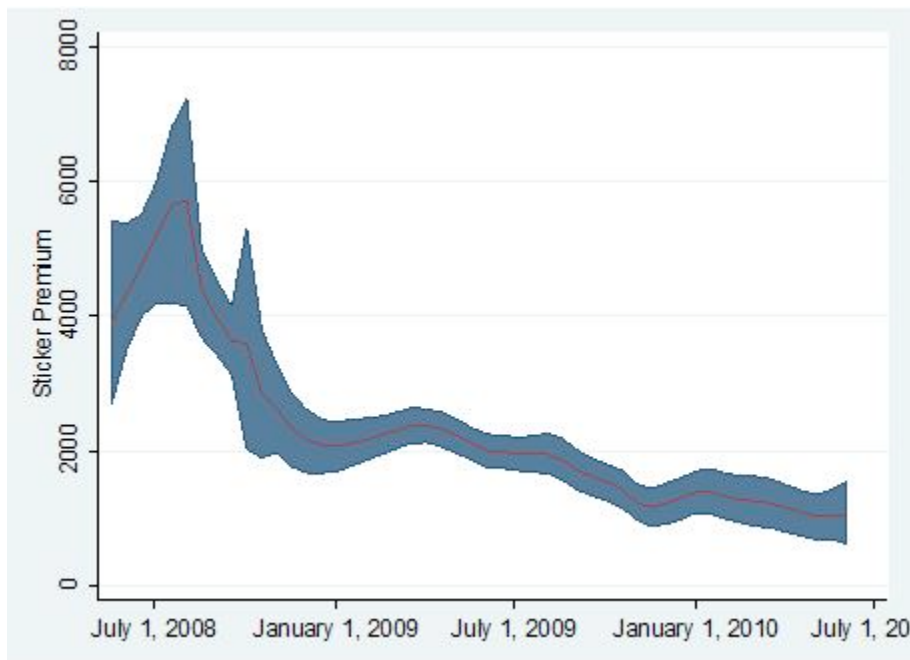
**Table D.1:** Linear Estimate of Sticker Price Over Time Using Ln(Price)

same patterns across the three models in Tables 4.4 and D.1. Transforming the natural log model using the Halvorsen-Palmquist adjustment (Halvorsen and Palmquist, 1980) we find that the average daily value of a sticker is \$3.85. This is eleven cents per day less than the additive model. The additive model is simpler and more intuitive than the multiplicative model. In discussions of the impact of a sticker value, on PriusChat, in Autotrader ads and in the popular press, a sticker was discussed as if it added something to the price, not multiplied the price by something.

## D.2 Regional Interactions with Sticker Price

Each observation from AutoTrader or Ebay came with a dealer address or a zip code in the case of a private seller. Since vehicles are an expensive but mobile expense, it is safe to assume some level of market integration across metropolitan regions, and so we grouped the observations into six regions: Los Angeles, San Diego, San Francisco, Sacramento, Northern California (north of the Bay Area and Sacramento) and the Central Valley. To test how location impacts the price of the sticker, we include an interaction term between sticker and the six regions in California. The dummy variables for each region are all negative, which shifts the non-parametric estimates of sticker value over time. Figure D.2 shows a function that has roughly the same shape as previous non-parametric

regressions but a slightly higher anchor point. The likely reason is that the anchor point is not precisely estimated in the data. The coefficients in Table D.2 are all negative, even if only one is statistically significant.



**Figure D.2:** Sticker Values Over Time Controlling For Regional Sticker Interactions

Modeling the evolution of the sticker over time as a linear function, we again look at whether or not the sticker's value depends on the metropolitan area. We find similar results in Table D.3. Additionally we look at the type of car being sold, the year and mileage of the car, and the types of options packages available with the sticker. We also run a model with both region and vehicle characteristics interacted with the presence of the sticker. These models are presented in the last two columns of Table D.3. In the last two regressions, Stata drops the HOV Intercept, which leaves all the region-HOV dummies as significant. This is not because adding the other car characteristics makes regional interactions suddenly significant but is an artifact of Stata's decision to drop the HOV intercept. None of the regions are statistically different from one another.

Region (Los Angeles Omitted)	Price	Ln(Price)
San Diego	-330.4 (318.5)	-0.0154 (0.0189)
San Francisco	-328.0 (206.5)	-0.0231* (0.0122)
Sacramento	-317.4 (457.7)	-0.04207 (0.0334)
Central Valley	-1034 (752.1)	-0.048 (0.0438)
Northern California	-669 (741.4)	-0.0245 (0.0547)

NOTE: Robust Standard Errors are in parentheses,  
Asterisks denote significant at the  
\*10%, \*\*5% and \*\*\*1% level.

**Table D.2:** Impact of Region on Sticker Price

Vehicle characteristics that did influence the value of the sticker were the model of the car, the mileage on the car, whether or not the car had leather seats and a premium JBL sound system. The sticker added the most value to an Insight, followed by the Prius. One significant difference between the natural log model and the additive model was the impact of mileage on sticker price. In the natural log model a sticker was worth less on a vehicle with high mileage than a vehicle with low mileage, but in the additive model there was no statistical difference. Interestingly a sticker on a vehicle with an upgraded sound system was worth less than a sticker on a vehicle without an upgraded sound system, but bluetooth and mp3 capabilities, as well as a navigation system or being listed as 'loaded' had no effect on the value of a sticker.

### D.2.1 Gas Prices and Vehicle Characteristics Interactions with Sticker Price

Gas prices may influence the price of a sticker so regressions were included to examine the impact of gas prices on both the value of a hybrid vehicle as well as the value of the Clean Air sticker. Statewide weekly retail prices for a gallon of California regular

Variable	Price	Ln(Price)	Price	Ln(Price)	Price	Ln(Price)
Intercept	4,414*** (697)	0.233*** (0.419)	6,293*** (1,125)	0.385*** (0.105)	dropped	dropped
HOV Slope	-28*** (6.56)	-0.0013*** (0.00037)	-20.1*** (6.12)	-0.0011*** (0.00344)	-19.7*** (6.64)	-0.0011*** (0.00038)
Los Angeles Region	-144 (556)	-0.00146 (0.404)			6,954*** (1,270)	0.427*** (0.113)
San Diego Region	-1177 (837)	-0.0612 (0.0543)			5,856*** (1,280)	0.362*** (0.121)
S.F. Region	-566.6 (571)	-0.0227 (0.0411)			6,450*** (1,211)	0.399*** (0.111)
Sacramento Region	dropped				7,245*** (1,289)	0.433*** (0.112)
Central Valley	-375.4 (768)	-0.00379 (0.0471)			7,039*** (1,373)	0.435*** (0.119)
Prius (Insight Omitted)			-1,640* (840.2)	-0.160* (0.0961)	-2,120** (921)	-0.188* (0.101)
Civic			-2,572*** (841.0)	-0.202** (0.0950)	-3,070 (925)	-0.231** (0.100)
Mileage			-0.014*** (0.00477)	-2.77e-07 (4.29e-07)	-0.014*** (0.00473)	-2.99e-07 (4.29e-7)
Leather			788* (455)	0.0418* (0.0240)	778* (463)	0.0407* (0.0243)
Loaded			503 (399)	0.0286 (0.0233)	491 (399)	0.0270 (0.0230)
JBL			-867** (429)	-0.050** (0.240)	-859** (433.6)	-0.0492* (0.024)
Bluetooth			-451 (520)	-0.0058 (0.0290)	-430 (522)	-0.00451 (0.0290)
MP3			-196 (485)	-0.00079 (0.268)	-172 (479)	0.00037 (0.0264)
Navigation			-236 (360)	-0.00768 (0.0208)	-246 (359)	-0.0078 (0.0204)
R-Squared	0.7197	0.7689	0.7221	0.7700	0.7228	0.7706

NOTE: Robust Standard Errors are in parentheses, Asterisks denote significant at the \*10%, \*\*5% and \*\*\*1% level.

**Table D.3:** Interactions Between Sticker Value and Vehicle Characteristics

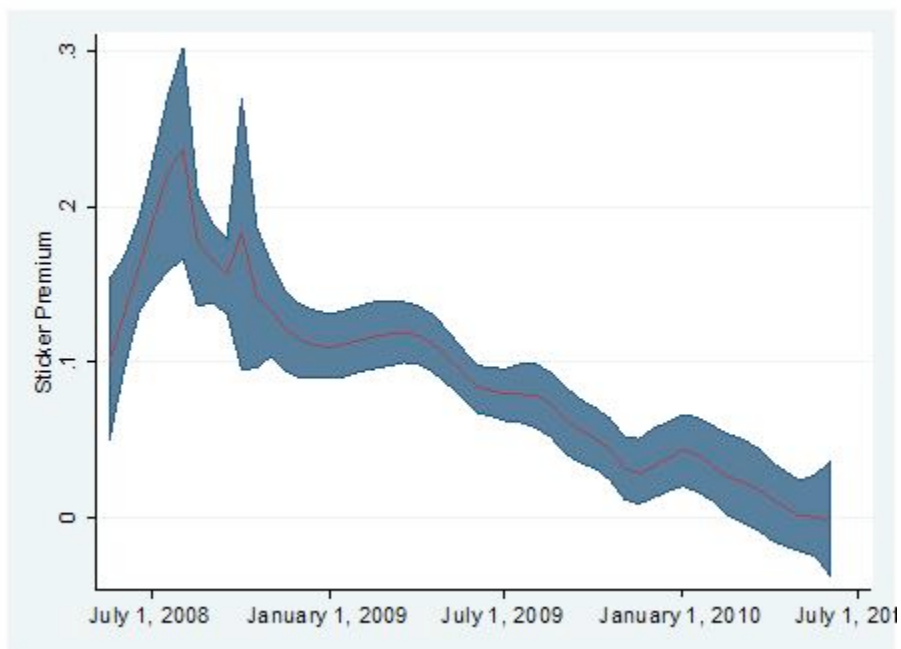
Variable	Price	Ln(Price)
Gas Price (cents)	25.02*** (3.03)	0.000571*** (0.000180)
Gas Price (cents) * Sticker	7.12*** (2.61)	0.000086 (0.00013)
HOV Intercept	1,156 (1,167)	0.1818*** (0.0677)
HOV Slope	-15.6** (6.30)	-0.00119*** (0.000391)
R-Squared	0.7201	0.7685

NOTE: Robust Standard Errors are in parentheses,  
Asterisks denote significant at the \*10%, \*\*5% and \*\*\*1% level.

**Table D.4:** The Impact of the Statewide Price of California Regular Gasoline on Vehicle and Sticker Value

gasoline were included both directly on the price of a car, as well as interacted with the Clean Air Sticker. These results are presented in Table D.4.

This regression shows the influence of gas prices (in cents) on the value of the California Clean Air sticker and the price of a hybrid car. The price of gas is from the Energy Information Agency. There exists considerable variation in the data, between \$1.73 and \$4.59. The price of gas has a significant impact on both the price of a vehicle and a smaller impact on the value of a sticker as can be seen in Table D.4. For every one cent increase in the price of gasoline, a hybrid vehicle is worth approximately \$0.25 more, while a sticker increases in price by \$0.07.



**Figure D.3:** Sticker Values Over Time Controlling For Regional Sticker Interactions Using  $\ln(\text{Price})$

## Appendix E

# Calculation of Air Pollution

## Benefits

In this section I analyze the two criteria pollutants that California is not yet in attainment for and which light duty passenger vehicles are a major contributor. These two pollutants are NO<sub>x</sub> and volatile organic compounds (VOC). I also examine greenhouse gases as measured by carbon dioxide equivalence, CO<sub>2e</sub>.

Hybrid cars produce 90 percent less NO<sub>x</sub> than the average car. The standard for 2000 model year passenger automobiles was 0.4 grams of NO<sub>x</sub> per mile<sup>1</sup>. If every driver who purchased a hybrid vehicle with a Clean Air sticker would not have purchased a sticker otherwise and those drivers would have driven the same amount<sup>2</sup> with their conventional car, 3,204 tons of NO<sub>x</sub> were reduced from 2006 to 2011. If we assume the vehicles lasted 10 years on average, this resulted in 6,408 tons of NO<sub>x</sub>.

$$\begin{aligned} \text{NO}_x &= 0.90 \times 0.4 \text{ grams/mile} \times 19,000 \text{ miles/year} \times 10 \text{ years} \times 85,000 \text{ cars} \\ &= 6,408 \text{ tons of NO}_x \end{aligned}$$

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<sup>1</sup>0.4 grams per mile of NO<sub>x</sub> was the standard for 2000 model year cars. The median model year for a car with a Clean Air sticker is 2005 which falls under a stricter standard.

<sup>2</sup>The average passenger car is driven approximately 12,000 miles per year, hybrid cars with a Clean Air Sticker in the sample are driven 19,000 miles per year. In general, increases in energy efficiency lead to more intensive uses, in this case a more efficient car will likely be driven more due to its lower per mile cost. This is known as the 'rebound effect'.



Using the 2000 standard of 0.090 grams/mile of VOCs, and assuming that hybrids emit 0.010 grams/mile of VOCs, then using the same assumptions below we have that this program prevented 1,420 tons of VOC from being emitted.

$$\begin{aligned} \text{VOC} &= (0.090 - 0.010) \text{ grams/mile} \times 19,000 \text{ miles/year} \times 10 \text{ years} \\ &\quad \times 85,000 \text{ cars} \\ &= 1,420 \text{ tons of VOCs} \end{aligned}$$

If each one of the cars with 45 miles per gallon (0.022 gallons/mile) had been replaced with a conventional car meeting CAFE requirements of 27.5 miles per gallon (0.036 gallons/mile) and each car lasted 10 years, then 604,000 tons of CO<sub>2e</sub> was reduced by this program.

$$\begin{aligned} \text{CO}_{2e} &= 8.788 \text{ kilograms CO}_{2e}/\text{gallon}^3 \times 0.014 \Delta \text{ in gallons gasoline/mile} \times \\ &\quad 19,000 \text{ miles/year} \times 10 \text{ years} \times 85,000 \text{ cars} \times 100/95^4 \\ &= 1.9 \text{ million tons of CO}_{2e} \end{aligned}$$

Another assumption is that every person buying a hybrid that obtained a Clean Air sticker did so because of the sticker. This assumption is clearly overly conservative as many people are buying hybrid cars even without the incentive and the research indicates the Clean Air sticker program did not increase the demand for hybrid cars. Even under these conservative assumptions, at \$50/ton of CO<sub>2e</sub>, \$15,00/ton of NO<sub>x</sub>, and \$4,100/ton of VOC<sup>5</sup>, the state could have reduced the same amount of air pollution for \$197 million.

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<sup>5</sup>Valuation of NO<sub>x</sub> and VOC from Small and Kazimi (Small and Kazimi, 1995)'s review of the costs of air pollution in California from motor vehicles, adjusted for 2009 dollars.