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Isolation or joining a mall? On the location choice of competing shops.*

Marielle C. Non†

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Abstract

I study the location choice of competing shops. A shop can either be isolated or join a mall. A fraction of consumers is uninformed about prices and incurs costs to travel between market places and to enter a shop. The equilibrium mall size is computed for several parameter values, showing that mall and isolated shops can coexist. Several effects play a role. Mall shops attract more consumers, but isolated shops set a higher maximum price. Moreover, numerical evaluations show that an increase in mall size decreases the average price level and increases the participation level of uninformed consumers.

Keywords: location choice, travel costs, pricing, consumer search

JEL codes: D43, D83, L11, L13

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1 Introduction

A city or town of reasonable size usually has multiple hairdressers, multiple grocery stores, multiple dry cleaners, and so on. Some of those hairdressers (or grocery stores, or dry cleaners, etc.) are located close to each other, for instance in a regional mall or in the city center. Others have no close competitors in their neighborhood, like a hairdresser in a small strip mall. In this paper I will use the term isolated shop for a shop that has no direct competitors nearby, whereas a shop that does have competitors nearby will be called a mall shop. Both types of shops coexist, and one might wonder how this is possible. At first sight, it seems unattractive to locate next to some direct competitors in a regional mall. On the other hand, once a regional mall with strong competition and low prices exists, how can isolated shops survive?

Most previous research only answers the question of why large malls exist, see e.g. Stahl (1982a, 1982b), Gehrig (1998) and Konishi (2005). None of these papers finds the existence of isolated shops. The intuition that these papers give revolves around the heterogeneity of goods. When goods are heterogeneous, consumers prefer to visit a mall with a large variety to increase the probability of finding a good match. This increases the volume of sales in a mall and makes it profitable to locate together. This effect of heterogeneous goods is however not the complete story. Even when goods are heterogeneous and malls are more attractive for consumers, isolated shops could survive by lowering their price. This does not occur in equilibrium because of a simplifying assumption that the aforementioned papers make: consumers can only visit one market place, independent of whether it is a mall or an isolated shop. Therefore, when a consumer decides to visit an isolated shop, he or she is stuck there and the isolated shop can ask a monopoly price. Consumers anticipate this and prefer to go to a mall, where prices are lower and the choice is greater. Consequently, isolated shops attract no consumers[1] and do not exist in equilibrium.

The current paper presents a three-stage location choice model where for a large range of parameters mall and isolated shops coexist in equilibrium. In the first stage shops choose a location that will maximize their individual profits, in the second stage shops simultaneously set prices and in the last stage consumers (who only know equilibrium price distributions, not the realized prices) decide whether and where to search and buy. An important difference with previous models is that consumers can search different shops in different market places. For a fraction γ of the consumers, called shoppers,

[1]Some of the papers mentioned above assume a spatial structure where consumers and shops are spread along a line or plane. In this case there are some consumers who prefer the isolated shop because it is much closer to their home than the mall. But because of the high price and limited choice in the isolated shop, the number of consumers they attract is very small and generally not enough to make an isolated shop profitable.

this search comes at no cost. Therefore these consumers search all shops and will buy at the cheapest shop. The other consumers, called non-shoppers, incur costs to search in different shops and different market places. There are costs to travel between market places (travel costs) and there are costs to enter a shop once in a market place (entering costs). This has important implications for isolated shops. First, setting a price that is higher than the expected mall price plus the entering and travel cost is not profitable, since non-shoppers who initially visited this isolated shop will not buy there. Instead they will continue to search in other shops. Isolated shops are thus forced to adjust their maximum price towards the mall prices. Next to that, there is fierce competition for the shoppers. This gives isolated shops an incentive to set even lower prices. Because of these two effects isolated shops attract a share of consumers and can survive in equilibrium.

Mall shops can also survive in equilibrium, even though I assume homogenous goods. Homogenous goods lead to more competition in the mall compared to when goods are heterogenous, but because of the entering costs that are incurred when searching another shop in the same market place the mall shops still make a positive profit. Moreover, as will be discussed later in more detail, mall shops attract more consumers per shop than isolated shops, especially when the mall is small. This increased sales volume makes locating in a mall attractive. Thus, even in a setting where variety does not play a role a mall can exist.

Apart from the joint existence of mall and isolated shops, the current paper adds some other interesting insights to the literature. First of all, the pricing behavior of shops has some special features. Mall shops compete for the shoppers, but have some power over the non-shoppers. The mall shops balance these two effects by randomizing over prices. For the same reason, isolated shops also randomize over prices, but they choose a different support than the mall shops. There is a strictly positive probability, smaller than one, that an isolated shop sets a price equal to the maximum mall price plus the travel costs (which in equilibrium equals the expected mall price plus the travel and entering costs). With the remaining probability an isolated shop will randomize over a continuous set of prices that is strictly below the maximum mall price. An isolated shop can profitably set a price above the maximum mall price because of the travel costs. When a non-shopper is currently in an isolated shop he or she has to incur entering and travel costs to visit another shop. When a non-shopper is in a mall shop, continuing search within the mall is free of travel costs. Therefore, when in an isolated shop, non-shoppers are willing to pay a somewhat higher price than when in a mall shop. On the other hand, an isolated shop also sometimes sets a relatively low price. If this price turns out to be the lowest price in the market the isolated shop will attract all shoppers. Therefore, setting a low price is as profitable as setting a high price. Importantly, the expected price in an isolated shop equals the expected mall price. This ensures that initially consumers are indifferent between visiting a mall or isolated shop. The pricing behavior of isolated shops could be interpreted as follows. Isolated shops generally set a price above the maximum mall price, but they also regularly offer a price that is low relative to the mall prices. An isolated shop will attract non-shoppers who hope to be lucky enough to find a low price. But even if the non-shopper does not find a low price he or she will stay at the isolated shop because to continue search the non-shopper has to incur travel costs.

Another interesting result is that mall shops attract more non-shoppers than isolated shops. The intuition behind this is fairly straightforward. If isolated shops would attract many non-shoppers, they would make more profits on the maximum price in their support than on the lower prices in their support, which cannot be an equilibrium situation. Put differently, isolated shops are only willing to randomize over prices when the shoppers are relatively important for them. For mall shops, the difference between the maximum and minimum price in their support is smaller and therefore they are willing to randomize over prices even when they attract many non-shoppers.

A simple consequence of the uneven distribution of non-shoppers over mall and isolated shops is that mall shops make more profits than isolated shops. Still, this does not imply that all isolated shops want to join the mall. If an isolated shop relocates and joins a mall, the mall size increases, which increases competition in the mall and decreases the expected prices in the mall. The remaining isolated shops also have to adjust their prices downwards, and as a result the profits of both the original mall shops and the remaining isolated shops decrease when an isolated shop relocates to a mall. Whether the profits of the relocating shop increase or decrease depends on the number of consumers that are gained by relocating to the mall and on the size of the decrease in prices. As the analysis will show, in some cases it is profitable to join a mall and capture additional non-shoppers, whereas in other cases the decrease in prices is too strong to make joining a mall profitable. When the costs of visiting a shop are high, a third effect plays an important role in the location choice of shops. As in Janssen, Moraga-Gonzalez and Wildenbeest (2005), when the costs to visit a shop are high, some non-shoppers stay at home and do not buy at all. When this happens and when more shops locate in the same mall, prices tend to decrease and the participation of non-shoppers increases. Thus, when an isolated shop joins a mall it will capture a larger share of non-shoppers *and* the total amount of non-shoppers will increase. The joint effect on the sales of the isolated shop that relocates is so strong that it is always profitable to join a mall. So for high enough search costs all isolated shops will want to join a mall and the only possible equilibrium has no isolated shops.

The second paragraph of this introduction already mentioned some previous work on location choice. Several other papers should also be mentioned.

First, as far as I know, Dudey (1990 and 1993) are the only papers on location choice that also assume homogenous goods. In these models, consumers can only visit one market place, but once in a mall a consumer can visit all shops in that mall at zero costs. To make sure mall prices are above zero, shops compete in quantities. Because consumers can visit only one market place, and because the largest mall has the lowest prices, all consumers visit the largest mall and isolated shops do not exist. Wolinsky (1983) is one of the few papers that assume consumers can visit more than one market place. Wolinsky however only analyzes conditions under which all shops locate in the same mall, and does not give any attention to the possibility of isolated shops. The paper that comes closest to the current paper is Fischer and Harrington (1996). In their model products are heterogeneous, consumers can visit more than one market place and once in the mall all shops in the mall can be visited for free. To keep their model tractable, Fischer and Harrington however need to assume that consumers expect an infinite number of isolated shops, even though in equilibrium there is a finite number of isolated shops. In the current paper, consumers know the (finite) number of isolated shops beforehand, and act accordingly. Moreover, the current paper shows that product heterogeneity is not necessary to find an equilibrium with both mall and isolated shops.

Although not written in terms of location choice, the model of Baye and Morgan (2001) is closely related to my model. In Baye and Morgan isolated shops selling a homogenous product have the option to join a platform. Consumers can at some cost visit their local isolated shop, or they can at some entrance fee visit the platform. Once a consumer has entered the platform, he can visit all shops on the platform for free. In equilibrium, shops randomize between joining the platform and staying isolated. This result resembles the main result in my paper, but the mechanism driving this result is completely different. In the setup of Baye and Morgan the competition in the platform ensures that the probability a shop joins the platform is strictly below one, while the existence of a profit maximizing platform owner ensures that the probability a shop joins the platform is strictly above zero. Next to this, the pricing strategy of the shops and the searching strategy of consumers is completely different from the equilibrium outcome in my model.

The remainder of the paper is organized as follows. The next section presents the model. Section 3 analyzes the second and third stage of the model for two extreme cases. In one case all shops are isolated and in the other case all shops are located in the same mall. This will build some intuition before heading on to section 4, which analyzes the second and third stage of the model for the case with both mall and isolated shops. Section 5 gives some comparative statics on the results found in Section 4. Finally, Section 6 analyzes the location choice of shops and Section 7 concludes. All proofs are in appendix A.

2 The model

The model has $n > 2$ shops in the market that sell a homogeneous good. Production costs are linear and without loss of generality they are assumed to be zero. As mentioned in the Introduction, the model has three stages. In the first stage shops choose a location that will maximize their individual future profits. I assume that only one mall can be formed. One can think of a town that has one regional mall with ample space for new shops and several much smaller in-town mini malls that have no space to expand and accommodate new shops. A shop thus has to choose between locating in the regional mall, next to some competitors, or locating in a mini mall without any direct competitors. In the remainder of this paper I will refer to the regional mall with several competitors as the 'mall'. A mall with k^* shops is an equilibrium when none of the mall shops can increase its profits by leaving the mall and none of the isolated shops can increase its profits by joining the mall.

In the second stage, the shops choose a price. I explicitly allow for a mixed strategy that depends on mall size k and on whether a shop is isolated or a mall shop. Therefore the price strategy of an isolated shop j is denoted by a price distribution $F^i_{kj}(p)$, where $F^i_{kj}(p)$ is the cdf of the price distribution. The strategy of a mall shop h is denoted by a price distribution $F^m_{kh}(p)$ The maximum price is denoted by $\overline{p}^i_{kj}$ or $\overline{p}^m_{kh}$ and the minimum price by $\underline{p}^i_{kj}$ or $\underline{p}^m_{kh}$. Note that if isolated shop j (mall shop h) chooses a pure price strategy with price p^* the price distribution is given by $F^i_{kj}(p) = 0$ ($F^m_{kh}(p) = 0$) for $p < p^*$ and $F^i_{kj}(p) = 1$ ($F^m_{kh}(p) = 1$) for $p \geq p^*$. In the next sections it will however become clear that there is no symmetric pure strategy equilibrium.

In the third stage of the model consumers decide on whether and where to search and buy. The model has a unit mass of consumers, all having unit demand and a valuation θ for the product. The consumers are aware of all the locations of the shops, but they do not know the prices in the shops. They however form rational price expectations and base their decisions on these expectations.

There are two different types of consumers. A fraction γ of consumers consists of shoppers who have zero entering and travel costs. As a consequence shoppers know all the prices and buy at the cheapest shop.[2] A fraction

[2] One could think of shoppers as consumers who obtain a strictly positive utility from the shopping experience, even if travel expenses are taken into account. For the results it is not strictly necessary that there are consumers with zero entering and travel costs who know all prices. The less restrictive assumption that some fraction γ of consumers gets to know the prices of two or more random shops without incurring entering and travel costs would be sufficient to obtain the results in this paper. For simplicity I however assume

$1-\gamma$ of consumers consists of consumers who incur strictly positive entering and travel costs. These consumers are referred to as non-shoppers. Non-shoppers incur entering costs $c_e > 0$ when entering a shop. These costs are incurred whenever a not previously visited shop is entered and do not depend on whether a shop is in a mall with several shops or is an isolated shop. The entering costs are equivalent to the continuation costs in a standard consumer search model. These costs reflect the time spent in the shop, finding the product on the shelf, finding the price of the product, waiting for a shop assistant to help you, etc. Note that positive entering costs are essential in the model. Without entering costs non-shoppers could without additional costs search all the shops in the mall. This would drive the mall prices to zero, and no shop would ever locate in a mall. In addition to the entering costs, non-shoppers incur travel costs $c_t > 0$ whenever they travel from their house to a market place or travel between market places, where a market place can be either a shopping mall with several shops selling the product or an isolated shop. The travel costs are incurred every time a non-shopper travels between market places, and therefore are also incurred when returning to a previously visited shop that is in a different market place than the market place where the non-shopper currently is.[3] The travel costs can be interpreted as the costs of, say, a bus ticket or petrol costs. The travel costs ensure that searching h shops in the same mall comes at less costs than searching h shops spread over different market places. Finally, the analysis is restricted to values of c_e and c_t for which $c_t + c_e < \theta$.
Non-shoppers search sequentially. This means that non-shoppers first decide on whether to stay at home, visit a mall shop or visit an isolated shop. Let μ_k denote the fraction of non-shoppers who decide to visit a shop (active non-shoppers) when the mall has size k and let $1 - \mu_k$ denote the fraction of non-shoppers who decide to stay at home. The fraction μ_k is determined in equilibrium. Based on the price found in the first shop, an active non-shopper decides on whether to search a second shop and whether this second search will be in the same market place as the first search (if possible) or in another market place. Then, based on the outcome of the second search, active non-shoppers decide on whether or not to search a third time and where the third search will be, etc.

In the analysis below I will derive a subgame perfect equilibrium of the three

the presence of a fraction of consumers with zero entering and travel costs.

[3]These return costs are necessary to prevent arbitrage. Imagine a situation of one mall with shops 1 and 2 and two isolated shops, 3 and 4. Now suppose a non-shopper's first search was in shop 1 and the second search was in shop 3. If there are no return costs and the non-shopper would like to visit shop 2 in his third search he could go there immediately at cost $c_t + c_e$, but he could also return to shop 1 at no costs and then visit shop 2 at cost c_e. To prevent such a situation return costs of at least c_t are necessary when returning to a shop in a different market place.

stage game described in this section. I will focus on symmetric equilibria in the sense that all shops in the same market place choose identical price distributions and market places of the same size have identical price distributions as well. Note that identical price distributions do not necessarily imply identical prices because realized prices could differ from each other. Because price distributions are symmetric where possible and because I only consider situations where there is at most one shopping mall I drop the shop indices j and h in $F^i_{kj}(p)$, $F^m_{kh}(p)$, $\underline{p}^i_{kj}$, $\underline{p}^m_{kh}$, $\overline{kj}^i$ and $\overline{p^m_{kh}}$. Also, for $k = 1$ and $k = n$ I drop the indices i and m because in those cases either all shops are isolated or all shops are in the mall.
Because shops choose symmetric pricing strategies, non-shoppers a priori have no preferences over shops that are located in the same mall. Moreover, non-shoppers a priori have no preferences over the isolated shops. Once a non-shopper has chosen to visit the mall he will therefore choose a random shop from this mall. In the same vein, once a non-shopper has decided to visit an isolated shop he will choose such a shop at random.

3 Two opposite cases: only isolated shops and only mall shops

Only isolated shops

This subsection gives some results on consumer behavior and pricing behavior of shops when all shops are isolated. In this case each visit to a shop comes at cost $c_t + c_e$, and each return visit to a previously visited shop comes at cost c_t. This model is equivalent to the model in Janssen, Moraga-Gonzalez and Wildenbeest (2005) (henceforth JMW) except for the return costs, which are absent in the JMW model. It is however relatively easy to show that the equilibrium derived in JMW also holds in a model with return costs and in this section I will focus on this equilibrium.[4] For the sake of brevity many details are omitted. See JMW for a more extensive discussion.

One key component of the equilibrium is the so-called reservation price r_1, implicitly defined by

$$\int_{\underline{p}_1}^{r_1} (r_1 - p)dF_1(p) = c_t + c_e.$$

The reservation price is defined in such a way that non-shoppers will stop searching as soon as they find a price at or below r_1. In JMW the unique equilibrium has $\overline{p}_1 \leq r_1$ and therefore I will concentrate on an equilibrium

[4]Return costs complicate a full analysis considerably and could potentially lead to multiple equilibria. See Janssen and Parakhonyak (2008) for an analysis of consumer behavior under the assumption of return costs.

with $\overline{p}_1 \leq r_1$. The derivation of the equilibrium shows that there is only one equilibrium with $\overline{p}_1 \leq r_1$, although there could also exist equilibria with $\overline{p}_1 > r_1$.
Note that $\overline{p}_1 \leq r_1$ implies that non-shoppers will immediately stop searching after they visited the first shop. This in turn implies that non-shoppers will only start searching when $\theta - Ep_1 - c_t - c_e \geq 0$, with $Ep_1 = \int_{\underline{p}_1}^{\overline{p}_1} p dF_1(p)$. Because $\overline{p}_1 \leq r_1$, the definition of r_1 can be rewritten as $r_1 - Ep_1 = c_t + c_e$. This gives that non-shoppers will only start searching when $\theta \geq r_1$. When $\theta > r_1$ all non-shoppers will search and $\mu_1 = 1$. This situation will be referred to as 'full search' and occurs if and only if $c_t + c_e$ is below some threshold C^*. When $\theta = r_1$, non-shoppers are indifferent between searching and not searching, and $0 < \mu_1 < 1$. This situation will be referred to as 'partial search' and occurs if and only if $c_t + c_e$ is above C^* and below θ. When $\theta < r_1$, non-shoppers do not search at all. Note that in that case shops only sell to the shoppers. This will drive the prices down to zero, and so $\theta < r_1$ can only occur when $c_t + c_e > \theta$. By assumption, $c_t + c_e < \theta$, and therefore at least some non-shoppers will search in equilibrium.
Deriving the optimal pricing behavior of shops is a fairly straightforward exercise. When all shops set a price at or below r_1 it is easy to see that deviating to a higher price leads to zero profits. Also, if $\overline{p}_1 < r_1$, it would be profitable to deviate to r_1, so in equilibrium $\overline{p}_1 = r_1$. Setting the profit function $\pi_1(p)$ equal to $\pi_1(r_1)$ gives the equilibrium price distribution. The next Propositions summarize.

Proposition 3.1 *(Full search equilibrium, $\mu_1 = 1$)*
If $c_t + c_e < \theta(1 - \int_0^1 \frac{1}{1+\frac{\gamma}{1-\gamma} n y^{n-1}} dy)$ then all non-shoppers are active. Non-shoppers will stop searching as soon as $\min(p^*, p^{min} + c_t) \leq r_1$, with p^* the price found in the shop that was last visited, p^{min} the lowest price found in previously visited shops (infinite when there are no previously visited shops) and with r_1 defined as

$$r_1 = \frac{c_t + c_e}{1 - \int_0^1 \frac{1}{1+\frac{\gamma}{1-\gamma} n y^{n-1}} dy}.$$

Shops randomize over $p \in [\frac{1-\gamma}{1+\gamma(n-1)} r_1, r_1]$ according to the price distribution

$$F_1(p) = 1 - (\frac{1-\gamma}{\gamma n} \frac{r_1 - p}{p})^{\frac{1}{n-1}}.$$

Expected profits are given by $\pi_1 = r_1 \frac{1-\gamma}{n}$.

Proposition 3.2 *(Partial search equilibrium, $0 < \mu_1 < 1$)*
If $c_t + c_e > \theta(1 - \int_0^1 \frac{1}{1+\frac{\gamma}{1-\gamma} n y^{n-1}} dy)$ a fraction $0 < \mu_1 < 1$ of the non-shoppers is active, whereas the remaining fraction $1 - \mu_1$ of non-shoppers is inactive. The fraction μ_1 is implicitly defined by

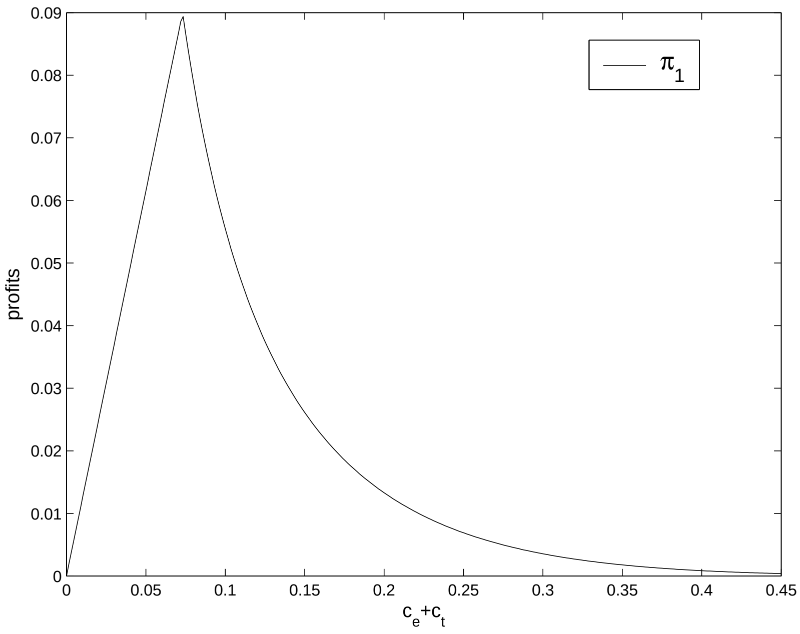

Figure 1: Expected profits as a function of the search costs when all shops are isolated. This figure is based on 10 shops, 10 % shoppers and a valuation of the product of 1.

$$h(\mu_1) \equiv \int_0^1 \frac{1}{1+\frac{\gamma n}{(1-\gamma)\mu_1} y^{n-1}} dy = \frac{\theta - c_t - c_e}{\theta}.$$

Active non-shoppers stop searching as soon as $\min(p^*, p^{min} + c_t) \leq \theta$. *Shops randomize over* $p \in [\frac{(1-\gamma)\mu_1}{\gamma n+(1-\gamma)\mu_1}\theta, \theta]$ *according to the price distribution*

$$F_1(p) = 1 - (\frac{(1-\gamma)\mu_1}{\gamma n} \frac{\theta - p}{p})^{\frac{1}{n-1}}.$$

Expected profits are given by $\pi_1 = \theta\mu_1 \frac{1-\gamma}{n}$.

Figure 1 shows the expected profits as a function of the search costs $c_t + c_e$. In this figure, the number of firms n equals 10, $\gamma = 0.1$ and $\theta = 1$. With these parameter values the full search equilibrium holds for $c_t + c_e < 0.073$ and the partial search equilibrium holds for $0.073 < c_t + c_e < 1$. Note that a search cost value of 0.073 implies that the search costs are 7.3% of the valuation of the product. The expected profits are plotted for $c_t + c_e < 0.45$; for higher values of $c_t + c_e$ the profits are decreasing and when $c_t + c_e$ approaches 1 the profits approach 0.

Only mall shops

When all the shops are in the same mall non-shoppers incur costs $c_e + c_t$ for the first search, they incur costs c_e for every next search and have no return costs. This model is equivalent to JMW except for the fact that the first search is more costly than every next search. There is a unique equilibrium, and once a non-shopper has visited one shop, the analysis is the same as in JMW. More specific, non-shoppers will stop searching as soon as they find a price at or below the reservation price r_n, which is defined by

$$\int_{\underline{p}_n}^{r_n} (r_n - p) dF_n(p) = c_e.$$

Note that the righthand side of this expression is c_e, instead of $c_e + c_t$ for the definition of r_1. As in JMW, the unique equilibrium has $\overline{p}_n = r_n$.
The analysis differs from JMW when it comes to full and partial search equilibria. All non-shoppers will search when $\theta - Ep_n - c_t - c_e > 0$. The definition of r_n gives $r_n - Ep_n = c_e$ and so all non-shoppers will be active when $r_n < \theta - c_t$. A partial search equilibrium occurs when $r_n = \theta - c_t$. Note that the travel costs explicitly occur in this expression. This is because non-shoppers incur travel costs when they search for the first time, whereas the expected prices $(r_n - c_e)$ are based on their behavior once they are in the mall, where travel costs do not play a role anymore.
Note that the maximum price that shops can ask is limited to $\theta - c_t$. Intuitively, the maximum price a shop can ask is limited to the expected price plus c_e. For a higher price, non-shoppers would continue searching and the shop setting the maximum price would not sell anything. On the other hand, the expected price can at most be $\theta - c_t - c_e$, otherwise no non-shopper would be active. Combined, this gives that the maximum price can at most be $\theta - c_t$.
The full search equilibrium has the following form.

Proposition 3.3 *(Full search equilibrium, $\mu_n = 1$)*
If $c_e < (1 - \int_0^1 \frac{1}{1+\frac{\gamma}{1-\gamma}ny^{n-1}}dy)(\theta - c_t)$ all non-shoppers are active and non-shoppers will stop searching as soon as they find a price at or below r_n, with r_n defined as

$$r_n = \frac{c_e}{1 - \int_0^1 \frac{1}{1+\frac{\gamma}{1-\gamma}ny^{n-1}}dy}.$$

Shops randomize over $[\frac{1-\gamma}{(1-\gamma)+\gamma n} r_n, r_n]$ according to the price distribution

$$F_n(p) = 1 - (\frac{(r_n - p)(1-\gamma)}{n\gamma p})^{\frac{1}{n-1}}.$$

Expected profits are $\pi_n = r_n \frac{1-\gamma}{n}$.

The partial search equilibrium is as follows.

Proposition 3.4 *(Partial search equilibrium, $0 < \mu_n < 1$)*
If $c_e > (1 - \int_0^1 \frac{1}{1+\frac{\gamma}{1-\gamma}ny^{n-1}}dy)(\theta - c_t)$ a fraction $0 < \mu_n < 1$ of non-shoppers is active, where μ_n is defined by

$$h(\mu_n) \equiv \int_0^1 \frac{1}{1 + \frac{\gamma n}{(1-\gamma)\mu_n} y^{n-1}} dy = \frac{\theta - c_t - c_e}{\theta - c_t}.$$

Active non-shoppers will stop searching as soon as they find a price at or below $\theta - c_t$. Shops randomize over $[(\theta - c_t)\frac{\mu_n(1-\gamma)}{\gamma n+\mu_n(1-\gamma)}, \theta - c_t]$ according to price distribution

$$F_n(p) = 1 - (\frac{(\theta - c_t - p)(1-\gamma)\mu_n}{n\gamma p})^{\frac{1}{n-1}}.$$

Expected profits are $\pi_n = (\theta - c_t)\frac{\mu_n(1-\gamma)}{n}$.

Figure 2 shows the expected profits as a function of the search costs $c_t + c_e$. Recall that the reservation value r_n depends only on the continuation costs of search, c_e. Moreover, the decision whether or not to search depends on c_t. Therefore, the expected profits do not depend on total costs $c_e + c_t$, but on c_t and c_e in isolation. To be able to make a plot of the expected profits as a function of the total costs $c_e + c_t$ I assume that c_e and c_t are related to each other in a fixed proportion, that is, $c_t = \beta(c_t + c_e)$ and $c_e = (1-\beta)(c_t + c_e)$, or consequently $c_t = \frac{\beta}{1-\beta}c_e$. In the figure, $\beta = 0.8$. As before, the number of firms n equals 10, $\gamma = 0.1$ and $\theta = 1$. The expected profits are plotted for $c_t + c_e < 0.8$. For higher values of $c_t + c_e$ the expected profits decrease to 0. The figure shows the same pattern as in the case where all the shops are isolated.

Comparing the two opposite cases

A comparison of the equilibria in the previous two subsections gives some new and interesting results. It can be shown that for low values of $c_t + c_e$ it is more profitable to have all shops isolated, whereas for high values of $c_t + c_e$ having all shops in the same mall leads to more profits. There are two opposing effects playing a role here. First, prices are higher when all shops are isolated. Second, more non-shoppers are active when all shops are in the same mall. When $c_t + c_e$ is low the second effect is either absent or small, but when $c_t + c_e$ is high, the second effect is stronger than the first. Figure 3 combines figures 1 and 2 by showing the expected profits as a function of the search costs $c_t + c_e$ in the case where all shops are isolated and in the case where all shops are located in the same shopping mall. Again, the number of firms n equals 10, $\gamma = 0.1$ and $\theta = 1$. In the figure

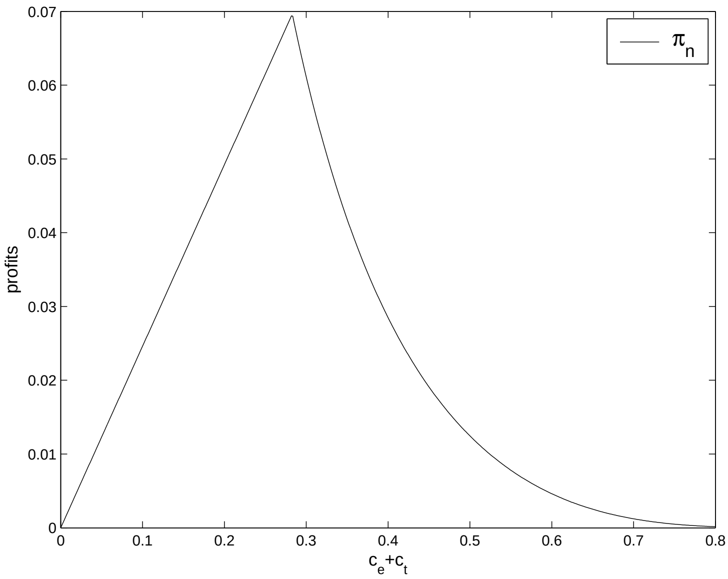

Figure 2: Expected profits as a function of the search costs when all shops are located in the same shopping mall. This figure is based on 10 shops, 10 % shoppers and a valuation of the product of 1. The travel costs c_t are set at 80% of the total search costs $c_t + c_e$

$c_t = 0.8(c_t + c_e)$ and $c_e = 0.2(c_t + c_e)$. The expected profits are plotted for $c_t + c_e < 0.5$.

The figure can be split in different parts. First, when the search costs $c_t + c_e$ are small enough (for the current parameter values $c_t + c_e < 0.073$) the full search equilibrium holds in both cases and $\pi_1 > \pi_n$. The intuition for this result is straightforward. By locating together in a single shopping mall shops decrease the costs to continue search from $c_e + c_t$ to c_e, leading to stronger competition and lower prices and profits. Second, when the search costs $c_t + c_e$ have an intermediate value (for the current parameter values $0.073 < c_t + c_e < 0.28$) the full search equilibrium holds in the case where all shops are located together and the partial search equilibrium holds in the case where all shops are isolated. The intuition for this is as before: when all shops are located together consumers expect lower prices and therefore consumers are more willing to search. This implies that when all shops are located together all non-shoppers are active and the expected profits increase in the search costs $c_t + c_e$. When all shops are isolated however only a fraction of the non-shoppers is active and expected profits decrease in the search costs $c_t + c_e$. When the search costs $c_t + c_e$ are high enough the expected profits when locating together are higher than the expected profits when all shops are isolated. Finally, when the search costs are high enough (for the current parameter values $c_t + c_e > 0.28$) the partial search

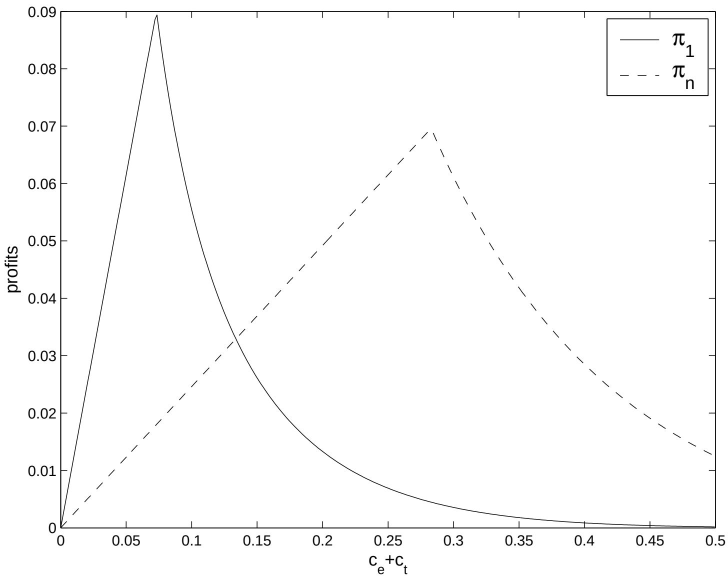

Figure 3: Expected profits as a function of the search costs when all shops are isolated and when all shops are located in the same shopping mall. This figure is based on 10 shops, 10 % shoppers and a valuation of the product of 1. The travel costs c_t are set at 80% of the total search costs $c_t + c_e$

equilibrium holds in both cases. The fraction of active consumers is however higher when all firms are located together and this leads to higher expected profits when all firms are located together.
The pattern shown in Figure 3 does not depend on the specific parameter values chosen. Name the value of $c_t + c_e$ where the full search equilibrium changes into a partial search equilibrium the inflection value. A close look at Propositions 3.1 and 3.3 shows that the inflection value is always higher when all shops are located together. It is also easy to see that when in both cases a full search equilibrium holds, that is, when the search costs $c_t + c_e$ are below the inflection value for the case when all shops are isolated, $\pi_1 > \pi_n$. With somewhat more effort it can be shown that $\pi_n > \pi_1$ when in both cases a partial search equilibrium holds, that is, when $c_t + c_e$ is at or above the inflection value for the case when all shops are located together. This gives Proposition 3.5.

Proposition 3.5 *Let* $c_t = \beta(c_t + c_e)$ *with* $0 < \beta < 1$. *Then there exists a number* c *such that for* $c_t + c_e < c$ $\pi_1 > \pi_n$ *and for* $c_t + c_e > c$ $\pi_1 < \pi_n$, *with*

$$\theta(1 - \int_0^1 \frac{1}{1 + \frac{\gamma}{1-\gamma} n y^{n-1}} dy) < c < \theta \frac{1 - \int_0^1 \frac{1}{1+\frac{\gamma}{1-\gamma} n y^{n-1}} dy}{1 - \beta \int_0^1 \frac{1}{1+\frac{\gamma}{1-\gamma} n y^{n-1}} dy}.$$

4 The intermediate case

In this section I will investigate the situation where $2 \leq k \leq n-1$ shops are located together in a shopping mall and the remaining $n-k$ shops are isolated. Recall that $F_k^m(p)$ is the price distribution used by the shops that are in a shopping mall with k shops, with $f_k^m(p)$ the corresponding probability density function. The support of $F_k^m(p)$ is defined by all prices for which $f_k^m(p) > 0$. Denote by π_k^m the expected profits of such a shop and define r_k^m as

$$\int_{\underline{p}_k^m}^{r_k^m} (r_k^m - p) dF_k^m(p) = c_e. \tag{1}$$

The same can be defined for the isolated shops: $F_k^i(p)$ is the price distribution used by them when k shops are located in the mall, with $f_k^i(p)$ the corresponding pdf. The support of $F_k^i(p)$ is defined by all prices for which $f_k^i(p) > 0$, π_k^i denotes the expected profits and r_k^i is defined as

$$\int_{\underline{p}_k^i}^{r_k^i} (r_k^i - p) dF_k^i(p) = c_e + c_t. \tag{2}$$

Note that the definition of r_k^m uses c_e whereas the definition of r_k^i uses $c_t + c_e$. The reason for this is that a non-shopper who is in an isolated shop and wants to continue search has to incur a search cost $c_e + c_t$, whereas a non-shopper who is in a mall can continue searching in the mall at cost c_e. As before, the reservation prices determine whether a consumer wants to continue search and moreover determine whether a full search or a partial search equilibrium holds. As in the previous section, I will concentrate on equilibria where $\overline{p}_k^m \leq r_k^m$ and $\overline{p}_k^i \leq r_k^i$.
Let x_k denote the total fraction of active non-shoppers who decide to first visit a mall shop and let $1 - x_k$ be the total fraction of active non-shoppers who first visit an isolated shop, with $0 < x_k < 1$. This implies that initially each mall shop attracts $\frac{x_k}{k}$ active non-shoppers, whereas each isolated shop initially attracts $\frac{1-x_k}{n-k}$ active non-shoppers. Note that if $x_k = \frac{k}{n}$ the active non-shoppers initially spread randomly over mall and isolated shops. It will be shown later that in equilibrium $\frac{x_k}{k} > \frac{k}{n}$.

The first step in deriving the equilibrium is to specify optimal consumer behavior. This is quite complex because of consumer choice being so rich. After one or more searches non-shoppers can decide to buy at the current shop, possibly return to a previously visited shop (incurring return costs), continue searching in the mall or continue searching in an isolated shop. The complete specification of optimal consumer behavior is only used in the formal proofs of the propositions in this section and to save space the complete specification of consumer behavior is therefore placed in Appendix B.

The optimal consumer behavior implies that $Ep_k^i = Ep_k^m$, with Ep_k^m the expected mall price, and Ep_k^i the expected price in an isolated shop. Intuitively, if $Ep_k^i < Ep_k^m$ all active non-shoppers prefer to search in an isolated shop and mall shops only attract shoppers. Because the number of mall shops, k, is at or above 2, this drives the prices in the mall shops down to zero and $Ep_k^i < Ep_k^m$ cannot hold. When there are at least two isolated shops ($k \leq n-2$), the reverse argument holds for $Ep_k^m < Ep_k^i$, showing that in equilibrium $Ep_k^m = Ep_k^i$ and non-shoppers are indifferent between first searching a mall shop and first searching an isolated shop. This also holds when there is only one isolated shop ($k = n-1$), but the argument is less intuitive. Appendix A provides more details. Note that when $\overline{p}_k^m \leq r_k^m$ and $\overline{p}_k^i \leq r_k^i$ definitions (1) and (2) can be rewritten as $r_k^m = Ep_k^m + c_e$ and $r_k^i = Ep_k^i + c_t + c_e$. This gives the following Proposition.

Proposition 4.1 *$r_k^i = r_k^m + c_t$ in any equilibrium with $\overline{p}_k^m \leq r_k^m$ and $\overline{p}_k^i \leq r_k^i$.*

Recall that we concentrate on equilibria with $\overline{p}_k^m \leq r_k^m$ and $\overline{p}_k^i \leq r_k^i$. The specification of optimal consumer behavior in Appendix B shows that when $\overline{p}_k^m \leq r_k^m$ and $\overline{p}_k^i \leq r_k^i$ all active non-shoppers will search only once. If a non-shopper would find a mall price above r_k^m or an isolated price above r_k^i he or she would continue searching. Moreover, all non-shoppers will be active ($\mu_k = 1$) when $r_k^i < \theta$ and only a fraction of non-shoppers will be active ($\mu_k < 1$) when $r_k^i = \theta$. Note that if all shops set $\overline{p}_k^m \leq r_k^m$ and $\overline{p}_k^i \leq r_k^i$, deviating to a higher price is not profitable. A deviating shop will not sell to any shoppers and all the active non-shoppers that it initially attracts will continue searching. Therefore, a deviating shop will have zero profits. For $p \leq r_k^m$ the profit function of mall shops is

$$\pi_k^m(p) = \gamma p(1 - F_k^m(p))^{k-1}(1 - F_k^i(p))^{n-k} + (1-\gamma)\mu_k \frac{x_k}{k} p.$$

For $p \leq r_k^i$ the profit function of isolated shops is

$$\pi_k^i(p) = \gamma p(1 - F_k^m(p))^{k}(1 - F_k^i(p))^{n-k-1} + (1-\gamma)\mu_k \frac{1 - x_k}{n-k} p.$$

Assume for the moment that $k < n-1$. The profit functions show that in equilibrium $\overline{p}_k^m = r_k^m$, because for a lower maximum price it would be profitable to deviate to r_k^m. Similarly, in equilibrium $\overline{p}_k^i = r_k^i$.[5] A standard undercutting argument also shows that atoms in $F_k^m(p)$ are only possible for those prices p^* at which $F_k^i(p^*) = 1$. Similarly, atoms in $F_k^i(p)$ are only possible for those prices p^* at which $F_k^m(p^*) = 1$. Exactly the same

[5] Note that this implies $\overline{p}_k^i \neq \overline{p}_k^m$ and therefore $F_k^i(p) \neq F_k^m(p)$.

results on maximum prices and atoms hold for $k = n-1$, but the proof is less intuitive and is placed in Appendix A. Equilibrium expected profits are $\pi_k^m = r_k^m \frac{x_k}{k} \mu_k (1-\gamma)$ and $\pi_k^i = r_k^i \frac{1-x_k}{n-k} \mu_k (1-\gamma)$.
Note that for $p \geq r_k^m$

$$\pi_k^i(p) = (1-\gamma)\mu_k \frac{1-x_k}{n-k} p.$$

This shows that isolated shops will never set a price between r_k^m and r_k^i and that there will be an atom at r_k^i. $F_k^i(p)$ should also have some probability mass below r_k^m because else the definition of r_k^i as given by (2) cannot hold. This probability mass is atomless, as well as $F_k^m(p)$.
For $p \leq r_k^m$ the price distributions can be derived by setting $\pi_k^m(p) = \pi_k^m$ and/or setting $\pi_k^i(p) = \pi_k^i$. Suppose that for all p in $[p_1, p_2]$ $f_k^i(p) = 0$ and $f_k^m(p) > 0$. That is, only mall shops set prices in $[p_1, p_2]$. Then $\pi_k^m(p) = \pi_k^m$ gives

$$F_k^m(p) = 1 - \left(\frac{(r_k^m - p)(1-\gamma)\mu_k \frac{x_k}{k}}{\gamma p (1 - F_k^i(p_1))^{n-k}} \right)^{\frac{1}{k-1}}. \quad (3)$$

Similarly, when for all p in $[p_1, p_2]$ $f_k^m(p) = 0$ and $f_k^i(p) > 0$ then $\pi_k^i(p) = \pi_k^i$ gives

$$F_k^i(p) = 1 - \left(\frac{(r_k^i - p)(1-\gamma)\mu_k \frac{1-x_k}{n-k}}{\gamma p (1 - F_k^m(p_1))^{k}} \right)^{\frac{1}{n-k-1}}. \quad (4)$$

Finally, when for all p in $[p_1, p_2]$ $f_k^m(p) > 0$ and $f_k^i(p) > 0$ then $\pi_k^m(p) = \pi_k^m$ and $\pi_k^i(p) = \pi_k^i$ jointly give

$$F_k^i(p) = 1 - \left(\frac{(r_k^i - p)(1-\gamma)\mu_k \frac{1-x_k}{n-k}}{\gamma p} \right)^{\frac{1}{n-1}} \left(\frac{\frac{x_k}{k}(r_k^m - p)}{\frac{1-x_k}{n-k}(r_k^i - p)} \right)^{\frac{k}{n-1}} \quad (5)$$

and

$$F_k^m(p) = 1 - \left(\frac{(r_k^i - p)(1-\gamma)\mu_k \frac{1-x_k}{n-k}}{\gamma p} \right)^{\frac{1}{n-1}} \left(\frac{\frac{1-x_k}{n-k}(r_k^i - p)}{\frac{x_k}{k}(r_k^m - p)} \right)^{\frac{n-k-1}{n-1}}. \quad (6)$$

The price distributions thus depend on the supports of $F_k^m(p)$ and $F_k^i(p)$. It can be shown that there are three types of supports.

Proposition 4.2 *In any equilibrium with $\overline{p}_k^m \leq r_k^m$ and $\overline{p}_k^i \leq r_k^i$, $\overline{p}_k^m = r_k^m$ and $\overline{p}_k^i = r_k^i$. $F_k^i(p)$ has an atom at $p = r_k^i$, $f_k^i(p) > 0$ for $\underline{p}_k^i \leq p \leq b$ and $f_k^i(p) = 0$ for $b < p < r_k^i$, with $b < r_k^m$. There are three possibilities for the support of $F_k^m(p)$:*

1. $f_k^m(p) > 0$ *for* $b \leq p \leq r_k^m$ *and* $f_k^m(p) = 0$ *elsewhere.*

2. $f_k^m(p) > 0$ *for* $\underline{p}_k^i \leq p \leq a$ *and for* $b \leq p \leq r_k^m$*, with* $a < b$*.* $f_k^m(p) = 0$ *elsewhere.*

3. $f_k^m(p) > 0$ *for* $\underline{p}_k^i \leq p \leq r_k^m$ *and* $f_k^m(p) = 0$ *elsewhere.*

When $k = n - 1$ *only possibility 3 can hold.*

Each equilibrium type has a full search variant with $\mu_k = 1$ and $r_k^i < \theta$ and a partial search variant with $0 < \mu_k < 1$ and $r_k^i = \theta$. For $k < n - 1$ this gives a total of six equilibria and for $k = n - 1$ this gives a total of two equilibria. The condition $\pi_k^i(\underline{p}_k^i) = \pi_k^i(r_k^i)$ gives that in all equilibrium types $\underline{p}_k^i = r_k^i \frac{(1-\gamma)\mu_k \frac{1-x_k}{n-k}}{\gamma+(1-\gamma)\mu_k \frac{1-x_k}{n-k}}$. In equilibrium type 1 $F_k^i(p)$ is given by (4) and $F_k^m(p)$ is given by (3). In equilibrium type 2 for $\underline{p}_k^i \leq p \leq a$ $F_k^m(p)$ is given by (6) and $F_k^i(p)$ is given by (5). For $a < p \leq b$ $F_k^i(p)$ is given by (4) and for $b \leq p < r_k^m$ $F_k^m(p)$ is given by (3). In equilibrium type 3 for $\underline{p}_k^i \leq p \leq b$ $F_k^i(p)$ and $F_k^m(p)$ are given by (5) and (6), whereas for $p > b$ $F_k^m(p)$ is given by (3).
The price distributions depend on r_k^m, r_k^i, x_k, b and possibly on μ_k and a. For each equilibrium type these variables are jointly determined by a system of equations. For each equilibrium type this system includes (1), (2), $r_k^i = r_k^m + c_t$ and (for partial search equilibria) $r_k^i = \theta$. On top of this, equilibrium type 1 has $F_k^m(b) = 0$, equilibrium type 2 has $F_k^m(\underline{p}_k^i) = 0$ and $F_k^m(a) = F_k^m(b)$, and equilibrium type 3 has $F_k^m(\underline{p}_k^i) = 0$. The resulting systems of equations are too complicated to solve analytically. This implies that it is impossible to analytically derive the parameter regions in which the different equilibria hold. Also, analytical expressions for profits, our main interest, cannot be obtained. In the next sections we will therefore resort to numerical methods.

Isolated shops randomize over a low price region $[\underline{p}_k^i, b]$ and a single high price r_k^i. Because of the shoppers isolated shops are willing to set a price in $[\underline{p}_k^i, b]$, but the fraction of shoppers that an isolated shop could attract should be large relative to the fraction of non-shoppers that an isolated shop attracts. This is because the difference between r_k^i and b is at least c_t. By setting a price at or below b an isolated shop foregoes profits of at least c_t per consumer, which should be made up by a relatively large increase in shoppers. Because the number of shoppers is fixed at γ, it should be that the number of non-shoppers per isolated shop, $\frac{1-x_k}{n-k}$, is small. This, in turn, affects the profits of an isolated shop. It can be shown that $\pi_k^i < \pi_k^m$.

Proposition 4.3 *In any equilibrium with* $\overline{p}_k^m \leq r_k^m$ *and* $\overline{p}_k^i \leq r_k^i$*,* $\frac{1-x_k}{n-k} < \frac{x_k}{k}$ *and* $\pi_k^i < \pi_k^m$*.*

5 Comparative statics

This section will give some comparative statics results on the equilibria that have been derived in the previous section, using numerical techniques. Recall that in section 3 β has been defined as a constant such that $c_t = \beta(c_t + c_e)$ and $c_e = (1-\beta)(c_t + c_e)$. It can be shown that in all full search equilibria for $2 \leq k \leq n-1$ the parameter x_k only depends on β, γ, n and k, and not on c_t and c_e. Moreover, all reservation prices and profits can be written as $c_t + c_e$ times some function of β, γ, n and k. The partial search equilibria for $2 \leq k \leq n-1$ are more complicated, in the sense that x_k depends not only on β, γ, n and k, but also on $c_t + c_e$. Moreover, the reservation prices and profits are nonlinear in $c_t + c_e$. This implies that when β, γ, n and k are fixed, the full search equilibria can be numerically calculated. To calculate the partial search equilibria, $c_t + c_e$ also needs to be specified. In this section I will therefore concentrate on the full search equilibria; partial search equilibria will be discussed in the next section.

Tables 1, 2 and 3 give simulation results for the full search equilibria. Table 1 uses a small value of γ; $\gamma = 0.05$. Table 2 has an intermediate value of γ ($\gamma = 0.1$) and table 3 has a large value of γ ($\gamma = 0.25$). Each table gives results for different values of k and β and in all tables $n = 10$. Each table gives a panel with results on $\frac{x_k}{k}$ and a panel with results on the reservation price. For $2 \leq k \leq n-1$, r_k^i is reported. Note that a full search equilibrium only holds when $r_k^i < \theta$, which translates to $c_t + c_e$ being small enough. For $k = 1$, the tables give the equivalence of r_k^i, r_1, and the full search equilibrium holds when $r_1 < \theta$. For $k = n$, the tables give $r_n + c_t$. This is the equivalence of r_k^i as a full search equilibrium holds when $r_n + c_t < \theta$.
Proposition 4.2 states that for $2 \leq k \leq n-2$ three equilibrium types are possible. The simulations suggest that these equilibria do not overlap and together fill the complete parameter space. In the tables lines denote when each equilibrium type holds. The equilibria in the upper right corner, for high β and low k, are of type 1. The equilibria in the lower left corner, for low β and high k, are of type 3. Note that also all equilibria with $k = n-1$ are of type 3. The intermediate equilibria are of type 2.

The tables suggest that $\frac{x_k}{k}$ increases in β and decreases in k. To understand this, recall that $\frac{1-x_k}{n-k}$ should be such that isolated shops are indifferent between only serving non-shoppers at a price r_k^i and serving both non-shoppers and shoppers (with strictly positive probability) at a price in $[\underline{p}_k^i, b]$. When β increases, $r_k^i - r_k^m$ increases because $r_k^i - r_k^m = c_t$. This implies that it is increasingly attractive for isolated shops to set a price r_k^i and sell only to non-shoppers. To counterbalance this effect, $\frac{1-x_k}{n-k}$ should decrease, or $\frac{x_k}{k}$ should increase.

(a) Values of $\frac{x_k}{k}$ for $n = 10$ and $\gamma = 0.05$.

		β								
		0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
	2	0.1010	0.1021	0.1032	0.1044	0.1105	0.1235	0.1434	0.1776	0.2484
	3	0.1009	0.1018	0.1028	0.1040	0.1052	0.1093	0.1208	0.1406	0.1826
	4	0.1008	0.1016	0.1025	0.1035	0.1047	0.1059	0.1107	0.1227	0.1499
k	5	0.1006	0.1014	0.1021	0.1030	0.1041	0.1053	0.1067	0.1125	0.1305
	6	0.1005	0.1011	0.1018	0.1026	0.1034	0.1045	0.1059	0.1076	0.1179
	7	0.1004	0.1008	0.1014	0.1020	0.1028	0.1037	0.1049	0.1065	0.1092
	8	0.1003	0.1006	0.1009	0.1014	0.1019	0.1027	0.1036	0.1050	0.1073
	9	0.1001	0.1003	0.1005	0.1007	0.1010	0.1014	0.1020	0.1029	0.1044

(b) Reservation prices for $n = 10$ and $\gamma = 0.05$. The first row gives r_1, the last row gives $r_n + c_t$ and the intermediate rows give r_k^i for $2 \leq k \leq 9$. For ease of notation, $c_t + c_e$ is denoted by c.

		β								
		0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
	1	23.93c	23.93c	23.93c	23.93c	23.93c	23.93c	23.93c	23.93c	23.93c
	2	23.47c	23.00c	22.54c	22.07c	21.51c	20.74c	19.68c	18.04c	14.81c
	3	23.24c	22.53c	21.82c	21.10c	20.38c	19.54c	18.36c	16.65c	13.54c
	4	22.99c	22.07c	21.12c	20.35c	19.17c	18.17c	17.00c	15.24c	12.26c
k	5	22.74c	21.56c	20.43c	19.22c	17.99c	16.73c	15.44c	13.79c	10.95c
	6	22.48c	21.06c	19.64c	18.19c	16.83c	15.33c	13.78c	12.17c	9.62c
	7	22.22c	20.57c	18.91c	17.24c	15.55c	13.87c	12.17c	10.29c	8.26c
	8	21.96c	20.07c	18.19c	16.30c	14.40c	12.47c	10.49c	8.54c	6.25c
	9	21.86c	19.79c	17.71c	15.62c	13.51c	11.38c	9.20c	6.96c	4.54c
	10	21.63c	19.34c	17.05c	14.76c	12.46c	10.17c	7.88c	5.59c	3.29c

Table 1: Values of $\frac{x_k}{k}$ and reservation prices for $n = 10$ and $\gamma = 0.05$.

(a) Values of $\frac{x_k}{k}$ for $n = 10$ and $\gamma = 0.1$.

k	β = 0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
2	0.1011	0.1024	0.1036	0.1050	0.1091	0.1220	0.1416	0.1750	0.2453
3	0.1010	0.1021	0.1033	0.1045	0.1059	0.1082	0.1195	0.1391	0.1805
4	0.1009	0.1018	0.1029	0.1040	0.1053	0.1068	0.1091	0.1216	0.1484
5	0.1007	0.1015	0.1025	0.1035	0.1047	0.1061	0.1077	0.1116	0.1293
6	0.1006	0.1013	0.1020	0.1029	0.1039	0.1052	0.1067	0.1087	0.1169
7	0.1004	0.1010	0.1016	0.1023	0.1031	0.1042	0.1055	0.1074	0.1101
8	0.1003	0.1007	0.1011	0.1016	0.1022	0.1030	0.1041	0.1056	0.1082
9	0.1002	0.1003	0.1005	0.1008	0.1012	0.1016	0.1022	0.1032	0.1049

(b) Reservation prices for $n = 10$ and $\gamma = 0.1$. The first row gives r_1, the last row gives $r_n + c_t$ and the intermediate rows give r_k^i for $2 \leq k \leq 9$. For ease of notation, $c_t + c_e$ is denoted by c.

k	β = 0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
1	13.67c	13.67c	13.67c	13.67c	13.67c	13.67c	13.67c	13.67c	13.67c
2	13.42c	13.16c	12.91c	12.66c	12.38c	12.00c	11.47c	10.65c	9.06c
3	13.29c	12.90c	12.51c	12.12c	11.72c	11.30c	10.70c	9.83c	8.28c
4	13.15c	12.65c	12.12c	11.59c	11.04c	10.49c	9.91c	9.00c	7.49c
5	13.02c	12.36c	11.73c	11.07c	10.38c	9.68c	8.97c	8.15c	6.70c
6	12.88c	12.09c	11.30c	10.50c	9.74c	8.90c	8.03c	7.13c	5.89c
7	12.73c	11.82c	10.90c	10.02c	9.08c	8.15c	7.14c	6.09c	4.96c
8	12.59c	11.59c	10.56c	9.52c	8.46c	7.38c	6.27c	5.12c	3.83c
9	12.53c	11.38c	10.23c	9.08c	7.91c	6.72c	5.52c	4.27c	2.91c
10	12.40c	11.14c	9.87c	8.60c	7.34c	6.07c	4.80c	3.53c	2.27c

Table 2: Values of $\frac{x_k}{k}$ and reservation prices for $n = 10$ and $\gamma = 0.1$.

(a) Values of $\frac{x_k}{k}$ for $n = 10$ and $\gamma = 0.25$.

k \ β	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
2	0.1015	0.1031	0.1048	0.1066	0.1086	0.1183	0.1373	0.1699	0.2386
3	0.1013	0.1028	0.1043	0.1060	0.1079	0.1099	0.1165	0.1355	0.1760
4	0.1011	0.1024	0.1038	0.1053	0.1071	0.1090	0.1112	0.1188	0.1449
5	0.1010	0.1020	0.1032	0.1046	0.1062	0.1080	0.1101	0.1128	0.1266
6	0.1008	0.1016	0.1026	0.1038	0.1052	0.1068	0.1088	0.1114	0.1148
7	0.1006	0.1013	0.1020	0.1029	0.1040	0.1054	0.1072	0.1095	0.1130
8	0.1004	0.1008	0.1014	0.1020	0.1028	0.1038	0.1052	0.1071	0.1102
9	0.1002	0.1004	0.1007	0.1010	0.1015	0.1020	0.1028	0.1039	0.1058

(b) Reservation prices for $n = 10$ and $\gamma = 0.25$. The first row gives r_1, the last row gives $r_n + c_t$ and the intermediate rows give r_k^i for $2 \leq k \leq 9$. For ease of notation, $c_t + c_e$ is denoted by c.

k \ β	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
1	7.13c	7.13c	7.13c	7.13c	7.13c	7.13c	7.13c	7.13c	7.13c
2	7.01c	6.89c	6.77c	6.65c	6.54c	6.39c	6.18c	5.85c	5.23c
3	6.95c	6.76c	6.57c	6.38c	6.19c	6.00c	5.77c	5.41c	4.78c
4	6.88c	6.63c	6.38c	6.12c	5.85c	5.58c	5.31c	4.96c	4.34c
5	6.81c	6.51c	6.19c	5.86c	5.53c	5.18c	4.83c	4.46c	3.89c
6	6.76c	6.38c	6.00c	5.61c	5.21c	4.80c	4.37c	3.92c	3.44c
7	6.69c	6.26c	5.82c	5.37c	4.90c	4.43c	3.93c	3.40c	2.83c
8	6.63c	6.14c	5.63c	5.13c	4.61c	4.08c	3.53c	2.94c	2.29c
9	6.58c	6.02c	5.47c	4.90c	4.33c	3.76c	3.17c	2.55c	1.88c
10	6.52c	5.90c	5.29c	4.68c	4.07c	3.45c	2.84c	2.23c	1.61c

Table 3: Values of $\frac{x_k}{k}$ and reservation prices for $n = 10$ and $\gamma = 0.25$.

To understand the effect of k recall that in equilibrium the expected isolated and the expected mall prices should be equal. The support of $F_k^i(p)$ consists of $[\underline{p}_k^i, b]$ and r_k^i. The support of the mall prices consists of $[b, r_k^m]$ and possibly some prices between $\underline{p}_k^i$ and b. To ensure that the expected prices are equal, $F_k^m(p)$ should not attach too much probability mass to prices in $[\underline{p}_k^i, b]$. In fact, it can be shown that in equilibrium $F_k^m(p) < F_k^i(p)$ for $p \in [\underline{p}_k^i, b]$. When k increases, an isolated shop gets less isolated competitors and more mall competitors. This increases the probability that all competitors set a price above b and therefore increases the profitability of setting a price b. To counter this effect and make a price r_k^i as attractive as a price b, $\frac{1-x_k}{n-k}$ should increase, or $\frac{x_k}{k}$ should decrease.
The effects of β and k on $\frac{x_k}{k}$ explain why equilibrium type 1 can only hold for high β and low k. For those parameter values $\frac{x_k}{k}$ is high, and mall shops have no incentive to capture all shoppers by deviating to $\underline{p}_k^i$. On top of that, when β is high, $r_k^i - r_k^m$ is high. To ensure that expected mall and isolated prices are equal, $\underline{p}_k^i$ should be strictly smaller than $\underline{p}_k^m$.
The tables also suggest that r_k^i decreases in β and k. To understand this, one first needs to understand the effect of β and k on r_k^m. Intuitively, when k increases, the competition in the mall increases, leading to a lower reservation price r_k^m. Because $r_k^i = r_k^m + c_t$, an increase in k also decreases r_k^i. When β increases, c_e decreases and c_t increases. Equation (1) shows that r_k^m only depends on c_e, and not on c_t. Therefore, when β increases r_k^m decreases. Because $r_k^i = r_k^m + c_t$ there are two effects on r_k^i: r_k^m decreases and c_t increases. The results in the tables suggest that the first effect is stronger.
When γ increases, it seems from the tables that r_k^i decreases, whereas $\frac{x_k}{k}$ can both increase and decrease. When γ increases there is a stronger competition for the shoppers. As a result, the isolated shops are less tempted to ask price r_k^i and the mall shops also prefer lower prices. The tables show that indeed equilibrium type 1 occurs less often when γ is larger: the mall shops are more tempted to set $\underline{p}_k^m = \underline{p}_k^i$. Because both types of shops prefer lower prices, it is no surprise that r_k^i decreases. The effect on $\frac{x_k}{k}$ is twofold. Isolated shops need to be indifferent between asking the high price r_k^i and lower prices. When the fraction of shoppers increases, isolated shops are less tempted to ask a high price, but at the same time the behavior of the mall shops leads to more competition in the lower price range, making a high price more attractive. When the first effect dominates, $\frac{1-x_k}{n-k}$ needs to increase and consequently $\frac{x_k}{k}$ needs to decrease to make sure that isolated shops still want to ask r_k^i. When the second effect dominates, $\frac{1-x_k}{n-k}$ needs to decrease and $\frac{x_k}{k}$ consequently increases to ensure that isolated shops still want to set a low price. Note that in equilibrium type 1 the second effect is absent and that indeed $\frac{x_k}{k}$ decreases in γ.

Even though a numerical analysis is needed to evaluate the equilibria, it is possible to analytically derive some limiting results.

Proposition 5.1 *Suppose* $2 \leq k \leq n-1$.

- *When* $\beta \to 0$, $\underline{p}_k^i = \underline{p}_k^m$, $r_k^i - r_k^m \to 0$, $F_k^i(p) - F_k^m(p) \to 0$ *and* $\frac{x}{k} \to \frac{1}{n}$.
- *When* $\beta \to 1$, $r_k^m \to 0$, $F_k^i(r_k^m) \to 1$ *and* $\frac{1-x_k}{n-k} \to 0$.
- *When* $\gamma \to 0$, $\mu_k \to 0$.
- *When* $\gamma \to 1$, $\underline{p}_k^m \to 0$, $\underline{p}_k^i \to 0$, $F_k^m(p) \to 1$ *and* $F_k^i(p) \to 1$

These results are in line with previous consumer search models, see e.g. JMW. Recall that β determines the relative sizes of c_t and c_e. When $\beta \to 0$, $c_t \to 0$ and in the limit consumers only incur entering costs. In that case, the difference between mall shops and isolated shops vanishes, which leads to equal price distributions and an equal distribution of non-shoppers over the firms. When $\beta \to 1$, $c_e \to 0$. In this case, once a consumer is in the mall, he can visit all mall shops almost for free. This leads to large competition between mall shops and consequently to prices of almost zero in the mall. To make mall and isolated shops equally attractive, isolated shops should set prices to almost zero as well. To make this possible, isolated shops should attract almost no non-shoppers. If they would attract too many non-shoppers, an isolated shop could set a price c_t and make a profit on the non-shoppers who visited the isolated shop in the first place.
When the fraction of shoppers, γ, vanishes, firms tend to focus completely on the captive consumers. This raises prices to monopoly levels, and consequently many non-shoppers drop out of the market. When the fraction of non-shoppers vanishes, firms compete strongly for the shoppers, leading to very low prices.

6 Location choice

In this section I will consider the equilibrium location choice of shops. A mall with k^* shops is considered an equilibrium when none of the mall shops has an incentive to leave the mall and when none of the isolated shops has an incentive to join the mall. Thus, a mall with k^* shops is an equilibrium when $\pi_{k^*}^m \geq \pi_{k^*-1}^i$ and $\pi_{k^*}^i \geq \pi_{k^*+1}^m$[6]. Note that if a mall shop would deviate and leave the mall, the mall size would decrease by one. Also, if an isolated shop would deviate and join the mall, the mall size would increase by one.

[6]Rental costs that differ between locations and relocation costs could easily be introduced, but will only change the equilibrium conditions with a constant.

	$\beta = 0.1$		$\beta = 0.4$		$\beta = 0.7$	
	π_k^m	π_k^i	π_k^m	π_k^i	π_k^m	π_k^i
$k = 1$	-	**1.2303c**	-	**1.2303c**	-	1.2303c
$k = 2$	1.2123c	1.2041c	1.1587c	1.1251c	1.3720c	0.9249c
$k = 3$	1.1990c	1.1909c	1.1025c	1.0693c	1.0751c	0.8821c
$k = 4$	1.1850c	1.1769c	1.0474c	1.0146c	**0.9046c**	**0.8374c**
$k = 5$	1.1710c	1.1629c	0.9935c	0.9611c	0.8012c	0.7448c
$k = 6$	1.1566c	1.1485c	0.9356c	0.9037c	0.7044c	0.6500c
$k = 7$	1.1421c	1.1340c	0.8854c	0.8540c	0.6118c	0.5597c
$k = 8$	1.1274c	1.1194c	0.8333c	0.8024c	0.5215c	0.4721c
$k = 9$	**1.1203c**	**1.1123c**	0.7872c	0.7569c	0.4432c	0.3969c
$k = 10$	1.1073c	-	0.7382c	-	0.3691c	-

Table 4: Profits for different values of k and β when a full search equilibrium holds. The number of firms, n, is fixed to 10 and $\gamma = 0.1$. For ease of notation $c_t + c_e$ is denoted by c. Bold profits indicate an equilibrium.

As mentioned in section 4, there is no analytical expression for the profits. This implies that there is no analytical expression for k^*. Therefore numerical methods are needed. I will first consider location choice when $c_t + c_e$ is small, such that a full search equilibrium holds. As mentioned in section 5, in a full search equilibrium profits can be written as $c_t + c_e$ times some function of β, γ, n and k. Table 4 gives the profits in a full search equilibrium when $n = 10$ and $\gamma = 0.1$. Three different values of β are considered and profits are given for every possible mall size k.

When comparing π_1 with π_2^i and π_2^m note that π_1 can both be below or above π_2^m whereas π_1 is always above π_2^i. When two shops decide to form a mall the reservation prices decrease. At the same time, the fraction of non-shoppers going to a mall shop, $\frac{x_k}{k}$, is clearly above $\frac{1}{n}$, the fraction of non-shoppers that a shop attracts when there is no mall. The two mall shops thus set lower maximum prices but sell more, and the total effect is ambiguous.[7] The isolated shops also set lower maximum prices, but on top of that lose customers. As a consequence, $\pi_2^i < \pi_1$.

Once a mall exists ($k \geq 2$), both mall and isolated profits seem to decrease in mall size. The reason for this again is that the reservation prices decrease in mall size. For mall shops it is also important that the fraction of captive consumers, $\frac{x_k}{k}$, decreases in mall size. This decreases the mall profits even more. Isolated shops attract more non-shoppers when the mall size increases, but this increase in non-shoppers does not offset the lower reservation prices. It does show however in the table that mall profits decrease faster in mall size than isolated profits.

[7] Note that $\frac{x_k}{k}$ is larger when β is large and thus the positive effect on mall profits is more pronounced for large values of β.

	$\gamma = 0.05$	$\gamma = 0.1$	$\gamma = 0.25$
$\beta = 0.1$	$k^* = 1$, $k^* = 9$	$k^* = 1$, $k^* = 9$	$k^* = 1$
$\beta = 0.2$	$k^* = 1$, $k^* = 9$	$k^* = 1$	$k^* = 1$
$\beta = 0.3$	$k^* = 1$	$k^* = 1$	$k^* = 1$
$\beta = 0.4$	$k^* = 1$	$k^* = 1$	$k^* = 1$
$\beta = 0.5$	$k^* = 1$	$k^* = 1$	$k^* = 1$
$\beta = 0.6$	$k^* = 3$	$k^* = 3$	$k^* = 1$
$\beta = 0.7$	$k^* = 4$	$k^* = 4$	$k^* = 3$
$\beta = 0.8$	$k^* = 6$	$k^* = 5$	$k^* = 4$
$\beta = 0.9$	$k^* = 7$	$k^* = 7$	$k^* = 6$

Table 5: Equilibrium mall sizes for several values of β and γ. The number of firms, n, is fixed to 10.

Once γ, β and n are fixed, a table with profits for different mall sizes is sufficient to find the equilibrium mall size. Take for example the case $n = 10$, $\gamma = 0.1$ and $\beta = 0.1$, which is the left panel in table 4. When no mall exists, profits are $1.2303(c_t + c_e)$. When a shop decides to deviate and join another shop, it will make profits $\pi_2^m = 1.2123(c_t + c_e)$. These profits are below $1.2303(c_t + c_e)$ and thus $k = 1$ is an equilibrium. This is indicated in the table by bold profits. For $2 \leq k \leq 8$, it is profitable for a mall shop to leave the mall: $\pi_k^m < \pi_{k-1}^i$. When $k = 9$, a mall shop has profits $1.1203(c_t + c_e)$. Leaving the mall would give smaller profits $(1.1194(c_t + c_e))$, so a mall shop has no incentive to deviate. An isolated shop has profits $1.1123(c_t + c_e)$ and joining the mall would give profits of $1.1073(c_t + c_e)$. An isolated shop therefore has no incentive to join the mall and consequently $k = 9$ is another equilibrium (denoted in bold). For $k = 10$, a mall shop would find it profitable to leave the mall, so $k = 10$ is not an equilibrium. In the middle and right panel of table 4 the same analysis gives the equilibria that are denoted by bold numbers.

Table 5 gives equilibrium mall sizes for different values of β and γ, keeping n fixed to 10. To understand the intuition behind the results in this table, first consider the incentives of mall and isolated shops to relocate. A mall shop that leaves the mall loses some of its captive consumers, but at the same time it can set a higher maximum price (r_k^i instead of r_k^m). On its own, this is not sufficient to leave the mall: in section 4 it has been shown that $\pi_k^m > \pi_k^i$. But when a shop leaves the mall, the mall size decreases. This lowers $\frac{1-x_k}{n-k}$ and increases the reservation prices. This magnifies the positive and negative effects mentioned before, such that $\pi_k^m < \pi_{k-1}^i$ is possible. For an isolated shop that joins the mall the effects on profits are reversed. After joining the mall, the shop attracts more captive consumers than before, but

the maximum price it can ask is lower. These two effects on their own would be sufficient to join the mall ($\pi_k^i < \pi_k^m$), but once again joining the mall changes the size of the mall. This will decrease $\frac{x_k}{k}$ and decrease the reservation prices. Both of these effects negatively affect the profits of a deviating isolated shop, such that $\pi_k^i > \pi_{k+1}^m$ is possible.
When β is high and k is small, $\frac{x_k}{k}$ is very high. Intuitively, when the travel costs c_t are large, many consumers prefer the mall. If joining the mall would not affect the mall size, isolated shops would have a large incentive to join the mall and capture this large share of non-shoppers, instead of the tiny share of non-shoppers they capture as isolated shop. As a counteracting effect, joining the mall increases the mall size and therefore decreases $\frac{x_k}{k}$ and the reservation prices. But because $\frac{x_k}{k}$ is very large from the outset and $\frac{1-x_k}{n-k}$ is very low, an isolated shop can still gain from joining the mall. This will only stop when the mall size has grown large and $\frac{x_k}{k}$ is relatively small. Thus, for high β the equilibrium mall size is fairly large.
When the fraction of shoppers, γ, is large, the only reason for an isolated shop to join the mall is not so important. An isolated shop will join the mall when it leads to a large increase in captive consumers, $\frac{x_k}{k}$. When γ is large, many consumers are shopping for the best deal and an isolated shop cannot gain many non-shoppers by joining the mall. As a consequence, the equilibrium mall size is smaller when γ is larger.
For small β there are two possible equilibria: $k^* = 1$ and $k^* = 9$. As proposition 5.1 shows, when $\beta \to 0$, the difference between mall and isolated vanishes, and shops are indifferent about their location.
An equilibrium with $k^* > 1$ in general gives lower profits for both firm types than a situation where there is no mall at all. To understand the intuition behind this, take a better look at the most right panel of table 4. Starting from a situation with no mall it is profitable to join two shops. These shops will attract much more non-shoppers than when they were isolated. But the remaining isolated shops suffer from this. Because they lose non-shoppers their profits are much lower than in the case no mall existed. Consequently, isolated shops find it profitable to join the mall. This drives down the mall profits to below the level when there were no mall at all, although the resulting mall profits are still higher than the isolated profits in the case of $k = 2$.

Thus far, I have only considered full search equilibria. In the remainder of this section I will also consider partial search equilibria. This equilibrium type is more complicated to analyze because the profits depend on c_t and c_e in a nonlinear way. Therefore, instead of tables, I will provide several plots of expected profits as a function of c_t+c_e. Simulations show that the plots of the profits as function of $c_t + c_e$ when both full and partial search equilibria are considered have the same pattern as in the extreme cases analyzed in

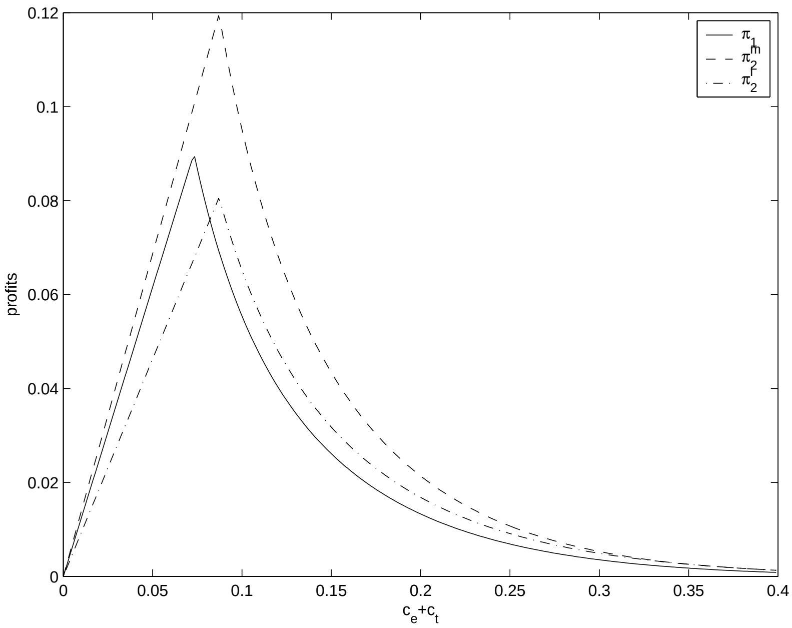

Figure 4: Expected profits as a function of the search costs when all shops are located separately and when there is a mall of two shops. This figure is based on 10 shops, 10 % shoppers and a valuation of the product of 1. The travel costs c_t are set at 70% of the total search costs $c_t + c_e$

Section 3. Again, there is a value of $c_t + c_e$, called the inflection value, such that for $c_t + c_e$ below this value the full search equilibrium holds and above this value the partial search equilibrium holds. The fraction of active non-shoppers, μ_k, is decreasing in $c_t + c_e$ and as a consequence the profits in the partial search equilibrium decrease in $c_t + c_e$.
Figures 4 and 5 show the expected profits for several values of k. In these figures γ is set at 0.1, $n = 10$, $\beta = 0.7$ and $\theta = 1$. Figure 4 depicts π_1, π_2^m and π_2^i and figure 5 depicts π_2^i, π_3^m and π_3^i. A first observation is that the inflection point shifts to the right when k increases. Note that this also can be inferred from table 2 because the inflection point is simply defined as the value of $c_t + c_e$ for which $r_k^i = \theta$. Intuitively, competition will be stronger when more shops are located in the mall. Therefore, more non-shoppers will be tempted to search, shifting the inflection point to the right. The numerical analysis also suggests that for $k \geq 2$ $\mu_k \geq \mu_{k-1}$. Intuitively, this is a consequence of the inflection point shifting to the right. Apart from this, recall that in a partial search equilibrium $r_k^i = \theta$ and $r_k^m = \theta - c_t$. This implies that a change in mall size does not affect the maximum prices the shops can ask. For an isolated shop, joining the mall therefore gives more captive consumers ($\frac{x_{k+1}}{k+1} > \frac{1}{n}$ instead of $\frac{1-x_k}{n-k} < \frac{1}{n}$) *and* increases the fraction of active consumers μ_k, at the same time not affecting r_k^i and r_k^m.

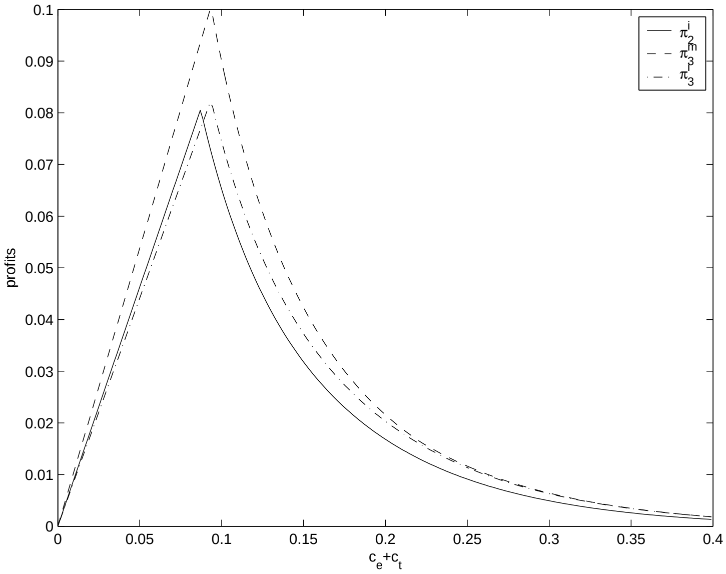

Figure 5: Expected profits as a function of the search costs when there is a mall of two shops and when there is a mall of three shops. This figure is based on 10 shops, 10 % shoppers and a valuation of the product of 1. The travel costs c_t are set at 70% of the total search costs $c_t + c_e$

Therefore, it seems that for large enough values of $c_t + c_e$ $\pi^m_{k+1} \geq \pi^i_k$. Figures 4 and 5 indeed show this. To save space, figures of the profits for $k > 3$ are not included in the paper, but they show the same pattern. Thus, for large enough values of $c_t + c_e$, $n = 10$, $\gamma = 0.1$, $\theta = 1$ and $\beta = 0.7$ isolated shops have an incentive to join the mall and the only possible equilibrium is one that has no isolated shops at all ($k = n$). Simulations for other parameter values give the same result.

7 Conclusion

This paper analyzes the incentives of a shop to locate together with similar shops in a shopping mall. In contrast with the other literature on location choice, this paper finds that in equilibrium isolated and mall shops can coexist. The main driver of this result is the assumption of consumer search costs. Non-shoppers incur costs when entering a shop, as in a standard sequential search model. On top of that, non-shoppers incur travel costs when traveling between shops that are not in the same mall, a novel feature in a sequential search setting. The addition of travel costs implies that searching in a shopping mall is more attractive than searching isolated shops. Finally, a part of the consumers are shoppers; they can visit all shops at zero costs.

In this setup, isolated shops will not raise their prices too high. If the difference between the isolated prices and mall prices gets too large, non-shoppers who initially visited an isolated shop will continue to search. Moreover, by setting a low price, an isolated shop can attract the shoppers. As a consequence of this, isolated shops attract some share of non-shoppers, which is a necessary condition for an isolated shop to exist in equilibrium.
The location choice of shops is driven by several different factors. First, mall shops attract more non-shoppers per shop than isolated shops. This gives an incentive for isolated shops to join a mall. Second, when an isolated shop joins the mall, the mall size increases and the number of non-shoppers per mall shop decreases. This dampens the first effect. Third, isolated shops can set a slightly higher maximum price than mall shops. This gives a mall shop an incentive to leave the mall. And, fourth, if a mall shop leaves the mall, the competition in the mall decreases and consequently all prices in the market (both mall and isolated prices) increase. These four factors work in different directions. The numerical results in this paper show that there is no dominant effect and mall and isolated shops can coexist.
When the search costs are large, there is a fifth effect playing a role. When the search costs are large, only a fraction of the non-shoppers is active. The other non-shoppers stay at home and do not buy at all. When an isolated shop joins the mall, the fraction of active non-shoppers increases, and this increases the profits of both mall and isolated shops. For high enough search costs the numerical results in this paper suggest that this effect is so strong that in equilibrium all shops want to join the mall.

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A Proofs

Proof of Propositions 3.1 and 3.2

To prove Propositions 3.1 and 3.2 first note that in an equilibrium where some non-shoppers search a shop will never ask a price above θ. If a shop would ask a price above θ it would not make any sales and profits would be 0. Asking a price $c_t + c_e$ however prevents non-shoppers from searching further and guarantees a strictly positive profit. Because prices are at or below θ, a non-shopper who is in a shop can always obtain a non-negative utility by buying from this shop.

To prove the optimality of the consumer behavior stated in the Propositions an induction argument will be used. Consider a non-shopper who expects the shops to price according to some price distribution $F_1(p)$ with $\overline{p} \leq \min(\theta, r_1)$, where r_1 is defined by

$$\int_{\underline{p}}^{r_1} (r_1 - p) dF_1(p) = c_e + c_t.$$

Denote by p^* the price the non-shopper found in his last search and denote by p^{min} the minimum price he found in previous searches, with p^{min} infinite when there are no previous searches. Let q denote $\min(p^* + c_t, p^{min} + c_t)$. If the non-shopper has already searched $n-1$ shops the utility from buying is $\theta - \min(p^*, p^{min} + c_t)$. If the non-shopper decides to search the nth shop as well and he finds a price below q he will buy in the nth shop. Else he will return to a previously visited shop. The expected utility from searching is given by

$$U(search) = -c_t - c_e + \int_{\underline{p}_1}^{q} (\theta - p) dF_1(p) + (1 - F_1(q))(\theta - q).$$

Note that the utility above holds even when $q > \theta$. For $p > \theta$ $F_1(p) = 1$ and therefore when $q > \theta$ the utility above reduces to $-c_t - c_e + \int_{\underline{p}_1}^{\theta} (\theta - p) dF_1(p)$, which is exactly the expected utility of search in case $q > \theta$. The utility from searching can be rewritten as

$$U(search) = -c_t - c_e + \theta - q + \int_{\underline{p}_1}^{q} (q - p) dF_1(p).$$

Now suppose that $\min(p^*, p^{min} + c_t) > r_1$. Then it must be that $q > r_1$. Using that $\overline{p} \leq r_1$, $\int_{\underline{p}_1}^{q} (q - p) dF_1(p) = \int_{\underline{p}_1}^{r_1} (q - p) dF_1(p) = q - r_1 + \int_{\underline{p}_1}^{r_1} (r_1 - p) dF_1(p) = q - r_1 + c_t + c_e$. This gives that the utility of searching equals $\theta - r_1$ and because the utility of buying immediately is $\theta - \min(p^*, p^{min} + c_t)$ searching is profitable for $\min(p^*, p^{min} + c_t) > r_1$.

If $\min(p^*, p^{min}+c_t) \leq r_1$ both $q > r_1$ and $q \leq r_1$ are possible. For $q > r_1$ the utility of search equals $\theta - r_1$ (see previous paragraph) and $U(buy) \geq \theta - r_1$. Search therefore is not profitable. For $q \leq r_1$, $\int_{\underline{p}_1}^{q}(q-p)dF_1(p) < c_t + c_e$ and $U(search) < \theta - q$. Because $U(buy) \geq \theta - r_1 \geq \theta - q$ search is not profitable. So for $\min(p^*, p^{min} + c_t) \leq r_1$ the non-shopper will stop searching whereas for $\min(p^*, p^{min} + c_t) > r_1$ he will continue to search.

This shows that the consumer behavior stated in the Propositions is indeed optimal when a consumer has searched $n-1$ shops. Now suppose that he has searched $h \geq 1$ shops and that the stated consumer behavior holds whenever he has searched $h+1$ or more shops. Because the consumer expects $\overline{p}$ to be at or below r_1 the optimal consumer behavior tells him to stop searching after searching the $h+1$th shop. Therefore, after searching h shops, if the consumer decides to continue searching he expects to search only one more shop and the utilities of continuing search and of stopping search are the same as before. After searching the hth shop the non-shopper will therefore continue his search if and only if $\min(p^*, p^{min} + c_t) > r_1$ and the stated consumer behavior also holds in the case $h \geq 1$ shops have been searched.

This leaves the case where no shops have been searched yet. Again, given the optimal consumer behavior for $h \geq 1$, the non-shopper expects to search only once and the utility of search equals

$$U(search) = -c_t - c_e + \int_{\underline{p}}^{\min(\theta, r_1)} (\theta - p)dF_1(p).$$

When $r_1 < \theta$ this reduces to $-c_t - c_e + \int_{\underline{p}}^{r_1}(\theta - p)dF_1(p) = -c_t - c_e + \theta - r_1 + \int_{\underline{p}}^{\min(\theta,r_1)}(r_1 - p)dF_1(p) = \theta - r_1 > 0$, so for $r_1 < \theta$ all non-shoppers will search. When $r_1 = \theta$ the expression above can be rewritten as $-c_t - c_e + \int_{\underline{p}}^{r_1}(r_1 - p)dF_1(p) = 0$ and so non-shoppers are indifferent between searching and staying home.

Before deriving an explicit expression for r_1, consider the pricing behavior of shops. First look at the full search case with $r_1 < \theta$. A standard undercutting argument shows that the price distribution has no atoms. For $p \leq r_1$ profits are given by

$$\pi_1(p) = p\gamma(1 - F_1(p))^{n-1} + p(1-\gamma)\frac{1}{n}$$

Under the assumption $\overline{p} \leq r_1$ it must be that $\overline{p} = r_1$. If $\overline{p} < r_1$ deviation to a price r_1 would be profitable. This gives that in equilibrium profits equal $\pi_1(r_1) = r_1(1-\gamma)\frac{1}{n}$ and equating this with $\pi_1(p)$ gives

$$F_1(p) = 1 - (\frac{1-\gamma}{\gamma n}\frac{r_1 - p}{p})^{\frac{1}{n-1}}.$$

Finally, the minimum price is the price $\underline{p}$ such that $F_1(\underline{p}) = 0$. This gives $\underline{p} = r_1 \frac{1-\gamma}{\gamma n + 1 - \gamma}$. Note that deviation to a price below $\underline{p}$ is not profitable and that deviation to a price above r_1 gives zero profits and therefore is not profitable as well.

Given $F_1(p)$ the reservation price r_1 can be derived. Rewriting the definition of r_1 gives $r_1 - \int_{\underline{p}}^{r_1} p dF_1(p) = c_t + c_e$. Rewriting $F_1(p)$ gives

$$p = \frac{r_1}{1 + \frac{\gamma n}{1-\gamma}(1 - F_1(p))^{n-1}}$$

and therefore

$$\int_{\underline{p}}^{r_1} p dF_1(p) = \int_0^1 \frac{r_1}{1 + \frac{\gamma n}{1-\gamma}(1-y)^{n-1}} dy.$$

This can be rewritten as

$$\int_{\underline{p}}^{r_1} p dF_1(p) = \int_0^1 \frac{r_1}{1 + \frac{\gamma n}{1-\gamma} y^{n-1}} dy.$$

The definition of r_1 then finally gives

$$r_1 = \frac{c_t + c_e}{1 - \int_0^1 \frac{1}{1 + \frac{\gamma n}{1-\gamma} y^{n-1}} dy}.$$

The full search equilibrium holds when $r_1 < \theta$.

Now look at the partial search case with $r_1 = \theta$. A standard undercutting argument shows that the price distribution has no atoms. For $p \leq r_1$ profits are given by

$$\pi_1(p) = p\gamma(1 - F_1(p))^{n-1} + p(1-\gamma)\frac{\mu_1}{n}$$

It must be that $\overline{p} = r_1 = \theta$. If $\overline{p} < \theta$ deviation to a price θ would be profitable. This gives that in equilibrium profits equal $\pi_1(\theta) = \theta(1-\gamma)\frac{\mu_1}{n}$ and equating this with $\pi_1(p)$ gives

$$F_1(p) = 1 - (\frac{(1-\gamma)\mu_1}{\gamma n}\frac{r_1 - p}{p})^{\frac{1}{n-1}}.$$

Finally, the minimum price is the price $\underline{p}$ such that $F_1(\underline{p}) = 0$. This gives $\underline{p} = r_1 \frac{(1-\gamma)\mu_1}{\gamma n + (1-\gamma)\mu_1}$. Note that deviation to a price below $\underline{p}$ is not profitable.

The condition $r_1 = \theta$ defines μ_1 and the condition $0 < \mu_1 < 1$ defines the parameter region for which the equilibrium holds. The definition of r_1 and $r_1 = \theta$ gives $\theta - \int_{\underline{p}}^{\theta} p dF_1(p) = c_t + c_e$. Using the same method as before, this can be rewritten as

$$\theta(1 - \int_0^1 \frac{1}{1 + \frac{\gamma n}{(1-\gamma)\mu_1} y^{n-1}} dy) = c_t + c_e$$

or,

$$h(\mu_1) = \int_0^1 \frac{1}{1 + \frac{\gamma n}{(1-\gamma)\mu_1} y^{n-1}} dy = \frac{\theta - c_t - c_e}{\theta},$$

defining μ_1. Note that $h(\mu_1)$ is increasing in μ_1, with $h(0) = 0$ and $h(1) = \int_0^1 \frac{1}{1+\frac{\gamma n}{(1-\gamma)} y^{n-1}} dy$. The condition $0 < \mu_1 < 1$ therefore gives

$$0 < \frac{\theta - c_t - c_e}{\theta} < \int_0^1 \frac{1}{1 + \frac{\gamma n}{(1-\gamma)} y^{n-1}} dy.$$

Recall that by assumption $\theta - c_t - c_e > 0$ and so the only relevant part is $\frac{\theta - c_t - c_e}{\theta} < \int_0^1 \frac{1}{1+\frac{\gamma n}{(1-\gamma)} y^{n-1}} dy$, or $c_t + c_e > \theta(1 - \int_0^1 \frac{1}{1+\frac{\gamma n}{(1-\gamma)} y^{n-1}} dy)$.

Proof of Propositions 3.3 and 3.4

Once a non-shopper has searched one shop he is in the situation described by Stahl (1989) with search costs c_e and so he will stop searching as soon as he finds a price at or below r_n, with r_n defined by

$$\int_{\underline{p}}^{r_n} (r_n - p) dF_n(p) = c_e.$$

Stahl (1989) also shows that the maximum price is at or below r_n. This implies that non-shoppers search at most once. The expected utility of the first search therefore is

$$-c_t - c_e + \int_{\underline{p}}^{r_n} (\theta - p) dF_n(p).$$

For $r_n \leq \theta$ this can be rewritten as

$$-c_t - c_e + \theta - r_n + \int_{\underline{p}}^{r_n} (r_n - p) dF_n(p),$$

which equals $\theta - r_n - c_t$. Therefore, for $r_n < \theta - c_t$ all non-shoppers will search and for $r_n = \theta - c_t$ non-shoppers are indifferent between searching and staying home. For $\theta - c_t < r_n \leq \theta$ searching clearly is not profitable and for $r_n > \theta$, $\int_{\underline{p}}^{r_n} (\theta - p) dF_n(p) < c_e$ and so the utility of searching is strictly negative as well.

In a full search equilibrium $r_n < \theta - c_t$ and the profits for $p \leq r_n$ are given by

$$\pi_n(p) = p\frac{1-\gamma}{n} + p\gamma(1 - F_n(p))^{n-1}.$$

This expression shows that $\overline{p} = r_n$ because else deviation to r_n would be profitable. Equilibrium profits are therefore $\pi_n(r_n) = r_n\frac{1-\gamma}{n}$ and equating $\pi_n(p)$ and $\pi_n(r_n)$ gives

$$F_n(p) = 1 - (\frac{(r_n - p)(1-\gamma)}{n\gamma p})^{\frac{1}{n-1}}$$

with $\underline{p}_n = r_n\frac{1-\gamma}{\gamma n + (1-\gamma)}$. It is clear that deviation to a price below $\underline{p}_n$ is not profitable. The same argument as in the proof of Propositions 3.1 and 3.2 finally shows that

$$r_n = \frac{c_e}{1 - \int_0^1 \frac{1}{1+\frac{\gamma}{1-\gamma}ny^{n-1}}dy}.$$

A full search equilibrium holds when $r_n < \theta - c_t$.

In a partial search equilibrium $r_n = \theta - c_t$ and a fraction μ_n of the non-shoppers searches. For $p \leq r_n$ the profits are

$$\pi_n(p) = p\frac{\mu_n(1-\gamma)}{n} + p\gamma(1 - F_n(p))^{n-1}.$$

This expression shows that $\overline{p}_n = r_n$ and equilibrium profits are $\pi_n(r_n) = r_n\frac{\mu_n(1-\gamma)}{n}$. Equating $\pi_n(p)$ with $\pi_n(r_n)$ gives

$$F_n(p) = 1 - (\frac{(r_n - p)(1-\gamma)\mu_n}{n\gamma p})^{\frac{1}{n-1}}$$

with $\underline{p}_n = r_n\frac{\mu_n(1-\gamma)}{\gamma n + \mu_n(1-\gamma)}$. It is clear that deviating to a price below $\underline{p}_n$ is not profitable. The fraction of searching non-shoppers, μ_n, is defined by the condition $r_n = \theta - c_t$. The same procedure as in the proof of Propositions 3.1 and 3.2 gives

$$h(\mu_n) \equiv \int_0^1 \frac{1}{1 + \frac{\gamma n}{(1-\gamma)\mu_n}y^{n-1}}dy = \frac{\theta - c_t - c_e}{\theta - c_t}.$$

Finally, because $h(\mu_n)$ is increasing in μ_n the condition $0 < \mu_n < 1$ gives

$$0 < \frac{\theta - c_t - c_e}{\theta - c_t} < \int_0^1 \frac{1}{1 + \frac{\gamma}{1-\gamma}ny^{n-1}}dy$$

where the first part, $\frac{\theta - c_t - c_e}{\theta - c_t} > 0$, is automatically satisfied because of the assumption $\theta - c_t - c_e > 0$.

Proof of Proposition 3.5

For ease of notation, let q denote $\int_0^1 \frac{1}{1+\frac{\gamma}{1-\gamma} n y^{n-1}} dy$.
For $c_t + c_e < \theta(1-q)$ the full search equilibrium holds both when all shops are located together and when all shops are isolated. Therefore $\pi_1 = \frac{1-\gamma}{n} \frac{c_t+c_e}{1-q} > \frac{1-\gamma}{n} \frac{c_e}{1-q} = \pi_n$.
Next, I show that for $c_t + c_e > \theta \frac{1-q}{1-\beta q}$ $\pi_1 < \pi_n$. In this case the partial search equilibrium holds both when all shops are located together and when all shops are isolated. Define the function $g(\mu) \equiv \int_0^1 \frac{1}{1+\frac{\gamma}{1-\gamma}\frac{n}{\mu} y^{n-1}} dy$ and note that μ_1 is defined by $g(\mu_1) = \frac{\theta - c_t - c_e}{\theta}$ and that μ_n is defined by $g(\mu_n) = \frac{\theta - c_t - c_e}{\theta - c_t}$. Expected profits are given by $\pi_1 = \theta \mu_1 \frac{1-\gamma}{n}$ and $\pi_n = (\theta - c_t)\mu_n \frac{1-\gamma}{n}$ and therefore $\pi_1 < \pi_n$ holds if and only if $\mu_n > \frac{\theta}{\theta - c_t}\mu_1$. Using that $g(\mu)$ is strictly increasing in μ this can be rewritten as $g(\mu_n) > g(\frac{\theta}{\theta - c_t}\mu_1)$ or

$$\frac{\theta - c_t - c_e}{\theta - c_t} > \int_0^1 \frac{1}{1 + \frac{\gamma}{1-\gamma}\frac{n}{\mu_1}\frac{\theta - c_t}{\theta} y^{n-1}} dy.$$

Because $\int_0^1 \frac{1}{1+\frac{\gamma}{1-\gamma}\frac{n}{\mu_1}\frac{\theta-c_t}{\theta} y^{n-1}} dy = \frac{\theta}{\theta - c_t} \int_0^1 \frac{1}{\frac{\theta}{\theta - c_t} + \frac{\gamma}{1-\gamma}\frac{n}{\mu_1} y^{n-1}} dy$, $\pi_1 < \pi_n$ if and only if

$$\int_0^1 \frac{1}{\frac{\theta}{\theta - c_t} + \frac{\gamma}{1-\gamma}\frac{n}{\mu_1} y^{n-1}} dy < \frac{\theta - c_t - c_e}{\theta}.$$

The definition of μ_1 gives $\frac{\theta - c_t - c_e}{\theta} = \int_0^1 \frac{1}{1+\frac{\gamma}{1-\gamma}\frac{n}{\mu_1} y^{n-1}} dy$ and so $\pi_1 < \pi_n$ if and only if

$$\int_0^1 \frac{1}{\frac{\theta}{\theta - c_t} + \frac{\gamma}{1-\gamma}\frac{n}{\mu_1} y^{n-1}} dy < \int_0^1 \frac{1}{1 + \frac{\gamma}{1-\gamma}\frac{n}{\mu_1} y^{n-1}} dy$$

and this always holds because $\frac{\theta}{\theta - c_t} > 1$.

For $\theta(1-q) < c_t + c_e < \theta \frac{1-q}{1-\beta q}$ a full search equilibrium holds when all shops are located together. This implies that $\pi_n = \frac{(1-\beta)(c_t+c_e)}{1-q} \frac{1-\gamma}{n}$ is linearly increasing in $c_t + c_e$. When all shops are located separately a partial search equilibrium holds with $\pi_1 = \theta \mu_1 \frac{1-\gamma}{n}$ and $g(\mu_1) = \frac{\theta - c_t - c_e}{\theta}$. Because $g(\mu)$ is strictly increasing in μ and $\frac{\theta - c_t - c_e}{\theta}$ decreases in $c_t + c_e$, μ_1 decreases in $c_t + c_e$ and therefore π_1 decreases in $c_t + c_e$. Because for $c_t + c_e < \theta(1-q)$ $\pi_1 > \pi_n$ and for $c_t + c_e > \theta \frac{1-q}{1-\beta q}$ $\pi_1 < \pi_n$ this implies that there exists a unique value c with $\theta(1-q) < c < \theta \frac{1-q}{1-\beta q}$ where $\pi_1 = \pi_n$, with $\pi_1 > \pi_n$ for

$c_t + c_e < c$ and $\pi_1 < \pi_n$ for $c_t + c_e > c$.

Proof of Proposition 4.1

Recall that in the model $c_t + c_e < \theta$ and therefore at least some non-shoppers will search. First assume that $r_k^i < r_k^m + c_t$. Proposition B.4 shows that under this assumption all searching non-shoppers will first search in an isolated shop. Using Proposition B.3 and using that $\overline{p}_k^i \leq r_k^i$ it is easy to see that the searching non-shoppers will stop searching after their first search and will buy from the isolated shop they visited. Consequently, shops in the mall will compete for the shoppers and mall prices will be zero. The definition of r_k^m in that case gives $r_k^m = c_e$, a contradiction of the initial assumption that $r_k^i < r_k^m + c_t$.

For $r_k^i > r_k^m + c_t$ all searching non-shoppers will first search in the mall, and because $\overline{p}_k^m \leq r_k^m$ they will stop searching after their first search and buy from the mall shop they visited. In case $k < n-1$ there are 2 or more isolated shops and these isolated shops would compete for the shoppers. Isolated prices would be zero and $r_k^i = c_t + c_e$, a contradiction of $r_k^i > r_k^m + c_t$.

To show that $r_k^i > r_k^m + c_t$ cannot hold when $k = n-1$, a lengthy argument is required. Here only a brief outline is given; full details are available on request. First, it can be argued that for $r_k^i > r_k^m + c_t$ and $k = n-1$ $\underline{p}_k^i \geq \underline{p}_k^m$. The next step then is to derive the equilibrium price distributions $F_k^m(p)$ and $F_k^i(p)$. It can be shown that $F_k^i(p)$ is strictly increasing for $0 < p < r_k^i$. If $\underline{p}_k^m < \underline{p}_k^i$ it thus should hold that $F_k^i(\underline{p}_k^m) < 0$. But then $\pi_k^i(\underline{p}_k^m) > \pi_k^i(\overline{p}_k^i)$. This shows that an equilibrium with $r_k^i > r_k^m + c_t$ and $k = n-1$ has $\underline{p}_k^i = \underline{p}_k^m$. Using this, a full equilibrium can be derived, including analytical expressions for the two reservation prices r_k^m and r_k^i. An analysis then shows that $r_k^i \leq r_k^m + c_t$ for all relevant parameter values, contradicting the initial assumption that $r_k^i > r_k^m + c_t$.

Proof of Proposition 4.2

To prove Proposition 4.2 it is convenient first to prove Proposition 4.3. Once Proposition 4.3 has been established, we can use in the proof of Proposition 4.2 that $\frac{x_k}{k} > \frac{1-x_k}{n-k}$.

Proof of Proposition 4.3

First note that

$$\pi_k^i = \pi_k^i(\underline{p}_k^i) = \gamma \underline{p}_k^i (1 - F_k^m(\underline{p}_k^i))^k + (1-\gamma)\frac{1-x_k}{n-k}\underline{p}_k^i \mu_k$$

and

$$\pi_k^m \geq \pi_k^m(\underline{p}_k^i) = \gamma \underline{p}_k^i (1 - F_k^m(\underline{p}_k^i))^{k-1} + (1-\gamma)\frac{x_k}{k}\underline{p}_k^i \mu_k.$$

Suppose contrary to the proposition that $\pi_k^m = \pi_k^i$, implying that $r_k^m \frac{x_k}{k} = r_k^i \frac{1-x_k}{n-k}$, or, using Proposition 4.1, $\frac{x_k}{k} > \frac{1-x_k}{n-k}$. This gives $\pi_k^i = \pi_k^i(\underline{p}_k^i) < \pi_k^m(\underline{p}_k^i) \leq \pi_k^m$, a contradiction to the assumption $\pi_k^m = \pi_k^i$.

Now suppose contrary to the proposition that $\pi_k^m < \pi_k^i$. Note that $\pi_k^m < \pi_k^i$ implies $\pi_k^i(\underline{p}_k^i) > \pi_k^m(\underline{p}_k^i)$ and therefore $\frac{1-x_k}{n-k} > \frac{x_k}{k}$. Moreover, $\pi_k^m = \pi_k^m(r_k^m) \geq \pi_k^m(\underline{p}_k^i)$ gives $(r_k^m - \underline{p}_k^i)(1-\gamma)\frac{x_k}{k}\mu_k \geq \gamma \underline{p}_k^i(1 - F_k^m(\underline{p}_k^i))^{k-1}$. Combining these two inequalities gives

$$\begin{aligned}
\pi_k^i(\underline{p}_k^i) &\leq \gamma \underline{p}_k^i (1 - F_k^m(\underline{p}_k^i))^{k-1} + (1-\gamma)\frac{1-x_k}{n-k}\underline{p}_k^i \mu_k \\
&\leq (r_k^m - \underline{p}_k^i)(1-\gamma)\frac{x_k}{k}\mu_k + (1-\gamma)\frac{1-x_k}{n-k}\underline{p}_k^i \mu_k \\
&< r_k^m(1-\gamma)\frac{1-x_k}{n-k}\mu_k \\
&< r_k^i(1-\gamma)\frac{1-x_k}{n-k}\mu_k = \pi_k^i(r_k^i),
\end{aligned}$$

a contradiction.

Because both $\pi_k^m = \pi_k^i$ and $\pi_k^m < \pi_k^i$ are not feasible, it should be that $\pi_k^m > \pi_k^i$, or $r_k^m \frac{x_k}{k} > r_k^i \frac{1-x_k}{n-k}$. Proposition 4.1 then gives that $\frac{x_k}{k} > \frac{1-x_k}{n-k}$. Q.E.D.

Now that Proposition 4.3 has been established, we will continue the proof of Proposition 4.2.

In the text it is already argued that for $k < n-1$ $\overline{p}_k^i = r_k^i$ and $\overline{p}_k^m = r_k^m$. Moreover, for $k < n-1$ atoms in $F_k^i(p)$ can only occur when $F_k^m(p) = 1$ and atoms in $F_k^m(p)$ can only occur when $F_k^i(p) = 1$.
When $k = n-1$ it is clear that atoms in $F_k^m(p)$ can only occur when $F_k^i(p) = 1$. Atoms in $F_k^i(p)$ can only occur when $F_k^m(p) = 1$. When $F_k^m(p) < 1$ and there would be an atom in $F_k^i(p)$ at price p^*, mall shops would undercut p^* and the isolated shop could increase profits by setting an atom at $p^* + \epsilon$ instead of p^*. Concerning the maximum prices, $\overline{p}_k^m = r_k^m$ because for $\overline{p}_k^m < r_k^m$ it would be profitable to deviate to r_k^m. Also, $\overline{p}_k^i = r_k^i$. If $\overline{p}_k^i < r_k^m$, then for prices between $\overline{p}_k^i$ and r_k^m, $f_k^m(p) = 0$. But then the isolated shop could profitably deviate to a price slightly above $\overline{p}_k^i$. If $overline p_k^i = r_k^m$ and there is no atom in $F_k^m(p)$ at $p = r_k^m$ then the isolated shop could profitably deviate to r_k^i. If $overline p_k^i = r_k^m$ and there is an atom in $F_k^m(p)$ at $p = r_k^m$

then the isolated shop could profitably deviate to $r_k^m - \epsilon$. And finally, if $r_k^m < \overline{p}_k^i < r_k^i$, deviating to r_k^i is profitable. This leaves $\overline{p}_k^i = r_k^i$.
It has already been argued in the main text that isolated shops have no probability mass for $r_k^m \leq p < r_k^i$ and that $F_k^i(p)$ has an atom at $p = r_k^i$. To finish the proof, the supports of $F_k^m(p)$ and $F_k^i(p)$ need to be specified for $p \leq r_k^m$. First note that there are no gaps in $[\min(\underline{p}_k^m, \underline{p}_k^i), r_k^m]$, where a gap is defined as a set of prices for which $f_k^m(p) = 0$ *and* $f_k^i(p) = 0$. If there would be a gap $[p_1, p_2]$, then $\pi_k^m(p_1) < \pi_k^m(p_2)$ and $\pi_k^i(p_1) < \pi_k^m(p_2)$, a contradiction. This implies that for prices in $[\min(\underline{p}_k^m, \underline{p}_k^i), r_k^m]$ either $f_k^m(p) > 0$ or $f_k^i(p) > 0$ or both. The corresponding price distributions have already been specified in the main text. I will now first show that in equilibrium $\underline{p}_k^m \geq \underline{p}_k^i$. Second, I will show that when $\underline{p}_k^m = \underline{p}_k^i$ and $k < n-1$ only equilibrium types 2 and 3 can hold. And, third, I will show that when $\underline{p}_k^m > \underline{p}_k^i$ and $k < n-1$ only equilibrium type 1 can hold. Finally, I will show that for $k = n-1$ only equilibrium type 3 can hold.

<u>$\underline{p}_k^m < \underline{p}_k^i$ cannot hold.</u>

Suppose that $\underline{p}_k^m < \underline{p}_k^i$. For $p \leq \underline{p}_k^i$

$$\pi_k^m(p) = \gamma p(1 - F_k^m(p))^{k-1} + (1-\gamma)\frac{x_k}{k}\mu_k p.$$

Because there are no atoms or gaps in $F_k^m(p)$ for $p \leq \underline{p}_k^i$, in equilibrium $\pi_k^m(\underline{p}_k^m) = \pi_k^m(\underline{p}_k^i)$. This gives

$$1 - F_k^m(\underline{p}_k^i) = \left[\frac{\gamma \underline{p}_k^m + (1-\gamma)\frac{x_k}{k}\mu_k(\underline{p}_k^m - \underline{p}_k^i)}{\gamma \underline{p}_k^i}\right]^{\frac{1}{k-1}}.$$

Note that $\pi_k^i(\underline{p}_k^i) = \gamma \underline{p}_k^i(1 - F_k^m(\underline{p}_k^i))^k + (1-\gamma)\frac{1-x_k}{n-k}\mu_k \underline{p}_k^i$. Plugging in the expression given above gives

$$\pi_k^i(\underline{p}_k^i) = (1 - F_k^m(\underline{p}_k^i))(\gamma \underline{p}_k^m + (1-\gamma)\frac{x_k}{k}\mu_k(\underline{p}_k^m - \underline{p}_k^i)) + (1-\gamma)\frac{1-x_k}{n-k}\mu_k \underline{p}_k^i.$$

Note that $\pi_k^i(\underline{p}_k^m) = \gamma \underline{p}_k^m + (1-\gamma)\frac{1-x_k}{n-k}\mu_k \underline{p}_k^m$. I will show that $\pi_k^i(\underline{p}_k^m) > \pi_k^i(\underline{p}_k^i)$ and that thus deviation to $\underline{p}_k^m$ is profitable. First rewrite $\pi_k^i(\underline{p}_k^m) > \pi_k^i(\underline{p}_k^i)$ as

$$\gamma \underline{p}_k^m + (1-\gamma)\frac{1-x_k}{n-k}\mu_k(\underline{p}_k^m - \underline{p}_k^i) > (1 - F_k^m(\underline{p}_k^i))(\gamma \underline{p}_k^m + (1-\gamma)\frac{x_k}{k}\mu_k(\underline{p}_k^m - \underline{p}_k^i)).$$

First note that $1 - F_k^m(\underline{p}_k^i) \geq 0$ implies that $\gamma \underline{p}_k^m + (1-\gamma)\frac{x_k}{k}(\underline{p}_k^m - \underline{p}_k^i) \geq 0$.

Now if $0 < 1-F_k^m(\underline{p}_k^i) \leq 1$, $(1-F_k^m(\underline{p}_k^i))(\gamma\underline{p}_k^m+(1-\gamma)\frac{x_k}{k}(\underline{p}_k^m-\underline{p}_k^i)) \leq \gamma\underline{p}_k^m+(1-\gamma)\frac{x_k}{k}(\underline{p}_k^m-\underline{p}_k^i)$. Because $\frac{x_k}{k} > \frac{1-x_k}{n-k}$ (Proposition 4.3) and $\underline{p}_k^m - \underline{p}_k^i < 0$, $\gamma\underline{p}_k^m+(1-\gamma)\frac{x_k}{k}(\underline{p}_k^m-\underline{p}_k^i) < \gamma\underline{p}_k^m+(1-\gamma)\frac{1-x_k}{n-k}(\underline{p}_k^m-\underline{p}_k^i)$. Combining gives $(1-F_k^m(\underline{p}_k^i))(\gamma\underline{p}_k^m+(1-\gamma)\frac{x_k}{k}(\underline{p}_k^m-\underline{p}_k^i)) < \gamma\underline{p}_k^m+(1-\gamma)\frac{1-x_k}{n-k}(\underline{p}_k^m-\underline{p}_k^i)$. If $1-F_k^m(\underline{p}_k^i) = 0$, the inequality reduces to $\gamma\underline{p}_k^m+(1-\gamma)\frac{1-x_k}{n-k}(\underline{p}_k^m-\underline{p}_k^i) > 0$. Because $\gamma\underline{p}_k^m+(1-\gamma)\frac{1-x_k}{n-k}(\underline{p}_k^m-\underline{p}_k^i) > \gamma\underline{p}_k^m+(1-\gamma)\frac{x_k}{k}(\underline{p}_k^m-\underline{p}_k^i) \geq 0$ this inequality always holds.

<u>When $\underline{p}_k^m = \underline{p}_k^i$ and $k < n-1$ only equilibrium types 2 and 3 can hold.</u>

Let p_j denote a set of increasing prices, that is, $p_1 < p_2 < p_3 < \ldots$. When $\underline{p}_k^m = \underline{p}_k^i$ there is a $p_1 > \underline{p}_k^i$ such that for $\underline{p}_k^i \leq p \leq p_1$ $f_k^m(p) > 0$ and $f_k^i(p) > 0$. Because $f_k^i(r_k^m) = 0$, $p_1 < r_k^m$.

Suppose first that for $p_1 < p \leq p_2$ $f_k^m(p) = 0$ and $f_k^i(p) > 0$. Then $p_2 < r_k^m$ because $f_k^m(r_k^m) > 0$. Also, for $p_2 < p \leq p_3$, $f_k^m(p) > 0$ and $f_k^i(p) = 0$. If for $p_2 < p \leq p_3$ both $f_k^m(p)$ and $f_k^i(p)$ would be positive, then according to (5) $F_k^m(p_1) < F_k^m(p_2)$. If $p_3 = r_k^m$ equilibrium type 2 holds.
A situation with $p_3 < r_k^m$ cannot hold. Suppose that $p_3 < r_k^m$. Using that

$$1 - F_k^i(p_2) = \left(\frac{(r_k^i - p_2)(1-\gamma)\mu_k\frac{1-x_k}{n-k}}{\gamma p_2(1-F_k^m(p_1))^k}\right)^{\frac{1}{n-k-1}}$$

one can write $F_k^m(p_1) = F_k^m(p_2)$ as $g(p_1) = g(p_2)$, with

$$g(p) = \frac{r_k^m - p}{r_k^i - p}\left(\frac{p}{r_k^i - p}\right)^{\frac{1}{n-k-1}}. \tag{7}$$

The function $g(p)$ is increasing for $p < \frac{r_k^i r_k^m}{(n-k)r_k^i-(n-k-1)r_k^m}$ and is decreasing for $p > \frac{r_k^i r_k^m}{(n-k)r_k^i-(n-k-1)r_k^m}$. Also note that when both $f_k^m(p) > 0$ and $f_k^i(p) > 0$, $(1-F_k^m(p))^{\frac{n-1}{n-k-1}}$ can be written as a constant divided by $g(p)$. Because $F_k^m(p)$ should increase in p, a situation with both $f_k^m(p) > 0$ and $f_k^i(p) > 0$ requires $p < \frac{r_k^i r_k^m}{(n-k)r_k^i-(n-k-1)r_k^m}$. Because $g(p_1) = g(p_2)$, $p_1 < \frac{r_k^i r_k^m}{(n-k)r_k^i-(n-k-1)r_k^m}$ and $p_2 > \frac{r_k^i r_k^m}{(n-k)r_k^i-(n-k-1)r_k^m}$. This implies that for prices above p_2 either $f_k^m(p) = 0$ or $f_k^i(p) = 0$. Because $p_3 < r_k^m$ this gives that there must be a $p_4 < r_k^m$ such that for $p_3 < p \leq p_4$ $f_k^i(p) > 0$ and $f_k^m(p) = 0$ and that for $p_4 < p \leq p_5$ $f_k^m(p) > 0$ and $f_k^i(p) = 0$, with $p_5 \leq r_k^m$. But $F_k^m(p_3) = F_k^m(p_4)$ can be written as $g(p_3) = g(p_4)$. Because $p_3 > \frac{r_k^i r_k^m}{(n-k)r_k^i-(n-k-1)r_k^m}$, $g(p)$ is strictly increasing for $p \geq p_3$ and

$g(p_3) = g(p_4)$ cannot hold.

Now suppose that for $p_1 < p \leq p_2$ $f_k^m(p) > 0$ and $f_k^i(p) = 0$. If $p_2 = r_k^m$, equilibrium type 3 holds. A situation with $p_2 < r_k^m$ cannot hold. To show this suppose that $p_2 < r_k^m$. It cannot be that for $p_2 < p \leq p_3$ both $f_k^m(p) > 0$ and $f_k^i(p) > 0$, because (5) would then give $F_k^i(p_1) < F_k^i(p_2)$. Thus, for $p_2 < p \leq p_3$, $f_k^m(p) = 0$ and $f_k^i(p) > 0$. Note that $p_3 < r_k^m$ as $f_k^m(r_k^m) > 0$. First suppose that for $p_3 < p \leq p_4$ $f_k^m(p) > 0$ and $f_k^i(p) > 0$. As mentioned in the previous part, to ensure that $F_k^m(p)$ is increasing in p, it should be that $p_4 < \frac{r_k^i r_k^m}{(n-k)r_k^i - (n-k-1)r_k^m}$. Also, rewriting $F_k^i(p_1) = F_k^i(p_2)$ gives $h(p_1) = h(p_2)$, with

$$h(p) = \frac{r_k^i - p}{r_k^m - p}\left(\frac{p}{r_k^m - p}\right)^{\frac{1}{k-1}}. \tag{8}$$

The derivative of $h(p)$ has the same sign as

$$r_k^i r_k^m + p((k-1)c_t - r_k^m).$$

If $(k-1)c_t - r_k^m \geq 0$, then $h'(p) > 0$ and $h(p_1) = h(p_2)$ cannot hold. If $(k-1)c_t - r_k^m < 0$ then $h(p)$ first increases in p for small p and decreases in p for large p. But $h'(\frac{r_k^i r_k^m}{(n-k)r_k^i - (n-k-1)r_k^m}) > 0$ and therefore for $p < \frac{r_k^i r_k^m}{(n-k)r_k^i - (n-k-1)r_k^m}$ $h(p)$ increases in p. Because $p_1 < p_2 < p_4 < \frac{r_k^i r_k^m}{(n-k)r_k^i - (n-k-1)r_k^m}$, $h(p_1) = h(p_2)$ cannot hold. Consequently, it cannot hold that for $p_3 < p \leq p_4$ $f_k^m(p) > 0$ and $f_k^i(p) > 0$.
Now suppose that for $p_3 < p \leq p_4$ $f_k^m(p) > 0$ and $f_k^i(p) = 0$. Then $F_k^i(p_1) = F_k^i(p_2)$ gives $h(p_1) = h(p_2)$, with $h(p)$ defined by (8), and $F_k^m(p_2) = F_k^m(p_3)$ gives $g(p_2) = g(p_3)$, with $g(p)$ defined by (8). As shown before, $g(p_2) = g(p_3)$ implies $p_2 < \frac{r_k^i r_k^m}{(n-k)r_k^i - (n-k-1)r_k^m}$ and $p_3 > \frac{r_k^i r_k^m}{(n-k)r_k^i - (n-k-1)r_k^m}$. But, as in the previous paragraph, when $p_2 < \frac{r_k^i r_k^m}{(n-k)r_k^i - (n-k-1)r_k^m}$, $h(p_1) = h(p_2)$ cannot hold.

<u>When $\underline{p}_k^m > \underline{p}_k^i$ and $k < n-1$ only equilibrium type 1 can hold.</u>

When $\underline{p}_k^m > \underline{p}_k^i$, (4) gives $\underline{p}_k^i = r_k^i \frac{(1-\gamma)\mu_k \frac{1-x_k}{n-k}}{\gamma + (1-\gamma)\mu_k \frac{1-x_k}{n-k}}$. Also, there is a $p_1 > \underline{p}_k^i$ such that for $\underline{p}_k^i \leq p \leq p_1$ $f_k^m(p) = 0$ and $f_k^i(p) > 0$. Because $f_k^i(r_k^m) = 0$, $p_1 < r_k^m$.

First suppose that for $p_1 < p \leq p_2$ $f_k^m(p) > 0$ and $f_k^i(p) > 0$. I will show that this gives mall shops an incentive to deviate. Note that $p_1 = \underline{p}_k^m$ is defined by (6); $F_k^m(\underline{p}_k^m) = 0$. This gives

$$g(\underline{p}_k^m) = \left(\frac{(1-\gamma)\mu_k}{\gamma}\frac{1-x_k}{n-k}\right)^{\frac{1}{n-k-1}}\frac{1-x_k}{n-k}\frac{k}{x_k}$$

with $g(p)$ defined by (7). Because for $p_1 < p \le p_2$ $f_k^m(p) > 0$ and $f_k^i(p) > 0$, $p_2 < \frac{r_k^i r_k^m}{(n-k)r_k^i-(n-k-1)r_k^m}$ (else (6) is decreasing in p). Also, $g(p)$ is increasing for $p < \frac{r_k^i r_k^m}{(n-k)r_k^i-(n-k-1)r_k^m}$. This implies that for $\underline{p}_k^i < \underline{p}_k^m$ to hold, $g(\underline{p}_k^i) < \left(\frac{(1-\gamma)\mu_k}{\gamma}\frac{1-x_k}{n-k}\right)^{\frac{1}{n-k-1}}\frac{1-x_k}{n-k}\frac{k}{x_k}$. Plugging in $\underline{p}_k^i = r_k^i\frac{(1-\gamma)\mu_k\frac{1-x_k}{n-k}}{\gamma+(1-\gamma)\mu_k\frac{1-x_k}{n-k}}$ and rewriting gives

$$\frac{r_k^i}{\gamma+(1-\gamma)\mu_k\frac{1-x_k}{n-k}} > \frac{r_k^m\frac{x_k}{k}}{\frac{1-x_k}{n-k}(\gamma+(1-\gamma)\mu_k\frac{x_k}{k})}.$$

Using $\underline{p}_k^i = r_k^i\frac{(1-\gamma)\mu_k\frac{1-x_k}{n-k}}{\gamma+(1-\gamma)\mu_k\frac{1-x_k}{n-k}}$ in $\pi_k^m(\underline{p}_k^i) = \gamma\underline{p}_k^i + (1-\gamma)\mu_k\frac{x_k}{k}\underline{p}_k^i$ gives $\pi_k^m(\underline{p}_k^i) > (1-\gamma)\mu_k\frac{x_k}{k}r_k^m = \pi_k^m(r_k^m)$. Therefore mall shops have an incentive to deviate.

Now suppose that for $p_1 < p \le p_2$ $f_k^m(p) > 0$ and $f_k^i(p) = 0$. If $p_2 = r_k^m$, equilibrium type 1 holds. I will show that $p_2 < r_k^m$ gives mall shops an incentive to deviate.
Suppose that $p_2 < r_k^m$. Note that $p_1 = \underline{p}_k^m$ is defined by (3); $F_k^m(\underline{p}_k^m) = 0$. As before, this gives

$$g(\underline{p}_k^m) = \left(\frac{(1-\gamma)\mu_k}{\gamma}\frac{1-x_k}{n-k}\right)^{\frac{1}{n-k-1}}\frac{1-x_k}{n-k}\frac{k}{x_k}$$

with $g(p)$ defined by (7), $g(p)$ increasing in p for $p < \frac{r_k^i r_k^m}{(n-k)r_k^i-(n-k-1)r_k^m}$ and $g(p)$ decreasing in p for $p > \frac{r_k^i r_k^m}{(n-k)r_k^i-(n-k-1)r_k^m}$. If there is a price region $[p_j, p_{j+1}]$ with $j \ge 2$ where $f_k^m(p) > 0$ and $f_k^i(p) > 0$, then $\underline{p}_k^m < p_{j+1} < \frac{r_k^i r_k^m}{(n-k)r_k^i-(n-k-1)r_k^m}$ because else (6) would decrease in p. If there is no such region, then for $p_2 < p \le p_3$ $f_k^m(p) = 0$ and $f_k^i(p) > 0$. Also, because $f_k^m(r_k^m) > 0$, for $p_3 < p \le p_4$ $f_k^m(p) > 0$ and $f_k^i(p) = 0$, with $p_4 \le r_k^m$. But then $F_k^m(p_2) = F_k^m(p_3)$ gives $g(p_2) = g(p_3)$, and therefore $\underline{p}_k^m < p_2 < \frac{r_k^i r_k^m}{(n-k)r_k^i-(n-k-1)r_k^m}$. So in either case $\underline{p}_k^m < \frac{r_k^i r_k^m}{(n-k)r_k^i-(n-k-1)r_k^m}$, $g(p)$ is increasing in $[\underline{p}_k^i, \underline{p}_k^m]$ and $\underline{p}_k^i < \underline{p}_k^m$ gives $g(\underline{p}_k^i) < \left(\frac{(1-\gamma)\mu_k}{\gamma}\frac{1-x_k}{n-k}\right)^{\frac{1}{n-k-1}}\frac{1-x_k}{n-k}\frac{k}{x_k}$. As before, using this inequality, $\pi_k^m(\underline{p}_k^i) > \pi_k^m(r_k^m)$, giving mall shops an incentive to deviate.

When $k = n-1$ only equilibrium type 3 can hold.

When $k = n-1$, $\pi_k^i(p) = \gamma p(1 - F_k^m(p))^{n-1} + (1-\gamma)\mu_k(1-x_k)p$. It has already been shown that $\underline{p}_k^m < \underline{p}_k^i$ cannot hold. Also $\underline{p}_k^m > \underline{p}_k^i$ cannot hold, because if $\underline{p}_k^m > \underline{p}_k^i$, $\pi_k^i(\underline{p}_k^m) > \pi_k^i(\underline{p}_k^i)$. Therefore, in equilibrium $\underline{p}_k^i = \underline{p}_k^m$. In equilibrium a price region in which $f_k^m(p) = 0$ (and consequently $f_k^i(p) > 0$) cannot occur. Suppose that for $p \in [p_1, p_2]$ $f_k^m(p) = 0$. Then $\pi_k^i(p_1) < \pi_k^i(p_2)$, a contradiction.

Because $\underline{p}_k^i = \underline{p}_k^m$ and $f_k^i(r_k^m) = 0$, there should be a $p_1 < r_k^m$ such that for $\underline{p}_k^i \leq p \leq p_1$ $f_k^m(p) > 0$ and $f_k^i(p) > 0$. Also, because $f_k^m(p) = 0$ cannot occur, there should be a p_2 such that for $p_1 < p \leq p_2$ $f_k^m(p) > 0$ and $f_k^i(p) = 0$. If $p_2 = r_k^m$, equilibrium type 3 holds. If $p_2 < r_k^m$, there should be a p_3 such that for $p_2 < p \leq p_3$ $f_k^m(p) > 0$ and $f_k^i(p) > 0$. But because $F_k^i(p)$ is given by (5), $F_k^i(p_2) > F_k^i(p_1)$, a contradiction.

Proof of Proposition 5.1

In this proof I will use that the definitions of r_k^m and r_k^i ((1) and (2)) can be rewritten using partial integration as

$$\int_{\underline{p}_k^m}^{r_k^m} F_k^m(p)dp = c_e$$

and

$$\int_{\underline{p}_k^i}^{r_k^i} F_k^i(p)dp = c_t + c_e$$

- Recall that $c_t = \beta(c_t + c_e)$, so $\beta \to 0$ implies $c_t \to 0$. Thus $r_k^i - r_k^m = c_t \to 0$.

  In equilibrium type 1, $\pi_k^i(b) = \pi_k^i(r_k^i)$ gives

  $$b\gamma(1 - F_k^i(b))^{n-k-1} = \frac{1-x_k}{n-k}(1-\gamma)\mu_k(r_k^i - b).$$

  Using that $F_k^m(b) = 0$, or $\gamma b(1 - F_k^i(b))^{n-k} = (r_k^m - b)(1-\gamma)\mu\frac{x_k}{k}$, we get

  $$(r_k^m - b)\frac{x_k}{k} = (r_k^i - b)\frac{1-x_k}{n-k}(1 - F_k^i(b)).$$

  Because $r_k^i - r_k^m \to 0$, it should be that $\frac{x_k}{k} - \frac{1-x_k}{n-k}(1 - F_k^i(b)) \to 0$. Because $\frac{x_k}{k} \geq \frac{1-x_k}{n-k}$ and $0 \leq F_k^i(b) \leq 1$, it should be that $\frac{x_k}{k} - \frac{1-x_k}{n-k} \to 0$ and $F_k^i(b) \to 0$. But then

$$\int_{\underline{p}_k^i}^{r_k^i} F_k^i(p)dp \to 0$$

and so equilibrium type 1 cannot hold. Note that equilibrium types 2 and 3 both have $\underline{p}_k^i = \underline{p}_k^m$.

Also note that μ_k should be strictly above 0. If $\mu_k \to 0$, both $F_k^i(p) \to 1$ and $F_k^m(p) \to 1$. Moreover, $\underline{p}_k^i \to 0$. Then $\int_{\underline{p}_k^i}^{r_k^i} F_k^i(p)dp = c_e + c_t$ gives $r_k^i = c_e + c_t$. But to have $0 < \mu_k < 1$, $r_k^i = \theta$ should hold, and $c_e + c_t < \theta$ by definition, a contradiction.

In equilibrium type 2, $\pi_k^m(b) = \pi_k^m(r_k^m)$ gives

$$\gamma b(1-F_k^m(a))^{k-1}\frac{(r_k^i - b)(1-\gamma)\mu_k \frac{1-x_k}{n-k}}{\gamma b(1-F_k^m(a))^k}(1-F_k^i(b)) = (1-\gamma)\mu_k \frac{x_k}{k}(r_k^m - b)$$

or

$$(r_k^i - b)\frac{1-x_k}{n-k}(1-F_k^i(b)) = (r_k^m - b)\frac{x_k}{k}(1-F_k^m(a)).$$

If $\frac{x_k}{k} - \frac{1-x_k}{n-k}$ does not approach 0, then $F_k^i(b) < F_k^m(a)$ and moreover for $p \le a$ $F_k^i(p) < F_k^m(p)$ (see (5) and (6)). But then

$$\int_{\underline{p}_k^i}^{r_k^i} F_k^i(p)dp = \int_{\underline{p}_k^i}^{a} F_k^i(p)dp + \int_a^b F_k^i(p)dp + (r_k^i - b)F_k^i(b) <$$

$$\int_{\underline{p}_k^m}^{a} F_k^m(p)dp + \int_a^b F_k^m(a)dp + \int_b^{r_k^m} F_k^m(p)dp = \int_{\underline{p}_k^m}^{r_k^m} F_k^m(p)dp,$$

a contradiction.

Therefore, $\frac{x_k}{k} - \frac{1-x_k}{n-k} \to 0$, or $\frac{x_k}{k} \to \frac{1}{n}$. But then for $p \le a$ $F_k^i(p) - F_k^m(p) \to 0$. Moreover, $\pi_k^m(b) = \pi_k^m(r_k^m)$ gives $F_k^i(b) \to F_k^m(a)$. Combining, this gives $F_k^i(b) - F_k^i(a) \to 0$. Because $\mu_k > 0$ $F_k^i(p)$ is strictly increasing in p for $a \le p \le b$, and so $b - a \to 0$. Because $\mu_k > 0$, $F_k^m(p)$ is strictly increasing for $b \le p \le r_k^m$. Therefore, if $b < r_k^m$,

$$\int_{\underline{p}_k^i}^{r_k^i} F_k^i(p)dp = \int_{\underline{p}_k^i}^{a} F_k^i(p)dp + \int_a^b F_k^i(p)dp + (r_k^i - b)F_k^i(b) <$$

$$\int_{\underline{p}_k^m}^{a} F_k^m(p)dp + (b-a)F_k^m(a) + \int_{b}^{r_k^m} F_k^m(p)dp = \int_{\underline{p}_k^m}^{r_k^m} F_k^m(p)dp.$$

Therefore, $b \to r_k^m$ and $F_k^i(p) \to F_k^m(p)$.

In equilibrium type 3, if $\frac{x_k}{k} - \frac{1-x_k}{n-k}$ does not approach 0, then for $p \le b$ $F_k^i(p) < F_k^m(p)$ (see (5) and (6)). This gives $\int_{\underline{p}_k^i}^{r_k^i} F_k^i(p)dp < \int_{\underline{p}_k^m}^{r_k^m} F_k^m(p)dp$, a contradiction. Therefore, $\frac{x_k}{k} - \frac{1-x_k}{n-k} \to 0$, or $\frac{x_k}{k} \to \frac{1}{n}$. But then for $p \le b$ $F_k^i(p) - F_k^m(p) \to 0$. Moreover, because $\mu_k > 0$, $F_k^m(p)$ is strictly increasing for $b \le p \le r_k^m$ and therefore if $b < r_k^m$

$$\int_{\underline{p}_k^i}^{r_k^i} F_k^i(p)dp = \int_{\underline{p}_k^i}^{b} F_k^i(p)dp + (r_k^i - b)F_k^i(b) <$$

$$\int_{\underline{p}_k^m}^{b} F_k^m(p)dp + \int_{b}^{r_k^m} F_k^m(p)dp = \int_{\underline{p}_k^m}^{r_k^m} F_k^m(p)dp,$$

a contradiction. This gives $b \to r_k^m$ and $F_k^i(p) - F_k^m(p) \to 0$.

- Note that $\beta \to 1$ implies $c_e \to 0$. Also note that (3) and (6) give that $F_k^m(p)$ is either strictly increasing in p or equal to 1. Because $c_e \to 0$, $\int_{\underline{p}_k^m}^{r_k^m} F_k^m(p)dp \to 0$, and this implies $r_k^m - \underline{p}_k^m \to 0$.

  In equilibrium type 1 $r_k^m - \underline{p}_k^m \to 0$ gives $r_k^m - b \to 0$. Note that b is defined by $F_k^m(b) = 0$, which gives $(r_k^m - b)(1-\gamma)\mu_k \frac{x_k}{k} = \gamma b(1 - F_k^i(b))^{n-k}$. Plugging in (4) for $F_k^i(b)$ and simplifying gives

$$(r_k^m - b)\frac{x_k}{k} = (r_k^i - b)\frac{1-x_k}{n-k}\left(\frac{(r_k^i - b)(1-\gamma)\mu_k \frac{1-x_k}{n-k}}{\gamma b}\right)^{\frac{1}{n-k-1}}$$

  Because $r_k^m - b \to 0$, this expression can only hold if $\mu_k \to 0$ or $\frac{1-x_k}{n-k} \to 0$. In either case, $F_k^i(p) \to 1$ (and therefore $F_k^i(r_k^m) \to 1$) and $\underline{p}_k^i \to 0$. Then $\int_{\underline{p}_k^i}^{r_k^i} F_k^i(p)dp \to c_t$ gives $r_k^i \to c_t$ and consequently $r_k^m \to 0$. To show that $\frac{1-x_k}{n-k} \to 0$, note that for $\mu_k < 1$ it should hold that $r_k^i = \theta$ and that by definition $c_t < \theta$. Therefore $\mu_k \to 0$ cannot hold and $\frac{1-x_k}{n-k} \to 0$ should hold.

  In equilibrium types 2 and 3, $\underline{p}_k^i = \underline{p}_k^m$ and therefore $\underline{p}_k^i \to r_k^m$. Then $\int_{\underline{p}_k^i}^{r_k^i} F_k^i(p)dp \to c_t$ gives $F_k^i(p) \to 1$ and therefore $F_k^i(r_k^m) \to 1$.

Note that $F_k^m(\underline{p}_k^m) = F_k^i(\underline{p}_k^m)$ gives $\frac{x_k}{k}(r_k^m - \underline{p}_k^m) = \frac{1-x_k}{n-k}(r_k^i - \underline{p}_k^m)$, or

$$\underline{p}_k^m = \frac{\frac{x_k}{k} r_k^m - \frac{1-x_k}{n-k} r_k^i}{\frac{x_k}{k} - \frac{1-x_k}{n-k}}.$$

Because $\underline{p}_k^m \to r_k^m$ and $r_k^i = r_k^m + c_t$ it should be that $\frac{1-x_k}{n-k} \to 0$. But then $\underline{p}_k^i = r_k^i \frac{(1-\gamma)\mu_k \frac{1-x_k}{n-k}}{\gamma+(1-\gamma)\mu_k \frac{1-x_k}{n-k}} \to 0$ and consequently $r_k^m \to 0$.

- In all equilibria, $\underline{p}_k^i = r_k^i \frac{(1-\gamma)\frac{\mu_k}{\gamma}(1-x_k)}{(n-k)+(1-\gamma)\frac{\mu_k}{\gamma}(1-x_k)}$. Note that $\underline{p}_k^i < r_k^m = r_k^i - c_t$, therefore $\frac{\mu_k(1-x_k)}{\gamma}$ cannot go to infinity. This implies that either $x_k \to 1$ or $\mu_k \to 0$, or both. Suppose that only $x_k \to 1$ holds. In equilibrium type 1

$$F_k^m(p) = 1 - \left(\frac{r_k^m - p}{p}\right)^{\frac{1}{k-1}} \left(\frac{(1-\gamma)\mu_k \frac{x_k}{k}}{\gamma}\right)^{\frac{1}{k-1}} \left(\frac{1}{1-F_k^i(p_1)}\right)^{\frac{n-k}{k-1}}.$$

Note that $\frac{(1-\gamma)\mu_k \frac{x_k}{k}}{\gamma} \to \infty$, implying that all mass of $F_k^m(p)$ is concentrated at $p = r_k^m$. But then the definition of r_k^m, given by (1), cannot hold.

In equilibrium types 2 and 3, for small enough prices

$$F_k^i(p) = 1 - \left(\frac{r_k^i - p}{p}\right)^{\frac{1}{n-1}} \left(\frac{(1-\gamma)\mu_k}{\gamma}\right)^{\frac{1}{n-1}} \left(\frac{\frac{x_k}{k}(r_k^m - p)}{(r_k^i - p)}\right)^{\frac{k}{n-1}} \left(\frac{n-k}{1-x_k}\right)^{\frac{k-1}{n-1}}.$$

Note that both $\frac{(1-\gamma)\mu_k}{\gamma} \to \infty$ and $\frac{n-k}{1-x_k} \to \infty$. This implies that $\underline{p}_k^i \to r_k^m$ and the definition of r_k^i, given by (1), cannot hold.

Concluding, if only $x_k \to 1$ holds, no equilibrium type can hold, implying that $\mu_k \to 0$.

- In all equilibria, $\underline{p}_k^i = r_k^i \frac{(1-\gamma)\mu_k \frac{1-x_k}{n-k}}{\gamma+(1-\gamma)\mu_k \frac{1-x_k}{n-k}}$. When $\gamma \to 1$, $\underline{p}_k^i \to 0$.

In equilibrium type 1, $F_k^i(p) = 1 - \left(\frac{(r_k^i - p)(1-\gamma)\mu_k \frac{1-x_k}{n-k}}{\gamma p}\right)^{\frac{1}{n-k-1}} \to 1$. Because $\underline{p}_k^i \to 0$ this gives $r_k^i \to c_t + c_e$ and consequently $r_k^m \to c_e$. But $r_k^m \to c_e$ can only occur when $\underline{p}_k^m \to 0$ and $F_k^m(p) \to 1$.

In equilibrium types 2 and 3, $\underline{p}_k^i = \underline{p}_k^m$ and so $\underline{p}_k^m \to 0$. Moreover, taking the limits of (5) and (6) gives $F_k^i(p) \to 1$ and $F_k^m(p) \to 1$.

B Optimal consumer behavior

A first useful result is the following.

Proposition B.1 *In equilibrium $\pi_k^m > 0$ and $\pi_k^i > 0$. Consequently, $\overline{p}_k^m \leq \theta$ and $\overline{p}_k^i \leq \theta$.*

Proof

Suppose to the contrary that $\pi_k^m = 0$ and $\pi_k^i > 0$. This implies that $\underline{p}_k^i > 0$ because for $\underline{p}_k^i = 0$ $\pi_k^i = \pi_k^i(\underline{p}_k^i) = 0$. If some of the non-shoppers visit the shopping mall in their first search then for a shop in the mall setting a price c_e will prevent the non-shoppers from continuing search, leading to positive profits, a contradiction. If none of the non-shoppers search in the shopping mall shops in the mall compete for the shoppers, leading to a maximum price of 0. But then non-shoppers would prefer to search in the shopping mall, a contradiction.
The case $\pi_k^m > 0$ and $\pi_k^i = 0$, with $k \leq n-2$ is the same as above, reversing the roles of the shops inside and outside the mall. For $k = n-1$, if the isolated shop attracts some non-shoppers, it can make a profit by setting $p = c_t + c_e$. If the isolated shop only attracts shoppers, it can make a profit by setting a price slightly below $\underline{p}_k^m$ because $\underline{p}_k^m > 0$.
This leaves the case $\pi_k^m = 0$ and $\pi_k^i = 0$. If non-shoppers would search then the shops attracting some non-shoppers could set a price c_e and make a strictly positive profit. If non-shoppers do not search the firms compete for the shoppers and all set a price 0. But in that case non-shoppers would find it optimal to search, a contradiction.
Because a price above θ would not lead to any sales, the profits of setting such a price are 0, which contradicts the fact that in equilibrium profits are strictly positive.

□

Using Proposition B.1 and assuming that $\overline{p}_k^m \leq r_k^m$ and $\overline{p}_k^i \leq r_k^i$ the behavior of non-shoppers can be derived. This behavior depends on the prices found in previous searches and on whether the consumer currently is in a shop inside the shopping mall or whether he currently is in a shop outside the shopping mall. First consider the case where the consumer currently is in the shopping mall and let $\tilde{p}_k^m$ denote the lowest price found on current and previous searches in the shopping mall. Let $\tilde{p}_k^i$ denote the lowest price found on previous searches outside the shopping mall, with $\tilde{p}_k^i = \infty$ when on previous searches no shops outside the mall have been visited. Note that if the non-shopper decides to stop searching and $\tilde{p}_k^m \leq \tilde{p}_k^i + c_t$ then he will buy from the cheapest shop in the shopping mall, at price $\tilde{p}_k^m$ (note that $\tilde{p}_k^m \leq \theta$ and therefore buying at $\tilde{p}_k^m$ is a better strategy than not buying at all). If the non-shopper decides to stop searching and $\tilde{p}_k^m > \tilde{p}_k^i + c_t$ then he will buy from the cheapest shop outside the shopping mall, incurring return costs c_t and buying at price $\tilde{p}_k^i$. In the proposition that follows I will use the term 'buy from the cheapest option' to denote this behavior.

Proposition B.2 *Consider a non-shopper who expects $\overline{p}_k^m \leq r_k^m$ and $\overline{p}_k^i \leq r_k^i$ and who currently is in a shop in the shopping mall.*
When $r_k^m \leq r_k^i$ ($r_k^m > r_k^i$) his optimal behavior is as follows. When $\min(\tilde{p}_k^m, \tilde{p}_k^i + c_t) \leq r_k^m$ ($\min(\tilde{p}_k^m, \tilde{p}_k^i + c_t) \leq r_k^i$) stop search and buy from the cheapest option. When $\min(\tilde{p}_k^m, \tilde{p}_k^i + c_t) > r_k^m$ ($\min(\tilde{p}_k^m, \tilde{p}_k^i + c_t) > r_k^i$) search further in (outside) the shopping mall, if possible. If there are no shops left to search in (outside) the shopping mall and $\min(\tilde{p}_k^m, \tilde{p}_k^i + c_t) \leq r_k^i$ ($\min(\tilde{p}_k^m, \tilde{p}_k^i + c_t) \leq r_k^m$) buy from the cheapest option. If there are no shops left to search in (outside) the shopping mall and $\min(\tilde{p}_k^m, \tilde{p}_k^i + c_t) > r_k^i$ ($\min(\tilde{p}_k^m, \tilde{p}_k^i + c_t) > r_k^m$) search further outside (in) the shopping mall, if possible. If there also are no shops left to search outside (in) the shopping mall buy from the cheapest option.

Now consider the case where the consumer currently is outside the shopping mall and let $\breve{p}_k^i$ denote the price found in the current shop. Let $\tilde{p}_k^i$ denote the lowest price found on previous searches outside the shopping mall, with $\tilde{p}_k^i = \infty$ when on previous searches no shops outside the mall have been visited. Let $\tilde{p}_k^m$ denote the lowest price found on previous searches inside the shopping mall, with $\tilde{p}_k^m = \infty$ when on previous searches no shops inside the mall have been visited. Note that if the non-shopper decides to stop searching and to buy and $\min(\breve{p}_k^i, \tilde{p}_k^i + c_t, \tilde{p}_k^m + c_t) = \breve{p}_k^i$ then he will buy from the shop he currently is, at price $\breve{p}_k^i$. If $\min(\breve{p}_k^i, \tilde{p}_k^i + c_t, \tilde{p}_k^m + c_t) = \tilde{p}_k^i + c_t$ then he will buy from the cheapest shop outside the shopping mall visited before, incurring return costs c_t and buying at price $\tilde{p}_k^i$. If $\min(\breve{p}_k^i, \tilde{p}_k^i + c_t, \tilde{p}_k^m + c_t) = \tilde{p}_k^m + c_t$ then he will buy from the cheapest shop inside the shopping mall, incurring return costs c_t and buying at price $\tilde{p}_k^m$. In the proposition that follows I will use the term 'buy from the cheapest option' to denote this behavior.

Proposition B.3 *Consider a non-shopper who expects $\overline{p}_k^m \leq r_k^m$ and $\overline{p}_k^i \leq r_k^i$ and who currently is in a shop outside the shopping mall.*
When $r_k^i \leq r_k^m + c_t$ ($r_k^i > r_k^m + c_t$) his optimal behavior is as follows. When $\min(\breve{p}_k^i, \tilde{p}_k^i + c_t, \tilde{p}_k^m + c_t) \leq r_k^i$ ($\min(\breve{p}_k^i, \tilde{p}_k^i + c_t, \tilde{p}_k^m + c_t) \leq r_k^m + c_t$) stop search and buy from the cheapest option. When $\min(\breve{p}_k^i, \tilde{p}_k^i + c_t, \tilde{p}_k^m + c_t) > r_k^i$ ($\min(\breve{p}_k^i, \tilde{p}_k^i + c_t, \tilde{p}_k^m + c_t) > r_k^m + c_t$) search further outside (in) the shopping mall, if possible. If there are no shops left to search outside (in) the shopping mall and $\min(\breve{p}_k^i, \tilde{p}_k^i + c_t, \tilde{p}_k^m + c_t) \leq r_k^m + c_t$ ($\min(\breve{p}_k^i, \tilde{p}_k^i + c_t, \tilde{p}_k^m + c_t) \leq r_k^i$) buy from the cheapest option. If there are no shops left to search outside (in) the shopping mall and $\min(\breve{p}_k^i, \tilde{p}_k^i + c_t, \tilde{p}_k^m + c_t) > r_k^m + c_t$ ($\min(\breve{p}_k^i, \tilde{p}_k^i + c_t, \tilde{p}_k^m + c_t) > r_k^i$) search further in (outside) the shopping mall, if possible.

If there also are no shops left to search in (outside) the shopping mall buy from the cheapest option.

Proof

A complete proof of Propositions B.2 and B.3 is available on request. Here I only give a short sketch of the proof.

Let h denote the number of shops that have not yet been searched, let h^m denote the number of shops in the mall that have not yet been searched and let h^i denote the number of isolated shops that have not yet been searched, with $h = h^m + h^i$. The proof uses several induction arguments. First, it is easy to see that both propositions hold when $h = 0$. A second step is to prove that both propositions hold when $h^m = 1$ and $h^i = 0$. Using a standard induction argument it can then be shown that both propositions also hold for $h^i = 0$ and $h^m > 1$. A third step is to prove that both propositions also hold when $h^m = 0$ and $h^i = 1$. Again using a standard induction argument it can then be shown that both propositions also hold for $h^m = 0$ and $h^i > 1$. These three steps together prove Propositions B.2 and B.3 for some corner cases. These cases together form the basis of one final induction step. This final step shows the following. If the propositions hold for $h = x - 2$ and for $h = x - 1$ then the propositions also hold for $h = x$, with $h^m \geq 1$ and $h^i \geq 1$. Because the propositions hold for $h = 0$ and $h = 1$, they will also hold for $h > 1$, $h^m \geq 1$ and $h^i \geq 1$. Note that steps two and three have already shown that the propositions hold for $h^m = 0$, $h^i > 1$ and for $h^i = 0$, $h^m > 1$.

□

Propositions B.2 and B.3 specify the optimal behavior of non-shoppers when they have searched at least one shop under the conditions $\overline{p}_k^m \leq r_k^m$ and $\overline{p}_k^i \leq r_k^i$. Proposition B.4 specifies the optimal behavior of non-shoppers when they have not yet searched any shop.

Proposition B.4 *Let shops price according to $\overline{p}_k^m \leq r_k^m$ and $\overline{p}_k^i \leq r_k^i$.*
If $r_k^i < r_k^m + c_t$ ($r_k^i > r_k^m + c_t$) non-shoppers prefer to first search an isolated shop (shop in the mall) above first searching a shop in the mall (an isolated shop). If $r_k^i < \theta$ ($r_k^m + c_t < \theta$) all non-shoppers will search, if $r_k^i = \theta$ ($r_k^m + c_t = \theta$) non-shoppers are indifferent between staying at home and searching an isolated shop (shop in the mall) and if $r_k^i > \theta$ ($r_k^m + c_t > \theta$) all non-shoppers prefer to stay at home.
If $r_k^i = r_k^m + c_t$ non-shoppers are indifferent between searching in an isolated shop or in a shop in the mall. When $r_k^i < \theta$ all non-shoppers will search, when $r_k^i = \theta$ non-shoppers are indifferent between searching and staying at home and when $r_k^i > \theta$ all non-shoppers prefer to stay at home.

Proof

First look at the case $r_k^i = r_k^m + c_t$. If a non-shopper would start his search in a shop in the mall he expects to find a price at or below r_k^m and as Proposition B.2 shows the non-shopper thus expects to stop searching after the first search. Expected utility of searching in the mall is

$$U(\text{mall}) = -c_t - c_e + \int_{\underline{p}_k^m}^{r_k^m} (\theta - p) dF_k^m(p),$$

which can be rewritten as

$$U(\text{mall}) = -c_t - c_e + \theta - r_k^m + \int_{\underline{p}_k^m}^{r_k^m} (r_k^m - p) dF_k^m(p) = \theta - r_k^m - c_t.$$

A non-shopper who starts his search in an isolated shop expects to find a price at or below r_k^i and as Proposition B.3 shows he expects to stop searching after the first search. Expected utility is

$$U(\text{isolated}) = -c_t - c_e + \int_{\underline{p}_k^i}^{r_k^i} (\theta - p) dF_k^i(p),$$

which can be rewritten as

$$U(\text{isolated}) = -c_t - c_e + \theta - r_k^i + \int_{\underline{p}_k^i}^{r_k^i} (r_k^i - p) dF_k^i(p) = \theta - r_k^i.$$

Because $r_k^i = r_k^m + c_t$, $U(\text{mall}) = U(\text{isolated})$ and non-shoppers are indifferent between searching in an isolated shop or in a shop in the mall. When $r_k^i < \theta$ $U(\text{isolated}) > 0$ and all non-shoppers will search. When $r_k^i = \theta$ $U(\text{isolated}) = 0$ and non-shoppers are indifferent between searching and staying at home. When $r_k^i > \theta$ $U(\text{isolated}) < 0$ and all non-shoppers prefer to stay at home.

The proof for the cases $r_k^i > r_k^m + c_t$ and $r_k^i < r_k^m + c_t$ follows the same arguments, but is mathematically slightly more complicated because non-shoppers sometimes expect to search twice instead of once. Details are available on request.

□