Mean, Median or Mode? A Striking Conclusion From Lottery Experiments

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Abstract

This paper deals with estimating data from experiments determining lottery certainty equivalents. The paper presents the parametric and nonparametric results of the least squares (mean), quantile (including median) and mode estimations. The examined data are found to be positively skewed for low probabilities and negatively skewed for high probabilities. This observation leads to the striking conclusion that lottery valuations are only nonlinearly related to probability when means are considered. Such nonlinearity is not confirmed by the mode estimator in which case the most likely lottery valuations are close to their expected values. This means that the most likely behavior of a group is fully rational. This conclusion is a significant departure from one of the fundamental results concerning lottery experiments presented so far.

JEL classification: C01, C13, C14, C21, C51, C81, C91, D03, D81, D87

Keywords: Lottery experiments, Certainty Equivalents, Least Squares, Quantile, Median, and Mode Estimators, Relative Utility Function, Prospect Theory.

1. Introduction

1.1. One of the most important questions posed when describing a set of data is how to capture important information about a random variable in a single quantity. Mean and median values are typically considered measures of central tendency. The median is typically preferred as it is more robust to outliers than the mean. However several other definitions of "average" value are also encountered. The mode is less often used despite being the most frequently occurring value in the data set.

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Similar question arises whenever a statistical model is built and an estimation technique has to be chosen. As the least squares estimation results in the conditional mean and the least absolute value estimation results in the conditional median, these two procedures are the most commonly used in practical applications. However, considerations between mean, median, and mode assume less importance when the distribution of a random variable is symmetric. In such a case, the mean, median and mode all coincide, and the sample mean can be used as an estimate of the population mode.

The situation is very different for skewed data, which are found in many applications (e.g. wages, prices, etc). For positively skewed data, the mode is generally less than the median, which is less than the mean. For negatively skewed data, the reverse holds true. Incorrect assumptions about random variable distribution can therefore invalidate statistical inference.

1.2. This paper analyzes data from experiments determining the certainty equivalents of lotteries. The typical approach presented by Tversky and Kahneman (1992), Gonzales and Wu (1999) and others, is to estimate the model parameters by using the median values of the certainty equivalents for specific lotteries and applying the nonlinear least squares procedure. This assumes a random (symmetric) distribution of errors. The results obtained by the authors point to the nonlinearity of lottery valuations: the lotteries are valued more than their expected values for low probabilities and less than their expected values for higher probabilities.

This paper shows that the distribution of data found in lottery experiments is not symmetric and varies with probability. In particular, the relative certainty equivalents for the two data sets are positively skewed for low probabilities, negatively skewed for high probabilities and not skewed for medium probabilities. This was first observed when using the standard measure of skewness. It was further confirmed by least squares (mean), quantile (including median) and mode estimations, where all these estimations were performed both parametrically and nonparametrically. The results led to the striking conclusion that the lottery valuation is only nonlinearly related to probability when the means and (in one of two sets) medians of certainty equivalents are considered. Such nonlinearity disappears once the mode estimator is used. This means that the most likely lottery valuations are close to their expected values. Another way of saying this is that the most likely behavior of the examined groups was fully rational. This conclusion is a significant departure from one of the fundamental results concerning lottery experiments presented so far.

1.3. This paper makes several contributions to the existing literature on analyzing lottery experiment results. To the best of the author's knowledge, this is the first paper to present: (i) an analysis of certainty equivalents distribution; (ii) a simple nonparametric estimation method; (iii) a quantile (including median) estimation; and (iv) a mode estimation.

All these contributions were made possible by the relative utility function model, which, in contrast to the Prospect Theory model, adopts the classical econometric approach when describing experimental data. This difference is explained in more detail in Point 2. Point 3 presents the data used for the analysis together with their characteristics obtained from standard measures of distribution moments. Point 4 covers nonparametric estimation methods and point 5 is devoted to parametric methods. Point 6 summarizes the results of the paper.

2. Prospect Theory and the Relative Utility Model

2.1. Certainty equivalents are typically determined in lottery experiments for several combinations of outcomes and for several probabilities of winning[3]. Usually, 20-30 subjects are examined in each experiment. An example of a data set can be found in Tversky and Kahneman (1992). These data served to derive the Cumulative version of Prospect Theory, for which Kahneman was awarded the Nobel Prize for Economics in 2002. Table 2.1 presents some of these data for illustrative purposes.

Probability / Lotttery	.01	.05	.10	.25	.50	.75	.90	.95	.99
{0, 100}		14		25	36	52		78	
{50, 150}		64		72.5	86	102		128	

Table 2.1. Median certainty equivalents for two lotteries from Tversky and Kahneman's experiment (1992). The given probabilities concern the greater of two outcomes.

2.2. Applying the classical econometric approach to the description of lottery experiments would require building a model in which certainty equivalents are explained by a nonlinear function of lottery parameters (outcomes and probabilities). Prospect Theory, however, assumes another approach. Certainty equivalents are related to lottery parameters through the so called lottery (prospect) value V, which is a product of two nonlinear functions – the value function $v(x)$ and the probability weighting function $w(p)$. The certainty equivalent has a value of $v(ce)$, as it is a specific form of lottery in which the probability of winning equals 1. In the simplest case, when a lottery has two outcomes, the lower of which has a

[3] A possible, but very rare, approach is the opposite one i.e. determining probability for different certainty equivalent values.

value of 0, the following relationship has to be resolved using the estimation procedure[4]:

$$v(ce) = v(P)\,w(p), \tag{2.1}$$

where P = Max(x) is the maximum lottery outcome, p denotes the probability of winning the prize, the two functions v and w are estimated, and the prospect value V is not present any more. This relationship is clearly not the most convenient estimation model[5]. Moreover, due to both the hidden representation of certainty equivalents and the product of the two estimated functions, the Prospect Theory model does not allow the joint presentation of experimental and estimation results on the one graph.

Tversky and Kahneman (1992) and Gonzales and Wu (1999) estimated the model parameters by using the median values of the certainty equivalents and applying the nonlinear least squares procedure. However, due to the model adopted "the standard nonlinear regression technique does not permit an examination of residuals for v and w separately" (Gonzales and Wu, 1999). This means that the Prospect Theory model makes it difficult, if not impossible, to notice the asymmetry of data. Almost every derivation of this model encountered in the literature therefore assumes a normal error distribution.

2.3. The model of the relative utility function provided by Kontek (2009a) results from the classical econometric approach to data description. This model assumes first that the certainty equivalent ce is a nonlinear function g of the lottery parameters:

$$ce = g(P, p). \tag{2.2}$$

As the certainty equivalent ce is a monotonic function of probability p, the following relationship should also hold:

$$p = h(ce, P), \tag{2.3}$$

where h is a nonlinear function. It is assumed that the certainty equivalent can be expressed in relation to the lottery prize P:

$$r = \frac{ce}{P}, \tag{2.4}$$

where r denotes the relative certainty equivalent. Eq. (2.3) then yields:

[4] If the lower payment is greater than 0, the formula is more complex (see Gonzales, Wu, 1992). The relationship becomes even more complicated for lotteries with more than two outcomes. However, as the details of Cumulative Prospect Theory digress from the main subject of the paper, they are not presented here.

[5] Gonzales and Wu (1999) state that "Estimation of the value function and weighting function in the context of utility function theory presents challenging problems. A major stumbling block is the need to use the inverse of the value function in estimation. In an experiment, however, one observes the ce rather than $v(ce)$."

$$p = Q(r), \tag{2.5}$$

where Q denotes a relative utility function, which should have the form of a cumulative density function defined over the range [0,1]. It should be emphasized that transforming the certainty equivalent to its relative form r is not just an artificial transformation, but is based on the observation that changes in wealth are perceived in relative rather than absolute terms (Kontek, 2009b). It therefore follows that the certainty equivalents of a lottery are perceived in relation to its prize.

As probability p is a single-variable function of the relative certainty equivalent r then r can be easily expressed as a function of p:

$$r = Q^{-1}(p), \tag{2.6}$$

where Q^{-1} is the inverse of the relative utility function. Because it is certainty equivalents, rather than probabilities, that are typically determined in experiments, the inverse form (2.6) of the relative utility function will be mostly used throughout the paper.

2.4. The assumed model may also be derived from the basic Prospect Theory model. According to Prospect Theory, the value function $v(x) = x^{\alpha}$, so (2.1) can be transposed to:

$$p = w^{-1}\left[\frac{v(ce)}{v(P)}\right] = w^{-1}\left[\frac{ce^{\alpha}}{P^{\alpha}}\right] = w^{-1}\left[\left(\frac{ce}{P}\right)^{\alpha}\right] = w^{-1}\left(r^{\alpha}\right), \tag{2.7}$$

from which is it clear that probability p is a single-variable function of the relative certainty equivalent r (cf. (2.5)). Consequently, rearranging (2.7) gives:

$$r = w(p)^{\frac{1}{\alpha}}, \tag{2.8}$$

from which it is clear that the relative certainty equivalent r is a single-variable function of probability p (cf. (2.6)). It is interesting to note that the relative certainty equivalent r also appears in Prospect Theory, although not in a transparent way.

2.5. According to the relative utility model, if the lower lottery outcome is greater than 0, then the value of r is expressed by:

$$r = \frac{ce - A}{P - A}, \tag{2.9}$$

where $A = Min(x)$ is the minimum lottery outcome. The relationship described by (2.9) ensures that r assumes values from the range [0, 1], even in the case of lotteries with a risk free component.

2.6. The main advantage of the relative utility approach is that it can explain the results of lottery experiments using only one function. Moreover, it is a single-variable model and should therefore allow both the data and the estimated results to be easily observed. The direct relationship between r and p makes the estimation procedure much easier and allows the model residuals to be examined. To illustrate the basic transformations (2.4) and (2.9), Table 2.2 presents the values of the relative certainty equivalents r for the certainty equivalents presented in Table 2.1.

Probability / Lottery	.01	.05	.10	.25	.50	.75	.90	.95	.99
{0, 100}		0.14		0.25	0.36	0.52		0.78	
{50, 150}		0.14		0.225	0.36	0.52		0.78	

Table 2.2. Values of the relative certainty equivalent r for the data from Table 2.1.

2.7. It is possible to propose several functional forms for the relative utility function Q. The aim of this paper, however, is not to compare specific types of functions, but to present the estimation methodology. Beta distribution is therefore the only one used in this paper, as it is the best known and most widely used distribution defined over the interval [0,1]. Hence, the function Q is described using Cumulative Beta Distribution as follows:

$$p = Q(r) = I(r;\alpha,\beta), \tag{2.10}$$

where I denotes the regularized incomplete beta function. The curve is S-shaped for $\alpha > 1$ and $\beta > 1$, inversed S-shaped for $0 < \alpha < 1$ and $0 < \beta < 1$, J-shaped for $\alpha > 1$ and $0 < \beta < 1$, and inverse J-shaped for $0 < \alpha < 1$ and $\beta > 1$. For $\alpha = 1$ and $\beta = 1$ the curve is linear. The inverse form of (2.10) is:

$$r = Q^{-1}(p) = I^{-1}(p;\alpha,\beta), \tag{2.11}$$

where I^{-1} denotes the inverse of the regularized incomplete beta function. More on the relative utility function approach, especially as applied to multi-prize lotteries, can be found in Kontek (2009a).

3. Characteristics of Data Sets

3.1. The full set of Tversky and Kahneman's data can now be visualized using the methodology presented in Point 2 (see Figure 3.1). These data are less useful for further research as only the medians of the lottery results were published. For this reason, this paper presents the estimation methodology using two other data sets.

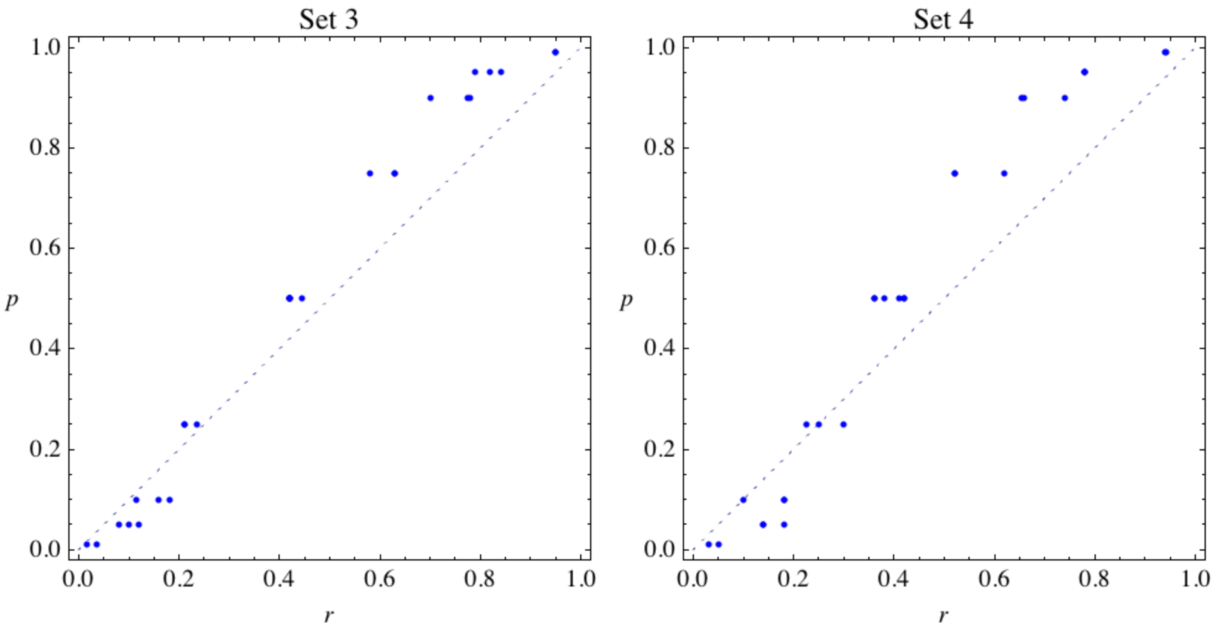

Figure 3.1. Tversky and Kahneman's data transformed using (2.9). The points represent the median values of certainty equivalents for specific lotteries. Set 3 consists of data for losses and Set 4 consists of data for gains.

3.2.1. Set 1 - the experimental data presented by Traub and Schmidt (2009), whose research concerned the relationship between WTP (Willingness to Pay) and WTA (Willingness to Accept). Twenty four subjects participated in the experiment. Only that subset of the data covering the certainty equivalents of two outcome lotteries was used in further analyses.

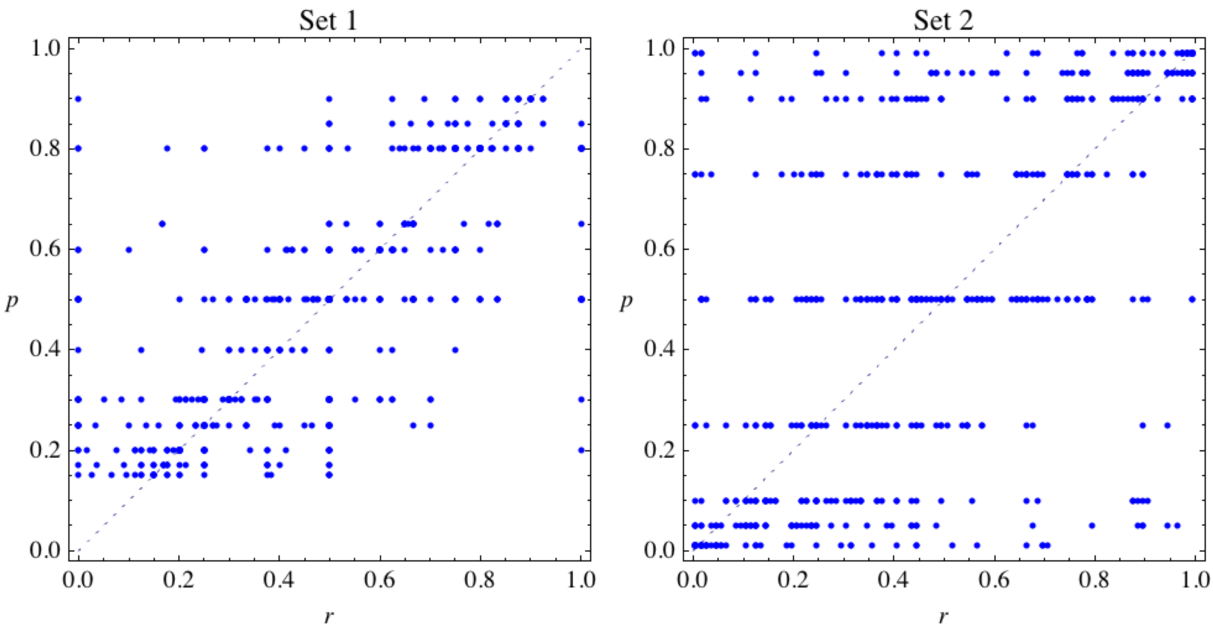

Figure 3.2. Experimental results transformed using (2.9) for Set 1 and Set 2.

3.2.2. Set 2 - the experimental data of Idzikowska (2009), whose research concerns the question of whether the form in which probability is presented has any impact on the shape of the probability weighting function. Twenty five subjects participated in the experiment and some of the responses were disregarded by Idzikowska on account of their inconsistency. The present research uses that subset of the data related to experimentally learned probabilities.

3.3. The data from both sets were used without correction. The only exception regarded Set 1, when the value of r was less than 0 or greater than 1. This resulted from providing certainty equivalent values outside the acceptable range (e.g. \$25 for a \$30-40 lottery or \$12 for a \$0-10 lottery). In such cases, the values of r were corrected to 0 and 1 respectively.

3.4. First and foremost, it should be noted that the data are scattered in both cases (in Set 2 even more than in Set 1). This results from the wide range of certainty equivalent values provided by the subjects which may, in turn, indicate very diverse risk attitudes within the examined group.

3.5. Careful analysis of Figure 3.2, and especially Set 2, leads to the conclusion that these data are positively skewed for low probabilities and negatively skewed for high probabilities. This observation is confirmed by the classical measures of variance, skewness and kurtosis determined for specific probabilities. The results are shown in Figure 3.3.

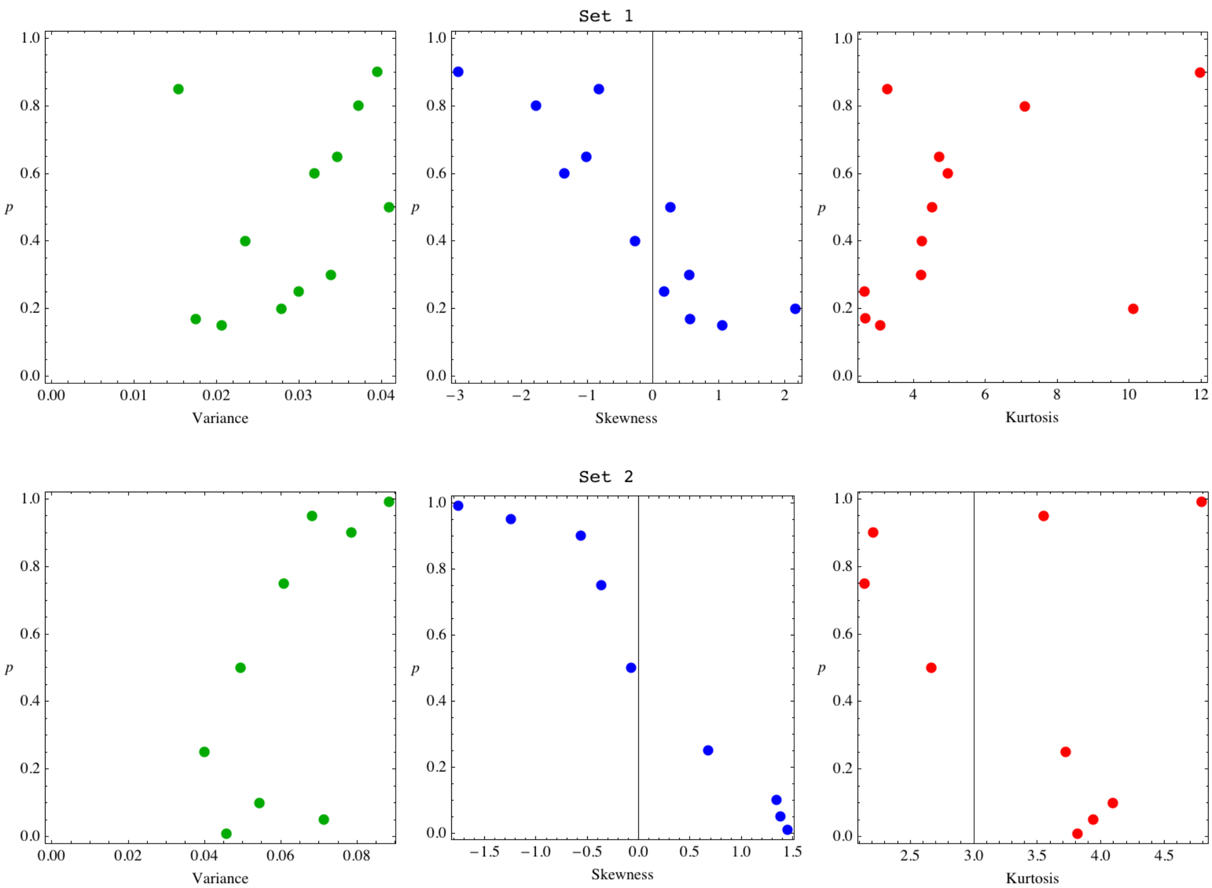

Figure 3.3. Distribution moments of relative certainty equivalents r for specific probabilities.

The calculated variances of r for specific probabilities differ in a ratio of 3:1. However, it is difficult to find any relationship between the probabilities and variances. Quite striking results may be observed for skewness. The values of r are clearly positively skewed for low probabilities and negatively skewed for high probabilities. Additionally, skewness changes almost linearly with probability. As a result, the values of r for medium probabilities are hardly skewed at all. Kurtosis values differ as well. In Set 1, kurtosis is greater than 3 for all probabilities. This demonstrates that the distribution of r is more peaked than a normal distribution. In Set 2, kurtosis assumes values both greater and less than 3. This means that the distribution of r is more peaked than a normal distribution for some probabilities and less peaked for others.

3.6. It is interesting to combine all the distributions and observe the aggregated distribution of r around its mean values. This is shown in Figure 3.4. The histograms presented give the impression that the distribution of the relative certainty equivalents r is roughly symmetric, and, in the case of Set 2, even normal. However, this is only true when the aggregated data are considered. In fact, the distribution of r is anything but symmetric or normal for most probabilities.

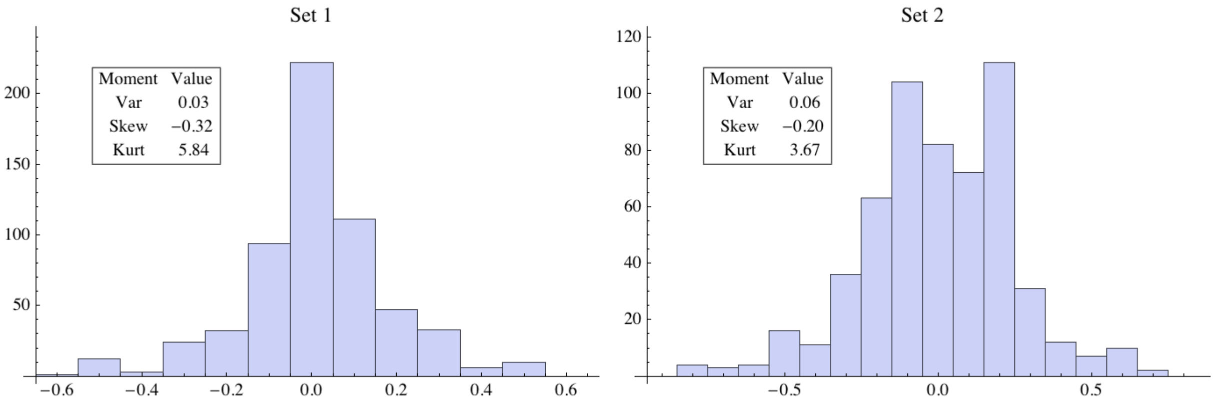

Figure 3.4. Aggregated distribution of r around its mean values.

4. Nonparametric Estimation

4.1. The advantage of nonparametric estimation methods is that they allow the approximate shape of an estimated relationship to be determined without specifying its functional form. Gonzales and Wu (1999) proposed a nonparametric estimation method for the Prospect Theory model, which is, however, very complex and relies on multiple, recursive interpolations of the v and w functions. In the case of the relative utility function, the nonparametric estimation is unusually simple and, in its simplest form, has already been presented in Figures 3.1 and 3.2. The full estimation procedure determines the mean, median, lower and upper quartiles, together with the mode of the relative certainty equivalent r for given probabilities. Determination of the first four values is obvious. Only the estimation of the mode, which in the case of a purely nonparametric approach means determining the most frequently occurring (i.e. the commonest) value or values of *r* for a given probability, requires some comment.

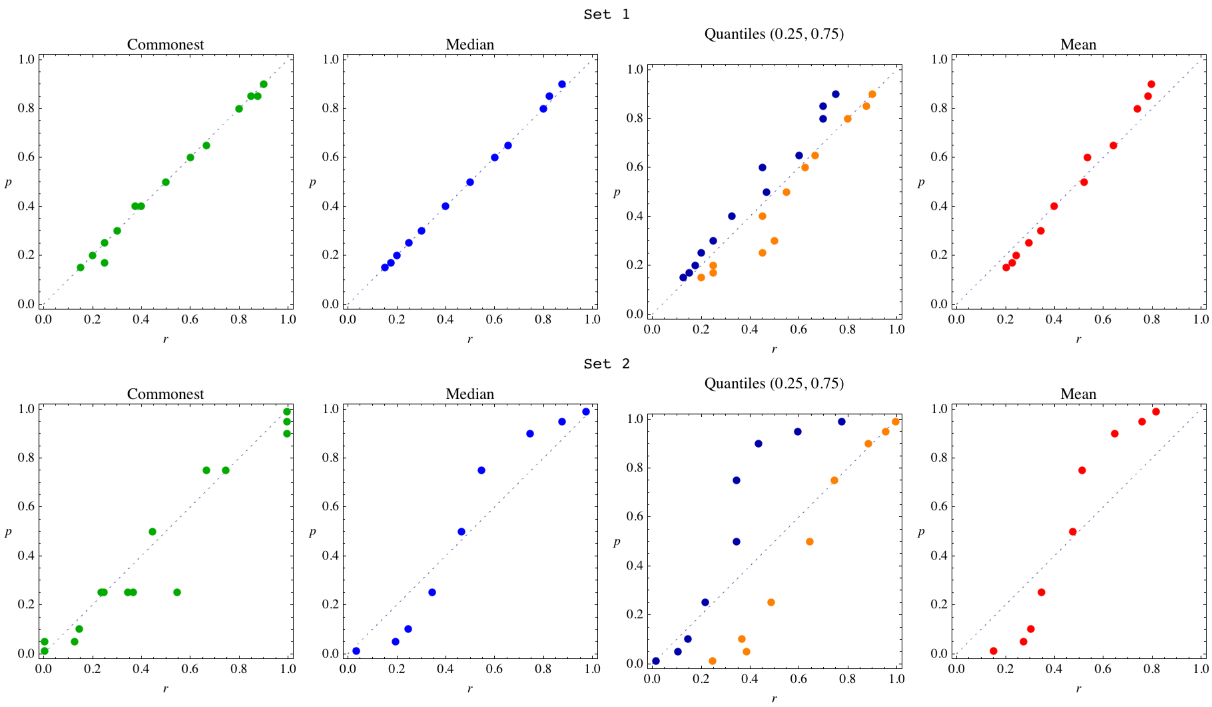

Figure 4.1. Nonparametric estimation of both data sets. In the case of quantile estimation, the lower quartile is marked with dark blue points, the upper quartile with orange points.

4.2. The graphs presented in Figure 4.1 give an instant image of the analyzed data. In the case of Set 1, both the commonest and median values are located on the $p = r$ line, which marks the expected value of the lottery. The values for the lower quartile line up above that curve and those of the upper quartile below it. Mean values create a slight S-shaped curvature. The data for Set 2 are more distinct. The median, quartile and mean values are clearly S-shaped, where this shape is more curved for means than for medians. Such curvature, however, is not that obvious for the commonest values, all the more so given that there are a few values *r* that appear most frequently for some probabilities (e.g. 0.25).

5. Standard Parametric Estimation

5.1. The general estimation model with additive errors is written in vector notation as (Cameron, Trivedi, 2005):

$$y = E[y \mid x] + e\,, \tag{5.1}$$

where E[$y|x$] denotes the conditional expectation of the observed variable y given explanatory variable x, and e denotes a vector of unobserved random errors. The nonlinear estimation model assumes E[$y|x$] to be a nonlinear function $g(x;\ \theta)$, where θ denotes a vector of parameters. The error term is then defined as the difference between the observed variable and its conditional expectation, $e_i = y_i - g(x_i;\ \theta)$. The loss function $L(e)$ defines the loss associated with the error e. The estimation procedure minimizes the expected value of the loss function (expected loss), and importantly, the type of the loss function determines the conditional expectation of the result.

5.2. If the loss function is the square of the error then the conditional expectation of y is the mean of x. In the case of the relative utility function Q, the least squares (mean) estimator minimizes the following function (cf. (2.6)):

$$S_{mean}(\theta) = \sum_{i=1}^{N} \left[r_i - Q^{-1}(p_i;\theta) \right]^2 , \tag{5.2}$$

where i denotes the next among all N data. Assuming the function Q has k parameters, the number of degrees of freedom is $N - k$. It follows that the average estimation error is given by:

$$err_{mean} = \sqrt{\frac{Min(S_{mean})}{N-k}} . \tag{5.3}$$

5.3. If the loss function is absolute error, then the conditional expectation of y is the median of x. In the case of the relative utility function, the median estimator minimizes:

$$S_{med}(\theta) = \sum_{i=1}^{N} \left| r_i - Q^{-1}(p_i;\theta) \right| . \tag{5.4}$$

The average absolute error is defined as:

$$err_{med} = \frac{Min(S_{med})}{N-k} . \tag{5.5}$$

5.4. If the loss function is asymmetric absolute error with a penalty of $q|e|$ on underprediction and a penalty of $(1 - q)|e|$ on overprediction, then the conditional expectation of y is the qth quantile of x. In the case of the relative utility function the qth quantile estimator minimizes:

$$S_q(\theta) = \sum_{i=1}^{N} q \left[r_i - Q^{-1}(p_i;\theta) \right]^+ + \sum_{i=1}^{N} (1-q) \left[r_i - Q^{-1}(p_i;\theta) \right]^- , \tag{5.6}$$

where a shorthand notation is used: $[a]^+ = a$ for positive a, and 0 for negative a; similarly $[a]^- = -a$ for negative a, and 0 for positive a. In the special case of $q = 0.5$, (5.6) reduces to (5.4).

The interpretation of the value S_q obtained by minimizing (5.6) may not be as clear as for median estimation, but (5.5) may be kept for convenience.

5.5. The loss function may be defined in terms of the kernel function (Lee, 1989, Kemp, Silva, 2009):

$$L(e) = 1 - 2K_R\left(\frac{e}{\sigma}\right), \tag{5.7}$$

where $K_R(u) = \frac{\mathbf{1}[|u| < 1]}{2}$ denotes the rectangular kernel, with **1**[A] being the indicator function for event A, and $\sigma > 0$ the bandwidth parameter. However, (5.7) may be presented in a simpler form:

$$L(e) = \begin{cases} 1 & if\ |e| > \sigma, \\ 0 & if\ |e| \le \sigma, \end{cases} \tag{5.8}$$

so that the loss function assumes a value of 1 outside a window of width 2 σ and a value of 0 inside it[6]. The conditional expectation of y is the mode of x in this case.

In the case of the relative utility function, there is one window per each probability considered in the experiment. The mode estimator minimizes the function:

$$S_{mode}(\theta) = \sum_{i=1}^{N} \mathbf{1}\left[\left| r_i - Q^{-1}(p_i;\theta)\right| > \sigma\right]. \tag{5.9}$$

The value S_{mode} obtained by the minimization specifies the number of data points located outside the windows and:

$$err_{mode} = \frac{Min(S_{mode})}{N}, \tag{5.10}$$

determines the percentage of such data. It might be more convenient to use $N - Min(S_{mode})$, which is the number points of data inside the windows. This value divided by 2 $M\ \sigma$, where M denotes the number of windows (probabilities considered), specifies the average data density in the vicinity of the estimated relative utility function:

$$\rho_m = \frac{N - Min(S_{mode})}{2M\sigma}. \tag{5.11}$$

It should be added that all measures given in this subsection may be sensitive to the chosen bandwidth parameter σ.

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[6] Cameron and Trivedi (2005, p. 67) state that the loss function for mode regression is the step function (i.e. $L(e) = 0$ if $e < 0$, and $L(e) = 1$ if $e \ge 0$). This cannot be correct, as it would result in the 100th quantile being estimated.

5.6. The results of all estimation procedures are presented jointly in Figure 5.1.

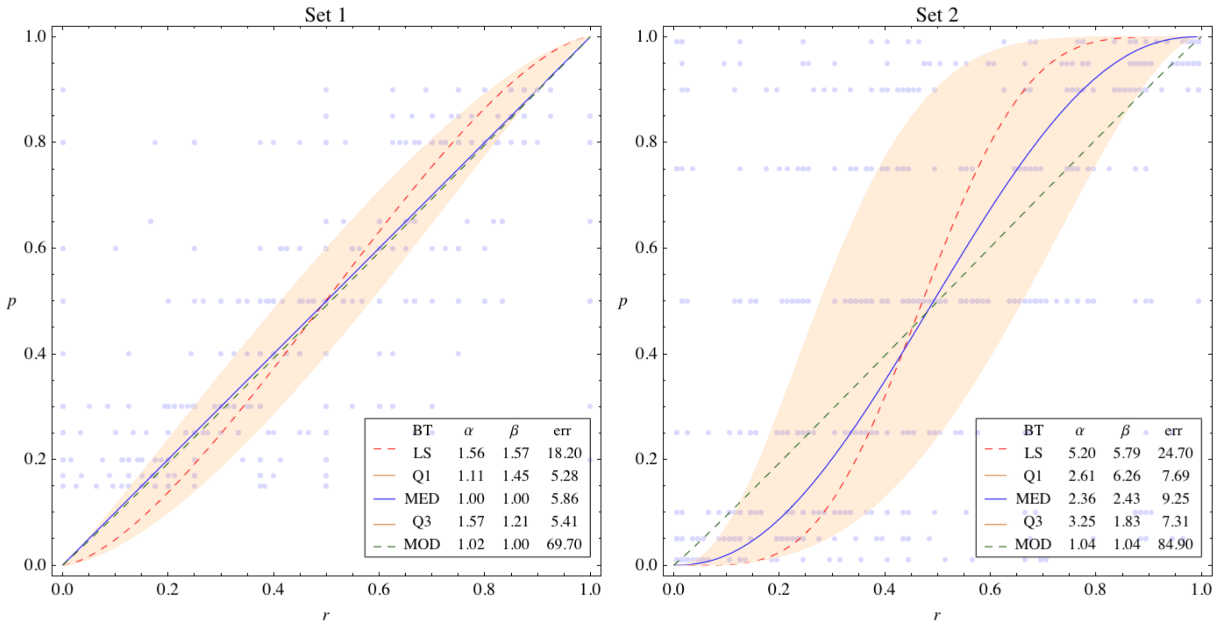

Figure 5.1. Estimation results for data sets using standard estimation methods: LS – Least Squares; Q1 – Lower Quartile; MED – Median; Q3 – Upper Quartile; MOD – Mode. The orange area marks data between the lower and upper quartiles. $\sigma = 0.01$ was assumed for the mode estimation. Errors are expressed as percentages.

To a large extent, these results confirm those obtained using nonparametric estimators. The difference between the mode, median and mean values (especially for Set 2) confirms the skewness of the data first noticed using standard measures of distribution moments. As this difference varies with probability and changes sign near where $p = 0.5$, the data clearly changes its character from positively to negatively skewed with probability.

In the first Set, the curves resulting from the mode and median estimations are practically superimposed on the line $p = r$. As the mode typically differs from the median value when the data are skewed, this can raise some objections as to the result. The mean estimator results in a slightly S-shaped curve.

In the second set, the functions obtained by the median, and especially the mean, estimator are much more curved. The area between the lower and upper quantiles is also much wider. However, the resulting curve of the mode estimator is almost a straight line $p = r$, as in the case of the first set.

5.7. Quantile, and especially mode, estimators are characterized by some computational inconveniences because estimated expected loss functions are not smooth and may have multiple minima (Figure 5.2). In the case of quantile estimators, this is caused by an estimated curve with two parameters attempting to pass through any two points from the dataset (similarly as the median value takes one of the values from the dataset with odd count). In the case of the mode estimator, this is an obvious corollary of definitions (5.7) – (5.9), and the number of local minima is really huge.

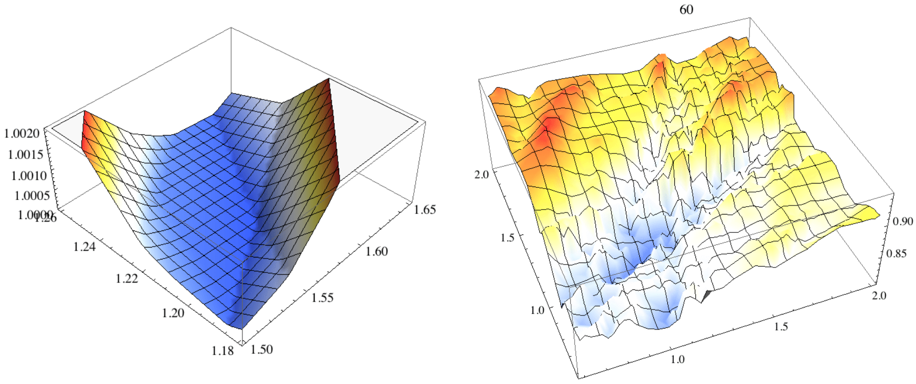

Figure 5.2. On the left, the shape of the expected loss function for the quantile estimator close to its minimum. On the right is the shape of the expected loss function for the mode estimator.

In regard to the above, the only practical method of finding the global minimum of the mode estimator is to use a contour plot and then make multiple attempts at finding the minimum in selected areas with different starting points. This process is clearly very burdensome. As a consequence, the mode estimators based on kernels with bounded support (as 5.7) "are difficult to implement and unattractive to practitioners, having seen little, if any, use in practice" (Kemp, Silva, 2009). If so, Figure 5.1 presents an original and at once very important result.

6. Conclusions

6.1. The results presented in this paper show that the distribution of data encountered in lottery experiments is not symmetric and varies with probability. Knowledge of this distribution is therefore essential in order to derive correct conclusions about the behavior of examined groups. As demonstrated, the lottery valuation is nonlinear with probability only when means and (in one of two sets) medians are considered. However, the relationship between the relative certainty equivalents and the probabilities is (almost) linear for modes in both data sets. This means that the most likely lottery valuation is close to its expected value. Another way of saying this is that the most likely behavior of the examined groups was fully rational. This is the main result of this paper.

6.2. The paper presented a wide range of estimation techniques in use for lottery experimental data. These include: least squares (mean), quantile (including median), and mode estimators, all performed in both parametric and nonparametric way. This is possibly the most wide-ranging coverage of estimation techniques for lottery experiment results ever undertaken. Such an extensive number of methods were made possible by using the relative utility model which, in contrast to the Prospect Theory model, adopts a classical econometric approach to data description.

6.3. The paper confirmed well known advantages and disadvantages of nonparametric and standard parametric estimators. The nonparametric methods easily determine the values of means, medians, quantiles and modes for given probabilities. The problem remains that the determined quantiles and modes take particular values and these can vary considerably with small changes to the examined sample. An unquestionable advantage of nonparametric estimators is their simplicity and computational ease. But the lack of any function describing the relationship prevents them being used in other applications and this is a distinct disadvantage.

On the other hand, the parametric estimators yield smoother estimations of the values sought, although data asymmetry which changes with probability may seriously affect the results of standard estimation procedures. This arises from the fact that the mean, median, qth quantile and mode estimations appear to be merely a consequence of using specific loss functions, viz. squared, absolute, asymmetric absolute and kernel errors. As a result, it is difficult to predict the influence that an unequal amount of data and all the changes in variance, skewness and kurtosis will have on the shapes of the estimated curves.

Computational problems also arise in the case of quantile and (especially) mode estimation. This is mostly caused by the type of expected loss functions, which are not smooth in either case.

6.4. Some topics are therefore left for further consideration. First, a more detailed analysis is required for the Set 1 results where the median and mode estimations are practically the same, but the mean estimation differs. As all these values should be different when the data are skewed, this can raise some objections as to the result. Second, in order to answer these objections, a more detailed analysis of data distribution is required as the standard measures of distribution moments only offer a partial description. Third, a more sophisticated estimation procedure might be required as the standard estimation techniques assume variance, skewness and other distribution moments to be constant over the estimation domain. Fourth, the standard median and mode estimators are characterized by computational inconveniences, which poses the risk that the global optimum will not be found. This raises the question of whether, and if so how, these inconveniences might be overcome. Finally, the important question of how to define the maximum likelihood estimator for the lottery experiment data has been left open.

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