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Abstract

In auctions where bidders are uncertain of their value and are fully liable for their bids, there exists the potential for losses if bids exceed realized values. Theoretically, bids will be higher if bidders are able to mitigate this downside loss through some form of limited liability. To determine the impact of differing forms of limited liability, this paper theoretically and experimentally examines a second price auction with uncertain private values in three environments: market-based limited liability, statutory limited liability, and full liability. Market-based limited liability is induced through inter-bidder resale following the auction. Statutory limited liability is created through a default penalty option in the event that a bidder would make a loss. Bids are theoretically shown to be higher under resale and the penalty default environments than under full liability. The experimental results confirm more aggressive bidding for resale and the low penalty default treatments, but not by as much as theory predicts. Notably, under the high default penalty bidders are not bidding significantly more than under full liability, despite the theoretical prediction that they should.

JEL Codes: D44 C90

Key Words: Auctions, Limited Liability, Resale, Experimental Economics

1 Introduction

Countless economic transactions, from housing sales and Ebay transactions to government procurement auctions, involve uncertainty and risk. Economic intuition would suggest that when individuals are facing a situation where risk is involved, they would be more willing to engage in risky choices if they are not fully liable for their choice in the event of a bad outcome. We may expect to

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see differences in the amount of risk an individual is willing accept, theoretically and behaviorally, depending on the level and form of liability rules.

All economic transactions where uncertainty and risk are involved must have some liability institution in place.[1] As an example, the recent surge in press coverage regarding the housing bubble and subsequent rise of foreclosures has highlighted the various liability settings available to homeowners. In addition to the popular press, important questions have developed in the academic literature, such as how are homeowners responding to the default options and various liability settings? This question was recently addressed by Ghent & Kudlyak (2009) who found an increase in the probability of default for states where lenders have recourse rights versus non-recourse states. While Ghent & Kudlyak provide important insights into the choice of default, an important question remains - if the loss can be mitigated in the future, does this factor into their initial bids? This paper will address this question in an auction setting with uncertain private values. Three primary liability types will be theoretically analyzed and implemented in a laboratory experiment to ascertain how bidding behavior differs across these liability rules.

In real markets there are a variety of forms of liability. The standard is, of course, full liability in which an agent is fully responsible for the entirety of any loss that he incurs. There are also a variety of bankruptcy rules, and other governmental policies, which can limit how much of a loss for which an individual or firm is liable. We will refer to this class of limited liability rules as statutory limited liability as these rules will typically be specified in the rules of any particular market or the relevant part of bankruptcy code. For simplicity in this context, we will assume this takes the form of an agent having to pay some fraction of the total price of the item promised in payment and having the agent forfeit the item. Depending on that fraction and the amount of the loss, though, the agent may not be best off by choosing to default this way and may prefer to keep the item suffering a small loss.

There is another form of limited liability that is perhaps not always thought of as a way of achieving limited liability, and that is the opportunity to resell an item. If an agent buys an item and realizes he overpaid relative to his value for the item, he may be able to find someone else to purchase the item from him and possibility limit the degree of loss or even potentially make a profit on the transaction. While resale opportunities may not always be thought of as a form of limited liability, it is an important form of it which should be examined. The main purpose of this paper is to examine how individuals, when confronted with uncertainty and risk, react to market-based limited liability induced through a secondary resale market and statutory limited liability compared to a full liability baseline. It is highly likely that while limited liability may lead to more aggressive behavior, statutory liability may lead to more aggressive behavior than market-based resale liability because of the subtle nature of resale. All limited liability forms are contrasted against full liability to determine if bidding

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[1]Even the lack of a statutory liability rule, still constitutes a liability institution - likely full liability.

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behavior does indeed become more aggressive.

To accomplish this, a second price auction is utilized where bidding takes place for the opportunity to play a lottery. This analysis examines how outcomes differ under varying liability rules. The baseline of analysis begins with the full liability scenario, where bidders are responsible for all losses. This is then extended to statutory and market-based limited liability scenarios. Statutory limited liability is created through default options, while market-based limited liability is shown to emerge from a post-auction resale opportunity.

The second price auction used in this paper is a modified version of the standard second price auction. This particular variant of the second price auction has a bid clock for withdrawal. The bid clock is typically used in the ascending auction, where the drop out bids are common knowledge. To maintain the second price format, the bids are not made public, but the clock is still used for withdrawal.[2] The dominant strategy in a standard second price or ascending auction is that all bidders bid their value. The inclusion of the clock is based on previous experimental results where over-bidding has been observed in second price auctions without the clock versus equilibrium bidding in an ascending auction. Adding the clock appears to reduce bidding mistakes, so differences in the observed bids for the current design can be attributed to changes in the bidding environment.[3]

An auction for a lottery is analogous to the situation where a bidder is bidding on an item of uncertain quality. After the auction this uncertainty is resolved and if the high state occurs, the value is high. If the low state occurs, the value is low. For example, in the recycling industry surplus manufacturing materials are often sold via auction. After the auction, it is sometimes revealed that the material purchased is no longer useful to the auction winner. This is a well-suited example for the particular theoretical environment created in this paper. The material either has value for the winner, or it does not. The information is revealed after the auction has taken place. The use of a lottery mimics this real world example of good and bad states.

How a bidder would choose to bid under uncertainty depends critically on the liability rules. In the above recycling example, if a bidder knew that default was an option, and not too costly, they would be more likely to bid aggressively. How the auctioneer sets up the default rules would determine how the bidder would bid. Alternatively, if the auctioneer set full liability as the statutory rule, limited liability is still possible. After the auction, the uncertainty is resolved. The winning bidder could resell the item won to another bidder if it were revealed

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[2] A second price auction with a clock begins at a price of zero, which gradually ticks up, based on the clock increment. A bidder chooses to drop out of the auction when the clock has reached the maximum amount that they are willing to pay for the lottery chance. The particular variant of the auction used for this paper is sealed, in that bidders are unaware of the other bids placed and the auction ends when all bids have been placed. The winning bidder pays a price equal to the second highest bid. The mechanism examined in this paper only has two bidders, so the information revelation is equivalent between the open ascending auction and this sealed version.

[3] See Kagel et al. (1987) for previous experimental results on ascending auctions and Cooper & Fang (2008) for results on second price auctions.

that they had a higher value for the item than the winning bidder. In this sense, the secondary market creates a situation of limited liability without the rule having been directly established.

When the auctioneer sets up the default rules, they must consider carefully how the bidder would respond. It is likely that if a bidder is facing limited liability instead of full liability, they would bid more aggressively. While intuitively it appears that more aggressive bidding would lead to higher revenue, this is not necessarily the case under limited liability. If limited liability takes the form of a default option (statutory), then this could potentially lower revenue as higher bids lead to a higher probability of default. Limited liability induced through a secondary resale market is, on the other hand, a very attractive outcome for the seller. Assume that the auction has a full liability rule for the winner, but also the possibility of resale. If bidders understand the resale market, and transfer the limited liability aspects of this market into higher bids, the seller will benefit. The aggressive bids raise revenue, and since default is not an option, the seller reaps all additional revenue.

A number of theoretical papers have addressed the issues of default and limited liability. Zheng (2001) focused on budget constraints including the possibility of default, and theoretically showed that low-budget bidders, given bankruptcy options, will bid more aggressively and declare bankruptcy with higher probability. Waehrer (1995) has a set-up similar to the construction analyzed in this paper examining exogenous statutory liability rules. In Waehrer, the default penalty takes the form of a deposit that is lost in the event of bankrupt bidders. Waehrer finds that bids become more aggressive as the deposit decreases. Board (2007) adds to the literature by examining default across multiple auction types and also adds insight into the perspective of the seller in the presence of defaulting bidders and various recovery scenarios.

This paper focuses on the bidding behavior, and analyzes revenue, but does not vary the seller's recovery options. The item is fully recovered in the event of default. Board examines the possibility of resale as one of the recovery options. This paper also looks at resale, but assumes that it is not the seller reselling, but the winning bidder who has just participated in a full liability auction. The secondary resale market creates market-based limited liability that does not statutorily exist in the auction.

Other experimental papers have addressed the possibility of default and limited liability, in a different sense. Limited liability created by budget constraints was examined by Hasen & Lott (1991) in a comment regarding the design of Kagel & Levin (1986). A typical constraint faced in economic experiments, and the issue raised by Hasen & Lott, is that subjects cannot make losses. Therefore, subjects with low cash balances have limited liability because the downside loss is capped at zero. Kagel & Levin (1991) respond to this comment with how they controlled for limited liability by providing cash endowments to the subjects that covered the maximum possible loss. They also "note the importance of not overlooking the potential effects of limited liability on bidding." Budget constraints can also be viewed as a form of market-based limited liability, but this type of liability is not examined in this paper and is different from

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market-based limited liability from resale that mitigates losses. Roelofs (2002) theoretically and experimentally examines a common value procurement auction with default. He finds evidence that bidders do bid more aggressively when default is allowed, as predicted, but this does not necessarily help the auctioneer's revenue, citing the winner's curse as a possible explanation. The approach used in this paper differs in the form of auction and extends the analysis from full default/full liability to limited liability/full liability.

To my knowledge, a unique contribution of this paper is the experimental test of resale as a form of market-based limited liability from the perspective of the bidder. Pagnozzi (2007) theoretically examines resale with strong and weak bidders, where the weak bidder enjoys limited liability due to a low initial wealth. The possibility of inter-bidder resale between the strong and weak bidder leads to more aggressive bidding by the limited liability weak bidder. The strong bidder, who does not enjoy limited liability, waits for resale to purchase the item. This paper differs from Pagnozzi in that all bidders have the same value distributions and wealth distribution. The resale possibility creates limited liability, not differing wealth constraints. Haile (2003) theoretically deals with uncertain values and resale opportunities, but the focus of Haile is not on the market-based limited liability aspects of resale.

The remaining sections of this paper include theoretical and behavioral predictions in section 2. The experimental design is discussed in section 3 with an analysis of the results in section 4. Section 5 concludes, including a discussion of the steps that can be taken to move forward.

2   Theory and Behavioral Predictions

2.1   Theoretical Predictions

In this section, I solve for the symmetric equilibrium bid strategies under limited liability induced through resale, statutory measures, and full liability. The model assumes a second price auction for a lottery with two risk neutral bidders. Each lottery consists of an upper and lower bound. The probability that the upper or lower bound from the lottery becomes the realized value is assumed to be .5. The lower bound, x, is drawn from the uniform distribution on the support $[0, 1]$. The upper bound of the lottery is equal to $x + y$, where $0 < y < 1$. The uncertainty is resolved immediately following the auction, when the lottery is played. The resulting realized value is denoted as v.

In the full liability case, neither the default option nor the resale option is available to the bidders. At the conclusion of the auction, all bidders realize their particular value that results from the 50/50 lottery. The winning bidder earns the difference between their realized value and the price that resulted from the lottery, with full liability in the event of a loss.

Statutory limited liability takes the form of a default option where the winning bidder pays a penalty cost in the event the bidder chooses to default. The penalty cost is equal to a percentage, α, of the price that results from the auc-

tion. All bidders are aware of the structure of default prior to the start of the auction.

Market-based limited liability is analyzed through a secondary resale market that occurs at the end of the auction. The resale transactions are assumed to only take place between the bidders that originally participated in the auction.[4] At the conclusion of the auction, all realized values become common knowledge for all bidders. The winner of the auction resells at a price equal to the highest value in the group of bidders.

While it is reasonable to assume that the resale price agreed upon in practice might exist below the value of the final buyer, this analysis is more focused on the limited liability aspects of resale, without the additional complication of alternative specifications of bargaining power over the final allocation. In this set-up, the reseller is assumed to have all of the bargaining power in resale. Noting this, under an alternative bargaining scheme the winning bidder would never resell for a price less than their realized value, therefore the final resale price must exist between the realized value of the winner and the realized value of the losing bidder. The limited liability aspects of resale would still hold with an alternative mix of bargaining power.

Construction of the bid functions involves assuming that the bid function is monotonically increasing and differentiable, then verifying that it is monotonic and differentiable. The remainder of this theoretical portion will solve for the symmetric equilibrium bid strategies under full liability first and then the statutory and market-based resale forms of limited liability.[5]

2.1.1   Full Liability

The lower bound of the lottery for bidder i is represented as x_i. Assume throughout that bid functions are symmetric, that is $b(x_i) = b_j(x_j)$, for all i and j. Assume all bidders, except i, bid using $b^*(x_j)$. The construction is designed to see how bidder i responds to $b^*(x_j)$, by allowing bidder i to choose to bid according to some other lower bound, r, which isn't necessarily his. Bidder i will win if $b(r) > b^*(x_j)$, which occurs with probability $F(r)$. The lower bound of the lottery, x, occurs with probability $\frac{1}{2}$. The upper bound of the lottery, equal to $x + y$, where $y < 1$, correspondingly also occurs with probability $\frac{1}{2}$. The parameter value, y, is known and common to all bidders. The realized value for bidder i is represented as v_i.

The second price auction problem is therefore defined as follows under the full liability case for bidder i:

$$\max_{r} U_i(v_i, r) = \int_0^r \left[\tfrac{1}{2}(v_i - b_j(t)) + \tfrac{1}{2}(v_i + y - b_j(t)) \right] dF(t) \qquad (1)$$

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[4] Inter-bidder resale is a limiting case of market-based limited liability where bidders can resell to others in the event of a possible loss. This analysis is restricted to inter-bidder resale to facilitate the experimental design. However, while not in the scope of this paper, limited liability would generally hold if the resale market was extended to potential buyers who did not participate in the auction.

[5] All asserted bid functions are explained in the appendix.

With the equilibrium condition

$$\frac{\partial(U_i(v_i,r))}{\partial r}\Big|_{r=v_i} = 0 \qquad (2)$$

The full liability (FL) bidding solution can be shown to be:

$$b_{FL}(v_i) = v_i + \frac{1}{2}y \qquad (3)$$

So, the equilibrium is to bid the expected value of the lottery.

2.1.2 Market-based Limited Liability through Resale

The assumptions made in the full liability case remain in the resale case. The important change in this model is that inter-bidder resale is now allowed. At the conclusion of the auction, the winning bidder has the option of reselling to the losing bidder, if the realized value of the losing bidder is higher than the realized value of the winning bidder.[6] The losing bidder is sold to at a price equal to his realized value. All realized values are common knowledge at the conclusion of the auction. The equilibrium bid function is derived below.

As in the full liability case, the second price auction problem is defined as follows under the market based resale liability for bidder i:

$$
\begin{aligned}
\max_r U_i(v_i,r) \quad = \quad & \int_0^{r-y} \left[\frac{1}{2}(v_i - b_j(t)) + \frac{1}{2}(v_i + y - b_j(t))\right] dF(t) + \\
& \int_{r-y}^r [\frac{1}{2}\{\frac{1}{2}(v_i - b_j(t)) + \frac{1}{2}(t + y - b_j(t))\} + \\
& \frac{1}{2}\{v_i + y - b_j(t)\}] dF(t) + \int_r^1 0 * d[1 - F(t)]
\end{aligned}
$$

$$(4)$$

With the equilibrium condition

$$\frac{\partial(U_i(v_i,r))}{\partial r}\Big|_{r=v_i} = 0 \qquad (5)$$

The resale liability (RL) bidding solution can be shown to be:

$$b^*_{RL}(v) = v + \frac{3}{4}y \qquad (6)$$

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[6]By construction, the equilibrium bid function is assumed to be increasing in the lower bound of the lottery. Additionally, the entire resale surplus is earned by the winning bidder. Therefore, bidders do not engage in strategic demand reduction. The resale option acts strictly as a limited liability option in the event that winning bidder's realized value is equal to their lower bound from the lottery, $v_i = x_i$, and is lower than the realized value of the losing bidder - which occurs when this bidder's realized value is $v_j = x_j + y$.

The first term in the equation represents what the bidder will earn when he wins the auction and keeps the item. The second term represents what the bidder will earn if he wins the auction and keeps the item, or wins the auction and resells. The last term represents the earnings if bidder i loses the auction, and purchases the item in resale. At the conclusion of the auction, all realized values become common knowledge across bidders, and the losing bidder is resold to at a price equal to their realized value. Therefore this last term represents zero potential earnings from buying in the resale market.

The exposition of bid functions for the full liability and resale market based liability cases leads to the first theoretical claim.

Claim 1 *Under a uniform distribution, market-based limited liability theoretically leads to more aggressive bidding, for all potential values, than what would result if bidders were not allowed to resell under full liability.*

The following subsection examines a limited liability scenario that is established through default rules in the auction. The above claim does not always hold for statutory liability.

2.1.3 Statutory Limited Liability

All general assumptions, not specific to resale, again hold for the statutory liability case. The key change from the full liability scenario is that now bidders are allowed to default. Additionally, it should be noted that resale possibilities are not included in this model. The specific default rule used in this model is if the winning bidder opts to default, they must pay a penalty percentage, $0 \leq \alpha \leq 1$, of the price that results from the auction.

A winning bidder would only choose to default, and not receive the value of the lottery, if the loss from default is less than the loss suffered under the auction. Therefore, if bidder i chooses to default, he would pay a default cost equal to $-\alpha b_j$. The range of α determines the extreme scenarios, one of which is the full liability case solved above where $\alpha = 1$. Likewise, if $\alpha = 0$, the bidder is not responsible for any losses and corresponds to full default (no liability).

A bidder will choose to default if the losses associated with default, $-\alpha b_j$, are less than the payoff from receiving the realized value from the lottery and paying the price that results from the auction, otherwise the bidder will accept the realized value. Therefore, the payoff resulting to the winning bidder in this second price auction set-up is defined as follows, under statutory default penalty rule for bidder i:

$$U_i(v_i, b_j, \alpha) = \tfrac{1}{2} \max \left\{ v_i - b_j, \, -\alpha b_j \right\} + \tfrac{1}{2} \max \left\{ v_i + y - b_j, \, -\alpha b_j \right\} \qquad (7)$$

A bidder choosing to default depends critically on the statutory choice of α. As the default penalty increases, the cost associated with default approaches full liability and therefore bidders are less likely to choose default as an option - behaving as if the default option does not exist. This can be seen formally, in the limit as α approaches 1, the problem reduces to the full liability problem as the

bidder would never choose to default for values greater than 0. For values equal to zero, the bidder is indifferent between default and no default, therefore it is assumed they will choose to not default.[7] As the default penalty decreases, the cost associated with default goes to zero, and bidders will default with higher frequency.

Choices of α that are above the full default case but less than the full liability case result in a cutoff value, v_c. This cutoff value kinks the equilibrium bid function. The intuition behind this cutoff value is that above some value, the cutoff value, the default cost is too high and they would never default. They then treat the auction with default as if it were an auction without the default possibility.

$$\frac{v_i}{1-\alpha} > b_j \qquad (8)$$

A bidder does not default as long as the payoff from the auction is greater than the penalty associated with default. This condition is seen in equation 8.

For the second price auction with stated rules, the symmetric statutory liability (SL) bidding solution can be shown to be:

$$b_{SL}(v_i) = \begin{cases} \frac{1}{1+\alpha}(v+y) & if \quad v \leq v_c \\ v + \frac{1}{2}y & if \quad v > v_c \end{cases} \qquad (9)$$

The bid function is equivalent to the full liability bid function for values above the cutoff value, v_c. For values below the cutoff value, the bidder is choosing to bid a percentage of the highest possible realized value. Notice also that if the default penalty α is low enough, the bidder would always choose to bid taking advantage of the default rule. Recall, the highest realized value that a bidder can achieve is $v + y$. Therefore, if a bidder was able to fully default, $\alpha = 0$, they would be willing to bid up to their highest possible value.

This analysis leads to the second theoretical claim.

Claim 2 *Under a uniform distribution, statutory limited liability theoretically leads to more aggressive bidding, for the subset of values that lie below the cutoff value, v_c, than what would result if bidders were not allowed to resell and faced full liability For values above the cutoff value, the bids under statutory limited liability and full liability without resale are equivalent.*

2.1.4 Revenue

Market-based limited liability leads to more aggressive bidding for all values over full liability. This aggressive bidding leads to higher revenue for the auctioneer, provided the opportunity for resale in the secondary market exists. This result holds assuming that the possibility of default is not allowed.

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[7] See appendix for details when $\alpha = 1$

Claim 3 *Under a uniform distribution, market-based limited liability theoretically leads to more aggressive bidding which results in higher revenue for the auctioneer than what would result if bidders were not allowed to resell under full liability.*

Without considering the probability of default, statutory limited liability leads to higher gross revenue through more aggressive bidding. The possibility of default under statutory limited liability does not have as clearly cut revenue predictions as the resale and full liability cases. Under statutory liability, the expectation of revenue depends on the highest and second highest lower bound draws, the spread between upper and lower bounds, and the default penalty. Given the sensitivity of revenue to the parameter set-up, the theoretical claims for revenue in the statutory default treatments are directly calculated for comparison in the results section.

2.2   Behavioral Predictions

The two key treatments involve varying the types of limited liability that the auction participants face. Behaviorally, the framing of the liability rules, market-based versus statutory, may lead to a differing mix of behaviors from the specific theoretical predictions made.

2.2.1   Market-based resale liability

Under market-based liability, the bidders are facing a secondary market. Prior evidence from ascending auctions with inter-bidder resale markets, reported in Saral (2009), has shown that when bidders face a secondary resale market where the reseller earns the majority of the surplus, bidders engage in speculative behavior by bidding above their value.

Speculation is not modeled specifically in the model above, as bidders never bid above the highest possible value in the lottery. However, it may be the case that bidders, in particular bidders with values that are on the lower end of the distribution, may choose to speculate by bidding above their highest possible value with the hopes of reselling.

Claim 4 *The presence of an inter-bidder resale market may lead to speculation where bidders choose to bid above the high bound of the lottery to resell to the losing bidder. This behavior is more likely when a bidder's lower bound is on the lower end of the distribution.*

This behavioral prediction, while valid, is likely muted in this environment by two factors. First, the bidders are facing a more complex environment with uncertainty. Second, bidding groups only involve two bidders. Speculation in this environment carries a higher risk of loss than a similar environment with more bidders. Behaviorally, we would expect bidders with a higher risk tolerance to engage in more frequently in speculation. If bidders engage in speculative behavior, we should expect revenue to increase with the increase in bids, over what would be expected without speculation.

2.2.2 Statutory liability

All theoretical models considered above assumed that bidders are risk neutral. It is highly likely that the risk attitudes of the subjects will vary. Katok and Salmon (2009), in a similar set-up, find that bidding behavior under uncertainty varies widely from the risk neutral prediction. Many bidders bid below the risk neutral prediction, indicating some form of risk or loss aversion. It is expected that these behavioral differences will impact all treatments, including the baseline full liability.

Claim 5 *Heterogeneity in risk attitudes is likely to be observed through changes in bidding behavior from the risk neutral prediction. Across bidders, those individuals who exhibit higher levels of risk tolerance should bid higher than individuals with lower risk tolerance, in all treatments.*

For the individual bidder, the prediction that statutory limited liability should increase bids is valid, but depends on the level of the default penalty cost, α. Bidders may respond differently to varying levels of the default, depending on their tolerance for loss and/or risk aversion. Behaviorally, we might expect that at higher levels of the default cost, some bidders may respond by bidding as if the default option does not exist, despite theoretical predictions that they should bid higher.

3 Design of Experiments

The experiment was designed to analyze bidding behavior in a second price clock auction with uncertain values, under three limited liability scenarios. Two of the three limited liability environments are statutory in nature, where bidders have a default option available that imposes a default penalty if the penalty is less than the full liability payment. The second limited liability environment is market-based limited liability induced through a resale opportunity.

A primary goal of this design was to observe how bidding behavior changes between the baseline environment with full liability to one with limited liability. A secondary goal was to compare behavioral differences in statutory and market-based liability. To accomplish these goals, three symmetric treatments of a second price clock auction were created. The baseline treatment was the full liability environment and using a within-subjects design, subjects also participated in one of the three limited liability treatments. A between-subjects design is used for comparisons across the limited liability treatments. The specific procedures are as follows:

Undergraduate students were recruited and brought into the laboratory at Florida State University where they participated in a series of second price clock auctions. A total of 96 subjects participated. Sixteen subjects participated in each of the 6 sessions run. In each session, the 16 subjects were randomly divided into two groups of 8.

Treatment	Sessions	# Subjects
Full Liability / Resale	2	32
Full Liability / 5% Default Liability	2	32
Full Liability / 25% Default Liability	2	32

Table 1: Experimental Sessions

The experiment was programmed using Z-tree software, Fischbacher (2007). Subjects were given instructions and run through the Holt & Laury (HL) (2002) risk tolerance procedure. One of the choices from the HL procedure was randomly chosen for payment. The second phase of the experiment consisted of 30 paid auction rounds. The first ten auction rounds had the subjects participate in the full liability baseline followed by one of the liability treatments, split equally between the 10 periods.

Prior to these rounds, subjects were given auction instructions for the full liability treatment only. These instructions included an example of bidding behavior, and the opportunity to participate in one unpaid practice period against a computerized bidder (robot), prior to the start of the paid periods. After the instructions and practice period, the subjects entered into the 5 paid rounds of the baseline treatment. Following the first five baseline periods, subjects were given new instructions for the particular limited liability treatment they were participating in.

Following the form of the baseline instruction, the subjects were given new instructions regarding the change in liability rules. These instructions again included an example of bidding behavior and an unpaid practice round against a robot. Upon conclusion of this second set of instructions, the subjects then participated in 5 paid auction rounds of the liability treatment.

The last twenty rounds of auctions were separated into 10 baseline rounds followed by 10 treatment rounds. In each auction round, the participants were rematched with a new partner within their initial group of 8. The subjects did not know they were randomly grouped into a group of 8, they were only informed that they would be randomly matched with a different subject in each round. The subjects were always notified on screen about the rule changes, and through verbal announcement. Tables 1 and 2 outline the experimental session and round structure.

In each period, subjects randomly drew their lower bound, x, for the lottery from a uniform distribution on the range [0,50]. The upper bound of the lottery was always equal to $x + 20$. The maximum bid allowed was 70 units, which is the maximum realized value the lottery could take. They participated in the auction through a computer interface, where they were able to see a bid clock gradually increasing from 0 in increments of 1.[8] The subjects chose to "drop out" when the bid clock reached a price they were no longer willing to pay. This auction was sealed, meaning the subjects would see the clock but they did not

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[8] Display screenshots can be found in the Appendix.

Auction Round	Treatment
Rounds 1-5	Full Liability Baseline
Rounds 6-10	Limited Liability Treatment
Rounds 11-20	Full Liability Baseline
Rounds 21-30	Limited Liability Treatment

Table 2: Treatments by Auction Rounds

see when the opposing bidder dropped out.

The auction ended when all bids had been placed, or when the bid clock hit 70. The winner of the auction was the subject with the highest bid and any ties were broken randomly by the computer program. At the conclusion of the auction, the outcome of the lottery was determined and the liability rule in effect would automatically be implemented in the event of a loss.

The theoretical bid functions are constructed assuming that bidders will choose the default option if it is the loss minimizing choice. This experimental design is structured to test the bidding predictions, so automatic default was chosen in line with the theory. The resale market was also automatically implemented. This was done to avoid bargaining between the winning and losing bidder. The theory assumes that the winner of the auction resells to the losing bidder at a price equal to their realized value, and automatic resale enforces this rule to clearly identify from the design if bidders are able to implement resale as limited liability.

The baseline and three treatments are summarily defined as follows:

Full Liability (Baseline) Treatment: The winner is fully liable for all losses in the event that earnings are negative.

Resale Treatment: In the event that the losing bidder has a higher realized value from the lottery than the winner, the winner automatically resells to the losing bidder at a price equal to the value of the winning bidder.

5% Liability Treatment: In the event that the winning bidder would make a loss, the bidder automatically defaults (if the loss is less under default) and pays a default cost equal to 5% of the losing bid.

25% Liability Treatment: In the event that the winning bidder would make a loss, the bidder automatically defaults (if the loss is less under default) and pays a default cost equal to 25% of the losing bid.

In each session, the subjects' earnings were denominated in experimental currency units (ECUs). These were exchanged into dollars at a rate of $0.04 per ECU. Subjects were given 150 ECUs as an endowment that losses and profits were added to as the experiment progressed. The earnings from the Holt Laury procedure in stage 1 were not included in this endowment. The endowment was

given to avoid biased bidding behavior for people close to bankruptcy. Subjects only participated in one session. Average earnings of the subjects were $28.16, including the show-up fee of $10 and Holt Laury earnings, with zero bankruptcies.

4   Results

Initially, I will discuss the impact of the various limited liability treatments on bidding behavior, using the full liability case as the baseline treatment. Revenue and efficiency results will be examined in the final sections. The first 5 periods of each treatment served primarily as a learning phase and were included so that the subjects would familiarize themselves with the auction format and the particular liability rule in place. The analysis that follows, unless explicitly stated, will use the data from the last 10 rounds of auctions that remained after the training phases.

4.1   Bidding behavior

Figure 1 charts out the observed bids against the lower bound draws. Included in each graph are 45 degree lines for where the bid would equal the lower bound and where the bid would equal the corresponding upper bound, regression lines, and theoretical predictions.[9]

The first theoretical claim proposed that bids would be higher under market-based resale limited liability than under full liability. The second theoretical claim stated that depending on the default penalty cost, α, bidding would be more aggressive in the lower range of values up to the cutoff and equal to full liability after the cutoff. From the regression and theoretical prediction lines, it is evident that when bidders have limited liability through a resale opportunity or with a low default cost, they are choosing to bid higher. It also appears that under a higher default cost of 25%, bidders are not bidding as high as predicted. Note that in the 25% default treatment, bids should theoretically be equivalent to the full liability bids after the cutoff of 30. Prior to this cutoff, bids should be higher under the 25% default rule. Interestingly, under full default, the regression indicates that bidders are choosing to bid higher than the prediction.

Regardless of the treatment imposed, the theoretical bid functions predicted bids between the upper and lower bounds of the lottery. It is evident from the four figures that in all treatments some bidders are choosing to bid above the upper bound of the lottery.

In the resale treatment this bidding behavior might be explained as speculation. However, because speculation can only occur in an auction where a secondary market exists, the presence of overbidding in the statutory liability treatments and full liability treatments dilutes the speculation hypothesis, and leaves open the question of why overbidding exists across all treatments.

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[9]The lower 45 degree line is where bid equals the fundamental value, x. The upper line graphs where bid equals the upper lottery draw, $x + 20$.
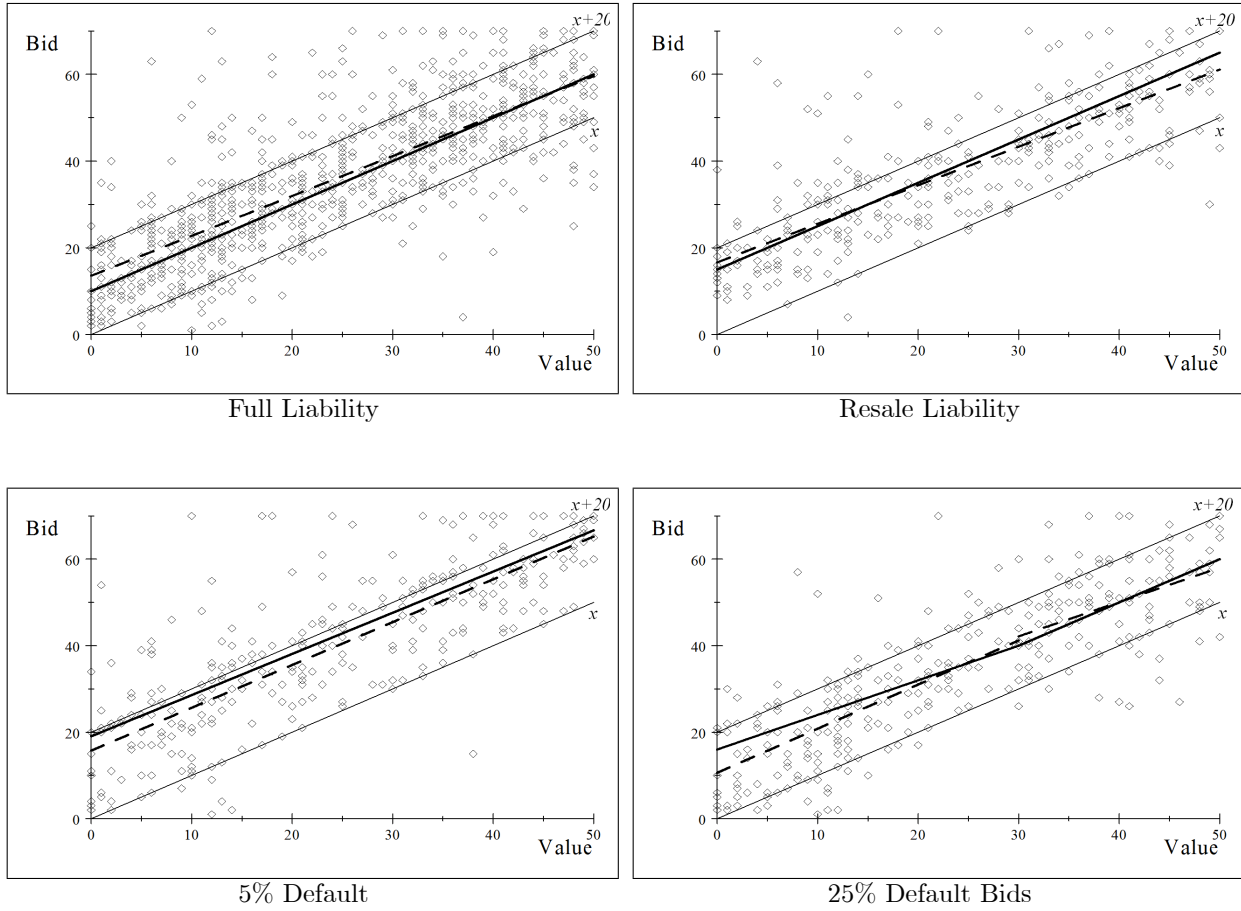
Figure 1: Observed Bids, Theoretical prediction (solid line), and Regression (dashed line)

The observed overbidding in all treatments, including the baseline, is similar to the bidding behavior that has been observed in multiple experiments for standard sealed-bid second price auctions, where bidders bid above their value, except here they are bidding above the upper bound of the lottery.[10]

Overbidding in the second price format was described by Kagel, Harstad, and Levin (1987) to be a result of "the illusion that it improves the probability of winning with no real cost to the bidder as the second-high-bid price is paid." Examples of over-bidding in a standard second price auction are also seen in Cooper & Hang (2008), who find evidence that bidders are less likely to overbid when their perception of the opponent's value is close to their own value. This result helps explain the overbidding, as the experimental design consisted of two-bidder groups. With a larger number of bidders, the overbidding would likely not have been as pronounced due to the higher probability that a competing bidder's lower bound was closer to their lower bound. Note that the observed overbidding should be differentiated from overbidding in auctions that are not second price, particularly the first price auction, where overbidding has been attributed to risk aversion, starting with Cox, Smith & Walker (1988).

Bidders are also bidding below the lower bound of the lottery. While under-bidding is not as pronounced as overbidding, it also exists across all treatments. This result of bidders choosing to bid below the fundamental value, was also observed by Katok and Salmon (2009) in a similar experimental design with full liability.

While bids do lie beyond the upper and lower bounds of the lottery, the number of bids outside of these bounds does not appear large when examined in relative terms against the total number of bids. Table 3, examines the percentage of under and over-bids in the treatments. The percentage of bids represented in table 3 matches the figures in the column listed Last 10. The remaining columns give the percentages for all 15 periods of auctions, and for the first 5 periods for comparison.

From table 3, it is seen that the overall percentage of bids that lie below the bounds of the lottery are decreasing. The first auctions that all subjects participated in were full liability for 5 periods. A high percentage of individuals are underbidding, while a modest percentage are overbidding. It appears that there is a treatment impact. This is especially evident in the observed percentage of overbidding. Increases in the percentage of overbidding occur in the 5% default and full liability treatments. Overbidding in the 5% default treatment is the highest, relative to all other treatments. The cost associated with overbidding in this treatment is quite low, so this result is not surprising.

The observed increase in overbidding for the full liability treatment is more surprising. The last 10 periods of the full liability treatment were played after the first 5 periods of the particular limited liability treatment that a subject was in. The limited liability treatment appears to be driving the results behind the full liability treatment. For the subjects that participated in the 5% default

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[10]Under and over-bidding is in reference to the upper and lower bounds of the lottery, respectively.

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treatment, overbidding in the subsequent full liability rounds occurred 9.06% of the time. The full liability percentages, after subjects had participated in 5 periods of the resale or 25% default sessions, were 6.56% and 5.31% respectively. The particular liability treatment had an impact on the bidding behavior observed under full liability, as more aggressive bidding from the treatment carried over into the full liability baseline.

Underbidding	All Periods	First 5	Last 10
Full Liability - Baseline	6.39%	12.50%	3.34%
Resale Liability	3.23%	5.31%	2.19%
5% Default Liability	2.40%	5.31%	0.94%
25% Default Liability	6.46%	10.63%	4.38%

Overbidding	All Periods	First 5	Last 10
Full Liability - Baseline	5.52%	2.60%	6.98%
Resale Liability	9.48%	9.69%	9.38%
5% Default Liability	10.73%	5.00%	13.59%
25% Default Liability	6.46%	7.19%	6.09%

Table 3: The percentage of bids below and above the fundamental value.

The theoretical bids are compared against the observed bids, averaged across sessions for the last ten rounds, in Table 4. Under full liability, bidders are bidding higher than the theoretical prediction. The baseline is the only treatment where bidders are bidding above the theory, and it is notably the only treatment where the bidders are not in a limited liability environment. Given these differences, the averages and regression lines presented in figure 1 show a large degree of conformity to the theoretical prediction, but there is a substantial amount of variance.

Treatment	Observed Bid	Theoretical Prediction	Difference
Full Liability - Baseline	35.60	33.96	1.64
Resale Liability	37.46	38.42	-0.96
5% Default Liability	38.93	41.36	-2.43
25% Default Liability	34.10	35.47	-1.37

Table 4: Average observed bids and theoretical predictions

In all of the limited liability treatments, bids are below the theoretical prediction but given the range of possible realized values, the difference between the theoretical bid and the observed bid does not appear to be substantial. The maximum difference occurs in the 5% default liability, and the observed bid is only approximately 6% less than the theoretical prediction. The resale liability is a slightly more difficult equilibrium bid for bidders to understand and implement, yet the difference between the theoretical prediction and observed bid is

minimal. This evidence, while promising, should be approached carefully. It is clear from figure 1 that the variation in bids is substantial. This implies substantial individual heterogeneity in bids, likely influenced by factors such as an individual's level of strategic thinking and risk attitudes. The most promising implication is that bids are increasing within limited liability environments.

Table 5 presents the bid regression results, using random effects at the individual level.[11] The predicted bid under full liability is $b_{FL}^* = Lb + 10$. The full liability specification gives a higher intercept of 13.55, which is significantly different from the predicted value of 10. Likewise, the coefficient on the lower bound is found to be significantly different from predicted value of 1. The predicted bid under resale liability is $b_{RL}^* = Lb + 15$. Under the resale limited liability scheme, the constant is found to be *not* significantly different from the predicted constant of 15, but the prediction on the lower bound does not hold. The 5% default rule had a predicted bid function of $b_{5\%}^* = .952Lb + 19.04$. In this model, the constant is found to be significantly different from the prediction at the 10% level, but the coefficient on the lower bound is *not* significantly different from the prediction. The last liability case, 25% default, involves the cutoff value in the bid function. For lower bounds below the cutoff value of 30, the predicted bid function is $b_{25\%}^* = .8Lb + 16$. The predicted bid function above the cutoff is equivalent to the full liability bid $b_{25\%}^* = Lb + 10$. To test these theoretical predictions, a dummy variable was created for lower bounds greater than the cutoff. This is then interacted with the lower bound to formulate observed bids for testing against the theoretical predictions. All of the observed coefficients are significantly different from the predictions.

	Full Liability		Resale		5% Default		25% Default	
Bid	Estimate	p-value	Estimate	p-value	Estimate	p-value	Estimate	p-value
Constant	13.55 (0.983)	< 0.001	16.64 (1.649)	0.319	15.74 (1.974)	0.094	10.61 (1.705)	0.002
Lower Bound (Lb)	0.92 (0.024)	< 0.001	0.89 (0.033)	< 0.001	0.99 (0.040)	0.346	1.02 (0.047)	< 0.001
Value > 30							7.73 (4.019)	0.031
Value > 30 * Lb							−0.23 (0.109)	0.036
R^2 (overall)	0.65		0.63		0.65		0.71	

Table 5: Panel random effects for bidding in all Treatments for the last 10 periods. Standard errors are in parentheses. The reported p-value represents the equality test of the coefficient to the theoretical prediction

The models that perform least well are the full liability and high default cost (25%) limited liability. The point predictions of the model do not explicitly hold, but this is not a surprise given the substantial heterogeneity observed in the figures. The theory also predicts directional shifts for bids based on the liability treatment. To test the directional consistencies of the theory and to

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[11] Clustering standard errors by subject.

analyze the heterogeneity in bidding behavior in greater depth, table 6 presents the regression results, using random effects at the individual level.

Bid	Model 1		Model 2		Model 3		Model 4	
	Estimate	p-value	Estimate	p-value	Estimate	p-value	Estimate	p-value
Constant	13.206	<0.001	15.014	<0.001	14.959	<0.001	12.810	<0.001
Lower Bound (Lb)	0.935	<0.001	0.935	<0.001	0.935	<0.001	0.933	<0.001
# Safe			-0.344	0.427	-0.334	0.436	0.085	0.860
Resale Liability	1.431	0.147	1.428	0.148	2.894	0.081	5.962	0.157
Lb x Resale					-0.063	0.184	-0.056	0.200
Safe x Resale							-0.620	0.275
5% Default Liability	3.263	0.001	3.261	0.001	2.560	0.089	2.625	0.445
Lb x 5% Default					0.030	0.466	0.032	0.434
Safe x 5% Default							-0.022	0.974
25% Default Liability	0.493	0.658	0.499	0.653	-0.251	0.863	9.902	0.001
Lb x 25% Default					0.032	0.379	0.040	0.268
Safe x 25% Default							-1.914	<0.001

Table 6: Panel random effects for bidding in all Treatments (Last 10 Periods)

In all model specifications, the # safe variable represents the result from the Holt & Laury (2002) test of risk preferences. The HL procedure presents each subject with 10 lottery choices. Within each of the ten lottery choices, one option is considered the safe option while the second option is considered the risky option. Of the ten choices, the variable used within the regression analysis above indicates the number of times the safe option was chosen.

The first theoretical claim stated that bidding behavior should be higher in the market-based resale limited liability environment than in the full liability environment. The direction of this claim is confirmed by the positive coefficients on resale liability, however it is only moderately significant in Model 3.

Result 1: The addition of the market-based limited liability through a secondary resale market increases bids above what is observed in a full-liability, no resale environment, but not by as much as theory would predict.

The first behavioral claim recognized that a secondary resale market creates limited liability, but it also creates an environment for speculation. It appears from figure 1 and the regression bidding results that there is some evidence for speculation, especially in the lower range of lower bound draws. However, the fact that the intercept shift is lower than what would be expected theoretically makes it less likely that bidders are learning to speculate, but more evidence is needed to differentiate speculation from higher bidding because of limited liability.

The second theoretical claim stated that statutory limited liability would lead to more aggressive bidding, for the subset of values that lie below the

cutoff value, v_c, than what would result if bidders were not allowed to resell under full liability. For values above the cutoff value, the bids under statutory limited liability and full liability are equivalent. In this particular environment, due to the range of values chosen, the cutoff value is not binding under the 5% default cost rule. Therefore, we should expect all bids, regardless of the value be more aggressive than the full liability treatment.

Under the 25% default cost rule, the cutoff value is 30. When a bidder has a fundamental value of 30 or higher, the bids are theoretically equivalent under the 25% default cost and full liability rules. The 5% default cost should have more aggressive bidding, across all values, than the 25% rule. Examining the results for the 5% default rule, it is evident that bidders are responding to the strong limited liability incentives. Across three of the four model specifications, the coefficients associated with this default rule are positive and significant.

Result 2: Under a statutory default option, where the default cost is low, bidders are bidding more aggressively than in the full-liability treatment, but not by as much as theory would predict.

Under high default costs, the evidence is mixed. The coefficients on the 25% default rule are positive in three of the four model specifications, as predicted, but the results are not robust across model specifications. In particular, it appears that the heterogeneity of risk attitudes across bidders determines how bidders respond to the 25% default rule, more so than in the other treatments. Model 4 examines risk attitudes interacted with the treatments. In the 25% default treatment, bidders who are more risk averse are bidding significantly less than bidders who are more risk tolerant.

Result 3: Under a statutory default option, where the default cost is high, bidders are NOT bidding more aggressively than in the full-liability treatment.

Result 3 rejects the hypothesis regarding statutory default for high penalty default rates. The theory was based on a risk neutral prediction, and the heterogeneity in risk attitudes is driving this result. Bidding is higher in all of the limited liability treatments, but the negative interaction of risk aversion across treatments indicates that risk averse individuals do not bid as aggressively under limited liability as others who are not as risk averse. While this effect exists in all treatments, it is only significant in the 25% default penalty environment.

Bidding higher occurs because of limited liability. Under statutory liability rules, the default option exists. If bidders are bidding higher, the probability of default is increasing. Table 7 examines the observed and predicted frequency of default by treatment for the statutory limited liability cases where default is an option.

Default in the statutory penalty cases was automatic. The cost of default was compared to the losses from the auction and if the former was less, the default option was exercised. The frequency of default is observed to be the highest under the 5% default rule, as would be expected due to the low default

cost and corresponding higher bids. In relative terms, the default percentage should be lower when the cost is higher, and this holds for the 25% default rule. The observed frequency of default is lower than predicted which implies that bidders are not bidding as high as predicted to take full advantage of the limited liability afforded by these rules.

Treatment Outcome	Observed	Predicted
Default 5% Penalty	0.13 (41)	0.16 (51)
Default 25% Penalty	0.04 (12)	0.06 (18)
Resale	0.15 (47)	0.09 (28)

Table 7: Observed and Predicted frequency of default and resale outcomes, by treatment. (Actual default/resale numbers in parenthesis).

The frequency of resale outcomes, observed and predicted, from the market-based limited liability treatment is also examined in table 7. The opportunity to resell was automatic. The losing bidder was automatically sold to at a price equal to their realized value if it was higher than the realized value of the winning bidder. Contrary to the default predictions where observed frequencies fall short of predictions, the observed frequency of resale outcomes is greater than the prediction. The predicted frequency of resale was based on a model that only considered the liability aspects of resale. Behaviorally, we know that speculation is something that bidders may engage in when resale is possible, and the higher observed frequency is evidence of that behavior.

4.2 Revenue and efficiency

The possibility of default clearly impacts the revenue collected by the auctioneer. In a second price auction without a default option, the revenue earned by the seller is equal to the second highest bid. The analysis of this paper assumes two bidders, so the revenue equates to the lowest bid placed in the auction in both the resale and full liability treatments. In the default treatments, the seller would earn 5% or 25% of the second highest bid in the event of default. Table 8 examines the average revenue, by treatment, achieved in the experimental auctions. It also examines the theoretical prediction for revenue. The predicted revenue is obtained by using the actual values bidders experienced in the experiment and the theoretical bid functions to generate the revenue that would have been observed had the subjects followed the theoretical bid functions.

The highest revenue occurred in the market-based resale treatment. This result matches the theoretical predicted rank. Higher bidding leads to higher revenue under the resale treatment, because default is not allowed. Under the statutory liability treatments, bidding was only observed to be more aggressive in the low default penalty case of 5%. In general, when default is allowed and the spread between the lower bounds is not substantial, revenue drops. Despite the

aggressive bidding under a low default penalty, the low penalty and frequency of default led to the lowest revenue in this treatment.

These theoretical predictions indicate that the lowest revenues should occur under full liability and the 5% default treatment, and the revenue from these two treatments should differ by less than 1%. Observationally, we see that revenue under full liability exceeds that of the low default cost treatment by approximately 36%. Instead of resulting in the second lowest revenue, full liability earns the second highest revenue. Theoretically, the second highest revenue should have occurred under the higher default penalty of 25%. Both statutory liability treatments earned less than the market-based resale liability and full liability cases. Interestingly, in all limited liability treatments the average observed revenue was less than what was predicted, and the revenue under full liability is higher than predicted.

Treatment	Average Net Revenue	Theoretical Prediction
Full Liability	26.03 (13.48)	25.76 (11.33)
Resale	28.31 (12.81)	29.79 (11.28)
5% Default	19.08 (15.26)	25.69 (15.29)
25% Default	23.01 (14.75)	27.45 (10.97)

Table 8: Revenue results by treatment (Standard Deviation)

To compare revenue to the theoretical prediction, Table 9 presents the regression results of theoretical predictions on the observed revenue, using random effects at the average group level.[12]

Observed Revenue	Full Liability Estimate	Full Liability p-value	Resale Estimate	Resale p-value	5% Default Estimate	5% Default p-value	25% Default Estimate	25% Default p-value
Constant	3.16 (1.331)	0.017	1.53 (2.490)	0.536	−6.87 (7.130)	0.334	−13.03 (3.325)	< 0.001
Theory Revenue	0.88 (0.039)	0.004	0.89 (0.073)	0.169	0.93 (0.254)	0.791	1.30 (0.085)	< 0.001
R^2 (overall)	0.72		0.81		0.35		0.75	

Table 9: Panel random effects for bidding in all Treatments for the last 10 periods. Standard errors are in parentheses. The reported p-value represents the equality test of the coefficient to the theoretical prediction

If the theoretical model predicts revenue accurately, we should see a coefficient of 1 on Theory Revenue with a zero constant. In the resale treatment and the 5% default treatment, the constant is positive and not significantly different from zero, and the coefficient on theory revenue is not significantly different

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[12]Each group of 16 bidders was divided into groups 8. The group of 8 is the level that the averages are formulated for.

from 1. These models were fairly accurate in predicting revenue. The remaining full liability and 25% default treatments do not predict as accurately. In both of these cases, the coefficient for theory revenue is significantly different from 1, and the constant is significantly different from zero.

Revenue results are examined in more depth in Table 10 to determine if the directional predictions of the theory hold with significance. These regression results use random effects at the group level. The variable $V_{(1)}$ represents the highest lower bound of the two bidders, while $V_{(2)}$ is the second highest lower bound.

Revenue	Model 1 (Last 10 periods)		Model 2 (Last 10 periods)	
	Estimate	p-value	Estimate	p-value
Constant	10.126	<0.001	9.921	<0.001
$V_{(1)}$	0.127	<0.001	0.131	0.009
$V_{(2)}$	0.749	<0.001	0.754	<0.001
Resale Liability	2.837	<0.001	2.918	0.001
5% Default Liability	-6.445	<0.001	-4.821	<0.001
25% Default Liability	-2.592	0.055	-2.016	0.023
Bidder Default			-6.470	<0.001

Table 10: Panel random effects for revenue

In both models, the average revenue results presented in Table 10 are confirmed by the coefficients on each treatment. The resale liability treatment results in higher revenue, represented by a positive significant intercept shift in models 1 and 2. The lowest revenue achieved was in the 5% default treatment, and this is confirmed by the significant negative intercept shifts, across both models. The 25% default rule also shows lower revenue than what occurs in the full liability treatment, but the effect is not as strong as the 5% default rule. For both statutory cases, there is a negative intercept shift in both models indicating that if the default option exists and it is exercised, revenue will decrease. Model 2 includes the binary variable for bidder default, clearly showing the impact it has on revenue. The second highest lower bound, $V_{(2)}$, has the greatest impact on revenue which is expected in a second price auction where the losing bidder's bid sets the auction price. These regression results lead to two key revenue findings.

Result 3: The addition of the secondary resale market increases revenue above what is observed in a full-liability, no resale environment, or a similar environment where default is allowed.

Result 4: Under a statutory default option, revenue is lower than what is observed under full-liability or a market-based resale environment.

The efficiency generated by the initial auction allocation is another way of characterizing outcomes from the auctions. Efficiency is generally measured as

the value generated by the auction divided by the maximum value the auction could have achieved, $\frac{V}{V^*}$. The uncertainty in this paper creates two interesting forms of efficiency. The first form, pre-lottery efficiency, results from the lower bounds of the lottery. Pre-lottery efficiency is defined as the lower bound of the winner of the auction divided by the maximum lower bound of the two bidders. The second form of efficiency, post-lottery efficiency, is calculated after the uncertainty of the lottery is resolved. Post-lottery efficiency is defined as the realized value of the winner divided by the maximum realized value of the two bidders in the bidding group.

In the resale treatment, the secondary market makes overall efficiency always equal to 1 as the final allocation in resale always goes to the bidder with the highest realized value. The efficiency forms considered above are the interim forms of efficiency before resale occurs. In statutory default treatments, the interim efficiency is calculated prior to the default decision. If default occurs, the item is given back to the auctioneer. The efficiency form above is again the interim efficiency, prior to the default decision.

Examining the efficiency of the initial auction is particularly significant in the case where there is a resale opportunity. If speculation is observed, then we should expect efficiency rates to decline. The standard efficiency of an ascending auction without uncertainty, or a resale opportunity, obtained from previous experimental results has resulted in high efficiency rates.[13]

The efficiency results of the current experiment are presented in Table 11, by treatment. Pre-lottery efficiency refers to the efficiency rate prior to the resolution of uncertainty of the lottery, while post-lottery efficiency is the interim efficiency rate following the resolution of the lottery but keeping the allocation from the auction. Post-lottery efficiency with default examines the efficiency of the final allocation, after the default option has been exercised. If a bidder defaults, the item is returned to the seller and efficiency is zero. The final allocation for the resale case is not examined, because by construction, this efficiency rate is always equal to 1, which is full efficiency.

Examining pre-lottery efficiency first, the rates across the full liability, resale and 5% default treatments are very similar, and relatively close to 1. It appears as if the addition of resale does not significantly lower overall efficiency compared to other treatments. The lowest pre-lottery efficiency resulted from the 25% default treatment. To test the validity of these numbers from a random occurrence, an additional efficiency measure was created from the average of the two values, for both pre-lottery and post-lottery values, divided by the maximum value. These measures hovered around .75 for pre-lottery efficiency, indicating that the observed efficiency rates are a 25% improvement over random. For post-lottery efficiency, the average random measure was approximately .78, so the observed efficiency rates are a 19% improvement over the random measure.

The post-lottery efficiency rates are lower for all treatments, except 25% default. This result is not surprising, given that the average spread between

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[13]High efficiency is close to 1. See Coppinger et al.(1980) for an example of experimental results of an ascending auction without resale. The second price clock auction is a similar format to the ascending auction, except that bids of the participants remained sealed.

the lower bounds of both bidders for all treatments was 16.4, which is less than 20. It is interesting to note that post-lottery efficiency rose, by a small amount in the 25% default treatment. High pre-lottery efficiency indicates that the bidder with the highest lower bound is winning the auction. This was anticipated theoretically. The lower pre-lottery efficiency for the 25% default treatment indicates that heterogeneity in response to the higher default cost led to bidders with a higher lower bound not necessarily winning the auction. The post-lottery efficiency increases slightly over the pre-lottery efficiency because of this heterogeneity and the fact that the 25% default treatment had the lowest spread average between the lower bounds of 15.6.

Treatment	Pre-Lottery Efficiency	Post-Lottery Efficiency	Post-Lottery Efficiency with Default
Full Liability	0.946	0.918	-
Resale	0.940	0.926	-
5% Default	0.943	0.935	0.725
25% Default	0.907	0.918	0.882

Table 11: Efficiency results by treatment

The high pre-lottery efficiency rate for the resale treatment is surprising, contrasted with the results in Saral (2009). Saral found strong inefficiency in auctions where a secondary market existed. This inefficiency resulted from bidders with low values speculating to win and resell to high value bidders who were shading their bids substantially because of the resale market. Saral found that the highest efficiency rate with resale was approximately .86, which is much lower than the pre-lottery efficiency rate of .94 observed in these auctions. The resale treatment efficiency results suggest that while a few bidders might be speculating, the majority of high bids are resulting from bidders taking advantage of the limited liability offered by a secondary market. The second price auction used a clock format, so the higher efficiency rates correspond to the high rates observed in ascending clock auctions without value uncertainty.

Value uncertainty is one component leading to slightly lowered efficiency rates, but it is also likely given the bidding data presented above, that risk heterogeneity is a factor in the lowered efficiency rates. Bidders with lower risk tolerance are bidding lower, evidenced with significance in the 25% default treatment. In the event that the two competing bidders had close lower bounds, it is highly likely that a risk averse bidder with a higher low bound placed a bid lower than their less risk averse opponent. This would lead to the result that the 25% default cost treatment had the lowest efficiency rates. In general, however, there are not significant differences in interim efficiency rates, by treatment.

The last efficiency rate examined is post-lottery efficiency with the default decision factored in. Efficiency in the 5% default treatment fell by 0.21 as these bidders exercised the default option once the uncertainty was resolved. This rate is notable in that it is actually lower than the random measure by approximately 7%. Default was not exercised as often in the 25% default treatment, so while the post-lottery efficiency rate with default does fall, it is only lower

by approximately 0.04.

5   Conclusion

This paper examines the impact of various liability settings in a second price clock auction environment to determine how liability rules influence the bidding behavior in the auction. Full liability rules were theoretically and experimentally contrasted against market-based limited liability induced through resale and statutory limited liability. It was predicted theoretically that bids would be more aggressive under all limited liability treatments than under full liability. The first fundamental experimental result is that bidding behavior becomes more aggressive under market-based limited liability and low statutory default penalties, but not by as much as theory would predict for a risk neutral bidder. Second, under statutory liability with a higher cost of default, bidders did not choose to bid more aggressively than under full liability, despite theoretical predictions that bids should be higher.

This paper also empirically addressed risk attitudes towards liability. With a very low default cost, all bidders chose to bid more aggressively, and risk attitudes did not factor significantly into the response to decreased liability. Similarly, when resale was allowed, risk attitudes did not factor substantially into this market-based limited liability environment. Under a secondary resale opportunity, it was observed that a majority of the bids were not speculative in nature. This is perhaps one explanation as to why risk attitudes are not more prevalent in this environment. Speculation is a risky endeavor as a speculating bidder is choosing to bid higher than their maximum possible value. If more speculation had been observed, then subjects with a higher risk tolerance would have likely appeared significant.

It is interesting to note that speculation is not observed as often was observed in Saral (2009). This is likely due to the added complexity of value uncertainty in two ways. First, bidders not only have to cognitively handle their expected value, but if they wanted to speculate, they must also account for the uncertainty of the other bidder's values. Second, it is also likely the result of the number of bidders participating. This environment only had two bidders, so the probability that the opposing bidder had a higher lower bound is less than it would be with more bidders. Given the lack of speculation, what this paper does show is that bidders are able to internalize the resale market as limited liability, shown by bids that are higher than under the corresponding full liability environment without resale.

When designing an auction the seller must carefully consider the statutory default conditions prior to holding the auction. It has been shown that a low default penalty does lead to more aggressive bidding, but this leads to default and actually lowers revenue. Under a higher default penalty, bidders are less likely to consider default as a viable option, and therefore bid as if default does not exist. To maximize revenue, these results suggest that an auctioneer would be best off enforcing full liability or a high statutory default option.

Market-based limited liability through resale, under an auction with no default, leads to more aggressive bidding that benefits an auction seller. The resale examined in this paper was strictly inter-bidder.[14] This is a narrow view of post-auction resale. It is highly likely, in a field setting, that potential buyers exist that did not know of or could not participate in the auction. These buyers could increase the probability that the winner bidder could resell, not only to mitigate losses, but to potentially make a profit. All of these factors highlight the positive benefit for the initial seller of resale on revenue.

In general, this paper examines how decisions with uncertainty are influenced by different liability conditions. The extensions of which go beyond the auction environment alone. Decisions may involve more risk as the loss potential is diminished through limited liability, but this is dependent on the risk attitudes of the subjects.

One additional topic not addressed by this paper, that could possible alter behavioral outcomes, is the stigma attached to default. Individuals may vary considerably in their perceptions of default. The experimental design considered in this paper made the default choice automatic if the losses from the auction exceeded the default payment. Behaviorally, we would expect that individuals have differing attitudes regarding default, and this may in turn substantially impact the default rates observed - and the bids leading to default. This would be an interesting idea for future research.

6   References

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[14]Inter-bidder resale implies that only auction participants are involved in the secondary market. Entry of potential buyers in the post-auction resale market is not examined in this paper.

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7 Appendix

7.1 Bid function under Full Liability

The second price auction problem is defined as follows under the full liability case for bidder i, assuming that the bid function, $b(v)$, is monotonic and differentiable:

$$\max_r U_i(v_i, r) = \int_0^r \left[\Pr(v_i = x_i) * (v_i - b_j(t)) + \Pr(v_i = x_i + y) * (v_i + y - b_j(t)) \right] dF(t)$$

Understanding that $\Pr(v_i = x_i) = \frac{1}{2}$ and $\Pr(v_i = x_i + y) = \frac{1}{2}$, the above simplifies to:

$$\max_r U_i(v_i, r) = \int_0^r \left[\frac{1}{2}(v_i - b_j(t)) + \frac{1}{2}(v_i + y - b_j(t)) \right] dF(t)$$

Again, repeating the equilibrium condition as $\frac{\partial(U_i(v_i, r))}{\partial r}\big|_{r=v_i} = 0$.

Taking the derivative leads to:

$$\left[\frac{1}{2}(v - b^*(r)) + \frac{1}{2}(v + y - b^*(r)) \right] \frac{\partial F(r)}{\partial r} = 0.$$

In equilibrium, $r = v$, giving the symmetric, increasing bid solution:

$$b^*_{FL}(v) = v + \frac{1}{2}y$$

7.2 Bid function under Resale

The second price auction problem is defined as follows under the full liability case for bidder i, assuming that the bid function, $b(v)$, is monotonic and differentiable:

$$
\begin{aligned}
\max_r U_i(v_i, r) &= \int_0^{r-y} \left[\Pr(v_i = x_i) * (v_i - b_j(t)) + \Pr(v_i = x_i + y) * (v_i + y - b_j(t)) \right] dF(t) + \\
&\int_{r-y}^r [\Pr(v_i = x_i)\{\Pr(v_j = x_j) * (v_i - b_j(t)) + \Pr(v_j = x_j + y) * (t + y - b_j(t))\} + \\
&\Pr(v_i = x_i + y)\{v_i + y - b_j(t)\}]dF(t) + \int_r^1 0 * d[1 - F(t)]
\end{aligned}
$$

Understanding that $\Pr(v_i = x_i) = \frac{1}{2}$ and $\Pr(v_i = x_i + y) = \frac{1}{2}$, the above simplifies to:

$$\max_r U_i(v_i, r) = \int_0^{r-y} \left[\frac{1}{2}(v_i - b_j(t)) + \frac{1}{2}(v_i + y - b_j(t))\right] dF(t) +$$

$$\int_{r-y}^r \left[\frac{1}{4}(v_i - b_j(t)) + \frac{1}{4}(t + y - b_j(t)) + \frac{1}{2}(v_i + y - b_j(t)\right] dF(t)$$

$$+ \int_r^1 0 * d[1 - F(t)]$$

Again, repeating the equilibrium condition as $\frac{\partial(U_i(v_i, r))}{\partial r}\big|_{r=v_i} = 0$.

Taking the derivative leads to:

$$\left[\frac{1}{2}(v - b(r - y)) + \frac{1}{2}(v + y - b(r - y))\right]\frac{dF(r)}{dr} +$$

$$\left[\frac{1}{4}(v - b(r)) + \frac{1}{4}(r + y - b(r)) + \frac{1}{2}(v + y - b(r)\right]\frac{dF(r)}{dr} -$$

$$\left[\frac{1}{4}(v - b(r - y)) + \frac{1}{4}(r + y - b(r)) + \frac{1}{2}(v + y - b(r)\right]\frac{dF(r - y)}{dr} = 0$$

In equilibrium, $r = v$, giving the symmetric, increasing bid solution:

$b_{RL}^*(v) = v + \frac{3}{4}y$

7.3   Bid function under Statutory Limited Liability

For the second price auction with stated rules, the symmetric statutory liability (SL) bidding solution can be shown to be:

$$b_{SL}^*(v_i) = \begin{cases} \frac{1}{1+\alpha}(v + y) & if \quad v \le v_c \\ v + \frac{1}{2}y & if \quad v > v_c \end{cases}$$

The cutoff value, v_c, is defined by equating the asserted bid functions.

$\frac{1}{1+\alpha}(v_c + y) = v_c + \frac{1}{2}y$
$\implies v_c = \frac{y}{2}(\frac{1-\alpha}{\alpha})$

Proof. Default will not be exercised as long as the payoff from not defaulting is greater than the payoff from defaulting, $v_i - b_j > -\alpha b_j$. This implies that default not occur as long as $\frac{v_i}{1-\alpha} > b_j$. In the first region, where $v \le v_c$, it is asserted that the bid function is $b = \frac{1}{1+\alpha}(v + y)$. The cutoff for default and the bid function combined give us that a bidder will not default as long as

$\frac{v_i}{1-\alpha} > \frac{1}{1+\alpha}(v_j + y)$. Bidder i will default when the reverse holds and their value, $v_i = x_i$, is the low draw. The above inequality, $\frac{v_i}{1-\alpha} > \frac{1}{1+\alpha}(v_j + y)$,gives us the interior limits of integration for the following maximization problem for bidder i in a second price auction.

$$\max_r U_i(v_i, r, \alpha) = \int_0^{(\frac{1+\alpha}{1-\alpha})r-y} [\Pr(v_i = x_i) * (v_i - b_j(t)) + \Pr(v_i = x_i + y) * (v_i + y - b_j(t))] \, dF(t) +$$

$$\int_{(\frac{1+\alpha}{1-\alpha})r-y}^{r} [\Pr(v_i = x_i) * (-\alpha b_j(t)) + \Pr(v_i = x_i + y) * (v_i + y - b_j(t))] \, dF(t)$$

Taking the derivative of the above maximization problem, and substituting in the basic lottery probabilities gives us the following:

$$\left[\tfrac{1}{2}(v_i - b_j((\tfrac{1+\alpha}{1-\alpha})r - y) + \tfrac{1}{2}(v_i + y - b_j((\tfrac{1+\alpha}{1-\alpha})r - y)\right] \frac{dF((\frac{1+\alpha}{1-\alpha})r-y)}{dr} +$$

$$\left[\tfrac{1}{2}(-\alpha b_j(r)) + \tfrac{1}{2}(v_i + y - b_j(r))\right] \frac{dF(r)}{dr} -$$

$$\left[\tfrac{1}{2}(-\alpha b_j((\tfrac{1+\alpha}{1-\alpha})r - y) + \tfrac{1}{2}(v_i + y - b_j((\tfrac{1+\alpha}{1-\alpha})r - y)\right] \frac{dF((\frac{1+\alpha}{1-\alpha})r-y)}{dr} = 0 \qquad (i)$$

The proposed bid function evaluated at the limits of integration

$$b_j((\tfrac{1+\alpha}{1-\alpha})r - y) = \tfrac{1}{1-\alpha}r$$
$$b_j(r) = \tfrac{1}{1+\alpha}(r + y) \qquad\qquad (ii)$$

Combining (i) and (ii), and simplifying results in the following:

$v = r$, which is the equilibrium condition.

For the region where $v > v_c = \frac{y}{2}(\frac{1-\alpha}{\alpha})$, a bidder would never default. First I will show that under either of the proposed bid functions, a bidder would never default for values greater than or equal to v_c. A bidder never defaults whenever $v_i - b_j > -\alpha b_j$. Assume the reverse, that a bidder would default when the value is given by the cutoff $v = \frac{y}{2}(\frac{1-\alpha}{\alpha})$. For a bidder to default, it must be the case that $v_i - b_j < -\alpha b_j$. Utilizing the default condition and the bid function, $b = \frac{1}{1+\alpha}(v+y)$, we can show that a bidder would never default by contradiction.

$v - b < -\alpha b$
$\frac{y}{2}(\frac{1-\alpha}{\alpha}) < (1 - \alpha)b$
$\frac{y}{2\alpha} < b = (\frac{1}{1+\alpha})(v + y)$
$\frac{y}{2\alpha} < (\frac{1}{1+\alpha})(\frac{y}{2}(\frac{1-\alpha}{\alpha}) + y)$
$1 + \alpha < \alpha$, a contradiction.

The same can be shown for the bid function, $b = v + \frac{1}{2}y$.

It is shown that default does not exist in the regions where $v > v_c$, so it remains to solve for the equilibrium bid function in this region. Define the second price auction problem as follows for bidder i, assuming that the bid function, $b(v)$, is monotonic and differentiable in this region:

$$\max_r U_i(v_i, r) = \int_{\frac{y}{2}(\frac{1-\alpha}{\alpha})}^{r} \left[\Pr(v_i = x_i) * (v_i - b_j(t)) + \Pr(v_i = x_i + y) * (v_i + y - b_j(t)) \right] dF(t)$$

Understanding that $\Pr(v_i = x_i) = \frac{1}{2}$ and $\Pr(v_i = x_i + y) = \frac{1}{2}$, the above simplifies to:

$$\max_r U_i(v_i, r) = \int_{\frac{y}{2}(\frac{1-\alpha}{\alpha})}^{r} \left[\frac{1}{2}(v_i - b_j(t)) + \frac{1}{2}(v_i + y - b_j(t)) \right] dF(t)$$

Again, repeating the equilibrium condition as $\frac{\partial (U_i(v_i, r))}{\partial r}|_{r=v_i} = 0$.

Taking the derivative leads to:

$$\left[\frac{1}{2}(v - b^*(r)) + \frac{1}{2}(v + y - b^*(r)) \right] \frac{\partial F(r)}{\partial r} - \left[\frac{1}{2}(v - b^*(\frac{y}{2}(\frac{1-\alpha}{\alpha}))) + \frac{1}{2}(v + y - b^*(\frac{y}{2}(\frac{1-\alpha}{\alpha}))) \right] \frac{\partial F(\frac{y}{2}(\frac{1-\alpha}{\alpha}))}{\partial r} =$$
$$0.$$

In equilibrium, $r = v$, giving the symmetric, increasing bid solution:

$$b_{SL}^*(v) = v + \frac{1}{2}y \quad \blacksquare$$

7.4 Limiting Case of Statutory Limited Liability

The payoff resulting to the winning bidder in this second price auction set-up is defined as follows, under statutory default penalty rule for bidder i:

$$U_i(v_i, b_j, \alpha) = \frac{1}{2} \max \{v_i - b_j, -\alpha b_j\} + \frac{1}{2} \max \{v_i + y - b_j, -\alpha b_j\}$$

If a bidder faces a default penalty $\alpha = 1$, the bidder is fully liable for bid payment. The above payoff function reduces to the following under this rule:

$$U_i(v_i, b_j, \alpha) = \frac{1}{2} \max \{v_i - b_j, -b_j\} + \frac{1}{2} \max \{v_i + y - b_j, -b_j\}$$

In the case that the realized value is the upper bound, the payoff is:
$\max \{v_i + y - b_j, -b_j\} = v_i + y - b_j$, because $v_i + y > 0$ by definition.
This implies that a bidder would never choose to default and accept a payment of $-b_j$.

In the case that the realized value is the lower bound, the payoff is:
$\max \{v_i - b_j, -b_j\} = v_i - b_j$, if $v_i > 0$

When $v_i = 0$,

$v_i - b_j = -b_j$, so there is no maximum and the bidder is indifferent between default and not defaulting.

It is assumed that the bidder will choose not to default, and accept the payoff of $v_i - b_j$.

The above analysis shows that in the limit, as $\alpha \to 1$, the statutory default payoff function reduces to that of full liability, and the second price auction problem is therefore defined as follows:

$$\max_r U_i(v_i, r) = \int_0^r \left[\Pr(v_i = x_i) * (v_i - b_j(t)) + \Pr(v_i = x_i + y) * (v_i + y - b_j(t)) \right] dF(t)$$

Which results in the asserted bid function:

$$b^*_{SL,\alpha=1}(v) = v + \tfrac{1}{2}y$$

7.5   Revenue comparisons

7.5.1   Resale vs. Full Liability

The bid functions under full liability and resale liability were shown to be

$$b^*_{FL}(v_i) = v_i + \tfrac{1}{2}y \qquad b^*_{RL}(v) = v + \tfrac{3}{4}y$$

Utilizing order statistics for the uniform distribution on the support $[0, 1]$, the prediction that revenue is higher under resale liability follows directly.

$$E_{FL}[revenue] = E_{FL}[b^*_{FL}(V_{N-1})] = \tfrac{1}{3} + \tfrac{1}{2}y < E_{RL}[revenue] = E_{RL}[b^*_{RL}(V_{N-1})] = \tfrac{1}{3} + \tfrac{3}{4}y$$