Eventology versus contemporary theories of uncertainty

Oleg Vorobyev

Institute of Mathematics, Siberian Federal University

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Oleg Yu. Vorobyev
Institute of Mathematics,
Siberian Federal University,
Krasnoyarsk State Trade Economic Institute,
Krasnoyarsk, Siberia, Russia
vorob@akadem.ru; www.eventology-theory.com

Abstract

The development of probability theory together with the Bayesian approach in the three last centuries is caused by two factors: the variability of the physical phenomena and partial ignorance about them. As now it is standard to believe [1], the nature of these key factors is so various, that their descriptions are required special uncertainty theories, which differ from the probability theory and the Bayesian credo, and provide a better account of the various facets of uncertainty by putting together probabilistic and set-valued representations of information to catch a distinction between variability and ignorance. Eventology [2], a new direction of probability theory and philosophy, offers the original event approach to the description of variability and ignorance, entering an agent, together with his/her beliefs, directly in the frameworks of scientific research in the form of eventological distribution of his/her own events. This allows eventology, by putting together probabilistic and set-event representation of information and philosophical concept of event as co-being [3], to provide a unified strong account of various aspects of uncertainty catching distinction between variability and ignorance and opening an opportunity to define imprecise probability as a probability of imprecise event in the mathematical frameworks of Kolmogorov's probability theory [4].

Keywords: *uncertainty, probability, event, co-being, eventology, imprecise event.*

1 Introduction

It is usually accepted to believe [1] that the development of contemporary uncertainty theories (the theory of imprecise probabilities, evidence, and possibility theory) was caused by juncture of two challenges: assimilating the variability of physical processes and mastering account of incompleteness of information in cognitive and decision processes. Juncture of these challenges are related by the fact: "in the face of variability and ignorance, it is impossible to know what the present state of the Being (physical together with mental) is".

At least there are two more challenges, which do not find the worthy answer in contemporary uncertainty theories: the assimilating set-event-based representations of partial ignorance and the entering human agents in the form of their set-event-based models directly in the framework of scientific research of uncertainty. The eventology has found answers to these challenges outside a quantitative level of researches (including a level of probabilities), considering and solving problems of uncertainty at higher and at more difficult level of events, at the set-event level.

Eventology [2], a new direction of probability theory and philosophy, offers the original event approach to the description of variability and ignorance, entering an agent, together with his/her beliefs, directly in the frameworks of scientific research in the form of eventological distribution of his/her own events. This allows eventology, by putting together probabilistic and set-event representations of information, and philosophical concept of event as co-being [3], to provide a unified strong account of various aspects of uncertainty catching distinction between variability and ignorance in the mathematical frameworks of Kolmogorov's probability theory [4].

Now eventology is a broad spectrum of scientific researches, which includes mathematical, philosophical, and practical eventology. The volume of this paper is not enough for a statement of fundamental bases of eventology (an event as a co-being; event and probability as two interconnected concepts, which cannot exist separately; an event as a Kolmogorov axiomatic event; a probability as a Kolmogorov axiomatic probability; various probability interpretations known now as different ways of assessment of Kolmogorov's axiomatic probability; each object (physical or mental) as a distribution of its set of events; each human agent as a distribution of his/her set of events). Therefore the paper contains only brief enumeration of basic concepts of this new direction of probability theory and philosophy, added by the detailed eventological analysis of the example of set-based representations of partial ignorance offered in [1]. The author hopes that analysis of this example will assist to illustrate and to understand opening opportunities of the new eventological description of uncertainty, variability, ignorance and impreciseness.

Partially to compensate a cable style of a statement of the paper there is a detailed list of references to papers that are sources of the ideas put in a basis of eventology; have the direct relation to eventological subjects; and represent areas of researches, where eventological methods can be effectively used.

2 Contemporary theories of uncertainty

Below the list of basic contemporary uncertainty theories together with references to leading authors is resulted:

- **theories of uncertainty** —
  - *fuzzy set theory* :
    Lotfi Ascer Zadeh [5],
  - *fuzzy logic* :
    Lotfi Ascer Zadeh [5, 6], Vilem Novak, Irina Perfilieva, Jiri Mockor [7],
  - intuitionistic fuzzy sets :
    Eulalia Szmidt, Janusz Kacprzyk [8],
  - *possibility theory* :
    Lotfi Ascer Zadeh [9], Didier Dubois, Henri Prade [10, 11, 12, 13, 14],
  - *evidence theory* :
    Arthur P. Dempster [15], Glenn Shafer [16, 17], Philippe Smets [18, 19, 20],

- *theory of imprecise probabilities* :
  Peter Walley [21, 22, 23], Gert de Cooman [24, 25], Arnold Neumaier [26],
- *generalized theory of uncertainty* : (Lotfi Ascer Zadeh [27], Didier Dubois [1]).

3 Ways of assessment of a probability

Now in eventology various probability interpretations existing are considered as various ways of an assessment of the same Kolmogorov axiomatic probability instead of as various concepts of probability. There are two broad categories of probability interpretations, which can be called *physical* and *evidential* probabilities. Physical probabilities, also called objective or frequency probabilities, are associated with random physical systems. Evidential probability, also called subjective or Bayesian probability, are associated with human agents and their beliefs. More detailed classification of probability interpretations known now with references to authors is resulted below:

- **physical** —
  - *classical and frequency*:
    Jacob Bernoulli [28], Pierre Simon Laplace [29], John Venn [30], Hans Reichenbach [31], Richard Edler von Mises [32], Sir Ronald Aylmer Fisher [33, 34], Jerzy Splawa Neyman [35] and Egon Sharpe Pearson [36],
  - *propensity*:
    Karl Raimund Popper [37, 38], David W. Miller [39], Ronald N. Giere [40] and James Henry Fetzer [41, 42],
  - *algorithmic*:
    Ray Solomonoff [43, 44];

- **evidential** —
  - *classical, epistemic or inductive*:
    Reverend Thomas Bayes [45], Pierre Simon Laplace [29], Frank Hyneman Knight [46], Richard Threlkeld Cox [47],
  - *logical*:
    John Maynard Keynes [48], Rudolf Carnap [49],
  - *subjective*:
    Frank Plumpton Ramsey [50], Bruno de Finetti [51, 52], Leonard Jimmie Savage [53], Francis John Anscombe and Israel Robert John Aumann [54], Daniel Kahneman and Amos Tversky [55, 56, 57, 58],
  - *intuitive*:
    Bernard Osgood Koopman [59], Irvin John Good [60], George Lennox Sharman Shackle [61, 62, 63], Kenneth Joseph Arrow and Gerard Debreu [64, 65, 66], Edi Karni [67],
  - *Bayesian*:
    Edwin Thompson Jaynes [68], Jose Miguel Bernardo [69],

- *informatively rationalistic* :
  John Charles Harsanyi [70],
- *imprecise*:
  Peter Walley [21, 22, 23], Gert de Cooman [24, 25], Arnold Neumaier [26].

So, eventology asserts that the listed authors offer only various ways of assessment of the same axiomatic probability, but not various concepts of probability. Moreover, eventology suggests to ascent of even-based level of analysis from a quantitative level of assessment of probability and to analyze similar ways of assessment of corresponding events: *physical* (classical, frequency, as propensity, and algorithmic) and *evidential* (classical, cognitive, inductive, logic, subjective, intuitive, Bayesian, informatively rationalistic, and imprecise). As a result various ways of assessment of the axiomatic probability, offered by the listed authors, it is possible to interpret as axiomatic probability of the events assessed in the corresponding various ways. For example, the imprecise probability can be interpreted as axiomatic probability of corresponding *imprecise event*, the new eventological notion which demands strict definition within eventology[1].

4 Probability as an axiomatic probability

Kolmogorov's axiomatics is a standard axiomatic approach to the mathematical description of event and probability; it is offered by Andrei Kolmogorov in 1929 and 1933 [4]; it has given probability theory the style accepted in the modern mathematics. Up to Kolmogorov attempts to axiomatize of probability theory were undertook by G. Bohlmann [72], S. N. Bernstein [73], R. Mizes [74], and also A. Lomnitsky [75] on the basis of E. Borel's ideas [76] about connection of concepts of probability and measure:

- **axiomatic probability** —
  - Andrei N. Kolmogorov [4],
  - G. Bolmann [72], Felix Edouard Justin Emile Borel [76], Sergei N. Bernstein [73], Richard Edler von Mises [74], and A. Lomnicki [75].

5 Event as a co-being

Among authors of modern theories of events three authors: M. Bakhtin [77, 78], A. Kolmogorov [4], and B. Russell [79, 80] have played a key role in becoming eventology as science. Bakhtin has proved a concept of event as a co-being; Kolmogorov has given an axiomatic definition of event in his axiomatics of probability theory; and Russell has emphasized: "matter is simply convenient way of linkage of events together". Below the list of authors of modern theories of events is resulted:

[1] In the same way, as in eventology the conditional probability is defined as probability *conditional event*, the new notion, which is introduced in [71] for the first time.

- **theories of events** —
  - Michael M. Bakhtin [77, 78],
  - Andrei N. Kolmogorov [4],
  - Bertrand Arthur William Russell [79, 80],
  - Nikolai O. Lossky [81], Henri Bergson [82], Martin Heidegger [83], Sigmund Freud [84], Gilles Deleuze [85], Donald Davidson [86], David Kellogg Lewis [87], Alain Badiou [88], and Jaegwan Kim [89].

6 Eventology

Eventology is the study on events, arisen from "unbearably light" observations: "event[2] is always co-being" [77, 3]; "matter is a simply convenient way of linkage of events together[3]" [79]; "mental behavior arises there and then, where and when an ability to make probabilistic choice is arisen[4]" [91]; and "mind is a simply convenient way of a probabilistic choice of perceiving and creating the sets of events[5]" [2].

- Authors of initial eventological ideas:
  - Michael M. Bakhtin [77, 3, 78],
  - Bertrand Arthur William Russell [79]
  - Vladimir A. Levebvre [91]
  - Oleg Yu. Vorobyev [2]

Mathematical eventology is a new section of probability theory based on Kolmogorov's axiomatics, already shown the efficiency in the mathematical description of eventological substantiation and development of existing uncertainty theories (fuzzy sets theory, possibility theory, and theory of evidence), and contemporary prospect theory also.

Step by step alongside with philosophical and mathematical questions of Being and co-being of human agents the eventology masters the impressing domain of economic, sociological and psychological questions [92], [93], [94], [95], [96], [97], [98], [99], [100], [101], [102], [103], [104], [105], [106], [107], [108], [109], [110], [2], [111], [112], [113], [114], [115], [116], [117], [118], [119], [120].

[2] "The Russian word used, "*sobytie*", is the normal word Russian would use in most contexts to mean what we call in English an "event". In Russian, "event" is a word having both a root and a stem; it is formed from the word for being — "bytie" — with the addition of the prefix implying sharedness, "so-", (or, as we should say in English, "co-" as co-operate or co-habit), giving "*sobytie*", event as co-being. "Being" for Bakhtin then is, not just an event, but an event that is shared. Being is a simultaneitly; it is always co-being [78, p. 25]".

[3] To his universal event-based definition of matter Russell has come as a result of the deep analysis of achievements of modern physics [79].

[4] At once from the surprising event-probability definition of mental behavior, which Lefebvre has given, basing on results of experimental psychology (see [90], for example), and from Bakhtin's brilliant definition of event as co-being follows the one of starting eventological idea: event as co-being and probability are two interconnected concepts nonexistent separately.

[5] The event-based definition of mind, which is key in eventology, follows from resulted above Bakhtin's definition of event, Russell's definition of matter, and Lefebvre's definition of mental behavior directly.

6.1 Random finite set of events (RFSE)

Let $\mathfrak{X}$ be a finite set of Kolmogorov's events ($|\mathfrak{X}| = N < \infty$), chosen from the algebra $\mathcal{F}$ of the eventological space $(\Omega, \mathcal{F}, \mathbf{P})$, where Ω is the set of all possible outcomes of Being, i.e. the set of all possible states of World; $2^{\mathfrak{X}}$ is the set of all subsets of $\mathfrak{X}$.

The random finite set of events (RFSE) is defined as a random element with values from $2^{\mathfrak{X}}$, i.e. as a measurable map $K : (\Omega, \mathcal{F}, \mathbf{P}) \to \left(2^{\mathfrak{X}}, 2^{\left(2^{\mathfrak{X}}\right)}\right)$. At an outcome of Being $\omega \in \Omega$ there come those events $x \in \mathfrak{X}$, for that $\omega \in x$. From here possible values of K are subsets $K(\omega) = \{x \in \mathfrak{X} : \omega \in x\}$ of such events $x \in \mathfrak{X}$ that come at the outcome ω, i.e. $K(\omega) \in 2^{\mathfrak{X}}$ (or $K(\omega) \subseteq \mathfrak{X}$), thus for any $X \subseteq \mathfrak{X}$

$$\{\omega : K(\omega) = X\} = \bigcap_{x \in X} x \bigcap_{x \in X^c} x^c, \tag{1}$$

where $x^c = \Omega - x$, $X^c = \mathfrak{X} - X$. The event $\mathrm{ter}(X) = \bigcap_{x \in X} x \bigcap_{x \in X^c} x^c$ is called the event-terrace. As a value of $K(\omega)$ a subset X is interpreted as the subset of events from $\mathfrak{X}$ that occur at the outcome $\omega \in \mathrm{ter}(X)$, and $X^c = \mathfrak{X} - X$ is interpreted as the subset of events from $\mathfrak{X}$ that do not occur at the ω.

By

$$p(X) = \mathbf{P}(K = X) = \mathbf{P}\left(\mathrm{ter}(X)\right) \tag{2}$$

denote the probability of the event-terrace $\mathrm{ter}(X)$, i.e. the probability of the event $\{K = X\}$. All of 2^N events-terraces, generated by the finite set of events $\mathfrak{X}$, form a finite partition of the certain event $\Omega = \sum_{X \subseteq \mathfrak{X}} \mathrm{ter}(X)$. From here $\sum_{X \subseteq \mathfrak{X}} p(X) = 1$, so probabilities $p(X)$ of events-terraces $\mathrm{ter}(X)$, taken all together for $X \subseteq \mathfrak{X}$, form something that can be named a probability distribution of RFSE K because of $K(\omega) = X$ as $\omega \in \mathrm{ter}(X)$. Here the function $p : 2^{\mathfrak{X}} \to [0, 1]$ is some *function of the set of events* (or *set-function*). Values of the set-function p on subsets of events $X \subseteq \mathfrak{X}$ are probabilities of corresponding events-terraces $\mathrm{ter}(X) \in \mathcal{F}_{\mathfrak{X}}$. It's clear that a set-function p, corresponding any RFSE, should satisfy to two obvious conditions:

1) to be non-negative: $p(X) \geq 0$, $X \subseteq \mathfrak{X}$;
2) to be normed: $\sum_{X \subseteq \mathfrak{X}} p(X) = 1$.

7 Eventological analysis of set-based interpretations of partial ignorance [1]

The ascent of an event-based level of thinking from a quantitative level of thinking assumes consecutive development of a set-based level of thinking and a level of thinking under uncertainty. During the ascention it is necessary to overcome step by step our inertia of quantitative thinking, an imperfection of our skills of set-thinking, and our aversion of uncertainty at last.

First, it is necessary to ascend: from a number up to a set of numbers; from a set of numbers up to a set of any elements; from a set of any elements up to an axiomatic event. Then, having taken this height, it is necessary to ascend from number up to a random variable and to continue the ascent: from a random variable up to a set of random variables; from a set of random variables up to a set of any random elements; from a set of any random elements up to a set of Kolmogorov's events.

Let's illustrate this process of our mental ascent of event-based level of thinking by eventological analysis of the example of set-based representations of partial ignorance offered in [1], where it is declared: "the basic tool for representing information incompleteness is set theory: an ill-known quantity is represented as a disjunctive set, i.e. a subset of mutually exclusive values, one of which is the real one".

Logically this assertion has no defects. However such logic can use only at a quantitative level of thinking. The matter is that the assertion silently bases on the property of order of numbers. But this property, which is characteristic for any quantitative structure, is not kept by offered set-based representation of an ill-known quantity by disjunctive set of mutually excluding values.

Then this truncated set-based representation is used for construction of the same truncated event-based representation of information incompleteness. The obtained event-based representation appears to be true only for embedded structures of events[6], instead of for any structures of events. As a result this leads to the following partial[7] set-based definition of "a possible event" and "a certain event" [1]:

- Any event A understand as the assertion $x \in A$ is "possible" (for this agent)[8] whenever $[a, b] \cap A$ is not empty.
- Any such event A is "certain" **for this agent**[9], whenever $[a, b] \subseteq A$.

The definition of "possible event" and "certain event" is key for set-based representations of partial ignorance and for all of set theoretic base of contemporary theories of uncertainty: possibility theory [9, 10, 11, 12, 13, 14], theory of evidences [15, 16, 17, 18, 19, 20], theory of imprecise probabilities [21, 22, 23, 24, 25, 26], and unified theory of uncertainty [27, 1]. Therefore let's analyze this definition from the eventological point of view in more detail.

Theoretical value and fruitfulness of characteristic replacement of any physical objects by events has been underlined by Russell [79]. The author has specified on theoretical inevitability of the same characteristic substitution events for any mental objects [2]: from the eventological point of view everything in World (physical and/or mental) can be characterized by events. However such characterization should be well defined eventologically in the sense that the resulted set of events should possess the characteristic properties: the event-based description should contain all characteristic properties of the physical and/or mental object.

[6]As the embedded structures at a set-based level keep characteristic properties of order structures at a quantitative level.

[7]Suitable only for a quantitative level of thinking.

[8]added by me.

[9]a bold is mine.

A set of numbers as a set of Kolmogorov's embedded events. Let's continue our analysis. In the example [1] "an event" A, understood as "$x \in A$", where x is an ill-known quantity, is placed on a numerical axis and is defined as some set of numbers: $A \subseteq \mathbf{R}$. Thus "the event" A is defined as a set of elements forming structure of the order. In the example such definition of "event" A is not well defined eventologically[10]: "the event" A does not characterize a set of ordered elements because of it does not contain the description of order structure. It is impossible "to restore" the order of elements of the any (disorder) set, "knowing" only what set is and "not knowing" that its elements possess the property of order (in the example these ordered elements are numbers).

More well defined from the eventological point of view "event" A is represented as the some *set* $A \subseteq \mathfrak{X}$[11] of *Kolmogorov's embedded events* $y = \{r \leq r_y\} \in \mathfrak{X}$, corresponding to numbers $r_y \in \mathbf{R}$.

So, the "event" A in the example is some set of numbers from $\mathbf{R}$. It is not well defined eventologically. While our *set of Kolmogorov's embedded events* A is well defined eventologically as $A = \Big\{\{r \leq r_y\} \subseteq \mathbf{R},\ y \in A\Big\}$, a set of subsets of numbers from $\mathbf{R}$, "storing the information" about the order structure of numbers[12].

An agent as a set of Kolmogorov's events. In eventology each object (physical and/or mental) is considered as a finite set of Kolmogorov's events, which characterizes this object, certainly, not entirely, but characterizes eventologically completely within the frameworks of some chosen finite set of events $\mathfrak{X}$. As the eventology reduces everything to events the behavior of some agent within the frameworks of the chosen set of events $\mathfrak{X}$ is eventologically described by this set of events.

Let's continue an event-based analysis of the example, in which a behavior of the human agent is exhausted by his/her choice of $[a, b] \subseteq \mathbf{R}$ for x: "asserting $x \in [a, b]$ comes down to declaring (by the agent)[13] any value outside $[a, b]$ as *impossible for x*". As above, the set of numbers $[a, b] \subseteq \mathbf{R}$ is eventologically characterized by the set of Kolmogorov's embedded events $[a, b] = \Big\{\{r \leq r_y\} \subseteq \mathbf{R},\ y \in [a, b]\Big\}$.

Let's limit a choice of our agent by the set $2^{\mathfrak{X}}$, the set of all subsets of finite set of Kolmogorov's embedded events $\mathfrak{X}$. This finite set characterizes an individual behavior of the agent eventologically completely in a context of our example. In each problem always eventology assumes that the same initial *eventological space* $(\Omega, \mathcal{F}, \mathbf{P})$ is defined, where Ω is the set of all possible outcomes of Being (physical together with mental), i.e. the set of all possible states of World, and $\mathcal{F}$ is the algebra of events from Ω.

The set of events $\mathfrak{X}$, characterizing our agent, consists of the events chosen from

[10] Given definition is quite well defined within the limits of a quantitative level, but cannot be applied without changes outside a quantitative level, at a set-based level and, especially, at an event-based level.

[11] $\mathfrak{X}$ is a some fixed set of the embedded events, which are of interest.

[12] Notice that while our analysis is spent exclusively at an event-based level and about any probability, which is so carefully avoided from all of theories of uncertainty, speech does not go. At the same time, a probability measure, defined on this set of Kolmogorov's embedded events, determines the corresponding distribution function, which characterizes some random variable on $\mathbf{R}$.

[13] added by me.

algebra $\mathcal{F}$: $\mathfrak{X} \subseteq \mathcal{F}$; each event $y \in \mathfrak{X}$ has probability $\mathbf{P}(y)$; and all of the set of events $\mathfrak{X}$ is characterized by the probability distribution $\{p(Y),\ Y \subseteq \mathfrak{X}\}$, where $p(Y) = \mathbf{P}\left(\bigcap_{y\in Y} y \bigcap_{y\in Y^c} y^c\right)$ is the probability of so-called event-terrace $\mathrm{ter}(Y) = \bigcap_{y\in Y} y \bigcap_{y\in Y^c} y^c$, "numbered" by the subset of events $Y \subseteq \mathfrak{X}$.

In eventology any finite set of events $\mathfrak{X} \subseteq \mathcal{F}$ corresponds to the random finite set of events $K : (\Omega, \mathcal{F}, \mathbf{P}) \to \left(2^{\mathfrak{X}}, 2^{\left(2^{\mathfrak{X}}\right)}\right)$ that is characterized by the same eventological distribution $\{p(Y),\ Y \subseteq \mathfrak{X}\}$ as the set of events $\mathfrak{X}$. Event-terraces $\mathrm{ter}(Y)$ and their probabilities $p(Y)$ are treated by one more equivalent way: $\mathrm{ter}(Y) = \{\omega : K(\omega) = Y\}$, $p(Y) = \mathbf{P}(K = Y),\ Y \subseteq \mathfrak{X}$.

Thus one can characterize any human agent by two equivalent eventological models: by the set of events $\mathfrak{X}$ and by the random set of events K, which have identical probability distributions written down in two following ways: $p(Y) = \mathbf{P}(\mathrm{ter}(Y)) = \mathbf{P}(K = Y),\ Y \subseteq \mathfrak{X}$. In eventology similar probability distributions, which have double event interpretation, are called *eventological distributions* usually. Therefore within a context of each concrete example or problem it is possible to use two equivalent expressions: "the agent $\mathfrak{X}$" or/and "the agent K" without any loss of generality of the event characteristization of the agent.

Let's return to the eventological analysis of definition of the "possible event" and "certain event" from the example. To everyone it is clear that only the agent is a source of his/her partial ignorance about value x. An uncertainty is incorporated in his/her choice of a set of events $[a, b] \subseteq \mathfrak{X}$. From the eventological point of view an agent is characterized by his/her individual random set of events K. This means that a choice by the agent K of this or that set of events $Y = [a, b]$ for an estimation of an ill-known quantity x is characterized by his/her individual probability distribution $p(Y) = \mathbf{P}(K = Y),\ Y \subseteq \mathfrak{X}$.

Let's give an eventological well defined variant of the following definition of "possible event" and "certain event" offered in [1]:

- Any set of events A understand as the assertion $x \in A$ is "possible" for the agent K whenever $K \cap A \neq \emptyset$.
- Any such set of events A is "certain" for the agent K, whenever $K \subseteq A$.

The individual probability distribution of the agent eventologically completely characterizes his/her behavior within the frameworks of the chosen set of events $\mathfrak{X}$ and defines the probability of any event connected with the agent K and with any subsets of events $A \subseteq \mathfrak{X}$. This concerns also the events appearing in the eventological variant of definition of "possible event" and "certain event". Probabilities of these events are equal to $\mathbf{P}(K \cap A \neq \emptyset) = \mathbf{P}\left(\bigcup_{y\in A} y\right) = u_A$, $\mathbf{P}(K \subseteq A) = \mathbf{P}\left(\bigcap_{y\in A^c} y^c\right) = p^A$ and are connected by the dual relation: $u_A = 1 - p^{A^c}$.

In eventology these two set-functions u_A and p^A have a strict probability sense. At the same time these set-functions possess the same set-theoretic properties as set-functions, used recently in the theory of possibilities (*possibility function* $\Pi(A) = u_A$ and *necessity function* $N(A) = p^A$)[14] and in the theory of evidences (*plausibility function* $\mathrm{Pl}(A) = u_A$

[14]Notice that for embedded events set-functional relations $u_A = \max_{y\in A} \mathbf{P}(y)$ and $p^A = 1 -$

and *belief function* $\mathrm{Bel}(A) = p^A$).

In eventology given two sets of probabilities $\{u_X,\ X \subseteq \mathfrak{X}\}$ and $\{p^X,\ X \subseteq \mathfrak{X}\}$ are considered as various forms of eventological distribution of the same set of events $\mathfrak{X}$. In total in eventology [2] the six basic forms of eventological distribution are used: $p(X), p_X, p^X, u(X), u_X, u^X,\ X \subseteq \mathfrak{X}$, all of which are connected in pairs by Möbius inversion formulas [2], for example,

$$p^X = \sum_{Y \subseteq X} p(Y), \quad p(X) = \sum_{Y \subseteq X} (-1)^{|X|-|Y|} p^Y,$$

$$1 - u_X = \sum_{X \subseteq Y} p(Y^c), \quad p(X) = \sum_{Y \subseteq X} (-1)^{|X|-|Y|} (1 - u_{Y^c}),$$

or by dual relations: $p(X) = 1 - u(X^c)$, $p_X = 1 - u^{X^c}$, $p^X = 1 - u_{X^c}$.

8 Conclusions

The eventology offers the event-based approach to description of variability and ignorance, entering agents together with their beliefs directly in frameworks of scientific research in the form of eventological distributions of their own events. This allows eventology to provide the uniform and strict account of various aspects of the uncertainty, catching distinction between variability, ignorance, and impreciseness in mathematical frameworks of Kolmogorov's probability theory.

We emphasize that contemporary uncertainty theories could pass without special efforts to eventologically correct use of the concepts of event and set of events and also to eventologically correct description of sets of the objects in various applications, having received in exchange a unique opportunity to improve its mathematical tools by methods of the mathematical eventology, based on Kolmogorov's probability theory, and also to develop philosophical interpretations of event, uncertainty, variability, ignorance, and impreciseness on the basis of achievements of the philosophical eventology.

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$\max_{y \in A^c} \mathbf{P}(y)$, which play a role of initial axioms of the possibility theory defining set-functions $\Pi(A)$ and $N(A)$, follows from probability properties of set-functions u_A and p^A automatically.

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