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# Transparency, Complementarity and Holdout

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*Abstract:* This paper characterizes the conditions under which holdout (i.e. bargaining inefficiency) may, or may not be significant in a two-sided, one-buyer-many-seller model with complementarity. We address this problem in a very general setup with a bargaining protocol that is symmetric and allows for both publicly observable, as well as secret offers, and a technology that allows for variable degrees of complementarity. The central insight is that the transparency of the bargaining protocol, formalized by whether offers are publicly observable or secret, as well as the extent of complementarity, play a critical role in generating efficiency. Even with perfect complementarity, holdout seems to be largely resolved whenever the bargaining protocol is public (but not if it is secret). Further, irrespective of the bargaining protocol, holdout is resolved if the marginal contribution of the last seller is not too large.

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# 1 Introduction

Many economic activities involve a single buyer seeking to acquire and combine objects from several sellers, e.g. drug development often requires separate patents, land developers have to combine separate plots of land and firms often purchase assets of other firms. Further, firms often bargain with multiple unions, and, in case of financial distress, with multiple creditors. Coase's (1960) famous railroad example considers a situation where a railroad has to acquire plots of land from several farmers.<sup>1</sup>

Given the complementarity inherent in all such activities, received wisdom suggests that the outcome is likely to exhibit *holdout*, with sellers refusing to transact until others have already done so, when commencing production becomes more profitable, allowing the sellers who holdout to extract a greater share of the surplus. In such a scenario holdout is expected to cause inefficiencies, viz. delay, or the implementation of an inefficient project, even, in the presence of strong complementarities, a complete breakdown of negotiation.<sup>2</sup>

We formalize such interactions as a non-cooperative bargaining problem with one buyer and many sellers, focusing on the tension between the complementarity intrinsic to such a setup and efficiency. Our central result is that the presence or absence of holdout (i.e. inefficiency) is critically dependent on the transparency of the bargaining protocol. Even with perfect complementarity, we find that *efficient* equilibria exist whenever buyer offers are publicly observable. In case buyer offers are secret (with a seller only observing her component of the offer), there is bargaining breakdown. Further, in case the technology is less than perfectly complementary, there may be efficient equilibria even if buyer offers are secret.

To this end we consider the interaction between one buyer and  $n \geq 2$  sellers, all of whom have an object to sell. These objects can be combined to produce value. In particular, all sellers have identical objects, with a project using  $m$  objects having value  $v(m)$ . We assume that  $v(m)$  is strictly super-additive, with  $v(0) = 0$  and  $v(n) = 1$ .<sup>3</sup> This formulation allows for different

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<sup>1</sup>Similarly, Cournot analyzed a problem where a brass manufacturer has to buy copper and zinc from two monopoly suppliers.

<sup>2</sup>In the context of land acquisition, many countries, including the USA, have promulgated *eminent domain* laws (that allow land acquisition for public purposes on payment of compensation), presumably to counter this holdout problem. One of our motivating examples comes from West Bengal, India, where the state government used the Land Acquisitions Act, 1894, to acquire land for building an automobile factory for Nano (the one lakh rupee car) in Singur (West Bengal). It has also been argued by some, e.g. Parisi (2002), that problems like excessive fragmentation can be traced, at least partially, to such holdout problems. In the patents literature, Shapiro (2001), suggests that holdout can be serious obstacle to R&D.

<sup>3</sup>Perfect complementarity thus arises as a special case where  $v(m) = 0, \forall m < n$ .

degrees of complementarity, with the buyer being allowed to implement a ‘partial’ project that does not require the use of all objects.

The negotiation process that we consider is a natural extension of the Rubinstein (1982) bargaining model in which the agents, the buyer, as well as the sellers, make simultaneous offers to the other side of the market in alternate periods. Note that this protocol is symmetric in the sense that at no point during negotiation, is an active seller shut out of the bargaining process. Further, the buyer can choose to exit at any period when he can implement a partial project involving the objects collected so far, or opt for an outside option of  $C$  which can be arbitrarily small. The question of interest is the possibility of obtaining equilibria that are *asymptotically efficient*, i.e. the grand project is implemented and delay costs, if any, goes to zero as the discount factor approaches 1.<sup>4</sup>

In order to focus on the holdout problem more sharply, we begin by analyzing the case of perfect complementarity, so that  $v(s) = 0, \forall s < n$ . When buyer offers are public, there exists equilibria that are asymptotically efficient and the buyer obtains a payoff close to  $1/2$ . When buyer offers are secret though, there is bargaining breakdown with the buyer opting out at the first period. These results are of interest for several reasons. First, these establish that protocol transparency is critical as far as efficiency is concerned. Second, in contrast to the literature on one-buyer-many-seller bargaining problems with complementarity, viz. Cai (2000, 2003) and Menezes and Pitchford (2004), which concludes that inefficiency is endemic in such setups (see detailed discussion later), we find that efficient equilibria exist whenever offers are public.

The intuition for these results depends on an interplay of two factors. Suppose the buyer has already acquired  $n - 1$  of the objects. Then the buyer has a strong incentive to conclude bargaining with the remaining seller also. Thus this seller (and thus *ex ante* all sellers) has some bargaining power. On the other hand, the buyer can opt out, which is a potential source of bargaining power for the buyer. When offers are secret however, the buyer cannot credibly commit to opt out of the game. This is because the buyer can make secret agreements with the other sellers, which may reduce his incentive to opt out. With public offers however, such secret agreements are not possible so that threats of opting out are credible. Hence the difference in results.

We then extend the analysis to the case where complementarity is less than perfect, so that  $v(s)$  is not necessarily zero for  $s < n$ . We find that the marginal contribution of the  $n$ -th seller, i.e.  $1 - v(n - 1)$  plays a critical role in the analysis. Whenever the marginal contribution of the last seller is not too large, in the sense that  $v(n - 1) < 1/2$ , we find that the results are

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<sup>4</sup>Given the folk-theorem like results in Chatterjee et al. (1993) and Herrero (1985), the most that we can hope for here is the existence of at least one equilibrium that is efficient, at least asymptotically. See Hyndman and Ray (2007) on this issue though.

completely analogous to those under perfect complementarity. However, when  $v(n - 1) > \frac{1}{2}$ , there is an *efficient* equilibrium where the grand project is implemented in the first period and, moreover, the buyer obtains a payoff of 1. This result holds under both bargaining protocols, public, as well as secret offers. These results show that the assumption that  $v(n - 1) < 1/2$ , provides a precise generalization of the perfect complementarity case.

Moreover, for  $v(n - 1) > 1/2$ , we find that there exists efficient equilibria even when buyer offers are secret. The intuition can be simply stated. If  $v(n - 1) > 1/2$ , then once the buyer reaches an agreement with  $n - 1$  of the sellers, in any continuation game the buyer must obtain at least  $v(n - 1)$ . In fact, given that  $v(n - 1) > \frac{1}{2}$ , the buyer has a strong incentive to terminate the project immediately and implement  $v(n - 1)$ . Since the seller then has a payoff of zero, this consideration considerably reduces the ability of the remaining seller, and *ex ante* of all the sellers, to holdout, allowing the buyer to extract all the surplus from trading.

### 1.1 Relation to Existing Literature

This paper traces its ancestry to one of the most important recent research areas, the theory of coalitional bargaining. Following the seminal work of Rubinstein (1982), as well as the literature on core implementation, researchers have studied the non-cooperative foundations of various cooperative solution concepts, in particular the core and the Shapley value. While Gul (1989) and Hart and Mas-colell (1996) are concerned with the Shapley value, Chatterjee et al. (1993), Serrano (1995) and Krishna and Serrano (1996) study implementing the core. Moreover, while Chatterjee et al. (1993) consider exogenous, but deterministic bargaining protocols, Okada (1996) examines a model with random proposers. There is also a relatively recent branch of this literature that tries to endogenize the process of coalition formation, e.g. Perry and Reny (1994), Bloch (1996), Ray and Vohra (1997, 1999) and Okada (2000), as well as allow for contractual renegotiation, e.g. Seidmann and Winter (1998), Hyndman and Ray (2007), etc.

Formal treatments of the holdout problem (using game theoretic arguments) were first provided in Eckart (1985) and Asami (1988). The theoretical literature was further developed in Cai (2000, 2003) and Menezes and Pitchford (2004). Like us, Cai (2000) and Menezes and Pitchford (2004) analyze a *cash-offer* model in which the seller is paid immediately after an agreement is arrived at. In contrast, Cai (2003) allows the buyer to offer a *contingent contract* that promises to pay the seller a given amount only when production is carried out. While Cai (2000) and Menezes and Pitchford (2004) find that inefficiency in the form of delays must occur in equilibrium, Cai (2003) finds that the buyer payoff is arbitrarily close to zero if the number of sellers is large. financing.

Our results extend the literature by clarifying the role of protocol transparency in obtaining

efficiency, showing that whether holdout is serious or not depends on two factors, first, the nature of the bargaining protocol, and second, on how complementary the sellers are. This paper also contributes to the literature by providing a framework that has some appealing features. First, the bargaining protocol adopted by us is symmetric, second, we allow for outside options (which however can be vanishingly small), and finally, we allow for general production technologies (which however does include perfect complementarity as a special case).

The rest of the paper is organized as follows. Section 2 describes the framework, and, in 2.1, also establishes some preliminary results. Under perfect complementarity, Section 3 examines the case where buyer offers are public, whereas Section 4 examines the case where buyer offers are secret. Section 5 extends the analysis to the case where complementarity is less than perfect. Section 6 concludes. Proofs of some of the propositions and lemmas are collected together in the Appendix.

## 2 The Framework

There are  $n + 1$  agents, one buyer and  $n \geq 2$  sellers. Every seller has an identical object that can be combined to generate returns for the buyer. We write  $v(s)$  to denote the return to the buyer when a project that combines  $s$  objects,  $0 \leq s \leq n$ , is implemented, where the grand project involves combining all the objects.  $v(s)$  is assumed to be non-decreasing in  $s$  and we normalize units such that  $v(0) = 0$  and  $v(n) = 1$ . We assume that  $v(s)$  is *strictly super-additive* in that  $v(n) > v(s) + v(n - s)$  for any  $s$ , where  $1 \leq s < n$ . Consequently implementing any project other than the grand project, is inefficient. The buyer is allowed to implement a project of size zero, which should be interpreted as the buyer exiting the game without acquiring any object. In that case the buyer obtains an outside option of  $C > 0$ .  $C$  can be arbitrarily small and in particular we assume for the rest of the analysis that  $C < 1/2$ .

To begin with, we focus on the case with *perfect complementarity* so that the return to the buyer is 1 if the project combines all  $n$  objects and  $v(s) = 0$ , for all  $s < n$ .<sup>5</sup>

The buyer and the sellers bargain over the price of the objects. The bargaining protocol that we use here is a simple variant of the standard Rubinstein (1982) procedure. Time is discrete and continues for ever, so that  $t = 1, 2, \dots$ . At the start of any period  $t$ , there is a set of ‘active’ sellers who are yet to sell their objects. Each period  $t$  is divided into three sub-stages.

We begin by describing the first two stages. The first stage of  $t$  consists of one side of the market making offer(s) to the other side, followed, in stage two, by the acceptance/rejection decisions of the other side. We assume without loss of generality that the buyer makes his offers

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<sup>5</sup>In Section 4 we extend the analysis to the case where complementarity is less than perfect, and show that some additional results of interest emerge, especially when  $v(n - 1) > 1/2$ .

in odd numbered periods, whereas the sellers make their offers in even numbered ones. Thus at  $t = 1, 3, \dots$ , the buyer offers a price vector to the set of sellers active at that point of time. In Section 3 we consider a bargaining protocol that involves ‘public offers’, i.e. each seller observes the entire vector of offers made by the buyer. In Section 4, we extend the analysis to allow for the case where buyer offers are secret.<sup>6</sup> These sellers then simultaneously decide whether to accept, or reject the offer made to each one. In even numbered periods,  $t = 2, 4, \dots$ , on the other hand, the active sellers simultaneously make their offers. After observing all the offers, the buyer decides which of these offers to accept, if at all. Finally, once a price is agreed upon between the buyer and any seller, the concerned seller immediately receives the agreed upon price and exits the game.

At the third stage of  $t$ , the buyer has the option of implementing a project of size  $k$ , where  $k$  denotes the number of objects that are acquired by the buyer till then, or opting out when he receives  $C$ . If the buyer either implements a project of size  $k$  or opts out, the game is over. When  $k < n$  however, the buyer may continue bargaining when the game goes to the next period, with the number of active sellers being  $n - k$ . We assume, without loss of generality, that if  $k = n$ , then the project is immediately implemented.

Finally, we assume that all agents are risk neutral and that they have a common discount factor  $\delta$ , where  $0 < \delta < 1$ .

For any given play of the game, the history at the beginning of stage  $i$  of date  $t$  includes information on all the past offers, the acceptance/rejection decision of the players and the set of active sellers that are yet to sell their objects. For the public offer game, all such information is common knowledge among the players. For the secret offer game, the set of active sellers at the beginning of stage  $i$  of date  $t$  is common knowledge.

Our focus in this paper is to study sequential equilibria in pure strategies. The central issue addressed here is if, for  $\delta$  large, one can support equilibrium in which the buyer implements the grand project and in which the ‘delay’ cost, if any, is arbitrarily close to zero. We term such equilibrium outcomes as ‘asymptotically efficient’.

## 2.1 Some Preliminary Results

In this sub-section we record some observations that will be used repeatedly in the rest of the paper. Consider a scenario where the buyer has already acquired  $n - 1$  of the objects. When  $\delta$  is large, it is straightforward to extend Osborne and Rubinstein (1990, 3.12.2) to show that the buyer will prefer to reach an agreement with the remaining seller as well, and implement

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<sup>6</sup>The assumption that the buyer makes an offer to every seller, is without loss of generality since the buyer can always make a negative offer to any seller that is surely to be rejected by the seller.

the grand project rather than opt for his outside option ( $v(n-1)$  is of course zero). From Rubinstein (1982), it also follows that in such a case, the continuation payoffs of the proposer and the responder are, respectively,  $\frac{1}{1+\delta}$  and  $\frac{\delta}{1+\delta}$ .

Let  $\delta^*$  satisfy

$$\frac{\delta}{1+\delta} = C.$$

**Lemma 1** *Let  $\delta > \max\{\delta^*, \frac{1}{2}\}$ .*

- (a) *In any equilibrium, if the buyer implements a project of size  $k$ , then  $k \neq n-1$ .*  
(b) *Consider any history that starts with exactly one seller. If  $t$  is odd, the buyer offers the price  $\frac{\delta}{1+\delta}$  which is accepted by the seller, while if  $t$  is even, the seller asks for  $\frac{1}{1+\delta}$ , which is accepted by the buyer.*

The next lemma puts a lower bound on seller payoffs when the buyer is making acceptable offers to all active sellers.

**Lemma 2** *Let  $\delta > \max\{\delta^*, \frac{1}{2}\}$ . Consider any history with  $m \geq 1$  active sellers at date  $t$ , where  $t$  is odd. If the equilibrium calls for the buyer to make an acceptable offer to all  $m$  sellers, then each seller must get at least  $\frac{\delta}{1+\delta}$ .*

*Proof.* Since at  $t$ , the buyer makes an acceptable offer to all existing sellers, if any of these sellers rejects, then at the end of period  $t$ , the buyer would have acquired exactly  $n-1$  objects. From Lemma 1, the negotiation will continue in the next period when the deviating seller has a payoff of  $\frac{1}{1+\delta}$ . Thus by rejecting the offer, given that the rest of the sellers are accepting their offers, any seller can assure himself a payoff of  $\frac{\delta}{1+\delta}$ . ■

Given  $\delta$ , let  $Y_B(m, \delta)$  be the supremum of buyer payoffs in any continuation equilibria beginning from a history with  $m$  active sellers. Similarly, let  $Y_i(m, \delta)$  denote the supremum of seller  $i$ 's payoffs in any continuation equilibria starting from a history with  $m$  active sellers, with seller  $i$  being one of these active sellers.

Our next lemma provides an upper bound on these payoffs.

**Lemma 3** *Let  $\delta > \max\{\delta^*, \frac{1}{2}\}$ . Fix an equilibrium and a history with  $m \geq 1$  active sellers, so that seller  $i$  is one of the sellers active at the point. Then,  $Y_B(m, \delta) \leq \frac{1}{1+\delta}$  and  $Y_i(m, \delta) \leq \frac{1}{1+\delta}$ .*

*Proof.* Please see Appendix A.

### 3 Perfect Complementarity: The Public Offers Case

The main result of this section is that for any sufficiently large  $\delta$ , there are equilibria that are asymptotically efficient and the *maximal* buyer payoff is arbitrarily close to  $1/2$ .



**Proposition 1** (a) If  $\delta > \delta^*$ , then in any equilibrium, the buyer's payoff is at most  $\frac{\delta}{1+\delta}$ .

(b) Fix  $\epsilon > 0$ , then there exists  $\delta(\epsilon) < 1$ , such that for all  $\delta > \delta(\epsilon)$ , there exists an equilibrium in which the buyer implements the grand project at  $t = 2$  and gets a payoff strictly greater than  $\frac{1}{2} - \epsilon$ .

*Proof of Proposition 1.* From Lemma 3, we know that the buyer's payoff in any equilibrium starting with any set of active sellers is at most  $\frac{1}{1+\delta}$ . Since  $\delta > 1/2$ , it also follows from Lemma 2 that at  $t = 1$ , if the buyer was to make an offer that will be acceptable to all sellers, the buyer's payoff will be  $\frac{\delta}{1+\delta}$  if  $n = 2$ , and negative if  $n \geq 3$ . Thus, part (a) of the proposition follows.

To prove part (b) of the proposition, we need

**Lemma 4** For  $\delta$  large, starting from any history  $h_t$  with exactly two active sellers, there is an equilibrium where the buyer obtains  $\frac{C}{\delta}$  at  $t$  even.

While the formal proof of this lemma is available in Appendix B, let us provide a sketch of the construction. Let  $\tilde{P} = \frac{\delta}{1+\delta} - \frac{C}{\delta}$ . At any  $t$  even with two active sellers, the sellers ask for  $\frac{1}{1+\delta}$  and  $\delta\tilde{P}$  respectively. The buyer accepts both these offers. No seller asks for more since, in that case, the buyer will reject both offers and there is transition to a phase where the buyer's present discounted payoff is  $\frac{\delta^3}{1+\delta}$ . Whereas if the sellers do not deviate, then the buyer's payoff is  $C/\delta$  if he accepts both offers,  $C$  if he opts out and  $\delta C$  if he rejects, but continues.

We now continue with the proof of part (b) of the Proposition. We number the sellers 1 through  $n$  and given any seller set  $S$ , the highest ranked seller is referred to as the first seller, while the lowest ranked seller is referred to as the last seller.

We first describe the action profile of the players along the equilibrium path: at  $t = 1$ , the buyer offers zero to all sellers. All sellers but the first one accept. In period 2, the first seller asks for  $\frac{1}{1+\delta}$  which is accepted by the buyer and the grand project is implemented at the end of period 2.

To describe the equilibrium strategy profile, suppose that these have been defined for all histories  $h_t$  that start with  $m$  active sellers where  $m = 1, 2, \dots, n-1$  and let  $m = n$ . If  $t$  is odd, the buyer offers zero to all active sellers.

To define the acceptance/rejection decision of a seller, consider any arbitrary offer vector  $(P_1, \dots, P_n)$ . Consider  $t$  odd. If  $P_1 \geq \frac{\delta}{1+\delta}$ , the first seller accepts  $P_1$ , while the acceptance decision of the sellers numbered 2 through  $n$  is the same as their decision for a history with seller set  $\{2, \dots, n\}$ , for the offer vector  $(P_2, \dots, P_n)$ . On the other hand, if  $P_1 < \frac{\delta}{1+\delta}$ , the first seller rejects while the rest of the sellers accept any non negative offer.

If  $t$  is even, every seller asks for a price of 1. The buyer rejects all these offers. Furthermore, if there is an unilateral deviation by one seller who asks for  $P > 0$ , the buyer continues to reject all of the offers.<sup>7</sup>

Finally, if at the end of any period, if the buyer has acquired  $n$  objects, the project is implemented. If he has acquired  $n - 1$  objects, he continues. In all other cases, he exits and collects his outside option  $C$ . ■

Since offers are publicly observed, it becomes possible for the buyer to make an acceptable offer to  $n - 1$  sellers in a given period and this increases the bargaining power of the buyer and, as Proposition 1 shows, allows him to capture approximately half of the total surplus.

**Remark 1** *While the preceding proposition focuses on establishing the maximum payoff that a buyer can obtain in an equilibrium, it leaves open the question as to whether, it is possible to support (in equilibrium) any buyer payoff in  $(C, \frac{1}{2})$ ? Given the folk theorem like results in Chatterjee et al. (1993) and Herrero (1985), this question is of natural interest. It turns out that a buyer payoff of  $x$  at  $t = 2$ , where  $C < x < 1/2$ , can be supported as follows. The equilibrium involves the buyer making unacceptable offers at  $t = 1$ , and the sellers all asking for  $\epsilon$ , where  $1 - (n - 1)\epsilon = x$  at  $t = 2$ . The buyer accepts all such offers. In case any seller asks for more, the buyer rejects all offers and plays the equilibrium prescribed in Proposition 1.*

**Remark 2** *Interestingly, inefficient equilibria exist as well. For example, for  $n = 2$ , we can sustain an equilibrium where the buyer makes unacceptable offers at  $t = 1$  and opts for his outside option at  $t = 1$ . This is sustained by using the idea in Lemma 4 where if the buyer continues negotiation at  $t = 1$ , his equilibrium payoff from  $t = 2$  perspective is exactly  $C/\delta$  at  $t = 2$ .*

Thus there is a range of equilibria all of which are asymptotically efficient. While inefficient equilibria exist as well, the results show that any inefficiency in outcome must be traced to coordination failures.

**Remark 3** *Interestingly,  $C$  can be provided an alternative interpretation in terms of  $\epsilon$ -rationality. Consider a scenario where there is no outside option, but the agents are indifferent between any*

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<sup>7</sup>To define the strategies of the buyer for any arbitrary history, let  $V(t, m)$  denote the continuation payoff of the players at the end of period  $t$  when  $m$  sellers remain and players play according to the strategies specified above. Consider now a history with  $m \geq 3$  sellers and the offer vector  $(P_1, \dots, P_m)$ . If the buyer accepts the offers made by the seller set  $S'$  with  $|S'| = s$ , then his continuation payoff is  $Y(S') = V(t, m - s) - \sum_{i \in S'} P_i$ . Let  $S^*$  maximize this payoff and let  $s^* = |S^*|$ . The buyer accepts the offers in  $S^*$  if and only if  $Y(S^*) \geq 0$ . He rejects all offers otherwise.

outcomes that involve a difference of less than  $C$ . It can be shown that Proposition 1 goes through under this alternative interpretation.<sup>8</sup>

We then show that when the buyer has no outside option, i.e.  $C = 0$ , the holdout problem is extremely severe as the buyer's payoff in any equilibrium approaches zero when  $\delta$  is close to 1.

**Proposition 2** *Suppose that  $C = 0$ . Fix  $\epsilon > 0$ , then there exists  $\delta(\epsilon) < 1$  such that if  $\delta > \delta(\epsilon)$ , the buyer's payoff in any equilibrium is strictly less than  $\epsilon$ .*

While we relegate the proof of this proposition to Appendix C, we provide the intuition for the result when  $n = 2$ . First, we note that if the buyer makes an offer of  $\frac{\delta}{1+\delta}$  to each of the sellers, these offers will be accepted by the sellers and thus when  $n = 2$ , the buyer's payoff can not be less than  $\frac{1-\delta}{1+\delta}$ , which is strictly positive. Thus, if  $C = 0$ , the requirement of subgame perfection implies that in any equilibrium, the buyer must implement the grand project. However because of Lemma 2, if the buyer has to make an acceptable offer at any date to both of the sellers, each seller must get at least  $\frac{\delta}{1+\delta}$ . Thus  $\frac{1-\delta}{1+\delta}$  is also the maximum payoff that the buyer can receive if he has to make an acceptable offer. Proposition 2 thus follows for  $\delta$  large.<sup>9</sup>

Comparing Propositions 1 and 2, we find that the results are dramatically different depending on whether the buyer has an outside option (however small), or not. While  $C > 0$  is clearly the case of interest, it is useful to understand the intuition behind this difference. When  $C = 0$ , the buyer has a strong incentive to conclude negotiations with the  $n$ -th seller whenever he has reached an agreement with  $n - 1$  sellers. This reduces the buyer's bargaining power, hence Proposition 2. However for  $C > 0$ , no matter how small, the buyer has a credible exit option which allows the buyer to extract approximately half of the surplus. The strength of Proposition 1 lies in that this intuition goes through for any positive outside option, no matter how small.<sup>10</sup>

**Remark 4** *It is important to stress that when  $n > 2$ ,<sup>11</sup> Proposition 2 is critically dependent on our assumption that the players (especially the sellers) are not allowed to randomize in their accept/reject decisions. To see why this is so, consider the situation with  $n \geq 3$  sellers. Assume*

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<sup>8</sup>The proofs for Remarks 1-3 are available on request.

<sup>9</sup>Of course, this argument is incomplete as it leaves open the possibility of an equilibrium outcome where the agreement with the sellers takes place sequentially. The proof in the Appendix formally rules out such possibilities in an equilibrium.

<sup>10</sup>Another paper (though in the context of bargaining with Myersonian obstinate agents) where the presence of even small outside options has a significant impact is Compte and Jehiel (2002).

<sup>11</sup>For  $n = 2$ , however, the proposition holds even when players are allowed to randomize in their choice of strategies.

that in period 1, the buyer makes an offer of  $P' < 1/n$  to each seller, where  $1 - nP' > C$ : each seller accepts this offer with probability  $r^*$  strictly less than 1. Furthermore, the buyer continues negotiation with probability one only if no fewer than two sellers have accepted his offer and he opts out otherwise. Given these strategies, if a seller rejects the current offer  $P'$ , she now faces the risk of getting zero if at least two other sellers reject because in that event, the buyer exits the game. On the other hand, when no more than one other sellers reject her offer, this seller's payoff will be strictly higher than  $P'$ . It thus follows that there exists a value of  $r^*$  for which each seller is indifferent between accepting the offer  $P'$ , or rejecting it. We, however, note that such outcomes are necessarily inefficient as with strictly positive probability, the buyer must exit the game without implementing any project and thus even if mixed strategies are available, when offers are secret, it is impossible to achieve asymptotic efficiency in any equilibrium.

## 4 Perfect Complementarity: The Secret Offers Case

In this section we consider an alternative bargaining protocol where each seller can observe only her component of the buyer's offer and does not know what offers are received by other sellers. We call this the 'secret offer' case. (Chatterjee and Dutta (1998) refer to this as the telephone bargaining setup.) While the public offers case is very natural (and widely adopted in the coalitional bargaining literature), it may be argued that in the context of holdout the secret offers protocol seems equally natural. It is thus of interest that in this case the results appear to be markedly different. In fact, for any sufficiently large discount factor there is complete breakdown of bargaining with the buyer opting for his outside option.

**Proposition 3** *Fix  $C$ . Then there exists  $\delta(C)$  such that if  $\delta > \delta(C)$ , then in any equilibrium, the buyer opts out in period 1.*

The intuition for this proposition can be understood by considering  $n = 2$ . First, observe that if the buyer obtains a payoff strictly greater than  $C$ , then because of strict complementarity, it must be true that the buyer implements the grand project. From Lemma 2, it then follows that if the buyer has to make an acceptable offer to both the sellers, then each seller must be given at least  $\frac{\delta}{1+\delta}$  leaving at most  $\frac{1-\delta}{1+\delta}$  for the buyer. Of course, for  $\delta$  close to 1, this payoff is strictly lower than  $C$ . It is interesting to note that since the offers are secret, there can not be an equilibrium, where the buyer first makes an acceptable offer to only one of the sellers (say seller 1) and then once it is accepted, negotiates with the the second seller. This is because the buyer can always offer the second seller  $\frac{\delta}{1+\delta}$  along with the equilibrium offer to the first seller. Since offers are secret, such an action will not affect the acceptance decision of the first seller. The buyer must be better off from this deviation since the project is implemented one period

earlier and thus saves on the discounting cost. Thus the buyer has the option of implementing the grand project at  $t = 1$  (when he obtains  $\frac{1-\delta}{1+\delta}$ ), or opting for his outside option and getting  $C$ . For  $\delta$  close to 1, opting out is the preferred choice.

*Proof of Proposition 3.* Given  $C$ , let  $\hat{\delta}$  satisfy  $C = \frac{1-\hat{\delta}}{1+\hat{\delta}}$ . Choose now  $\delta > \max\{\delta^*, 1/2, \hat{\delta}\}$ . Since  $\delta > \delta^*$ , Lemmas 1-3 hold. Let  $Y_B(m, \delta)$  denote the supremum of the continuation payoff to the buyer in any equilibrium, starting from a history when  $m$  sellers are active. We first show that  $Y_B(m, \delta) \leq \max\{\frac{1-\delta}{1+\delta}, C\}$  for  $m \geq 2$ . We first note that if  $t$  is odd, then the buyer can always make an offer of  $\frac{\delta}{1+\delta}$  to each of the sellers. By Lemma 3, both sellers must accept yielding a payoff of  $\frac{1-\delta}{1+\delta}$  to the buyer. Opting for the outside option of course gives  $C$  to the buyer.

We first prove this result for  $m = 2$ .

If the claim is false, then there exists an equilibrium in which the buyer's payoff is arbitrarily close to  $Y_B(m, \delta)$ , which in turn is strictly greater than  $\max\{\frac{1-\delta}{1+\delta}, C\}$ . Given strict complementarity, it must be that the buyer implements the grand project in equilibrium.

Clearly, in such an equilibrium, the payoff to at least one of the remaining active sellers (label her seller 1) must be strictly less than  $\frac{\delta}{1+\delta}$ . Let  $t$  be the date at which an agreement with seller 1 takes place. Clearly both sellers must be present at that date by Lemma 1. Now if at  $t$ , seller 1 herself was making an offer, she could have asked for a slightly higher price, the buyer could not have rejected since his payoff in this equilibrium was arbitrarily close to  $Y_B(m, \delta)$ . Therefore, this offer was made by the buyer. Furthermore, at that date, the buyer could not have made an acceptable offer to seller 2 as well. Since then by Lemma 2, seller 1 could have rejected his offer and obtained  $\delta/(1+\delta)$  in the next period. The proof is now complete because if the buyer were to make an immediate offer of  $\frac{\delta}{1+\delta} + \epsilon$  to the second seller along with the prescribed equilibrium offer to seller 1, seller 1 will continue to accept and, because of Lemma 3, seller 2 must accept it as well. Since the agreement now takes place in that period itself, the buyer will save on the discounting cost and thus for  $\epsilon$  small, this must be a profitable deviation. This proves that  $Y_B(m, \delta) \leq \max\{\frac{1-\delta}{1+\delta}, C\}$  for  $m = 2$ .

Assume now as an induction hypothesis that the result is true for all  $m = 2, \dots, n-1$  and let  $m = n$ . If the claim is false, then there is an equilibrium in which the grand project is implemented at some date  $t$  giving the buyer a payoff strictly greater than  $\max\{\frac{1-\delta}{1+\delta}, C\}$ . Because of the induction hypothesis, the number of active sellers at that date must be either  $m = n$ , or  $m = 1$ .

First consider the case  $m = n$ . Since  $n \geq 3$ , and  $\delta > 1/2$ , it follows from Lemma 2 that the buyer could not have made an acceptable offer at that date. Thus, it is the sellers who are making the offers. But then any seller can ask for a slightly higher price which must be accepted

by the buyer.

Thus,  $m$  must equal 1. Label this seller 1. Now at  $t - 1$ , all  $n$  sellers must have been present, otherwise, the induction hypothesis would have applied. Now if  $t$  is odd, then at  $t - 1$ , seller 1 was making an unacceptable offer and the rest were making acceptable offers. If seller 1 at  $t - 1$  asked for  $P \in (\frac{\delta^2}{1+\delta}, \frac{\delta}{1+\delta})$ , the buyer would have accepted this offer. This would thus have been a profitable deviation for seller 1. Therefore  $t$  must be even and at  $t - 1$ , the buyer is making an acceptable offer to only  $n - 1$  sellers. This, however is impossible since the buyer could have secretly offered  $\frac{\delta}{1+\delta}$  to seller 1 along with specified equilibrium offers to the rest of the sellers. All sellers must accept and this will be a profitable deviation for the buyer as he implements the project a period earlier and saves on the discounting cost.

Thus, the continuation payoff to the buyer in any equilibrium is at most  $C$ . ■

Propositions 1 and 3 together provide an important insight for the holdout problem:

*Holdout is severe when buyer offers are secret, but much less so if buyer offers are transparent, i.e. public.*

## 5 Complementarity: The General Case

We now extend our earlier analysis to allow for technologies other than perfect complementarity, so that  $v(s)$  is not necessarily zero whenever  $s < n$ . Given that in this case the buyer has an additional threat of implementing a partial project, it is important to enquire whether the results for the perfect complementarity case is robust to this extension. We first show that our results are completely analogous to those for the perfect complementarity case when  $v(n - 1) < 1/2$ .

### 5.1 $v(n - 1) < 1/2$

When  $v(n - 1) < 1/2$  and offers are public, an exact analogue of Proposition 1 holds; for  $\delta$  close to 1, there exists an equilibrium that is asymptotically efficient and in which the buyer's payoff is arbitrarily close to  $1/2$ . The results, however are somewhat modified when offers are secret. As the following Proposition shows that unlike in the perfect complementarity case, there exists equilibrium outcome which are asymptotically efficient.

**Proposition 4** (a) *Suppose that  $C \geq v(n - 2)$ , then there exists  $\delta(C)$  such that for  $\delta > \delta(C)$ , in any equilibrium the buyer opts out at  $t = 1$ .*

(b) *Suppose that  $C < v(n - 2)$ , then for any  $\epsilon > 0$ , there exists  $\delta(\epsilon)$  such that if  $\delta > \delta(\epsilon)$ , there exists an equilibrium, in which the buyer implements the grand project at  $t = 2$  and in which his payoff is greater than  $v(n - 2) - \epsilon$ .*

The intuition behind Proposition 4(b) is that with  $v(n-2) > C$ , the buyer has the credible threat of implementing a project of size  $n-2$ . This allows us to support an asymptotic efficient equilibrium in which no offers are accepted in period 1, but in period 2, all sellers ask for  $x$  such that  $1-nx = \delta v(n-2)$  which are accepted by the buyer and the grand project is implemented at  $t=2$ . If any seller asks for more, the buyer rejects all offers and makes acceptable offer to exactly  $n-2$  sellers next period and obtain  $v(n-2)$ . The formal proof follows.

*Proof of Proposition 4.* To prove this proposition, one first establishes that for discount factor close to 1, the maximum payoff to the buyer in the secret offer can be no more than  $\max\{C, v(n-2)\}$ . The proof for this follows the same line as that of Proposition 3. Thus, when  $C > v(n-2)$ , in any equilibrium, the buyer must opt out at  $t=1$ . However, when  $v(n-2) > C$ , there exists an equilibrium outcome (for  $\delta$  close to 1) in which along the equilibrium path, the buyer makes an offer that is rejected by every seller. In period 2, each seller then asks for  $x$  where  $1-nx = \delta v(n-2)$ . The buyer accepts all these offers and implements the grand project at  $t=2$ .

We now formally describe the equilibrium strategy profiles for the players.

Fix a history  $h_t$  that ends with the active seller set  $S$  and  $|S| = m$ . When  $m = 1$ , the strategies are specified in Lemma 1 and thus assume that  $m \geq 2$ .

If  $t$  is odd, the buyer offers zero to all sellers. The first two sellers accept an offer  $P_i$  if and only if  $P_i \geq \frac{\delta}{1+\delta}$ . All other sellers accept any nonnegative offer for all  $t \geq 3$ . For  $t = 1$ , however, these sellers accept an offer  $P_i$  if and only if  $P_i \geq \frac{1-\delta}{1+\delta}$ .

If  $t$  is even and  $t \neq 2$ , each seller asks for  $P$  such that  $1-mP = v(n-m)$ . While at  $t = 2$ , each seller asks for  $P^0$  where  $1-nP^0 = v(n-2)$ . The buyer is supposed to accept these offers. If any seller asks for more, the buyer rejects all offers.

Finally, consider the implementation decision of the buyer at the end of any period  $t$  where the buyer has acquired  $k$  objects. The buyer implements the project of size  $k$  if and only if  $v(k) > 0$ . It continues negotiation otherwise.

We observe that given the specified strategies, all sellers in period 1 rejects the buyer's offer. In period 2, each seller offers  $P^0$  and these offers are accepted by the buyer and the grand project is implemented in period  $t = 2$ .

We note that at  $t = 2$ , if any seller demands  $P > P^0$ , the buyer is better off rejecting such an offer. This is because by rejecting all offers at that period, the buyer hopes to get  $v(n-2)$  next period since his offer of zero will be accepted by the last  $n-2$  sellers. Thus, rejection yields a payoff of  $\delta v(n-2)$  to the buyer. If the buyer were to accept the deviating seller's offer of  $P > P^0$ , his payoff is necessarily less than  $\delta v(n-2)$ . Thus, no seller at  $t = 2$  has a profitable deviation. It is also straightforward to check that given the acceptance strategies of the sellers,

the buyer can not hope to make an acceptable offer to a group of sellers and get a higher payoff.

Finally, we note that for  $\delta$  large,  $v(n-2) > \frac{1-\delta}{1+\delta}$  and thus given the offer/acceptance decision of the sellers, for any  $t \geq 3$ , it is always optimal for the buyer to implement the project of size  $n-2$  rather than continuing negotiation with the remaining two sellers. ■

## 5.2 $v(n-1) > \frac{1}{2}$

If  $v(n-1) > 1/2$ , then, an efficient equilibrium exists where the grand project is implemented in period 1 itself, with the buyer extracting the entire surplus. The intuition for this result is the following. Given that  $v(n-1) > 1/2$ , in any continuation game with exactly one active seller, the buyer can credibly threaten to exit, resulting in a zero payoff for the remaining seller (see Lemma 5). Thus, no seller wants to hold out. This allows the buyer to come to an immediate agreement with all of the sellers.

The argument critically relies on the buyer having a credible exit threat once he has acquired  $n-1$  of the objects. The following lemma does exactly that.

**Lemma 5** *Suppose that  $v(n-1) > \frac{1}{2}$  and  $\delta > v(n-1)$ . Consider any history that starts at  $t$  with exactly one active seller. Then there is a continuation equilibrium such that at every  $t$  even, the buyer's payoff is  $\frac{v(n-1)}{\delta}$ .*

While the formal proof can be found in Appendix D, an informal discussion of the action profiles in this equilibrium may be useful. At every  $t$  even, the seller asks for  $1 - \frac{v(n-1)}{\delta}$ , which the buyer accepts, whereas at every  $t$  odd, the buyer offers the seller  $\delta - v(n-1)$ , which the seller accepts. If the seller asks for an amount higher than  $1 - \frac{v(n-1)}{\delta}$ , the buyer rejects and there is transition to a state where at every  $t$  odd the buyer asks for 1 which the seller accepts since otherwise the buyer exits from the game and implements the project of size  $n-1$ .

Using this lemma, one can now show the existence of an equilibrium in which negotiation is concluded in period 1 with the buyer receiving the entire surplus from trade. The strength of this proposition arises from the fact that it holds for *all*  $v(n-1) > \frac{1}{2}$ , and not just for  $v(n-1)$  close to 1.

**Proposition 5** *Suppose  $v(n-1) > 1/2$  and  $\delta > v(n-1)$ . For both public, as well as secret offers, there exists an equilibrium where the buyer implements the grand project at date  $t = 1$  and receives a payoff of 1.<sup>12</sup>*

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<sup>12</sup>Observe that if  $v(s)$  is super-additive,  $v(n-1) > 1/2$  must imply that  $n \geq 3$  since  $v(n) = 1$ . The proof of the proposition however does not rely on this implication.



While the proof can be found in Appendix D, here we outline the structure of the equilibrium profile. At every  $t$  even, the sellers all make unacceptable offers, whereas at every  $t$  odd, the buyer offers a payoff of zero to all sellers. At  $t$  odd, the sellers all accept. If the buyer faces exactly one active seller, he exits the game and implements a project of size  $n - 1$ . Lemma 5 plays a critical role here in ensuring that in this case the buyer is actually indifferent between exiting the game, and continuing to bargain.

**Remark 5** *Can any payoff in  $(0, 1)$  be supported in an equilibrium when  $v(n - 1) > 1/2$ ? The answer is in the affirmative as long as  $\frac{v(s)}{s}$  is increasing in  $s$ . When this condition fails however, say when  $n = 3$ ,  $C = v(1) = 2/5$  and  $v(2) = 3/5$ , a payoff of  $1/15$  is impossible to support in an equilibrium where the grand project gets implemented.*

## 6 Conclusion

This paper characterizes the conditions under which holdout (i.e. bargaining inefficiency) may, or may not be significant in a two-sided, one-buyer-many-seller model with complementarity. The central insight is that the transparency of the bargaining protocol, formalized by whether offers are publicly observable or secret, as well as the extent of complementarity, play a critical role in generating efficiency. Holdout seems to be largely resolved whenever the bargaining protocol is public, and/or the marginal contribution of the last seller is not too large, but not otherwise. Interestingly, this view finds some support in the empirical literature on land acquisition. Benson (2005), for example, discusses examples where private railroads managed to collect the required plots without any government intervention. In the Indian context, for example, while the Nano project in Singur, West Bengal ran into problems, around the same time there were many instances of trouble free land acquisition by private agents, even in West Bengal.

In a somewhat broader context, this paper has some implications for the Coase theorem. While it is well known that informational problems can lead to inefficiencies, the literature on coalitional bargaining has identified strategic issues that may, even in the absence of informational issues, cause the Coase theorem to fail.<sup>13</sup> While one response to such strategic inefficiency has been to study random bargaining protocols, e.g. Okada (1996), another line of research examines bargaining protocols with renegotiation, Seidmann and Winter (1998), Hyndman and Ray (2007), etc. In this paper, however, we examine a *deterministic* (though symmetric) bargaining protocol that does *not* allow for renegotiation. Remarkably enough, even then we find

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<sup>13</sup>Chatterjee et al. (1993), Bloch (1996) and Ray and Vohra (1997, 1997), among others, have pointed out the role of renegotiation in this context.

that there is *some* equilibrium that is *asymptotically efficient*, as long as the buyer offers are publicly observable. Further, this holds even with perfect complementarity.

The analysis in this paper focuses on simple ‘unconditional cash offer contracts’ in which if a seller accepts an offer by the buyer, the seller sells the object and exits the game. This leads to the holdout problem. The results (for the public offer case) are thus strong in that even in the class of these simple contracts, it is possible to support efficiency, as well as a strictly positive payoff for the buyer. Clearly, in such situations, use of complex contracts is unnecessary. For situations where the holdout problem does bite (as in the secret offer case with perfect complementarity) however, it may be of interest to know whether alternative contractual forms may restore efficiency. The answer to this question is in the affirmative. Consider a situation where the buyer can make ‘conditional offer’ to the sellers. Under a conditional offer, the buyer buys only when every seller agrees to sell, and not otherwise. Such contracts completely mitigate the holdout problem as one can show that for sufficiently patient players, for any  $x \in [0, 1] > 0$ , it is possible to support an equilibrium where the grand project is implemented in period 1 and the buyer’s payoff is  $x$ .<sup>14</sup>

## 7 Appendix

### 7.1 Appendix A: Lemma 3

*Proof of Lemma 3.* By Lemma 1, the result is clearly true for  $m = 1$ . So assume an induction hypothesis that the result is true for histories that begin with  $m = 1, \dots, n - 1$  active sellers. Consider now an history that begins with  $m = n$  active sellers.

(i) We first argue that  $Y_i(n, \delta) \leq \frac{1}{1+\delta}$  for any seller  $i$ . Suppose not. Then there exists some seller  $i$  for whom  $Y_i(n, \delta) > \frac{1}{1+\delta}$ . This implies that there is an equilibrium outcome in which an offer  $P_i$  is agreed upon by the buyer and seller  $i$  at some date  $t$  (following this history) giving seller  $i$  a payoff that is arbitrarily close to  $Y_i(n, \delta)$  which in turn is strictly greater than  $\frac{1}{1+\delta}$ . Now because of the induction hypothesis, all sellers must be active at this date.

Now if the buyer was making this offer, then by Lemma 2, each of the remaining seller  $j$  must be getting an offer  $P_j \geq \frac{\delta}{1+\delta}$ . If the buyer deviates by offering  $P_i - \eta$  to seller  $i$ , seller  $i$  must accept for  $\eta$  small. Now if the remaining sellers were offered  $P_j + \epsilon$  where  $(n - 1)\epsilon < \eta$ , then all such sellers must accept it also. Clearly, this will constitute a profitable deviation for the buyer.

Therefore, it is seller  $i$  who is asking for  $P_i$  which is accepted by the seller. We first claim that the game must be over after this date with the buyer implementing a project. Otherwise,

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<sup>14</sup>A proof is available on request.

the game continues in the next period, when, by our induction hypothesis, the buyer's payoff in the continuation equilibrium is at most  $\delta/(1 + \delta)$ . Since  $P_i > \frac{1}{1+\delta}$ , the buyer's payoff in this equilibrium is thus negative. Now we claim that all of the remaining sellers must be making an acceptable offer at this date as well. Otherwise, the buyer is implementing a partial project, and moreover, since  $v(n - 1) < 1/2$ , the buyer's payoff then is negative in this equilibrium. Thus, at this date, all sellers make acceptable offers of  $P_j$  resulting in a payoff of  $1 - \sum_j P_j$ . Suppose the buyer deviates, rejects seller  $i$ 's offer, but accepts the rest of the offers. Then in period  $t + 1$ , the buyer will offer  $\frac{\delta}{1+\delta}$  to seller  $i$  which, by Lemma 1, must be accepted by seller  $i$ . Such a deviation will yield a payoff of  $\frac{\delta}{1+\delta} - \sum_{j \neq i} P_j$ . Since  $P_i > \frac{1}{1+\delta}$ , the buyer will be better off from this deviation. This establishes that the supremum of equilibrium seller payoff for any seller is  $\frac{1}{1+\delta}$ .

(ii) We now show that  $Y_B(n, \delta) \leq \frac{1}{1+\delta}$ . Otherwise,  $Y_B(n, \delta) > \frac{1}{1+\delta}$  and thus there is an equilibrium outcome in which an agreement is reached in period  $t$  with the buyer getting a payoff arbitrarily close to  $Y_B(n, \delta)$ , which in turn is strictly greater than  $\frac{1}{1+\delta}$ . Since  $v(n - 1) < 1/2$  and the payoff to each seller must be non-negative, it must be that in this equilibrium, the buyer implements the grand project. Furthermore, at  $t$ , the number of active sellers must be  $n$ , otherwise, the induction hypothesis will apply. Now if  $t$  is odd, then it is the buyer who is making an acceptable offer to all of the  $n$  active sellers. Since  $n \geq 2$ , by Lemma 2, the buyer's payoff is no more than  $\frac{1-\delta}{1+\delta}$ , a contradiction. Therefore, at  $t$ , it is the sellers who are making these acceptable offers. Since the payoff to the buyer is more than  $\frac{1}{1+\delta}$  and  $n \geq 2$ , at least one of the sellers is making an offer  $P_j$  that is strictly less than  $\frac{\delta}{1+\delta}$ . If this seller deviates and asks for a slightly higher price, the buyer must accept. This is because by accepting all the offers, his payoff will be arbitrary close to  $Y_B(n, \delta)$  while if he rejects all offers, then his payoff is at most  $\delta Y_B(n, \delta)$ . Finally, if he accepts only a subset and continues, by induction hypothesis, his payoff from tomorrow is at most  $\frac{\delta}{1+\delta}$ . Thus, seller  $j$  has a profitable deviation. ■

## 7.2 Appendix B: Lemma 4

*Proof of Lemma 4.* Choose  $\delta$  such that  $\frac{\delta^3}{1+\delta} > C$ .

For any history with  $m = 1$  active seller, the strategy is given by Lemma 1. Consider now any history  $h_t$  that starts with  $m = 2$  active sellers and at the start of  $t - 1$ , the number of active sellers is greater than two.

Let  $\tilde{P}$  satisfy  $\frac{\delta}{1+\delta} - \tilde{P} = \frac{C}{\delta}$ . Since  $C < 1/2$ , for  $\delta$  close to 1,  $\tilde{P}$  is strictly positive. The strategies of the players in the continuation equilibrium will depend on the phase it is in,  $A$  or  $B$ . The first period always starts in phase  $A$ .

### Strategies in Phase A.

If  $t$  is odd, the buyer offers zero to the first seller and  $\delta\tilde{P}$  to seller 2. The first seller accepts an offer  $P_1$  if and only if  $P_1 \geq \frac{\delta}{1+\delta}$ . Seller 2 accepts an offer  $P_2$  if and only if  $P_2 \geq \delta\tilde{P}$  and  $P_1 < \frac{\delta}{1+\delta}$ .

If  $t$  is even, the first seller asks for  $\frac{1}{1+\delta}$ , while seller 2 asks for  $\tilde{P}$ . These offers are accepted by the buyer. If any seller asks for more, the buyer rejects both the offers.

In this phase, the buyer always continues negotiation if he has not acquired all objects and the period is odd.

If however, the period is even and the buyer has less than  $n - 1$  objects, he will continue negotiation only if one of the sellers in that period have deviated and asked for a higher price. He will opt out otherwise. Thus, if the sellers did not deviate from their offer strategies but the buyer rejected both of the offers, the buyer will opt out.

**Transition.**

If  $t$  is even, and one of the sellers deviate from the above strategies and the buyer rejects both of the offers, there will be a transition from phase  $A$  to phase  $B$ .<sup>15</sup> By construction, in phase  $B$ , it is the buyer who has to make an offer.

**Strategies in Phase B.**

The buyer makes an offer of zero to both players. Seller 2 accepts any non-negative offer if  $P_1 < \frac{\delta}{1+\delta}$ . While seller 1 accepts an offer if and only if  $P \geq \frac{\delta}{1+\delta}$ .

In stage  $B$ , at the end of the period, the buyer continues negotiation only if he has acquired  $n - 1$  objects. He will opt out otherwise.

The state stays in phase  $B$  for precisely one period and will revert to phase  $A$  in the following period.

Observe that in phase  $A$ , the payoff to the buyer is exactly  $C$  when  $t$  is odd, and it is  $\frac{C}{\delta}$ , if  $t$  is even. Thus, at the end of an odd period, the buyer's discounted payoff by continuing is exactly  $C$ . He also gets  $C$  by opting out, and thus it is optimal for him to continue in odd periods. If  $t$  is even, however and the sellers did not deviate from their equilibrium strategies, phase  $A$  will continue and thus if the buyer continues he will get exactly  $\delta C$  whereas by opting out he gets  $C$ , thus, he is better off opting out. Finally, if  $t$  is even and the state is going to be in phase  $B$ , then by rejecting all offers today and continuing next period, the buyer expects to get  $\frac{\delta^3}{1+\delta}$  which is strictly greater than  $\frac{C}{\delta}$  and thus the buyer will be better off continuing negotiation.

To check that the strategies of the sellers are optimal, it is sufficient to consider seller 2's offer when  $t$  is even. If she asks for any price greater than  $\tilde{P}$ , given that seller 1 is asking for

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<sup>15</sup>It is important to note that the transition to phase  $B$  takes place only if one or both sellers deviate from their prescribed strategies.

$\frac{1}{1+\delta}$ , phase  $A$  will transit to phase  $B$ . In phase  $B$ , the buyer will offer zero to her and the buyer will exit at the end of phase  $B$  if the seller rejects. Note that if at the end of phase  $B$ , the set of active seller continues to number two, the buyer will opt out. ■

### 7.3 Appendix C: Proposition 2

*Proof of Proposition 2.* We will show that  $Y_B(m, \delta)$ , the supremum of buyer's payoff in any equilibrium is in fact bounded above by  $\frac{1-\delta}{1+\delta}$  for all  $\delta > \delta^*$  and  $m \geq 2$ .

So let  $m = 2$ . Since  $\delta > \delta^*$ , Lemmas 1-3 hold. In particular, by Lemma 1, we have that in any equilibrium with a positive payoff for the buyer, the grand project must be implemented. We will show that  $Y_B(2, \delta) \leq \frac{1-\delta}{1+\delta}$  for all  $\delta > \delta^*$ . We first note that if  $t$  is odd, then the buyer can always make an offer of  $\frac{\delta}{1+\delta}$  to each of the sellers. By Lemma 3, both sellers must accept and the buyer has a positive payoff. Thus, in any equilibrium, the buyer can never exit without acquiring any object as that will give him a zero payoff.

If the claim is false, then there exists an equilibrium in which the buyer's payoff is arbitrarily close to  $Y_B(2, \delta)$ , which in turn is strictly greater than  $\frac{1-\delta}{1+\delta}$ .

Clearly, in such an equilibrium, the payoff to at least one of the sellers (label him seller 1) must be strictly less than  $\frac{\delta}{1+\delta}$ . Let  $t$  be the date at which an agreement with seller 1 takes place. Clearly both sellers must be present at that date by Lemma 1. Now if at  $t$ , seller 1 herself was making an offer, she could have asked for a slightly higher price, the buyer could not have rejected since his payoff in this equilibrium was arbitrarily close to  $Y_B(2, \delta)$ . Therefore, this offer was made by the buyer. Furthermore, at that date, the buyer could not have made an acceptable offer to seller 2 as well. Since then by Lemma 2, seller 1 could have rejected his offer and obtained  $\delta/(1+\delta)$  in the next period. Thus, agreement with seller 2 takes place at  $t+1$ . If  $P$  is the price offer made to seller 1, the payoff to the buyer in this equilibrium, then, is given by

$$K = -P + \frac{\delta^2}{1+\delta}.$$

This follows since at  $t+1$ , the seller makes the offer and by Lemma 1, the buyer obtains  $\delta/(1+\delta)$ . Since  $K$  is arbitrarily close to  $Y_B(2, \delta) > 0$ , we must have

$$P < \frac{\delta^2}{1+\delta}. \tag{1}$$

Suppose seller 1 rejects the buyer's offer. Since the buyer can not exit the game without acquiring the object, at  $t+1$ , let seller 1 make a counter offer of  $P' = \frac{P}{\delta} + \epsilon$ . Clearly, acceptance of this offer will be a profitable deviation for seller 1. We now argue that this offer will be accepted by the buyer for  $\epsilon$  small. If the buyer were to accept seller 1's offer, his overall payoff

(from period  $t + 1$  perspective) can not be less than

$$Y = \frac{-P}{\delta} - \epsilon + \frac{\delta}{1 + \delta}.$$

This is because the buyer can always reject seller 2's offer and in  $t + 2$  offer  $\delta/(1 + \delta)$  to her which, by Lemma 1, will be accepted by seller 2. On the other hand, if he were to reject this offer, then the maximum that he can obtain is  $\delta P_\delta^*$  which is arbitrarily close to  $\delta K$ . It is easy to check that  $Y > \delta K$  if and only if  $P < \frac{\delta^2}{1 + \delta}$  which is true because of (1).

This proves that  $Y_B(2, \delta) \leq \frac{1 - \delta}{1 + \delta}$ .

Assume now as an induction hypothesis that the result be true for all  $m = 2, \dots, n - 1$ . If the claim is false, then there is an equilibrium in which the grand project is implemented at some date  $t$  giving the buyer a payoff arbitrarily close to  $Y_B(m, \delta)$ , which in turn is strictly greater than  $\frac{1 - \delta}{1 + \delta}$ . Because of the induction hypothesis, the number of active sellers  $m$  can be either  $m = n$ , or  $m = 1$ .

First consider the case  $m = n$ . Since  $n \geq 3$ , and  $\delta > 1/2$ , it follows from Lemma 2 that the buyer could not have made an acceptable offer at that date. Thus, it is the sellers who are making the offers. But then any seller can ask for a slightly higher price which must be accepted by the buyer.

So it must be that  $m = 1$ . Label this seller 1. Now at  $t - 1$ , all  $n$  sellers must have been present, otherwise, the induction hypothesis would have applied. Now if  $t$  is odd, then at  $t - 1$ , seller 1 was making an unacceptable offer. If seller 1 at  $t - 1$  asked for  $P \in (\frac{\delta^2}{1 + \delta}, \delta/(1 + \delta))$ , the buyer would have accepted this offer. This would thus have been a profitable deviation for seller 1. Therefore  $t$  must be even. Thus seller 1 is asking for  $P = \frac{1}{1 + \delta}$  at date  $t$ . The buyer's payoff from  $t - 1$  onwards is thus  $\frac{\delta^2}{1 + \delta} - \sum_{i \neq 1} P_i$ . Since  $P_i \geq 0$  and the buyer's payoff is strictly greater than  $\frac{1 - \delta}{1 + \delta}$ , for every seller  $i \neq 1$ , we have

$$P_i < \frac{\delta^2}{1 + \delta} - \frac{1 - \delta}{1 + \delta}. \quad (2)$$

If any of these sellers (say seller 2) rejected the buyer's offer at period  $t - 1$ , given that the other sellers are accepting their respective offers, the next period would begin with exactly two active sellers. In the continuation game, with exactly two sellers present, the buyer will offer exactly  $\delta/(1 + \delta)$  to each of them. Thus, the worst payoff to seller 2 following this deviation is that the agreement takes place in  $t + 1$  with the buyer making an offer of  $\delta/(1 + \delta)$  to seller 2. Thus, from the perspective of period  $t - 1$ , seller 2 can assure himself a payoff of  $\frac{\delta^3}{1 + \delta}$ . Since  $\delta < 1$ , it follows that  $\frac{\delta^3}{1 + \delta} > \frac{\delta^2}{1 + \delta} - \frac{1 - \delta}{1 + \delta} > P_2$ . The last inequality follows from (2). This will thus be a profitable deviation for player 2. ■

## 7.4 Appendix D: Lemma 5 and Proposition 5

*Proof of Lemma 5.* The proof involves constructing an equilibrium where at every  $t$  even, the seller asks for  $\frac{v(n-1)}{\delta}$ , which the buyer accepts. The strategies are conditional on whether the game is in either of two phases A, or B.

In *phase A*, at every  $t$  odd the buyer offers  $\delta - v(n-1)$  to the seller, and the seller accepts if and only if she obtains at least  $\delta - v(n-1)$ . Whereas at every  $t$  even, the seller asks for  $1 - \frac{v(n-1)}{\delta}$ . The buyer accepts if and only if he obtains at least  $\frac{v(n-1)}{\delta}$ .

In this phase, the buyer always continues negotiation in case an offer is rejected.

Finally, there is *transition* to *phase B* if the seller asks for more than  $1 - \frac{v(n-1)}{\delta}$ .

In *phase B*, at every  $t$  odd the buyer offers  $(1, 0)$ , and the seller accepts if and only if she obtains at least 0. Whereas at every  $t$  even, the seller offers  $(\delta, 1 - \delta)$ . The buyer accepts if and only if he obtains at least  $\delta$ ,

If  $t$  is even and an offer is rejected by the buyer, the buyer continues negotiation.

If however,  $t$  is odd and the seller rejects buyer's offer, the buyer implements the project of size  $n-1$ . In this case, if the buyer fails to implement the project and continues, phase *B* transits to phase *A*. ■

*Proof of Proposition 5.* The proof involves constructing equilibrium profiles such that at every  $t$  even, the sellers all make unacceptable offers, whereas at every  $t$  odd, the buyer offers a payoff of zero to all sellers. At  $t$  odd, the sellers all accept since otherwise the other sellers accept, and the buyer exits the game and implements a project of size  $n-1$ . We now formally describe the strategies.

For any history that starts with exactly one active seller, the strategies of the players are as specified in the proof of Lemma 5. For any history that starts with  $m$  sellers,  $m > 1$ , the strategies are as follows:

If  $t$  odd, the buyer offers zero to each of the active sellers. Each seller accepts any non-negative offer.

At the end of period  $t$  where  $t$  is odd, the buyer implements the project only if he has acquired at least  $n-1$  objects. He continues otherwise.

If  $t$  even, each seller asks for  $P = 1$ . Given any offer vector  $P = (P_1, P_2, \dots, P_m)$ , let  $Z = 1 - \sum_{i \in M} P_i$ , where  $M$  is the set of active sellers. The buyer accepts every offer if  $Z \geq \delta$ . If  $Z < \delta$ , he rejects all offers.

At the end of period  $t$ , where  $t$  is even, the buyer implements the project only if he has acquired all the objects. He proceeds to the next period otherwise.

Consider a subgame with  $t$  odd, where  $n-1$  of the offers have been accepted by the sellers. The buyer's payoff from implementing a project is  $v(n-1)$ , whereas if he continues to negotiate,

then he obtains  $\frac{v(n-1)}{\delta}$  in the next period (Lemma 5), so that opting out immediately is optimal. Further, note that these strategies work for the secret, as well as the public offer game. ■

## 8 References

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