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Abstract

An approach to incorporate spatial dependence into Stochastic Frontier analysis is developed and applied to a sample of 215 dairy farms in England and Wales. A number of alternative specifications for the spatial weight matrix are used to analyse the effect of these on the estimation of spatial dependence. Estimation is conducted using a Bayesian approach and results indicate that spatial dependence is present when explaining technical inefficiency.

Key words: Spatial dependence, technical efficiency, Bayesian, spatial weight matrix

JEL classification: C11, C13, C23, C51, Q12

1 Introduction

Despite many economic phenomena being driven by spatial processes, spatial relationships have rarely been exploited in economic literature before the late 1990s' (Bockstael 1996; Anselin 2001). Disregarding spatial aspects of the data may produce inefficient or biased estimates and consequently, misleading inference (Anselin 2001). However, interest increased recently, it was in the 1990s when there were the first calls for the introduction of spatial econometrics in agricultural economics (Bockstael 1996; Weiss 1996). Weiss (1996) stresses, as does Bockstael (1996), that an economic process such as agricultural production is a spatial phenomenon and factors such as yield, soil characteristics, landscape configurations and pest populations show spatial variability. Weiss (1996) calls for the use of spatial information in agricultural economics, and points out that the results obtained from spatial analysis applied to agricultural economics will have implications for farm management and for agricultural and environmental policy. For instance, spatial information can reveal where fertiliser can be profitable and where counterproductive (Weiss 1996), or where to put in place policies aiming to increase efficiency.

Spatial econometrics models have their roots in regional science which, through theoretical formulations on human spatial behaviour, attempts to solve issues faced by cities and regions (Anselin 1988). According to Anselin (1988) the term spatial econometrics was coined in the 1970s by Jean Paelinck, but the lineage of spatial econometrics can be traced further back to the 19th century economist Johann Heinrich von Thünen who explained the effect of transport costs on production location through his rings model (von Thünen 1826). Von Thünen's model shows that production is distributed into different areas (i.e. concentric rings), the most profitable production being the closest to the city (i.e. the market). Other relevant authors were Christaller (1966) and Lösch (1954) who studied the spatial organisation of markets and market centres.

Farrell (1957) showed early concerns about how spatial aspects may be correlated with efficiency. He applied his method of measuring efficiency to agricultural production in the United States and stated "...the apparent differences in efficiency...reflect factors like climate, location and fertility that have not been included in the analysis, as well as genuine differences in efficiency" (Farrell 1957, p. 270). Then Farrell investigated the correlation between efficiencies and variables representing location, temperature and rainfall finding little correlation (Farrell 1957). Despite these early concerns spatial dependence has not yet been incorporated into the Stochastic Frontier analysis. Efficiency literature usually considers spatial heterogeneity, which refers to the fact that efficiency levels may differ depending on the location, whereas spatial dependence refers to the correlation between the efficiency level at the farm and the efficiency levels of the "neighbouring farms". Spatial heterogeneity in technical efficiency literature is controlled (if controlled at all) by introducing dummy variables for political divisions of the land such as regions, counties and provinces. For example, Hadley (2006) introduced dummy variables to account for regional heterogeneity. The introduction of dummy variables is also used to account for spatial heterogeneity in certain areas of interest such as less favoured areas (Hadley 2006; Iraizoz et al. 2005). Contrary to what may be expected spatial heterogeneity and spatial dependence do not necessarily go together. This may happen when spatial dependency occurs at a different spatial level than the one studied, which is usually a political division. For instance, it may be the case that no spatial heterogeneity is found (i.e. no differences in efficiency levels between regions, counties) but there is spatial dependence within the region or across regions (i.e. the efficiency levels of a farm are correlated to the efficiency levels of the farms around). Since spatial dependency leads to heterogeneity at the same spatial level, the study of spatial dependency or heterogeneity should not be restricted to political divisions of the land.

As for agricultural production, there are a number of potential sources of spatial dependence in efficiency including soil quality, climatic conditions, socio-economic aspects and other location-specific attributes. For instance, spatial dependence in technical efficiency can be found because farmers in an area may emulate each other; it may be due to the level of infrastructure in the area; it may be because of the climatic and topographic conditions of the area where the farm is located. All these are unobservable latent variables that may be spatially correlated.

In recent years spatial econometric models have been developed and used in a wide number of areas of research including economics, sociology, geography, biology, meteorology and political science. In agricultural economics and environmental and resource economics a number of publications have reviewed and applied either spatial econometric techniques or geographic information system (GIS) techniques. Many of the applications using spatial econometric techniques are used in the context of hedonic price functions or production functions.

Bokstael (1996) used an hedonic model for residential transactions in which spatial characteristics were included. The results were used to predict land use conversion. Anselin (2002) discusses a number of issues related to the implementation of spatial models covering different model and weight matrix specifications.

Two special issues have been devoted to the subject of spatial econometrics in agricultural economics journals in recent years. Firstly the special issue of Agricultural Economics (2002) and secondly the special issue of the Journal of Agricultural Economics (2007).

Holloway et al. (2007) provide an excellent review of recent literature in which spatial econometrics techniques have been used. The authors focused their review of the spatial econometrics literature on those papers dealing with spatial bio-economic modelling and land use modelling and categorise articles according to the two criteria above: those that explicitly use spatial econometric methods and those that use geographic information systems (GIS) techniques.

A number of models have been developed to account for spatial dependence such as the spatial autoregression model (SAR) (Anselin 1988), the spatial error model (SEM) and its variant the higher order contiguity model or spatial Durbin model which allows for explanatory variables from neighbouring observations (LeSage 1999; Bell and Bockstael 2000). None of these models cover or discuss the incorporation of technical efficiency. Despite these advances in the econometric application of spatial analysis (Anselin 1988; LeSage 1999) very little research can be found in the literature on how to incorporate spatial dependence into technical efficiency analysis (Druska and Torrace, 2004; Schmidt *et al.* (2009).

We incorporate spatial dependence into technical efficiency analysis by using an autoregressive specification in the *inefficiency term* of a compound error term of the stochastic efficiency analysis differing from Druska and Torrace (2004) work, which based on a standard fixed effects model used an autoregressive specification in the *error term* to estimate the spatial dependence. We also differ from previous work done by Schmidt *et al.* (2009) that make farm inefficiency depend on a parameter that captures the unobserved spatial characteristics and assigning prior distributions to it. Our work differs in both the specification of the model used and the scope of the analysis. We directly integrate the unobserved spatial characteristics in the stochastic frontier model by specifying the inefficiency to be spatially autoregressive and including a parameter that measures the level spatial dependence. Schmidt *et al.* (2009) examined the unobserved local characteristics in each municipality by incorporating them to the analysis assuming that a) either they follow a CAR distribution (i.e. they incorporate the assumption that neighbour municipalities hava a similar level of unobserved local characteristics) or b) a Normal distribution (i.e. unobserved local characteristics are independent of the neighbours). On the other hand, our work analysed the presence of spatial dependency at varous area sizes by estimating the relevance of spatial location of the farm in farm inefficiency levels.

Regarding the scope of the analysis Schmidt *et al.* (2009) examine unobserved spatial effects at relatively small levels (i.e. municipalities), whereas we take different specifications, some not restricted by political boundaries. By examining different spatial structures we are able to discern how spatial dependence varies with different characterisations of neighbourhood, which is an aspect the Scmidt *et al.* (2009) conclude as worth to be investigated. Our approach enable us to obtain both the degree and significance of spatial dependence in the whole area studied for different characterisations of neighbour farms, whereas Schmidt *et al.* (2009) provide information only on the significance of spatial dependence at municipality level.

The following sections are dedicated to the description of the data used, and the methodology and empirical approaches for integrating spatial dependence into Stochastic Frontier analysis are presented. The empirical section includes a description of the data used and the results obtained. The article ends with a section devoted to conclusions.

2 Data

The analysis uses balanced panel data from the Farm Business Survey (FBS) for the years 2000-2005. A total of 215 dairy farms in England and Wales are included in the dataset. The FBS data include a large amount of information related to the farm enterprise. We classify farm output data into: i) milk and other dairy products, ii) leasing out quota, and iii) other products. Laspeyres and Paasche quantity indices were calculated in order to calculate a Fisher quantity index which aggregated the output in milk and other milk output into one variable and other products also into one variable. The base for price and output indices was calculated as the average of prices and outputs. With regard to inputs included, these are the utilised agricultural area (UAA) in ha; herd size (number of cows); labour (£); machinery and general farming costs (£), which includes contract work, machinery rental, machinery and equipment valuation[1], machinery and equipment repairs, vehicle fuel and oil, electricity, heating fuel for all purposes, water for all purposes, insurance excluding labour and farm buildings, bank charges professional fees, vehicle tax and other general farming costs; livestock costs (concentrate feedstuff, coarse fodder, veterinary services and medicines).

Spatial information on the farms was provided by the Department for Environment Food and Rural Affairs (Defra) as part of the Farm Business Survey. The FBS includes a grid reference which provides information on the location of the farm at a 10 km grid square level. This information was used to build a number of connectivity (or spatial weight) matrices W which gather the relative spatial information of the farms.

3 Methods

Spatial dependence refers to how much the level of technical inefficiency for a farm i depends on the level of the technical inefficiency set by other farms $j = 1, \ldots, n$. Spatial dependence implies that the inefficiency (z) of farms at

[1] Note that machinery (a flow) is not added to valuation (a stock) since equipment valuation is depreciated in the FBS. For instance, cars valuation accounts for 17% depreciation.

location i depend on how inefficient farms are at locations $j \neq i$. The joint density is not the product of the marginals for z_i and z_j at locations $j \neq i$.

3.1 *The spatial weight matrix*

Although the use of political divisions of the land in efficiency analysis may capture some effects associated with policies at regional, county or provincial levels there may be factors such as climatic and topographic conditions which differ within those political divisions. In order to account for those factors that may be present on a smaller or larger scale, a quantification of the structure of spatial dependence between farms for the efficiency term in the stochastic multi-output production function is introduced.

The spatial information of the farms can be introduced into a connectivity matrix or spatial weight matrix. A connectivity matrix can be defined in different ways depending on the researcher's views about what constitutes a neighbourhood, which will depend on previous information on the particular issue studied, or due to the type of spatial data that the researcher has (i.e. scale). The way in which the spatial weight matrix is specified is relevant. Two questions usually arise when analysing technical efficiency; how spatial dependence is specified and what size is considered adequate to specify which farms are close. This is especially problematic in micro data environments where observations are scattered throughout a landscape (Holloway and Lapar 2007; Bell and Dalton 2007). There are two main ways in which W can be defined:

a) A common specification for quantifying the structure of spatial dependence used in literature relies on a $n \times n$ spatial weight matrix W with elements $W_{ij} \geq 0$ (after being row standardised such as the row elements add up to one, as it facilitates the interpretation of model coefficients) for observations $j = 1, \ldots, N$ sufficiently close to observation i and $W_{ij} = 0$ otherwise (LeSage, 1999). Let's consider 4 farms where farm 1 is close enough to farm 2; farm 2 is close to farms 1, 2 and 3; farm 3 is close to farms 2 and 4; while farm 4 is close to farms 2 and 3. The spatial weight matrix based on this spatial example takes the form

$$W = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 \end{pmatrix} \tag{1}$$

The diagonal elements W_{ii} are set to 0 in order to preclude an observation of the efficiency z_i from directly predicting itself. The spatial weight matrix is row standardised so each element in the standardised matrix W^S, $w_{ij} = \frac{w_{ij}}{\sum_j w_{ij}}$, is between 0 and 1 as shown below

$$W^S = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 0.33 & 0 & 0.33 & 0.33 \\ 0 & 0.5 & 0 & 0.5 \\ 0 & 0.5 & 0.5 & 0 \end{pmatrix} \quad (2)$$

Close proximity can have different interpretations. Thus, it can mean adjacent neighbours or neighbours within a given distance. For the latter the elements of the W matrix are given by: $w_{ij} = 1$ if $0 <$ distance between $i, j \leq h$ (h is the distance beyond which no dependence is assumed); otherwise $w_{ij} = 0$ before being row standardised.

Effectively when we estimate a model such as $z = \rho W z + \varepsilon$ the parameter ρ measures the correlation between z and the weighted average of z. Under this approach all neighbours have the same weight in the average. If z referred to farm efficiency then ρ would represent the correlation between individual farm efficiency and the mean efficiency of the neighbouring farms.

b) An alternative approach to the one shown above is the use of weight matrix based on distance (Anselin 2002). In this case neighbours have different weights in the average, those with higher weights being the closest in distance. Therefore ρ would be the correlation between farm efficiency and adjusted by distance mean efficiency of neighbouring farms. This approach is also arbitrary in the sense that the cutoff distance is arbitrarily selected. The distance weight specification used here is one of a power form

$$w_{ij} = \exp\left(-d_{ij}^2/h^2\right) \tag{3}$$

where d_{ij} is the distance between a farm in location i and a farm in location j; h is the distance around a given observation over which other observations are likely to be dependent.

The cutoff distance chosen to determine the distance beyond which spatial effects are not relevant is a key issue. Bell and Bockstael (2000) found that their results were more sensitive to the specification of the spatial weight matrix (i.e. choice of the cutoff distance) than to the estimation technique used. They found that the spatial dependence estimate changed with the distance associated to the cutoff distance, increasing first at a small cutoff distance and falling afterwards as the cutoff distance was increased. They applied a higher order contiguity model and showed how spatial dependence diminishes with distance.

Roe et al. (2002) highlight that the appropriate cutoff distance is an empirical issue. They estimated their models using different cutoff distances. Kim et al. (2003) used SAR and SEM hedonic price models to measure the benefits of air quality improvement. The spatial weight matrix was specified based on distances between district centroids with a cutoff distance of 4 km chosen after experimenting with a series of different cutoff distances. These articles show that a cutoff distance exists where spatial dependence reaches a maximum.

3.2 *Scope*

Milk producer farm efficiencies in England and Wales are studied in this paper. Milk producers have an annual milk quota that partially binds production since producers can lease in and/or lease out milk during the production year. Therefore we include in the analysis the fact that production is partially constrained by the annual quota Q which includes the initial quota $\pm$ quota bought/sold, leasing in quota qui and leasing out quota quo. Not accounting for such constraints may lead to wrongly attributing the effects of such constraints to the farmer being unsuccessful in optimising production (Färe et al. 1994).

Assuming that producers optimise their production by not wasting resources leads them to operate near their production possibilities set. However there may be an array of motives for why not all producers are successful in optimising production. In this article we focus on developing a way to explain technical inefficiency through spatial dependency. The departure point of any technical efficiency analysis is the definition of the production technology of a firm. This can be characterised in terms of a technology set, the output set of production technology, and the production frontier.

3.3 *Output distance function*

We use a distance function approach since it describes technology in a way that allows efficiency to be measured for multi-input and multi-output enterprises (Coelli et al. 2005). An output distance function describes the degree to which a firm can expand its output given its input vector. We start from a producible output set, which is the set of all outputs that can be feasibly produced using the set of all inputs. The output set for production technology is defined as

$$\begin{aligned} P(x,Q) &= \left\{ y \in R_{+}^{M} : x\ can\ produce\ y\ given\ y = Q + qui - quo \right\} = \\ &= \{ y : (x,y) \in T \} \end{aligned} \tag{4}$$

where y refers to all M outputs of the farm including milk, the leasing out of quota (quo) and other outputs, which take only positive real numbers R_{+}^{M}, and x refers to all K inputs used in the farm, which take only positive real numbers R_{+}^{K}, including the leasing in quota (qui) and the annual allocation of quota Q which includes $\pm$ amount of quota bought/sold in the current year. The output set is included within the technological set T.

The output distance function is defined on the output set $P(x,Q)$ as

$$
\begin{aligned}
D_O(x,y,Q) &= \min\left\{\theta : \left(\frac{y}{\theta}\right) \in P(x,Q)\right\} \\
for\ all\ x \in R_+^K & \qquad (5)
\end{aligned}
$$

which means that the inital allocation of quota Q, the leasing in qui and leasing out quota quo are treated in the same way as conventional inputs (x) and outputs (y).

Assuming a translog functional form for the parametric distance function with M outputs and K inputs offers several attractive properties including flexibility, as well as making it easy to derive and permit the imposition of homogeneity, which makes it the preferred form in the literature (Coelli and Perelman 1999; Lovell et al. 1994; Brümmer et al. 2002; Brümmer et al. 2006).

$$
\begin{aligned}
\ln D_{Oi} &= \alpha_0 + \sum_{m=1}^{M} \alpha_m \ln y_{mi} + \frac{1}{2}\sum_{m=1}^{M}\sum_{n=1}^{M} \alpha_{mn} \ln y_{mi} \ln y_{ni} + \sum_{k=1}^{K} \beta_k \ln x_{ki} + \\
&+ \frac{1}{2}\sum_{k=1}^{K}\sum_{l=1}^{K} \beta_{kl} \ln x_{ki} \ln x_{li} + \sum_{k=1}^{K}\sum_{m=1}^{M} \delta_{km} \ln x_{ki} \ln y_{mi} \\
& i = 1, ..., N \qquad (6)
\end{aligned}
$$

where i denotes the ith farm in the sample; qui and Q are included in x as inputs; and quo are part of y as an output. By using linear homogeneity of the output distance function in outputs, equation (3) can be transformed into an estimable regression model by normalising the function by one of the outputs (Brümmer et al. 2006; Brümmer et al. 2002; Coelli and Perelman 1999; Coelli and Perelman 2000; Lovell et al. 1994; Orea 2002; O'Donnell and Coelli 2005). From Euler's theorem, homogeneity of degree one in output implies:

$$\sum_{m=1}^{M} \alpha_m + \sum_{m=1}^{M} \sum_{n=1}^{M} \alpha_{mn} \ln y_{ni} + \sum_{m=1}^{M} \sum_{k=1}^{K} \delta_{km} \ln x_{ki} = 1 \tag{7}$$

which will be satisfied if $\sum_{m=1}^{M} \alpha_m = 1$, $\sum_{m=1}^{M} \alpha_{mn} = 0$ for all n, and $\sum_{m=1}^{M} \delta_{km} = 0$ for all k. Substituting these constraints is equivalent to normalising by one of the outputs, which leads to the following expressions:

$$\ln D_O \left(\frac{y_i}{y_{2i}}, x \right) = \ln D_o \frac{1}{y_{2i}} \left(y_i, x \right) \tag{8}$$

and

$$\begin{aligned} -\ln y_2 \quad = \quad & \alpha_0 + \sum_{m=1}^{M} \alpha_1 \ln \frac{y_{mi}}{y_{2i}} + \frac{1}{2} \sum_{m=1}^{M} \sum_{n=1}^{M} \alpha_{mn} \ln \frac{y_{mi}}{y_{2i}} \ln \frac{y_{ni}}{y_{2i}} \\ & + \sum_{k=1}^{K} \beta_k \ln x_{ki} + \sum_{k=1}^{K} \sum_{l=1}^{K} \beta_{kl} \ln x_{kl} \ln x_{li} \\ & + \sum_{k=1}^{K} \sum_{m=1}^{M} \delta_{km} \ln x_{ki} \ln \frac{y_{mi}}{y_{2i}} + \varepsilon_i - z_i \end{aligned} \tag{9}$$

where ε_i is a symmetric random error term that accounts for statistical noise and z_i is a non-negative random variable associated with technical inefficiency.

Monotonicity constraints involve constraints on functions of the partial derivatives of the distance function. As pointed out by O'Donnell and Coelli (2005) the elasticities of distance with respect to inputs and outputs are important derivatives.

$$\frac{\partial \ln D_o}{\partial \ln x_k} = \beta_k + \sum_{k=1}^{K} \beta_{kl} \ln x_{li} + \sum_{m=1}^{M} \delta_{km} \ln \frac{y_{mi}}{y_{2i}} \tag{10}$$

$$\frac{\partial \ln D_o}{\partial \ln y_m} = \alpha_m + \sum_{m=1}^{M} \alpha_{mn} \ln \frac{y_{ni}}{y_{2i}} + \sum_{k=1}^{K} \delta_{km} \ln x_{ki} \tag{11}$$

For D_o to be non-increasing in x, $\frac{\partial \ln D_o}{\partial \ln x_k} \leq 0$ while for D_o to be non-decreasing in y $\frac{\partial \ln D_o}{\partial \ln y_m} \geq 0$. The data were normalised so that each variable had a sample mean of one. This means that the monotonicity conditions can be expressed as $\alpha_m \geq 0$ and $\beta_k \leq 0$. It is worth noting that coefficient results have been changed the sign and therefore the expected coefficients should be $\alpha_m \leq 0$ and $\beta_k \geq 0$.

We used the spatial information contained in the dataset to create a number of specifications of the W matrix and investigate the effect of these on the results. One involves the introduction of a spatial connectivity matrix whose common specification is $n \times n$ matrix W with elements

$W_{ij} = 1$ for farms $j = 1, \ldots, n$ within 10 square km grid to farm i and $W_{ij} = 0$ for those farms that are not close. Once W is row standardised this effectively accounts for the average efficiency of the farms surrounding the farm within the 10 Km square grid. Another alternative specifies that the spatial connectivity matrix W has elements $W_{ij} = 1$ for farms $j = 1, \ldots, n$ within the GOR of farm i and $W_{ij} = 0$ for those farms that are in the same GOR. Finally, four more alternatives were used by specifying a spatial distance matrix W with elements $W_{i,j} = d_{i,j}$ where d_{ij} is the Euclidean distance. The weight specification used was the power form (equation 11) and four cutoff distances were used (h =20 Km; h =100 km; h =180 km and h =240 km). As pointed out above, the selection of which definition of close proximity to use is arbitrary as is the size of the spatial effect (i.e. cutoff distance). The distance between farms is calculated using the Euclidean distance

$$d_{1,2} = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2} \tag{12}$$

where x_i, y_i are the coordinates of the points.

3.4 *Estimation*

A Bayesian procedure involving the use of a Gibbs sampler and two Metropolis-Hastings steps is used to estimate the spatial dependence of farm efficiency. We start with the standard stochastic output distance function model which is specified as

$$y_{it} = x_{it}\beta + \varepsilon_{it} - z_i \tag{13}$$

where y_{it} is a vector of the logarithm of milk and other milk products for each farm i in year t; x_{it} is a matrix of the logarithm of other outputs and inputs of the farm i in year t; β is a vector of parameters associated with the outputs and inputs of the farm to be estimated; ε_{it} is the random error and z_i represents the inefficiency of the farm. Note that efficiency here is understood to be the maximum output each farm can obtain with the given inputs. Stacking all the variables into matrices we obtain

$$y_i = x_i\beta + \varepsilon_i - z_i 1_T \tag{14}$$

where y_i, x_i and ε_i denote vectors of T observations. Or in matrix form

$$y = x\beta + \varepsilon - (z \otimes 1_T) \tag{15}$$

This standard model can be transformed to account for spatial dependence in the inefficiency term. The spatially dependent inefficiency term is

$$z = \rho W z + \tilde{z} \tag{16}$$

$$\tilde{z} = (I - \rho W)\, z \tag{17}$$

$$z = (I - \rho W)^{-1}\, \tilde{z} \tag{18}$$

where W is a connectivity matrix that includes the relative spatial information of the farms and ρ is the spatial coefficient and z and $\tilde{z}$ are latent variables whose distributional form is unknown. By plugging (18) into (15) we obtain the following expression

$$y = x\beta + \varepsilon - \left((I - \rho W)^{-1}\, \tilde{z}\right) \otimes 1_T \tag{19}$$

The parameter ρ is assumed to be between 0 and 1, although we will break this assumption in order to evaluate the robustness of our results.

3.4.1 *The conditional likelihood function*

The distributional assumptions determine the form of the likelihood function. Here, it is assumed that the prior distributions for the latent errors are normal and gamma distributed (Koop 2003; Koop et al. 1995). In this case normality is assumed. Note that we have $i = 1, \ldots, N$ farms observed during T years ($t = 1, \ldots, T$). Here $p()$ refers to the density and $p(|)$ is the conditional density.

$$p\left(y|\beta,h,\rho,\mu_z^{-1},\tilde{z}\right)=\prod_{i=1}^{N}\frac{h^{\frac{T}{2}}}{(2\pi)^{\frac{T}{2}}}\exp\left(-h\frac{\varepsilon_i'\varepsilon_i}{2}\right) \tag{20}$$

$$p\left(y|\beta,h,\rho,\mu_z^{-1},\tilde{z}\right)\propto h^{\frac{TN}{2}}\prod_{i=1}^{N}\exp\left(-h\frac{\varepsilon_i'\varepsilon_i}{2}\right) \tag{21}$$

noting that $p\left(y|\beta,h,\rho,\mu_z^{-1},\tilde{z}\right)=p\left(y|\beta,h,\rho,\mu_z^{-1},z\right)=p\left(y|\beta,h,\mu_z^{-1},\tilde{z}\right)$.

Defining $\tilde{y}_i=\left[y_i+(I-\rho W)^{-1}\tilde{z}_i\iota_T\right]$ the following expression is obtained

$$p\left(y|\beta,h,\mu_z^{-1},\tilde{z}\right)\propto h^{\frac{TN}{2}}\exp\left[-\frac{h}{2}\left(\tilde{y}_i-x_i\beta\right)'\left(\tilde{y}_i-x_i\beta\right)\right] \tag{22}$$

The expression above is of a standard form used for efficiency analysis (Koop 2003; Koop et al. 1995) with the spatial element being the extension of the model.

3.4.2 *The priors*

The likelihood function must be complemented with a prior distribution on the parameters $(\beta,h,\mu_z^{-1},\rho)$ and the latent variable z in order to conduct Bayesian inference. The matrix W is predefined rather than estimated. An independent Normal-Gamma prior is used for the coefficients in the production frontier and the error precision. The priors for β and h are

$$\beta\sim N\left(0,V_0\right) \tag{23}$$

$$p\left(h\right)=h^{\frac{v_0-2}{2}}\exp\left(\frac{-hs_0^2}{2}v_0\right) \tag{24}$$

where V_0, s_0^2 and v_0 are hyper parameters set prior to estimation.

The distribution of the inefficiency term is determined by the distribution of z, which is a latent variable. We define $p\left(\tilde{z}|\mu_z^{-1}\right)$ instead of $p\left(z|\mu_z^{-1}\right)$, which is defined given ρ, W and $p\left(\tilde{z}|\mu_z^{-1}\right)$. The prior for the latent variable $\tilde{z}$ is

$$p\left(\tilde{z}_i|\mu_z^{-1}\right)=f_G\left(\tilde{z}_i|\alpha,\mu_z^{-1}\right)=\frac{z_i^{\alpha-1}}{\mu^j\Gamma\left(\alpha\right)}\exp\left(-\mu_z^{-1}\tilde{z}_i\right) \tag{25}$$

where $\Gamma\left(.\right)$ is the gamma function; and $f_G\left(\tilde{z}_i|\alpha,\mu_z^{-1}\right)$ indicates the Gamma density with parameters α and μ_z^{-1}. This prior is commonly used in literature (Fernández et al. 2000; Koop et al. 1995; van Den Broeck et al. 1994). Assuming $\alpha=1$, the inefficiency distribution is exponential and the inefficiency prior becomes

$$p\left(\tilde{z}_i|\mu_z^{-1}\right)\propto\exp\left(-\mu_z^{-1}\tilde{z}_i\right) \tag{26}$$

The prior for μ_z^{-1} is assumed to be gamma with parameters 2 and $-\ln\left(r^*\right)$

$$p\left(\mu_z^{-1}\right)\propto f_G\left(\mu^{-1}|2,-\ln\left(r^*\right)\right) \tag{27}$$

$$p\left(\mu_z^{-1}\right)\propto\mu_z^{-1}\exp\left(\mu_z^{-1}\ln\left(r^*\right)\right) \tag{28}$$

where r^* is the median of the prior distribution.

Finally, the prior for ρ is assumed to be an indicator function.

$$f(\rho) = I(\rho \in [0,1]) \tag{29}$$

The expression above is a uniform distribution and its applicability depends on the appropriate construction of the weight matrix. The indicator function $I(\cdot) = 1$ if $\rho \in [0,1]$ or otherwise $I(\cdot) = 0$. This means that the parameter ρ that accounts for spatial dependence is expected to have a positive impact on the efficiency scores.

3.4.3 *The joint posterior*

The joint posterior distribution can be broken down into as the multiplication of the conditional likelihood function and the priors. The joint posterior in terms of z is

$$\begin{aligned} p\left(\beta, h, \mu_z^{-1}, z, \rho | y\right) &= p\left(y|\beta, h, \mu_z^{-1}, z\right) \times p(\beta) \times p(h) \times p\left(z|\mu_z^{-1}\right) \\ &\times p\left(\mu_z^{-1}\right) \times I(\rho \in [0,1]) \end{aligned} \tag{30}$$

3.4.4 *The conditional posteriors*

The Gibbs sampler is based on conditional distributions which describe the probabilities of a combination of values for parameters of interest which are conditional on the observables. The use of conditional distributions facilitates obtaining posterior distributions of the parameters of interest. In order to estimate the model it is useful to have the conditional distributions in order to employ the Gibbs sampling method (Geman and Geman 1984; Casella and George 1992). The conditional posterior for β is a Normal distribution after extracting the kernel for β from expression (30). For the full derivation of the conditional posteriors the reader is referred to the Appendix.

$$p\left(\beta|h, \mu_z^{-1}, \tilde{z}, \rho, y\right) \sim N\left(b, \bar{V}\right) \tag{31}$$

As in Koop (2003) the conditional posterior density for h is

$$p\left(h|\beta, \mu_z^{-1}, \tilde{z}, \rho, y\right) \sim G\left(\bar{s}^{-2}, \bar{v}\right) \tag{32}$$

In order to obtain the conditional posterior for μ_z^{-1} it is more useful to use $\tilde{z}$ rather than z. The joint conditional posterior density for μ_z^{-1} and $\tilde{z}$ is the kernel from expression (30) that involves μ_z^{-1} and $\tilde{z}$ (see Appendix for full derivation).

$$p\left(\tilde{z}, \mu_z^{-1}|\beta, h, y, \rho\right) \propto \prod_{i=1}^{N} \exp\left(-h\frac{\varepsilon_i^{'}\varepsilon_i}{2}\right) \times p\left(\tilde{z}|\mu_z^{-1}\right) \times p\left(\mu_z^{-1}\right) \tag{33}$$

from which the conditional posterior for μ_z^{-1} is

$$p\left(\mu_z^{-1}|\tilde{z},\beta,h,y,\rho\right) \propto p\left(\tilde{z}|\mu_z^{-1}\right) \times p\left(\mu_z^{-1}\right) \tag{34}$$

$$p\left(\mu_z^{-1}|\tilde{z},\beta,h,y,\rho\right) \sim G\left(m,\eta\right) \tag{35}$$

which is a Gamma distribution with parameters $m = \frac{N+1}{\sum_{i=1}^{N}\tilde{z}_i - \ln(r^*)}$ and $\eta = 2N+2$.

Recalling that z and $\tilde{z}$ are related as in expression (17) the conditional posterior distribution for $\tilde{z}_i$ is

$$p\left(\tilde{z}_i|\beta,h,\mu_z,y,\rho\right) \propto \exp\left[-\frac{hT}{2}\left[z_i - \left(\overline{x}_i^{'}\beta - \overline{y}_i + \frac{\mu_z^{-1}}{Th}\right)\right]^2 + \left(\tilde{z}_i - z_i\right)\mu_z^{-1}\right] \tag{36}$$

where $\overline{X}_i = \sum_{t=1}^{T}\frac{x_{i,t}}{T}$ and $\overline{y}_i = \sum_{t=1}^{T}\frac{y_{i,t}}{T}$.

The previous equation is not of a recognisable form. Therefore a posterior simulator (i.e a random number generator) needs to be used, such as a Metropolis-Hastings algorithm (Metropolis 1970; Hastings et al. 1953). We use a random walk algorithm proposal whereby a new set of $\tilde{z}_i$ are proposed using a Metropolis based on the posterior above. Given a new draw of $\tilde{z}_i$ then the entire z needs to be updated in each iteration. This is done using expression (18) above.

In order to obtain the conditional posterior of ρ the spatial problem can be represented in matrix form as

$$y + (z \otimes 1_T) = X\beta + \varepsilon \tag{37}$$

$$\left(y + \left((I - \rho W)^{-1} \tilde{z}\right) \otimes 1_T\right) - X\beta = \varepsilon \tag{38}$$

It follows that the conditional posterior for ρ is

$$\begin{aligned} p\left(\rho | \beta, h, \mu_z, y, \tilde{z}_i\right) &\propto \exp\left(-h\frac{\varepsilon' \varepsilon}{2}\right) \times p\left(\rho\right) \\ &= \exp\left(-h\frac{\varepsilon' \varepsilon}{2}\right) \times I\left(\rho \in (0,1)\right) \end{aligned} \tag{39}$$

which provides the basis for the use of a second Metropolis-Hastings step. A random walk Metropolis-Hastings algorithm is used to draw ρ with probability of acceptance of the proposed ρ^* being

$$prob = \min\left(1, \frac{p\left(\rho^* | y, \beta, h, z\right)}{p\left(\rho^{old} | y, \beta, h, z\right)}\right) \tag{40}$$

Recall that expresions (37-39) are for the case that $\rho > 0$. In the case of $\rho = 0$ (i.e. there is no spatial component) note that $z = \tilde{z}$.

4 Results

We expect the nature of the connectivity matrix will determine the results and for this reason we wish to explore alternative specifications for the weight matrix. We would expect ρ to increase with the cutoff distance for the spatial effects up to a distance and then decrease. We would expect the spatial dependence to be lower for small neighbourhoods since such areas may not include the whole area which has a spatial incidence on efficiency. In addition, we would expect that once we reach a given cutoff distance the spatial effect should decrease indicating that the spatial dependence has a limit. Two spatial models for inefficiency were estimated, one where the weight or connectivity matrix is specified regarding neighbours as farms within a 10 km square grid (SM1); one where neighbours are those farms in the same GOR (SM2); and another where the connectivity matrix is specified as a distance matrix (SM3). The SM3 was estimated using 4 cutoff distances, 20, 100, 180 and 240 km (SM3-20; SM3-100; SM3-180 and SM3-240).

Results for the parameters associated with inputs and outputs of the production function are shown in table 1 for models SM1 and SM2; table 2 for models SM3-20 and SM3-100 and table 3 for models SM3-180 and SM3-240. All signs are as expected with the exception of the coefficient for the leasing quota in, which is negative but the 90% coverage posterior region shows that there is no clear evidence that supports the belief that this coefficient is negative. The number of cows and milk quota allocated at the beginning of the year are the two most important inputs in terms of milk production whereas the production of other outputs by the farm reduces the production of milk, holding everything else constant.

Figures 1 and 2 show the kernel distributions for farm efficiency for models SM1, SM2 and SM3. Results suggest that the way in which the connectivity matrix is defined has an impact on the levels of efficiency. The efficiency average is 0.86 when neighbours are considered to be those within a 10 km grid square and 0.78 when neighbours are considered to be those farms in the same region. Figure 2 shows smaller differences between the alternatives. The mean efficiencies are 0.84, 0.81, 0.80 and 0.80 for SM3-20, SM3-100, SM3-180 and SM3-240 respectively.

Regarding the results for the conditional posterior distribution for the spatial dependence parameter ρ, these are shown in Figures 3 and 4. Results for SM1 and SM2 are shown in figure 3 whereas the four alternatives of MS2 are shown in Figure 4. Spatial models SM1 and SM2 show similar results for the spatial parameter ρ with averages of 0.13 and 0.18 respectively which suggests that efficiency is farm determined rather than spatially determined. The parameter ρ is 59.2% more likely to be higher using SM2 than SM1 which suggests that the spatial dependence is larger than just a 10 km square grid.

Models SM3 were run to investigate the effect of the cutoff distance chosen on the correlation ρ between efficiency and the adjusted by distance mean efficiency. Results show that the spatial dependence parameter ρ increases with the cutoff distance up to a point between 100 km and 240 km and then decreases. The probability that ρ using SM3-180 is higher than using SM3-240 is 53% whereas the probability that ρ using SM3-180 is higher than SM3-100 is 59%. These results indicate that the spatial parameter ρ may increase with the cutoff distance but will decrease once the cutoff reaches a distance between 100 and 240 km. These results are similar to those obtained by Bell and Bockstael (2000) where the spatial estimate increases and then falls. A reason for this is that spatial matrix W at small distances may not contain enough observations that help to obtain a good estimate of the mean efficiency in the neighbourhood. The spatial estimate will start to fall once farms that are not related in terms of efficiency with the farm of interest start to be included in the spatial distance weight matrix W. This will occur at a given distance. With regard to the mean of ρ this is 0.14, 0.31, 0.35 and 0.34 for the 20, 100 and 180 and 240 km alternative models respectively.

5 Conclusions

The work outlined in this article has shown how spatial dependence can be accounted for within a stochastic frontier model, thus filling a gap in the literature. The application of these techniques gives insightful information on whether there is spatial dependence in technical efficiency.

Results for the conditional posterior of the spatial dependence parameter ρ are sensitive to the specification of the spatial weight matrix. It may not be only due to whether we use a connectivity matrix or a distance based spatial matrix but also due to the cutoff size chosen. Thus results from the connectivity matrix raise the question of how big the size of the spatial effect is. Mean spatial dependence reaches its maximum over a 100 km distance from the farm. Therefore, an examination of how sensitive results are to the type of weight put to individual farms as well as to the cutoff size chosen must be conducted in order to present meaningful results.

Results suggest that there is a spatial dependence aspect in technical efficiency in dairy farms in England and Wales, and not accounting for it may produce biased results for the efficiency distribution. Farm technical efficiency depends to some degree on where the farm is located and therefore policies aiming to improve efficiency should take this into account.

When analysing spatial heterogeneity there is not a strong reason to support this being analysed at the political division level. In fact, usually heterogeneity occurs due to the geographical and climatic characteristics of the area, which do not necessarily have to coincide with the political divisions of the land. Therefore it should not be surprising that heterogeneity is not found at political division level and it should be analysed taking this into account. The consequences of studying heterogeneity at the wrong spatial level may be important as policy decisions would be based on misleading information. For example, based on an analysis whose results show that no heterogeneity is found between a number of regions the same policy may be applied for these regions. However, if heterogeneity is in fact present at other smaller or larger spatial levels a more appropiate policy would be to apply different policies within those regions or covering various regions. Results shown in this article are important for policy makers as they highlight that policies devoted to improving farm performance do not have necessarily to be applied at the national or regional level. Spatial dependency or heterogeneity may cross political borders or differ within the same political region. This represents a challenge to policy makers on how to implement policies at the "right" geographical level. Governments would like to see production allocated to those areas where efficiency is higher and/or help to increase efficiency in those areas where efficiency can be improved. This article has shown that farm specific inefficiency associated with spatial dependence can be identified as well as those farms which may need help in improving their performance. Most importantly, since farm efficiency was found to be spatially dependent this means that there are drivers behind technical efficiency that are correlated with where farms are located. Identification of these drivers can have a major impact on designing policies aiming to improve farm performance.

Two are the areas on which future research should focus. Firstly, research should focus on developing ways to estimate the distance at which the dependence parameter reaches its maximum. This would be helpful to design more accurately the spatial level of policies that aim to improve farm efficiency. Secondly, once it has been identified that spatial dependence exists, research should concentrate on identifying and incorporating into the analysis potential explanatory factors for such spatial dependence.

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Appendix

The conditional densitites

The derivation of the conditional posterior for β. Defining $b = \left(V_0^{-1} + \sum X_i X_i\right)^{-1} \sum X_i \left(y_i + 1_T z_i\right)$ leads to the following result

$$p\left(\beta | h, z, \mu_z^{-1}, y\right) \propto \exp\left(-h \frac{(\beta - b)' \left(V_0^{-1} + \sum_i X_i X_i'\right)(\beta - b)}{2}\right) \tag{41}$$

$$p\left(\beta | h, z, \mu_z^{-1}, y\right) \sim N\left(b, h^{-1}\left(V_0^{-1} + \sum_i X_i X_i'\right)^{-1}\right) \tag{42}$$

The derivation of the conditional posterior for h. The kernel for h from expression (30) is

$$p\left(h | z, \mu_z^{-1}, y, \beta\right) \propto h^{\frac{TN}{2}} \prod_{i=1}^{N} \exp\left(-h \frac{\varepsilon_i' \varepsilon_i}{2}\right) \times h^{\frac{v_o - 2}{2}} \exp\left(\frac{-h s_0^2}{2} v_0\right) \tag{43}$$

thus

$$p\left(h|z,\mu_z^{-1},y,\beta\right) \propto h^{\frac{TN}{2}+\frac{v_0}{2}-1}\prod_{i=1}^{N}\exp\left(-h\left(\frac{\varepsilon_i'\varepsilon_i+\frac{s_0^2 v_0}{n}}{2}\right)\right) \quad (44)$$

Using

$$\frac{\sum_{i=1}^{N}\varepsilon_i'\varepsilon_i+s_0^2 v_0}{2}=\frac{TN+v_0}{2\bar{s}^{-2}} \quad (45)$$

where

$$\bar{s}^{-2}=\frac{TN+v_0}{\sum_{i=1}^{N}\varepsilon_i'\varepsilon_i+s_0^2 v_0} \quad (46)$$

It follows that the precision h has a posterior gamma distribution using Koop's notation (Koop, 2003).

$$p\left(h|z,\mu_z^{-1},y,\beta\right)\sim G\left(\frac{TN+v_0}{\sum_{i=1}^{N}\varepsilon_i'\varepsilon_i+s_0^2 v_0},v_o+TN\right) \quad (47)$$

The derivation of the conditional posterior for μ_z^{-1} is as follows. From expression (34) in the text:

$$\begin{aligned}
p\left(\mu_z^{-1}|\beta,h,\rho,\tilde{z},y\right) &\propto \left[\prod_{i=1}^{N}\mu_z^{-1}\exp\left(-\tilde{z}_i\mu_z^{-1}\right)\right]\times\exp\left(\mu_z^{-1}\ln\left(r^*\right)\right)\\
&= \mu_z^{-N}\exp\left(\sum_{i=1}^{N}-\tilde{z}_i\mu_z^{-1}+\mu_z^{-1}\frac{\ln\left(r^*\right)}{N}\right)\\
&= \left(\mu_z^{-1}\right)^{\frac{2(N+1)}{2}-1}\exp\left(-\mu_z^{-1}\left(\sum_{i=1}^{N}z_i-\ln\left(r^*\right)\right)\right) \quad (48)
\end{aligned}$$

Using

$$\left(\sum_{i=1}^{N} \tilde{z}_i - \ln\left(r^*\right)\right) = \frac{2N+2}{2m} \tag{49}$$

$$m = \frac{N+1}{\left(\sum_{i=1}^{N} \tilde{z}_i - \ln\left(r^*\right)\right)} \tag{50}$$

we obtain

$$p\left(\mu_z^{-1}|\beta, h, \rho, \tilde{z}, y\right) \sim G\left(\frac{N+1}{\sum_{i=1}^{N} \tilde{z}_i - \ln\left(r^*\right)}, 2N+2\right) \tag{51}$$

The derivation of the conditional posterior for $\tilde{z}$ is as follows. From expression (34) in the text:

$$p\left(\tilde{z}, |\mu_z^{-1} y, \beta, \rho, h\right) \propto \left[\prod_{i=1}^{N} \exp\left(-h\frac{\varepsilon_i^{'}\varepsilon_i}{2}\right)\right] \times p\left(\tilde{z}|\mu_z^{-1}\right) \tag{52}$$

The ith inefficiency has the posterior

$$p\left(\tilde{z}_i, |\mu_z^{-1} y, \beta, \rho, h\right) \propto \exp\left(-h\frac{\varepsilon_i^{'}\varepsilon_i}{2} - \tilde{z}_i \mu_z^{-1}\right) \times I\left(\tilde{z}_i > 0\right) \tag{53}$$

Using:

$$\left(e_{i,t} + \frac{\mu_z^{-1}}{Th}\right)^2 = e_{i,t}^2 + \frac{\mu_z^{-2}}{T^2 h^2} + 2e_{i,t}\frac{\mu_z^{-1}}{Th} \tag{54}$$

and

$$\varepsilon_i^{'}\varepsilon_i = \sum_{t=1}^{T} e_{i,t}^2 = \sum_{t=1}^{T}\left(e_{i,t} + \frac{\mu_z^{-1}}{Th}\right)^2 - \frac{\mu_z^{-2}}{Th^2} - 2\bar{e}_i\frac{\mu_z^{-1}}{h} \tag{55}$$

where $\bar{e}_i = \sum_{t=1}^{T}\frac{e_{i,t}}{T}$.

It follows that

$$\begin{aligned} p\left(\tilde{z}_i, |\mu_z^{-1}y, \beta, \rho, h\right) &\propto \exp\left(-h\frac{\sum_{t=1}^{T}\left(e_{i,t} + \frac{\mu_z^{-1}}{Th}\right)^2}{2} + \bar{e}_i\mu_z^{-1} - \tilde{z}_i\mu_z^{-1}\right) \\ &\times I\left(\tilde{z}_i > 0\right) \end{aligned} \tag{56}$$

Assuming $\rho \neq 0$ and recalling that $\tilde{z} = (I - \rho W)\, z$

$$\begin{aligned} \bar{e}_i\mu_z^{-1} - \tilde{z}_i\mu_z^{-1} &= \bar{e}_i\mu_z^{-1} - z_i\mu_z^{-1} + (\tilde{z}_i - z_i)\,\mu_z^{-1} \\ &= \bar{y}_i - \bar{x}_i\beta + (\tilde{z}_i - z_i)\,\mu_z^{-1} \end{aligned} \tag{57}$$

$$\begin{aligned} p\left(\tilde{z}_i, |\mu_z^{-1}y, \beta, \rho, h\right) &\propto \exp\left(-Th\frac{\left(z_i - \left(x_i^{'}\beta - \bar{y}_i + \frac{\mu_z^{-1}}{Th}\right)\right)}{2} + (\tilde{z}_i - z_i)\,\mu_z^{-1}\right) \\ &\times I\left(\tilde{z}_i > 0\right) \end{aligned} \tag{58}$$

Table 1 Slope parameters for models SM1 and SM2

		SM1		SM2
	Coeff.	**90% posterior**	**Coeff.**	**90% posterior**
α_0	-0.03	$(-0.10, 0.04)$	0.04	$(-0.06, 0.18)$
Leasing quota out	-0.12	$(-0.18, -0.05)$	-0.11	$(-0.18, -0.05)$
Other output	-0.28	$(-0.33, -0.24)$	-0.29	$(-0.34, -0.25)$
Utilised Agricultural Area	0.05	$(0.00, 0.10)$	0.05	$(0.00, 0.12)$
Milk Quota	0.39	$(0.28, 0.50)$	0.35	$(0.24, 0.46)$
Number of cows	0.42	$(0.31, 0.54)$	0.46	$(0.34, 0.58)$
Leasing quota in	-0.02	$(-0.06, 0.02)$	-0.02	$(-0.06, 0.02)$
Machinery&General costs	0.10	$(0.03, 0.18)$	0.10	$(0.02, 0.17)$
Labour costs	0.03	$(-0.03, 0.09)$	0.036	$(-0.03, 0.10)$
Livestock costs (per cow)	0.16	$(0.10, 0.22)$	0.181	$(0.12, 0.25)$

Table 2 Slope parameters for models SM3-20 and SM3-100

		SM3-20		**SM3-100**
	Coeff.	**90% posterior**	**Coeff.**	**90% posterior**
α_0	0.02	$(-0.06, 0.11)$	0.17	$(-0.03, 0.51)$
Leasing quota out	-0.11	$(-0.18, -0.05)$	-0.11	$(-0.17, -0.05)$
Other output	-0.29	$(-0.34, -0.24)$	-0.29	$(-0.34, -0.25)$
Utilised Agricultural Area	0.06	$(0.00, 0.12)$	0.06	$(0.00, 0.12)$
Milk Quota	0.35	$(0.24, 0.46)$	0.31	$(0.17, 0.43)$
Number of cows	0.45	$(0.34, 0.57)$	0.50	$(0.36, 0.65)$
Leasing quota in	-0.02	$(-0.05, 0.02)$	-0.01	$(-0.05, 0.02)$
Machinery&General costs	0.10	$(0.02, 0.18)$	0.09	$(0.00, 0.16)$
Labour costs	0.03	$(-0.04, 0.10)$	0.04	$(-0.03, 0.12)$
Livestock costs (per cow)	0.18	$(0.12, 0.25)$	0.20	$(0.13, 0.29)$

Table 3 Slope parameters for models SM3-180 and SM3-240

		SM3-180		**SM3-240**
	Coeff.	**90% posterior**	**Coeff.**	**90% posterior**
α_0	0.21	$(-0.02, 0.57)$	0.19	$(-0.02, 0.52)$
Leasing quota out	-0.11	$(-0.17, -0.04)$	-0.11	$(-0.17, -0.04)$
Other output	-0.29	$(-0.34, -0.25)$	-0.29	$(-0.34, -0.25)$
Utilised Agricultural Area	0.06	$(0.00, 0.13)$	0.06	$(0.00, 0.12)$
Milk Quota	0.29	$(0.15, 0.42)$	0.30	$(0.16, 0.43)$
Number of cows	0.51	$(0.37, 0.66)$	0.51	$(0.37, 0.66)$
Leasing quota in	-0.01	$(-0.05, 0.03)$	-0.01	$(-0.05, 0.02)$
Machinery&General costs	0.09	$(0.00, 0.17)$	0.08	$(0.00, 0.16)$
Labour costs	0.04	$(-0.03, 0.12)$	0.05	$(-0.03, 0.12)$
Livestock costs (per cow)	0.21	$(0.13, 0.29)$	0.20	$(0.13, 0.28)$

Fig. 1 Efficiency distributions for SM1 vs SM2 models

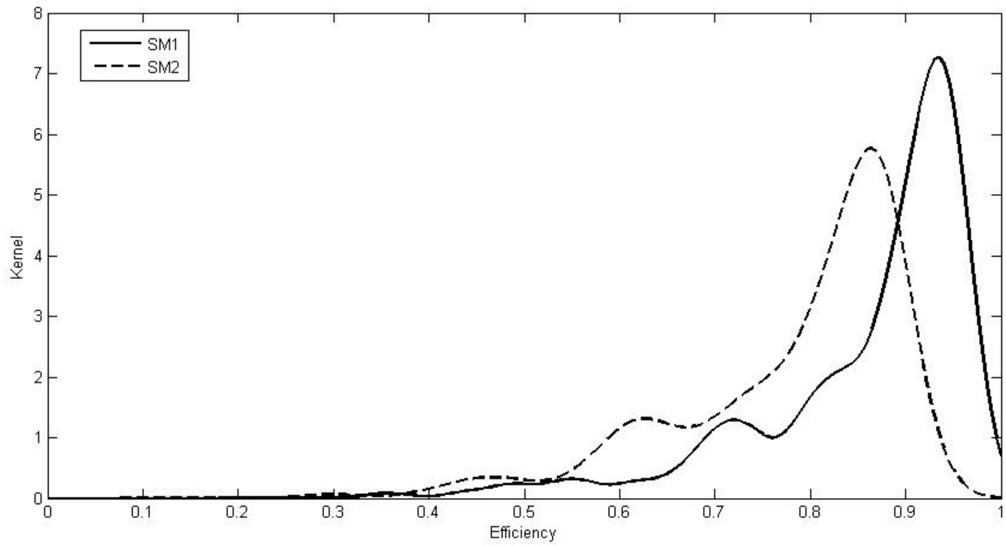

Fig. 2 Efficiency distributions for SM3 models

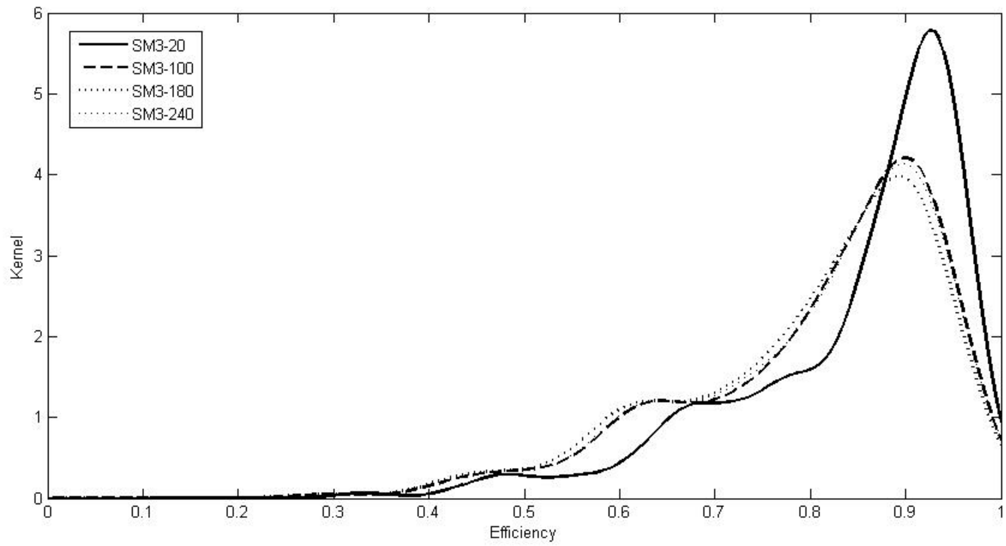

Fig. 3 Kernel distribution for ρ: 10 km grid square vs. GOR

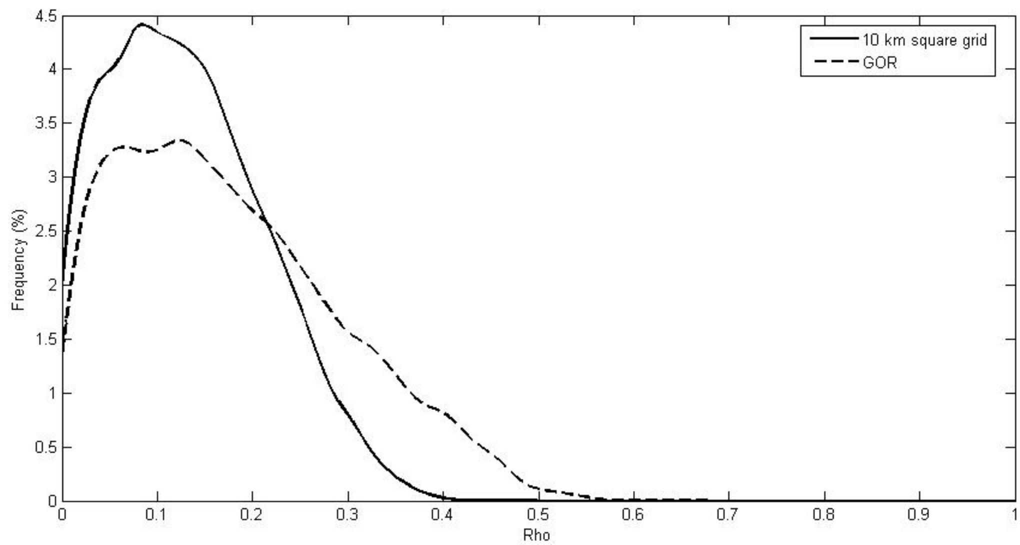

Fig. 4 Kernel distribution for ρ for $h = 20$ km; $h = 100$ km; $h = 180$ km; $h = 240$ km