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'NOISE-TRADER RISK' AND BAYESIAN MARKET MAKING IN FX DERIVATIVES: ROLLING LOADED DICE?

CARLOS A. ULIBARRI[a,*,†], PETER C. ANSELMO[b], KAREN HOVESPIAN, JACOB TOLK[a] and IONUT FLORESCU[b]
[a]*Department of Management, New Mexico Institute of Mining and Technology, USA*
[b]*Department of Mathematical Sciences, Stevens Institute of Technology, USA*

ABSTRACT

This paper develops and simulates a model of a Bayesian market maker who transacts with noise and position traders in derivative markets. The impact of noise trading is examined relative to price determination in FX futures, noise transmission from futures to options, and risk-management behaviour linking the two markets. The model simulations show noise trading in futures results in wider bid–ask spreads, increased price volatility, and greater variation in hedging costs. Above all, the Bayesian market maker manages price-risk by trend chasing not for speculative purposes, but to avoid being caught on the wrong side of the market. The pecuniary effects from this risk-management strategy suggest that noise trading tends to constrain the market maker's capacity to arbitrage; particularly when the underlying price is mean averting as opposed to a Martingale and trading sessions exhibit significant price volatility.

JEL CODE: G13; G14; G15

KEY WORDS: Noise trading; market making; FX derivatives; Bayesian agent; noise transmission

This paper examines the impact of 'noise-trader risk' on market-making behaviour. Noise trading relates to agents buying and selling an asset regardless of its fundamental value (Kyle, 1985). During bullish conditions noise traders continue buying an overvalued asset believing that its price will continue rising. Conversely, during bearish periods noise traders continue selling an undervalued asset believing that its price will continue falling (Shleifer, 2003). This trend chasing tends to drive price away from fundamental value, creating potential arbitrage opportunities.

Rational arbitrageurs with sufficient patience and capital endowments can profit from inefficient asset pricing by taking contrarion positions, which in the aggregate tend to move price towards 'fair value' (Friedman, 1953; Fama, 1965, 1970, 1991, 1998). This process reflects a commitment of resources to exploit price differentials over space and time, under conditions of market disequilibrium (Hirshleifer and Reilly, 2003). Yet as argued by Shleifer (2003) and DeLong *et al*. (1990, 1991), arbitrageurs assume significant risks in taking contrarion positions against noise traders. Perhaps foremost is the risk that noise-trader beliefs will become more extreme, resulting in deeper mispricing, i.e. 'noise-trader risk.' Also see Schleifer and Vishny (1997), and Schleifer and Summers (1990).

The argument that noise-trader risk discourages arbitrage may explain the persistence of inefficient asset valuations under a variety of market conditions. Indeed, an arbitrageur who bets against noise traders runs

*Correspondence to: C.A. Ulibarri, Department of Management, New Mexico Institute of Mining and Technology, 801 Leroy Place, Socorro, NM 87801, USA.
†E-mail: ulibarri@nmt.edu

the risk of losing the bet if forced to liquidate his/her position prior to a correction in market sentiments. Thus, by discouraging arbitrage, noise-trader risk may forestall efficient price reversions and contribute to the persistence of asset price bubbles, i.e. when an asset price rises at a rate beyond what can be explained by market fundamentals (Kindleberger, 1978).

Early empirical studies of arbitrage opportunities in FX derivative markets provide strong support for the absence of arbitrage conditions once transactions costs are accounted.[1] For example Galai (1978) finds apparent arbitrage opportunities vanished for a CBOE options trader facing a 1% transaction cost. In addition, more extensive tests by Bhattacharya (1983) confirm this view. Nonetheless, both authors suggest that arbitrage opportunities may exist for a 'very low-cost trader', such as a market maker (Kolb, 1991).

Certainly market-making behaviour is fundamental to the efficient operation of a market, both through the provision of liquidity and the promotion of price discovery (Silber, 1984; Sarno and Taylor, 2001).[2] However, it would seem that market makers face unique challenges in attempting to exploit noise-driven arbitrage opportunities. A key concern in such operations is the inventory price-risk from maintaining a contrarion market position—either briefly, in hopes of profiting from intraday price differentials, or over a longer period.

The present study focuses on the impact of noise trading on market making in FX derivatives. Market makers maintain two-sided markets by matching incoming buy–sell orders through market-clearing adjustments in their bid–ask spreads. If they are unable to match incoming orders, they become 'buyer–seller of the last resort'. Consequently under intense noise trading market makers add inventory as prices get hammered, and otherwise lose inventory as they inflate.

Clearly noise trading exposes market makers to inventory price-risk from taking contrarion positions. Here we consider the use of delta-hedging in managing the inventory price-risks posed by noise trading in the underlying market. We maintain that noise trading increases hedging costs in derivative markets, thus discouraging market makers from taking arbitrage positions in FX derivatives—particularly over volatile trading periods when their trading capital is at greater risk.

Our framework contributes to a growing literature on 'noise-trader risk' by examining its effect on market-making behavior in FX derivatives. Following Lyons (1991, 1996), we take a Bayesian approach in modelling the effect of order flow on bid–ask prices and maintaining inventory balance.[3] We assume that bid–ask prices are determined through an 'information channel' and an 'inventory control channel.' The market maker supplies liquidity by quoting bid and ask prices so as to match incoming orders and generate profitable inventory turnover. This objective entails buying low and selling high while managing the inventory price-risk from quoting too high a bid-price or too low an asked-price (Garman, 1976).[4]

Our framework incorporates features from other theoretical models of FX market microstructure, i.e. 'sequential-trade models' of a single dealer and Kyle's (1985) auction-market model.[5] Similar to these models, we focus on an optimizing agent who uses available information in determining price, thereby influencing asset liquidity and market efficiency. Also we adapt a trading protocol wherein the agent quotes prices on one side of the transaction and then trades sequentially at these prices with other agents selected from a pool at random. This protocol results in a 'quote-driven market,' wherein private information is gained from the sequential arrival of distinct orders and is impounded in the asset price (Lyons, 2001). The resulting price discovery process has a unique game-like outcome in our model: either a Martingale or mean-averting price prevails, depending on the odds of drawing a noise trader from the pool as opposed to a position trader.

The study proceeds in the following manner. Section 2 uses a heuristic options pricing model to describe arbitrage opportunities from noise trading in futures. Section 3 models a Bayesian market maker who forms beliefs over market sentiments based on order flow from noise and position traders. This information conditions liquidity and pricing in the futures market. Section 4 describes pricing and hedging in the options market using Black's (1976) framework for pricing European-style options on FX futures. Section 5 simulates the impact of intraday noise trading in the futures market on pricing and delta-hedging in the options market. Section 6 concludes.

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2. OPTION VALUE WITH NOISE TRADING

Consider a single-period binomial options pricing framework (Ross, 2003), where the underlying asset is an FX futures contract. A representative trader takes a position at time 0 in both the futures and options markets, buying (or selling) F futures contracts and O_c call option contracts at 'fair values,' p_0 and c_c. At expiration the call option gives the trader the right to buy the underlying asset at the strike price p_e. Assume that the futures price follows a binomial random walk in terms of moving up (u) or down (d) as described by the random event $Z = \{p_f^u, p_f^d\}$. Accordingly, the value of the portfolio P when the call option expires is given by

$$P = \begin{cases} p_f^u F + (p_f^u - p_e)O_c & \text{if } p_f^u \\ p_f^d F & \text{if } p_f^d \end{cases} \tag{1}$$

This portfolio would have the same value under p_f^u or p_f^d if the following position in options were taken at time 0:

$$O_c = -F\left[\frac{(p_f^u - p_f^d)}{(p_f^u - p_e)}\right] \tag{2}$$

yielding a portfolio worth Fp_f^d. Given the original cost of taking long positions in both the futures and options markets $(p_0F + c_cO_c)$, portfolio profits π are given by $Fp_f^d - (p_0F + c_cO_c)$, or in view of equation (2),

$$\pi = F\left\{p_f^d - p_0 + c_c\left(\frac{p_f^u - p_f^d}{p_f^u - p_e}\right)\right\} \tag{3}$$

Consequently, the only option price that exhausts arbitrage profits is given by

$$c_c = \frac{(p_0 - p_f^d)(p_f^u - p_e)}{(p_f^u - p_f^d)} \tag{4}$$

i.e. the 'fair value' of a call option. This value can also be derived on the basis of the expected return from taking the long position in call options, $E(R) = v(p_f^u - p_e) - c_c$, where v denotes the probability that the random event Z will realize p_f^u and therefore that the call option will be exercised. Assuming that expected returns are driven to zero, the 'fair value' option price is

$$c_c = v(p_f^u - p_e) \tag{5}$$

where v denotes the probability that event Z will realize the price p_f^u and therefore that the call option will be exercised.

Similar reasoning yields the fair value corresponding to a put option, i.e. an option to sell futures at the strike price p_e. In the case of put options the 'fair value' is given by

$$c_p = \frac{(p_f^u - p_0)(p_e - p_f^d)}{(p_f^u - p_f^d)} \tag{6}$$

or the equivalent

$$c_p = (1 - v)(p_e - p_f^d) \tag{7}$$

where $1-v$ denotes the probability that event Z will realize the price p_f^d and therefore that the put option will be exercised.

Note that the only values for v and $1-v$ consistent with zero expected returns, and hence the perceived absence of arbitrage opportunities, must satisfy

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Table 1. Noise-trader sentiments ($v_N, 1 - v_N$) and price expectations

Vars		Bearish			No-arbitrage			Bullish	
v	0.0000	0.1250	0.2500	0.3750	0.5000*	0.6250	0.7500	0.8750	1.0000
$1-v$	1.0000	0.8750	0.7500	0.6250	0.5000*	0.3750	0.2500	0.1250	0.0000
$E(p)$	1.16	1.17	1.19	1.19	$P_o = 1.20$	1.21	1.22	1.23	1.24
C_c	0.0000	0.0000	0.0000	0.0000	0.0200	0.0250	0.0300	0.0350	0.0400
C_p	0.0400	0.0350	0.0300	0.0250	0.0200	0.0200	0.0000	0.0000	0.0000

*The no-arbitrage parameter values $v = 1-v = 0.5$ assume a strike price $p_e = 1.20$ and max–min $p_u = 1.24$, $p_d = 1.16$. See Barnes and Logue (1975), Bollerslev and Domowitz (1993), Bollerslev and Melvin (1994), and Lyons (1998).

$$v = \frac{(p_0 - p_f^d)}{(p_f^u - p_f^d)} \text{ and } 1 - v = \frac{(p_f^u - p_0)}{(p_f^u - p_f^d)} \tag{8}$$

Under rational expectations the values for v and $1-v$ given in (8) are internally consistent with the 'fair value' option prices (5) and (7). This is seen by considering the process by which rational traders form expectations over the fundamental value of the underlying asset, p_0. Note that the expected returns from taking a long position in the futures contract are given by

$$\begin{aligned} E(R_f) &= vp_f^u + (1 - v)p_f^d - p_0 \\ &= E(p_f) - p_0 \end{aligned} \tag{9}$$

No-arbitrage conditions imply zero expected returns from increased buying or selling of futures, or in other words, the expected futures price is equivalent to its fundamental value at the beginning of the period. Thus, FX derivative traders who form 'rational expectations' consistent with this 'no-arbitrage property' will take the price at the beginning of the period as an unbiased predictor of the expected futures price at the end of the period, $E(p_f) = p_0$.[6] Otherwise, traders would overvalue or undervalue futures, and thus perceive option values different from the 'fair values' given by (5) and (7).

To set the stage for later work, assume that position traders have unbiased beliefs denoted by the random parameter v_o, while noise traders have biased beliefs denoted by v_N. Thus, under bullish conditions noise traders believe that the underlying asset is undervalued with probability $v_N > v_o = 0.5$, and under bearish conditions they believe that it is overvalued with probability $1 - v_N > 1 - v_o > 0.5$. Accordingly, these biased sentiments form non-Bayesian expectations among noise traders, i.e. agents who trade on noise rather than information (Black, 1986).[7]

Table 1 gives the effect of bullish and bearish market sentiments relative to price expectations and option values. The no-arbitrage future price is defined by $P_o = 1.20$, consistent with unbiased sentiments ($v = 1-v = 0.5$). Deviations from the no-arbitrage price reflect biased valuations: bullish traders who overvalue futures will buy more calls and sell more puts; bearish traders who undervalue futures will sell more calls and buy more puts. These transactions create apparent arbitrage opportunities for a contrarion trader, such as a market maker.

3. BAYESIAN MARKET MAKING

Initially, the sentiments implied by the parameters v_N and v_o reflect private information to noise and position traders. Assume that a *Bayesian* market maker assigns binomial probability distributions to these valuation parameters. Thus, at the market open, let $P_M(v_N)$ and $P_M(v_o)$ denote the market maker's prior beliefs over the valuation parameters v_N and v_o,[8] where $P_M(v_N) + P_M(v_o) = 1$. Assume that the market maker continually revises his/her priors on the basis of incoming market orders, $n = 1, 2, 3, \ldots$, of which j represent buy orders and $n-j$ represent sell orders.[9] Given this information (Φ),

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the market maker's posterior probabilities are denoted by $P_M(v_N|\Phi)$ and $P_M(v_o|\Phi)$, expressing revised beliefs conditional upon buy–sell order flows. These posterior probabilities are obtained applying Bayes' rule:

$$P_M(v_N|\Phi) = \frac{P(\Phi|v_N)P_M(v_N)}{P(\Phi|v_N)P_M(v_N) + P(\Phi|v_o)P_M(v_o)} \quad \text{and} \quad P_M(v_o|\Phi) = 1 - P(v_N|\Phi) \tag{10}$$

where the conditional probability terms $P(\Phi|v_N)$ and $P(\Phi|v_o)$ denote the *likelihood* of biased and unbiased asset valuations (v_N and v_o) given incoming order flow information, i.e.

$$P(\Phi|v_N) = \left(\frac{n!}{j!(n-j)!}\right) v_N^j (1 - v_N)^{n-j} \quad \text{and} \quad P(\Phi|v_o) = \left(\frac{n!}{j!(n-j)!}\right) v_o^j (1 - v_o)^{n-j} \tag{11}$$

Note how the valuation parameters v_N and v_o are weighted by the j buy orders and $n{-}j$ sell orders. Consequently, the terms $v_N^j(1 - v_N)^{n-j}$ and $v_o^j(1 - v_o)^{n-j}$ reflect the weighted sentiments of noise and position traders, and the expression $n!/j!(n - j)!$ reflects the number of possible combinations of j buy orders out of n orders received.

Table 2 describes the market maker's Bayesian learning process over 5 intraday time steps under bullish and bearish conditions. For illustrative purposes the number of buy and sell orders at each time step is represented by a random integer value of 1 or 0 orders, assuming valuation parameters $v_0 = 0.5$ for position traders and $v_N = 0.65$ for noise traders.

Panel (a) describes bullish sentiments among noise traders. These conditions increase the chance that buy orders will outnumber sell orders. Thus, in time step 1 the market maker receives $j = 1$ buy orders and $n{-}j = 0$ sell orders. This information increases the likelihood of a bull market from 0.5 to 0.65, and moves the market maker's posterior probability of bullish valuation from 0.5 to 0.57. Additional order flow information arrives in time step 2 via $j = 1$ buy orders and $n{-}j = 1$ sell orders. The arrival of new information dampens the likelihood of a bull market from 0.65 to 0.46, thus decreasing the market maker's posterior probability of bullish valuation from 0.57 to 0.54.

Panel (b) describes bearish noise-trader sentiments with sell orders outnumbering buy orders. In time step 1 the market maker receives $n{-}j = 1$ sell orders and $j = 0$ buy orders. This information increases the likelihood of a bear market from 0.5 to 0.65, thus increasing the market maker's posterior probability of bearish valuation from 0.5 to 0.57. Additional market orders arrive in time step 2 via $j = 1$ buy orders and $n{-}j = 0$ sell orders. This new information dampens the likelihood of a bear market from 0.65 to 0.35, thus decreasing the market maker's posterior probability of bearish valuation from 0.57 to 0.48.

From the standpoint of market making, this Bayesian learning has implications in how noise trading affects market-making behaviour; specifically, the determination of bid and ask prices, and their adjustment

Table 2. Bayesian learning process

	j Buy orders	$n{-}j$ Sell orders	$n!/j!(n{-}j)!$	$P(I\|V_n)$ likelihood	$P(I\|V_o)$ likelihood	$P_m(V_n\|I)$ revised	$P_m(V_o\|I)$ revised
Panel (a): bull market: $v_N = 0.65$ noise traders; $v_0 = 0.5$ position traders							
$T = 1$	1	0	1	0.65	0.50	0.57	0.43
$T = 2$	1	1	2	0.46	0.50	0.54	0.46
$T = 3$	1	0	1	0.65	0.50	0.61	0.39
$T = 4$	1	1	2	0.46	0.50	0.58	0.42
$T = 5$	1	0	1	0.65	0.50	0.65	0.35
Panel (b): bear market: $1 - v_N = 0.65$ noise traders; $v_0 = 0.5$ position traders							
$T = 1$	0	1	1	0.65	0.50	0.57	0.43
$T = 2$	1	0	1	0.35	0.50	0.48	0.52
$T = 3$	0	1	1	0.65	0.50	0.54	0.46
$T = 4$	0	1	1	0.65	0.50	0.61	0.39
$T = 5$	0	1	1	0.65	0.50	0.67	0.33

Table 3. Market-making behaviour

	j Buy orders	$n-j$ Sell orders	Inventory	P_b	P_a	P_a–P_b
Panel (a): bull market: $v_N = 0.65$ noise traders; $v_0 = 0.5$ position traders*						
$T=1$	0	0	0	1.1995	1.2005	0.0010
$T=2$	1	0	−1	1.2008	1.2019	0.0010
$T=3$	0	1	0	1.1995	1.2005	0.0010
$T=4$	1	0	−1	1.2008	1.2018	0.0010
$T=5$	1	0	−2	1.2021	1.2032	0.0011
Panel (b): bear market: $1-v_N = 0.65$ noise traders; $v_0 = 0.5$ position traders*						
$T=1$	0	1	1	1.1984	1.1994	0.0010
$T=2$	0	1	2	1.1973	1.1984	0.0011
$T=3$	0	0	2	1.1973	1.1984	0.0011
$T=4$	1	0	1	1.1984	1.1994	0.0010
$T=5$	0	1	2	1.1973	1.1984	0.0011

*The unbiased values $v_o = 1 - v_o = 0.5$ assume a strike price $p_e = 1.20$ and max–min prices $p_u = 1.24$ and $p_d = 1.16$.

in maintaining inventory control. To describe this behaviour, we assume that bid and ask prices (P_b and P_a) are quoted at each intraday time step relative to a market-clearing price for the underlying asset (P_o), the market maker's perception of market sentiments (a_N) and $(a_N)^{-1}$, and the extant inventory position (I), i.e.

$$P_b = P_o - \frac{T}{2}(a_N)^{-1} + f_b(I) \quad \text{and} \quad P_a = P_o + \frac{T}{2}(a_N) + f_a(I) \tag{12}$$

where T denotes the minimum tick size; the terms (a_N) and $(a_N)^{-1}$ denote the odds of bullish and bearish sentiments, i.e.

$$(a_N) = \frac{P_m(v_N|\Phi)}{1 - P_m(v_N|\Phi)} \quad \text{and} \quad (a_N)^{-1} = \frac{1 - P_m(v_N|\Phi)}{P_m(v_N|\Phi)}$$

and the terms $f_a(I)$ and $f_b(I)$ describe inventory control behaviour, where $f_a'(I) < 0, f_b'(I) < 0$ and $f_a(0) = f_b(0) = 0$.[10]

Table 3 gives the effect of noise-driven order flow on market-making behaviour over 5 intraday time steps. Panel (a) assumes bullish sentiments and thus an increased chance that buy orders will exceed sell orders ($j > n-j$) at each time step. This tendency appears over time steps 3–5 as shown by an increasing odds ratio a_N and higher bid–ask prices. Panel (b) assumes bearish sentiments with net selling pressure over all 5 intraday time steps. These market perceptions increase a_N^{-1} and lower the bid–ask prices. These price adjustments reflect the operation of both an information and inventory control channel in determining market-making behaviour by the optimizing agent.[11]

4. HEDGING 'NOISE-TRADER RISK'

Suppose that noise traders dominate position traders in the underlying asset market (FX futures). Depending on the degree of noise trading, the market maker may be unable to correct order flow imbalances through intraday adjustments in bid and ask price quotes. Consequently, noise-driven order flow may result in inefficient liquidity supply, as reflected by undesired inventory positions. Here we consider how the resulting noise-induced inventory price-risk can be delta-hedged (Silber, 1990).

Table 4 summarizes the qualitative relationship between noise-driven order flow in futures and the market-maker's inventory and hedging positions. Bearish sentiments tend to lower the futures price, provoking an increased number of sell orders for calls and buy orders for puts. The market maker therefore ends trading with a long position in calls and a short position in puts. This inventory position carries the risk that the futures price will continue falling overnight. Conversely, bullish futures trading leaves the

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Table 4. Inventory and hedging positions from noise-driven order flow

State-of-the-world	Order flow	Inventory	Inventory price-risk	Hedge
Bear market	↑ Sell orders for calls	Long-in-calls	↓Futures price	Sell futures
↓Futures price	↑ Buy orders for puts	Short-in-puts	↓Call price ↑Put price	
Bull market	↑ Buy orders for calls	Short-in-calls	↑Futures price	Buy futures
↑Futures price	↑ Sell orders for puts	Long-in-puts	↑Call price ↓Put price	

market maker short in calls and long in puts; the risk here is that the underlying price will be bid-up even further overnight. In either case, the inventory price-risk is due to the mean-averting tendency of the noise-driven futures price.

Delta-hedging can be used to neutralize the risk posed by noise-driven price movements in the underlying asset. For example, suppose intraday trading reduces the futures price by 4%, resulting in a 2% change in both call and put prices. Thus, at the market close the delta-hedge ratios are 0.5 for calls ($-0.02/-0.04$) and -0.5 for puts ($0.02/-0.04$). If the inventory position in puts and calls is balanced then no hedging is required, i.e. the options portfolio is delta-neutral. Otherwise delta-hedging is required to neutralize inventory price-risk, either by selling or buying the underlying asset.

We examine delta-hedging using Black's model for pricing European-style options on FX futures. The pricing formulas for European call and put options on FX futures are given by

$$c = [FN(d_1) - XN(d_2)]e^{-r(T-t)} \quad \text{and} \quad p = [XN(d_2) - FN(-d_1)]e^{-r(T-t)} \tag{13}$$

with parameters

$$d_1 = \frac{\ln(F/X) + 0.5\sigma^2(T-t)}{\sigma\sqrt{T-t}} \quad \text{and} \quad d_2 = d_1 - \sigma\sqrt{T-t} \tag{14}$$

The corresponding hedge ratios for call and put options on FX futures are given by

$$\Delta_c = N(d_1)e^{-r(T-t)} \quad \text{and} \quad \Delta_p = [N(d_1) - 1]e^{-r(T-t)} \tag{15}$$

where $N(^\circ)$ is the cumulative probability distribution function for a normally distributed variable with mean zero and standard deviation 1.

5. SIMULATION ANALYSIS

In order to simulate pricing and hedging behaviour we assume that a trading day covers 20 time steps over which the market maker revises his/her probability beliefs based on the arrival of buy and sell orders. Order flow and bid–ask pricing are determined simultaneously each time step conditional on extant noise-trader sentiments, i.e. the V_n parameter. Intraday trading is simulated $n = 5000$ times for each specific V_n-parameter value. In total we obtain 1.5 million simulation trials, reflecting varying degrees of noise-trader sentiments ranging from $V_n = 0.35$ (bearish) to $V_n = 0.65$ (bullish), with step increments of size 0.001. The results from these simulations shed light on the impact of noise trading on liquidity and pricing in the underlying market, and the prices and delta-hedge ratios for puts and calls on the underlying asset.

Figures 1(a)–(d) illustrate price and inventory in the underlying market based on descriptive statistics drawn from the entire sample of simulation trials. Figure 1(a) plots the mean values of the closing price relative to noise-trader sentiments. With neutral sentiments the closing price remains near its initial value (1.000), and otherwise becomes mean averting as noise traders lean in one direction or the other. This tendency reappears in Figures 1(b) and (c), which plot mean values of the bid–ask spread and price volatility (measured by the %-change between open–close prices). The spreads increase as noise-trader sentiments grow stronger, adjusting to information arrival and inventory imbalances. Figure 1(d) shows

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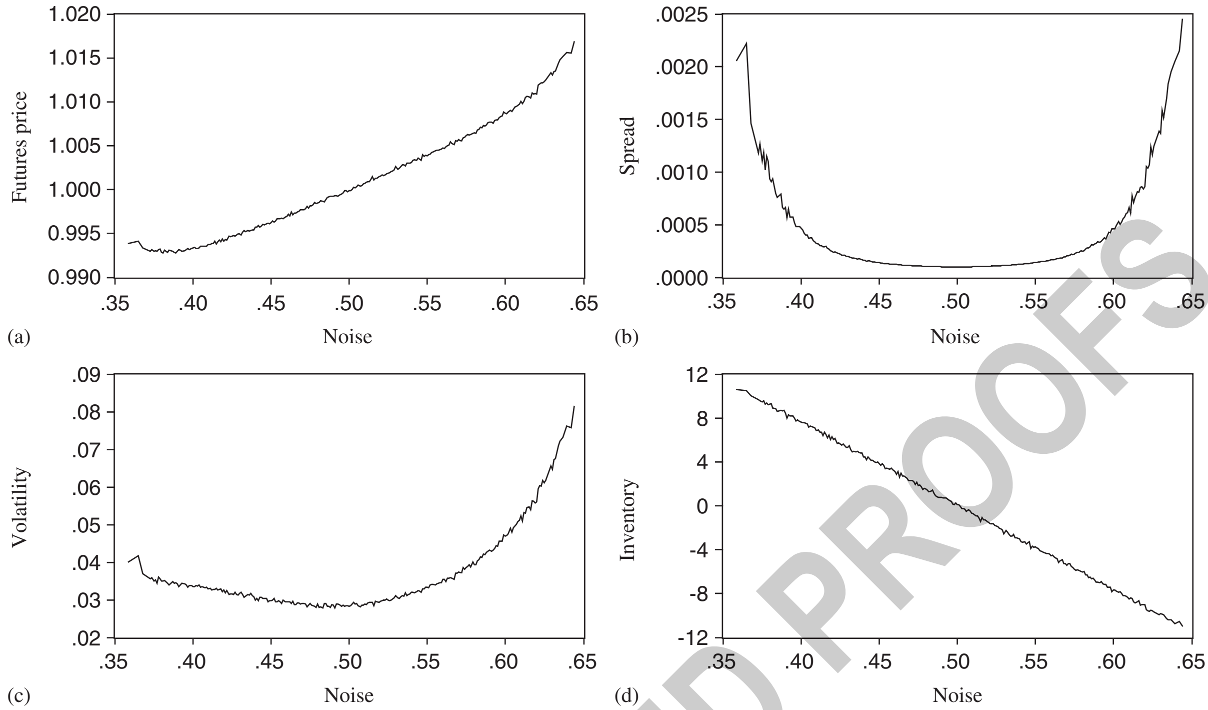

Figure 1. Noise-trading effects in futures market. *Noise-trader sentiments vary from 0.350 (bearish) to 0.650 (bullish), with 0.500 representing neutral. Five thousand simulations were made at each value, using step sizes of 0.001. Thus, each figure reflects the output from 1.5 million simulations, i.e. $5000 \times (0.65-0.35)/0.001$. Figure 1(a) shows the impact of noise on mean values of the futures price (measured by the midpoint of the bid–ask spread at the end of trading); Figure 1(b) shows the effect of noise on the mean bid–ask spread; Figure 1(c) shows the impact of noise on price-volatility (measured by the percentage change between open and closing prices); and Figure 1(d) shows the impact of noise on inventory in the underlying asset (measured at the end of the trading session).

Table 5. Simulation results for futures market

	Mean	Std. dev.	Min	Max	CV
$V_n = 0.35$; $N = 4380$					
Closing futures price	0.9917	0.0041	0.9406	1.0008	0.0041
Bid–ask spread	0.0009	0.0015	9.91E−05	0.0098	1.6667
Futures price volatility	0.0357	0.0182	1.03E−06	0.2654	0.5098
Futures inventory	10.42	6.4306	−12	26	0.6171
$V_n = 0.50$; $N = 5000$					
Closing futures price	0.9999	0.0001	0.973	1.0310	.0001
Bid–ask spread	1E−04	1.13E−08	9.73E−05	0.0001	1.13E−04
Futures price volatility	0.0283	0.0003	0.0000	0.1386	0.0106
Futures inventory	0.0854	0.1129	−31	27	1.3220
$V_n = 0.65$; $N = 4881$					
Closing futures price	1.0159	0.0169	0.8624	1.0920	.0166
Bid–ask spread	0.0019	0.0042	0.0001	0.0487	2.211
Futures price volatility	0.0772	0.0670	1.62E−05	0.6154	0.8679
Futures inventory	−11.22	7.3664	−29	17	−0.6565

*Descriptive statistics are reported for 5000 simulation trials under varying degrees of noise-trader sentiment: 0.350 (bearish), 0.500 (neutral), and 0.650 (bullish). The futures price is measured by the midpoint of the bid–ask spread at the end of trading. Price observations above 2 standard deviations were purged from the sample simulations, resulting in sample sizes of $n = 4380$ and 4881 under bearish and bullish market sentiments. Futures price-volatility is measured by the percentage change in open–close prices. The futures inventory is measured at the end of the trading session. CV denotes the coefficient of variation.

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inventory positions at the end of trading. Under neutral conditions the market maker ends trading with minimal inventory. Otherwise, noise trading results in inventory imbalances at the end of trading.

Table 5 reports descriptive statistics for a sub-sample of the simulations represented in Figures 1(a)–(d). Under neutral sentiments ($V_n = 0.5$; $n = 5000$ trials) the mean closing price (0.9917) and the pre-trade price (1.0000) are virtually the same. In addition, the mean value of the bid–ask spread is consistent with a 1-tick market (1E−04), wherein the market maker maintains efficient inventory turnover and market liquidity. Not surprisingly, the mean value of inventory at the end of trading is practically zero (0.0854). On the contrary, noisy sentiments ($V_n = 0.35$, 0.65; $n = 4380$, 4881 trials) correlate with increased price volatility (0.0357 or 0.0772), wider bid–ask spreads (0.0009 or 0.0019), and non-zero inventory positions at the end of trading (10.42 or −11.22).

Table 6 reports descriptive statistics for option prices and delta-hedge ratios for the same sub-sample of simulation trials described in Table 5. The mean values of the noise-driven option prices and delta ratios vary from the neutral-sentiment values according to the direction of the noise and the degree of price volatility in the underlying market. In addition, the standard deviations corresponding to these variates increase under noisy conditions in the underlying market. This noise-dependency is observed more generally in Figures 2(a)–(d) over the entire sample of simulation trials.

Market sentiments also impact the cost of hedging inventory positions in the derivatives market. For example, neutral sentiments are more likely to result in balanced inventory positions at the end of trading (puts ≈ calls), and thus minimal exposure to inventory price-risk if these positions are maintained overnight. Accordingly, the cost of hedging options inventory will depend on the degree of noise trading in the underlying market and the market maker's overnight inventory constraint.

Table 7 reports descriptive statistics for delta-neutral hedging subject to an overnight position constraint of $1 mm. Under neutral sentiments the market maker opens and closes trading with a balanced position in puts and calls (four puts and four calls). No hedging is required in this case because the inventory position is delta-neutral. Under bearish sentiments the market maker ends trading with a long position in calls and short position in puts. In this case delta-hedging overnight inventory requires selling futures with a mean cost of −$991 613 and a standard deviation of $55 943. Under bullish sentiments the market maker ends trading short in calls and long in puts. In this case delta-hedging overnight inventory requires buying futures with a mean cost of $902 313 and a standard deviation of $64 350.

Table 6. Simulation results for options market

	Mean	Std. dev.	Min	Max	CV
$V_n = 0.35$; $N = 4380$					
Midpoint call price	0.0442	0.0019	0.0234	0.0487	0.0430
Call delta	0.9918	0.0041	0.9406	1.0008	0.0041
Midpoint put price	0.0522	0.0020	0.0479	0.0816	0.0383
Put delta	0.4927	0.0129	0.6552	0.4639	0.0262
$V_n = 0.50$; $N = 5000$					
Midpoint call price	0.0483	5.808E-05	0.0355	0.0657	1.20E-03
Call delta	0.9999	0.0001	0.973	1.0310	0.0001
Midpoint put price	0.0484	5.276E-05	0.0353	0.0620	1.09E-03
Put delta	0.4669	0.0004	0.5529	0.3715	0.0009
$V_n = 0.65$; $N = 4881$					
Midpoint call price	0.0573	0.0098	0.0063	0.1079	0.1710
Call delta	1.0159	0.0169	0.8624	1.0921	0.0166
Midpoint put price	0.0416	0.0068	0.0176	0.1413	0.1635
Put delta	0.4179	0.0499	0.8559	0.2149	0.1194

*As in Table 5, the descriptive statistics reflect 5000 separate simulation trials at values 0.350 (bearish), 0.500 (neutral), and 0.650 (bullish). Reduced sample sizes are reported under bearish and bullish market sentiments, i.e. $n = 4380$ and 4881. The noise-driven option prices and deltas are measured at the end of trading assuming expiration in 250 days, a risk-free interest rate of 5%, and mean volatility values described in Table 5. CV denotes the coefficient of variation.

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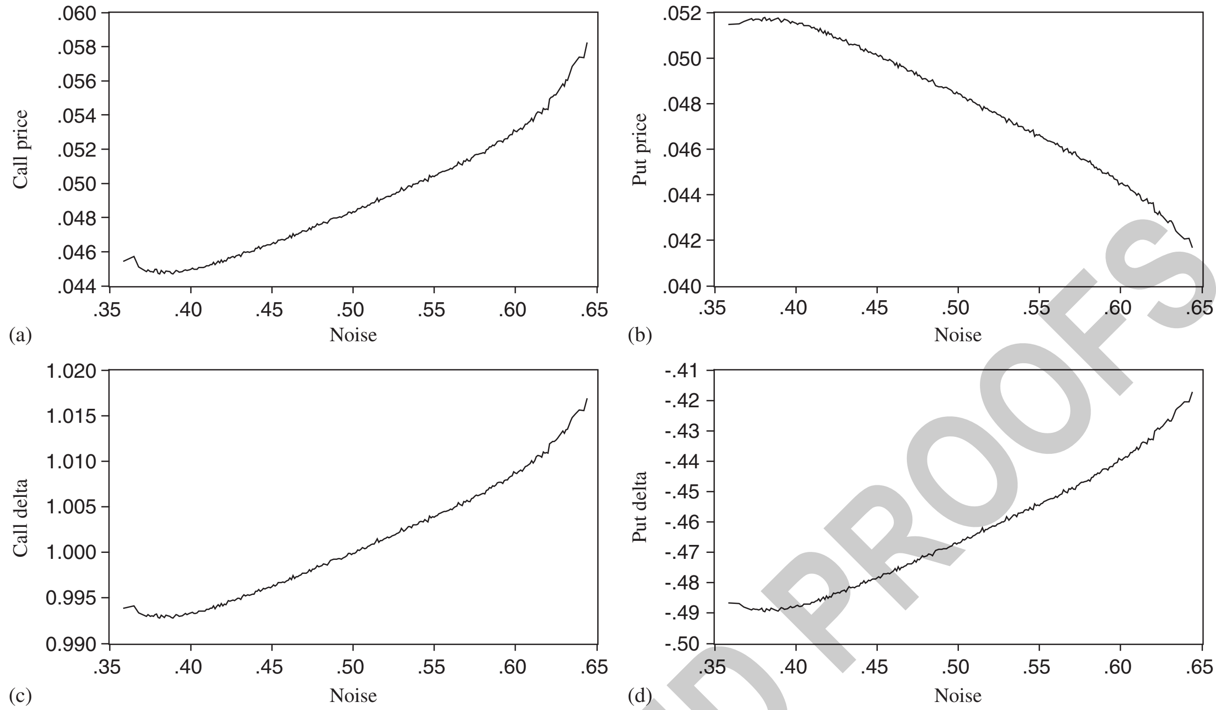

Figure 2. Noise transmission from futures-to-options-market. *As in Figures 1(a)–(d), noise-trader sentiments reflect a variation from 0.350 (bearish) to 0.650 (bullish) using step sizes of 0.001. Figures 2(a) and (c) show the impact of noise on mean values of the call price and call delta, while Figures 2(b) and (d) measure the effect of noise on the mean values of the put price and put delta. These noise-driven option prices and deltas are measured at the end of trading assuming expiration in 250 days, a risk-free interest rate of 5%, and the mean volatility values shown in Figure 2(c).

For simplicity the simulation trials assume that the market maker rebalances his hedged position at the end of the trading day subject to an overnight inventory constraint. Here one sees that noise trading has a notable impact on the variation in hedging costs. Moreover, the impact on hedging cost increases if the hedged position is rebalanced less frequently under a less restrictive position constraint.

6. CONCLUDING REMARKS

'Noise-trader risk' impacts the pricing and risk-management behaviour of a Bayesian market maker. We model and simulate this behaviour assuming that order flow from noise and position traders gives a noisy signal of market sentiments. Our theoretical framework identifies the price discovery process following either a Martingale or a mean-averting process, depending on the degree of noise trading. Moreover, the model predicts that noise transmission from futures to options will have pecuniary effects on delta-hedging behaviour. To this end 'noise-trader risk' may keep market makers from taking arbitrage positions.

Under noisy market conditions a Bayesian market maker privately benefits from a risk-management strategy, which coincides with trend chasing; that is, selling futures as their price gets hammered, or buying them as it inflates—not for speculative purposes, but to avert the risk of being caught on the wrong side of the market. This hedging behaviour tends to push the FX futures price further away from the fair value, and is motivated by the argument that supplying liquidity in a noise-driven derivatives market engenders significant inventory price-risk. In such an environment one is reminded that 'it may pay more for smart money to follow dumb money rather than to lean against it'—a perspective argued some time ago by Haltiwanger and Waldman (1985) and Russell and Thaler (1985).

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Table 7. Noise effects on delta-hedging options

	Neutral market close	Bear market close	Bull market close
Options position at close of trading	Long 4 puts	Long 12 calls	Short 4 puts
(open market long 4 puts and 4 calls)	Long 4 calls	Short 4 calls	Long 12 puts
Futures hedging transaction	–	Short futures	Long futures
Interday delta-neutral hedging cost	−\$25 481	−\$991 613	\$902 313
	(\$23 792)	(\$55 943)	(\$64 350)

*Option pricing and delta ratios are calculated from the distribution of closing prices for FX futures, with each observation reflecting the mid-point estimate of the closing bid and ask price quotes. Daily returns for FX futures are calculated using the percentage change in open-to-close prices. The mean closing price and the standard deviation of returns are then used in the Black model to simulate the option prices and delta-hedge ratios. Option prices and deltas are measured at the end of trading assuming expiration in 250 days, a risk-free interest rate of 5%, and mean volatility values described in Table 5.

NOTES

1. Various empirical studies examine the efficiency of FX options in satisfying the no-arbitrage conditions, e.g. Tucker (1985), Bodurtha and Courtadon (1986), Shastri and Tandon (1986a,b), and Ogden and Tucker (1987). See Kolb for full citations on these papers.
2. Silber provides a general review of options market making, while Sarno and Taylor review the literature specific to FX market making.
3. As noted in Sarno and Taylor (2001), Lyons models the formation of price expectations using a Bayesian model in the tradition of Amihud and Mendelson (1980), Cohen *et al.* (1981), Conroy and Winkler (1981), Glosten and Milgrom (1985), and Madhaven and Smidt (1991). See Flood (1994) for a non-technical discussion of this literature.
4. The literature explains bid–asked spreads on the basis of order-processing costs, asymmetric information costs, and inventory-carrying costs. In the case of FX markets it has been suggested that inventory-carrying costs play the major role in explaining spreads, where the costs of maintaining open positions will vary depending on the degree of price-risk and trading activity facing market makers. Conversely, order-processing costs and asymmetric information costs are viewed as less significant in FX markets given the large scale of transactions involved, efficiencies in order execution, and the absence of ‘inside information’ to trade on.
5. See Lyons (2001) for an insightful discussion of the characteristics underlying the Kyle (1985) model and sequential-trade models, esp. Chapter 4, pp. 63–93.
6. As advanced by Muth (1961), rational expectations imply that agents make estimates of unknown variables (e.g. an asset’s price) in the best possible manner (unbiased, on average), using all information currently available.
7. This behaviour is inconsistent with the use of Bayes’ rule in predicting uncertain outcomes, as described in the seminal paper by Kahneman and Tversky (1973) on the ‘psychology of prediction.’
8. Recall from (5) and (7) that the parameter v_o (or $1 - v_o$) reflects an unbiased probability of a rising (or falling) futures price, and therefore that call (put) options will be exercised.
9. Intuitively, consider a dice roll: one is red, representing sell orders; and the other is green, representing buy orders. Once rolled, the resulting difference in red-*versus*-green values represents the market maker’s net position in maintaining a two-sided market. Thus, noise trading is tantamount to rolling loaded dice.
10. Options market makers quote option prices based on the bid–ask spreads in the underlying futures market, which is both deeper and more liquid (Silber, 1990).
11. These implications are consistent with standard inventory-cost models insofar as assuming that the market maker seeks to balance order flow while maintaining inventory control, e.g. Amihud and Mendelson (1980) and Flood (1994).

REFERENCES

Amihud Y, Mendelson H. 1980. Dealership market: market-making with inventory. *Journal of Financial Economics* **8**(1): 31–53.

Barnea A, Logue D. 1975. The effects of risk on the market maker’s spread. *Financial Analysts Journal* **26**(4): 45–69.

Bessembinder H, Hendrik K, Serguin P. 1996. An empirical examination of information, differences of opinion and trading activity. *Journal of Financial Economics* **40**(1): 105–134.

Bhattacharya M. 1983. Transaction data tests of the Chicago Board of Options Exchange. *Journal of Financial Economics* **12**(2): 161–185.

Black F. 1976. The pricing of commodity contracts. *Journal of Financial Economics* **3**(1/2): 167–179.

Black F. 1986. Noise. *Journal of Finance* **41**(3): 529–543.

Bodurtha J, Courtadon G. 1986. Efficiency tests of the foreign currency options market. *Journal of Finance* **41**(1): 151–162.

Bollerslev T, Domowitz I. 1993. Trading patterns and prices in the interbank foreign exchange market. *Journal of Finance* **48**(4): 1421–1443.

Bollerslev T, Melvin M. 1994. Bid-ask spreads and volatility in the foreign exchange market: an empirical analysis. *Journal of International Economics* **36**(3–4): 355–372.

Cohen K, Maier S, Schwartz R, Whitcomb D. 1981. Transaction costs, order placement strategy, and existence of the bid-ask spread. *Journal of Political Economy* **89**(2): 287–305.

Conroy R, Winkler R. 1981. Informational differences between limit and market orders for a market maker. *Journal of Financial and Quantitative Analysis* **16**(5): 703–724.

DOI: 10.1002/ijfe

De Long J, Shleifer A, Summers L, Waldman R. 1990. Noise-trader risk in financial markets. *Journal of Political Economy* **98**(4): 703–738.

De Long J, Shleifer A, Summers L, Waldman R. 1991. The survival of noise-traders in financial markets. *Journal of Business* **64**(1): 1–19.

Fama E. 1965. The behavior of stock market prices. *Journal of Business* **38**(1): 84–105.

Fama E. 1970. Efficient Capital Markets: A review of theory and empirical work. *Journal of Finance* **25**(2): 383–417.

Fama E. 1991. Efficient Capital markets: II. *Journal of Finance* **46**(5): 1575–1617.

Fama E. 1998. Market efficiency, long-term returns, and behavioral finance. *Journal of Financial Economics* **49**(3): 283–306.

Flood M. 1994. Market structure and inefficiency in the foreign exchange market, *Journal of International Money and Finance* **13**(2): 131–158.

Friedman M. 1953. The case of flexible exchange rates. *In Essays in Positive Economics*. University of Chicago Press: Chicago.

Galai D. 1978. Empirical tests of boundary conditions for CBOE options. *Journal of Financial Economics* **6**(2/3): 187–211.

Garman M. 1976. Market Microstructure. *Journal of Financial Economics* **3**(3): 257–275.

Glosten L, Milgrom P. 1985. Bid, ask and transaction prices in a specialist market with heterogeneously informed traders. *Journal of Financial Economics* **14**(1): 71–100.

Haltiwanger J, Waldman M. 1985. Rational expectations and the limits of rationality: an analysis of heterogeneity. *American Economic Review* **75**(3): 326–340.

Hirshleifer J, Riley J. 1992. The Analytics of Uncertainty and Information. Cambridge University Press: Cambridge.

Kindleberger C. 1978. Manias, Panics and Crashes. New York: Basic Books.

Kolb R. 1991. Understanding Futures Markets. Kolb Publishing Company: Miami.

Kyle, Robert S. 1985. Continuous auctions and insider trading. *Econometrica* **53**(6): 1315–1335.

Lyons R. 1991. Private beliefs and informational externalities in the foreign exchange market. *National Bureau of Economic Research Working Paper No. 3889*. National Bureau of Economic Research: Cambridge.

Lyons R. 1996. Foreign exchange volume: sound and fury signifying nothing? In The Microstructure of Foreign Exchange Markets, Frankel J *et al*. (eds). University of Chicago Press: Chicago.

Lyons R. 1995. Tests of microstructural hypotheses in the foreign exchange market. *Journal of Financial Economics* **39**(2–3): 321–351.

Lyons R. 1998. Profits and position control: a week of FX dealing. *Journal of International Money and Finance* **17**(1): 97–115.

Lyons R. 2001. The Microstructure Approach to Foreign Exchange Rates. MIT Press: Cambridge.

Madhaven A, Smidt S. 1991. A bayesian model of intraday specialist pricing. *Journal of Financial Economics* **30**(1): 99–134.

Muth J. 1961. Rational expectation and the theory of price movements. *Econometrica* **29**(3): 315–335.

Ogden J, Tucker A. 1987. Empirical tests of the efficiency of the currency futures options market. *Journal of Futures Markets* **17**(6): 695–703.

Ross S. 2003. Introduction to Probability Models, 8th edn. Academic Press: Boston.

Russell T, Thaler R. 1985. The relevance of quasi rationality in competitive markets. *American Economic Review* **75**(5): 1071–1082.

Sarno L, Taylor M. 2001. The microstructure of the foreign exchange market: a selective survey of the literature. *Princeton Studies in International Economics*, vol. **89**. Princeton University.

Shastri K, Tandon K. 1986a. An empirical test of a valuation model for American options on futures contracts. *Journal of Financial and Quantitative Analysis* **21**(4): 377–392.

Shastri K, Tandon K. 1986b. Valuation of foreign currency options: some empirical tests. *Journal of Financial and Quantitative Analysis* **21**(2): 145–160.

Shleifer A. 2000. Inefficient Markets: An Introduction to Behavioral Finance. Oxford University Press: Oxford.

Shleifer A, Summers L. 1990. The noise trader approach to finance. *Journal of Economic Perspectives* **41**(2): 19–33.

Shleifer A, Vishny R. 1997. The limits of arbitrage. *Journal of Finance* **52**(1): 35–55.

Sliber W. 1984. Marketmaker behavior in an auction market: an analysis of scalpers in futures markets. *Journal of Finance* **39**(4): 937–953.

Silber W. 1990. Marketmaking in options: principles and implications. In Financial Options: From Theory to Practice, Figlewski S *et al*. (eds). Irwin: Homewood, IL.

Tucker A. 1985. Empirical tests of the efficiency of the currency option markets. *Journal of Financial Research* **8**(4): 275–285.

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