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December 2009

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December 10, 2009

Abstract

This paper works on an adaptation of the standard Pissarides model to incorporate an exponentially distributed match specific job destruction rate. We disscuss the characterization of equilibrium, equilibrium wage, number of equilibria, stead state unemployment, welfare and comparative statics problems in this adapted model.

1 Introduction

Pissarides model of unemployment (1990) is one of the baseline models for search theory. Various extension has been made upon this fundamental model. In Pissarides (1990), one basic parameter is the job destruction rate λ, which is the main issue this paper will focus on.

In the basic model of Pissarides (1990), the transition between employment and unemployment is modeled as a trading process. Workers with labor services and firms with job vacancies want to trade. And this trading process is uncoordinated, time-consuming and costly for both parties. Equilibrium in this system is defined by a state in which both parties maximize their own objective functions under given matching and production technologies, and in which the flow of workers into unemployment is equal to that of out of unemployment.

For a worker, his problem is to maximize his present-discounted value of the expected income flow when he is unemployed and when he is employed. In the basic model, employed workers earn a wage w and lose their jobs and become unemployed at the exogenous rate

λ, however, this uniform job destruction rate assumption is far from realistic. Different workers may face different probability of losing their jobs. And people may only know the distribution of their job destruction rate. In this paper, we generalize the Pissarides (1990) model by using a randomly chosen job destruction rate λ_i which distributing exponentially for each worker i from a unit mass of population 1. It turns out that in the equilibrium, expectation plays a much important role in matching decision than a single worker's specific job destruction rate λ_i.

The rest of the paper is organized as follows. Section 2 introduces the assumption and the characterization of the equilibrium of the standard Pissarides model. Section 3 presents the scenario in which for each work, match specific job destruction rate follows an exponential distribution, and addresses the new characterization of equilibrium, equilibrium wage, number of equilibria, stead state unemployment, welfare and comparative statics problems. Section 4 concludes.

2 Revisit Pissarides model of unemployment (1990)

First, we introduce some basic assumptions from Pissarides(1990):

For the demography, we assume that trade and production are strictly separated from each other. In each firm, there exists filled jobs and vacant jobs, denote mass of vacancies as v. Similarly, a worker may be employed or unemployed, but only unemployed worker will search for vacant jobs. Suppose there are mass 1 of workers, and unemployed ratio is denoted as u.

For the preferences, suppose both workers and firms are risk neutral, and they share common discount rate r. Also assume that during job search, workers enjoy some real return b.

In this model, there are two kinds of technologies: first we consider production technology, let each matched pair produce output worth p and firms engage in hiring at a fixed cost a per unit of time. For example, a can be imagined as the cost for job advertising. For matching technology assumptions. Suppose a matching function which satisfies

$$mL = M(uL, vL), \tag{2.1}$$

for L workers.

It is assumed that Equation (2.1) to be increasing and twice differentiable in both its arguments, concave, and constant return to scale. Introducing notation θ to denote the ratio $\frac{v}{u}$, then, by constant return to scale, we get transmission rate α_w at which unemployed workers become employed as

$$\alpha_w = \frac{M(u,v)}{u} = M(1,\theta), \tag{2.2}$$

and transmission rate α_f at which vacant jobs become filled as

$$\alpha_f = \frac{M(u,v)}{v} = \frac{M(1,\theta)}{\theta}. \tag{2.3}$$

Furthermore, for notation convenience,

$$m(\theta) = M(1,\theta) \tag{2.4}$$

is used thereafter. Assume

$$m(\theta) - \theta m'(\theta) > 0$$

then

$$\alpha_f'(\theta) = [\frac{m(\theta)}{\theta}]' = -\frac{m(\theta) - \theta m'(\theta)}{\theta^2} < 0. \tag{2.5}$$

We also have assumptions about information and institutions: in this model only aggregate variables are known to both parties. And terms of trade determined by Ex post bargaining. In one case, generalized Nash bargaining solution for optimal wage w is determined by

$$w = \arg\max_{w_i}(V_{f_i} - U_{f_i})^{\phi}(V_{w_i} - U)^{1-\phi}, \tag{2.6}$$

in which V_{f_i} denotes value to filed job and U_{f_i} denotes value to holding vacancy for firm $i \in [0,1]$, U is value to unemployment and V_{w_i} is value to employment given wage w_i for worker i, parameter ϕ stands for the bargaining power of firms' and $\phi \in [0,1]$.

Based on these assumptions, we consider the following questions:

First of all, for Firms: We now have one asset value equations for firms when holding vacancy and the other when job is occupied by a worker, in which λ_i stands for job destruction rate:

$$rU_{f_i} = -a + \alpha_f(V_{f_i} - U_{f_i}) - \lambda U_{f_i}, \tag{2.7}$$

$$rV_{f_i} = p - w_i - \lambda V_{f_i}. \tag{2.8}$$

At the same time, for workers: We establish asset value equation when a worker is unemployed as

$$rU = b + \alpha_w(V_{w_i} - U), \tag{2.9}$$

and similarly, if a worker is holding a job with wage w_i, his asset value equation is

$$rV_{w_i} = w_i + \lambda(U - V_{w_i}). \tag{2.10}$$

From the Nash bargaining condition, since U_{f_i} and U are exogenous options, we solve the Nash bargaining problem by the first order condition to get

$$V_{f_i} - U_{f_i} = \phi[(V_{f_i} - U_{f_i}) + (V_{w_i} - U)]. \tag{2.11}$$

And because in equilibrium, all profit opportunities from new jobs are exploited, we know

$$U_{f_i} = 0, \tag{2.12}$$

which is called free-entry condition in equilibrium.

By asset value equations for firms and workers (from (2.7) to (2.10)), wage determine function (2.11) and free-entry condition (2.12), we have characterization of equilibrium as

$$\frac{m(\theta)\phi(p-b)}{\theta[r + \lambda + (1-\phi)m(\theta)]} = a. \tag{2.13}$$

3 An adaptation of Pissarides (1990) on job destruction rate

Now, keeping the other assumption unchanged, if job destruction rate is match specific and distributed exponentially, what will happen to the unemployment model?

Matching specific job destruction rate means that both firms and workers do not know λ_i they could possibly face until they are actually in the matching process(subscript $i \in [0,1]$ indexes a random worker i.). The only information about λ_i its distribution function $F(\lambda) = 1 - \exp(-\eta\lambda)$, where $\eta \in R_{++}$ is a parameter.

As a result, when an unemployed worker i meets a firm with a job opening, he can only use the expectation of asset value of employment across the distribution of λ_i to evaluate the asset value of unemployment, then if we define $E_{\lambda_i} V_{w_i}$ as the expectation of flow value of employment of worker i given wage w_i across λ_i,(2.9) is turned into:

$$rU = b + \alpha_w (E_{\lambda_i} V_{w_i} - U), \tag{3.1}$$

By the same logic, we define $E_{\lambda_i} V_{f_i}$ as expectation of flow value of filed job for firm i across λ_i, and modify asset value equation for holding vacancies (2.7) as (imposing job destruction rate $\lambda_i = 0$ to simplify this problem):

$$rU_{f_i} = -a + \alpha_f (E_{\lambda_i} V_{f_i} - U_{f_i}), \tag{3.2}$$

As for equation (2.10) and (2.8), since both of these two equations were evaluated after the matching process when λ_i and w_i are available to both parties (w_i is the corresponding wage for λ_i), hence, we have

$$rV_{w_i} = w_i + \lambda_i (U - V_{w_i}), \tag{3.3}$$

$$rV_{f_i} = p - w_i - \lambda_i V_{f_i}. \tag{3.4}$$

Given (3.1) to (3.4), we are now able to establish a new characterization for the equilibrium.

New equilibrium:

From free entry condition for equilibrium, consider (3.2), we know

$$\alpha_f (EV_{f_i}) = a, \tag{3.5}$$

As long as we can express EV_{f_i} with model parameters, the above equation can lead to a characterization for the new equilibrium.

Adding (3.3) and (3.4), we get

$$(r+\lambda_i)(V_{w_i}+V_{f_i}) = p+\lambda_i U, \tag{3.6}$$

By calculating expectation across λ_i, we have

$$E_{\lambda_i}(V_{w_i}+V_{f_i}) = E_{\lambda_i}(\frac{p}{r+\lambda_i}) + UE_{\lambda_i}(\frac{\lambda_i}{r+\lambda_i}), \tag{3.7}$$

Thus, suppose

$$k = E_{\lambda_i}(\frac{1}{r+\lambda_i})$$

$$E_{\lambda_i}V_{w_i} + E_{\lambda_i}V_{f_i} = pk + U(1-rk), \tag{3.8}$$

From wage determine function (2.11), we have

$$\frac{1-\phi}{\phi}V_{f_i} + U = V_{w_i}, \tag{3.9}$$

If we take expectation over λ_i for both sides of this equation, then

$$\frac{1-\phi}{\phi}E_{\lambda_i}V_{f_i} + U = E_{\lambda_i}V_{w_i}, \tag{3.10}$$

After that, combine this equation with (3.1) and (3.8), we get

$$E_{\lambda_i}V_{f_i} = \frac{\phi(p-b)}{k^{-1}+\alpha_w(1-\phi)}, \tag{3.11}$$

and

$$U = \frac{\alpha_w E_{\lambda_i}V_{f_i}\frac{1-\phi}{\phi}+b}{r}, \tag{3.12}$$

Finally, the characterization of new equilibrium follow (3.5) is

$$a = \frac{m(\theta^*)\phi(p-b)}{\theta^*[k^{-1}+m(\theta^*)(1-\phi)]}, \tag{3.13}$$

for

$$k = E_{\lambda_i}(\frac{1}{r+\lambda_i})$$

If we compare this characterization with (**??**), which is

$$a = \frac{m(\theta^*)\phi(p-b)}{\theta^*[r+\lambda+(1-\phi)m(\theta^*)]},$$

We can conclude that the main difference is that in the new equilibrium, we use $[E_{\lambda_i}(\frac{1}{r+\lambda_i})]^{-1}$ to take the place of $r+\lambda$ in the old one.

Equilibrium wage:

From (3.3),(3.4) and (3.9), we have the following three equations that determine the equilibrium wage w_i:

$$(r+\lambda_i)V_{w_i} = w_i + \lambda_i U, \tag{3.14}$$

$$(r+\lambda_i)V_{f_i} = p - w_i, \tag{3.15}$$

$$\frac{1-\phi}{\phi}V_{f_i} + U = V_{w_i}. \tag{3.16}$$

In equilibrium,

$$w_i = \phi r U + (1-\phi)p. \tag{3.17}$$

From equations (3.12) and (3.11), we can induce that

$$w_i = \phi r \frac{\alpha_w E V_f \frac{1-\phi}{\phi} + b}{r} + (1-\phi)p = \frac{\alpha_w \phi(p-b)(1-\phi)}{k^{-1}+\alpha_w(1-\phi)} + \phi b + (1-\phi)p \tag{3.18}$$

Thus, we find that equilibrium wage w_i is uniquely determined by model parameters, random job destruction rate does not affect wage rate at all.

Next, in order to induce unemployed worker to accept possible job offer, we must have

$$w_i \geq b. \tag{3.19}$$

Considering (3.18), this will lead to

$$\frac{\alpha_w \phi(p-b)(1-\phi)}{k^{-1}+\alpha_w(1-\phi)} + \phi b + (1-\phi)p \geq b. \tag{3.20}$$

Or equivalently,

$$\frac{\alpha_w \phi(p-b)}{k^{-1}+\alpha_w(1-\phi)}+p \geq b.$$

It is obvious that any $\eta > 0$ will satisfy this relation given a reasonable assumption that $p > b$.

Uniqueness of the equilibrium:

From condition (2.5), we know

$$d\frac{m(\theta)}{\theta}/d\theta < 0, \tag{3.21}$$

together with

$$m'(\theta) > 0,$$

we know that the right hand side of equation (3.13) is decreasing in θ.

On the other side, when $\theta \to 0$, we know $m(\theta) \to 0$, and right hand side of (3.13)$\to$ $\frac{m(\theta)\phi(p-b)}{\theta(k^{-1})}$, which is a positive constant.

In general, as long as $\lim_{\theta\to 0}\frac{m(\theta)}{\theta}$ is large enough, say larger than $\frac{a(k^{-1})}{\phi(p-b)}$, we can safely conclude that right hand side of equation (3.13) is larger than a when $\theta \to 0$.

Therefore, since $\lim_{\theta\to\infty}\frac{m(\theta)\phi(p-b)}{\theta[k^{-1}+m(\theta)(1-\phi)]} = 0$ and $\lim_{\theta\to 0}\frac{m(\theta)\phi(p-b)}{\theta[k^{-1}+m(\theta)(1-\phi)]} > a$ if $\lim_{\theta\to 0}\frac{m(\theta)}{\theta} > \frac{a(k^{-1})}{\phi(p-b)}$ and the continuity of matching function, together with the fact that right hand side of (3.13) is decreasing in θ, we know that equilibrium vacancy-unemployment ratio θ exists and unique.

Steady state unemployment:

Since people who are employed losing their job at the rate of λ_i, $i \in [0, 1-u]$, while people with mass u who are currently unemployment matching a position at the rate of α_w, these two will yield a stead state condition for unemployment in this model:

$$\int_0^{1-u}\lambda_i di = u\alpha_w.$$

Since

$$\int_0^{1-u}\lambda_i di = (1-u)E\lambda_i,$$

given each λ_i i.i.d, we have stead state condition similar to that of Pissarides (1990) and by the distribution of λ_i, we know $E\lambda_i = \frac{1}{\eta}$:

$$\frac{1-u}{\eta} = u\alpha_w.$$

or equivalently,

$$u = \frac{1/\eta}{m(\theta) + 1/\eta} = \frac{1}{m(\theta)\eta + 1} \tag{3.22}$$

Welfare:

If w is present value of all future utility, then social planner will solve

$$\max_{\theta} rw$$

to find the optimal θ which maximize social welfare.

In steady state, since

$$rw = ub + p(1-u) - av,$$

and plug (3.22) in,

$$v = \theta u = \frac{\theta}{m(\theta)\eta + 1},$$

we finally get

$$\begin{aligned} rw &= \frac{b}{m(\theta)\eta + 1} + p(1 - \frac{1}{m(\theta)\eta + 1}) - a\frac{\theta}{m(\theta)\eta + 1} \\ &= \frac{b + pm(\theta)\eta - a\theta}{m(\theta)\eta + 1}. \end{aligned}$$

Now planner's problem is

$$\max_{\theta} \frac{b + pm(\theta)\eta - a\theta}{m(\theta)\eta + 1}$$

From F.O.C

$$\frac{m'(\hat{\theta})(p-b)}{m(\hat{\theta}) - \hat{\theta}m'(\hat{\theta}) + \frac{1}{\eta}} = a. \tag{3.23}$$

where $\hat{\theta}$ is planner's choice of θ.

When will decentralized optimal θ^* coincide $\hat{\theta}$? We have already know that decentralized economy optimal vancancy-unemployment ratio θ^* satisfies (3.13), and it is OK for the planner to pick $r = 0$ in (3.13), in that circumstances, if bargaining power of firms'

$$\phi = \theta \frac{m'(\theta)}{m(\theta)}$$

(3.13) coincides (3.23).

Comparative statics:

Similar to the model of Pissarides (1990),

based on (3.13), we have the following comparative statics:

If b increases, the right hand side of (3.13) falls for every value of θ, as a result, θ^* will decrease, $m(\theta^*)$ will decrease, u will increase and v will decrease. Welfare will increase.

If a decreases, means θ^* will increase, $m(\theta^*)$ will increase, v will increase and u will decrease. Welfare will increase since a is a total cost.

4 Concluding remarks

In this paper, we adapt the stanard Pissardes (1990) model to a randomly distributed job destruction rate. We discuss the characterization of the equilibrium and welfare from the social planner's view. we describe the stead state of unemployment and comparative statics in equilibrium, and we prove that equilibrium is unique.

5 Reference

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