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# **The role of Regime Shifts in the Term Structure of Interest Rates: Further evidence from an Emerging Market**

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## **Abstract**

In this paper, we investigate the interrelations among Turkish interest rates with different maturities by using a regime switching Vector Error Correction (VECM) model. We find a long run equilibrium relationship among interest rates with various maturities. Furthermore we conclude that term structure dynamics exhibit significant nonlinearity. Forecasting experiment also reveals that the nonlinear term structure models do fare better than other linear specifications. However, we cannot conclude that interest rate adjustments are made in an asymmetric way in the long run equilibrium.

**JELClassification:** E43, C52, C53, C32

**Key Words:** Term Structure of Interest Rates, Regime Switching, Forecasting, Forecast Evaluation, Cointegration

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## 1. Introduction

Studies on term structure dynamics have always been at the core of macroeconomics and finance research. Campbell and Clarida (1986), Campbell and Shiller (1987) and Hall et al (1992) studied the long run dynamics of term structure of interest rates. Recently, Clarida et. al. (2006) proposed a nonlinear multivariate vector error correction (VECM) model to investigate the term structure of interest rates. Clarida et al. (2006) also incorporated the potential asymmetries in the error correction mechanism. They also studied the weekly forecasting performance of the nonlinear dynamic interest rate model against some linear benchmark models. Despite the importance of these developments, relatively few studies addressed the dynamics of term structure of interest rates in emerging markets<sup>1</sup>. However, none of these studies, except Guillen and Tabak (2008) for the Brazilian case, explicitly addressed the nonlinearity of term structure of interest rates. During the last two decades, Turkish economy has experienced a number of sharp downturns and economic crises<sup>2</sup>. These have a direct impact on the interest rates and term structure of interest rates. As a result, term structure dynamics of the Turkish economy can be better investigated under nonlinear models. It is also interesting to see whether the short run adjustments towards equilibrium is symmetric or not.

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<sup>1</sup> For instance, Alper et al. (2007) provided an analysis of term structure of interest rates for Turkey. Telatar et al. (2003) examined the information content in term structure of interest rate about future inflation by using time-varying-parameter model. Kaya and Yazgan (2009) emphasized the effect of monetary policy change on the nature of this information content. Gonzales et al. (2000) investigated the predictive power of term structure of interest rate for different macroeconomic variables including inflation. Ogaki and Santealla (2000) analyzed the effects of term structure of interest rate on exchange rate. Although it is a small European economy, we can also include Greece in our short list by quoting Drakos (2001) who investigated the impact of monetary policy on term structure of interest. Guillen and Tabak (2008) studied Brazilian term structure of interest rates and characterized how the term premia had changed over time.

<sup>2</sup> See Yilmazkuday and Akay (2008) for a brief account of these developments and an analysis of business cycles of Turkish economy in regime switching approach.

In this paper, we wish to fill these gaps in empirical macroeconomics. Following Clarida et al (2006) we analyze the term structure dynamics of the Turkish interest rates by using the weekly Turkish interest rate data between 1993 and 2009. We empirically test the existence of nonlinearity in the term structure of interest rates. We conduct a weekly forecasting experiment on the Turkish interest rates with different maturities. In addition to these, we extend the regime switching specification by allowing the speed of adjustment coefficients to change across regimes. Furthermore, we adopt the reality check methodology of White (2000) to test the adequacy of forecasts generated by the various alternative nonlinear and asymmetric models.

Consequently, we obtain three main results: First we conclude that long run relationships among various interest rates exist, which supports the predictions of expectation hypothesis. We also demonstrate that there exists a nonlinear regime switching structure in the weekly interest rate data we study. Finally, forecasts generated by symmetric nonlinear regime switching model beats the other alternative linear vector time series models. This result suggests that an asymmetric adjustment in the interest rate modeling do not exist in our data. To the best of our knowledge, this is the first comprehensive attempt on analyzing the term structure dynamics on an emerging market economy.

The organization of the paper is as follows: In the following section, we discuss the theories of term structure and in section three we discuss its estimation through

nonlinear dynamic time series models. In the fourth section we present our empirical results. We conclude in the final section.

## 2. Cointegration and the expectations hypothesis

Expectations hypothesis (EH) can be formulated as follows (see Clarida et al., 2006 and Campbell and Shiller, 1991)

$$R_{k,t} = \frac{1}{k} \sum_{j=1}^{k-1} E_t(R_{1,t+j-1}) + \gamma_{k,t} \quad (1)$$

where  $R_{k,t}$  is the yield to maturity obtained from a  $k$ -period pure discount bond. As is known, this equation can be interpreted as the longer term spot rates are equal to the average expected short rate of interests. If we subtract  $R_{1,t}$  from both sides of equation (1) we obtain the following equation

$$R_{k,t} - R_{1,t} = \frac{1}{k} \left( \sum_{m=1}^{k-1} \sum_{j=1}^m \Delta E_t R_{1,t+j} \right) + \gamma_{k,t} \quad (2)$$

where  $\Delta$  is the first difference operator and  $E_t$  refers to the expectation operator conditioned on information available time  $t$ . The last term in this equation refers to the time varying term premia. More specifically, the term spread between long and short term of maturities should be explained by first difference of interest rates with various maturities. Therefore, equation above has testable implications. In this formulation, if

we allow a time varying and stationary term premia,  $\gamma_{k,t}$  and if we assume that the interest rates are integrated order one,  $I(1)$ , the above equation implies a cointegrating relationship between the term spread (i.e. the difference between interest rate of maturity  $k$  and  $l$ ), In other words, if the theoretical predictions of the above model is correct then term spread should follow  $I(0)$  i.e.  $R_{k,t} - R_{l,t} \sim I(0)$ . More concretely, the interest rates of maturity  $k$  and  $l$  are cointegrated with a vector  $[1, -1]'$ . Hence, according to EH, if we have  $n$  interest rates of different maturities, there must be exactly  $n-1$  distinct co-integrating relationships among them. Each of these cointegrating vectors are given by stationary spreads between  $R_{k,t} - R_{l,t}$  for  $k = 2, \dots, n$ . As is well known, given the existence of cointegrating relationships between a set of interest rates of different maturities, the dynamic relationships between them can be formulated within a vector error correction model (VECM).

### **3. Modeling Term Structure Nonlinearities via Regime-Switching Vector Error Correction Model (RSVECM)**

Term structure of interest rates is very much affected by economic growth and business cycles. Consequently, the levels and the term structure of interest rates have varying dynamics in different economic regimes. Recent studies on Regime Switching models by Hamilton (1983), Krolzig (1997) have investigated the properties of regime switching econometric models both in univariate and multivariate contexts.

Consider the MS-VAR process in its most general form:

$$\mathbf{y}_t = \mathbf{v}(s_t) + \mathbf{\Pi}_1 y_{t-1} + \dots + \mathbf{\Pi}_k y_{t-k} + \boldsymbol{\varepsilon}_t, \quad t = 1, 2, \dots, T \quad (3)$$

where  $\mathbf{y}_t$  is an  $n$  dimensional time series vector observed at time  $t$  and  $T$  is the sample size. In our specific example  $n$  is equal to 5 and the vector  $\mathbf{y}$  contains interest rates at 90, 120, 180, 270 and 360 days maturities i.e  $y_t = (R_{90,t}, R_{120,t}, R_{180,t}, R_{270,t}, R_{360,t})'$ .  $\mathbf{v}$  is the vector of intercepts,  $\mathbf{\Pi}_1, \dots, \mathbf{\Pi}_p$  are the matrices containing the autoregressive parameters and  $\boldsymbol{\varepsilon}_t$  is a white noise vector process such that  $\boldsymbol{\varepsilon}_t | s_t \sim NID(\mathbf{0}, \boldsymbol{\Sigma}(s_t))$ . The regime generating process is assumed to be an ergodic Markov chain with a finite number of states  $s_t \in [1, \dots, M]$  governed by transition probabilities  $p_{ij} = \Pr(s_{t+1} = j / s_t = i)$  and  $\sum_{j=1}^M p_{ij} = 1$  for all  $i, j \in \{1, \dots, M\}$ . The MS-VAR setting also allows for a variety of specifications.

Krolzig (1997) established a common notation to provide simplicity in expressing the models in which various parameters are subject to shifts with the varying. In Equation (3) the intercept term is assumed to vary with state beside other parameters. Intercept switch specification is used in cases where the transition to the mean of the other state is assumed to follow a smooth path. An alternative representation is obtained by allowing the mean to vary with the state. This specification is useful in cases where a one-time jump is assumed in the mean after a change in regime.<sup>3</sup>

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<sup>3</sup> Note that the intercept  $\mathbf{v}$  controls the mean of  $\mathbf{y}_t$  through the relationship  $\boldsymbol{\mu}(s_t) = \mathbf{v}(s_t) \{ \mathbf{I} - \mathbf{\Pi}_1(s_t) - \dots - \mathbf{\Pi}_p(s_t) \}^{-1}$ .

This type of MS(M)-VAR(p) model, which allows regime shifts, both in intercept and variance and covariance matrix, is termed as Markov-switching-intercept-heteroskedastic- VAR (MSIH-VAR) after Krolzig (1996). Then, the VEC representation of the MSIH-VAR(p) model, or MSIH-VECM(p-1) can be written as

$$\Delta \mathbf{y}_t = \mathbf{v}(s_t) + \mathbf{\Pi} \mathbf{y}_{t-1} + \sum_{i=1}^{p-1} \mathbf{\Gamma}_i \Delta \mathbf{y}_{t-i} + \boldsymbol{\varepsilon}_t \quad (4)$$

Where

$$\mathbf{\Pi} = -(\mathbf{I} - \mathbf{\Pi}_1 - \dots - \mathbf{\Pi}_p) \text{ and}$$

$$\mathbf{\Gamma}_i = -(\mathbf{\Pi}_{j+1} + \mathbf{\Pi}_{j+2} + \dots + \mathbf{\Pi}_p) \text{ for } j = 1, 2, \dots, p-1$$

Given that  $\mathbf{\Pi}$  is not full rank, it can be written as the product of two rectangular matrices  $\boldsymbol{\alpha}$  and  $\boldsymbol{\beta}$  of order  $n \times r$  such that  $\mathbf{\Pi} = \boldsymbol{\alpha} \boldsymbol{\beta}'$ . The vector  $\boldsymbol{\beta}$  is the co-integration vector and the vector  $\boldsymbol{\alpha}$  is the factor-loading (or speed of adjustment) vector. Hence  $r$  is the number of co-integrating vectors. Therefore MSIH-VECM in (4) can be written as

$$\Delta \mathbf{y}_t = \mathbf{v}(s_t) + \boldsymbol{\alpha} \boldsymbol{\beta}' \mathbf{y}_{t-1} + \sum_{i=1}^{p-1} \mathbf{\Gamma}_i \Delta \mathbf{y}_{t-i} + \boldsymbol{\varepsilon}_t \quad (5)$$

As indicated by Clarida et.al. (2006) the asymmetric adjustment in interest rates, can be modeled within this framework. To capture the asymmetries in the data they write the above MSIH-VECM model by allowing differing speeds of adjustment to equilibrium depending on whether interest rates are above or below equilibrium, i.e. whether the  $\boldsymbol{\beta}' \mathbf{y}_{t-1}$  is negative or positive.



$$\Delta \mathbf{y}_t = \mathbf{v}(s_t) + \mathbf{\Psi}_t \boldsymbol{\alpha}^+ \boldsymbol{\beta}' \mathbf{y}_{t-1} + (\mathbf{I}_t - \mathbf{\Psi}_t) \boldsymbol{\alpha}^- \boldsymbol{\beta}' \mathbf{y}_{t-1} + \sum_{i=1}^{p-1} \boldsymbol{\Gamma}_i \Delta \mathbf{y}_{t-i} + \boldsymbol{\varepsilon}_t \quad (6)$$

Where  $\mathbf{I}_t$  is a  $r \times r$  identity matrix, and  $\mathbf{\Psi}_t$  is a  $r \times r$  diagonal matrix whose  $j$ th diagonal at time  $t$  taking the value of unity or zero according to whether the lagged  $j$ th deviation from the equilibrium, i.e. the  $j$ th element of  $\boldsymbol{\beta}' \mathbf{y}_{t-1}$  is positive or negative respectively. The model in (6) is termed as MSIH Asymmetric VECM

Note that specifications in (5) and (6) do not allow regime dependent behavior neither for speed of adjustment nor the autoregressive coefficients (or short-run parameters). We can enrich the models considered by Clarida et al. (2006) by allowing both type of regime switching and we can, first, rewrite MSIH-VECM in (5) as

$$\Delta \mathbf{y}_t = \mathbf{v}(s_t) + \boldsymbol{\alpha}(s_t) \boldsymbol{\beta}' \mathbf{y}_{t-1} + \sum_{i=1}^{p-1} \boldsymbol{\Gamma}_i(s_t) \Delta \mathbf{y}_{t-i} + \boldsymbol{\varepsilon}_t \quad (7)$$

This model can be noted as Markov-switching-intercept-autoregressive-heteroskedastic (MSIAH) VECM. In this model we retain the usual assumption of this literature by supposing that whereas the long-run parameters contained in the cointegration vector  $\boldsymbol{\beta}$  is regime-invariant, speed of adjustment coefficients of vector  $\boldsymbol{\alpha}$  are regime dependent.

Then considering asymmetric behavior defined in (6) within this model leads to the following MSIAH Asymmetric VECM.

$$\Delta \mathbf{y}_t = \mathbf{v}(s_t) + \mathbf{\Psi}_t \mathbf{\alpha}^+(s_t) \boldsymbol{\beta}' \mathbf{y}_{t-1} + (\mathbf{I} - \mathbf{\Psi}_t) \mathbf{\alpha}^-(s_t) \boldsymbol{\beta}' \mathbf{y}_{t-1} + \sum_{i=1}^{p-1} \mathbf{\Gamma}_i(s_t) \Delta \mathbf{y}_{t-i} + \boldsymbol{\varepsilon}_t \quad (8)$$

In forecasting exercises provided in Section 4 we concentrate the following 9 models outlined in Table 1.

Table 1 is about here

Estimation of MSIAH-VECM models in (5)-(8) can be carried out in two steps as suggested by Krolzig (1996), and as applied, among others, by Clarida et. al. (2003, 2006), Krolzig (2002), Sarno and Valente (2005, 2006). First, cointegration tests and the estimation of the parameters of the long-run relations can be accomplished by the maximum likelihood (ML) approach to the problem of estimation and hypothesis testing in the context of VECMs as outlined in Johansen's (1991, 1996). Second step, the long-run parameter matrix,  $\boldsymbol{\beta}$  estimated (and identified) in the first step is embedded in the above MS-VECMs. Then, the remaining parameters can be estimated by using the expectation maximization algorithm as in Krolzig (1996).

### **3. Long Run Equilibrium Relationship and term structure of interest rates**

#### **3.1 Data and Time Series Properties**

In this section we analyze the time series properties of the variables that are included in our analysis<sup>4</sup>. We use weekly data covering the period 1993w1-2009w5 for interest rates at 90, 120, 180, 270 and 360 days maturities:  $R_{90}, R_{120}, R_{180}, R_{270}, R_{360}$ <sup>5</sup>. As interest rates we use Treasury bond rates with maturities 90, 120, 180, 270 and 360 days. These data are obtained from Istanbul Stock Exchange database on a daily basis<sup>6</sup> and weekly averages are used in the estimation. In order to proceed with the co-integration analysis, we first test for presence of unit roots<sup>7</sup>. Table 2 displays the results of different unit root tests of the interest rate levels and their first differences at different maturities. Since the interest rate data do not contain a trend term, unit root test regressions are run without a trend term by only including an intercept term. As can be seen from the results presented in the following table, for all maturities we cannot reject the existence of unit roots in the levels. But this is not true for the first differences, hence the conclusion that all of our series are I(1).

Table 2 is about here

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<sup>4</sup> This is important for us since the multivariate cointegration test applied here requires that variables are firmly established as I(1).

<sup>5</sup> In fact, we first perform the following transformation to our data:  $R = \ln(1 + i)$ , where  $i$  is the interest rate.

<sup>6</sup> The interest rate data has been obtained by Riskturk ([www.riskturk.com](http://www.riskturk.com)). In constructing the yield curve official bond market data has been collected from Istanbul Stock Exchange. Since the Turkish Fixed Income Bill and Bonds are traded in an official exchange (more information can be found at [www.ise.org](http://www.ise.org)) a reliable official data exists and the market is rather liquid for an emerging market. Once the official data is obtained from the ISE, the spot yields are solved. Then a simple interpolation scheme is used to construct the yield curve. We believe a more advanced yield curve construction technique such as Nelson and Siegel (1987) (see Diebold and Li (2006)) do not make a big difference since the correlation between these two techniques was very high. More details can be found in <http://www.riskturk.com>.

<sup>7</sup> Unit root tests are computed by using Eviews 6.1. The computations in section 3.2 are carried out by using CATS Version 2. MS VECM models are estimated by using the GAUSS routine, MSVARlib, developed by Benoit Bellone (<http://bellone.ensae.net/MSVARlib.html>). The codes for remaining calculations, forecasts and forecasts test statistics, are written in GAUSS. Our GAUSS code and data are available upon request

### 3.2. Cointegration Tests and Long-Run Identification

As mentioned above, in the first stage of our estimation process, we work in a symmetric linear VAR model in levels (i.e. Equation (3) with  $M = 1$ ) to accomplish our cointegration analysis within the Johansen's framework. Prior to cointegration tests, the decision about the lag length ( $p$ ) of underlying (linear) vector autoregressive (VAR) model must be accomplished. However, as is well known (e.g. Cheung and Lai (1993)) Johansen's cointegration tests are rather sensitive to different parameterizations in the lag length. Therefore we report the results for different lag specifications up to 6 lags. It should be mentioned that the results outlined below is highly robust to higher lag orders of the VAR model. When these tests are performed, the intercept term is constrained into cointegration space. Since as mentioned above, the level variables are not trended, this formulation ensures that the solution of the model in terms of level variables does not contain linear trends<sup>8</sup>.

Trace statistics<sup>9</sup>, reported in Table 3, indicate that the interest rate series are cointegrated with the co-integration vector dimension of 4. In other words we conclude that the interest rate series have a long run equilibria with a co-integration dimension of 4 out of five variables (i.e.  $r = 4$ )<sup>10</sup>. This finding is consistent with the EH hypothesis which suggests that interest rates with various maturities should move together in the long run.

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<sup>8</sup> Following Clarida et al. (2006), we think the cointegration vectors should not contain a constant term, as opposed to Tilmann et al (2006). Therefore we further test whether intercept terms to be equal to zero or not.

<sup>9</sup> We only report trace statistics as suggested by Cheung Lai (1993) and Lutkepohl et al. (2001).

<sup>10</sup> Small sample corrected trace test statistics (Trace \* statistics) of Johansen (2002) are qualitatively indicate the same result.

Table 3 is about here

We also wish to test whether the over-identifying restrictions imposed by EH can be supported by the data. More specifically, EH, as outlined above, implies the following 4 over-identifying restrictions on  $\beta$  matrix:

$$\beta'y_t = \begin{bmatrix} -1 & 1 & 0 & 0 & 0 & 0 \\ -1 & 0 & 1 & 0 & 0 & 0 \\ -1 & 0 & 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} R_{90,t} \\ R_{120,t} \\ R_{180,t} \\ R_{270,t} \\ R_{360,t} \end{bmatrix}$$

The resulting Likelihood ratio (LR) test has a chi-square value  $\chi(8) = 79.868$ <sup>11</sup> which leads the rejection of these over-identifying restrictions with an associated p-value of 0.000. However, Johansen (2000) argues LR tests over-reject over identifying restrictions and suggests a Barlett correction factor to overcome this problem. If we use correction we obtain an LR statistic that is equal to  $\chi(8) = 9.509$  together with a p-value 0.301. The underlying correction factor is equal to 8.399. Therefore the restrictions implied by EH hypothesis cannot be rejected by the data at any conventional significance levels.

In addition to the use of Barlett correction, we proceed to examine whether the departure from the null hypothesis were large by imposing the following restrictions:

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<sup>11</sup> The degrees of freedom is equal to 8 since we also restrict all the 4 intercepts to be equal to zero as is mentioned above.

$$\beta'y_t = \begin{bmatrix} -1 & \beta_{12} & 0 & 0 & 0 & 0 \\ -1 & 0 & \beta_{13} & 0 & 0 & 0 \\ -1 & 0 & 0 & \beta_{14} & 0 & 0 \\ -1 & 0 & 0 & 0 & \beta_{15} & 0 \end{bmatrix} \begin{bmatrix} R_{90,t} \\ R_{120,t} \\ R_{180,t} \\ R_{270,t} \\ R_{360,t} \end{bmatrix}$$

In this case  $\beta$  parameters are left unrestricted. This approach yields the following estimates<sup>12</sup>

$$\beta_{12} = 1.015, \beta_{23} = 1.025, \beta_{34} = 1.014, \beta_{45} = 1.001$$

It can be observed from the estimated values the departure from the over-identifying restrictions on  $\beta$  s can be considered small in magnitude.

### 3.2. Tests of Asymmetry and Linearity

As we have show that interest rate series have long run equilibrium, it is interesting to investigate the short run dynamic adjustments. One major question regarding the term structure modeling is whether the short run error dynamics exhibit an asymmetric pattern. In other words, what we wish to distinguish is whether the sign of the shock causes a different adjustment speed towards the equilibrium. One may expect that the negative shocks might take longer to adjust than that of positive shocks. In this subsection we test the error correction asymmetries (III, VIII and IX in Table 1) against their symmetric alternatives (see Table 4 below), by using LR tests. Similarly, we test our five nonlinear models against their relevant linear alternatives (see Table 5 below). All LR tests indicate that both asymmetries are nonlinearities are present in the data and

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<sup>12</sup> The LR statistics is equal to  $\chi(4)=8.619$  with an associated p-value of 0.071. Hence the zero constraints on the intercepts terms are accepted in this case.

asymmetric MSIAH VECM should be the preferred model as it holds the highest LR test statistics.

Table 4 is here

Table 5 is here

#### 4. Forecasting the Term Structure of Interest Rates out of Sample with MSIAH VECMs

The approach developed by Krolzig (1996, 2002) is used to predict multiple time series subject to Markovian shifts in the regime. The k-step ahead predictor for symmetric MSIH-VECM in (5) is given by

$$E(\Delta \mathbf{y}_{t+k} | \Delta \mathbf{y}_t, \dots, \Delta \mathbf{y}_0) = \mathbf{M} \mathbf{P} \hat{\boldsymbol{\xi}}_{t|t} + \mathbf{N} \mathbf{P} \hat{\boldsymbol{\xi}}_{t|t} \boldsymbol{\beta}' \mathbf{y}_{t-1} + \sum_{i=1}^{p-1} \boldsymbol{\Gamma}_i \Delta \mathbf{y}_{t-i} \quad (9)$$

Where  $\mathbf{M} = [\mathbf{v}_1 : \dots : \mathbf{v}_M]$  and  $\mathbf{N} = [\boldsymbol{\alpha}_1 : \dots : \boldsymbol{\alpha}_M]$ .  $\mathbf{P}$  is the transposed matrix of transition probabilities,  $\hat{\boldsymbol{\xi}}_{t|t}$  is the vector of filtered regime probabilities at time  $t$ . The forecast for the other models can be constructed in similar manners.

The out-of-sample forecasts for a given horizon  $k$  are constructed by using (9). The coefficients in (9) are estimated by running regressions with data up through the date

$t_0 < T$ . The first k-horizon forecast is obtained by using the coefficient estimates from this first regression. Next, the time subscript is advanced, and the procedure is repeated for  $t_0 + 1, t_0 + 2, \dots, T - k$  to obtain  $N_f = T - t_0 - k + 1$  k-step distinct forecasts<sup>13</sup>. By using these  $N_f$  k-horizon forecasts we evaluate the forecasting performance our models by using Root Mean Square Error (RMSE)<sup>14</sup>.

In Table 6 we report the average of RMSE over  $N_f$  (number of forecasts at k-horizon) and over  $n$  variables (5 interest rates at different maturities).

Table 6 is here

The table compares different models at different forecast horizons (k). The emboldened numbers show the smallest of the corresponding criteria, therefore the best model, at the corresponding forecast horizon (k). The table reveals the fact that the symmetric MSIAH-VAR of Equation (7) with  $\mathbf{\Pi} = \mathbf{0}$  is the “best” model in terms of forecast accuracy at all horizons.

#### **4.1 Assessing the forecast accuracy: Diebold and Mariano test**

In order to assess the relative accuracy of forecasts derived from two competing models, we first employ the Diebold and Mariano (1995). The suggested DM statistics is distributed as standard normal under the null hypothesis of equal forecast accuracy as

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<sup>13</sup> The number forecasts differ between 89 and 37 for different k values between 1 and 52

<sup>14</sup> We also use Mean Absolute Error (MAE) in all the forecasting and forecast evaluation procedures that we use in this paper. To save the space we only report the results associated with RMSE. The results are qualitatively identical with MAE which is available upon request.



shown by Diebold and Mariano. When we test the equal forecast accuracy hypothesis we first use asymmetric MSIAH VECM as our benchmark model, which is the best model according to results of LR tests above. Then symmetric MSIAH VAR is used as the benchmark, which is the best model in terms of forecast accuracy.

Table 7 is here

The results on DM statistics displayed in Table 7 generally indicate that our benchmark model MSIAH-VAR is found superior than the other competing models. Even though we notice the forecast superiority, at the lower forecast horizons are less clear. Therefore, we further investigate the forecast precision of our benchmark model with a newer test developed by White (2000).

#### **4.2 Assessing the forecast accuracy: Reality Check**

White (2000) developed an elegant test of superior unconditional predictive ability among multiple models built on Diebold and Mariano (1995) and West (1996). Our interest is to compare all the models with our benchmark of symmetric nonlinear term structure model (MSIAH-VAR). An appropriate null hypothesis is that our competing model is no better than our benchmark. Our objective function is, once more RMSE<sup>15</sup>. We report the results on White (2000) test in Table 8 where Prc1 is the bootstrap p-value for comparing a single model with the benchmark model, and Prc2 is the bootstrap reality check p-value for comparing m models with the benchmark model.

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<sup>15</sup> As with the results of DM tests the results with MAE are identical qualitatively.

The first number for Prc2 is the bootstrap reality check p-value for the null hypothesis that the best of the first  $m$  models has no superior predictive power over the (MSIAH-VAR) model. The last number for Prc2 checks if the best of all the models under comparison has superior predictive ability over the benchmark model. As in most of the cases the loss differential is high and Reality Check probabilities are small we conclude that our benchmark has a superior forecasting precision than its alternatives. In this testing method we observe a more decisive forecast superiority in our symmetric nonlinear model. Therefore, we conclude that on the basis of reality check of White (2000), a symmetric regime switching model has a better predictive power for the weekly interest rate series.

Table 8 is here

### **4.3 Comments on Empirical Findings**

We can summarize our findings as follows. First we observe a long run relationship in the interest rate data. We show that interest rates with different maturities are moving together in the long run. Furthermore, the dynamics of these interest rate series can be better modeled in a non-linear environment. For instance, nonlinear time series model fits the data better than that of linear benchmark models. This may be expected since the economy faces different growth and inflationary states, a regime switching model can mimic interest rates data more successfully. Finally, unlike Clarida et al (2006), we cannot obtain asymmetry in our data set.

## **5. Conclusion**

In this paper we study the nonlinearity and asymmetry in the weekly Turkish interest rates. The interest rate data with various maturities move together which is in line with the predictions of the expectations hypothesis theory. In addition, we also show that the interest rate data exhibits nonlinear time series properties. Both in the in-sample and out of sample data, we demonstrate that nonlinear regime switching models have better predictive power. However, we cannot reach a decisive conclusion regarding the asymmetric adjustment properties of the interest rate data. Linking the term structure with macroeconomic factors is left for future research.

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**Tables:**

**Table 1 Models used in Forecasting**

<b>(I)</b>	Linear Symmetric VAR, i.e. (5) with $M = 1$ and $\Pi = 0$
<b>(II)</b>	Linear Symmetric VECM, i.e. (5) with $M = 1$
<b>(III)</b>	Linear Asymmetric VECM, i.e. (6) with $M = 1$
<b>(IV)</b>	MSIH Symmetric VAR, i.e. (5) with $\Pi = 0$
<b>(V)</b>	MSIAH Symmetric VAR, i.e. (7) with $\Pi = 0$
<b>(VI)</b>	MSIH Symmetric VECM, i.e. (5)
<b>(VII)</b>	MSIAH Symmetric VECM, i.e. (7)
<b>(VIII)</b>	MSIH Asymmetric VECM, i.e. (6)
<b>(IX)</b>	MSIAH Asymmetric VECM, i.e. (8)

**Table 2: Augmented Dickey-Fuller Unit Root Tests**

variable	ADF	DF-GLS	Variable	ADF	DF-GLS
$R_{90}$	-2.103 (-2.865)	-1.611 (-1.941)	$\Delta R_{90}$	-8.043 (-2.865)	-8.036 (-1.941)
$R_{120}$	-2.144 (-2.865)	-1.604 (-1.941)	$\Delta R_{120}$	-7.826 (-2.865)	-7.793 (-1.941)
$R_{180}$	-1.859 (-2.865)	-1.476 (-1.941)	$\Delta R_{180}$	-8.497 (-2.865)	-7.432 (-1.941)
$R_{270}$	-1.606 (-2.865)	-1.249 (-1.941)	$\Delta R_{270}$	-7.577 (-2.865)	-5.734 (-1.941)
$R_{360}$	-1.480 (-2.865)	-1.112 (-1.941)	$\Delta R_{360}$	-10.131 (-2.865)	-9.061 (-1.941)

**Notes:**

- ADF denotes augmented Dickey-Fuller test. DF-GLS denotes GLS detrended Dickey-Fuller by Elliot, et.al. (1996). Lag order selection of these tests is carried out by Akaike Information Criterion by setting the maximum number of lags being equal to 8.

-Critical values are given in parentheses.

- Since interests rate variables are not trended at the levels, unit root test include an intercept term only. Accordingly we do not include an intercept term for the first differences.



**Table 3: Cointegration Rank Statistics (Trace Statistics)**

$H_0$	$H_1$	VAR (1)	VAR (2)	VAR (3)	VAR (4)	VAR(5)	VAR(6)
$r = 0$	$r \geq 1$	1111.028 (0.000)	711.115 (0.000)	566.529 (0.000)	467.667 (0.000)	421.127 (0.000)	380.881 (0.000)
$r \leq 1$	$r \geq 2$	675.351 (0.000)	429.186 (0.000)	301.664 (0.000)	235.302 (0.000)	242.769 (0.000)	219.308 (0.000)
$r \leq 2$	$r \geq 3$	336.036 (0.000)	228.539 (0.000)	146.009 (0.000)	133.126 (0.000)	129.978 (0.000)	125.420 (0.000)
$r \leq 3$	$r \geq 4$	82.944 (0.000)	74.651 (0.000)	52.540 (0.000)	52.134 (0.000)	50.656 (0.000)	55.603 (0.000)
$r \leq 4$	$r \geq 5$	2.689 (0.646)	2.656 (0.656)	2.196 (0.738)	1.951 (0.783)	1.948 (0.784)	2.049 (0.765)

**Note:** p-values are given parentheses.

**Table 4: Asymmetry Tests**

$H_0 \downarrow$	$H_1 \rightarrow$	Linear Asymmetric VECM	MSIH Asymmetric VECM	MSIAH Asymmetric VECM
Linear Symmetric VAR		805.267	3114.302	3201.502
Linear Symmetric VECM		140.892	2449.928	2537.128
MSIH Symmetric VAR			318.842	406.042
MSIAH Symmetric VAR				305.941
MSIH Symmetric VECM			133.423	220.623
MSIAH Symmetric VECM				96.833

**Note:** LR is likelihood ratio tests and associated p-values of the symmetry null hypothesis are indicated in cells, where the unrestricted and restricted models being tested are indicated in columns and rows respectively. The tests are constructed as  $2(\ln L^* - \ln L)$ , where  $L^*$  and  $L$  represent the unconstrained and the constrained maximum log likelihood respectively. These test statistics are asymptotically distributed as  $\chi^2(g)$  under the null hypotheses, where  $n$  is the number of restrictions. We do not report p-values since they all are very close to 0.

**Table 5: MSIH-VECM Estimation: Linearity Tests**

$H_0 \downarrow$	$H_1 \rightarrow$	MSIH Symmetric VAR	MSIAH Symmetric VAR	MSIH Symmetric VECM	MSIAH Symmetric VECM	MSIH Asymmetric VECM	MSIAH Asymmetric VECM
<b>Linear Symmetric VAR</b>		2795.459	2895.560	2980.878	3104.668	3114.302	3201.502
<b>Linear Symmetric VECM</b>				2316.504	2440.294	2449.928	2537.128
<b>Linear Asymmetric VECM</b>						2309.035	2396.235

**Note:** LR is likelihood ratio tests of the linearity null hypothesis are indicated in cells, where the unrestricted and restricted models being tested are indicated in columns and rows respectively. The tests are constructed as  $2(\ln \hat{L} - \ln L)$ , where  $\hat{L}$  and  $L$  represent the unconstrained and the constrained maximum log likelihood respectively. These test statistics are asymptotically distributed as  $\chi^2(g)$  under the null hypotheses, where  $n$  is the number of restrictions. We do not report p-values since they all are very close to 0.

**Table 6: Forecast Accuracies of Different Models (RMSE)**

Forecast Horizon	Linear Symmetric VAR	Linear Symmetric VECM	Linear Asymmetric VECM	MSIH Symmetric VAR	MSIAH Symmetric VAR	MSIH Symmetric VECM	MSIAH Symmetric VECM	MSIH Asymmetric VECM	MSIAH Asymmetric VECM
k=1	0.29138	0.38243	0.30934	0.28232	<b>0.27765</b>	0.28467	0.28692	0.29718	0.29380
k=2	0.31454	0.41523	0.32930	0.31193	<b>0.30906</b>	0.31680	0.33228	0.33393	0.32468
k=4	0.33082	0.40946	0.33756	0.33197	<b>0.32906</b>	0.33858	0.36380	0.36509	0.34153
k=12	0.35362	0.38973	0.36592	0.35419	<b>0.34913</b>	0.37109	0.39693	0.38542	0.38141
k=24	0.33243	0.34974	0.36580	0.33266	<b>0.32727</b>	0.35431	0.37593	0.38950	0.35371
k=36	0.32586	0.33678	0.34400	0.32602	<b>0.32012</b>	0.35399	0.36896	0.39133	0.33764
k=48	0.33176	0.33815	0.34501	0.33178	<b>0.32548</b>	0.36063	0.37294	0.39808	0.33987
k=52	0.33660	0.34191	0.35008	0.33660	<b>0.33054</b>	0.36547	0.37737	0.40350	0.34556

**Note:** RMSE is given by.  $RMSE = \sqrt{\frac{1}{k} \sum_{i=1}^k (y_{t+i} - \hat{y}_{t+i})^2}$ . Where  $\hat{y}_{t+i}$  is the i period-ahead forecast of  $y_{t+i}$ .

**Table 7: DM Statistics**

**(I) Benchmark: MSIAH Asymmetric VECM**

Forecast Horizon	Linear Symmetric VAR	Linear Symmetric VECM	Linear Asymmetric VECM	MSIH Symmetric VAR	MSIAH Symmetric VAR	MSIH Symmetric VECM	MSIAH Symmetric VECM	MSIH Asymmetric VECM
k=1	-0.1687	<b>3.7017</b>	1.2187	-0.9575	-1.5434	-0.8477	-0.5593	0.3771
p-value	0.5670	0.0001	0.1115	0.8308	0.9386	0.8017	0.7120	0.3531
k=2	-0.8223	<b>3.3691</b>	0.4743	-1.0187	-1.3474	-0.6530	0.4935	1.0608
p-value	0.7946	0.0004	0.3176	0.8458	0.9111	0.7431	0.3108	0.1444
k=4	1.0608	-0.8673	<b>2.7713</b>	-0.4055	-0.7280	-1.0406	-0.2117	1.2518
p-value	0.1444	0.8071	0.0028	0.6574	0.7667	0.8510	0.5838	0.1053
k=12	<b>2.7337</b>	-1.9158	0.4984	-2.0167	-1.8404	-2.3512	-0.6138	0.9050
p-value	0.0031	0.9723	0.3091	0.9781	0.9671	0.9906	0.7303	0.1827
k=24	-1.6027	-0.2799	<b>2.6855</b>	-1.5754	-2.1856	0.0440	1.7687	<b>5.7759</b>
p-value	0.9455	0.6102	0.0036	0.9424	0.9856	0.4824	0.0385	0.0000
k=36	-1.0850	-0.0802	<b>2.5888</b>	-1.0695	-1.7589	1.6376	<b>3.9367</b>	<b>11.4224</b>
p-value	0.8610	0.5319	0.0048	0.8576	0.9607	0.0508	0.0000	0.0000
k=48	-1.2755	-0.2852	<b>2.1593</b>	-1.2765	-2.4360	<b>6.1650</b>	<b>8.0147</b>	<b>9.4276</b>
p-value	0.8989	0.6123	0.0154	0.8991	0.9926	0.0000	0.0000	0.0000
k=52	-1.5250	-0.6492	<b>2.0859</b>	-1.5420	-2.7472	<b>4.3913</b>	<b>6.2722</b>	<b>9.6387</b>
p-value	0.9364	0.7419	0.0185	0.9385	0.9970	0.0000	0.0000	0.0000

## (II) Benchmark: MSIAH Symmetric VAR

Forecast Horizon	Linear Symmetric VAR	Linear Symmetric VECM	Linear Asymmetric VECM	MSIH Symmetric VAR	MSIH Symmetric VECM	MSIAH Symmetric VECM	MSIH Asymmetric VECM	MSIAH Asymmetric VECM
k=1	1.1962	<b>5.8954</b>	<b>2.3084</b>	0.8441	1.0972	1.3021	1.3951	1.5434
p-value	0.1158	0.0000	0.0105	0.1993	0.1363	0.0964	0.0815	0.0614
k=2	0.8074	<b>5.4061</b>	<b>2.1280</b>	0.8467	1.6046	<b>3.0252</b>	1.5482	1.3474
p-value	0.2097	0.0000	0.0167	0.1986	0.0543	0.0012	0.0608	0.0889
k=4	1.3474	0.6022	<b>4.5272</b>	1.2605	1.2830	<b>2.0031</b>	<b>3.3167</b>	<b>2.2480</b>
p-value	0.0889	0.2735	0.0000	0.1037	0.0998	0.0226	0.0005	0.0123
k=12	1.0406	<b>2.9063</b>	<b>7.0143</b>	<b>2.1043</b>	<b>3.0998</b>	<b>4.8956</b>	<b>8.5199</b>	<b>2.4565</b>
p-value	0.1490	0.0018	0.0000	0.0177	0.0010	0.0000	0.0000	0.0070
k=24	<b>3.0276</b>	<b>5.8220</b>	<b>3.2034</b>	<b>3.2061</b>	<b>7.4784</b>	<b>13.9689</b>	<b>4.1089</b>	<b>2.1856</b>
p-value	0.0012	0.0000	0.0007	0.0007	0.0000	0.0000	0.0000	0.0144
k=36	<b>4.4275</b>	<b>8.0188</b>	<b>2.3081</b>	<b>4.7427</b>	<b>9.5559</b>	<b>11.0714</b>	<b>5.6382</b>	<b>1.7589</b>
p-value	0.0000	0.0000	0.0105	0.0000	0.0000	0.0000	0.0000	0.0393
k=48	<b>6.0629</b>	<b>6.2269</b>	<b>2.7994</b>	<b>6.5315</b>	<b>7.1073</b>	<b>7.1068</b>	<b>6.8312</b>	<b>2.4360</b>
p-value	0.0000	0.0000	0.0026	0.0000	0.0000	0.0000	0.0000	0.0074
k=52	<b>6.6473</b>	<b>5.5813</b>	<b>3.2062</b>	<b>7.2346</b>	<b>5.9137</b>	<b>6.3549</b>	<b>7.0817</b>	<b>2.7472</b>
p-value	0.0000	0.0000	0.0007	0.0000	0.0000	0.0000	0.0000	0.0030

**Note:** DM statistics is given as  $DM = \frac{\bar{d}}{\sqrt{\widehat{LRV}_{\bar{d}} / N_j}}$ . Where  $\bar{d}$  is an average (over  $N_j$  observations) of

the loss differential function of the RMSE, and  $\widehat{LRV}_{\bar{d}}$  is a consistent estimate of the asymptotic variance of the loss differential function, which is defined as  $LRV_{\bar{d}} = \gamma_0 + 2 \sum_{j=1}^{\infty} \gamma_j$ , where  $\gamma_j = \text{cov}(d_t, d_{t-j})$ . In the table we report the results associated with  $j = 5$ . The qualitatively similar results are obtained with the smaller  $j$ s. The emboldened numbers indicate the best models when compared with the given benchmark.

**Table 8: Reality Check**

Forecast Horizon	Linear Symmetric VAR	Linear Symmetric VECM	Linear Asymmetric VECM	MSIH Symmetric VAR	MSIH Symmetric VECM	MSIAH Symmetric VECM	MSIH Asymmetric VECM	MSIAH Asymmetric VECM
<b>k=1</b>	-0.2914	-0.3824	-0.3093	-0.2823	-0.2847	-0.2869	-0.2972	-0.2938
Prc1	0.0837	<b>0.0000</b>	<b>0.0000</b>	<b>0.0000</b>	<b>0.0000</b>	<b>0.0000</b>	<b>0.0000</b>	<b>0.0000</b>
Prc2	0.0818	<b>0.0000</b>	<b>0.0052</b>	0.1813	0.0993	0.0529	0.0571	0.0884
<b>k=2</b>	-0.3145	-0.4152	-0.3293	-0.3119	-0.3168	-0.3323	-0.3339	-0.3247
Prc1	0.1669	<b>0.0000</b>	<b>0.0000</b>	<b>0.0000</b>	<b>0.0000</b>	<b>0.0000</b>	<b>0.0000</b>	<b>0.0000</b>
Prc2	0.1628	<b>0.0000</b>	<b>0.0057</b>	0.1595	<b>0.0223</b>	<b>0.0001</b>	<b>0.0231</b>	0.0652
<b>k=4</b>	-0.3308	-0.4095	-0.3376	-0.3320	-0.3386	-0.3638	-0.3651	-0.3415
Prc1	0.2838	<b>0.0000</b>	<b>0.0000</b>	<b>0.0000</b>	<b>0.0000</b>	<b>0.0000</b>	<b>0.0000</b>	<b>0.0000</b>
Prc2	0.2747	<b>0.0000</b>	<b>0.0428</b>	<b>0.0414</b>	<b>0.0011</b>	<b>0.0000</b>	<b>0.0003</b>	0.0739
<b>k=12</b>	-0.3536	-0.3897	-0.3659	-0.3542	-0.3711	-0.3969	-0.3854	-0.3814
Prc1	<b>0.0003</b>	<b>0.0000</b>	<b>0.0000</b>	<b>0.0000</b>	<b>0.0000</b>	<b>0.0000</b>	<b>0.0000</b>	<b>0.0000</b>
Prc2	<b>0.0000</b>	<b>0.0000</b>	<b>0.0003</b>	<b>0.0000</b>	<b>0.0000</b>	<b>0.0000</b>	<b>0.0000</b>	<b>0.0000</b>
<b>k=24</b>	-0.3324	-0.3497	-0.3658	-0.3327	-0.3543	-0.3759	-0.3895	-0.3537
Prc1	<b>0.0000</b>	<b>0.0000</b>	<b>0.0000</b>	<b>0.0000</b>	<b>0.0000</b>	<b>0.0000</b>	<b>0.0000</b>	<b>0.0000</b>
Prc2	<b>0.0000</b>	<b>0.0000</b>	<b>0.0000</b>	<b>0.0000</b>	<b>0.0000</b>	<b>0.0000</b>	<b>0.0000</b>	<b>0.0001</b>
<b>k=36</b>	-0.3259	-0.3368	-0.3440	-0.3260	-0.3540	-0.3690	-0.3913	-0.3376
Prc1	<b>0.0000</b>	<b>0.0000</b>	<b>0.0000</b>	<b>0.0000</b>	<b>0.0000</b>	<b>0.0000</b>	<b>0.0000</b>	<b>0.0000</b>
Prc2	<b>0.0000</b>	<b>0.0000</b>	<b>0.0000</b>	<b>0.0000</b>	<b>0.0000</b>	<b>0.0000</b>	<b>0.0000</b>	<b>0.0007</b>
<b>k=48</b>	-0.3318	-0.3382	-0.3450	-0.3318	-0.3606	-0.3729	-0.3981	-0.3399
Prc1	<b>0.0000</b>	<b>0.0000</b>	<b>0.0000</b>	<b>0.0000</b>	<b>0.0000</b>	<b>0.0000</b>	<b>0.0000</b>	<b>0.0000</b>
Prc2	<b>0.0000</b>	<b>0.0000</b>	<b>0.0000</b>	<b>0.0000</b>	<b>0.0000</b>	<b>0.0000</b>	<b>0.0000</b>	<b>0.0000</b>
<b>k=52</b>	-0.3366	-0.3419	-0.3501	-0.3366	-0.3655	-0.3774	-0.4035	-0.3456
Prc1	<b>0.0000</b>	<b>0.0000</b>	<b>0.0000</b>	<b>0.0000</b>	<b>0.0000</b>	<b>0.0000</b>	<b>0.0000</b>	<b>0.0000</b>
Prc2	<b>0.0000</b>	<b>0.0000</b>	<b>0.0000</b>	<b>0.0000</b>	<b>0.0000</b>	<b>0.0000</b>	<b>0.0000</b>	<b>0.0000</b>

**Note:** The number associated with forecasts horizons are the loss differential function of the RMSE. Prc1 and Prc2 are the bootstrap reality check p-values.