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## **Assessing the influence of spot price predictability on electricity futures hedging**

### **Abstract**

A common feature of energy prices is that spot price changes are partially predictable due to weather and demand seasonalities. This paper follows the Ederington and Salas (2008) framework and considers the expected change in spot prices when minimum variance hedge ratios are computed. The poor effectiveness of hedging strategies obtained in previous studies on electricity was because the standard hedging approach underestimates the effectiveness of hedging. In the empirical study made in this paper, weekly spot price risk is hedged with weekly futures in the Nord Pool electricity market. In this case, the optimal selection of the futures contract may produce risk reductions whose values vary between 60% and 80% – depending on the hedging duration (one to three weeks) and the analysed sub-period (in-sample and out-of-sample sub-periods).

**Key words:** electricity markets, futures, hedging ratio, and electricity price risk.

## Assessing the influence of spot price predictability on electricity futures hedging

### 1. Introduction

A common feature of energy prices is that spot price changes are partially predictable due to weather and demand seasonalities. The influence of weather variables on electricity load and prices has been studied in the literature by many authors. These variables are especially important at Nord Pool, the largest and most liquid European electricity market.<sup>1</sup>

Recently, Ederington and Salas (2008) have adapted the standard minimum variance hedge ratio approach (Ederington, 1979) to the case where spot price changes are partially predictable. In this context, they show that the riskiness of the spot position is overestimated, the achievable risk reduction underestimated, and more efficient estimates of the hedge ratios are obtained. Ederington and Salas (2008) propose to use the basis (futures price minus the spot price) at the beginning of the hedge as the information variable to approximate the expected spot price change. If futures prices are unbiased predictors of futures spot price, the basis will be a measure of the expected change in the spot price until maturity (Fama & French, 1987). This new approach is followed in this paper, because it is very suitable for hedging electricity price risk at Nord Pool.

Futures contract valuation and its use for risk management are more difficult than usual when dealing with a non-storable commodity, such as electricity. The lack of a cash-and-carry arbitrage mechanism produces a looser relationship between spot and futures prices, especially as futures maturity becomes more distant. In addition, electricity spot price behaviour has some well-known characteristics: jumps, positive skewness, very high volatility, mean-reversion, seasonalities, and heteroscedasticity (see, for example, Koopman *et al.* (2007) for daily frequency data from European markets). Both effects combined produce a lower than usual correlation between spot and futures prices, and might produce a poor performance when hedging spot price risk with futures contracts.

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<sup>1</sup> Engle *et al.* 1992; Peirson & Henley, 1994; Li & Sailor, 1995; Sailor & Muñoz, 1998; Henley & Peirson, 1998; Pardo *et al.* 2002; Koopman *et al.* 2007.

The main criticism of the existing literature in electricity hedging is the poor effectiveness obtained in reducing spot price risk (see Moulton (2005) for California-Oregon-Border and Palo Verde futures traded at NYMEX, and Bystrom (2003) for futures traded at Nord Pool).<sup>2</sup> In Moulton (2005), the underlying spot to the NYMEX futures was the average of peak hour spot prices in a month. In this case, the poor effectiveness of hedging strategies was due to the mismatch between the hedging period of the spot position (one day) – and the underlying settlement period in the futures used as a hedging vehicle (one month). In Bystrom (2003), weekly spot price risk is hedged with weekly futures; but only one-week hedge durations were considered.

To obtain an acceptable performance when electricity futures are used to hedge spot positions, two important and well-known considerations must be kept in mind – and these points are especially relevant in electricity price hedging.

Firstly, the period of the spot position to hedge, and the underlying spot period in the futures contracts chosen for hedging should be identical – or at least similar. This consideration prevents the harmful effects of cross-hedging. The following example may be useful to aid understanding. The use of weekly futures will probably be fairly unsuccessful (monthly futures would be the worst) in hedging daily spot price risk – as the underlying price of weekly futures will be some sort of average of the contained daily spot prices; and so will not cope with day-of-the-week seasonal effects. Furthermore, daily and weekly prices will differ in their display of the typical statistical features of electricity prices.<sup>3</sup>

Secondly, futures positions ought to be held until maturity, or as close to maturity as possible. If futures positions are cancelled early, basis risk will appear and the hedging result will be uncertain.

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<sup>2</sup> The risk reduction obtained in Moulton (2005) varies between -2% and 20%. In Bystrom (2003) the best performing hedge ratio strategy obtains risk reductions that range between 7% and 29% for the whole ‘out-of-sample’ period.

<sup>3</sup> As weekly prices are computed as some kind of average of seven daily prices, the number and size of the spikes will decrease with weekly prices. Consequently, descriptive statistical values will be dampened in weekly prices and lower volatility, kurtosis, and skewness will be expected.

However, hedging performance will be satisfactory if the futures hedge is held as near as possible to the futures maturity.<sup>4</sup>

Taking the above two considerations into account, a direct hedge is the most advisable hedging strategy in this commodity. That is, hedging until maturity and with a perfect match between the futures underlying settlement period, and the period length in which the electricity is going to be bought or sold on the spot market. This desirable perfect match between the spot position to hedge and the futures underlying asset probably explains the wide range of maturities and delivery periods offered in derivatives markets. Nord Pool, for example, trades daily and weekly futures and monthly, quarterly, and yearly forward contracts.

This paper presents empirical results about hedging electricity price risk with futures when an early cancellation of futures positions is made. The empirical study is made with data from one of the oldest and most important deregulated electricity markets in the world, the Nord Pool. Using weekly futures contracts and the weekly spot price for the period 1998 to 2008, several combinations of hedging period lengths (one to three weeks) and ‘times to maturity’ when futures positions are cancelled (one to three weeks) are examined. Results can be summarised in the following points: (i) hedging performance improves as hedging duration increases. That is, two-week hedges perform better than one-week hedges and so on; (ii) hedging strategy performances worsen as ‘time to maturity’ increases when futures positions are cancelled early. For example, those hedges whose futures positions are cancelled two weeks prior to futures settlement perform worse than those whose futures positions are held until one week prior to futures settlement, and so on; (iii) minimum variance hedge ratios are unconditionally estimated with the new approach proposed in Ederington and Salas (2008) and conditionally estimated with the multivariate GARCH model proposed by Kroner and Ng (1998), and known as the Asymmetric Dynamic Covariance model (ADC, hereafter) – but using a bivariate *t*-Student distribution. Results are not conclusive in favour of any method, and consequently it does not seem that improving statistical price modelling

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<sup>4</sup> Typically, basis value variation will be constricted by the maturity effect, as at maturity, futures and spot prices are forced to be equal. This effect means that uncertainty about a futures hedging result decreases as maturity approaches (Hull, 2006; chapter 3).

will guarantee better performance;(iv) it is found that the basis has an important predictive power for explaining spot price changes (between 25% and 50%), consequently, the Ederington and Salas (2008) framework perfectly suits to our experiment and unexpected spot prices changes must be computed using the information contained in the basis; (v) it is shown that very large risk reductions, unprecedented in electricity markets, are achievable by using the new approach proposed in Ederington and Salas (2008) and optimizing the futures contract selection as described above. Specifically, risk reduction values vary between 60% and 80% – depending on the hedging duration (one to three weeks) and the analysed sub-period (in-sample and out-of-sample sub-periods).

This article is divided into seven sections. In section 2, hedging ratios and their effectiveness measure are defined. In section 3, the econometric model used to obtain conditional estimates of hedging ratios is presented. Section 4 contains the data description and some preliminary analysis. Estimation and hedging results are shown in section 5. The paper finishes with conclusions and cited references.

## **2. The Minimum Variance Hedge Ratio**

The conventional minimum variance hedge ratio is defined in a one-period model. At the beginning of the period, or ‘ $t$ ’, an individual is committed to a given position in the spot market. To reduce the risk exposure, the individual may choose to hedge at time ‘ $t$ ’ in the futures market with the same underlying asset. At the end of the period, say, ‘ $t + 1$ ’, the hedger’s result per unit of spot is calculated as follows

$$x_{t+1} = \Delta S(t) - b_t \Delta F(t,T) \quad (1)$$

where  $x_{t+1}$  is the value variation between  $t$  and  $t+1$ ,  $\Delta S(t)=\log(S(t+1)/ S(t))$  is the spot value log variation,  $\Delta F(t,T)=\log(F(t+1,T)/ F(t,T))$  the futures value log variation of a futures contract maturing at  $T$  and  $b_t$  the hedging ratio. If  $b_t$  is positive (negative), short (long) positions are taken in futures. The hedger will choose  $b_t$  to minimize the risk associated with the random result  $x_{t+1}$ . A standard way to measure risk in economics is by the variance conditional on the available information. The risk of a hedge strategy is calculated as the variance of  $x_{t+1}$ ,

$$VAR[x_{t+1}|\psi_t] = VAR[\Delta S(t) - b_t \Delta F(t,T)|\psi_t] \quad (2)$$

The most used definition for the optimal hedge ratio<sup>5</sup> is the Minimum Variance Hedge Ratio that can be obtained by minimizing equation (2)

$$b_t = \frac{cov(\Delta S(t), \Delta F(t,T)|\psi_t)}{var(\Delta F(t,T)|\psi_t)} \quad (3)$$

where second moments are conditioned to the information set available at the beginning of the hedging period,  $\psi_t$ . When an unconditional probability distribution is used, the hedge ratio in equation (3) can be estimated from a linear relationship between spot and futures returns. That is, estimating by OLS the linear relationship appearing in equation (1), but adding an intercept and white noise

$$\Delta S(t) = a + b\Delta F(t,T) + \varepsilon(t) \quad (4)$$

In this case, the OLS estimator of  $b$  is the unconditional definition of the optimal hedge ratio appearing in equation (3) (Ederington, 1979).

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<sup>5</sup> For an excellent revision on futures hedging see Lien and Tse (2002).



Recently, Ederington and Salas (2008) have adapted the above approach to the case where spot price changes are partially predictable and futures prices are unbiased estimators of future spot prices. In this context, they show that the riskiness of the spot position is overestimated and the achievable risk reduction underestimated. Under this new approach, the unexpected result of the hedge in equation (1) can be reformulated as

$$x_{t+l} = (\Delta S(t) - E[\Delta S(t) | \psi_t]) - b'_t \Delta F(t, T) \quad (5)$$

The risk of the hedge strategy in equation (2) is reformulated as

$$VAR[x_{t+l} | \psi_t] = VAR[(\Delta S(t) - E[\Delta S(t) | \psi_t]) - b'_t \Delta F(t, T) | \psi_t] \quad (6)$$

and the Minimum Variance Hedge Ratio obtained after minimizing equation (6) is

$$b'_t = \frac{cov((\Delta S(t) - E[\Delta S(t) | \psi_t]), \Delta F(t, T) | \psi_t)}{var(\Delta F(t, T) | \psi_t)} \quad (7)$$

Ederington and Salas (2008) propose to use the basis (futures price minus the spot price) at the beginning of the hedge as the information variable to approximate the expected spot price change. If futures prices are unbiased predictors of futures spot price, the basis will be a measure of the expected change in the spot price until maturity (Fama & French, 1987). An unconditional estimate of the hedge ratio in equation (7) can be obtained by estimating the following linear regression using OLS

$$\Delta S(t) = a' + b' \Delta F(t, T) + \lambda(\log(F(t, T)/S(t))) + \varepsilon'(t) \quad (8)$$

where  $\lambda(\log(F(t,T)/S(t)))$  is used to estimate  $E[\Delta S(t)|\psi_t]$ . Ederington and Salas (2008) show that OLS estimation of equation (8) obtains an unbiased and more efficient estimation of the unconditional minimum variance hedge ratio ( $b'$ ) than that obtained by using equation (4). This is providing that the expected change in the spot price is perfectly approximated with the product between the basis at the beginning of the hedge – and its estimated coefficient (namely  $\hat{\lambda}(\log(F(t,T)/S(t))) = E[\Delta S(t)|\psi_t]$ ).

### Measuring hedging effectiveness

The risk reduction is computed to compare the hedging effectiveness of each strategy. Furthermore, *ex post* and *ex ante* results are distinguished by splitting the data sample into two parts. In the first part, the hedging strategies are compared *ex post*, whereas in the second part, an *ex ante* approach is used. That is, in the *ex ante* study, strategies are compared using forecasted hedge ratios, and models are estimated every time a new observation is considered. The variance of a hedge strategy is calculated as the variance of the hedged portfolio – as equation (6) shows. In this equation, the OLS estimated approximation of the expected spot price change using the basis is introduced ( $\hat{\lambda}(\log(F(t,T)/S(t))) = E[\Delta S(t)|\psi_t]$ ). The risk reduction achieved for each strategy is computed by comparison with the variance of the spot position ( $b_t = 0$  for all  $t$  in equation (6)).

In the empirical application presented in sections 4 and 5, futures with different maturities ( $F(t,T_i)$  with  $i = 1, 2, 3$  and  $4$ ; and  $T_i = t + i$ ) are considered to hedge the spot price variation. Furthermore, three hedging lengths are considered: one, two, and three weeks. Table 1 shows the six types of hedges carried out in this paper, one per row. This typology enables a study of the influence of the hedging length, and the ‘time to maturity’ effect on hedging performance (first and second columns, respectively in Table 1). The ‘time to maturity’ is computed as the time remaining to futures maturity when the hedge is finished. The ‘time to maturity’ is one (two or three) week(s) in those hedges finished one (two or three) week(s) prior to maturity. It is expected that hedging

performance improves as hedging length increases and time to maturity decreases.<sup>6</sup> The third and fourth columns contain the spot and futures price variations implied in each hedging operation. Finally, the last column in Table 1 reports the basis used to approximate the expected spot price change in equations (6) and (8). It is important to note that only one basis is used per hedging period. This practice enables a comparison to be made of the hedging effectiveness of different futures contracts for the same hedging period – as the variance of the spot position to hedge is the same ( $b_t = 0$  for all  $t$  in equation (6)).<sup>7</sup>

In the empirical application in section 5, four hedging strategies are compared. The hedging ratio obtained after estimating equation (4) is labelled ‘OLS without basis’ – and the hedging ratio obtained after estimating equation (8) is identified as ‘OLS with basis’. In the following section, a conditional covariance model enables the estimation of the hedging ratio appearing in equation (7). This hedging strategy is identified as ‘ADC’. Hedging analysis is completed with the ‘Naive’ hedging ratios, that is, a hedge where futures positions have the same size, but the opposite sign than the position held in the spot market (*i.e.*  $b_t = 1$  for all  $t$ ).

[Insert Table 1 about here]

### 3. The econometric framework

One of the objectives of this paper is to compare the hedging effectiveness of conditional and unconditional minimum variance hedge ratio estimates. To obtain conditional estimates of the second moments, a two-step estimation procedure is followed. Firstly, a model in means is

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<sup>6</sup> Lindahl, 1992.

<sup>7</sup> The unhedged spot price risk will be measured as  $VAR[\Delta^k S(t) - \hat{\lambda}(\log(F(t, T_k)/S(t)))]$  after estimating  $\lambda$  by OLS from the adapted equation (8):  $\Delta^k S(t) = a + b' \Delta^k F(t, T_i) + \lambda(F(t, T_k) - S(t)) + \varepsilon'(t)$  for  $k=1, 2$  and  $3$ ,  $i = 1, 2, 3$  and  $4$ ; and  $i > k$ . In the *ex ante* study, the unhedged spot price risk measure is computed by repeating this procedure each time a new observation is considered, obtaining a vector of  $\lambda$  coefficients that is as large as the out-of-sample period.

estimated and then the residuals of this model are taken in the second step as an input to model the conditional variance. To clean up any autocorrelation behaviour, a vector autoregressive regression model (VAR) is estimated in the first step. The model for the means is

$$\begin{aligned}\Delta^k S(t) &= \gamma_1 + \gamma_{10} \log(F(t, T_k)/S(t)) + \sum_{\tau=1}^p \gamma_{11\tau} \Delta^k S(t-\tau) + \sum_{\tau=1}^p \gamma_{12\tau} \Delta^k F(t-\tau, T_i) + \varepsilon_{1,t+k} \\ \Delta^k F(t+\tau, T_i) &= \gamma_2 + \gamma_{20} \log(F(t, T_k)/S(t)) + \sum_{\tau=1}^p \gamma_{21\tau} \Delta^k S(t-\tau) + \sum_{\tau=1}^p \gamma_{22\tau} \Delta^k F(t-\tau, T_i) + \varepsilon_{2,t+k}\end{aligned}\quad (9)$$

where  $\Delta^k S(t) = \log(S(t+k)/S(t))$  with  $k = 1, 2, \text{ and } 3$ ;  $\Delta^k F(t, T_i) = \log(F(t+k, T_i)/F(t, T_i))$  with  $T_i = t+i$ ;  $k = 1, 2, \text{ and } 3$  and  $i = 1, 2, 3, \text{ and } 4$  and  $k < i$ ; represents the  $k$  log differences in futures prices when ‘ $i$ ’ periods remain to ‘delivery’ or settlement (note that  $F(t+k, T_i) = S(t+k)$  when  $k = i$ ); the gammas are the parameters to estimate,  $p$  is the lag of the VAR and is chosen by minimizing the Akaike information criteria, so eliminating any autocorrelation patterns. The VAR model is estimated by OLS (Engle & Granger, 1987). The vector of residuals,  $\varepsilon_{t+k} = (\varepsilon_{1t+k}, \varepsilon_{2t+k})'$ , are saved and used as observable data to estimate multivariate GARCH models. This two-step procedure (Kroner & Ng, 1998; Engle & Ng, 1993) reduces the number of parameters to estimate in the second step, decreases the estimation error, and enables a faster convergence in the estimation procedure. In the VAR model in equation (9), the basis described in the last column of Table 1 appears as an external variable. The basis can be seen as an error correction term when spot and futures prices are cointegrated, but this is not the case (Viswanath, 1993; Lien, 1996). The inclusion of the basis in the VAR specification implies an efficient conditional estimation of the minimum variance hedge ratio (see equation (7)) as it contains important information for anticipating spot price changes.

The number of published papers modelling conditional covariance is quite small compared to the enormous bibliography on time-varying volatility. The three most widely used models are: (1) the VECH model proposed by Bollerslev *et al.* (1988); (2) the constant correlation model, CCORR, proposed by Bollerslev (1990) and; (3) the BEKK model of Engle and Kroner (1995).

Each model imposes different restrictions on the conditional covariance and can lead to substantially different conclusions in any application that involves forecasting conditional covariance matrices. Recently, Kroner and Ng (1998) derived another multivariate GARCH model, the Asymmetric Dynamic Covariance Matrix model, ADC. This model encompasses the above models in the sense that, under certain restrictions, any particular model can be obtained.<sup>8</sup> This is a good framework to compare the existing models, and the significance of the restrictions imposed by each. Kroner and Ng (1998) introduce asymmetries following the Glosten *et al.* (1993) approach. This is the most common method for introducing asymmetries in multivariate GARCH modelling (Gagnon & Lypny, 1995; Hendry & Sharma, 1999; Bekaert & Wu 2000).

Kroner and Ng (1998) adopt a structured approach, similar to Hentschel (1995). They introduce a General Dynamic Covariance (GDC) matrix model nesting the existing models. This model can be generalized to include the asymmetric effects, the ADC. Under this framework, model selection is made easier by testing restrictions on the ADC. Kroner and Ng (1998) proved that with certain restrictions in the ADC model, the other models discussed above could be derived. The bivariate ADC and the restrictions imposed to obtain the other models can be written as

$$\begin{aligned}
\begin{bmatrix} h_{11,t+k} & h_{12,t+k} \\ \cdot & h_{22,t+k} \end{bmatrix} &= \begin{bmatrix} \sqrt{\theta_{11,t+k}} & 0 \\ 0 & \sqrt{\theta_{22,t+k}} \end{bmatrix} \begin{bmatrix} 1 & \rho_{12} \\ \rho_{12} & 1 \end{bmatrix} \begin{bmatrix} \sqrt{\theta_{11,t+k}} & 0 \\ 0 & \sqrt{\theta_{22,t+k}} \end{bmatrix} \\
&+ \begin{bmatrix} 0 & \phi_{12} \\ \phi_{12} & 0 \end{bmatrix} \circ \begin{bmatrix} \theta_{11,t+k} & \theta_{12,t+k} \\ \cdot & \theta_{22,t+k} \end{bmatrix} \\
&= \begin{bmatrix} \theta_{11,t+k} & \phi_{12}\theta_{12,t+k} + \rho_{12}\sqrt{\theta_{11,t+k}}\sqrt{\theta_{22,t+k}} \\ \cdot & \theta_{22,t+k} \end{bmatrix}
\end{aligned} \tag{10}$$

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<sup>8</sup> Myers (1991) and Baillie and Myers (1991) have used the VECH specification, without the asymmetric extension, in spot-futures covariance modelling for hedging purposes for various agricultural commodities. The CCORR model has been often used for modelling spot-futures covariance dynamics. Some examples are Cecchetti *et al.* (1988) in public debt; Kroner and Sultan (1993) in currencies; and Park and Switzer (1995) in stock indexes. The BEKK model has been used in Baillie and Myers (1991) (without asymmetries), and Gagnon and Lypny (1995), in modelling spot-futures covariance for agricultural commodities and interest rates, respectively. Finally, the ADC model has been used by Meneu and Torró (2003) to estimate conditional hedge ratios in stock indexes, and obtains the best *ex ante* performance when compared to the above-mentioned conditional specifications.

where

$$\begin{aligned} \begin{bmatrix} \theta_{11,t+k} & \theta_{12,t+k} \\ \cdot & \theta_{22,t+k} \end{bmatrix} &= \begin{bmatrix} \omega_{11} & \omega_{12} \\ \cdot & \omega_{22} \end{bmatrix}' \begin{bmatrix} \omega_{11} & \omega_{12} \\ \cdot & \omega_{22} \end{bmatrix} + \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix}' \begin{bmatrix} h_{11,t} & h_{12,t} \\ \cdot & h_{22,t} \end{bmatrix} \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix} \\ &+ \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}' \begin{bmatrix} \varepsilon_{1,t}^2 & \varepsilon_{1,t}\varepsilon_{2,t} \\ \cdot & \varepsilon_{2,t}^2 \end{bmatrix} \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} + \begin{bmatrix} g_{11} & g_{12} \\ g_{21} & g_{22} \end{bmatrix}' \begin{bmatrix} \eta_{1,t}^2 & \eta_{1,t}\eta_{2,t} \\ \cdot & \eta_{2,t}^2 \end{bmatrix} \begin{bmatrix} g_{11} & g_{12} \\ g_{21} & g_{22} \end{bmatrix} \end{aligned}$$

and  $\circ$  is the Hadamard product operator (element-by-element matrix multiplication) and  $\omega_{ij}$ ,  $b_{ij}$ ,  $a_{ij}$ , and  $g_{ij}$  for all  $i, j = 1, 2$  are parameters,  $\varepsilon_{1t}$  and  $\varepsilon_{2t}$  are the unexpected shock series obtained from equation (1).  $\eta_{1t} = \max [0, -\varepsilon_{1t}]$  and  $\eta_{2t} = \max [0, -\varepsilon_{2t}]$  are the Glosten *et al* (1993) dummy series collecting a negative asymmetry from the shocks, and  $h_{ijt}$  for all  $i, j = 1, 2$  are the conditional second moment series. The specification test proposed by Kroner and Ng (1998) is as follows: (1) If  $\rho_{12} = b_{12} = b_{21} = a_{12} = a_{21} = g_{12} = g_{21} = 0$ , a restricted asymmetric VECH is obtained with the conditional covariance equation having coefficients  $b_{11}$   $b_{22}$ ,  $a_{11}$   $a_{22}$  and  $g_{11}$   $g_{22}$ ; (2) if  $\phi_{12} = b_{12} = b_{21} = a_{12} = a_{21} = g_{12} = g_{21} = 0$ , the asymmetric CCORR model is derived; (3) if  $\phi_{12} = 1$  and  $\rho_{12} = 0$  the asymmetric BEKK model is obtained.

Normality assumption is not a realistic assumption for log-price variation in electricity. One empirical fact that characterises electricity price distribution is its leptokurtosis due to the presence of many extreme values. For this reason, a natural alternative to normality is the bivariate  $t$ -Student<sup>9</sup>

$$\varepsilon_{t+k} | \mathcal{W}_t \sim t(0, H_{t+k}, \nu) \quad (11)$$

Parameter estimation is carried out by maximizing the sample log-likelihood function  $L_N(\beta, \nu)$  for  $N$  observations, with respect to the vector of parameters ( $\beta$ ) in equation (5) and the

<sup>9</sup> Koopman et al. (2007) use this distribution in the univariate case for daily electricity spot prices.

degrees of freedom ( $\nu$ ) parameter of a conditional bivariate  $t$ -Student distribution. That is, by maximizing

$$L_N(\beta, \nu) = \sum_{t=1}^N \log f(\varepsilon_{t+k} | \beta, \nu, \psi_t) \quad (12)$$

where

$$f(\varepsilon_{t+k} | \beta, \nu, \psi_t) = |H_{t+k}|^{-1/2} \frac{\Gamma\left(\frac{\nu+2}{2}\right)}{\Gamma\left(\frac{\nu}{2}\right) [\pi(\nu-2)]} \left[ 1 + \frac{(H_{t+k})^{-1/2} \varepsilon_{t+k}}{\nu-2} \right]^{-\frac{\nu+2}{2}}$$

and  $\Gamma(\cdot)$  is the Gamma function. The degrees of freedom parameter,  $\nu$ , must be positive and it is convenient to assume that  $\nu > 2$ . When  $\nu$  tends to infinity, the Student density tends to the normal density (see Bauwens *et al.* (2006) for more details). The standard errors and their associated critical significance levels are calculated using the quasi-maximum likelihood method of Bollerslev and Wooldridge (1992) – and which is robust to the non-normality assumption.

#### 4. Data and preliminary analysis

Electricity futures prices and spot prices are directly obtained from Nord Pool's FTP server files. In the spot market, hourly power contracts are traded daily for physical delivery in the next 24-hour period. This price is known as the system price and it is computed and published at midday the day before delivery. The system price is the spot reference for derivative contracts traded at the Nord Pool market and those contracts traded OTC – but settled by Nord Pool clearing services. There is a wide range of electricity derivative contracts (forward, futures, and options) traded at the Nord Pool exchange. At the moment, the most important are daily and weekly futures; monthly, quarterly and yearly forwards; and European type options on the quarter and year forwards.

To select which futures/forward contracts can be included in this study two important considerations are necessary: (i) firstly, a large number of observations are required to obtain insightful results; (ii) secondly, non-overlapping futures contracts are preferable in order to avoid artificially introducing autocorrelation in the data series. Therefore, it is necessary to balance the data frequency and delivery period length of the contracts to avoid introducing autocorrelation in the data series. For example, if yearly forwards are selected, no more than one price per year can be introduced; otherwise, expectations on the underlying commodity cannot be completely renewed. As a result, well-designed data series of yearly forward prices contain very few observations and no significant study can be carried out. Similar reasons can be argued for quarterly and monthly forward contracts. Therefore, the present study focuses on weekly futures (i.e. futures with delivery periods of one week), taking one price per week, with a closing price each Friday, or the day before if non-tradable.

Futures prices in the Nord Pool database started to be collected at the end of 1995. Important changes in the contractual conditions and trading system were introduced in 1996 and 1997. Electronic trading was initiated at the end of 1996 and contracts with delivery periods longer than a week were changed from futures to forwards by the end of 1997. These changes were important enough to preclude the present study from using these years, and they were used instead as a learning period. As a result, the data period analysed is from January 1, 1998, until December 28, 2008; that is, 574 weeks.<sup>10</sup> During the sample period, eight weekly futures contracts could be traded daily, but only the four contracts nearest to the delivery period are free from non-trading problems. With the four contracts nearest to delivery weekly futures contracts, four data series of futures prices are built by maintaining the time to delivery constant.

In Nord Pool, settlement of futures contracts involves both daily mark-to-market settlement and a final cash settlement for those positions remaining open at maturity. Final settlement covers

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<sup>10</sup> Futures maturing after December 31, 2005 are quoted in euros. In order to have a homogenous currency, those futures prices maturing in 2006 and 2007 are expressed in Norwegian kroner, or NOKs, by using the exchange rate appearing in Nord Pool files. All of the empirical study was repeated for the data sample finishing on December 31, 2005, and identical conclusions were obtained.



the difference between the last closing price of the futures contract and the system price in the ‘delivery period’. The system price is the hourly spot reference of the physical market. Consequently, in weekly futures contracts, the clearing spot reference is the average of the 168 system prices (24 hours  $\times$  7 days) of the week, Monday to Sunday of the ‘delivering’ week.<sup>11</sup> This is the spot reference used in this paper. Figure 1 exhibits this time series jointly with each of the above presented futures price time series.

[Insert Figure 1 about here]

Futures prices are taken on Fridays, or the day before if non-tradable. This point might be very important when the effectiveness of several hedging strategies is compared, especially for the electricity case. As futures closing prices are computed at 15:30 and only one price is used each week for the preparation of the weekly spot price time series, an acceptable synchronization between spot and futures prices computation time is achieved when the closing price of the last trading day of the week is used.<sup>12</sup> Furthermore, if another futures price is used, for example, the Wednesday closing price, the maturity effect in the analysed hedging strategies would not be measured as exactly as if the last trading day of the week was used.

A preliminary analysis follows. Table 2 displays the basic statistics of spot and futures log price differences. Mean values deserve the first important comment. Whereas spot mean values are not significantly different from zero, futures means are negative and significantly different from zero in six out of seven cases. Specifically, the mean values of  $\Delta^k F(t, T_i)$  in the  $i=k$  case, take values varying between -1.53% and -5.40%. In the classical view of hedging pressure as a determinant of futures premiums (also known as a *forward bias* or *forward premium*), when a significant declining pattern is found in futures prices (futures prices above expected spot prices) it would be said that the

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<sup>11</sup> Each year, there is a week in spring with 167 hours and a week in autumn with 169 hours because of daylight saving time.

<sup>12</sup> The weekly spot price is known at Saturday midday. More specifically, Monday to Saturday system prices of each week will be already known at midday Friday. Nevertheless, to compute the weekly spot price, the Sunday system prices remain, but these prices are not published until Saturday midday.

futures market is in *contango* (long hedging pressure). The Kruskal-Wallis test contrasts the null of median equality between spot and futures time series. Results show that the null is rejected in six out of seven cases.

Table 2 also displays the standard deviation of the analysed series. A pair-wise comparison between spot and futures standard deviation shows that the former is always higher. The Levene test contrasts the null of variance equality between spot and futures differenced series. Results show that the null is rejected in all the cases at 5% of significance level.

The four spot time series analysed in Table 2 do not display significant skewness, whereas futures time series have significant skewness in all cases. Furthermore, when futures differences imply final settlement at maturity, that is when  $i=k$ , skewness is negative; but the skewness is positive in the remaining reported cases. The kurtosis results indicate that all the time series appearing in Table 2 have significant excess kurtosis. In accordance with the above results, normality distribution hypothesis is clearly rejected in all cases. Maximum and minimum values help to explain the above results, especially the high kurtosis. Finally, the Ljung-Box test with twenty lags detects significant autocorrelation and heteroscedasticity.

The statistical behaviour of futures and spot log differences have some significant discrepancies that might be critical obstacles to overcome in order to design a successful hedging strategy. The two most insightful results are that futures have a declining pattern as maturity approaches, and that spot prices are more volatile than futures prices. This disparity produces a lower correlation than usual for linking futures and their underlying spots. This correlation appears in Table 3 and varies between 0.44 and 0.80. The highest correlation between spot and futures is obtained for those futures positions held until maturity, and steeply decreases as futures cancellation dates are increasing far from maturity dates.

[Insert Table 2 about here]

[Insert Table 3 about here]

Obviously, these results are important for electricity price risk management, as they show that only those futures contract positions held until maturity ensure a good risk reduction for hedgers. If futures positions are cancelled before, then the statistical differences between spot and futures prices will probably cause poor performance. These ideas are corroborated in the following section.

## 5. Results

In Table 5 some evidence of the predictive ability of the basis for the spot and futures price changes is presented. As this table shows, the basis has an important predictive power for explaining unexpected spot price changes (between 25% and 50%). However, the basis has little, or no ability to forecast futures price changes (between -0.16% and 1.16%). These results perfectly coincide with the Ederington and Salas (2008) approach where spot price changes are partially predictable; but futures prices are martingale.

The estimation of the conditional covariance model (see equations (10) and (11)) is carried out by maximizing the sample log-likelihood function (see equation (12)). The estimation output is reported in Table 4. It is interesting to look at the last row of Panel (A) where the estimated values of the degrees of freedom parameter ( $\nu$ ) are displayed. This parameter is significantly different from zero in the six estimated models and takes values varying between 4.56 and 6.83; consequently, the assumed distribution is supported by data. Looking at the results appearing in Panel (B), it can be said that the estimated ADC models cannot be reduced to any of the nested models.<sup>13</sup>

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<sup>13</sup> The standardized residuals of the model show no evidence of autocorrelation and heteroscedasticity. Results are omitted to maintain space.

Figures 2 and 3 display, respectively, the estimated conditional second moment and hedging ratios. It can be seen from Figure 3 that conditional hedging ratio values move around their unconditional values and, consequently, their performance is expected to be quite similar.<sup>14</sup>

[Insert Table 4 about here]

[Insert Figure 2 about here]

[Insert Figure 3 about here]

Table 6 displays the variance reduction of the different hedging methods.<sup>15</sup> This table contains three panels (A, B, and C) each corresponding to a different hedging length period (one to three weeks). The middle column in each panel reports *in-sample* results for the period December 29, 1997 to October 5, 2003 (300 weeks).<sup>16</sup> The last column in each panel reports *out-of-sample* results for the period October 6, 2003 to December 28, 2008 (274 weeks).<sup>17</sup> Results can be

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<sup>14</sup> The average values of the OLS minimum variance hedge ratios appearing in Figures 3(a) to 3(f) are respectively 0.65, 0.65, 0.68, 0.87, 0.90 and 0.93. This hedge ratio follows the classical pattern in the literature: as duration of the hedge increases, the hedging ratio and its performance increases (Lindahl, 1992). It is interesting to note that a perfect hedge is possible when hedges are held until maturity and a naive hedge is adopted. Adapting equation (8) to the notation in Table 1, if futures are held until maturity, a perfect hedge will be obtained as the relationship  $\Delta^k S(t) = a' + b' \Delta^k F(t, T_i) + \lambda(\log(F(t, T_i)/S(t)))$  becomes an identity for  $i=k$ ,  $F(t+k, T_i)=S(t+k)$  with  $a'=0$ ,  $b'=1$  and  $\lambda=1$ . This simple explanation is not possible using the equation (4) of the standard model. Furthermore, hedging effectiveness will be far below the expected 100% when spot price changes are partially predictable.

<sup>15</sup> Transaction costs are not considered when comparing hedging methods as the hedging theoretical framework is a one-period model for all hedging methods. Within this framework, the individual (see section 2) must take futures positions at the beginning of the period and cancel them at the end of the period. As hedging ratio values are quite similar in the three considered methods, the three methods will have similar transaction costs. The average trading fees for an additional trade on the Nord Pool futures has been less than 0.1% of the underlying asset value for the considered period. In December 2008, transaction costs represent approximately 0.007% of the underlying asset value.

<sup>16</sup> The autumn of 2002 was a dry season that pushed the hydro reservoirs into a sharp reduction (54% of average inflow for the preceding 20 years). In the late autumn and winter of the period 2002-2003 spot prices registered a very high level (twice to three times the normal level, with 850 NOK/MWh in January 2003). Further to the severe drought suffered, other factors could be important for such price behaviour, see von der Fehr et al. (2005) for more details. In order to split the total sample in two sub-periods, it was preferred to include the turmoil period in the first sub-period where an *ex post* view is adopted. By doing this, the second sub-period, where an *ex ante* view is adopted, will be free of such unusual price behaviour. Additionally, Lucia and Torró (2008) results support the view that circumstances changed in the Nord Pool market after the shock period.

<sup>17</sup> Results on Table 6 were repeated for the standard approach and similar conclusions to Ederington and Salas (2008) were obtained. First, when the unhedged spot variance is computed as  $VAR[\Delta^k S(t)]$  instead of  $VAR[\Delta^k S(t) - \hat{\lambda}(\log(F(t, T_k)/S(t)))]$ , the riskiness of the unhedged positions will be overestimated by 60% on average. Second, the estimates of the percentage risk reduction in the standard approach measured as:  $100 \times (1 - VAR[\Delta^k S(t) - b_i \Delta^k F(t, T_i)] / VAR[\Delta^k S(t)])$ , underestimate the risk reduction obtained in Table 6 by about 30% on average. The risk reduction obtained within the standard approach is below 50% in most cases. It is interesting to note that obtaining risk reductions below 50% is quite common when futures hedging is carried out on commodities and

summarised in the following points: (i) hedging performance improves as hedging duration increases. That is, two-week hedges perform better than one-week hedges and so on. (ii) Hedging strategy performances worsen as ‘time to maturity’ increases when futures positions are cancelled early. That is, those hedges whose futures positions are cancelled two weeks prior to futures settlement have worse performances than those whose futures positions are held until one week prior to futures settlement, and so on. (iii) Differences in the risk reduction obtained by OLS methods (with and without the basis) are lower than 1% and inconclusive. Consequently, a more efficient hedge ratio estimate will not imply an improvement in the performance of the hedging strategy. (iv) Finally, when OLS and ADC hedge ratio performances are compared, results are again inconclusive in favour of any method as differences are quite small between both strategies. This result implies that the better statistical performance of the ADC model does not imply a better hedging strategy performance. Furthermore, the Naive strategy obtains a similar performance to the above remaining strategies in those hedges with durations of two and three weeks; but in the one-week hedges, the Naive strategy clearly obtains the worst score.

[Insert Table 5 about here]

## 6. Conclusions

This paper follows the Ederington and Salas (2008) framework when considering the expected change in spot prices when minimum variance hedge ratios are computed. The use of this new approach enables a significant improvement on the poor effectiveness measures of hedging strategies obtained in previous studies on electricity (Bystrom, 2003; Moulton 2005). Specifically, previous studies have overestimated the unexpected shocks in spot prices as a large part these

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the standard approach is used. This is especially true for non-storable commodities (Carter, 1999; section 3.2). Finally, Newey-West standard errors of the hedge ratios estimated using equation (8) are 25% lower on average than those obtained after using equation (4). Consequently, the introduction of the basis in the model allows more efficient minimum variance hedge ratio estimates. All these results are not reported to save space, but are available on request.

shocks (between 25% and 50%) can be anticipated using the information contained in the basis. Consequently the riskiness of the spot position in previous studies was overestimated and the achievable risk reduction underestimated. This poor effectiveness was also due to the special statistical features of electricity prices. There are two facts in electricity markets that explain the difficulty in obtaining a good performance when hedging spot price risk with futures contracts. Firstly, the no-storability property of electricity avoids the cash-and-carry connection between spot and futures, and so simultaneous spot and futures price liaison are less tight than is usual between futures and their underlying assets. Secondly, the special statistical features of electricity prices, specifically their high volatility and kurtosis. Both effects combined produce a low correlation between spot and futures prices and, consequently, a poor performance of hedging strategies can be expected.

Further to the use of the new approach proposed by Ederington and Salas (2008), the empirical study carried out reveals that hedging performance can be significantly improved by increasing hedging duration and maintaining futures positions as near as possible to their final settlement. In this paper, weekly spot price risk is hedged with weekly futures, so the underlying asset in the futures contract and the asset in the spot position are practically identical. This identification is almost completed when futures positions are held as close as possible to their maturity. In this case, risk reduction attains its maximum values and better results are obtained by increasing the hedge duration. In this case, depending on the hedging duration (one to three weeks), and the analysed sub-period (in-sample and out-of-sample sub-periods), risk reduction attains values of between 60% and 80%.

Finally, minimum variance hedge ratios that take into account the fact that spot price changes are partially predictable offer a similar performance to the unconditional version based on simple linear regressions, and the conditional version based on multivariate GARCH models. Consequently, it does not seem that improving statistical price modelling guarantees better performance.

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**Table 1. Type of hedges.**

This table displays the type of hedges and helps clarify the notation. Spot log variations are computed as  $\Delta^k S(t) = \log(S(t+k)/S(t))$  with  $k = 1, 2$  and  $3$ ; and represent the Nord Pool System Price log variation in  $k$  weeks, where the weekly system price ( $S(t)$ ) is computed as the average price from Monday to Sunday of the total weekly hours (24 hours per 7 days).  $\Delta^k F(t, T_i) = \log(F(t+k, T_i)/F(t, T_i))$  with  $T_i = t+i$ ;  $i = 1, 2, 3$ , and  $4$ ;  $k = 1, 2$  and  $3$ ; and  $k < i$ , represent the  $k$  weeks log variation in the weekly futures closing prices – ‘ $i$ ’ weeks remaining to ‘delivery’ traded at Nord Pool on the last trading day of the week  $t$ . Note that  $F(t+k, T_i) = S(t+k)$  when  $k = i$ . ‘Duration’ column reports the number of weeks in each hedging period. ‘Time to maturity’ column is computed as ‘ $i$ ’ minus ‘ $k$ ’ and represents the time remaining to futures maturity when the hedge is finished. Last column reports the basis used to approximate the expected spot price change in equation (8).

Duration ( $k$ weeks)	Time to maturity ( $i-k$ weeks)	Spot log variation $\Delta^k S(t)$	Futures log variation $\Delta^k F(t, T_i)$	Basis approximating $E[\Delta^k S(t) \psi_t]$
1	1	$\Delta S(t) = \log(S(t+1)/S(t))$	$\Delta F(t, T_2) = \log(F(t+1, T_2)/F(t, T_2))$	$\log(F(t, T_1)/S(t))$
1	2	$\Delta S(t) = \log(S(t+1)/S(t))$	$\Delta F(t, T_3) = \log(F(t+1, T_3)/F(t, T_3))$	$\log(F(t, T_1)/S(t))$
1	3	$\Delta S(t) = \log(S(t+1)/S(t))$	$\Delta F(t, T_4) = \log(F(t+1, T_4)/F(t, T_4))$	$\log(F(t, T_1)/S(t))$
2	1	$\Delta^2 S(t) = \log(S(t+2)/S(t))$	$\Delta^2 F(t, T_3) = \log(F(t+2, T_3)/F(t, T_3))$	$\log(F(t, T_2)/S(t))$
2	2	$\Delta^2 S(t) = \log(S(t+2)/S(t))$	$\Delta^2 F(t, T_4) = \log(F(t+2, T_4)/F(t, T_4))$	$\log(F(t, T_2)/S(t))$
3	1	$\Delta^3 S(t) = \log(S(t+3)/S(t))$	$\Delta^3 F(t, T_4) = \log(F(t+3, T_4)/F(t, T_4))$	$\log(F(t, T_3)/S(t))$

**Table 2**

The variables appearing in the heading of each column are described in Table 1. The *Kruskal-Wallis* and *Levene* statistics test median and variance equality, respectively, between  $\Delta^k S(t)$  and  $\Delta^k F(t, T_i)$  for  $k=1$  in Panel (A) and  $k=i$  and  $i=2, 3$  and  $4$  in Panels (B), (C) and (D), respectively. *Skewness* means the skewness coefficient and has the asymptotic distribution  $N(0, 6/T)$  under normality, where  $T$  is the sample size. The null hypothesis tests whether the skewness coefficient is equal to zero. *Kurtosis* means the excess kurtosis coefficient and it has an asymptotic distribution of  $N(0, 24/T)$  under normality. The hypothesis tests whether the kurtosis coefficient is equal to zero. The *Jarque-Bera* statistic tests for the normal distribution hypothesis. The Jarque-Bera statistic is calculated as  $T[\text{Skewness}^2/6 + (\text{Kurtosis})^2/24]$ . The Jarque-Bera statistic has an asymptotic  $\chi^2(2)$  distribution under the normal distribution hypothesis.  $Q(20)$  and  $Q^2(20)$  are Ljung Box tests for twentieth order serial correlation in the differentiated and its squared series, respectively. Marginal significance levels of the statistical tests are displayed as [.]

Panel (A): One week log variations										
	$\Delta S(t)$		$\Delta F(t, T_1)$		$\Delta F(t, T_2)$		$\Delta F(t, T_3)$		$\Delta F(t, T_4)$	
<i>Mean</i> × 100	0.16	[0.74]	-1.53	[0.00]	-1.96	[0.00]	-1.23	[0.00]	-0.60	[0.06]
<i>Median</i> × 100	0.16		-1.43		-1.36		-0.94		-0.38	
<i>Kruskal-Wallis</i>			11.42	[0.00]	15.60	[0.00]	7.34	[0.00]	2.48	[0.11]
<i>S. D.</i>	0.11		0.08		0.10		0.09		0.08	
<i>Levene</i>			34.95	[0.00]	4.86	[0.02]	11.92	[0.00]	31.53	[0.00]
<i>Skewness</i>	-0.15	[0.14]	-0.50	[0.00]	0.48	[0.00]	0.46	[0.00]	0.41	[0.00]
<i>Kurtosis</i>	3.49	[0.00]	4.09	[0.00]	6.80	[0.00]	5.55	[0.00]	5.92	[0.00]
<i>Jarque-Bera</i>	286.86	[0.00]	424.79	[0.00]	1129.33	[0.00]	755.69	[0.00]	853.40	[0.00]
<i>Minimum</i>	-0.50		-0.43		-0.47		-0.41		-0.42	
<i>Maximum</i>	0.52		0.31		0.71		0.62		0.55	
$Q(20)$	44.20	[0.00]	44.66	[0.00]	36.21	[0.01]	51.64	[0.00]	39.77	[0.00]
$Q^2(20)$	106.51	[0.00]	138.15	[0.00]	80.23	[0.00]	73.74	[0.00]	97.22	[0.00]

**Table 2 (continued)**  
**Summary statistics of spot and futures prices log-differences**

	Panel (B)				Panel (C)				Panel (D)			
	Two-week log variations		$\Delta^2 F(t, T_2)$		Three-week log variations		$\Delta^3 F(t, T_3)$		Four-week log variations		$\Delta^4 F(t, T_4)$	
	$\Delta^2 S(t)$				$\Delta^3 S(t)$				$\Delta^4 S(t)$			
<i>Mean</i> × 100	0.31	[0.65]	-3.51	[0.00]	0.49	[0.55]	-4.77	[0.00]	0.66	[0.48]	-5.40	[0.00]
<i>Median</i> × 100	-0.06		-2.37		-0.03		-3.40		0.60		-5.13	
<i>Kruskal-Wallis</i>			21.57	[0.00]			24.61	[0.00]			23.32	[0.00]
<i>S. D.</i>	0.17		0.13		0.20		0.17		0.23		0.19	
<i>Levene</i>			10.57	[0.00]			4.64	[0.03]			9.16	[0.00]
<i>Skewness</i>	-0.16	[0.12]	-0.70	[0.00]	-0.25	[0.01]	-0.70	[0.00]	-0.13	[0.20]	-0.58	[0.00]
<i>Kurtosis</i>	2.96	[0.00]	3.52	[0.00]	2.51	[0.00]	2.94	[0.00]	1.62	[0.00]	2.34	[0.00]
<i>Jarque-Bera</i>	211.13	[0.00]	343.35	[0.00]	155.71	[0.00]	251.94	[0.00]	63.94	[0.00]	161.72	[0.00]
<i>Minimum</i>	-0.80		-0.76		-0.83		-0.94		-0.80		-0.97	
<i>Maximum</i>	0.63		0.43		0.76		0.49		0.81		0.56	
<i>Q(20)</i>	179.33	[0.00]	174.03	[0.00]	365.99	[0.00]	344.73	[0.00]	481.33	[0.00]	583.55	[0.00]
<i>Q<sup>2</sup>(20)</i>	180.00	[0.00]	228.06	[0.00]	256.19	[0.00]	283.89	[0.00]	332.55	[0.00]	403.05	[0.00]

**Table 3****Correlation matrix of the spot and futures prices log variations**

The variables appearing in the heading of each row and columns are described in Table 1. For a sample size of  $T$  observations, the asymptotic distribution of the  $\sqrt{T}$  times the correlation coefficient is a zero-one normal distribution. \* indicates significance at the 1% significance level.

Panel (A): One-week log variations

	$\Delta S(t)$	$\Delta F(t, T_1)$	$\Delta F(t, T_2)$	$\Delta F(t, T_3)$	$\Delta F(t, T_4)$
$\Delta S(t)$	1.0	0.72*	0.54*	0.47*	0.44*
$\Delta F(t, T_1)$		1.0	0.76*	0.68*	0.62*
$\Delta F(t, T_2)$			1.0	0.93*	0.89*
$\Delta F(t, T_3)$				1.0	0.96*
$\Delta F(t, T_4)$					1.0

Panel (B): Two-week log variations

	$\Delta^2 S(t)$	$\Delta^2 F(t, T_2)$	$\Delta^2 F(t, T_3)$	$\Delta^2 F(t, T_4)$
$\Delta^2 S(t)$	1.0	0.80*	0.65*	0.60*
$\Delta^2 F(t, T_2)$		1.0	0.87*	0.80*
$\Delta^2 F(t, T_3)$			1.0	0.96*
$\Delta^2 F(t, T_4)$				1.0

Panel (C): Three-week log variations

	$\Delta^3 S(t)$	$\Delta^3 F(t, T_3)$	$\Delta^3 F(t, T_4)$
$\Delta^3 S(t)$	1.0	0.79*	0.72*
$\Delta^3 F(t, T_3)$		1.0	0.91*
$\Delta^3 F(t, T_4)$			1.0

Panel (D): Four-week log variations

	$\Delta^4 S(t)$	$\Delta^4 F(t, T_4)$
$\Delta^4 S(t)$	1.0	0.79*
$\Delta^4 F(t, T_4)$		1.0

**Table 4**  
**Multivariate GARCH model estimates and restrictions tests**

The headings appearing in the first and second rows of each column are described in Table 1 and represent the pair of variables used to fit the model. This pair of variables are the input of a VAR model as described in equation (9). From each VAR, an innovation vector  $(\varepsilon_{1t}, \varepsilon_{2t})'$  is obtained without autocorrelation problems. Panel (A) of this table displays the quasi maximum likelihood estimates of the ADC model in equation (2), assuming a conditional  $t$ -Student distribution for the innovation vector  $(\varepsilon_{1t}, \varepsilon_{2t})'$ . Significant coefficients at the 1%, 5%, and 10% of significance level are highlighted with one (\*), two (\*\*) and three (\*\*\*) asterisks, respectively. Panel (B) displays the Wald test for the restrictions imposed on the ADC model to obtain the encompassed models. The specification test proposed by Kroner and Ng (1998) is as follows: (1) If  $\rho_{12} = b_{12} = b_{21} = a_{12} = a_{21} = g_{12} = g_{21} = 0$ , a restricted asymmetric VECH is obtained with conditional covariance equation having coefficients  $b_{11} \times b_{22}$ ,  $a_{11} \times a_{22}$  and  $g_{11} \times g_{22}$ ; (2) if  $\phi_{12} = b_{12} = b_{21} = a_{12} = a_{21} = g_{12} = g_{21} = 0$ , the asymmetric CCORR model is derived; (3) if  $\phi_{12} = 1$  and  $\rho_{12} = 0$  the asymmetric BEKK model is obtained. Significant rejection of the null hypothesis at 1% of significance level is highlighted with an asterisk (\*).

Panel (A). Multivariate GARCH model estimates						
	$\Delta S(t)$	$\Delta S(t)$	$\Delta S(t)$	$\Delta^2 S(t)$	$\Delta^2 S(t)$	$\Delta^3 S(t)$
	$\Delta F(t, T_2)$	$\Delta F(t, T_3)$	$\Delta F(t, T_4)$	$\Delta^2 F(t, T_3)$	$\Delta^2 F(t, T_4)$	$\Delta^3 F(t, T_4)$
$\omega_{11}$	0.03*	0.00	0.01*	0.03*	0.02*	0.04*
$\omega_{22}$	-0.01*	0.00	0.01*	-0.01*	-0.02*	0.00
$\omega_{12}$	0.03*	0.04*	-0.01*	0.04*	0.02*	0.05*
$a_{11}$	0.54*	0.61*	0.15*	0.32*	0.53*	0.61*
$a_{12}$	0.27*	0.49*	0.11*	-0.03*	0.10***	0.19*
$a_{21}$	-0.03	-0.50*	0.09*	0.08*	-0.01	-0.19*
$a_{22}$	0.32*	-0.24*	0.16*	0.54*	0.40*	0.31*
$b_{11}$	0.78*	0.16*	0.99*	0.93*	0.78*	0.97*
$b_{12}$	0.07	-0.90*	-0.17*	0.40*	0.02	0.75*
$b_{21}$	-0.09*	0.62*	-0.07*	-0.21*	0.01	-0.32*
$b_{22}$	0.69*	1.12*	1.03*	0.40*	0.77*	0.14*
$g_{11}$	-0.98*	-0.59*	-0.23*	1.14*	0.51*	-0.13*
$g_{12}$	-1.31*	-0.29*	-0.31*	1.28*	0.57*	-0.75*
$g_{21}$	0.48*	0.04	0.28*	-0.66*	-0.52*	0.51*
$g_{22}$	0.94*	-0.23*	0.28*	-0.94*	-0.34*	0.82*
$\rho_{12}$	0.04*	0.11*	0.43*	-0.03*	0.04	-0.13*
$\phi_{12}$	0.92*	0.96*	0.45*	0.94*	0.89*	1.06*
$\nu$	4.56*	5.34*	5.15*	6.15*	6.83*	6.12*
Panel (B). Testing restrictions for nested models						
VECH	119.84*	3.35×10 <sup>3*</sup>	2.47×10 <sup>7*</sup>	7.43×10 <sup>11*</sup>	11.20*	208.46*
CCORR	7.44×10 <sup>3*</sup>	3.34×10 <sup>3*</sup>	2.07×10 <sup>7*</sup>	6.37×10 <sup>11*</sup>	321.39*	2.32×10 <sup>3*</sup>
BEKK	17.85*	58.36*	8.53×10 <sup>7*</sup>	1.4721×10 <sup>5*</sup>	16.08*	154.91*

**Table 5. The basis as a predictor of the change in spot and futures prices**

This table displays the results of the regression between spot and futures changes appearing in the first column on the basis as defined in the second column for the whole sample period (1998-2008). Between brackets  $t$ -statistic values computed with Newey-West standard errors are reported. Significant coefficients at the 1%, 5%, and 10% of significance level are highlighted with one (\*), two (\*\*) and three (\*\*\*) asterisks, respectively.

Dependent variable	basis	intercept	Basis coefficient	Adjusted R <sup>2</sup>
$\Delta S(t)$	$\log(F(t,T_1)/S(t))$	-0.01 (-4.01) *	1.02 (17.26) *	50.55%
$\Delta F(t,T_2)$	$\log(F(t,T_1)/S(t))$	-0.02 (-3.80) *	0.02 (0.44)	-0.15%
$\Delta F(t,T_3)$	$\log(F(t,T_1)/S(t))$	-0.01 (-2.44) *	-0.01 (-0.14)	-0.14%
$\Delta F(t,T_4)$	$\log(F(t,T_1)/S(t))$	-0.01 (-1.57)	0.01 (0.19)	-0.16%
$\Delta^2 S(t)$	$\log(F(t,T_2)/S(t))$	-0.03 (-4.39) *	0.99 (14.33) *	34.89%
$\Delta^2 F(t,T_3)$	$\log(F(t,T_2)/S(t))$	-0.03 (-3.14) *	-0.12 (-1.48)	0.57%
$\Delta^2 F(t,T_4)$	$\log(F(t,T_2)/S(t))$	-0.02 (-1.88) ***	-0.09 (-1.19)	0.35%
$\Delta^3 S(t)$	$\log(F(t,T_3)/S(t))$	-0.04 (3.24) *	0.83 (7.95) *	25.55%
$\Delta^3 F(t,T_4)$	$\log(F(t,T_3)/S(t))$	-0.03 (-2.54) **	-0.16 (-1.76) ***	1.16%



**Table 6. Hedging effectiveness**

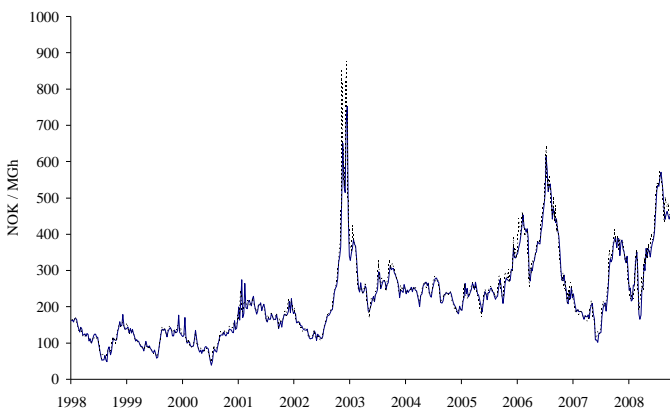
This table displays the risk reduction achieved by each hedging strategy: Naive, OLS without the basis (see equation (4)), OLS with the basis (see equation (8)) and the ADC conditional estimate. The second column in each panel reports *in-sample* results for the period December 29, 1997, to October 5, 2003, (300 weeks). The third column in each panel reports *out-of-sample* results for the period October 6, 2003, to December 28, 2008, (274 weeks). In the first row of each panel, the unhedged spot position variance is reported. This variance is computed as  $VAR[\Delta^k S(t) - \hat{\lambda}(\log(F(t, T_k)/S(t)))]$  and constitutes the base to calculate the risk reduction achieved with each hedging strategy. Variance of each hedging strategy is computed as  $VAR[\Delta^k S(t) - \hat{b}_t \Delta^k F(t, T_i) - \hat{\lambda}(\log(F(t, T_k)/S(t)))]$  where spot and futures log-variations are defined as in Table 1 and  $b_t$  represents the hedging ratio. *Ex ante* hedging ratios are forecasted values in  $t$  and each time a new observation is added the model is estimated again in the ADC and OLS hedging strategies. Those strategies with largest risk reduction are indicated with an asterisk (\*).

<b>Panel (A). Hedging one-week spot risk (<math>\Delta S(t)</math>)</b>		
	<b>In the sample</b>	<b>Out of the simple</b>
Spot variance (no hedged)	0.00791	0.00431
<b>Hedging Strategy</b>	<b>Risk reduction (%)</b>	<b>Risk reduction (%)</b>
Hedging with the second to 'delivery' ( $\Delta F(t, T_2)$ )		
Naive ( $b=1$ )	43.58	27.62
OLS w/o basis	58.46	58.93
OLS with basis	58.47 (*)	59.07 (*)
ADC	57.68	57.69
Hedging with the third to 'delivery' ( $\Delta F(t, T_3)$ )		
Naive ( $b=1$ )	35.50	13.05
OLS w/o basis	48.13	46.40
OLS with basis	48.20	45.84
ADC	49.80 (*)	47.76 (*)
Hedging with the fourth to 'delivery' ( $\Delta F(t, T_4)$ )		
Naive ( $b=1$ )	33.70	6.63
OLS w/o basis	40.77	33.94
OLS with basis	40.79	34.31 (*)
ADC	42.97 (*)	33.97

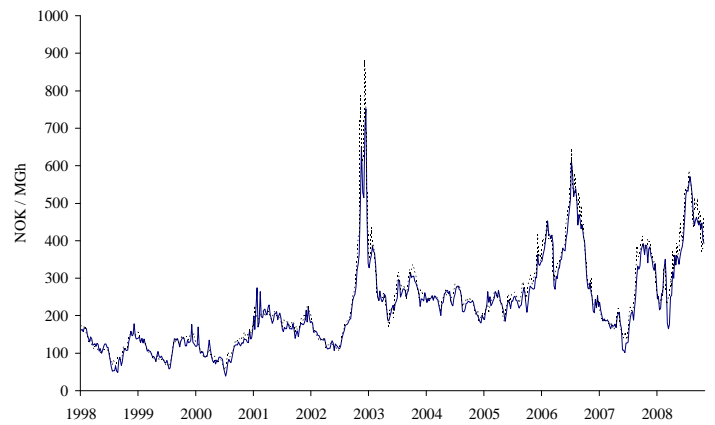
**Table 6. Hedging effectiveness (continued).**

<b>Panel (B). Hedging two-week spot risk (<math>\Delta^2 S(t)</math>)</b>		
Spot variance (not hedged)	0.02278	0.01295
Hedging with the third to 'delivery' ( $\Delta^2 F(t, T_3)$ )		
Naive ( $b=1$ )	75.52	69.09
OLS w/o basis	76.29	75.83 (*)
OLS with basis	76.90 (*)	74.98
ADC	71.11	70.26
Hedging with the fourth to 'delivery' ( $\Delta^2 F(t, T_4)$ )		
Naive ( $b=1$ )	66.19	58.15
OLS w/o basis	66.48	63.20 (*)
OLS with basis	66.76 (*)	62.31
ADC	57.81	57.15
<b>Panel (C). Hedging three-week spot risk (<math>\Delta^3 S(t)</math>)</b>		
Spot variance (not hedged)	0.03549	0.02286
Hedging with the fourth to 'delivery' ( $\Delta^3 F(t, T_4)$ )		
Naive ( $b=1$ )	81.42	81.71
OLS w/o basis	80.84	82.60
OLS with basis	81.73 (*)	82.86 (*)
ADC	68.05	74.48

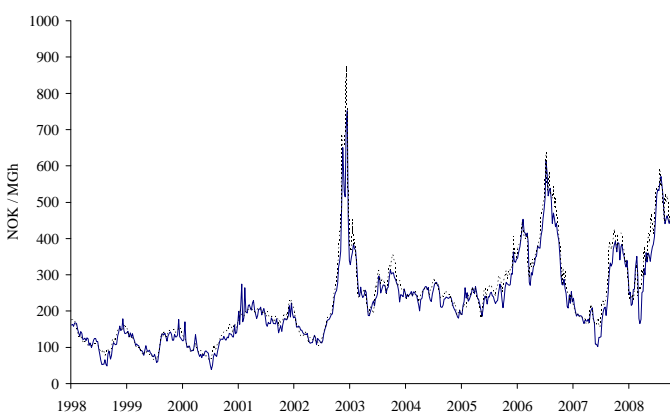
**Figure 1.** System and weekly futures prices.



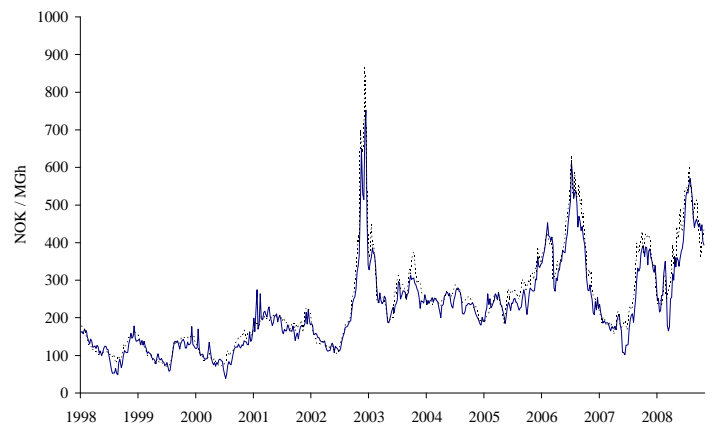
**Figure 1(a).** System price (—) and the first to 'delivery' futures price (- - -)



**Figure 1(b).** System price (—) and the second to 'delivery' futures price (- - -)



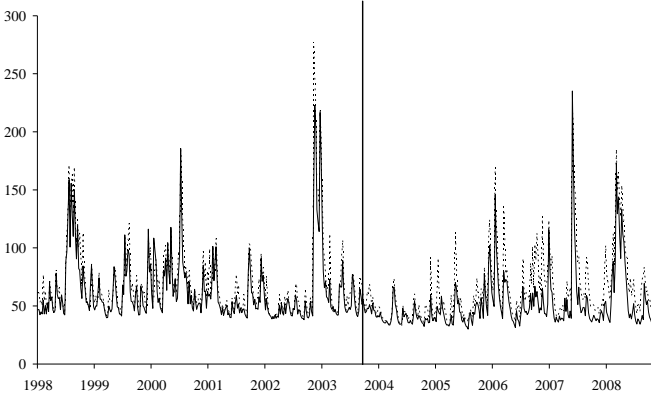
**Figure 1(c).** System price (—) and the third to 'delivery' futures price (- - -)



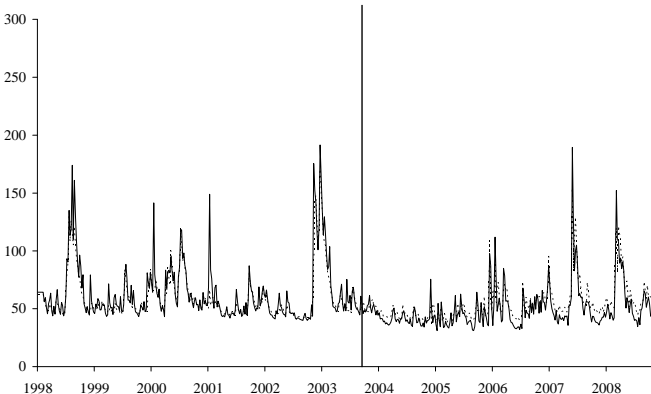
**Figure 1(d).** System price (—) and the fourth to 'delivery' futures price (- - -)

**Figure 2. Annualized conditional volatilities.**

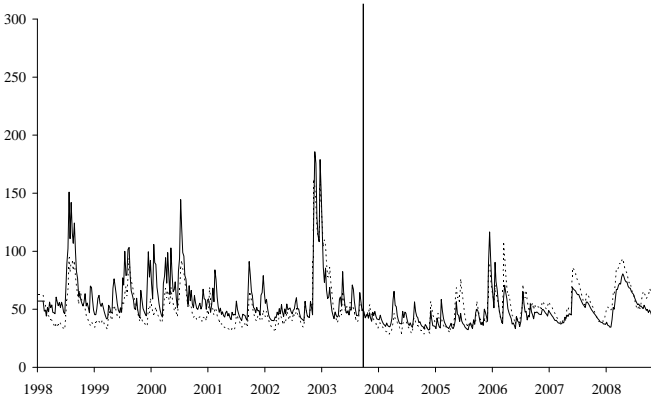
Notes. In each graph, the solid line (—) and the dashed line (- - -) correspond to the spot and futures annualized conditional volatility (in percentage), respectively. The displayed conditional volatilities are estimated in the ‘one-week’ hedging period models. The vertical line separates the *ex post* and *ex ante* hedging periods.



**Figure 2(a).** Second to ‘delivery’ futures



**Figure 2(b).** Third to ‘delivery’ futures price



**Figure 2(c).** Fourth to ‘delivery’ futures

### Figure 3. Hedging ratios.

Notes. The vertical line separates the *ex post* and *ex ante* hedging periods. ADC hedging ratios are represented with continuous lines (—) and OLS hedging ratios estimated with equation (8) are represented with dashed lines (- - -). Spot and futures log-variations are defined as in Table 1.

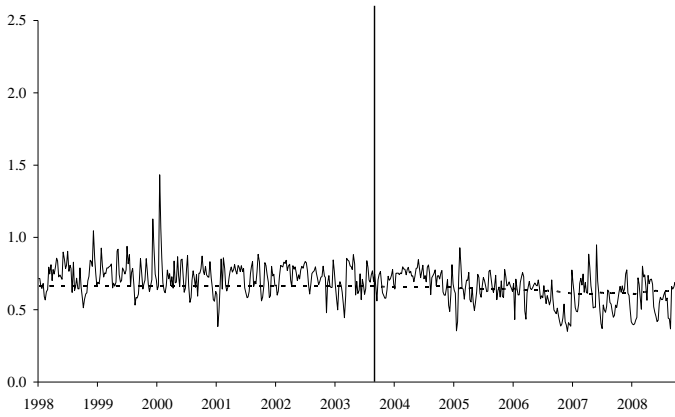


Figure 3(a). Hedging  $\Delta S(t)$  risk with  $\Delta F(t, T_2)$ .

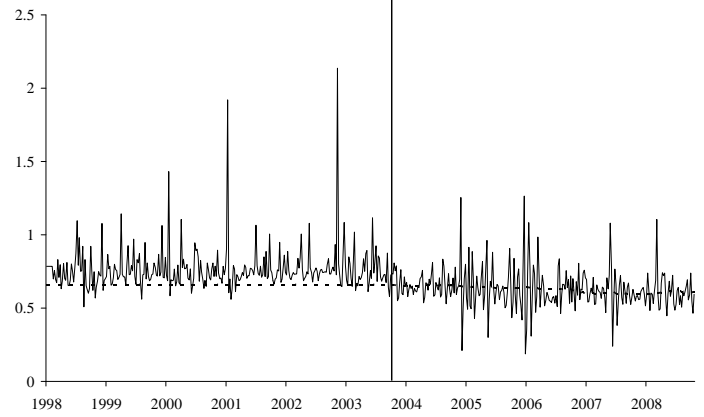


Figure 3(b). Hedging  $\Delta S(t)$  risk with  $\Delta F(t, T_3)$ .

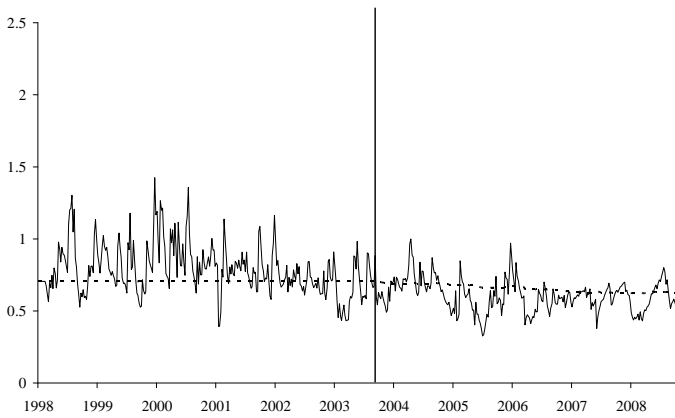


Figure 3(c). Hedging  $\Delta S(t)$  risk with  $\Delta F(t, T_4)$ .

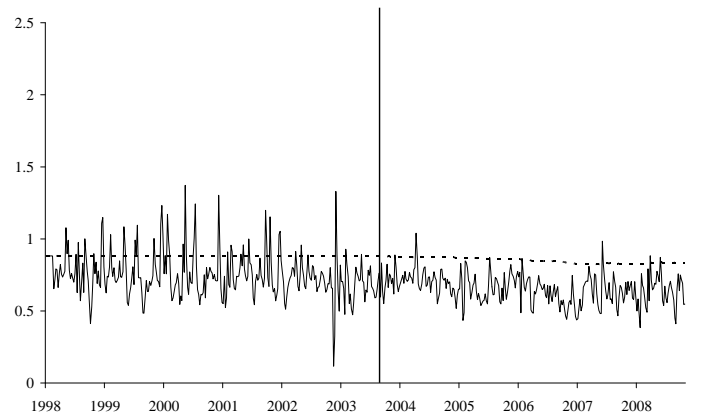


Figure 3(d). Hedging  $\Delta^2 S(t)$  risk with  $\Delta^2 F(t, T_3)$ .

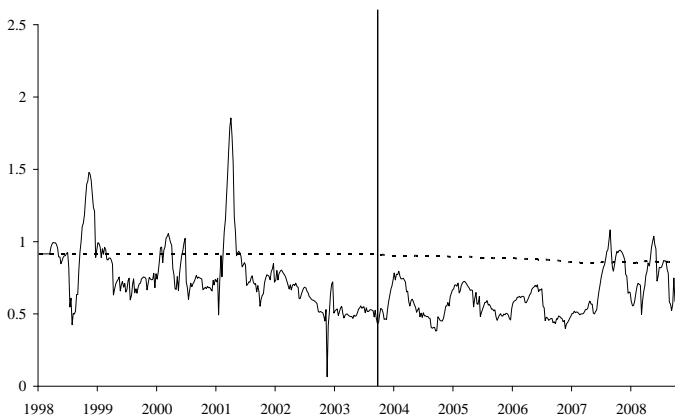


Figure 3(e) Hedging  $\Delta^2 S(t)$  risk with  $\Delta^2 F(t, T_4)$ .

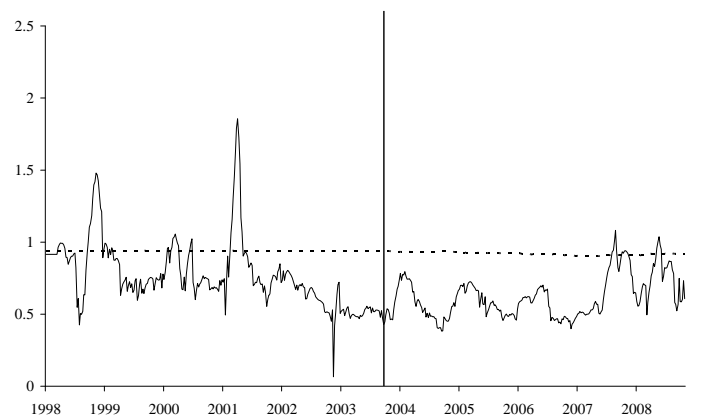


Figure 3(f) Hedging  $\Delta^3 S(t)$  risk with  $\Delta^3 F(t, T_4)$ .