View metadata, citation and similar papers at core.ac.uk
brought to you by CORE
provided by Munich Personal RePEc Archive

MPRA
Munich Personal RePEc Archive

Growth and social capital: an evolutionary model

L Correani and F Di Dio and G Garofalo

University of Tuscia in Viterbo - Department Distateq

2009

Online at http://mpra.ub.uni-muenchen.de/17043/
MPRA Paper No. 17043, posted 2. September 2009 20:29 UTC

GROWTH AND SOCIAL CAPITAL:
AN EVOLUTIONARY MODEL

Abstract: In this paper, we analyze the role of cooperation between firms through a model of growth and social capital. In a growth model *à la* Solow we incorporate the set of resources that a relational network has at its disposals, as a distinct production factor, and thus examine its dissemination through evolutionary type processes in firm interactions. Dynamic analysis of the model demonstrates that cooperation is able to increase the productivity of factors, fostering a higher rate of growth in the long term. The most significant result is that scarcity of social capital can produce a general collapse of the economic system in areas in which long term growth is usually sustained by the *learning by doing* and *spillover of knowledge* phenomena. This conclusion leads to reconsider the role of local development economic policies that should concentrate on activities that promote repeated interaction between firms proven to be cooperative or that encourage the formation of technological consortia.

Growth and social capital: an evolutionary model

1. Introduction

The promotion of efficient social and institutional structures facilitates an exchange of knowledge and skills that in turn promotes cooperation between economic actors (North, 1990). An efficient network of relations between production units may, in fact, create momentum for the productive sector and improve the performance of firms within it with respect to those outside of it. The amount of additional resources provided by the relational network identifies the *social capital* that is able to promote the dissemination of trust and cooperation between economic agents.

A concept spreading in more recent literature is that the study of the growth of an economic system must also take into consideration social capital as a separate production factor, together with physical, human and technological capital.

An example of this new approach can be found in Chou [2006] who includes social capital in the growth model *à la* Lucas. This acts on growth through accumulation of human capital, financial development and the dissemination of new technologies. Conversely, Antoci-Sacco-Vanin [2007] and Francois-Zabojnik [2006] develop evolutionary models to explain the dynamics of social capital and trust. However, the conclusions drawn in the latter two studies are discordant: while in Francois-Zabojnik's study capital has positive effects on growth, in Antoci-Sacco-Vanin's it provokes a reduction, inasmuch as it subtracts resources from private consumption, even if equilibrium with a high level of social capital is Pareto-dominant.

In our contribution we propose an implementation of the approaches just mentioned, by merging the traditional growth analysis with the study of the dynamics of cooperation. To achieve this we incorporate the fundamental elements of the Solow growth model in a context of strategic interaction between firms based on a game of coordination[1]. Each firm belonging to the network system can adopt cooperative or non-cooperative strategies; the strategic approach that on average ensures the best outcome tends to be adopted (imitated) by a growing number of firms. Each disposes the same social capital that is however only usable in mutual collaboration with another firm of the system.

The model therefore integrates elements from the sociological approach (Bordieu [1986]), which interprets social capital as a phenomenon that springs from collective cooperation (*actual* social capital), with aspects of North's [1990] institutionalist theory, according to which (*potential*) social capital is the set of institutions and rules of an economic system.

The model, while maintaining the basic growth structure *à la* Solow is developed in two different versions. In the first, a traditional sector with diminishing returns of physical capital is considered. The model has two possible

[1] We selected the Solow model because it is the most appropriate to focus attention on the role of firm cooperative strategies in a context of interaction, omitting considerations on aspects related to intertemporal consumption, human capital etc..

equilibria in the long term, both characterized by nil per capita growth rates but with different levels of cooperation. Given that the latter allows using the additional social capital factor, the equilibrium in which it is more widespread dominates the other in a Paretian sense; convergence towards Pareto-dominated equilibrium is not excluded, which occurs when the level of cooperation is below a threshold value.

The second version hypothesizes the existence of *learning by doing* and *spillover* knowledge processes, hypotheses that are particularly plausible if the subject of the study is a system of firms that share resources and information due to their geographical proximity and affinity of production activities. In this case, the model is able to generate constant returns of physical capital and therefore long-term endogenous growth[1]. In this second variant an unexpected fact emerges: the economic system, although having long term growth potential, may find itself in a 'poverty trap' and experience a general collapse with the annulment of output and disintegration of the network of firms when the levels of potential social capital and cooperation are not sufficiently high. This is a significant consequence that grants social capital and cooperation a leading role in sustaining the more advanced productive sectors where the roles played by learning processes and dissemination of knowledge are key.

In conclusion, the models developed demonstrate that a traditional production system can continue to exist in a steady-state with low levels of social capital and cooperation, while the most technologically advanced sectors consider these factors as indispensable to their survival.

2. A model of growth and social capital

Let us assume that the economic system is composed of a set of production units, each of which can achieve output by acting individually or sharing resources with another production unit (cooperative strategy). This is a strategy based on the trust of the other economic agent with whom interaction is invoked. If cooperation is reciprocal, output by each will be greater than that which can be obtained by acting individually. The greater output is essentially due to an additional production factor that we define as social capital $K_s \geq 0$, whose level depends on the quality of the firm's institutional, cultural and technological fabric.

However, two levels of social capital exist: *potential* social capital, distributed evenly between firms, which expresses all the resources available to the network, and *actual* social capital, which is that actually used in production processes through cooperation.

The potential social capital will thus equal $K_s n$, the actual capital $K_s nx$ with n as the number of firms present in a local network and x the part of the potential social capital transformed into actual capital, or rather, the share of firms that agree to cooperate ($0 \leq x \leq 1$). Even if the system can potentially use a level of actual social capital equal to $K_s n$ only a fraction x of it is active i.e. used in the production process.

[1] From literature emerges (see Barro and Sala-i-Martin, 1995) that the absence of diminishing returns of capital allows obtaining endogenous growth.

The amount $K_s n(1-x)$ thus remains unexpressed as a result of a share $(1-x)$ of non-cooperative firms. Non-cooperation produces a social cost that is quantifiable in the amount of unused social resources.

The population of economic agents can thus be divided into two sub-populations based on the type of strategic behavior adopted. This is a typical game of coordination, as illustrated in Table 1.

G_1 / G_2	Cooperative	Non-cooperative
Cooperative	$y_1 ; y_1$	$0; y_2$
Non-cooperative	$y_2 ;0$	$y_2 ; y_2$

Table 1. Game of coordination

In this context the payoff of the game represents the output of the firm itself obtained through a function of production: $y_1 = y_1(A_1, k)$ and $y_2 = y_2(A_2, k)$ are, respectively, the production functions of cooperative and non-cooperative firms, in which $A_1 = A_2 + K_s$, $y_1 = y_2$ if $K_s = 0$ and $y_1 = y_2 = 0$ if $k = 0$, while A_2 is the exogenous level of technology. Thus, by construction $y_1 > y_2$.

The physical capital is essential to production while the social capital simply improves productivity. In the absence of social capital, the cooperative strategy is weakly dominated by the individualistic strategy. By calculating the game's average payoff equal to $y_m = x^2 y_1 + (1-x) y_2$, the dynamic equation of physical capital available in each single firm can be written as:

(1) $$\dot{k} = s\left(x^2 y_1 + (1-x) y_2\right) - \delta k$$

with s as the share of output invested in the production processes, δ the rate of depreciation of the physical capital and x the share of cooperative firms. To (1) the dynamic equation of the share of the cooperative population is added, based on replicator logic (for further information see Weibull (1998)). Indicating the cooperative firm's expected payoff with $\Pi_1 = x y_1$ and the average payoff of the entire system with $\Pi_{av} = x^2 y_1 + y_2(1-x)$, the replicator equation is $\dot{x} = x\left\{\Pi_1 - \Pi_{av}\right\}$ which, following some simple steps, becomes:

(2) $$\dot{x} = x(1-x)\left\{x y_1 - y_2\right\}$$

Variations in the increase of value x entail increases in the social capital, in the sense that the full use of the local firm network's potentiality (hereinafter referred to as the local system), defined by K_s, spreads to a growing number of production units, with positive effects on the accumulation of physical capital and growth of output.
In the following sections we proceed with the study of the dynamic system described with equations (1) and (2), first hypothesizing diminishing returns and then constant returns of physical capital.

2.1 A model with diminishing returns of physical capital

Let us hypothesize that capital returns are diminishing, or rather, that:

$$\frac{\partial y_i}{\partial k} > 0 \text{ e } \frac{\partial^2 y_i}{\partial k^2} < 0 \ \ (i = 1,2)$$

We further assume that the Inada conditions holds and that the production factors are first-degree homogenous in A_i and k. We also assume that:

$$\frac{\partial y_1}{\partial k}\frac{k}{y_1} = \frac{\partial y_2}{\partial k}\frac{k}{y_2} \quad \forall k$$

With these assumptions, it is possible to demonstrate (Proposition 1) that, as in Solow's model, diminishing returns of capital determine conditioned convergence and a gradual reduction of output increments. In the long term, very different transition dynamics accompany the reduction of capital growth rates, according to the initial state. Therefore, local systems with identical technological-institutional characteristics but with different levels of cooperation can converge towards totally different states, highlighting in time a gap of wealth produced and capital accumulated that can persist or deteriorate when not intervening with targeted economic policies.

Proposition 1: the system described in equations (1) and (2) has the following stationary points in space (x, k):

1. ($x = 0,\ k = \breve{k}$) locally stable;
2. $\left(x = 1, k = \hat{k}\right)$ locally stable;
3. $\left(x = \frac{y_2}{y_1},\ k = \tilde{k}\left(\frac{y_2}{y_1}\right)\right)$ saddle point, where $\tilde{k}(x)$ is a convex function with $\partial\tilde{k}/\partial x > 0$ if $x > y_2/2y_1$ and $\partial\tilde{k}/\partial x < 0$ if $0 < x < y_2/2y_1$.
4. ($0 \le x \le 1,\ k = 0$) unstable.

Furthermore, if $x < \frac{y_1}{y_2}$, the system converges towards the attractor $(0, \breve{k})$; if $x > \frac{y_1}{y_2}$, the system converges towards the attractor $(1, \hat{k})$. **Proof (see Appendix A)**

With diminishing returns of physical capital the long term growth rate is zero since the system converges to one of the two stationary points $(0, \breve{k})$ and $(1, \hat{k})$ in which $\dot{x} = \dot{k} = 0$ (see figure 1).

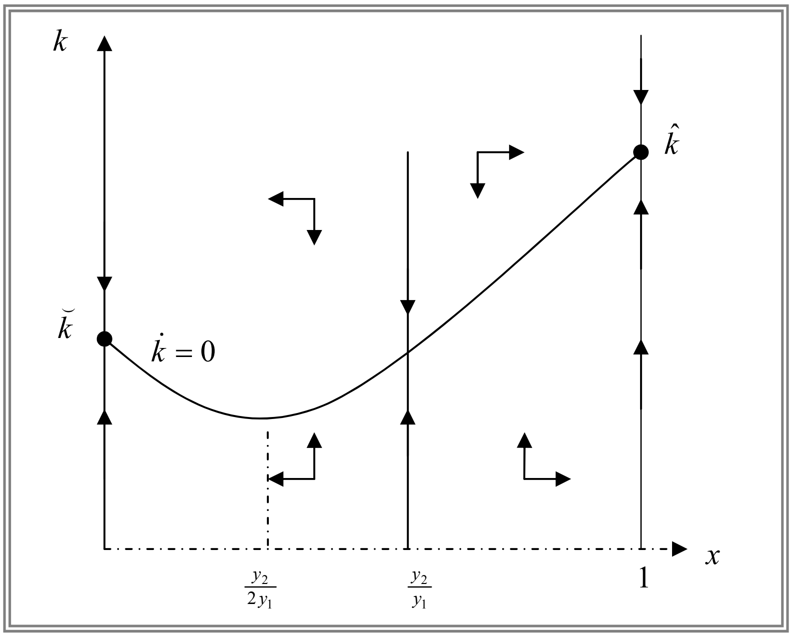

Fig. 1. Phase diagram in the case of diminishing returns of physical capital

Preempting topics we discuss further on, we consider it opportune to emphasize at this point that economic policy interventions are generally designed to reduce parameter δ or increase share s of reinvested output through traditional fiscal policy levers. These measures are not decisive, precisely on account of the presence of social capital. They in fact create an upwards shift of function $\tilde{k}(x)$ without determining significant effects on long-term trends. The trajectories continue to converge towards the same level of social participation even if, in its new steady-state, the level of physical capital is now higher.

On the other hand, if it is true that technological innovation facilitates an increase in the levels of steady-state capital, allowing longer economic growth, under certain conditions it produces a secondary effect, this time negative, on the dynamics of cooperation. In fact, if output elasticity with respect to technological progress is such that:

$$\frac{\partial y_2}{\partial A_2}\frac{A_2}{y_2} > \frac{\partial y_1}{\partial A_2}\frac{A_2}{y_1}$$

then $\frac{\partial\left(\frac{y_2}{y_1}\right)}{\partial A_2} > 0$, or rather, an increase in the level of technology would cause a shift towards the right of the line $\dot{x} = 0$, thus increasing the basin of attraction of the fixed point $(0,\ \breve{k})$ that is Pareto dominated. The fact is that technological improvements produce output increments for all firms, even for non-cooperative ones, and nothing precludes that the effect (in terms of elasticity) is greatest precisely for the non-cooperative firms.

2.2 The model with constant returns of physical capital, *learning by doing* and *spillover* of knowledge

In this second version of the model we assume that the investment activity of a firm generates types of positive externalities in relation to others: a firm increasing its physical capital simultaneously learns to use the technology available in a more efficient way (*learning by doing*); this capability tends to also spread to all other firms in the economy (*spillover* of knowledge). As is well known, the processes described above allow to counter the diminishing returns of physical capital and to generate long-term endogenous growth.

The production function of the generic firm therefore becomes $y_i = y_i(A_i K, k)$ with $K = nk$, where n is the number of production units in the network. As is usual in *learning by doing* growth models, we have $\frac{\partial y_i}{\partial k} > 0$, and $\frac{\partial^2 y_i}{\partial k^2} < 0$, considering K constant.

In practice, the individual firm's capital return is diminishing while the average product of capital is constant; in fact, from the first-degree homogeneity of the production function we obtain $\frac{y_i}{k} = y_i(A_i n)$. As concerns the cooperative population dynamic, (2) does not undergo changes.

Instead, the physical capital dynamic and the share of cooperative firms change, which the following proposition defines precisely:

Proposition 2: From the capital equation (1), we obtain:

$$\dot{k} = 0 \Leftrightarrow \left\{ k = 0 \vee s\left(x^2 \frac{y_1}{k} + (1-x)\frac{y_2}{k} \right) - \delta = 0 \right\}$$

where $\bar{y}_i = \frac{y_i}{k}$ implies: $\dot{k} = 0$ if $x = \frac{\bar{y}_2}{2\bar{y}_1}\left[1 \pm \sqrt{\Delta}\right]$ with $\Delta = 1 - 4\frac{\bar{y}_1}{\bar{y}_2}\left(1 - \frac{\delta}{s\bar{y}_2}\right)$.

We conclude that:

1. if $\Delta < 0$ then $\dot{k} > 0 \;\; \forall x \in [0,1]$;

2. if $\Delta = 0$ then: $\dot{k} = 0$ if $x = \frac{\overline{y}_2}{2\overline{y}_1}$; $\dot{k} > 0$ if $x \neq \frac{\overline{y}_2}{2\overline{y}_1}$;

3. if $\Delta > 0$ then, given that $x = \frac{\overline{y}_2}{2\overline{y}_1}\left[1 \pm \sqrt{\Delta}\right]$, we have three cases:

   - if $0 < \Delta < 1$, $0 < x_2 < x_1 < \frac{\overline{y}_2}{\overline{y}_1} < 1$ and $\dot{k} < 0$ if $x_2 < x < x_1$;
   - if $\Delta = 1$, $x_2 = 0$, $x_1 = \frac{\overline{y}_2}{\overline{y}_1}$ and $\dot{k} < 0$ if $x < \frac{\overline{y}_2}{\overline{y}_1}$;
   - if $\Delta > 1$ and $\overline{y}_1 > \frac{\delta}{s}$, $\frac{\overline{y}_2}{\overline{y}_1} < x_1 < 1$ and $x_2 < 0$ from which $\dot{k} < 0$ if $x < x_2$ and $\dot{k} > 0$ if $x_1 < x \leq 1$.

Proof (see Appendix B)

On the basis of that established with Propositions 1 and 2, we represent the dynamics of the system in the two relevant cases, or rather, with $0 < \Delta < 1$ and $\Delta > 1$. In the long-term average product y_m and capital tend to grow at the same rate. By limiting the analysis to the cases where $x = 0$ and $x = 1$, and taking into account that the rate of growth of the average product is equal to $\frac{\dot{y}_m}{y_m} = \frac{\dot{x}(2xy_1 - y_2) + \dot{k}(x^2 y_1' + (1-x)y_2')}{x^2 y_1 + (1-x)y_2}$, from Proposition 2 we obtain:

$$lim_{x \to 0} \frac{\dot{y}_m}{y_m} = lim_{x \to 0} \frac{\dot{k}}{k} = sy_1' - \delta$$
$$lim_{x \to 1} \frac{\dot{y}_m}{y_m} = lim_{x \to 1} \frac{\dot{k}}{k} = sy_2' - \delta$$

if $0 < \Delta < 1$;

$$lim_{x \to 0} \frac{\dot{y}_m}{y_m} = lim_{x \to 0} \frac{\dot{k}}{k} = 0$$
$$lim_{x \to 1} \frac{\dot{y}_m}{y_m} = lim_{x \to 1} \frac{\dot{k}}{k} = sy_1' - \delta$$

if $\Delta > 1$ and $\overline{y}_1 > \frac{\delta}{s}$;

where $y_i^{'}$ is the marginal productivity of physical capital at social level ($\overline{y}_i = y_i^{'}$). Dividing by k and given that $y_1^{'} > y_2^{'}$ we obtain $\left(\frac{\dot{y}_m}{y_m}\right)_{x=1} > \left(\frac{\dot{y}_m}{y_m}\right)_{x=0}$.

In the first case referred to in point 3 ($0 < \Delta < 1$) the dynamics of transition may converge on two different states of endogenous growth, one Pareto-dominant with maximum levels of cooperation, the other with the absence of cooperation (see Figure 2).

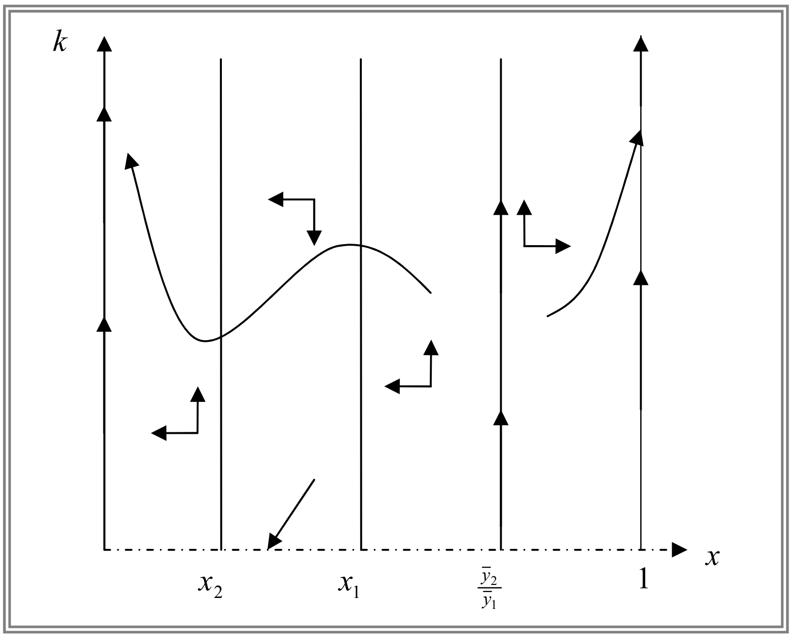

Fig. 2. Phase diagram in the case of constant returns of capital and $0 < \Delta < 1$.

The transition dynamics are absent if the initial level of cooperation is $x = \frac{\overline{y}_2}{\overline{y}_1}$. In this case, the economy would immediately grow at a constant rate $\frac{\dot{k}}{k} = \frac{\dot{y}_m}{y_m} = sy_2^{'} - \delta$. The possibility of a general collapse of the system is not excluded with $k \to 0$ when the trajectory finds itself at the basin of attraction of fixed points $(x, 0)$ with $x_2 < x < x_1$. The risk of a collapse of the economy is of a much more relevant nature if $\Delta > 1$ (see Figure 3)

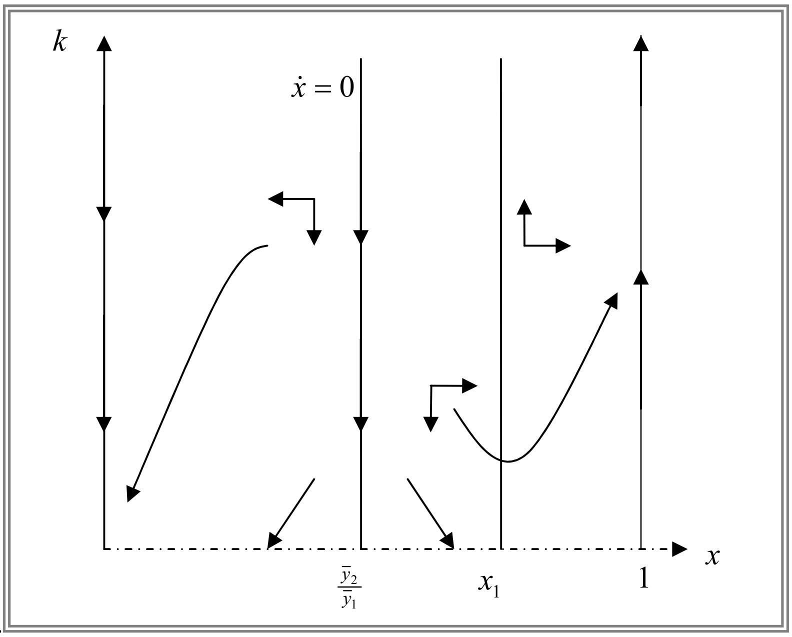

Fig. 3. Phase diagram in the case of constant returns of capital and $\Delta > 1$.

Low levels of cooperation $\left(0 < x < \frac{\bar{y}_2}{\bar{y}_1}\right)$ compromise the possibility of future growth of the system irrespective of the level of accumulated capital up to such time, while for $\frac{\bar{y}_2}{\bar{y}_1} < x < x_1$ only a sufficiently high level of capital ensures endogenous long-term growth. The latter option induces us to sustain that growing local economies, and therefore with low levels of physical capital, in a context in which $\Delta > 1$, should be able to count on widespread cooperation between firms and hope to grow in the future. As in all growth models, the technological innovation element, consisting in a rise in parameter A_2, is the system's driving force that allows exiting from the 'poverty trap'. Let us suppose that $\Delta > 1$, $x < \frac{\bar{y}_2}{\bar{y}_1}$ and $k > 0$; Proposition 2 indicates that the system is in a poverty trap, with continuously diminishing levels of cooperation, capital and output. An increase of the technological progress would produce a reduction of Δ; if the latter were to become negative (or less than the unit), the system would regain growth, even if the long-term growth rate would be lower than the maximum expressible by the economy. Technological improvements, in fact, allow eliminating the danger of the poverty trap but do not condition the cooperation dynamic, which would therefore, even after the increase of parameter A_2, continue to fall. The possibility that technological progress can produce perverse effects on the spread of cooperation remains valid, as mentioned previously in the case of diminishing returns.

We hypothesize an economic system in which $\frac{\overline{y}_2}{\overline{y}_1} < x < x_1$, and k is sufficiently high; in Proposition 2, cooperation would tend to spread to all businesses and capital would tend to grow. In such a situation, it could be that even after an increase of A_2, we continue obtaining $\Delta > 1$.

At this point, the only effect produced by rising technological progress is a shift to the right of the $\dot{x} = 0$ line with the risk that $x < \frac{\overline{y}_2}{\overline{y}_1}$ will occur. If this were the case the economy, which was previously in growth, would now be in the area of the poverty trap. The phenomenon just described is more relevant the higher the $\frac{\delta}{s}$ ratio is.

Also in this case, increases in the potential social capital can be decisive, inasmuch as:

- if $\Delta > 1$, it allows the system to exit the poverty trap, shifting the $x = \frac{\overline{y}_2}{\overline{y}_1}$ line towards the left:
- if $0 < \Delta < 1$, it consents a genuine change in the system by converging Δ to negative values. In Proposition 2, if $\Delta < 0$ then the economy will grow regardless of the value, however positive, of physical capital and cooperation.

Unlike technological progress, social capital can condition the dynamics of cooperation and guarantee the highest rates of growth that the system can express in the long-term. These considerations remain fully valid if we generalize the model by considering simpler production functions of the AK type (see paragraph 3.2).

3. Economic policy implications

In a context where social capital has an effect on the productivity of the system, it is no longer sufficient to relaunch the economy of a local system through 'only' a technological/qualitative reconversion of processes and products or by opening up international markets; such intervention may worsen the situation if not properly supported by policies that encourage firms towards cooperation and to improve the functioning of local institutions.; in practice, policies that directly or indirectly increase the level of the *potential* social capital. Acting directly on the levels of potential social capital through institutional reforms and legislation may however be protracted and costly. It is therefore advisable to encourage cooperation through interventions that allow firms to better exploit the available relational resources, favoring the repetition of cooperation relations through their involvement in long-term projects requiring the sequential development of basic and applicative research, or promoting the formation of multiple relations through the creation of technological consortia. This type of intervention allows increasing the share of cooperative firms, facilitating the convergence of the system towards Pareto-dominant equilibria.

This consideration propels us to carefully evaluate the nature of economic policy interventions. We certainly need to promote technological innovation but it is essential to take measures to encourage cooperation at the same time. On the basis of the model developed, cooperation can grow if action is taken on the allocation of the potential social capital K_s, or by directly creating a favorable social climate and institutional environment for firms or by indirectly organizing the network of firms in order to allow cooperative firms the continued use of the potential capital they have at their disposal. In the latter case, the K_s provision remains unchanged but the firm has the opportunity to repeatedly employ it in the production process.

It is with such interventions, mainly of a qualitative nature, that cooperation becomes a winning strategy, stimulating non-cooperative firms to adopt it and giving a notable boost to the economic growth of the system, since firm performances improves the more widespread cooperation is.

Unlike physical capital, social capital does not wear down with use: indeed, it is through use that its consistency increases; its deterioration is linked to non-use. In our model, the accumulation of social capital is through the transformation from the potential to the actual, which is only possible if a growing number of firms decide to use it through cooperation. Without cooperative behaviors, the relational potentiality of the system remains unused and the actual capital accumulated up to that time begins to diminish.

In the following subsections we examine two possible measures which are able to sustain the spread of cooperation and long-term growth:

1. encourage repeated interactions between firms having proved cooperative, promoting long-term projects that are able to involve them;

2. promote the formation of technological consortia (Baumol, 2001) that allow cooperative firms to use the relational resources at their disposal in several simultaneous transactions. In developing this latter point, we make recourse to a generalization of the model mentioned in the previous section, using AK type production functions.

3.1 Repeated interactions and cooperation

We hypothesized that each cooperative firm repeats the game with other cooperative firms. This means that if it interacts with a non-cooperative firm it will immediately discover its defection and will interrupt the game in the first round. Repeated interactions between two cooperative firms can come to an end with probability p. We indicate with m the number of repetitions of the game before the relation is definitively discontinued. We can thus write $pr(m=k)=p(1-p)^{k-1}$, i.e. m is a random variable with geometric distribution and the expected value $E(m)=1/p$. With each repetition of the game, cooperative firms can reuse the potential social capital provision; the average potential resources used are therefore equal to K_s/p, and the production function, in the case of mutual cooperation, becomes $y_1=(A_2+\frac{K_s}{p},k)$. It is immediately evident that the reduction of the probability of discontinuation of cooperative relations improves the performances of cooperative firms, encouraging the dissemination of cooperation. With reference to figures 2 and 3, the reduction of p generates a shift towards the left of the line $\dot{x} = 0$ reducing the risk of convergence towards Pareto-dominated equilibria.

We need to now understand what the institutional intervention (among others, regional authorities, *meta-management* institutions and universities) must consist of in order to exploit the system's potential. We believe that an important method of promoting the recurrence of relations between firms is based on the promotion of long-term research projects that involve firms for more than one round of the game. One way to achieve this result is to encourage an active role of universities, which typically perform an activity requiring long-term interaction between subjects. Firms (with the exception perhaps of large firms) usually do not undertake basic research activities on their own, since by definition this has no immediate economic return. Universities can play an important role by making their competences available and involving local firms. Universities can not only bring knowledge to firms of the state of progress of research in a particular field, but also propose agreements for the transference of basic research on a level of application.

If research is designed to reach higher levels of technology and to introduce new products, then close cooperation between firms is necessary, especially since the finished product will require the development of a large number of complementary activities difficult to organize in a single firm. The initial input provided by universities is the propulsion for the development of cooperation between firms, which, once these are mutually recognized as cooperative firms, will spontaneously carry the successive phases of the process forward. This may occur, for example, with the promotion of a common brand that only firms following a common path can take advantage of .

3.2 Multiple interactions and technological consortia

The actualization of economic policies aimed at local economic growth through the promotion of active cooperation between firms in the exchange of information, technologies and advanced research is complex and protracted. This concerns acting on structural aspects of the local economy and on consolidated behavioral models, in a context where cooperation is probably a dominated strategy. According to Baumol's (2001) proposal, the formation of technological consortia is much more suited to organizational models characterized by small firms. This form of association in fact not only allows individual firms access to technology developed by other members in exchange for their own, or via payment of royalties, but also provides a valid incentive to conduct R&D activities in order to obtain goods to exchange. These agreements are concluded separately with each (or even only some) of the other members of the consortium and can therefore also take different forms with very different contents.

Baumol, having proposed some significant examples of technological consortia, underlined how mutual exchange of technology permits the spread of knowledge within an economic system, with a clear benefit to social welfare.

In a technological consortium, the technology and knowledge are the currency of exchange to obtain other knowledge and allow innovative/cooperative firms to obtain a profit from their innovative efforts in the form of faster and cheaper access to technology and innovations developed by others. In a local system organized in consortia, cooperation becomes the predominant strategy and tends to spread to all firms, enabling them to exploit the potential social capital they have at their disposal.

Referring back the growth model suggested earlier, we integrate this with the hypothesis of the existence of technological consortia assuming that each cooperative firm simultaneously interacts with n firms. With each of these, it establishes a bilateral exchange of information and technology: the exchange is only profitable if it takes place with another cooperative firm (in accordance with the game's schema in figure 2). Non-cooperative firms do not activate any consortia but can take part in it if involved by a cooperative firm.

The schema in Figure 4 clarifies the concept of the network of consortia created by the initiatives of cooperative firms. In the example, each cooperative firm has activated links with another three firms in the system that it is part of, while non-cooperative firms are involved only in one interaction, which may be within the consortium or external to it.

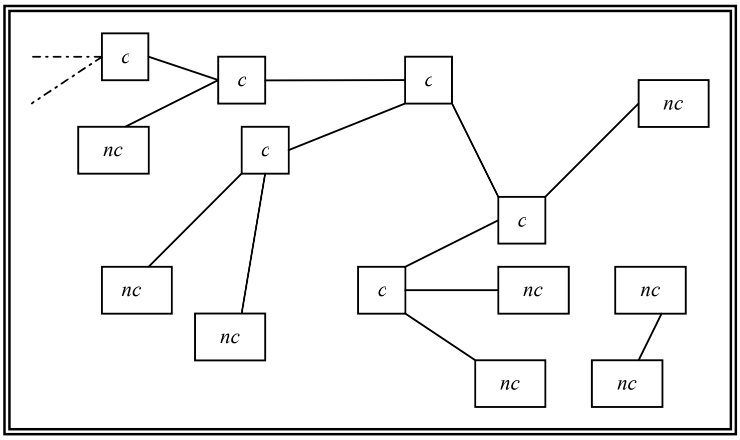

Fig. 4. Network of technological consortia between firms in the same economy and n=3

The accumulation of social capital is conducted through the search for profitable exchange opportunities. The formation of technological consortia increases the probability of success, encouraging the spread of cooperative attitudes among firms. A similar idea, even if developed in a much more complex formal framework than this, is referred to in Vega-Redondo (2006).

In our model, the composition of the group of firms composing the consortium is incidental: at the time of the exchange, the cooperative firm is not aware of the nature of its interlocutors. By indicating with z the number of cooperative firms in the group of partners n of the consortium, we can easily ascertain that this number is a hypergeometric aleatory variable with an expected value $E(z) = nx$. Multiple cooperation enables cooperative firms to repeatedly use the potential social capital, which will therefore be multiplied by a value equal to the number of cooperative firms present in the consortium. Indicating with i the actual number of cooperative partners, we can express the cooperative firm's expected payoff as:

$$\sum_{i=1}^{n} y_1(A_2 + iK_s) \cdot p(z = i)$$

recalling that $y_1 = 0$ if $i = 0$.

Taking into account that the expected payoff of non-cooperative firms remains equal to y_2, we can rewrite the differential equations of the model in the following way:

$$\dot{x} = x(1-x)\left[\sum_{i=1}^{n} y_1(A_2 + iK_s, k)p(z = i) - y_2\right]$$

$$\dot{k} = s\left[x\sum_{i=1}^{n} y_1(A_2 + iK_s, k)p(z = i) - (1-x)y_2\right] - \delta k$$

In order to make the analysis more manageable, let us assume that the production functions are of the Ak type, precisely that $y_1 = (A_2 + iK_s)k$ and $y_2 = A_2 k$.

After some algebraic passages, the dynamic equations of the model become:

$$\dot{x} = x(1-x)\left[kK_s nx - A_2 kp(z = 0)\right]$$

$$\dot{k} = s\left[A_2 k(1 - xp(z = 0)) + kK_s nx^2\right] - \delta k$$

The model shows constant returns of capital, and is thus able to generate long-term endogenous growth. The growth rates are not conditioned by the hypothesis of technological consortia while the dynamics of transition are. It is enough to study the effects of consortia on the dynamics of cooperation to verify this. It is clear that $\dot{x} > 0$ if $K_s nx > A_2 p(z = 0)$. Given that $\frac{\partial P(z=0)}{\partial x} < 0$ and $\frac{\partial P(z=0)}{\partial n} < 0$, an $\tilde{x}$ exists such that $\dot{x} > 0$ if $x > \tilde{x}$ and $\dot{x} < 0$ when $x < \tilde{x}$. An increase in n (keeping x constant) reduces the limit value $\tilde{x}$ allowing the system to converge with greater probability towards cooperative equilibria.

In practice only few cooperative firms, assuming they have the necessary institutional, legal and technological instruments available to create a network of relations of mutual technological and information exchange, can produce a significant change in the dynamics and make a decisive contribution to the spread of cooperation.

The role of local institutions is essential to promote and coordinate the formation and subsequent management of local consortia. Interventions may take different forms, amongst which the assumption of a part of the costs of the creation and management of the consortium, access to particular resources granted not to individual firms but only to the consortium, the priority of evasions of practices concerning the consortium with respect to those relating to individual firms. If the local institution does not want to enter directly into the consortium, it can still promote its creation by facilitating communication between firms in such a way as to translate the willingness to cooperate in some of them into concrete initiatives. Cooperative firms, in fact, have no way of deploying their cooperative nature if they do not come into contact with other cooperative firms, a contact that is unlikely if the transaction costs and information asymmetries are not reduced. The task of institutions is to strengthen the information network that makes information on the activity of local businesses accessible in good time and generates a reputational effect able to promote the dissemination of cooperation.

4. Conclusions

The model proposed in this work develops the fundamental elements of neoclassical growth à la Solow in a context of strategic interaction between firms belonging to a local network. Cooperation is able to increase the productivity of factors (specifically of physical capital) promoting, in the long-term, a higher rate of growth (in the case of constant productivity of capital) or a higher level of physical capital in a steady-state (in the case of diminishing productivity of capital). Cooperative strategies foster better results through the accumulation of a production factor that we define as social capital. In particular, the spread of cooperation allows transforming the social capital from a potential to an actual resource and, therefore, directly usable in the production process.

Given the nature of the return of physical capital, the technological progress (extended to all firms) does not condition the dynamics of cooperation, in the sense that an economic system converging to a Pareto-dominated equilibrium would not alter such a tendency even following a technological improvement. It is even possible that the opposite occurs, that is, that an economy with increasing levels of cooperation and output experiences a reversal of this trend due to an increase of the productivity factor A_2. For non-cooperative firms, since the reactivity of output to technological progress is higher, an increase of A_2 produces increments of higher output thus rendering the individualistic strategy stronger in evolutionary terms.

These conclusions have led us to reconsider the nature of local development economic policies. We believe that the promotion of investment in new technology continues to be the strategy that should be followed, but we need to review the substance of the forms of organization of this type of investment, to be integrated with interventions that create the right incentives for cooperation. These concern the direct growth of potential social capital or the repeated use of this resource, through participation in long-term research projects, or exchange of technology and information within technological consortia.

Appendix A: *Proposition 1*

First step: From (2) we obtain $\dot{x}=0 \Leftrightarrow \left\{ x=0 \vee x=1 \vee x=\frac{y_2}{y_1} \right\}$. With $x=\frac{y_2}{y_1}$, we define a function $x(k)$ that is a vertical line on the point $x=\frac{y_2}{y_1}<1$. Moreover if $0<x<\frac{y_2}{y_1}$ then $\dot{x}<0$, and if $x>\frac{y_2}{y_1}$ then $\dot{x}>0$.

Second step: From (1) we obtain $\dot{k}=0 \Leftrightarrow \{k=0 \vee s(x^2 y_1+(1-x)y_2)-\delta k=0\}$. With $s(x^2 y_1+(1-x)y_2)-\delta k$ we define a function $\tilde{k}(x)$ with $\frac{\partial \tilde{k}}{\partial x}>0$ if $x>\frac{y_2}{2y_1}$ and $\frac{\partial \tilde{k}}{\partial x}<0$ if $x<\frac{y_2}{2y_1}$. In fact, from the homogeneity of the production function $\frac{\dot{k}}{k}=0 \Rightarrow s\left(x^2 y_1\left(\frac{A_1}{k}\right)+(1-x)y_2\left(\frac{A_2}{k}\right)\right)-\delta=0$.

Using the implicit function theorem we obtain $\left(\frac{\partial k}{\partial x}\right)_{\dot{k}=0}=-\frac{2xy_1(A_1/k)-y_2(A_2/k)}{-x^2 y_1'(A_1/k^2)-(1-x)y_2'(A_2/k^2)}$ from which we define a function $\tilde{k}(x)$ and it is easy to show that $\frac{\partial \tilde{k}}{\partial x}>0$ if $x>\frac{y_2}{2y_1}$. Moreover $\tilde{k}(0)=\breve{k}$ and $\tilde{k}(1)=\hat{k}$ with $\hat{k}>\breve{k}>0$. In fact with $x=0$ a $\breve{k}>0$ exists such that $sy_2\left(\frac{A_2}{k}\right)-\delta=0$ and, with $x=1$ a $\hat{k}>0$ exists such that $sy_1\left(\frac{A_1}{k}\right)-\delta=0$. Since, keeping k constant, it must be $y_1\left(\frac{A_1}{k}\right)>y_2\left(\frac{A_2}{k}\right)$, as a consequence we have $\hat{k}>\breve{k}>0$.

Furthermore, keeping x constant, $k<\tilde{k}(x) \Rightarrow \dot{k}>0$ and $k>\tilde{k}(x) \Rightarrow \dot{k}<0$.

Third step: From the linearization of the equations (1) and (2) we show the local stability of points ($x=0$; $k=\breve{k}$) and ($x=1$; $k=\hat{k}$), and the instability of the point $(x=y_2/y_1, k=\tilde{k}(y_2/y_1))$. With $k=0+\varepsilon$ we have that $k<\tilde{k}(x)$, from which $\dot{k}>0$; trajectories move away from the point (x; 0). Given that we have not cyclic paths around the inner equilibrium point, using the theorem of Poincarè-Bendixson the system will converge to a point that, given steps 1 and 2, will be $(0;\breve{k})$ if $x<\frac{y_2}{y_1}$ and $(0;\hat{k})$ if $x>\frac{y_2}{y_1}$.●

Appendix B: *Proposition 2.*

Points 1 and 2 of the Proposition derive directly from the manipulation of the formula $s\left(x^2\frac{y_1}{k}+(1-x)\frac{y_2}{k}\right)-\delta=0$. We therefore demonstrate only that described under point 3. The demonstration proceeds by steps.

First step: From $\dot{k}=0 \Leftrightarrow \left\{k=0 \vee s\left(x^2\frac{y_1}{k}+(1-x)\frac{y_2}{k}\right)-\delta=0\right\}$ we conclude that $\dot{k}=0$ if $x=\frac{\overline{y}_2}{2\overline{y}_1}\left[1\pm\sqrt{\Delta}\right]$ with $\overline{y}_1=\frac{y_1}{k}$, $\overline{y}_2=\frac{y_2}{k}$ and $\Delta=1-4\frac{\overline{y}_1}{\overline{y}_2}\left(1-\frac{\delta}{s\overline{y}_2}\right)$.

Second step: About point 3 ($\Delta>0$) we have three cases:

a. $0<\Delta<1$: in this case we have *i)* $0<1-\sqrt{\Delta}<1$ and *ii)* $1+\sqrt{\Delta}<2$ from which $x_2>0$, $x_1<\frac{\overline{y}_2}{\overline{y}_1}$ and so $0<x_2<x_1<\frac{\overline{y}_1}{\overline{y}_2}<1$.

b. $\Delta>1$: in this case we have: *i)* $1-\sqrt{\Delta}<0$ and *ii)* $1+\sqrt{\Delta}>2$. From these we obtain $x_2<0$ and $x_1>\frac{\overline{y}_2}{\overline{y}_1}$. Then, adding the condition $\overline{y}_1>\frac{\delta}{s}$ we have $x_1<1$, from which $x_2<0<\frac{\overline{y}_2}{\overline{y}_1}<x_1<1$

c. $\Delta=1$: in this case we have: *i)* $1-\sqrt{\Delta}=0$ and *ii)* $1+\sqrt{\Delta}=2$ from which $x_2=0$ and $x_1=\frac{\overline{y}_2}{\overline{y}_1}$. •

NOTE: The case $\overline{y}_1<\frac{\delta}{s}$ is not explicitly considered, inasmuch as it produces a general collapse of the system with $\dot{k}<0$ for each $0<x<1$. If, in fact, $\overline{y}_1<\frac{\delta}{s}$ then we would have $x_2<0$ and $x_1>1$.

References

Antoci A., Sacco L., Vanin P. (2007), Social capital accumulation and the evolution of social partecipation, *The Journal of Socio-Economics*, 36 (1), 128-143.

Barro R., Sala i Martin X. (1995), *Economic growth*, The MIT Press.

Baumol W. (2001), When is inter-firm coordination beneficial? The case of innovation, *International Journal of Industrial Organization*, 19 (5), 727-737.

Bourdieu, P. (1986), The forms of capital, in John G Richardson (ed.), Handbook of theory and research in sociology education, N.Y. Greenwald Press.

Chou Y.K. (2006), Three simple models of social capital and economic growth, *The Journal of Socio-Economics*, 35 (5), 889-912.

Francois P., Zabojnik J. (2005), Trust, social capital and economic development, *Journal of the European Economic Association*, 3 (1), 51-94.

North D.C. (1990), Institutions, institutional changes and economic performances, Cambridge University Press.

Sabatini, F. (2008), Social capital and the quality of the economic development, *Kyklos*, 61(3), 466-499.

Vega Redondo F. (2006), Building up Social Capital in a changing world, *Journal of Economic Dynamics and Control*, 30, 23-51.

Weibull J., Evolutionary game theory, The MIT press, Cambridge, 1998.