A fundamental power price model with oligopolistic competition representation

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June 2009

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First version: March 2007. This version: January 2009

Most popular approaches for modeling electricity prices rely at present on microeconomics rationale. They aim to study the interaction between decisions of agents in the market, and usually represent the impact of uncertainty in such decisions in a simplified way. The usual methodology of microeconomics models is the study of the interaction between the profit-maximization problems faced by each of the firms. On the other hand, there is a growing literature that describes the power price dynamics from the financial standpoint, through the statement of a more or less complex stochastic process. However, this theoretical framework is based on the assumption of perfect competition, and therefore the stochastic process may not capture important features of price dynamics. In this paper, we suggest a mixed approach, in the sense that the price is thought of as the composition of a long-term component, where the strategic behavior is represented, and a short-term source of uncertainty that agents cannot take into account when deciding their strategies. The complex distributional implications of the oligopolistic behavior of market players are then given by the long-term-component dynamics, whereas the short-term component captures the uncertainty related to the operation of power systems. In addition, this modeling approach allows for a direct description of the long-term volatility of power markets, which is usually hard to estimate through statistical models.

Key words: power markets; pricing models; market power; long-term/short-term decomposition

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1. INTRODUCTION

One of the main consequences of the liberalization process that has taken place in the electricity and gas industry during last decades, and ultimately one of its main motivations, is the fact that market players have become responsible for the risks associated with the economic activity. In this context, market agents face the need to participate in financial markets that allow them to manage the risks involved in

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The authors gratefully acknowledge discussions and comments provided by C. Vázquez, P. Rodilla and F. Leuthold. The authors also benefited from discussions provided by seminar participants at the Comillas University, the 2008 CLAIO conference, and the 2008 Madrid meeting of the Young Energy Engineers and Economists Seminar.

their activity. Not surprisingly, the liberalization process has promoted a considerable amount of trading activity, which is in part similar to any other financial market. However, several particular characteristics motivate the special attention paid to energy markets. On the one hand, the specific risks in the energy industry promote the existence of products that cannot be found in any other market. In fact, they are standard products such as forwards or options, but their highly physical nature makes them to have unique characteristics. Moreover, the underlying asset is different of most of the usually traded products. For example, despite the apparently homogeneity of electricity, power produced today is essentially a different product of the electricity produced tomorrow. In addition, energy price evolution is far from usual. Probably, the most visible feature is the exceptional volatility of energy prices. In fact, it is not rare that the volatility of crude or natural gas prices is one order of magnitude higher than the, for example, the interest rates.

The need for pricing and hedging the particular risks of power markets has motivated an unprecedented amount of research concerned with the description of the better strategies when dealing with uncertain results in the future. In particular, modeling the electricity price by means of a certain stochastic process has recently attracted a considerable part of the effort in the power markets description. Financial theory provides the theoretical underpinning for this approach. That is, the market equilibrium implies the allocation of agents' risk, and such risk bearing can be described by a stochastic process. If one further assumes that the market is liquid enough, so that any risk can be hedged away, then there is a unique stochastic process for the price dynamics that represents the risk preferences of market players, and consequently can be used to price any financial contract –see for instance Duffie (2001) for a detailed discussion–. This approach has drawbacks, however, since although it represents the uncertainty in detail, the representation of the agents' decision-making process is based on disregarding the strategic behavior. When dealing with power markets, one should take into account an additional feature that causes their departure from usual financial markets: they are highly concentrated. Probably, this is the main motivation for the fact that an important part of the literature on power price modeling is based on the description of market agents' behavior.

The usual methodology of such models is the study of non-cooperative games. It builds on the idea that the market can be modeled as a game where players try to maximize their own profit, and that the market price is part of the solution of the game. Under the perfect competition paradigm, when the number of competitors is high enough, there is no relationship between the firms' profit-maximization problems, and the game is equivalent to the cost-minimization problem typical of regulated systems. However, most of power markets are not perfectly competitive. Consequently disregarding the oligopolistic behavior may not be adequate, and thus the need for game-theoretic models. One typical consequence of an oligopolistic quantity game is the existence of a strategy where market players withhold production to raise the price. This behavior cannot be captured by a cost-minimization program. On the negative side, this kind of model simplifies the impact of uncertainty in the market behavior, which usually complicates the task of recovering the price dynamics in a proper quantitative way.

Most of the financial models proposed in the literature to deal with power prices belong to the family usually called reduced-form models. This modeling strategy consists of the selection of a parameterized family of stochastic processes, and the calibration of the values of the requisite parameters –either through estimation from historical prices or through calibration from actual market data-. They usually consider the seasonality as a deterministic function of time –see for instance Deng (1999), Lucia and Schwartz (2002), Escribano, Peña and Villaplana (2002), Geman and Roncoroni (2002)–. We interpret this modeling strategy as a decomposition of the price in a long-term factor, which represents the long-term equilibrium level and short-term stochastic deviations from the equilibrium. The long-term/short-term decomposition can be found, among others, in Pilipovic (1997). In addition, Schwartz and Smith (2000) interpret these factors as the combination of an equilibrium price and a short-term factor.

This paper proposes a mixed approach, aimed to benefit from the combination of both families of models. The basic idea is the following: the price is the sum of two components, a long-term level of the price described by means of an oligopolistic equilibrium, and a second factor describing short-term, stochastic deviations from such long-term equilibrium. In addition, as we represent the equilibrium component by means of its fundamental drivers, the methodology allows us for extending the previous literature in power prices by including the uncertainty of the equilibrium component. In particular, the long-term factor uncertainty will be considered through the uncertainty related to its fundamentals. Moreover, from the statistical viewpoint, this modeling strategy has the advantage of mitigating the difficulties caused by the usual lack of data of power markets, because the fundamental representation of the electricity price allows us to make use of the available data in other markets to describe electricity prices –e. g. natural gas market–. That is, the ability to exploit new sources of information is an important advantage of the model. Finally, the model proposed also allows for qualitative analysis of the properties of the distribution of power prices. In particular, the strategic behavior provides a new factor responsible for the skewness observed in power prices. In this regard, price spikes induces positive skewness in the price distribution –see for instance Geman and Roncoroni (2002)–, and thus, according to Bessembinder and Lemon (2002), risk premium in power markets depends on the probability of a price spike. In addition, we will show that the exercise of market power may result in another source of asymmetry. Actually, we will study in section 8 the prices in the Spanish wholesale market, where the implementation of a price cap precludes the existence of price spikes, but power prices remain skewed.

We begin in section 2 with the description the general description of the decomposition of power prices in two factors, an equilibrium component and stochastic perturbations around the equilibrium model. Section 3 develops the description of the oligopolistic competition effects on market prices, that is, the effects considered in the equilibrium factor. Section 4 is devoted to the equilibrium perturbation factor. Section 5 contains the concrete description of the power price model. Section 6 contains the analysis of the statistical properties of the model, while 7 discusses the calibration of the model parameters. Finally, section 8 develops an empirical study of the model applied to Spanish power prices, and section 9 states our conclusion.

2. General framework

Seasonality is one of the most typical characteristics of power prices, and consequently any realistic model must incorporate this characteristic. Probably, the most direct approach is modeling power prices as stochastic perturbations of explicitly defined seasonal averages. Pilipovic (1997), for instance, proposes a model based on this strategy, which consists of a mean-reverting stochastic process –for the log-prices– reverting to a long-term price that evolves stochastically. This model is in turn close to the model in Schwartz (1997), where the proposal is a model for commodity prices that includes the convenience yield. In addition, Pindyck (1999) studies a long time series of natural gas, coal and oil, concluding that energy prices exhibit mean-reversion to a stochastically fluctuating trend, according to the parameterization of Pilipovic (1997). Geman and Roncoroni (2002) and Deng (1999) also model the price behavior as a stochastic process representing the variations of the price around a deterministic seasonal trend, but in these models the stochastic process includes a jump component as well.

However, the direct modeling of the price process is a difficult task in power markets. On the one hand, the complex evolution of prices suggests the use of high-dimensional models, and thus the need for the estimation of a large number of parameters. At the same time, power markets are still rather immature, so that the available price data is not always enough for the correct specification of the model. The seasonal-plus-stochastic decomposition suggests an alternative way of describing power prices: one may interpret the seasonal component as a certain long-term factor, and consider stochastic movements reverting to it –such an interpretation can be found in Schwartz and Smith (2000)-. From this viewpoint, it seems reasonable to make use of the information contained in the fundamental drivers of power prices to describe the long-term component. This is the rationale of a family of models that we call fundamental models. They build on the idea that the long-term component of prices may be described as a transformation of the underlying variables that determine the price formation –fuel prices, demand...–. The description of the long-term component through its fundamentals may be tackled from two different methodologies.

One the one hand, it is possible to calibrate the transformation function from historical data. This strategy has been discussed in a number of works, including Eydeland and Geman (1998), Pirrong and Jermakyan (1999) or Skantze and Illic (2001). These models make use of the idea that the parameters of the price process can be described as functions of the underlying factors, so that the historical data regarding the fundamental drivers can be used to calibrate such functions, expanding the available data set. However, they still face the need of extensive historical information to estimate the transformation.

Alternatively, in order to avoid the use of scarce price data, one can use the structure of the market to derive the transformation function, or in other words, the transformation of the fundamental variables should reflect the price-formation process. Consequently, the function mapping the power-price drivers to actual prices is given by a description of the market-clearing mechanism, so it need not be estimated. In this paper, we propose a model that follows this strategy. When doing so, we will be close to the proposal in Eydeland and Wolyniec (2003). Their proposal builds on the classical merit order of regulated power systems, where the unit commitment is the result of a cost-minimization problem. In this context, the cost of the system may be visualized as a stepwise function representing the marginal cost of the plants in increasing order. The approach, however, faces a critical drawback. Since it is based on the solution of the cost-based dispatch, it disregards the strategic interaction between market players. One of the most commented consequences of the oligopolistic competition is that the merit order used to construct the model is unknown *a priori*, and therefore it cannot be used to specify the structure of the transformation. In addition, the calibration of such models cannot capture the dynamics introduced by the exercise of market power. We will study these issues in detail in section 3.

The previous proposals suggest the link between financial models for power prices and the models based on the microeconomics literature. Actually, the process of power prices may be described as a static transformation of the processes of the fundamental drivers. However, it is important to highlight that this static transformation cannot be cost based, but it should be a model that takes into account the market behavior. From this standpoint, a reasonable candidate for such a transformation is a model representing the decision-making process of market players, which ultimately determines the price formation.

Therefore, we propose to describe the transformation function of the underlying drivers of power prices by means of the solution of a static non-cooperative game.

Our reasoning has led us from financial models, based on the calibration of stochastic processes of the price, to a fundamental model aimed to describe the market price as a function of its drivers. This model is a static transformation of the underlying variables that ultimately determine the power prices. We have showed, in addition, that a cost-based approach is not suitable to describe the evolution of prices, since it disregards the effects of the oligopolistic competition. However, the static model cannot represent the dynamic dimension of the true game. In fact, power markets are a sequence of spot markets, and consequently the strategies of players are conditioned by the fact that the game is repeated. Our approach builds on the idea that the equilibrium of such a repeated game may be modeled as a stable long-term equilibrium, represented by the static game, and short-term deviations from the equilibrium path reverting to the long-term equilibrium. The rationale behind this approach is that when one firm deviates from the equilibrium strategy, the rest of the firms will punish the deviation in the next markets, creating the incentive for the first firm to recover the equilibrium strategy. Nevertheless, this kind of behavior results in extremely complex effects, which are in general rather difficult to define in advance. In this paper, we propose an alternative way of accounting for dynamic effects, based on an aggregated representation. Actually, the power price will be a combination of the static equilibrium and a stochastic process reverting to the equilibrium factor, which accounts for the short-term deviations from the static game strategies –in addition to short-term contigencies, such as network failures...–.

Therefore, let p_t represent the price of electricity at time t. We propose to describe power prices by the following expression:

$$p_t = \pi_t + y_t \qquad (2.1)$$

We propose a decomposition of the price in an equilibrium component π_t and stochastic perturbations of the equilibrium component y_t. The next two sections discuss in depth these two components.

3. THE STRATEGIC BEHAVIOR OF MARKET PLAYERS

The aim of this section is to describe the strategic component of the price π_t. We propose to model the equilibrium factor as the solution of a non-cooperative game, where the players decide their output in order to maximize their profit. The solution of the game is the Nash equilibrium. Thus, the solution of the quantity game provides the strategic behavior of market players, represented by their production decisions. Formally, the game is defined by the profit-maximization problems of each firm, taking into account that their decisions effectively can modify the market price. In addition, the market operator clears the market, and provides the power price. First, we will study a stylized version of the profit-maximization problem that every firm must solve. Then, let g_t^i be the total production of the firm i, at time t. In addition, the output decision is characterized by a maximum output $\overline{g}^i$ and the generation cost $c_t\left(g_t^i\right)$.

Therefore, each firm solves the following problem:

$$\begin{aligned} \max_{g_t^i} \quad & \sum_t \pi_t g_t^i - c_t\left(g_t^i\right) \\ s.t. \quad & g_t^i \leq \overline{g}^i \qquad\qquad \perp \mu_t^i \end{aligned} \tag{3.1}$$

where π_t is the market clearing price at time t, and $g_t^i \leq \overline{g}^i$ represents the maximum output constraint. The optimality conditions of the firm i's problem are the following:

- The optimality with respect to output decisions

$$\pi_t + \frac{\partial \pi_t}{\partial g_t^i} g_t^i - \frac{\partial c\left(g_t^i\right)}{\partial g_t^i} + \mu^i = 0 \quad \forall i \tag{3.2}$$

- The maximum output constraint

$$g_t^i \leq \overline{g}^i \quad \forall i \tag{3.3}$$

- The complementarity condition

$$\left(g_t^i - \overline{g}^i\right)\mu_t^i = 0 \quad \forall i \tag{3.4}$$

To solve this set of equations, it is necessary to define the market price, and consequently, there is a need for a model of the market-clearing process. Thus, we model a market operator that receives the bids of market players and clears the market. The problem may be represented, assuming inelastic demand, by the equation

$$\sum_i g_t^i = D_t \tag{3.5}$$

where D_t is the system demand at time t. Consequently, the solution of the game is provided by the optimality conditions (3.2)-(3.4) for each of the firms i and the market-clearing condition (3.5). This is probably one of the most discussed problems in electricity markets –see for instance Scott and Read (1996), Metzler, Hobbs and Pang (2003), Yuan and Smeers (1999)–. It is worth to analyze the results implied by the equilibrium model in some detail, since they play a key role in the characteristics of the price process proposed in this paper. In particular, equation (3.2) represents the relationship between the firms' decisions and the market price. The expression $-\frac{\partial c\left(g_t^i\right)}{\partial g_t^i} + \mu_t^i$ is the marginal cost of the firm i. If the output of the firm is below its limits, then the maximum output constraint is not active, and its Lagrange multiplier is equal to zero $\mu_t^i = 0$. Therefore, the production of the plant is at the margin. If the maximum output constraint is binding, the Lagrange multiplier is not equal to zero, and the firm is below the margin.

Consider first that there is no opportunity to manipulate the price, or equivalently, the market is perfectly competitive. Then, $\frac{\partial \pi_t}{\partial g_t^i} = 0$ and the equation becomes $\pi_t = \frac{\partial c\left(g_t^i\right)}{\partial g_t^i} - \mu_t^i$, the traditional "price is equal to marginal cost" result. The term $\frac{\partial \pi_t}{\partial g^i} g^i$ shows the incentives for price manipulation that arises in the market. This is a value that makes the price result higher than the marginal cost –note that $\frac{\partial \pi_t}{\partial g_t^i}$ is negative–. $\frac{\partial \pi_t}{\partial g_t^i}$ can be interpreted as the ability of the firm to modify the prices, while g_t^i measures how much does the firm benefits from that increment.

The strategic term shows the most critical issue that the oligopolistic competition introduces in the modeling of power markets. Actually, it results in a complex transformation of the ordered generation costs of the system. In other words, the term $\frac{\partial \pi_t}{\partial g^i} g_t^i$ not only raises the marginal cost of the firms, but also may change the merit order. A big firm, owning a generation portfolio with a high maximum output, will face a high market power term, because the production below the margin will be high. By contrast, smaller firms will face a smaller incentive to raise the price, since their infra-marginal production is low. Consider two plants: the first belongs to the big firm, and the second to the small firm. In addition, the first plant has lower generation costs than the second. The problem introduced by the strategic interaction is that it may cause the bid price of the second plant to be lower than the bid price of the first, because the incentive of the big firm to exercise market power is higher than the incentive of the small firm. In other words, the merit order is not known *a priori*, but it is determined through the solution of the game. Therefore, representing the market-clearing price as a certain calibration of the generation cost of the marginal plant, and consequently simplifying the game between producers considering no market power, may be not approximate enough. The strategic term implies that the bid price of the marginal plant depends not only on its own cost, but also on the production of the rest of the generation portfolio.

Therefore, the oligopolistic competition plays a key role in the evolution of power prices, and consequently the equilibrium component of the price must reflect these effects. Thus, the model for the equilibrium factor should be an oligopolistic market model. We argue that any fundamental model that disregards the strategic interaction, including those built on an a priori merit order, will not be able to capture, in general, the characteristics of the evolution of power prices in a suitable manner.

4. THE EQUILIBRIUM PERTURBATION FACTOR

The previous section describes the effects that a static fundamental model, based on a quantity game, can describe. It is necessary, in addition, to analyze what kind of effects it cannot capture. First, it is not easy to define the generation cost of the plants in advance. In fact, agents in power markets use to purchase in advance a large part of the supplies required for the production of electricity. For example, it is common practice to purchase natural gas supplies through long-term contracts –say 10 or 20 years in advance–, whose prices may have more to do with the particular negotiation of the contract than with international indexes, such as Henry Hub or ICE prices. From this point of view, it is difficult to estimate the real cost of producing electricity. However, since we assume no strategic behavior with respect to power purchases, the price of the contract is usually strongly related to liquidly traded indexes.

Furthermore, it is in general computationally expensive to model the short-term operation of the system in detail. The representation of the technical characteristics of power systems operation usually results in complex optimization problems for the firms, computationally very expensive. As an instance of complicating short-term operation issues, including the start-up costs of the power plants transform the profit-maximization problem in a non-convex program. However, market players must internalize these costs in their bids, so that they will be included in the final price. The price to pay for including start-up costs, on the other hand, is the increased difficulty of the resulting model.

Nevertheless, the most delicate issue concerns the fundamental model used to find the transformation of the underlying variables. The equilibrium framework studied above captures the effect of the strategic interaction between market players by means of a static equilibrium model. However, the quantity game is just an approximation of the real behavior of a firm. A more detailed description would take into account that the bidding process involves not only the quantity decision, but the determination of the bid price as well. This fact would lead us to the statement of more sophisticated games, which calculates the profit-maximization problem deciding a complete curve relating production decisions and bid prices, which results in the Supply Function Equilibrium, Klemperer and Meyer (1989). In addition, we model the oligolistic competition by means of a static game. The real decision-making process involves a sequence of stages where the acquisition of information with respect to competitors' decisions plays a key role. However, the static game disregards the effects of dynamic competition. Moreover, electricity markets are very often more subject to regulatory decisions than any other commodity market. From this point of view, the true game is not only a game between market players, but additionally it should contain the strategic interaction between market players and the regulator.

Therefore, the equilibrium model cannot capture all the relevant features of power price dynamics. There is an uncertainty about the equilibrium price of the system that is difficult to anticipate through the behavior of market players. The perturbation component aims to capture the uncertainty around the equilibrium marginal price. Therefore, one should expect several features derived from this definition. First, the impact of these random shocks in the electricity price should tend to disappear. Consider the effect of an unexpected high price, in the sense that it is not explained as a consequence of a random shock in the fundamentals. It may be motivated by a punctual bidding strategy of one or several agents, or even by a mistake in the bidding process. The idea is that the source of uncertainty is not explained through the observation of the underlying drivers. It will probably result in higher prices the next periods, which can be interpreted as a reaction of market players to punish the rivals' strategy, or a reaction motivated by the uncertainty in the nature of the movement. However, it seems reasonable to expect that the price will tend to the long-term equilibrium of the system as the unexpected high price is forgotten. Therefore, the short-term component should revert to the long-term equilibrium. Thus, the perturbation component will be modeled as stochastic process that will exhibit equilibrium reversion, that is, it will revert to the equilibrium factor.

5. THE MODEL

The next step consists of the proposal of a specific model, based on the analysis carried out in previous sections. Therefore, we will first describe the model aimed to capture the equilibrium component, and then we will state the model for the equilibrium perturbation factor.

5.1. The equilibrium model

The oligopolistic model that introduced Barquín, Centeno and Reneses (2004) is of particular interest for the purpose of our methodology. One of the fundamental insights pointed out by this methodology is that the first order equilibrium conditions are equivalent to a quadratic program, which is easier to solve. We will show next how the equilibrium can be found through an equivalent quadratic problem.

We begin with the quantity game defined in section 3. The solution of the game is characterized by the first order optimality conditions of the profit-maximization programs provided by equations (3.2)-(3.4), and the market clearing condition defined in equation (3.5). As shown in Barquín, Centeno and Reneses (2004), these equilibrium conditions are the same as the first order optimality conditions of the following quadratic program:

$$
\begin{array}{lll}
\min & \sum_{i,t} \left\{ \frac{1}{2} \frac{\partial \pi_t}{\partial g_t^i} \left(g_t^i \right)^2 + c_t \left(g_t^i \right) \right\} & \\
s.t. & g_t^i \le \overline{g}^i & \mu_t^i \\
 & R\left(g_t^i \right) = 0 & \lambda_t^i \\
 & \sum_i g_t^i = D_t & \pi_t
\end{array}
\qquad (5.1)
$$

where $R\left(g_t^i\right)=0$ represents the set of system constraints considered. Since the program (5.1) is based on a quadratic optimization, it can take advantage of a well-developed theoretical corpus as well as very powerful algorithms and software. Therefore, this model can deal with real-size systems with little computational effort. In fact, we think that the computational efficiency of the model is an important advantage with respect to complementarity-based approaches, such as Day, Hobbs and Pang (2002). A comparison between different approaches can be found Neuhoff, Barquín, Boots, Ehrenmann, Hobbs, Rijkers and Vázquez (2005).

Besides, the program (5.1) allows for a rather general representation of the demand behavior. In fact, it is worth to note that we have assumed no price elasticity in the behavior of the system demand. However, it is not difficult to represent demand reactions in the proposed model, by means of the use of the utility of the demand. The quadratic program will be –see Barquín, Centeno and Reneses (2004) for details-:

$$\begin{aligned} \min \quad & \sum_{i,t} \left\{ \frac{1}{2} \frac{\partial \pi_t}{\partial g_t^i} \left(g_t^i \right)^2 + c_t \left(g_t^i \right) \right\} + \sum_t U\left(D_t \right) & & \\ s.t. \quad & g_t^{i,j} \leq \overline{g}^i & \mu_t^i & \\ & R\left(g_t^i \right) = 0 & \lambda_t^i & \end{aligned} \qquad (5.2)$$

where $U\left(D_t\right) = \int_0^D \pi_t\left(D_t\right) dD_t = \frac{1}{\alpha_t}\left(D_t D_{0,t} - \frac{1}{2} D_t^2 \right)$ is the utility of the demand. However, it is usual to consider that competitors' reaction comes before the demand's, so that the demand elasticity is disregarded. On the other hand, the parameter $\frac{\partial \pi_t}{\partial g_t^i}$, representing the price responses of market players, must be calibrated in order to solve the equilibrium. This problem will be analyzed in section 7. Nevertheless, it is important to note that the price responses must be non-negative, so that the quadratic problem is well defined. In addition, removing the quadratic term in the objective function allows us to recover the classical unit-commitment problem. In other words, the quadratic term describes the adjustment of the bid prices from the generating costs.

Finally, note that in general the problem can be stated using the output of each power plants. That is:

$$\begin{array}{lll}
\min & \sum_{i,j,t}\left\{\frac{1}{2}\frac{\partial \pi_t}{\partial g_t^{i,j}}\left(g_t^i\right)^2+c_t\left(g_t^{i,j}\right)\right\} & \\
s.t. & g_t^i=\sum_j g_t^{i,j} & \kappa_t^i \\
 & g_t^{i,j}\le \overline{g}^{i,j} & \mu_t^{i,j} \\
 & R\left(g_t^{i,j}\right)=0 & \lambda_t^{i,j} \\
 & \sum_{i,j} g_t^{i,j}\le D_t & \pi_t
\end{array} \tag{5.3}$$

where $g_t^{i,j}$ is the production of the plant j belonging to the producer i at time t.

5.2. The equilibrium perturbation factor

The model for the short-term component y_t will be a discrete-time autoregressive process. This model allows for capturing the equilibrium-reverting behavior of power prices. The model is given by the following expression:

$$y_t=\sum_{i=1}^{t} h_{t-i+1}u_i \tag{5.4}$$

where $u_i \sim N\left(0,\sigma_t\right)$ are random shocks, and h_{t-i+1} are the series of parameters. This quite general model can represent many of the models proposed to describe energy prices. In particular, we propose to fit an autoregressive model of the form:

$$H\left(q^{-1}\right)y_t=u_t \tag{5.5}$$

where q^{-1} is the lag operator and $H\left(q^{-1}\right)$ is an autoregressive polynomial, or possibly the product of several polynomials. The rationale for several polynomials is the fact that the equilibrium perturbation component y_t should capture any seasonal behavior left by the long-term component. For instance, power prices typically present a weekly periodical component that is mainly related to different demands on working days and weekends, and it is not part of the long-term component. Therefore, this behavior is described in the model through the product of two autoregressive polynomials. That is,

$$H\left(q^{-1}\right)=H_d\left(q^{-1}\right)H_w\left(q^{-7}\right) \tag{5.6}$$

where the base lag of $H_d\left(q^{-1}\right)$ is a day, and the base lag of $H_w\left(q^{-7}\right)$ is a week. This model is fit using the Akaike criterion –see for instance Lütkepohl (1993)–. Then, we obtain the truncated polynomial $H^{-1}\left(q^{-1}\right)$ to get the coefficients $\left\{h_{\ t-i+1}\right\}_{i=1,\dots,t}$ of (5.4).

Turning to the model for the random shocks u_i , it should be taken into account that power prices usually are characterized by stochastic volatility. That is, the volatility of the price is higher during some periods, but then they revert to lower volatility levels –volatility clustering–. We will model the innovations u_i to have GARCH dynamics –Bollerslev (1986)–, in order to take into account the stochastic volatility:

$$\sigma_i^2=\sum_{j=1}^{P}\alpha_j\sigma_{i-j}^2+\sum_{k=1}^{Q}\beta_k\varepsilon_{i-k}^2 \tag{5.7}$$

$$\varepsilon_i \sim N\left(0,1\right) \tag{5.8}$$

The approach proposed in this paper allows for handling the power price volatility related to the volatility in its fundamentals in a convenient manner. Assume that the only risk factor affecting power prices is the natural gas price. Then it is possible to calculate the power prices for a sample of gas price scenarios, and so to obtain the volatility caused by gas price dynamics, capturing the effect of oligopolistic behavior.

6. ANALYZING ELECTRICITY PRICES

One of the most discussed properties of power prices is the presence of spikes. Our model provides a fundamental explanation for these large random movements. That is, the scarcity conditions of the system in a certain period is well defined by the demand model and the production structure. Let us consider a certain period when demand is so high that most of the plants are required to produce. The opportunity costs of the peaking plants would produce non-served energy prices..

The above reasoning is the usual scarcity pricing of power systems. There is, however, another effect related to oligopolistic markets. During scarcity periods, most of the plants have to produce, and thus the effective competitors are reduced to a small number of plants. In a model disregarding market power, this case may be represented by a scaling factor representing an increased generation cost for such periods. But consider two different peaking plants, one belonging to a firm that owns just this unit, and the other belonging to a firm with a large amount of production. The incentive to raise the price above their generation costs is not the same for these firms. Assume that only one of the two will be cleared. If the small firm bids the maximum price, and the large firm just a little bit lower price, the plant owned by the large firm will be cleared. Otherwise, the plant cleared will be the smallest firm's one. Nevertheless, the small firm needs more run-time hours. Therefore, the firm owning the marginal plant crucially determines the magnitude of the spike. This fact is, from our point of view, a central advantage of our modeling approach.

Another effect that is difficult to explain without considering the market-power term is the impact of variability in demand levels and fuel prices. Considering just a scaling factor in the generation costs is equivalent to assume that, price sensitivities to changes in fundamental drivers' values are independent of the actual production. In other words, if the gas price is doubled the electricity price will be doubled as well, when the marginal plant is a gas-fired unit. Additionally, consider an increase in the generation cost of the marginal plant, possibly caused by an increased fuel price. According to (3.2), the increase of the market price is proportional to the increase in the generation cost. However, the impact of the change in the fuel price is measured by a change in the equilibrium of the game, which is in general different of such a proportional change. In fact, the merit order may change so that there is no effect at all.

Regarding asymmetries of power prices, we have shown how the opportunities to exercise market power make the power price distribution to be structurally different of the marginal cost of the system. The term $\frac{\partial \pi_t}{\partial g_t^i} g_t^i$ in (3.2) is an increasing function of the total generation of each firm, so it may be approximated as an increasing function of the system demand. Furthermore, the model also implies a connection with theoretical results regarding the risk premium of electricity prices. In particular, it implies a relationship between risk premia and market power. As shown in Bessembinder and Lemon (2002), power price skewness produce a risk premium in forward contracts: informally, the higher probability of positive deviations motivates that market players ask for compensation when trading electricity in advance. When market power is considered, the forward price is increased to account for the expected opportunities to exercise market power. Now, the additional term does not describe a symmetric factor. The total generation of the firm that is setting the price defines the increased cost, so the probability of generation determines the shape of the price distribution at a certain instant. Another effects, such as the power network, may be taken into account as well.

7. THE CALIBRATION METHODOLOGY

Let us begin by describing the methodology to estimate the equilibrium factor from a set of prices.

7.1. Equilibrium factor calibration

In addition, let us denote the equilibrium model defined by a certain scenario e of fuel prices and system demand as

$$V^e\left(\theta_{i,t}\right)=\begin{cases}\min & \sum_{i,t}\frac{1}{2}\theta_{i,t}\cdot\left(g_t^{e,i}\right)^2+c_t^e\left(g_t^{e,i}\right)\\ s.t. & 0\leq g_t^{e,i}\leq\overline{g}^i\\ & \sum_i g_t^{e,i}=D_t^e\end{cases}\qquad(6.1)$$

where the superscript e denotes the variables in the scenario considered. We have substituted the term $\frac{\partial\pi_t}{\partial g_t^i}$ by $\theta_{i,t}$. The rationale behind this change is that, in order to solve the model, we use the conjectured price response approach. That is, we assume that the price sensitivity is part of the problem data, and will be denoted by $\theta_{i,t}$. Thus, we can express the equilibrium factor of the market price as $\pi_t^e = \arg\max\left\{V\left(\theta_t^i\right)\right\}$. The equilibrium model is then defined by the strategic parameters θ_t^i. From the statistical point of view, there are two many parameters to determine embedded in the equilibrium factor. Actually, since θ_t^i represents one parameter for each market player and each point in time, the equilibrium factor can match any set of price values –the overfitting problem–. Hence, the approach adopted in this paper is to rely again on fundamental considerations: the strategic behavior of market players should not change each point in time. Consider, for instance, a set $\{t_1,...,t_s\}$ representing the period when the strategic behavior does not change. We impose an additional constraint so that $\theta_{t_1}^i = ... = \theta_{t_s}^i$, which can be understood as a version of the parsimonious principle, or alternatively as an assumption concerning the behavior of market players.

The approach adopted in this paper to represent the above constraint is to state an aggregated version of the equilibrium model. The subscript t in the problem (6.1) is substituted by τ, so that we have an aggregate production G_τ^i of the firm i in the set τ, and the aggregate price in τ, π_τ. In addition, let us denote by m the set firms owning a marginal plant at τ. From the optimality conditions of the problem, we have that, for the marginal firm(s) m, it must be fulfilled the following equation:

$$\pi_\tau - \theta_\tau^m G_\tau^m - \frac{\partial c\left(G_\tau^m\right)}{\partial G_\tau^m} = 0 \tag{6.2}$$

From (6.2) we can obtain the strategic parameter of the marginal firm(s) as a function of model price:

$$\theta_\tau^m = \frac{1}{G_\tau^m}\left(\pi_\tau - \frac{\partial c\left(G_\tau^m\right)}{\partial G_\tau^m}\right) \tag{6.3}$$

Therefore, we state an iterative procedure that works as follows:

- First, we assume that every agent bids competitively, or in other words, no market power opportunities are taken into account, $\theta_\tau^{i,0} = 0$. With this assumption, we obtain a collection of productions $G_\tau^{m,0}$ and an equilibrium price π_τ^0. The second superscript denotes the iteration 0

- The new strategic parameters $\theta_\tau^{m,k}$ are obtained using (6.3), but substituting π_τ by the mean price observed in the market in the set τ, π_τ^{real}. The superscript k denotes the iteration number. Note that the new parameters correspond only to the marginal firms. The rest of parameters remain zero.

- The new parameters $\theta_\tau^{m,k}$ changes the output decisions of the power producers, so that we obtain a new set of productions $G_\tau^{m,k+1}$ and a new price π_τ^k

- The algorithm stops when the difference between equilibrium prices of two consecutive iterations is less than a tolerance ε, $\pi_\tau^{k+1} - \pi_\tau^k \leq \varepsilon$

7.2. Calibration of the model

We will study in this section three alternative approaches to calibrate the complete model. The first approach that will be analyzed is based on recovering the risk-neutral measure from market prices. The second approach relies on the statistical estimation of the model parameters. Finally, we propose a mixed approach.

The market approach

Essentially, there are two general strategies to calibrate the model. The first strategy, which might be referred to as the market approach. The idea behind this methodology is that the model should be able to recover the price of the instruments available in the market, since they contain the information used by the market agents to price the contracts. Therefore, one can use the direct approach of finding the model parameters that provide the prices of liquidly traded products. The method can be described as follows.

It begins by the simulation of a set of price scenarios, $e \in \{1,...,E\}$. Let us assume that the market price obtained from the model, for the scenario e, is given by $p_t^e = \pi_t^e + y_t^e$. The market approach to adjust the model consists of finding the parameters that match actual prices of liquidly traded instruments. Then, there is a set of prices, which will be denoted by $\widehat{P}_k$, $k = \{1,...,N\}$, assuming that we observe N products. On the other hand, with the simulation of the model we can obtain values for the instruments based on the prices given by the model. Let us denote them as P_k. Therefore, we can state the following least squares problem:

$$\min_{\theta_M^i, \xi, \sigma} \quad \sum_{k=1}^{N} \left(P_k - \widehat{P}_k \right)^2 \tag{6.4}$$

The key condition for the procedure stated above is that the process for power prices can calculate the value of the contracts. For example, a the value of a forward contract provided by our model is defined by $P_F = \frac{1}{E} \sum_{e=1}^{E} p^e$, and consequently allows for the computation of the model parameters that match market prices.

The statistical approach

There are, however, several limitations of the previous approach, which ultimately motivate the use of an alternative calibration strategy. First, the problem (6.4) is usually ill-conditioned. In addition, many power markets do not have a large enough number of liquidly traded instruments. Finally, the actively traded products may not contain all the relevant information for pricing any other contract. For example, monthly options account only for monthly volatility. If the contract studied is a daily option, the volatility involved in the calculation is the daily volatility. Using only monthly contracts for the calibration of the price model will not capture the appropriate dynamics of daily volatility of market prices. Therefore, the calibration of the model just based on actual market data is not possible in most of the power markets – the Spanish market, described in the case study of section 8, is a typical example of such a market–.The statistical approach for the calibration problem is the estimation by means of a set of historical spot prices. This alternative is based on a direct statistical estimation of the parameters, likely by the maximization of the likelihood, as in the reduced-form models. The procedure would work as follows.

We can express the equilibrium factor of the market price as $\pi_t^e = \arg\max\left\{V\left(\theta_t^i\right)\right\}$. Using a set of historical prices, we can define the strategic parameters required, by means of the algorithm described in section 7.1. In addition, the estimated strategic parameters allow us to compute a set of historical values for the equilibrium factor, and consequently to calculate the differences between historical prices and historical equilibrium-factor values – Note that the model obtains the mean of the real price during a certain period, the set τ. But it does not match the price at each instant t –. Actually, these errors are supposed to be represented by the equilibrium perturbations, which we model by the expression

$$y_t^e = \sum_{i=1}^{t} h_{t-i+1} u_i^e \tag{6.5}$$

Therefore, we can use the previously computed historical errors to estimate the time series model (6.5). Therefore, the strategy to estimate the model would be:

- Find the strategic parameters required for the definition of the equilibrium factor using the methodology described in section 7.1.

- Calculate the errors between real prices and equilibrium prices. Note that the model obtains the mean of the real price during a certain period –the set τ –. But it does not match the price at each instant t

- Estimate the model **¡Error! No se encuentra el origen de la referencia.** to describe historical errors

A hybrid approach

Furthermore, we propose a hybrid approach for the calibration process. It is based on estimating as many parameters as possible using actual market data. Assume, for example, that we study where weekly forward contracts are liquidly traded. A possible strategy is to use the contracts to determine the strategic parameters of the equilibrium factor, and to estimate a time series model to describe historical errors. This is the methodology used in the case study of section 8.

8. CASE STUDY

One of the most important features of the Spanish wholesale market is that, at present, it is quite illiquid. Since July 2007, there is an organized exchange (OMIP[1]), although there is still little trading done in the exchange. Most of the trading activity is done over-the-counter. Additionally, the regulator has imposed the obligation, since 2007, for the two major power producers of the market –currently Endesa and Iberdrola– to negotiate a percentage of their production capacity (Virtual Power Plants). They have the obligation to sell European call options, which give the right to the owner to buy a certain amount of electricity for each hour of a period of three months. The regulator fixes the strike price, and a public auction settles the price. In addition, a small amount of the demand (10% approximately) is allocated through quarterly products: approximately ten days before the beginning of each quarter –e. g. the first quarter, from January to March– there is a public auction (CESUR auction[2]) where the demand is sold through quarterly products with delivery each hour of the quarter. Summing up, the main product traded in the Spanish market is the forward contract, essentially through over-the-counter agreements.

In this context, we study the daily price corresponding to the first eight months of 2008.

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[1] http://www.omip.pt/.

[2] http://www.subasta-cesur.eu.

8.1. The model

The first step in the simulation process is the definition of the power price model. The general scheme of the model is presented in Figure 1.

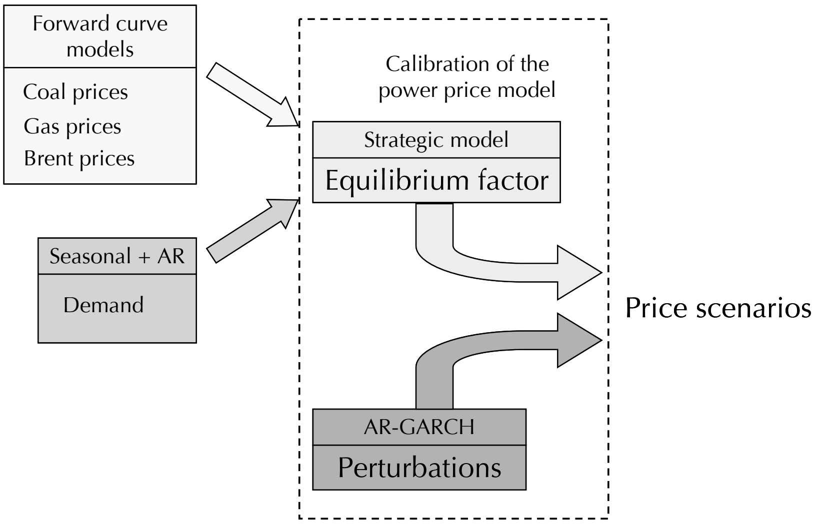

Figure 1. The model scheme

In this section, we outline the description of the evolution of the primary drivers, represented in the left part of Figure 1, which are the basic input for the model. In addition, we provide a description of the methodology to obtain a set of price scenarios generated from the model proposed in this paper.

Fuel price model

The approach considered in this study is to split fuels in two categories. The feature that distinguishes between the two groups is whether the fuel is liquidly traded or not. Thus, we consider separate models for the evolution of coal, heating-oil and gas prices. The rest of costs, namely the corresponding to nuclear plants, are modeled as a known variable cost. The model for the evolution of coal, oil and gas prices is the model of forward curves proposed in Clewlow and Strickland (1999). Basically, it can be represented by the following expression:

$$\frac{dF_{t,T}}{F_{t,T}} = \sum_{i=1}^{N} \phi^i (T-t) dW_t^i \qquad (6.6)$$

where the changes in the forward curve are explained by means of N random shocks, T is the maturity of the contract and t is the quotation date. Each of these perturbations is specified as a Gaussian factor dW_t^i multiplied by a deterministic function of the time-to-maturity $\phi^i(T-t)$, which is defined through principal components analysis.

Demand model

Models for power demand have attracted a considerable amount of research activity –see for instance Weron (2006)–. In addition, utilities and system operators have been developing these models for many years to improve short-term forecasts of power loads. Typically, they are complex non-linear transformations of weather variables, and macroeconomic and demographic data. This detailed representation follows from the need for accurate forecasts of power demand in the short term. However, when the objective of the model is to provide a description of the longer-term distributions – as in the pricing model proposed in this paper–, there is little advantage in capturing each factor affecting the power demand, when there is not enough information to describe accurately the evolution of such factors. Therefore, it is in many cases preferable to choose a simplified representation to represent the distribution of power demand. We choose to model in this case study the Spanish power demand directly, instead of as a function of the temperature or humidity. The main reason is that there is no trading activity concerning the primary drivers, and then there is no market information available. Thus, the model for power demand is based on an autoregressive process, combined with a deterministic seasonal component:

$$D_t = M_t + \alpha_1 D_{t-1} + \ldots + \alpha_n D_{t-n} + \varepsilon_t \tag{6.7}$$

The model parameters $\{\alpha_1,\ldots\alpha_n\}$ are defined by maximizing the maximum likelihood using historical data, and the number of parameters *n* is chosen following the Akaike criterion, see for instance Lütkepohl (1993). The Linear Hinges model, following the methodology in [Vázquez, 2008 # 429], estimates the seasonal component M_t.

In addition, hydro and wind production are modeled in the study as known production in the system. Although in a general study they should be stochastic input as well, we choose to model them as deterministic to highlight the effects of fuel prices in power markets. Therefore, the demand values faced by equilibrium model should be thought of as the thermal demand of the system. That is, the system demand discounting the hydro production and the wind generation.

Equilibrium model

The input data required to solve the equilibrium model stated in section 5.1 are the system demand, and the variable costs of the power plants. Therefore, there is a need to transform the fuel prices provided by equation (6.6) into variable costs. We model such transformation, in the study, as the price of just one forward contract of the curve, multiplied by the efficiency of the plant. In particular, the variable cost will be the forward price of the contract expiring in three months, multiplied by the efficiency. The rationale behind is that power producers need at least three months to get additional fuel, and their variable cost is the cost of refueling. In addition, we describe the procedure to calibrate the strategic parameter in section 8.2.

Equilibrium deviations model

The last step is the definition of the short-term component. We will assume that the equilibrium component is calibrated. In this study, we use historical deviations from the long-term equilibrium to fit the short-term model. In other words, with the historical values of fuel prices and demands, we obtain the historical price provided by the long-term model, and then we calculate historical errors as differences between the model and the real price. That is, we obtain the set of errors $\left\{y_1,...y_N\right\}$ provided by $y_i = p_i - \pi_i$, where i represents each of the historical dates, p_i is the real price at date i and π_i is the equilibrium price at date i. The set of dates used in this study corresponds to year 2007.

8.2. Calibration of the equilibrium model

The procedure to calibrate the model in this study is ultimately defined by market data available in the Spanish market. The only forward-looking contract traded in the market is the monthly forward. Thus, to use only the market approach to calibrate the model one should assume a unique, monthly strategic parameter. On the contrary, we use an hybrid approach. First, we use historical prices to find out the pattern of the strategic parameter. That is, we fit the strategic parameter to describe historical prices. To do so, we assume the same parameter for every peak hour in a certain month, and the same parameter for the off-peak hours in a month –the off-peak period is defined as the first twelve hours in a day–. This provides the variation pattern of a monthly strategic parameter, from peak to off-peak periods. Then, we use the forward contracts with expiration in 2008 to estimate a monthly parameter. Finally, peak and off-peak parameters are defined by applying the historical pattern to the monthly parameters.

8.3. Justification of the model

In order to justify the model, we need to show that it can recover appropriately the empirical characteristics of the electricity prices. To do so, we study of the behavior of the fundamental transformation proposed in this paper. That is, we will test the power price distribution assuming that the evolution of primary drivers is perfectly known, so instead of simulating fuel prices and demand values, we will take actual realizations of the variables. Thus, with actual values for the input variables, we calibrate the model and analyze the price distributions obtained. Figure 2 shows the power price in the test time scope, compared with the real prices.

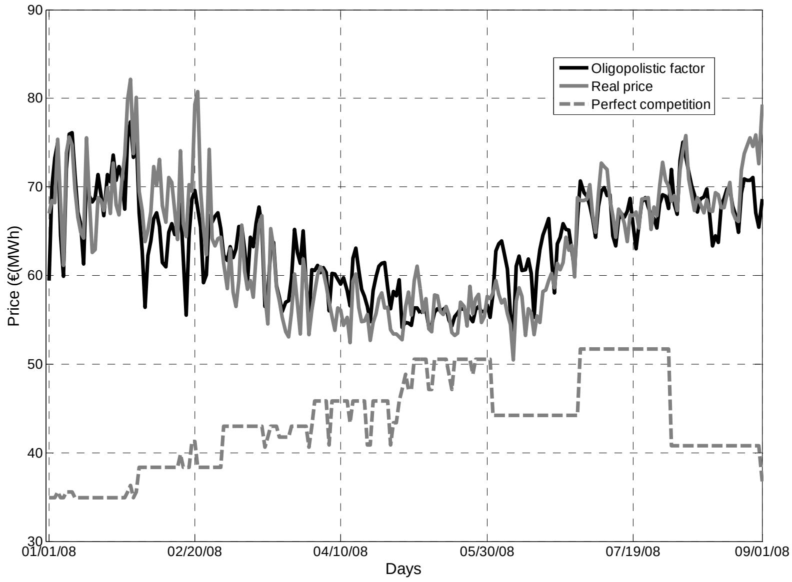

Figure 2.- Model vs. real prices

In addition, Figure 2 shows the price obtained assuming that there are no opportunities to exercise market power. In such a case, the power price is the variable cost of the marginal plant, i. e. the most expensive plant that is producing. In addition, Figure 3 shows the resulting strategic parameters for the firms in the system at each month of the simulation scope –firm number one is the largest one, and firm number 6 is the 6th largest firm–.

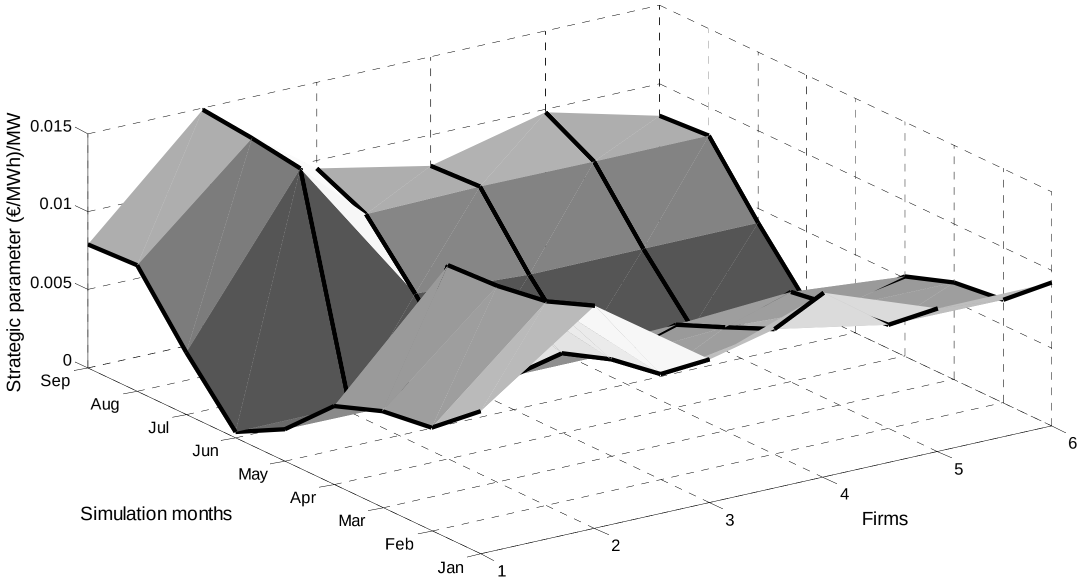

Figure 3.- Calibrated strategic parameters for each of the firms in the system.

Regarding price volatility, Figure 4 compares the annualized volatility of the prices obtained with the model and the actual volatility. It is important to note that the parametric decomposition of this paper, describing the price as made up of long- and short-term factors describes the volatility of real prices accurately. Actually, the price volatility is the sum of a long-term level, defined by the volatility induced by the volatility of the primary drivers, and a short-term level that reverts to the long-term volatility.

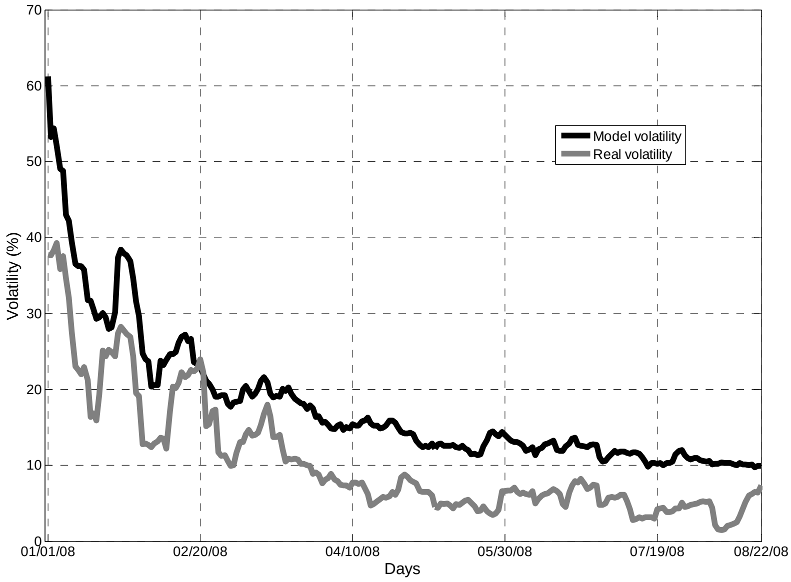

Figure 4.- Model volatility

Turning to higher order moments, Table 1 compares the model and empirical values of the third and fourth normalized moments for different periods. Q1 represents the first quarter of 2008, Q1+Q2 the first and second quarters, and Sept. is the time scope of the simulation, from January to August of 2008. The numbers in the table shows the agreement of the higher normalized moments. This is particularly important because we did not use historical values to match price distributions, but we used the fundamental representation of the model to capture the market characteristics. Matching higher order moments, then, is an additional justification of our modeling approach.

	Skewness		**Kurtosis**	
	Empirical	Strategic	Empirical	Strategic
Q1 (2008)	0.1675	0.1793	-0.3663	-0.4976
Q1+Q2 (2008)	0.8834	0.8926	2.3922	2.3426
Sept. (2008)	0.2768	0.2531	-0.8308	-1.0032

Table 1.- Model and empirical third and fourth moments of the price distribution.

In addition, there are important features of power prices that should be represented by the model. First, the volatility of power prices depends on the system demand. That is, when the system demand is high, the volatility of prices is higher than when system demand is lower. Figure 5 shows the dependence of the volatility weekly log-prices on the system demand. This is a typical phenomenon of electricity prices.

High levels of demand imply a great variability in the number of plants actually producing. When a peaking plant starts to produce, the inframarginal production of the owner is typically high, and thus the market power incentive. Therefore, each peaking plant that starts to produce forces a steep variation in the price.

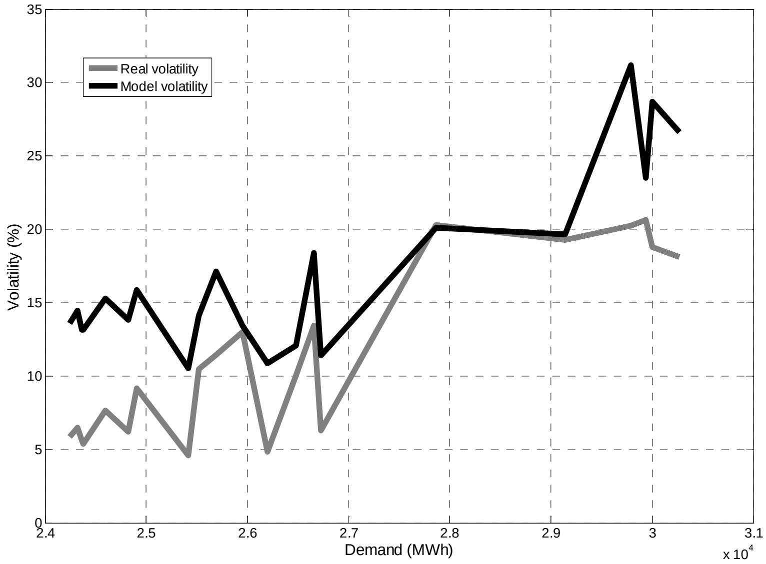

Figure 5.- Volatility of weekly log-prices increases with the demand level.

Another important effect, strongly related to the previous dependence of volatility on demand levels, is the dependence of the volatility on the implied heat rate of the system –the implied heat rate is the ratio between power and natural gas prices–. As shown in Figure 6, volatility is low for small values of the implied heat rate, and it increases as the implied heat rate increases. This effect can be explained with the dependence on the system demand. High levels of demand, in the Spanish system, correspond to high values of the implied heat rate. When the implied heat rate is low, the volatility is low, because in the Spanish system this corresponds to a case where the marginal plant is always a coal unit, and there is little incentive to exercise market power. In a system with more nuclear production, so that sometimes the marginal plant is a nuclear unit, and sometimes is coal-fired, we would expect that the volatility of the left side of the curve would be higher.

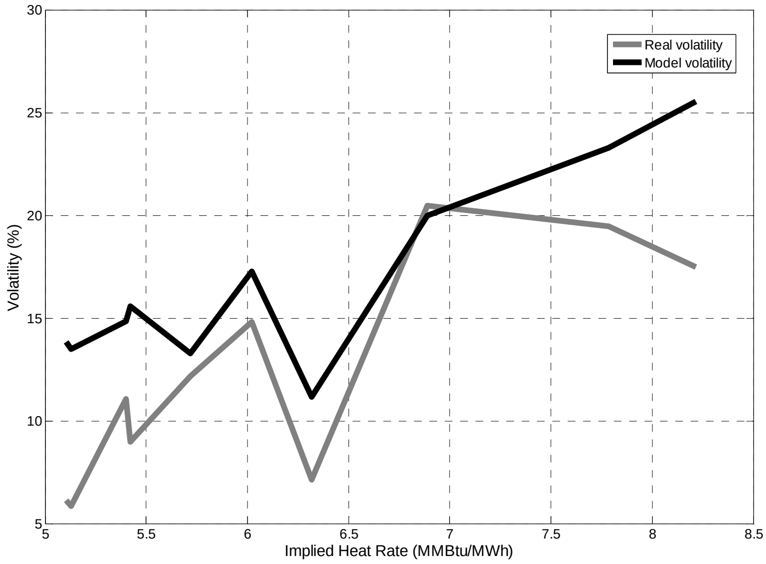

Figure 6.- Volatility of weekly log-prices increases with the implied heat rate.

The correlation structure of power prices is another important feature that must be captured by the price model. The power plant is essentially a spread option between power and fuel prices, and consequently the correlation is central for hedging and pricing purposes. Again, the correlation structure has not been calibrated, so the agreement between empirical and model results can be considered as a powerful out-of-sample test. Figure 7 shows that the correlation structure between gas and power prices obtained from empirical data and the one obtained from the model simulation follows the same pattern. When the implied heat rate is high, the correlation coefficients are low. This effect is related to the fact that for high heat rates, the marginal plant is probably a peaking unit, and for such demand values, the number of units producing has a stronger effect on power prices than the cost of the fuel used to produce –see comments on Figure 5–. The mid part of the curve shows high correlation coefficients for intermediate values of the heat rate. In this part of the curve, the plant setting the price is probably a gas- fired unit, and consequently the correlation between gas and power prices increases. The left part of the curve shows that, for low values of the heat rate the correlation decreases, representing the fact that low gas/power rates implies that a lower-cost plat, typically coal fired, is setting the price.

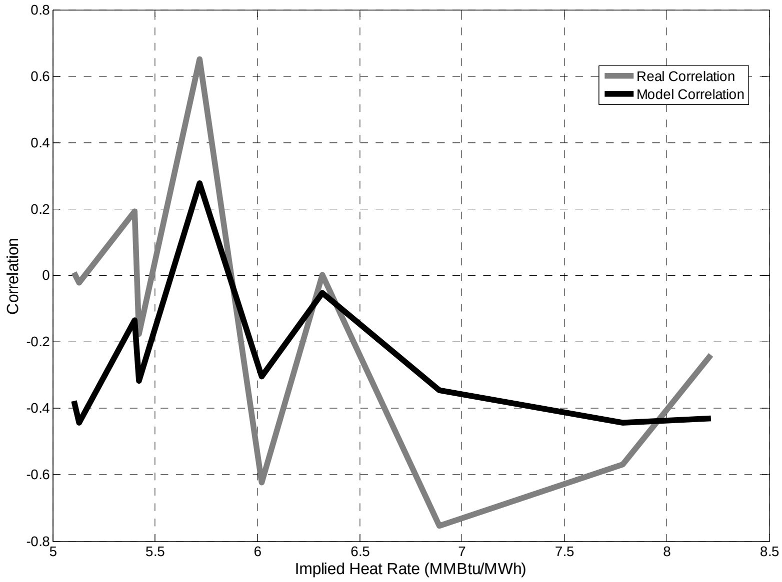

Figure 7.- Gas correlation vs. implied heat rate

Figure 8 shows the correlation between power prices, and gas and coal prices, in the lower panel, and the system demand in the upper panel. Both panels consider a time scope from April 1st to May 20th, to show in detail the dependence of the correlation pattern on the demand pattern. It can be observed that when the system demand is low, the correlation with gas prices is lower, and the correlation with coal prices is higher. Moreover, we can explain this correlation structure from a fundamental viewpoint. When the system demand is low, the marginal plant will be probably a coal-fired unit, and on the contrary, when the system demand is high, the marginal plant will be a gas-fired unit.

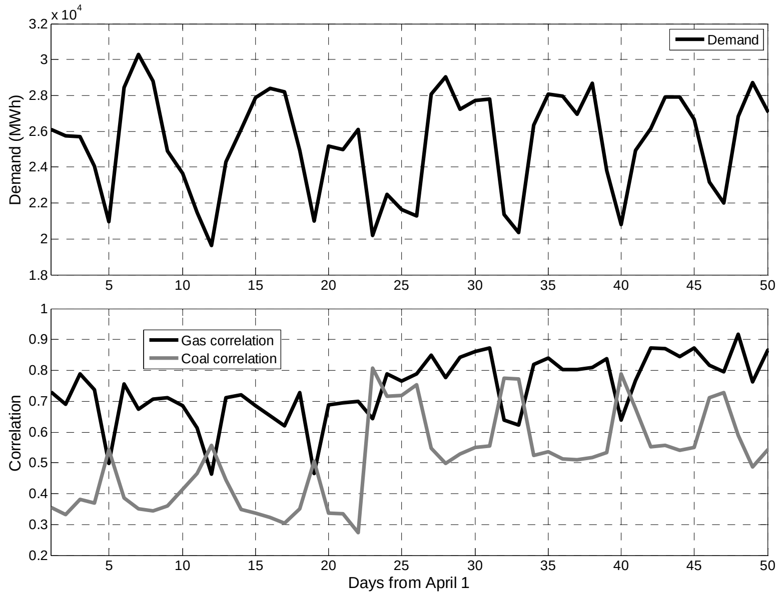

Figure 8.- Evolution of system demand and fuel correlations.

9. CONCLUSION

We have proposed in this paper a new family of models to describe the evolution of electricity prices. It combines a fundamental model to describe the price formation process with an explicit model of random deviations around the fundamental description. The fundamental model builds on oligopoly analysis, and we have shown that the strategic behavior of spot market players plays a central role in the definition of the price distribution, both under the real probability and under the risk-neutral measure. When the ability of power producers to raise the spot price withholding production is taken into account, generators' output decisions define the incentive to increase the spot price, and this incentive induces asymmetries in the power price distribution. Consequently, opportunities to exercise market power are the source of an additional risk premium that compensates for the possibility of obtaining an increased price in the spot market.

Furthermore, we have proposed a particular model to describe the spot price evolution, and we have proved it in the context of the Spanish wholesale market. The case study shows that the model proposed in this paper successfully captures both the features of the risk-neutral probability and the correlation structure of the spot price. The volatility and correlation structure, as we have shown, strongly depends on the characteristics of the generation mix of each particular system. Therefore, projecting this structure into the future using only price data is a difficult task.

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