Balanced growth and structural breaks: Evidence for Germany

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Abstract One of the central hypotheses of the neoclassical growth literature is the balanced-growth hypothesis, which predicts that output, consumption, and investment grow at the same rate. Empirically, this implies that the consumption-to-output ratio and the investment-to-output ratio must be stationary and that consumption and investment must be cointegrated with output. This paper tests these implications with respect to Germany, using unit root tests and cointegration techniques that allow for an endogenously determined structural break. We find that the long-run growth path of the German economy is consistent with the balanced-growth hypothesis if we allow for a structural break associated with the worldwide productivity slowdown of the early 1970s.

Keywords Balanced growth · Unit roots · Cointegration · Endogenous structural breaks

JEL-Classification E23 · E32 · C32

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1 Introduction

Since its inception, modern growth theory (e.g., Harrod 1939, and Solow 1956) has focused on balanced growth paths. Along such paths, an economy's endogenous variables grow at constant, though not necessarily equal, rates; factors shares and the interest rate are constant, as is the capital-output ratio. In particular, from the economy's resource constraint, according to which the sum of consumption and investment is limited by output, it follows that consumption, investment, and output share the same steady-state growth rate. If these economic aggregates grow at the same rate, their ratios must be constant, or stationary, over time, in turn implying that both consumption and investment must be cointegrated with output.[1]

Even though balanced growth characterises a variety of both exogenous and endogenous growth models, tests of balanced growth have generally been presented as tests of the neoclassical exogenous growth model (Attfield and Temple 2006). This literature originates with King et al. (1991), who find the theoretically expected cointegrating relationships using time-series data for the United States (US), and who interpret this result as evidence supporting the neoclassical growth model. Their conclusion, however, has been questioned by Neusser (1991) who, by applying unit root and cointegration tests to time-series data for Austria, Canada, Western Germany, Japan, the United Kingdom (UK), and the US, finds clear evidence in favour of the balanced-growth hypothesis solely for the US. Also, Harvey et al. (2003), in a unit-root and cointegration analysis for Canada, France, Germany, Italy, Japan, the UK, and the US, conclude that their findings are generally not consistent with balanced growth. This conclusion is in line with the results by Serletis and Krichel (1995), which reject the balanced growth hypothesis for Canada, France, Germany, Italy, the Netherlands, Norway, Switzerland, Japan, the UK, and the US.

A common feature of these studies is the assumption that the determinants of the steady-

[1] For earlier discussions of 'great ratios' in macroeconomics see Klein and Kosobud (1961) and Ando and Modigliani (1963).

state consumption and investment ratios are constant for the period of consideration. Other studies consider the possibility of structural breaks in these determinants, thereby finding more evidence for the balanced-growth hypothesis. Clemente et al. (1999), for example, analyse the stationarity of the great ratios of consumption and investment to output for 21 OECD countries, and find that allowing for one or two structural breaks substantially increases the number of rejections of the unit-root null hypothesis. Specifically, their unit-root test results suggest that the two ratios are stationary for Australia, Austria, Canada, Denmark, Finland, Portugal, Spain, Sweden, Switzerland, and the US. Similarly, Attfield and Temple (2006) examine the balanced-growth hypothesis for the US and the UK. Using cointegration analysis with structural breaks, they find the cointegrating vectors predicted by theory for both countries. Finally, Li and Daly (2009) apply unit root and cointegration tests to time-series data for China. Allowing for a structural break in the late 1970s, they find evidence of balanced growth in the pre-break period.

Following this line of research, this paper examines whether the great ratios are stationary for Germany. Germany is an interesting case since it is the largest economy in Europe. Moreover, to date, there is no evidence to support the stationarity of the two great ratios for Germany.[2] In fact, the first impression that emerges from Figure 1 is that the two ratios are not stationary; the German consumption-to-output ratio appears to have an upward trend, while the investment-to-output ratio appears to exhibit a downward trend. We argue that this first impression is due to an un-modelled structural change in the rate of technical progress which is associated with worldwide productivity slowdown of the early 1970s, and which caused a reduction in the steady state value of the investment ratio and—as the two are mirror images of each other—an increase in the steady state value of the consumption ratio.

Specifically, this paper makes the following contributions. First, in the theoretical part, we

[2] Clemente et al. (1999) find that the investment-to-output ratio is stationary for Germany; they find no evidence of a stationarity consumption-to-output ratio. The latter could be due to their relatively small sample size. In fact, unit root tests have low power with short time spans of data and, therefore, failure to reject the unit root null should be interpreted with caution.

introduce a CES technology into a standard neoclassical growth model to derive the steady-state investment ratio as a function of solely structural parameters. Second, we derive the restrictions required for a drop in productivity growth to yield a reduction in the investment rate, and we verify that such conditions apply to Germany. Third, because standard unit-root tests are biased towards a non-rejection of the null of a unit root in the presence of structural breaks, we use the Perron and Vogelsang (1992) approach in our empirical analysis. This approach permits a formal evaluation of the time-series properties in the presence of an endogenously determined structural break. Fourth, since standard cointegration tests too often incorrectly fail to reject the null of no cointegration when there is a break in the cointegrating vectors, we examine the cointegrating rank using the Johansen et al. (2000) maximum likelihood method. This method is explicitly designed to allow for a structural break. Fifth, taking into account the endogenously determined structural break, we estimate the long-run relationship between consumption and output, as well as between investment and output, and test whether the estimates are consistent with their theoretically predicted values. To preview the main result: We find that the long-run growth path of the German economy is consistent with the predictions of the neoclassical growth model if we allow for a structural break associated with the worldwide slowdown in productivity at the beginning of the 1970s.

The rest of the paper is organised as follows: Section 2 discusses the theoretical framework, the empirical analysis is presented in Section 3, and Section 4 concludes.

2 Theoretical considerations

In this section we outline a discrete-time, deterministic, rational-expectations model of neoclassical exogenous growth. We adopt the framework proposed by King, Plosser and Rebelo (2002) (KPR hereafter) and we apply it to the case of CES production functions. After analysing the conditions under which CES technology and neoclassical growth are compatible, we derive the steady state investment rate of the economy as a function of solely structural (preferences and technology) parameters. We use this derivation to obtain a restriction for the investment rate to react negatively to a drop in the rate of growth of technical change, and, finally, we verify that this restriction applies to the German case.

1.1 The economy

The economy is populated by a large number (normalised to 1) of identical agents whose preferences over consumption streams $\{C_t\}_{t=0}^{\infty}$ can be ordered according to $U = \sum_{t=0}^{\infty} \beta^t u(C_t)$, where $\beta \in (0,1)$.

Output at time *t* is produced through a constant-returns technology by employing capital and labour as inputs, $Y_t = F_t(K_t, N_t)$. For the moment, we do not impose all of the standard regularity conditions of neoclassical production functions upon $F(.)$; we only assume differentiability, leaving a more thorough discussion of technology to Section 2.2. Output can either be consumed or invested to increase the capital stock available for production in the next period. In turn, we have $Y_t = C_t + I_t$ and $K_{t+1} = (1-\delta)K_t + I_t$, where *I* represents gross investment and δ is the rate of capital depreciation. Finally, each agent is endowed with one unit of labour input, which is supplied inelastically ($N_t = 1$).

Since we focus on balanced growth, we make the following assumptions to ensure that steady states (i.e. paths along which endogenous variables grow at constant rates) do exist:

- the inter-temporal elasticity of substitution is constant, so that the instantaneous utility function is given by

$$u(C) = \begin{cases} \dfrac{C^{1-\vartheta}}{1-\vartheta} & for \quad 0 < \vartheta < 1, \quad \vartheta > 1 \\ \log C & for \quad \vartheta = 1 \end{cases},$$

- technical change is Harrod-neutral, so that the production function can be written as $F_t(K_t, 1) = F_t(K_t, X_t)$, where X represents labour efficiency.

Let g_j be the steady-state growth factor of variable *j*. It follows from the constraint on output uses and from the constraint on capital accumulation that $g_Y = g_C = g_I = g_K$. Moreover, constant returns to scale technology with no population growth imply $g_Y = g_X$ (see KPR 2002, 92).

2.2 Equilibrium

The assumption of a representative agent implies that in competitive equilibrium there will be no exchange. In turn, equilibrium allocation will coincide with the sequence $\{C_t, K_{t+1}\}_{t=0}^{\infty}$ which, for an initial K_0, maximises U subject to the economy's resource constraints. Following KPR (2002, 89), we can write the Lagrangian of this problem as: $L = \sum_{t=0}^{\infty} \beta^t \frac{C_t^{1-\vartheta}}{1-\vartheta} + \sum_{t=0}^{\infty} \lambda_t [F(K_t, X_t) - C_t - K_{t+1} + (1-\delta)K_t]$, where λ_t is the Lagrangian multiplier at time *t*. An equilibrium path can be found by solving a system made up of the transversality condition, $\lim_{t\to\infty} \lambda_t K_{t+1} = 0$, and the first order conditions with respect to (C_t, K_{t+1}, λ_t):

$$\beta^t C_t^{-\vartheta} = \lambda_t, \tag{1}$$

$$\lambda_{t+1}[F_{K_{t+1}} + (1-\delta)] = \lambda_t, \tag{2}$$

$$F(K_t, X_t) - C_t - K_{t+1} + (1-\delta)K_t = 0, \tag{3}$$

for $t = 0,1,...,\infty$. Eq.s (1) and (2) can be used to derive the Euler equation:

$$\left(\frac{C_{t+1}}{C_t}\right)^{\vartheta} = \beta[F_{K_{t+1}} + (1-\delta)]. \tag{4}$$

Notice that since constant returns imply $F(K, X) = XF(\overline{K},1)$, then $F_K = F_{\overline{K}}$, where 'bar' variables denote variables divided by *X*. Such normalisation is necessary to move to steady states as there is no steady state level of $\overline{K}$ in Eq. (4). On the contrary, since *K* and *X* grow at the same rate, there exists a stationary level of their ratio $\overline{K}$. In what follows we will denote $j*$ as the steady-state value of variable *j*. In turn, focusing on steady states in Eq. (4), we have:

$$\left(\beta\left[F_{\overline{K}}\big|_{\overline{K}=\overline{K}*}+(1-\delta)\right]\right)^{\frac{1}{\vartheta}} = g_C = g_X. \tag{5}$$

From Eq. (3) we obtain $I_t / K_t + (1-\delta) = K_{t+1} / K_t$, which calculated in steady states yields $\bar{I}*/\overline{K}* = g_K - (1-\delta) = g_X - (1-\delta)$. Finally, the steady-state investment rate is given by $s_i* = \frac{\bar{I}*}{\overline{Y}*} = \frac{\bar{I}*}{\overline{K}*}\frac{\overline{K}*}{\overline{Y}*} = \left[g_X - (1-\delta)\right]\frac{\overline{K}*}{\overline{Y}*}$. We now aim at calculating the steady-state capital-output ratio when a CES technology is assumed.

2.3 Exogenous growth with CES production function

CES production functions can be written as $F(K_t,\, X_tN_t) = A[aK_t^{\rho} + (1-a)X_tN_t]^{\frac{1}{\rho}}$, with $\rho \le 1$ and with $\sigma = 1/(1-\rho) \ge 0$ being the constant elasticity of substitution between capital and labour. In per-effective-worker terms, we can write $F(\overline{K}_t, 1) \equiv f(\overline{K}) = A[a\overline{K}_t^{\rho} + (1-a)]^{\frac{1}{\rho}}$.[3] We want to impose

[3] We have adopted the original formulation of CES function proposed by Arrow et al. (1961). In recent years a renewed interest in the relation between CES productions function and growth theory has arisen; in this context, the idea of normalised CES functions elaborated by Klump and de La Grandville (e.g., de La Grandville 1989, and Klump and de

restrictions which assure the existence of steady states on ρ. The marginal product of capital is $f'(\overline{K}) = Aa[a+(1-a)\overline{K}^{-\rho}]^{\frac{1-\rho}{\rho}}$. First, consider the case $\rho > 0$. Since $\lim_{\overline{K}\to\infty} f'(\overline{K}) = Aa^{\frac{1}{\rho}}$ the second Inada condition is violated and a steady state may not exist. In particular, since $f''(\overline{K}) \le 0$ for $0 < \rho \le 1$, if $Aa^{\frac{1}{\rho}} > g_X{}^{\vartheta}/\beta-(1-\delta)$ Eq. (5) can never be satisfied: accumulation will never come to an end and neoclassical exogenous growth will not be feasible. In the Appendix we show that this event requires $A > g_X{}^{\vartheta}/\beta-(1-\delta)$ (intuitively, capital productivity must be high) and $\rho >$ $\log a/[\log(g_X{}^{\vartheta}/\beta-(1-\delta))-\log A] \equiv \rho_+ > 0$. If, to the contrary, $\rho < 0$, then $\lim_{\overline{K}\to 0} f'(\overline{K}) = Aa^{\frac{1}{\rho}}$; the first Inada condition is violated. Returns to capital may be so low that the accumulation process does not even begin and that no $\overline{K}^* > 0$ exists. Again, using Eq. (5) and $f''(\overline{K}) < 0$ for $\rho < 0$, we conclude that we will face a growth trap when $Aa^{\frac{1}{\rho}} < g_X{}^{\vartheta}/\beta-(1-\delta)$. For this condition to be satisfied, $A < g_X{}^{\vartheta}/\beta-(1-\delta)$ and $\rho < \log a/[\log(g_X{}^{\vartheta}/\beta-(1-\delta)) - \log A] \equiv \rho_- < 0$ must hold (see Appendix).[4]

This analysis restricts the values of the elasticity of substitution compatible with the neoclassical exogenous growth to[5]

$$\sigma = \begin{cases} \le 1/(1-\rho_+) & \text{if} \quad A > g_X{}^{\vartheta}/\beta-(1-\delta) \\ \ge 1/(1-\rho_-) & \text{if} \quad A < g_X{}^{\vartheta}/\beta-(1-\delta) \end{cases}.$$

La Grandiville 2000) has found wide support. They normalise the CES function in such a way that the technological parameter A and the distributional parameter b can be expressed as functions of σ. Since we are not interested in the effects of changes in the elasticity of substitution we can develop our analysis by adopting the original CES functional form (see Klump and Saam (2008) for a discussion of normalised CES functions and calibration).

[4] We have denoted ρ_+ and ρ_- as the two threshold values for endogenous growth and for no growth to emphasise that even though they are characterised by the same analytical expression, the former is positive and the latter is negative.

[5] We also rule out $\rho \to -\infty$ otherwise the function may tend to a fixed coefficient Leontief technology, which is not differentiable.

Restricting our attention to the values of the elasticity of substitution, for which steady states exist, we look for the steady state capital-output ratio. In the Appendix, we show that substituting $f'(\overline{K}) = Aa[a+(1-a)\overline{K}^{-\rho}]^{\frac{1-\rho}{\rho}}$ into Eq. (5) and solving for $\overline{K}*$ yields

$$\overline{K}* = \left(\frac{1-a}{\left(\frac{\frac{g_X{}^{\vartheta}}{\beta} - (1-\delta)}{Aa} \right)^{\frac{\rho}{1-\rho}} - a} \right)^{\frac{1}{\rho}}, \text{ hence } f(\overline{K}*) = A \left(\frac{(1-a)\left(\frac{\frac{g_X{}^{\vartheta}}{\beta} - (1-\delta)}{Aa} \right)^{\frac{\rho}{1-\rho}}}{\left(\frac{\frac{g_X{}^{\vartheta}}{\beta} - (1-\delta)}{Aa} \right)^{\frac{\rho}{1-\rho}} - a} \right)^{\frac{1}{\rho}},$$

and, finally, the capital-output ratio is $\dfrac{\overline{K}*}{f(\overline{K}*)} = \dfrac{A^{\frac{\rho}{1-\rho}} a^{\frac{1}{1-\rho}}}{\frac{g_X{}^{\vartheta}}{\beta} - (1-\delta)}$.

In turn, we can go back to the steady-state investment ratio to obtain[6]

$$s_i* = \frac{A^{\frac{\rho}{1-\rho}} a^{\frac{1}{1-\rho}} [g_X - (1-\delta)]}{\left(\frac{g_X{}^{\vartheta}}{\beta} - (1-\delta) \right)^{\frac{1}{1-\rho}}}.$$

The model is consistent with Germany reducing its investment ratio in response to a slowdown in productivity growth if $ds_i*/dg_X > 0$. This condition does not always hold; it can be easily verified that it requires $1-\vartheta\sigma+\vartheta\sigma(1-\delta)/g_X > (1-\delta)\beta g_X{}^{-\vartheta}$.[7] In order to check the inequality we have assumed standard values such as $\delta = 0.1$, $\beta = 0.96$, $g_X = 1.16$. We have also used two alternative

[6] Smetters (2003) and Gòmez (2008) derive an analogous result for the continuous time case.
[7] We have proceeded analogously to Attfield and Temple (2006, pp 11-12) but assuming CES technology allows us to consider explicitly the influence of the elasticity of substitution on the investment rate.

estimates of the factors elasticity of substitution McAdam and Willman (2008) provided for Germany: $\sigma_1 = 0.87$, $\sigma_2 = 0.55$.[8]

If we calculate the ratio between investment rate and capital share, ω, in steady state we obtain $s_i * / \omega * = \left(g_X - (1-\delta)\right) / \left(g_X{}^{\vartheta} / \beta - (1-\delta)\right)$ (see the Appendix for a derivation). We set $s_i * = 0.24$ since it is the average value of the investment ratio for the period 1953 through 1972; in the next section, we find that the break of the investment ratio series is 1972. If we assume a steady-state value of capital share $\omega * = 0.4$, we can solve for ϑ as the only unknown left in the equation.[9] The last equation returns $\vartheta = 1.66$, which for both values σ_1 and σ_2 of the elasticity of substitution, satisfies the condition required for $ds_i * / dg_X > 0$. In turn, the negative response of the German investment rate to a negative shift in the growth rate of productivity appears consistent with the prediction of the neoclassical growth model.

3 Empirical analysis

3.1 Preliminary considerations

If the great ratios of consumption and investment to output are constant along the steady-state growth path, both the difference between the logarithm of consumption and the logarithm of output, $c_t - y_t$, and the difference between the logarithm of investment and the logarithm of output, $i_t - y_t$, become stationary processes. If the logarithms of output, consumption, and investment behave as

[8] The two results are obtained under different assumptions on technical change. The authors find σ_1 = 0.87 when assuming factors augmenting technical change, and σ_1 = 0.55 with labour augmenting. It is a widely known result in production theory, specifically, the 'impossibility theorem', and due to Diamond and MacFadden (see Diamond, MacFadden and Rodriguez 1978), that it is impossible to simultaneously estimate the elasticity of substitution and the bias of technical change. The usual way out of this impasse consists in assuming some restriction on the structure of technical change.

[9] The result is robust against the choice of smaller values of capital share.

random walks or integrated processes of order 1, stationarity of the great ratios, in turn, implies two linearly independent cointegrating vectors and thus the following matrix of cointegrating vectors when the variables are ordered c_t, i_t, y_t:[10]

$$\beta' = \begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & -1 \end{bmatrix} \tag{6}$$

Important to note is that these considerations apply to the case of constant g_X. However, the dramatic worldwide productivity slowdown of the early 1970s indicates that productivity growth has not been constant. The relatively sudden drop in productivity growth in the early 1970s could have, in fact, caused a major structural break in the data. Therefore, it might be more reasonable to assume a structural shift in g_X has occurred. Possible explanations for the drop in productivity growth are: (i) the two oil shocks in 1973 and 1979, (ii) the recessions in 1974 to 1975 and 1980 to 81, (iii) the end of the post-war golden age of the 1950s and 1960s, (iv) the general slowdown in the process of tariff reduction compared to the substantial reduction in tariffs between 1950 and 1970, and (v) a tendency towards increased government regulations from the beginning of the 1970s (Maddison 1987). Thus, the exact break date is not known a priori and we accordingly model the average growth rate of total factor productivity with a structural break at an unknown break date.

Let $D_{t\tau}$ denote a dummy variable defined by

$$D_{t\tau} = \begin{cases} 0, & if \quad t < \tau \\ 1, & if \quad t \geq \tau, \end{cases} \tag{7}$$

where the unknown parameter τ, $\tau \in T$, denotes the time at which the change occurs. Average productivity growth with a structural break can then be expressed as:[11]

$$g_X - 1 = \mu + \theta D_{t\tau}, \tag{8}$$

where μ is the average growth rate of total factor productivity before the break, and θ denotes the

[10] We have named β both the discount factor and the matrix of cointegrating vectors. We made this choice as no possible confusion can arise and the symbol is standard in the respective literature for each.
[11] Average productivity growth rate is $g_X - 1$, because we have denoted steady state growth factors by g (= 1 + growth rate), while in Eq. (8) we talk about rates.

change in average productivity growth at the time of the break.

From Eq. (8), it follows that output, consumption and investment still share a common growth rate, but the growth rate of the pre-break period differs from that of the post-break period. Empirically, this implies

(i) that the logarithmic differences between output, consumption, and investment, $c_t - y_t$ and $i_t - y_t$, are stationary with a structural break,

(ii) that in the trivariate system of c_t, i_t, and y_t, there are two cointegrating relationships with a structural break, and

(iii) that the cointegrating vectors in the system are $\beta_1' = (1, 0, -1, \theta_1)$ and $\beta_2' = (0, 1, -1, \theta_2)$, where θ_1 and θ_2 are the parameters for the structural break, which is assumed to coincide with the worldwide productivity slowdown in the early 1970s.

In the following, we test these implications using German time-series data.

3.2 Data

The data are from the *International Financial Statistics* of the International Monetary Fund. Output is measured by real GDP, the consumption variable is represented by real private and government consumption, and real gross fixed capital formation is the measure of investment employed. Given that only nominal data are available over a sufficiently long time span, the values are converted into real terms using the consumer price index. All data are annually reported and cover the period from 1953 to 2007, implying that our analysis includes 55 annual observations ($T = 55$). Indeed, quarterly series are also available, but only for a shorter time period. Because, however, the power of unit root and cointegration tests depends far more on the time span than on the number of observations

(Shiller and Perron 1995, Hakkio and Rush 1991, Lahiri and Mamingi 1995), we have chosen to use annual data.

3.3 Univariate analysis

Standard unit root tests as well as Perron structural change tests suggest that c_t, i_t, and y_t are integrated of order 1 (results are not reported here to save space). Thus, the first step is to investigate the stationarity properties of the great ratios $c_t - y_t$ and $i_t - y_t$. As already noted, the most striking feature of the plots in Figure 1 is that $c_t - y_t$ and $i_t - y_t$ do not, at first glance, appear to be stationary. This first impression is examined formally using a simple ADF test. As can be seen from Table 1, the ADF test fails to reject the null hypothesis of a unit root in the great ratios, thus seeming to confirm our first impression.

[Figure 1 about here]

[Table 1 about here]

However, standard unit root tests are biased in favour of identifying data as integrated if there is a structural break, as noted at the beginning of the paper. In fact, Figure 1 shows a change in both the consumption-to-output and the investment-to-output ratio in the early 1970s, suggesting a major structural break. Therefore, we use the Perron and Vogelsang (1992) procedure, which allows us to test the unit root null hypothesis against the alternative of stationarity except for an endogenously determined change in the mean of the series. More specifically, we estimate the innovational outlier model where the structural change occurs gradually rather than suddenly:

$$\Delta z_t = v + \gamma D_{t\tau} + \delta d_t + \alpha z_{t-1} + \sum_{j=1}^{k} c_j \Delta z_{t-j} + \zeta_t \, , \tag{9}$$

where z_t stands for the great ratios (z_t = $c_t - y_t$ or $i_t - y_t$) and $D_{t\tau}$ and d_t are indicator dummy variables for the break at time τ. The step dummy $D_{t\tau}$ is defined as in Section 3.1, while the impulse dummy d_t is constructed as: d_t = 1 if t = τ and 0, otherwise. The break point is determined by estimating the model for each possible break date in the data set, and τ is selected as the value which minimises the t-statistics for testing $\alpha = 0$:

$$\text{Min}\, t_{\hat{\alpha}}\, (\tau, k),$$

where $t_{\hat{\alpha}}$ (τ, k) is the t-statistic for testing $H_0 : \alpha = 0$ against $H_0 : \alpha < 0$ with a break date τ and truncation lag parameter k. Alternatively, the break year can be determined by maximising the absolute value of the t-statistics on the coefficient of $D_{t\tau}$:

$$\text{Max}\left| t_{\hat{\gamma}} \right| (\tau, k).$$

The computed t-values of $\hat{\gamma}$ and $\hat{\alpha}$ are presented in Figure 2. For both $c_t - y_t$ and $i_t - y_t$, the maximum and minimum values, respectively, are attained in 1972, implying a shift in the mean of the series associated with that year. Accordingly, the timing of the structural break coincides with the worldwide productivity slowdown of the early 1970s. The associated t-statistics for testing the unit root null against the alternative of stationarity with a break in 1972 are presented in Table 2. Given that $t_{\hat{\alpha}}$ (τ, k) exceeds in absolute value the five-percent critical value for both Min $t_{\hat{\alpha}}$ (τ, k) and Max$\left| t_{\hat{\gamma}} \right|$ (τ, k), the null hypothesis of a unit root is rejected. Thus, the great ratios are stationary around a broken mean, suggesting that the balanced-growth hypothesis is valid for Germany.

[Figure 2 about here]

[Table 2 about here]

Finally, to exclude the possibility of further regime shifts in the mean of $c_t - y_t$ and $i_t - y_t$, we compute the CUSUM-of-squares test statistics for the corresponding mean shift models

$c_t - y_t = v + \phi D72 + e_t$ and $i_t - y_t = v + \phi D72 + e_t$, where $D72$ is a step dummy for the break in 1972. The results are presented in Figure 3. The CUSUM-of-squares statistics do not cross the five-percent significance lines, suggesting that the estimated models are stable. Thus, there is no reason to assume that there is more than one break. This break obviously occurred in 1972, as the Perron-Vogelsang procedure suggests, so that in the following, the date of the break can be assumed to be known.

[Figure 3 about here]

3.4 Multivariate analysis

To provide further evidence in support of the balanced-growth hypothesis, we test for the number of cointegrating vectors among the three variables c_t, i_t, and y_t, and estimate the cointegrating parameters. We use the Johansen et al. (2000) maximum likelihood approach for this purpose,[12] which is based on reformulating an n-dimensional and kth-order vector x_t to a vector error correction model (VECM):

$$\Delta x_t = v + \alpha(\beta' x_{t-1} - \theta D72_{t-1}) + \sum_{i=1}^{k-1} \Gamma_i \Delta x_{t-i} + \sum_{i=0}^{k-1} \Phi_i d72_{t-i} + \varepsilon_t, \tag{10}$$

where x_t is an $n \times 1$ vector of endogenous variables ($x_t = (c_t, i_t, y_t)'$, n = 3), $D72$ is, as before, a step dummy variable for the break in 1972, $d72$ is an impulse dummy for the year 1972, β is a $n \times r$ matrix whose r columns represent the cointegrating vectors among the variables in x_t, α is a $n \times r$ matrix whose n rows represent the error correction coefficients, Γ_i is a $n \times r$ matrix of short-run

[12] The Johansen et al. (2000) approach treats the break point as known. Because the Perron-Vogelsang procedure suggests that the break occurred in 1972, and because theory suggests that the worldwide productivity slowdown of the early 1970s did have a major impact, the dating of the break point can be assumed to be known. Thus, the Johansen et al. (2000) approach should be applicable, although, admittedly, the endogenous selection of the break date may affect the distribution of the likelihood ratio test for cointegration.

coefficients, and Φ_i represents a $n \times r$ matrix of coefficients on $d72_{t-i}$. In order to test for cointegration, we use the trace test, which tests the rank r of the $n \times n$ product matrix $\alpha\beta'$ such that the reduced rank, $0 < r < n$, implies cointegration. The corresponding critical values can be calculated using the response surface estimates of Trenkler (2008).

The lag length of the VECM, $p = k - 1$, is determined by the Schwarz criterion (p = 1); the trace statistics are adjusted by the small-sample correction factor proposed by Reinsel and Ahn (1992), $(T - n \times p) / T$, to account for the small size of the sample.

Both the adjusted and the unadjusted trace statistics are reported in the top part of Table 3. They indicate the presence of two cointegrating vectors, as theory predicts; these vectors are presented in the middle of Table 3. The point estimates are close to $(1, 0, -1, \theta_1)'$ and $(0, 1, -1, \theta_2)'$, and the dummy variables are highly significant, indicating that a structural break in 1972, in fact, exists. Finally, in the bottom of the table, we report the results of a Wald test for the hypothesis that the parameters on y_t are –1. As can be seen, this restriction is not rejected at the five-percent level, implying that our results are broadly consistent with balanced growth.

[Table 3 about here]

5 Conclusion

This paper has examined the balanced-growth hypothesis using time-series data for Germany. We found that the long-run growth path of the German economy is consistent with the balanced growth hypothesis if we allow for an endogenously determined structural break in 1972. This finding can be interpreted as evidence of a drop in the common average growth rate of output, consumption, and investment due to the worldwide productivity slowdown of the early 1970s. Of course, these results are not necessarily representative of other countries. Further country-studies that account for possible structural breaks are needed before definitive conclusions about the general validity of the balanced-growth hypothesis can be drawn.

Appendices

A Threshold values for *ρ*

We start by discussing $\rho > 0$. Notice that $a^{\frac{1}{\rho}} < 1$, as $a < 1$. $A > {g_X}^{\vartheta} / \beta - (1-\delta)$ is therefore a necessary condition for the inequality, $Aa^{\frac{1}{\rho}} > {g_X}^{\vartheta} / \beta - (1-\delta)$, to be satisfied. Taking logs, $\log A + 1/\rho \log a > \log[{g_X}^{\vartheta} / \beta - (1-\delta)]$, hence $\log a > \rho[\log[{g_X}^{\vartheta} / \beta - (1-\delta)] - \log A]$ and, since $\log[{g_X}^{\vartheta} / \beta - (1-\delta)] - \log A < 0$, multiplying both sides by it yields $\rho > \log a / [\log[{g_X}^{\vartheta} / \beta - (1-\delta)] - \log A] > 0$ as $\log a < 0$.

When $\rho < 0$, $a^{\frac{1}{\rho}} > 1$. In turn, $Aa^{\frac{1}{\rho}} < {g_X}^{\vartheta} / \beta - (1-\delta)$ can occur only if $A < {g_X}^{\vartheta} / \beta - (1-\delta)$. Taking logs yields $\log A + 1/\rho \log a < \log[{g_X}^{\vartheta} / \beta - (1-\delta)]$, or $1/\rho \log a < \log[{g_X}^{\vartheta} / \beta - (1-\delta)] - \log A$. Since ρ is negative, multiplying both sides by ρ and rearranging yields $\rho < \log a / [\log[{g_X}^{\vartheta} / \beta - (1-\delta)] - \log A] < 0$ as $\log[{g_X}^{\vartheta} / \beta - (1-\delta)] - \log A > 0$.

B Steady state capital-output ratio and capital share

In order to solve for $\overline{K}*$ from Eq. (5), we set $Aa[a+(1-a)\overline{K}*^{-\rho}]^{\frac{1-\rho}{\rho}} = g_X{}^{\vartheta}/\beta-(1-\delta)$. Raise both sides to $\frac{\rho}{1-\rho}$ and rearrange to get $\frac{1-a}{\overline{K}*^{\rho}} = \left(\frac{g_X{}^{\vartheta}/\beta-(1-\delta)}{Aa}\right)^{\frac{\rho}{1-\rho}} - a$, hence the result

$$\overline{K}* = \left(\frac{1-a}{\left(\frac{g_X{}^{\vartheta}/\beta-(1-\delta)}{Aa}\right)^{\frac{\rho}{1-\rho}} - a}\right)^{\frac{1}{\rho}}.$$

Plugging $\overline{K}*$ into $f(\overline{K}*)$ yields $f(\overline{K}*) = A\left(\frac{a(1-a)}{\left(\frac{g_X{}^{\vartheta}/\beta-(1-\delta)}{Aa}\right)^{\frac{\rho}{1-\rho}} - a} + 1 - a\right)^{\frac{1}{\rho}}$

$$= A\left(\frac{(1-a)\left(\frac{g_X{}^{\vartheta}/\beta-(1-\delta)}{Aa}\right)^{\frac{\rho}{1-\rho}}}{\left(\frac{g_X{}^{\vartheta}/\beta-(1-\delta)}{Aa}\right)^{\frac{\rho}{1-\rho}} - a}\right)^{\frac{1}{\rho}}$$

Finally, the capital-output ratio is

$$\frac{\overline{K}*}{f(\overline{K}*)} = \frac{1}{A}\left(\frac{\left(\frac{g_X{}^{\vartheta}/\beta-(1-\delta)}{Aa}\right)^{\frac{\rho}{1-\rho}} - a}{(1-a)\left(\frac{g_X{}^{\vartheta}/\beta-(1-\delta)}{Aa}\right)^{\frac{\rho}{1-\rho}}}\right)^{\frac{1}{\rho}} \left(\frac{1-a}{\left(\frac{g_X{}^{\vartheta}/\beta-(1-\delta)}{Aa}\right)^{\frac{\rho}{1-\rho}} - a}\right)^{\frac{1}{\rho}} = \frac{A^{\frac{\rho}{1-\rho}} a^{\frac{1}{1-\rho}}}{\frac{g_X{}^{\vartheta}}{\beta} - (1-\delta)}.$$

.

Since $f'(\overline{K}^*) = g_X{}^{\vartheta} / \beta - (1-\delta)$, we can calculate the steady state capital share as

$$\omega^* = \frac{f'(\overline{K}^*)\overline{K}^*}{f(\overline{K}^*)} = \frac{A^{\frac{\rho}{1-\rho}} a^{\frac{1}{1-\rho}}}{\left(\frac{g_X{}^{\vartheta}}{\beta} - (1-\delta)\right)^{\frac{\rho}{1-\rho}}}$$, and the ratio

$$\frac{s_i^*}{\omega^*} = \frac{A^{\frac{\rho}{1-\rho}} a^{\frac{1}{1-\rho}} \left(g_X / \beta - (1-\delta)\right)}{\left(g_X{}^{\vartheta} / \beta - (1-\delta)\right)^{\frac{1}{1-\rho}}} \frac{\left(g_X{}^{\vartheta} / \beta - (1-\delta)\right)^{\frac{\rho}{1-\rho}}}{A^{\frac{\rho}{1-\rho}} a^{\frac{1}{1-\rho}}} = \frac{\frac{g_X}{\beta} - (1-\delta)}{\frac{g_X{}^{\vartheta}}{\beta} - (1-\delta)}.$$

Notice that calibrating ϑ through this equation is particularly convenient as the technological and distributive parameters A and a disappear from the equation and, in turn, the normalisation issues discussed in footnote 2 do not apply.

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Fig. 1 Great ratios

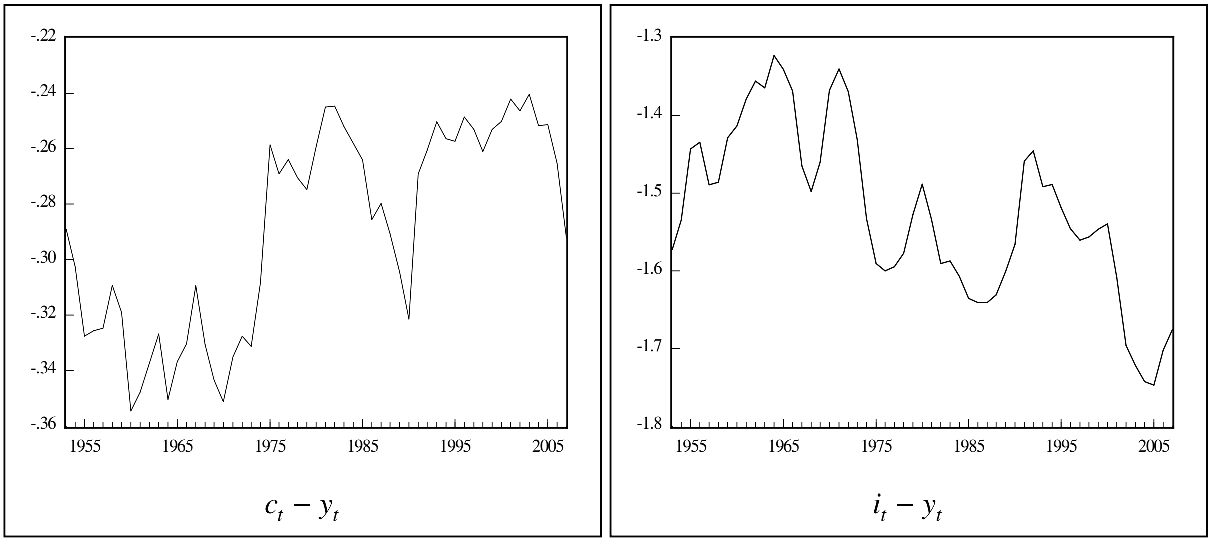
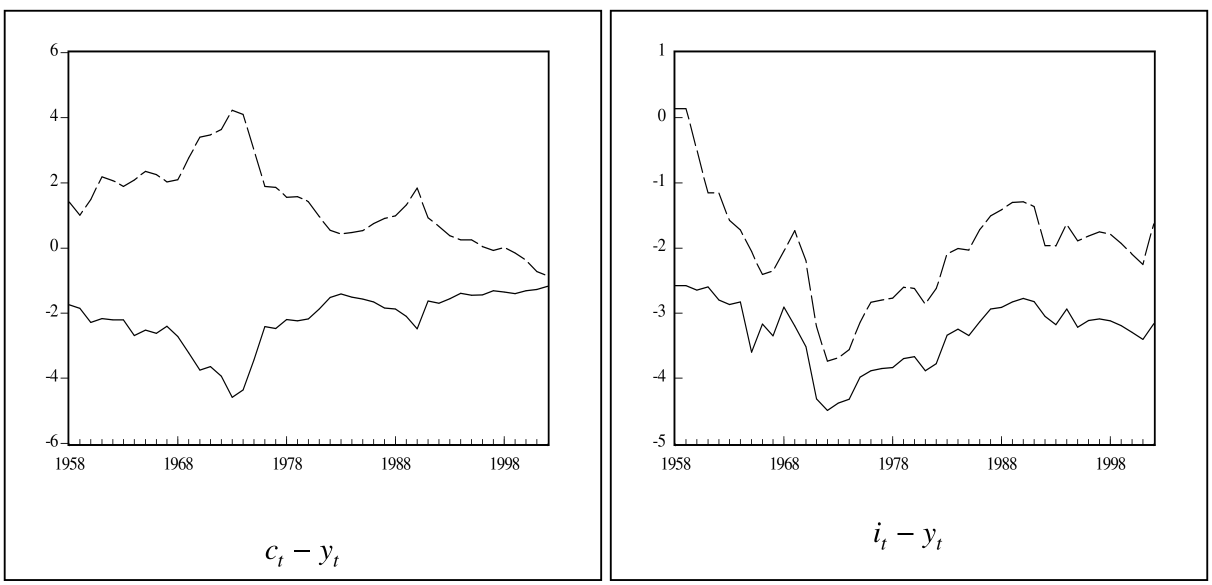


Fig. 2 Sequential unit root tests



t-values of $\hat{\gamma}$ (---) and $\hat{\alpha}$ (—). Following common practice, we computed the t-statistics for each breakpoint in the interval $0.10T - 0.90T$.

Fig 3 Stability analysis: CUSUM of squares and 5% significance bounds

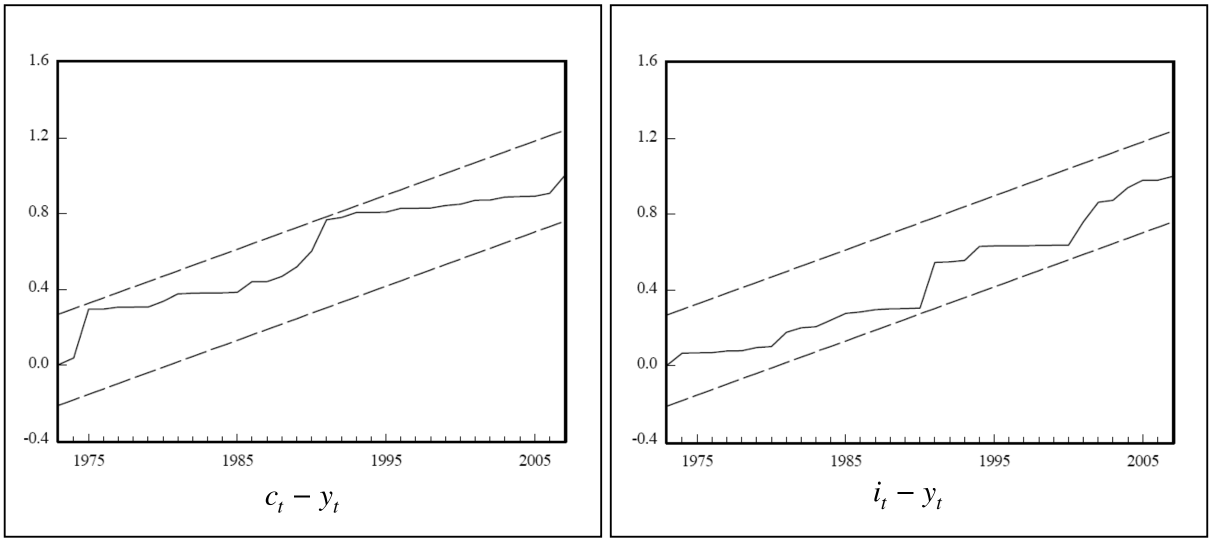


Table 1 ADF unit root test

Variables	Test statistic	5% critical value	Number of Lags
$c_t - y_t$	-1.62	2.91	0
$i_t - y_t$	-2.53	2.91	1

The number of lags was determined by the Schwarz criterion. Critical values are from MacKinnon (1996).

Table 2 Perron and Vogelsang (1992) unit root tests

Variables	Break point	Test statistic $t_{\hat{\alpha}}(\tau, k)$	5% (1%) critical value Min $t_{\hat{\alpha}}(\tau, k)$	5% (1%) critical value Max $\lvert t_{\hat{\gamma}} \rvert (\tau, k)$	Number of lags (k)
$c_t - y_t$	1972	-4.58	-4.44 (-4.95)	-4.19 (-4.73)	0
$i_t - y_t$	1972	-4.49	-4.44 (-4.95)	-4.19 (-4.73)	1

The number of lags was determined by the Schwarz criterion. Critical values are from Perron and Vogelsang (1992).

Table 3 Johansen et al. (2000) approach

Cointegration rank test			
Null hypothesis	Trace statistics	Adjusted trace statistics	1% (5%) critical value
$r = 0$	159.88***	150.99***	45.87 (40.9)
$r = 1$	33.75***	31.87***	29.01 (24.77)
$r = 2$	12.62	11.91	16.62 (12.73)
Estimated cointegrating vectors			
Variables	$\hat{\beta}_1$	$\hat{\beta}_2$	
c_t	1	0	
i_t	0	1	
y_t	-0.982***	-0.992***	
	(125.81)	(29.86)	
$D72$	0.200**	-0.054***	
	(2.75)	(-3.18)	
Wald test of balanced growth restrictions			
$\chi^2(2)$	(p-value)		
5.82	0.055		

t-statistics in parenthesis beneath the estimated coefficients. *** (**) denote the 1% (5%) level of significance. *D*72 is 1 from 1972 onwards and 0 otherwise. The number of degrees of freedom υ in the $\chi^2(\upsilon)$ tests correspond to the number of restrictions.