Structural Analysis of Fertility in Russia

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"Structural Analysis of Fertility in Russia"

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I would like to express my gratitude to Alexis Belyanin for my economic upbringing and for his explanations of dynamic programming. I am also very thankful to Anna Lukyanova, for her unvaluable help in mastering Stata and understanding RLMS database. This paper would unlikely to exist without help of Vladimir Sukhomlinov, who contributed very much to the research grant “Determinants of Fertility in Russia”, which became the base for this paper.

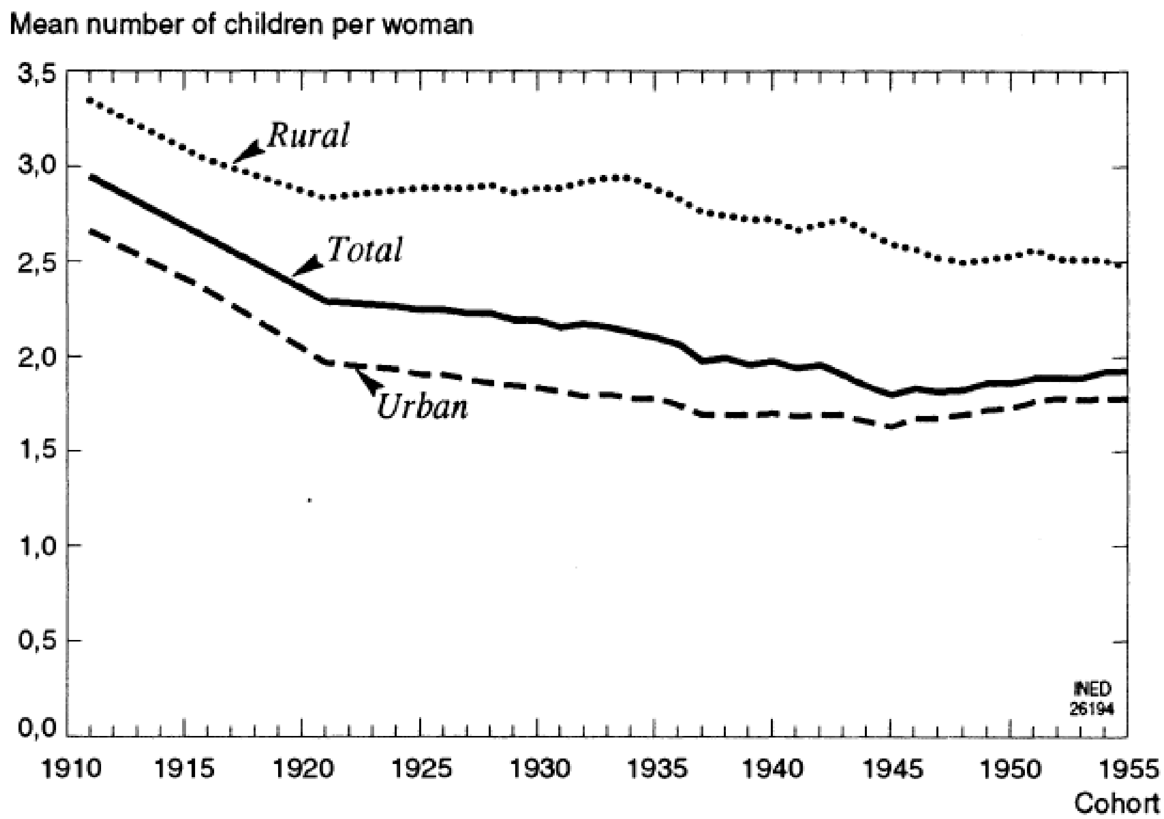
Content.

Chapter 1. Demographic situation in Russia.

1.1. Introduction.Historical Trends.

From the 1992 natural increase of Russian population became negative. Although it is generally accepted that decline of the population was caused by political and economic reasons of 90-s, data show that Russia completed its second demographic transition in 50-s and 60-s. From the table 1 it can be found that even for women born in 1935 average number of children was lower than 2.3, which is required for the stabilization of the population.

Pic 1. Mean Number of Children per Woman for each cohort from 1910-1950[1]



Is it possible to increase rate of fertility in Russia? What kind of measures should be implemented?

1.2. Russia in the context of international tendencies.

In comparison with other developed countries Russian low fertility is absolutely typical. Russia as ordinary catching up country moved to the new model of family a little later but significantly faster than most developed countries. For instance, in France that period took about 120 years, for typical European countries – only 40-60 years (Vishnevsky, 2006). At the end of 90-s Russia was supposed to complete its second demographic transition from the high birth and death rates to the low birth and death rates.

There was a rise in fertility rates in 80-s thanks to simulative demographic policy in Russia. However, those measures had only short run effect. Families just moved planned births to earlier ages, therefore, and that contributed to the decrease of the birth rate in 90-s. Today Russian demographic rates are similar to those in Europe.

[1] Table 1 was taken from "A Survey of Modern Russian Fertility" Avdeev A. and Monnier A (1995).

1.3. Demographic Situation in 90-s and Empiric Research on Russian Fertility.

Micro-data in the analysis of Russian fertility are used not more than 15 years. Basic database for such research is Russian Longitudinal Monitoring Survey (Kohler and Kohler,2001; Grogan, 2003; Roshina and Boykov, 2005; Roshina, 2006). Another researches used data of population micro-cencus (Kharkova and Andreev, 2000) or data of research “Parents and children, men and women in family and society” (Zaharov S., Ibragimova M. and Kartseva M., 2007).

The major puzzle that researchers were interested in was sudden decrease in the birth rate from the beginning of 90-s. There are several explanations provided for that puzzle.

1. Demographic explanation. Decrease of fertility rate is a long-run tendency and economic crisis just accelerated tendencies that anyway would take place, but a little slower in its absence (Kharkova and Andreev, 2000).
2. Adaptation explanation. Philipov and Kohler (2001) show that model of reaction to economic and social factors is becoming closer to the behavior of European families. With the transition to the market economy typical families are organized later than in the Soviet Union. These authors forecasts that fertility rate would decrease in the short run and than in the long-run return to current values.
3. Deferring explanation (Heleniak,1995; Rimashevskaya, 1997). That authors blame economic crisis and destruction of social and economic support of the big families as a main reason of the decrease in the birth rate. That factors caused delaying of births till the end of crisis.

Reproductive behavior in Russia has several peculiarities.

1. Social norms (e.g. Roshina, 2006). It is generally accepted in Russia to have at least one child and at most two.
2. Rural and urban behavior continues to be closer to each other, although fertility rate is still slightly higher in the country (Avdeev and Monnier, 1995)
3. Housing conditions are extremely important (Maleva and Sinyavskaya, 2006).

Usually there was no correlation between the income of woman and number children. However, Buhler (2004) found negative correlation between households income and probability of birth of the first child. Additional income (temporary, or from second or third job) positively influences fertility.

Contribution of this paper is that structural and reduced-form analysis on the Data RLMS is used and that helps to understand decision-making process in the intertemporal framework. Stochastic dynamic programming approach can explain the nature of birth-planning and role of uncertainty in that process. Structural dynamic discrete modeling with a permanent heterogeneity is probably to be used for the first time on Russian data.

Chapter 2. Data.

2.1. Data Description[2].

For estimation of fertile behavior RLMS (Russian Longitudinal Monitoring Survey) database is used. It includes data on 10 years from 1994 to 2005 or, respectively from the 5 to 14 rounds. Database includes 20 files: 2 files for each year, one is corresponds to data on individuals and another one contain information about households. Individual file includes very detailed information on each person: age, nationality, education, job, health, smoking and alcohol preferences… What is especially interesting, it has a part for women, e.g. whether woman would like to have a new child. Household file contains information about housing conditions, short information about each member of households and relationships between different members.

Unfortunately, the list of questions changed from year to year. Moreover, question set about reproductive behavior shortens sharply from 1999 in the individual files. From the 2004 in the individual file we can't even get information about quantity of children born by particular woman. However, that information is still available by indirect methods from household files (in the household we can get information about children only living within the same household of their mother).

The sample was restricted to women from 16 to 39 years in the reduced form models and from 16 to 35 years in the structural form models. The specific problem was to create dependent variable. Dependent variable birth was equal to one in year T if the baby was conceived between surveys at T and T+1, otherwise it is equal to zero. Although such procedure was a little more complicated and asks interaction between household file and different individuals due to incomplete information in each separate file. In other papers it was generally accepted to make dependent variable birth in the period T equal to one if number of children recorded in the survey at year T+1 exceeds corresponding value at T. Such procedure may lead to the misleading results: suppose that three surveys were conducted on November at each 2004, 2005 and 2006 year. If a particular woman decided to have a baby on March 2005 and conceived it at the same month, then she would give a birth at January 2006 and it would be recorded on November 2006, so as a result birth at 2005 year would be equal to one, although regressors at that moment could be affected as the woman is pregnant. She is likely to smoke and drink less and probably quiet her job.

The list of typical independent variables included

1) Woman characteristic: age, education, number of children, employment status, income, being in maternity leave, alcohol and smoking preferences, satisfaction of life, self-estimate health.
2) Characteristics of her husband (only for married): age, employment status, income, alcohol and smoking preferences, satisfaction of life, self-estimate health.
3) Characteristics of the household: size, status of the settlement, average living space per one member of the household, number of facilities in the apartment, other revenues of the household.

All incomes are deflated to the level in November 1994 year and logarithms of the real incomes were used (in the reduced form models, in structural form models logarithms were not used).

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[2] Descriptive results are given in the full-edition of the thesis

Chapter 3.Reduced Form Models.

3.1. Methodology.

It is supposed that the woman can get different utilities from different decisions. She compares two situations: when she has additional child in the family and when everything stays the same. The fact that at the given period woman doesn't give birth tells that utility from additional child lower than in the case of its absence. Utility depends on observable and unobservable factors. Using econometric method it is possible to build such utility functions that best fit for chosen decisions.

Observable factors include age (five categories); quantity of already born children and square of that number; employment status (three categories); education (four categories); drinking (five categories); health (three categories); real income; additional income of the household; dummy variables for equality of woman real income and additional income to zero; average living space per person in the household (five categories); number of other members of the household excluding woman; her husband and children; number of additional desirable children and its square; settlement status (four categories), federal region (7 categories); year dummy variables. Models are estimated on three samples: total sample, subsamples of unmarried and married.

Extended number of variables leaded to the decrease of observations, thus, they some of them with a lot of missing observation were not used (e.g. nationality). There was an additional work on fixing measurement errors. If total income of the household was lower than the sum of woman and her husband income its value was replaced by this sum. Observations with highest 0.5 % income were excluded from the sample.

Relatively all explanatory variables can be divided into three groups: (1) variable that affect costs of upbringing child, (2) variables that characterizes financial and timing restriction of the household, (3) factors that details preferences in relationships to the number of children. Some of variables can be referred to several groups. Dummy-variable of being married influence on the costs of upbringing, as sometimes husband can help in that process, also it affects financial opportunities of the household as non-labor revenues of women increase, preference for the number of children are also affected through the preferences of the husband. Education also can be referred to all groups through higher opportunity costs of upbringing, higher financial opportunities (second group) and non-traditional preferences and demand for higher quality of children instead of quantity (last group). In the first group number of other members of the household, personal income of an employed woman can be listed. Husband income and additional income of the household, living space measures belong to the second group. Desirable number of children is closer to the last group of variables.

In given research panel probit random effect approach is used. It supposes that unobservable equal to the sum of the individual effect and random shock $\varepsilon_i + e_{it}$. Individual effect is permanent across periods for a given individual. It is supposed to be normally distributed across individuals. Random shock e_{it} is also normally distributed random variables independent across periods and individuals and also uncorrelated with individual effect. Then probability of a given individual to have a child equal

$$\Pr(birth=1\,|\,X_i)=\Pr(U_{birth=1}=\beta_0+\sum\beta_{ik}X_{ik}+\varepsilon_i+e_{ik}>0)=F(\frac{\beta_0+\sum\beta_{ik}X_{ik}+\varepsilon_i}{\sigma_e}) \quad (1)$$

All coefficients are determined by using MLE estimation, which is given by formula

$$\sum_{i=1}^{N}\ln\int_{-\infty}^{\infty}\prod_{t=1}^{n_i}(F(\frac{\beta_0+\sum\beta_{ik}X_{ik}+\varepsilon_i}{\sigma_e}))^{d_{it}}*(1-F(\frac{\beta_0+\sum\beta_{ik}X_{ik}+\varepsilon_i}{\sigma_e}))^{1-d_{it}}\frac{1}{\sigma_\varepsilon}f(\frac{\varepsilon_i}{\sigma_\varepsilon})d\varepsilon_i\rightarrow\max_{\beta,\sigma_e,\sigma_\varepsilon} \quad (2)$$

Coefficients of the ordinary pooled probit model were also estimated.

3.2. Estimation Results[3].

Estimation results of the pooled probit model probit random effect model are presented in the table 4 and 5 respectively. In the lowest row of the table 5 there are tests that check, whether the contribution of individual effect in total variation is significant from zero. For the 5 % confidence interval zero-hypothesis about equality of individual effect to zero cannot be rejected only for the sub-sample of married women. Sign and significance of the coefficient is exactly the same for pooled probit model and probit random effect model even for married women.

Age, marriage, employment, number of children and number of additional desirable children, number of additional members of family, settlement status were significant. Contraception, living space, health, education, income, year-dummy variables were not significant.

Negative influence of age can be seen only for 30-34 and 35-39 age groups. Therefore, for groups from 16 to 29 years fertility level is approximately the same. Country and small towns have higher rate of births than big cities.

Employed women have higher probability of giving a birth than not employed. Positive significance of the employment can explain why education and were insignificant. Employed women usually are more educated and have higher income, therefore they all may correlate, what decreases probability of their significance. Another explanation lies in social rules, employed woman can get additional financial opportunities on her maternity leave or if it is a social work than working experience may give respective ability to place a child in a good kindergarten.

Probability of having additional child non-linearly depends on already born children. Second child is less likely to be born than the first, the third one is less likely to be born than the second, although difference in probabilities is smaller, but the forth and next one have higher probabilities to be born, than previous ones. Number of desirable children is positively significant what is natural. Although it has to be noticed as it is also correlated with other regressors it could decrease number of the significant variables.

Generally patterns of fertilities are similar between sub-samples. However, in sub-sample of unmarried women decrease of births begins from later age, only group of 35-39 has negatively significant coefficient. For married fertility decreased from 20-24 years. Thus, early marriage is likely to lead to the birth of the first child. Among unmarried women relationship between probability of the new child and

[3] Includes all described table in Appendix 1.

number of already born ones is unclear. It can be explained by the fact that 70 % of unmarried women don't have children and only 8 % have two or more children.

Contraception is negatively significant only for married women. Consumption of alcohol positively influences probability of a having baby only for unmarried women. Indirectly that explains non-planned character of births for that group.

Economic factors are significant for married women on the 10 % confidence level. The same is true for a high average living space. For unmarried women that variables are not significant.

Significance of marriage may be explained by the fact that women with higher demand for children prefer being married. Other characteristic of the husband are not significant, even under joint F-test (although it is simply asymptotic).

Chapter 4. Dynamic discrete choice structural models.

4.1. Basic algorithms.

In dynamic discrete choice structural models, agents are forward-looking and maximize expected intertemporal payoffs. The parameters to be estimated are structural in the sense that they describe preferences of agents and beliefs about technological and institutional constraints.

Dynamic discrete choice models are equivocal. They can deal with almost any question of the labor Economics. Decision to retire, to accept or reject new job, to give a birth – all them include estimation of not only current but also future benefits by individual. That is where stochastic dynamic programming problem comes. Individual is maximizing expected sum of his discounted utilities:

$$\underset{\{d\}_{t=0..T}}{Max}\, E_0\left[\sum_{t=0}^{T}\beta^t(d_t u_{1,t}+(1-d_t)u_{0,t})\right] \tag{3}$$

where d_t is the strategy in t and $\{d\}_{t=0..T}$ is the sequence of strategies over the life time.

The basic papers of dynamic discrete choice structural models includes classical papers: Wolpin (1984) on mortality and fertility, Keane and Wolpin(1994) on occupational choice, Rust (1987) that estimated optimal bus engine replacements, Miller (1984) on patent renewal, Hotz and Miller (1993).

4.1.1. Rust (1987) Engine Replacement Model.

Rust has made the following assumptions[4]:

"ASSUMPTION AS (Additive separability): The one-period utility function is additively separable in the observable and unobservable components:

$$U(a,x_{it},\varepsilon_{it})=u(a,x_{it})+\varepsilon_{it}(a) \tag{4}$$

where $\varepsilon_{it}(a)$ is a

zero mean random variable with support the real line.12 That is, there is one unobservable state variable for each choice alternative, so the dimension of ε_{it} is (J + 1) × 1.

ASSUMPTION IID (iid unobservables): The unobserved state variables in ε_{it} are independently

[4] WP. V. Aguirregabiria and P. Mira (2007) pp. 7.

and identically distributed over agents and over time with CDF $G_\varepsilon(\varepsilon_{it})$ which has finite first moments and is continuous and twice differentiable in ε_{it} .13

ASSUMPTION CI-X (Conditional independence of future x): Conditional on the current values of the decision and the observable state variables, next period observable state variables do not depend on current ε: i.e., $CDF(x_{i,t+1} | a_{it}, x_{it}, \varepsilon_{it}) = F_x(x_{i,t+1} | a_{it}, x_{it})$. We use θ_f to represent the vector of parameters that describe the transition probability function F_x.

ASSUMPTION CI-Y (Conditional independence of y): Conditional on the values of the decision and the observable state variables, the value of the payoff variable y is independent of ε: i.e., $Y(a_{it}, x_{it}, \varepsilon_{it}) = Y(a_{it}, x_{it})$. The vector of parameters that describe Y is θ_Y.

ASSUMPTION CLOGIT: The unobserved state variables { ε_{it} (a) : a = 0, 1, ..., J} are independent across alternatives and have an extreme value type 1 distribution.

ASSUMPTION DIS (Discrete support of x): The support of x_{it} is discrete and finite: $x_{it} \in X$ = {x(1), x(2), ..., x(|X|)} with |X| < ∞".

Here assumption about conditional independence of future observable variables and iid deduce that *Emax* can be expressed as a function of only observable variables.

$$\overline{V}(x_{it}) = \int \max_{a \in A}\{u(a, x_{it}) + \varepsilon_{it}(a) + \beta \sum_{x_{i,t+1}} V(x_{i,t+1} | a, x_{i,t}) f_x(x_{i,t+1} | a', x_{it})\} dG_\varepsilon(\varepsilon_{it}) \tag{5}$$

Then probability accepted decision a is

$$\begin{aligned} P(a | x, \theta) = \int I\{u(a, x_{it}) + \varepsilon_{it}(a) + \beta \sum_{x_{i,t+1}} \overline{V}(x_{i,t+1} | a, x_{i,t}) f_x(x_{i,t+1} | a, x_{it}) > \\ > u(a', x_{it}) + \varepsilon_{it}(a') + \beta \sum_{x_{i,t+1}} \overline{V}(x_{i,t+1} | a', x_{i,t}) f_x(x_{i,t+1} | a', x_{it})\} dG_\varepsilon(\varepsilon_{it}) \end{aligned} \tag{6}$$

Moreover, assuming that ε have extreme value type one distribution

$$\overline{V}(x_{it}) = \log(\sum_{a=0}^{J} \exp\{u(a, x_{it}) + \beta \sum_{x_{i,t+1}} V(x_{i,t+1} | a, x_{i,t}) f_x(x_{i,t+1} | a, x_{it})\}) \tag{7}$$

and correspondingly, probability equals

$$P(a | x_{it}, \theta) = \frac{\exp\{v(a, x_{it})\}}{\sum_{j=0}^{J} \exp\{v(j, x_{it})\}} \tag{8}$$

where

$$v(a, x_{it}) = u(a, x_{it}) + \beta \sum_{x_{i,t+1}} \overline{V}(x_{i,t+1} | a, x_{i,t}) f_x(x_{i,t+1} | a, x_{it}). \tag{9}$$

Although Rust's model was the most popular among economists because of its simplicity it is not able to deal with unobserved heterogeneity. In practice although iid assumption fit well for bus engines (for which Rust's framework was implemented) is likely to oversimplify the world of humans.

4.1.2. Backward induction algorithm.

It was developed by Keane and Wolpin(1994). It is assumed that individuals live T (finite) periods

$$V(S(t),t)=\underset{\{\{d_k\}_{\tau=t..T}\}_{k\in K}}{Max}\ E\left[\sum_{t=\tau}^{T}\beta^t(\sum_{k\in K}d_{kt}u_{kt})\,|\,S(t)\right] \quad (10)$$

where d_{kt}=1 if alternative k is chosen at time t, otherwise 0; u_{kt} is a reward from the chosen strategy; $S(\tau)$ is a space that consists of all factors, known to the individual but not the researcher, that affect current rewards or the probability distribution of any of the future rewards.

The value function can be written as

$$V(S(t),t)=\max_{k\in K}(V_k(S(t),t)) \quad (11)$$

and, respectively, $V_k(S(t),t)$ - alternative-specific expected lifetime reward or value function, expressed according to the Bellman equation (Bellman, 1957)

$$V_k(S(t),t)=U_k(S(t),t)+\beta E(V(S(t+1),t+1)\,|\,t,d_k(t)=1) \quad (12)$$

Here $E(V(S(t+1),t+1)\,|\,t,d_k(t)=1)$ will be called *Emax* function and it represents particular interest. Let's calculate *Emax* for the last period,

$$E(V(S(T),T)\,|\,T-1,d_k(T-1)=1)=\int_{\varepsilon_{kT}}...\int_{\varepsilon_{1T}}\max(U_1(T)..U_k(T))\,f(\varepsilon_{1T}..\varepsilon_{kT})d\varepsilon_{1T}..d\varepsilon_{kT} \quad (13)$$

Using backward induction and formulas (11), (12) and (13), we can find *Emax* for any point in the state space.

Although calculation of *Emax* has some problems with a limited computational capacities. Keane and Wolpin provided solution to this through simulation and interpolation. First, they compute integrals using simulation (although Bingley and Lanot (2004) found that for given distribution of ε all them can be expressed analytically) and interpolation. They calculate *Emax* for a subset of sample space and then regress that function on given factors and assign other points predicted values of a regression.

The most important advantage of Keane and Wolpin method is its ability to deal with a permanent unobserved heterogeneity. Keane and Wolpin(1994) propose, that there n types of agents and for each type dynamic programming problem should be solved. After that there are taken exogenous weights for each type and weighted likelihood is maximized.

4.2. Structural fertility models.

Applying standard dynamic model to the process of fertility, we observe that individual maximizes his expected utility over life time at each period from τ to T.

$$\max E_\tau\sum_{t=\tau}^{T}\beta^t U(N_t,M_t,X_t,H_t,u_t,\theta)$$

As before β is a discounting factor, U -current utility, M_t - number of children, X_t -consumption, H_t - leisure time, θ - individual-specific random term, N_t -dummy variable equal to one if the birth was given, u_t - contraceptive efficiency. Utility is maximized under constraint

$$I_t+w_t(\bar{H}-H_t)=X_t+P_tM_t+p_tu_t \quad (14)$$

by choosing X_t, H_t and u_t. Then N_t is a random variable dependent on u_t. However in many papers, e.g. in Wolpin (1984) or Leung (1991) u_t is omitted in the constraint. Some of the specifications ignore $w_t(\bar{H}-H_t)$ (Wolpin (1984) and all listed papers in the table except Hotz and Miller (1984) and Rosenzweig and Schultz (1985)).

Table 1[5]. Different Specifications of The Utility Functions.

Heckman and Willis (1976):

$$U = W(\psi N_t, X_t) - f(u_t)$$

Wolpin (1984):

$$U = W(M_t, X_t, \theta) = (\alpha_1 + \theta)M_t - \alpha_2 M_t^2 + \beta_1 X_t - \beta_2 X_t^2 + \gamma M_t X_t;\ \gamma \text{ any sign}$$

Hotz and Miller (1984):

$$U = W(M_t, Z_t);\ Z_t = Z(H_t, X_t, \zeta_t)$$

where Z_t is household production

ζ_t is a random error

Rosenzweig and Schultz (1985):

$$U = W(N_t, M_t, X_t, H_t, \theta) = \phi_1 N_t - \phi_2 N_t^2 + \alpha_1 M_t - \alpha_2 M_t^2 + \beta_1(\theta) X_t - \beta_2 X_t^2 + \delta_1 H_t - \delta_2 H_t^2 + \gamma H_t M_t;\ \gamma \text{ any sign}$$

Newman (1988):

$$U = W(M_t, X_t, u_t) = \alpha_1 M_t - \alpha_2 M_t^2 + \beta_1 X_t - \beta_2 X_t^2 + \gamma M_t X_t + \rho_1 u_t - \rho_2 u_t^2;\ \gamma \text{ any sign}$$

Leung (1991):

$$U = W(M_t, X_t) - f(\pi(u_t))$$

where $\pi(.)$ is the probability of a birth

Ordinary, system evolutes by the following way

$$M_{t+1} = M_t + N_t \quad (15)$$

$$N_t = N(\pi_b(1-u_t)) \quad (16)$$

$$V(M_t;t) = \max\{U(M_t, X_t, H_t, u_t, \theta) + \beta E_t V(M_{t+1}; t+1)\} = \\ = \max\{U(M_t, X_t, H_t, u_t, \theta) + \beta[\pi_b(1-u_t)V(M_t+1; t+1) + (1-\pi_b(1-u_t))V(M_t; t+1)]\} \quad (17)$$

Unfortunately data on contraception in RLMS and restriction of computational capacities make author to omit u_t and $w_t(\bar{H}-H_t)$ in the model.

4.3. Model.

Current utility function is supposed to depend on woman consumption and number of children:

$$U_t = f(M_t, X_t) + eN_t \quad (18)$$

To be concrete

$$U_t = (\alpha_1 + w)M_t + \alpha_2 M_t^2 + \alpha_3 M_t^3 + \beta_1 X_t + \beta_2 X_t^2 + eN_t \quad (19)$$

[5] This table was taken from Arroyo (1997)

Where M_t - number of children, X_t - consumption, N_t - dummy equal to 1 if woman gave a birth, e – random shock independent across individuals and periods, w - index of being fond of children, it is a random variable across individuals and constant across periods. Consumption is defined by

$$X_t = I_t - P_t M_t - cN_t \tag{20}$$

Where I_t - income of the household, P_t - consumption of one child, c - costs of birth.

$$M_{t+1} = M_t + N_t \tag{21}$$

Individual maximizes the value

$$\max_{\{N_t\},t=1..T} E\{\sum \delta^t[(\alpha_1 + w)M_t + \alpha_2 M_t^{\;2} + \alpha_3 M_t^{\;3} + \beta_1(I_t - P_t M_t - cN_t) + \beta_2(I_t - P_t M_t - cN_t)^2 + eN_t]\} \tag{22}$$

Or using principle of Bellman

$$\max_{N_t}(-\beta_1 c + \beta_2 c^2 - 2\beta_2(I_t - P_t M_t))N_t + \delta(E_t V(M_t + N_t, S_{it})) \tag{23}$$

Taking into account finiteness of time horizon backward induction algorithm is used. For the last period

$$\begin{aligned} V(T,M) = \int_{-\infty}^{\infty} f(e)[\max_{N_T}\{(\alpha_1 + w)M_T + \alpha_2 M_T^{\;2} + \alpha_3 M_T^{\;3} + \beta_1(I_T - PM_T - cN_T) + \\ +\beta_2(I_T - PM_T - cN_T)^2 + \gamma M_T(I_T - PM_T - cN_T) + eN_T]de \end{aligned} \tag{24}$$

For earlier periods:

$$\begin{aligned} V(t,M) = \int_{-\infty}^{\infty} f(e)[\max_{Nt}\{(\alpha_1 + w)M_t + \alpha_2 M_t^{\;2} + \alpha_3 M_T^{\;3} + \beta_1(I_t - PM_t - cN_t) + \\ +\beta_2(I_t - PM_t - cN_t)^2 + \gamma M_t(I_t - PM_t - cN_t) + eN_t + \delta V(M_{t+1}, t+1)]de \end{aligned} \tag{25}$$

That integrals are calculated using Monte-Carlo simulation. In such a way matrix of Value function for all space is created. This is part is traditionally called Nested Fixed Point Algorithm and they are done in the file IMPVALUE.PRC. It can be used for the calculation of likelihood:

$$\begin{aligned} \sum_{i=1}^{N} \ln \int_{-\infty}^{\infty} \prod_{t=1}^{n_i} (F(\frac{\beta_1 c + \beta_2 c^2 - 2\beta_2 c(I_{it} - PM_{it}) - \gamma M_{it} c + \delta \Delta V(\beta x_{it}, \delta, w_i, \sigma_e)}{\sigma_e}))^{d_{it}} * \\ *(1 - F(\frac{\beta_1 c + \beta_2 c^2 - 2\beta_2 c(I_{it} - PM_{it}) - \gamma M_{it} c + \delta \Delta V(\beta x_{it}, \delta, w_i, \sigma_e)}{\sigma_e}))^{1-d_{it}} * \\ * \frac{1}{\sigma_\varepsilon} f(\frac{w_i}{\sigma_\varepsilon}) dw_i \rightarrow \max_{\beta,\delta,\sigma_e,\sigma_\varepsilon} \end{aligned} \tag{26}$$ [6]

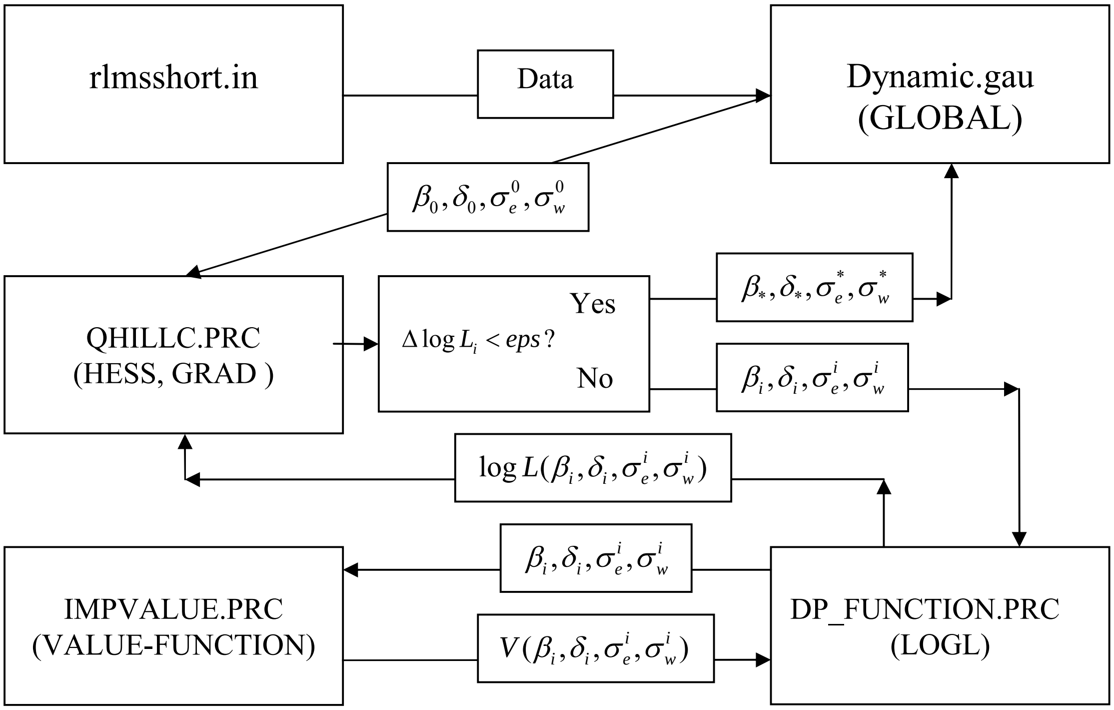
Where $F(.)$ is a culmulative standard normal distribution, $f(.)$ is a density of the standard normal distribution. This procedure is realized in the file DP_FUNCTION.PRC.

Realization of the whole algorithm is shown in the Pic. 2[7].

Pic. 2. Algorithm.

[6] Here integration is taken by trapezoidal sums, as all other methods, including quadratures and Monte-Carlo simulation generally make us to change space of the Value-function.

[7] Idea of the algorithm and structure of files are based on Duan (2000)



Implementing this algorithm we get the following results:

Table 2. MLE Parameters.

σ_w	σ_e	δ	α_1	α_2	α_3	β_1	β_2	γ	P	c
0.972	0.998	0.688	0.526	-0.262	0.0362	0.487	0.486	0.505	0.495	0.526

Log-Likelihood=-3067.94447553

Table 3. Actual and Predicted Probabilities of Birth Given Number of Children.

Number of Children	0	1	2	3	4	5	6	7
Actual Probability of Birth	0.0789	0.0487	0.0192	0.0193	0.1052	0	0.2500	1.0000
Predicted Probability of Birth	0.0785	0.0487	0.0533	0.0810	0.1272	0.2202	0.7089	0.7946

All coefficients appears to be significant (jointly and separately),. Looking at the results of the model it can be seen that probability of the second child is minimal, however true probability values show that third child is most unlikely. It is important that high value of variances of unobserved effects contribute to the major non-panned character of children.

Conclusions.

Both structural and reduced form support hypothesis that probability of birth initially decreases with the increase of the number of children and then increases (quadratic nature). Structural form helps to understand the crucial importance of “taste for children” and that should be taken into account in the demographic policy, which should concentrate on propaganda of traditional values and big families.

Individual-specific effect was significant only in the structural form model, while probit random effect model has shown insignificant contribution of individual unobserved heterogeneity to the total variance and its coefficients are similar to those in pooled probit model. Moreover, high value of variance in random shock in the structural model demonstrates large proportion of non-planned births. In whole estimation results don’t contradict economic intuition and results from previous researches.

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Appendix 1. Estimation Results.

Table 4. Results of the Estimation of Pooled Probit Model, Marginal Effects

Variables	Total		Unmarried		Married	
	df/dx	SE	df/dx	SE	df/dx	SE
Age(16-19)						
20-24	0.002	0.005	0.010	0.008	-0.016***	0.005
25-29	-0.003	0.006	0.007	0.011	-0.020***	0.005
30-34	-0.017***	0.005	-0.002	0.011	-0.029***	0.006
35-39	-0.030***	0.004	-0.019**	0.008	-0.037***	0.007
Married	0.035***	0.004				
Use of Contraception	-0.003	0.004	0.011	0.007	-0.007*	0.004
Eduacation (Compulsory Education)						
General Secondary Education	-0.001	0.004	0.004	0.007	-0.005	0.006
Vocational education	-0.002	0.005	-0.006	0.008	-0.005	0.006
University degree	-0.003	0.006	-0.007	0.009	-0.003	0.006
Employment Status (Unemployed of Out of Labor Force)						
Employed	0.013***	0.004	0.015**	0.007	0.011***	0.004
On Maternity Leave	-0.002	0.007	0.037	0.046	-0.009*	0.006
Number of Children	-0.029***	0.004	0.000	0.014	-0.024***	0.005
Number of Children^2	0.006***	0.001	-0.009	0.008	0.005***	0.001
Health (Bad)						
Average	0.004	0.008	-0.001	0.012	0.005	0.008
Good	0.005	0.008	-0.002	0.012	0.008	0.009
Drinking (abstain)						
1 Time a Month	0.009*	0.005	0.013	0.009	0.005	0.005
2-3 Times a Month	0.007*	0.004	0.020**	0.008	0.001	0.005
1 Time a Week	0.008	0.006	0.022*	0.012	0.001	0.007
2 Times a Week and More	0.006	0.008	0.033*	0.019	-0.005	0.008
Real Income/100	0.001	0.001	0.001	0.002	0.000	0.002
Real Income=0	0.003	0.004	0.013*	0.007	-0.002	0.004
Additional Income of The Household /100	0.000	0.000	-0.001	0.001	0.001*	0.001
Additional Income of The Household=0	0.003	0.005	-0.005	0.007	0.005	0.004
Living Space (1-st Quintile)						
2-nd Quintile	0.002	0.005	0.006	0.009	-0.002	0.005
3-rd Quintile	0.001	0.005	0.000	0.008	0.003	0.006
4-th Quintile	0.006	0.005	-0.003	0.008	0.012*	0.007
5-th Quintile	0.010	0.006	-0.004	0.008	0.015*	0.008
Number of Other Members of the Household	0.004***	0.001	0.003*	0.002	0.002	0.001
Number of Desirable Additional Children	0.044***	0.005	0.022***	0.007	0.053***	0.007
Number of Desirable Additional Children ^2	-0.014***	0.002	-0.007***	0.003	-0.017***	0.002
Settlement Status (City with a population more than 1 mln.)						
Regional Center	0.001	0.005	-0.001	0.008	-0.002	0.006
Small Town	0.012**	0.006	0.011	0.009	0.006	0.006
Country	0.017***	0.006	0.019*	0.010	0.006	0.006
Federal Region (Central)						
North-Western	-0.000	0.006	-0.002	0.009	0.004	0.007
Southern	-0.001	0.005	-0.004	0.008	0.007	0.007
Privolgskiy	0.001	0.004	0.002	0.008	0.003	0.005
Ural	0.012	0.008	0.008	0.011	0.008	0.009
Siberian	-0.004	0.005	0.001	0.009	-0.006	0.006
Far-Eastern	0.030***	0.011	0.031*	0.017	0.029**	0.015
Year (1994)						
1995	0.006	0.007	-0.006	0.009	0.014	0.010
1996	0.003	0.006	-0.004	0.009	0.007	0.009
1998	-0.001	0.006	-0.009	0.008	0.005	0.009

Variables	Total		Unmarried		Married	
	df/dx	SE	df/dx	SE	df/dx	SE
2000	-0.001	0.006	-0.003	0.009	0.001	0.008
2001	-0.002	0.006	-0.004	0.008	0.003	0.007
2002	0.000	0.006	-0.003	0.009	0.003	0.007
2003	0.000	0.006	-0.014**	0.007	0.009	0.009
Husband, Age					-0.001*	0.000
Husband, Employment Status (Unemployed)					0.002	0.005
Husband, Health (Bad)						
Average					-0.014	0.011
Good					-0.009	0.011
Husband, Drinking (Abstain)						
1 Time a Month					0.004	0.007
2-3 Times a Month					-0.003	0.005
1 Time a Week					0.000	0.005
2 Times a Week and More					-0.009*	0.005
Husband, Real Income/100					0.000	0.001
Husband, Real Income=0					0.001	0.005
N	11768		4504		6361	
Statistical Coefficients	Wald χ2 (46)=471.52 Prob>χ2= 0.0000 Pseudo R2=0.1256		Wald χ2(45)=121.47 Prob>χ2= 0.0000 Pseudo R2=0.0708		Wald χ2(55)=508.98 Prob>χ2= 0.0000 Pseudo R2=0.2016	

Significance of the Coefficient under Respective Confidence Level .01 - ***; .05 - **; .1 - *

Table 5. Panel Probit Random Effect Model, Marginal Effects.

Variables	Total		Unmarried		Married	
	df/dx	SE	df/dx	SE	df/dx	SE
Age(16-19)						
20-24	0.027	0.074	0.145	0.105	-0.343***	0.127
25-29	-0.032	0.088	0.104	0.145	-0.411***	0.143
30-34	-0.289***	0.103	-0.025	0.180	-0.648***	0.170
35-39	-0.607***	0.124	-0.373	0.230	-0.867***	0.207
Married	0.588***	0.064				
Use of Contraception	-0.036	0.053	0.156*	0.094	-0.122*	0.069
Eduacation (Compulsory Education)						
General Secondary Education	-0.007	0.067	0.072	0.098	-0.084	0.102
Vocational education	-0.024	0.079	-0.091	0.136	-0.103	0.111
University degree	-0.049	0.093	-0.115	0.169	-0.068	0.127
Employment Status (Unemployed of Out of Labor Force)						
Employed	0.197***	0.059	0.219**	0.100	0.204**	0.081
On Maternity Leave	-0.042	0.108	0.386	0.316	-0.200	0.135
Number of Children	-0.462***	0.062	-0.021	0.240	-0.435***	0.078
Number of Children^2	0.088***	0.011	-0.136	0.143	0.088***	0.013
Health (Bad)						
Average	0.055	0.115	-0.017	0.184	0.098	0.161
Good	0.077	0.117	-0.024	0.186	0.142	0.165
Drinking (abstain)						
1 Time a Month	0.126**	0.062	0.184*	0.109	0.092	0.086
2-3 Times a Month	0.113*	0.061	0.273***	0.102	0.014	0.088
1 Time a Week	0.115	0.078	0.278**	0.131	0.018	0.115
2 Times a Week and More	0.095	0.107	0.387**	0.165	-0.083	0.168
Real Income/100	0.008	0.021	0.013	0.037	0.005	0.030
Real Income=0	0.040	0.058	0.187**	0.093	-0.037	0.084
Additional Income of The Household /100	0.006	0.006	-0.008	0.011	0.018*	0.010
Additional Income of The Household=0	0.039	0.078	-0.078	0.118	0.080	0.070
Living Space (1-st Quintile)						
2-nd Quintile	0.035	0.073	0.091	0.125	-0.040	0.102
3-rd Quintile	0.018	0.076	0.002	0.130	0.055	0.103
4-th Quintile	0.097	0.076	-0.038	0.131	0.189*	0.103

Variables	Total		Unmarried		Married	
	df/dx	SE	df/dx	SE	df/dx	SE
5-th Quintile	0.138*	0.082	-0.070	0.143	0.228**	0.110
Number of Other Members of the Household	0.060***	0.018	0.053*	0.029	0.039	0.027
Number of Desirable Additional Children	0.684***	0.074	0.338***	0.118	0.945***	0.113
Number of Desirable Additional Children ^2	-0.216***	0.024	-0.110***	0.042	-0.299***	0.035
Settlement Status (City with a population more than 1 mln.)						
Regional Center	0.016	0.078	-0.014	0.128	-0.024	0.110
Small Town	0.179**	0.076	0.163	0.126	0.107	0.106
Country	0.244***	0.079	0.273**	0.130	0.115	0.113
Federal Region (Central)						
North-Western	-0.004	0.087	-0.032	0.151	0.065	0.121
Southern	-0.010	0.080	-0.064	0.136	0.121	0.110
Privolgskiy	0.012	0.071	0.034	0.117	0.047	0.099
Ural	0.167*	0.088	0.123	0.150	0.125	0.127
Siberian	-0.071	0.086	0.019	0.140	-0.109	0.121
Far-Eastern	0.347***	0.103	0.361**	0.163	0.376**	0.148
Year (1994)						
1995	0.086	0.099	-0.092	0.169	0.210	0.137
1996	0.043	0.098	-0.060	0.159	0.114	0.140
1998	-0.010	0.098	-0.144	0.163	0.083	0.139
2000	-0.015	0.095	-0.047	0.152	0.013	0.136
2001	-0.037	0.092	-0.059	0.147	0.041	0.132
2002	0.001	0.090	-0.037	0.146	0.047	0.129
2003	0.006	0.090	-0.240	0.155	0.149	0.127
Husband, Age					-0.014*	0.008
Husband, Employment Status (Unemployed)					0.038	0.088
Husband, Health (Bad)						
Average					-0.255	0.175
Good					-0.166	0.175
Husband, Drinking (Abstain)						
1 Time a Month					0.070	0.111
2-3 Times a Month					-0.059	0.095
1 Time a Week					-0.002	0.099
2 Times a Week and More					-0.170	0.110
Husband, Real Income/100					0.003	0.013
Husband, Real Income=0					0.019	0.091
N	11768		4504		4922	
Statistical Coefficients	Wald $\chi2(46)$=444.88 Prob>$\chi2$= 0.0000 LR ρ=0: $\chi2(01)$=1.79 Prob>=$\chi2$=0.091		Wald $\chi2(45)$=77.41 Prob>$\chi2$= 0.0019 LR ρ=0: $\chi2(01)$=0.27 Prob>=$\chi2$=0.301		Wald $\chi2(55)$=396.49 Prob>$\chi2$= 0.0000 LR ρ=0: $\chi2(01)$=2.74 Prob>=$\chi2$=0.049	

Significance of the Coefficient under Respective Confidence Level .01 - ***; .05 - **; .1 - *