

# Imperfect credit markets: implications for monetary policy

Gertjan W Vlieghe

London School of Economics

20. March 2007

Online at http://mpra.ub.uni-muenchen.de/12957/ MPRA Paper No. 12957, posted 28. January 2009 03:25 UTC

## Imperfect credit markets: implications for monetary policy\*

Gertjan W. Vlieghe<sup>†</sup>

20 March 2007

#### Abstract

I develop a model for monetary policy analysis that features significant feedback from asset prices to macroeconomic quantities. The feedback is caused by credit market imperfections, which dynamically affect how efficiently labour and capital are being used in aggregate. I then analyse what implications this mechanism has for monetary policy. The paper offers three insights. First, the monetary transmission mechanism works not only via nominal rigidites but also via a reallocation of productive resources away from the most productive agents. Second, following an adverse productivity shock there is a dynamic trade-off between the immediate fall in output, which is an effcient response to the productivity fall, and the fall in output thereafter, which is caused by a reallocation of resources away from the most productive agents. The more the initial output fall is dampened with a temporary rise in inflation, the more the adverse future effects of the reallocation of resources are mitigated. Third, in a full welfare-based analysis of optimal monetary policy I show that it is optimal to have some inflation variability, even if the only shocks in the economy are productivity shocks. The optimal variability of inflation is small, but the costs of stabilising inflation too aggressively can be large.

<sup>&</sup>lt;sup>\*</sup>I thank Nobuhiro Kiyotaki for invaluable advice and encouragement, and David Aikman, Kosuke Aoki, and Kalin Nikolov for many useful and detailed discussions. For helpful comments and suggestions I also thank Wouter Den Haan, Martin Ellison, Matthias Paustian, Thomas Sargent, Andrei Sarychev, Lars Svensson, Tony Yates and seminar participants at the London School of Economics, the Bank of England, the Bank of Spain, the University of Sussex, CREI, the EEA and the Money, Macro and Finance Annual conferences.

<sup>&</sup>lt;sup>†</sup>Brevan Howard, London. email: gertjan.vlieghe@brevanhoward.com

## 1 Summary

This paper aims to address the following questions. If credit markets do not work perfectly, how does that affect the overall economy? Furthermore, if monetary policy can influence the level of economic activity in the short run, how should monetary policy be set optimally in the presence of credit market imperfections?

It is thought that credit markets may not operate perfectly because of limitations on how much information a lender has about the quality of the borrower, or limitations on how well contracts between lenders and borrowers can be enforced. One consequence of such credit market imperfections might be that borrowing can only take place (or take place more cheaply) against collateral, such as land, buildings, machines. If that is the case, then changes in the value of collateral will affect the ability of firms and households to borrow. This could have important consequences for aggregate economic activity.

I consider in particular a case where there are two types of firms, those with high productivity and those with low productivity. Ideally, those with low productivity would lend all their resources to high productivity firms, so that high productivity firms can carry out all production. But when there are collateral constraints, some production is also carried out by low productivity firms. The total level of output is therefore determined by how much of the economy's productive resources are held by the high productivity firms. High productivity firms still end up borrowing from low productivity firms, but not as much as would be desirable in the absence of borrowing constraints.

Following a shock that reduces current output and/or the price of capital (which is used as collateral), the net worth of high productivity firms falls by more than the net worth of low productivity firms, because high productivity firms are highly leveraged. This means that high productivity firms can only afford to buy a lower share of the total capital stock for production in the following period. Because capital shifts to those with lower productivity, this reduces expected future returns on capital, which depresses the value of capital today, and exacerbates the initial redistribution of net worth. Output falls further in the subsequent period, as the economy's resources are now used much less efficiently. It takes time for the high productivity firms to rebuild their share of the capital stock, and output is therefore depressed for many periods, even if the initial disturbance only lasted a single period.

How does this mechanism interact with monetary policy? First, the transmission mechanism of interest rates in this model works through sticky goods prices as well as a reallocation of resources to less efficient producers. So the output response to monetary shocks is larger than in a model without borrowing constraints. Second, when responding to productivity shocks, the monetary policy-maker faces a trade-off. It is efficient for output to fall immediately following an adverse productivity shock. So, considered in isolation, there is no reason for a monetary policy-maker to offset the inial output fall. But the presence of borrowing constraints means that there is a trade-off between current inflation and future output. The larger the immediate fall in output, the larger the reallocation of resources away from the most productive firms, which will lead to future output being inefficiently low. By allowing inflation to rise temporarily and thereby dampening the initial output fall, monetary policy can mitigate inefficiently large future output fluctuations in subsequent periods.

But monetary policy cannot accomodate inflation too far, as inflation expectations must remain anchored, and inflation variability itself is costly too. So this begs the question of how much inflation variability it is optimal to tolerate. I answer this question formally by assuming that the monetary policy-maker maximises the welfare of the private sector. There are two frictions in the economy: credit market frictions and sticky prices. The policy-maker has a single instrument available, the nominal interest rate, to off-set the inefficiencies generated by these frictions. I find that the cost of responding to inflation too aggressively can be large, by creating excessive variability in output. By allowing only a small amount of inflation variability, policy can achieve a large reduction in output variability.

## 2 Introduction

This paper aims to address the following questions. If credit market imperfections are an important feature of the economy, how might they affect the economy's response to shocks? Furthermore, if monetary policy can influence real outcomes in the short run, how should monetary policy be set optimally in the presence of credit market imperfections?

The model economy consists of ex-ante identical entrepreneurs who can produce intermediate goods using capital, which is in fixed supply (e.g. land), a variable input (e.g. inventories) and labour. Using the approach of Kiyotaki (1998), I assume that some entrepreneurs are more productive than others, but spells of high productivity do not last, and arrive randomly. While an entrepreneur is highly productive, he will want to invest as much as possible in his own technology. Entrepreneurs with low productivity, on the other hand, would rather invest in the technology of others, as this generates superior returns. Let us therefore call the entrepreneurs that currently have high productivity 'producers', and the entrepreneurs with low productivity 'investors'. In principle, investors could lend to producers so that producers end up applying their technology to the entire capital stock. This would be the first-best outcome. But it is assumed that there are credit market imperfections, so borrowing can only take place against collateral. The larger the net worth of the borrower, the more capital he can buy. Moreover, since capital serves as collateral as well as a factor of production, an increase in the value of capital will increase the net worth of a producer who already had some capital installed and will therefore allow him to invest more.

To allow monetary policy to influence aggregate real outcomes, there has to be some friction, or non-neutrality, preventing instantaneous adjustment of prices, wages, debt contracts or asset portfolios. My approach is to assume that product prices cannot fully adjust, but the results of the paper do not hinge crucially on this particular choice of monetary non-neutrality.

In this model economy the wealth distribution has important effects on aggregate output. Following a shock that reduces current output and/or the price of capital, the net worth of producers falls by more than the net worth of investors, because producers are highly leveraged. This means that producers can only afford to buy a lower share of the total capital stock for production in the following period. Because capital shifts to those with lower productivity, this reduces expected future returns, which depresses the value of capital today, and exacerbates the initial redistribution of wealth from producers to investors. Output falls further in the subsequent period, as the economy's resources are now used much less efficiently. It takes time for the producers to rebuild their share of the wealth distribution to its steady-state level, and output is therefore below its steady-state level for many periods, even if the initial disturbance only lasted a single period.

How does this mechanism interact with monetary policy? First, the transmission mechanism of monetary shocks works through nominal rigidities as well as a reallocation of resources to less efficient producers. So the output response to monetary shocks is larger than in a model without credit imperfections. Moreover, the fall in output will manifests itself as a fall in measured total factor productivity, even though firm-level productivity has remained unchanged. Second, when responding to productivity shocks, the monetary policy-maker faces a trade-off. It is efficient for output to fall immediately following an adverse productivity shock. So, considered in isolation, there is no reason for a monetary policy-maker to offset the inial output fall by allowing inflation to rise temporarily. But the presence of credit frictions means that there is a dynamic trade-off between dampening the immediate fall in output by allowing inflation to rise temporarily, and reducing future falls in output. The larger the immediate fall in output, the larger the reallocation of resources away from the most productive agents, which will lead to future output being inefficiently low. By allowing inflation to rise temporarily and using nominal rigidities to dampen the initial output fall, monetary policy can mitigate inefficiently large future output fluctuations.

But inflation variability is costly too, so this begs the question of how much inflation variability it is optimal to tolerate. I answer this question by assuming that the monetary policy-maker tries to maximise the welfare of the private sector agents, which is commonly referred to as a Ramsey problem. There are two frictions in the economy: credit market frictions and sticky prices. The policy-maker has a single instrument available, the nominal interest rate, to off-set the inefficiencies generated by these frictions. I find that, by allowing a small amount of inflation variability, policy can achieve a large reduction in output variability. In other words, the cost of responding to inflation too aggressively can be large.

The remainder of this paper is organised as follows. Section 3 reviews the literature that relates to the questions studied in this paper. Section 4 presents the model in detail. Section 5 outlines the competitive equilibrium. Section 6 presents quantitative results. Section 7 describes how monetary policy should optimally be set. Section 8 analyses how sensitive the results are to variations in parameter choices and section 9 concludes.

## **3** Related literature

There is a vast theoretical and empirical literature that investigates the qualitative and quantitative importance of credit frictions in the propagation of shocks. Gertler (1988) gives a useful overview of the literature up to that date, and Schiantarelli (1996) and Hubbard (1998) specifically review the empirical micro-evidence. A first clear statement of how the financial health of borrowers could influence the propagation of shocks was made by Fisher (1933), who emphasised that the fall in inflation following a downturn in the economy could exacerbate the downturn by increasing the real burden of debt faced by borrowers, which would trigger fire sales of assets and bankruptcies. Important contributions in making the theoretical case for credit frictions at the micro-level were Jaffee and Russell (1976) and Stiglitz and Weiss (1981), who showed that credit rationing could occur as an equilibrium if lenders have insufficient information about borrowers. At the macrolevel, Scheinkman and Weiss (1986) showed that in a model where each agent faces exogenous borrowing constraints and idiosyncratic productivity, aggregate output and asset price dynamics will depend on the entire distribution of wealth. Their model also features more volatile asset prices than the complete markets version of that model, but not necessarily an amplification of output effects. Bernanke and Gertler (1989) showed that credit market imperfection could lead increased persistence of the effects of shocks on aggregate output. Carlstrom and Fuerst (1997) and (1998) embed a costly-state verification mechanism into an otherwise standard real business cycle model, and analyse to what extent this modifies the properties of the real business cycle model. They find that the effect of shocks on output can be either amplified or dampened, depending on which sector of the economy the financial constraint applies to. They also find that in their particular set-up there is either amplification or increased persistence, but not both. Kocherlakota (2000) constructs a useful, highly simplified version of a credit constrained economy to show that the amount of amplification is related to the share in production of the collateralisable asset, and that the degree of amplification that can plausibly be achieved in his setting is small.

Kiyotaki and Moore (1997) also examine the effect of credit market frictions on business cycle dynamics, but instead of putting constraints on information, they put constraints on contracting, in the sense that borrowers cannot commit to repay. Following an adverse shock, there is a redistribution of capital from highly productive agents to less productive agents, and this results in an amplified and persistent drop in output following a small and temporary drop in productivity. Kiyotaki (1998) extends this mechanism by considering a situation where agents are not permanently stuck in a high or low productivity state, but their productivity state changes stochastically. This leads to added richness in the dynamics, as the persistence of the stochastic productivity switching process affects the dynamics of aggregate output. A set of papers that includes Kehoe and Levine (1993), Kocherlakota (1996), Alvarez and Jerman (2000) use a more general constraint on contracting, where multi-period and state-contingent financial contracts are possible as long as the borrower has the incentive to repay in every state of the world. However, earlier models generally feature exogenous income processes in order to make them tractable. Cooley, Marimon and Quadrini (2004) who embed this contracting structure in a model with production. They find that productivity shocks cause highly amplified output fluctuations when there are incentive constraints on financial contracting. Jermann and Quadrini (2002) propose a model with limitations on contracting where an endogenous firm size distribution interacts with borrowing constraints to produce aggregate fluctuations. Expectations about future productivity causes a rise in asset prices, which eases borrowing constraints. That concentrates capital in smaller, constrained firms, which are more productive due to diminishing returns to scale, leading to increased aggregate output.

There is some empirical literature that finds evidence for a Kiyotaki and Moore type mechanism of reallocation across different producers. Eisfeldt and Rampini (2005) find that the amount of capital reallocation across firms is procyclical, and that the dispersion of productivity across firms is countercyclical. These two facts are consistent with a model where capital needs to flow to the producers with the highest productivity, but these flows can more easily happen during cyclical upturns, when informational or contractual frictions are smaller. A second empirical paper that is directly relevant to this framework is Barlevy (2003), who shows that highly productive firms tend to borrow more, again consistent with a framework where credit needs to flow from low to high productivity firms, making highly productive firms highly indebted.

All the models discussed so far are real models. There is no role for money or monetary policy. Bernanke, Gertler and Gilchrist (1999) introduce the costly-state verification mechanism into a New Keynesian business cycle model, i.e. into a real business cycle framework with nominal rigidities added. They use this model to analyse macroeconomic dynamics resulting from a wide range of shocks including monetary policy shocks, and find that, compared to a version of the model that has no financial frictions, the investment response to shocks is amplified and more persistent, leading to an amplified and more persistent response of aggregate output. Bernanke and Gertler (2001) then use the Bernanke, Gertler and Gilchrist (1999) framework to ask whether monetary policy should respond to asset prices as well as to inflation and the output gap. In practice, this means analysing whether, in the class of ad hoc monetary policy rules, a rule that includes asset prices performs better than one that does not. To stack the cards in favour of finding a strong role for asset prices in monetary policy, the authors add to their model non-fundamental movements in asset prices, or bubbles, which, via the collateral effect, have real effects on investment and output. They find that there is very little benefit to be had, in terms of minimising an ad hoc loss function, from letting monetary policy-makers respond to asset prices. Iacoviello (2005) carries out a similar analysis, based on the credit frictions framework of Kiyotaki and Moore (1997), and also concludes that there is little benefit to be derived from monetary policy that responds directly to asset prices. Faia and Monacelli (2004) extend the analysis of Bernanke and Gertler (2001) by examining a wider class of monetary policy rules, and by evaluating an approximation of welfare, rather than an ad hoc loss function. They find the optimal coefficients on various arguments of a monetary policy rule, and then experiment with changing those coefficients, and analyse the resulting welfare loss. They too find that including asset prices in the monetary policy rule does not improve welfare much.

A shortcoming of these analyses is that the class of monetary policy rules that is considered is rather arbitrary, and even if changing or restricting coefficients on asset prices has only a small effect on welfare, there is potentially a large welfare loss from using the restricted monetary policy rule relative to a fully optimal monetary policy <sup>1</sup>. So it is not clear at all what one can conclude from the statement that asset prices do not have a big coefficient in optimised ad hoc policy rules, and that changes in such coefficients do not have a large effect on welfare or a measure of loss.

Gilchrist and Leahy (2002) use a different methodology to analyse the problem, and get closer to what is probably the more interesting question: if credit frictions are important, does that mean monetary policy makers should try to achieve a significantly different path for macroeconomic variables compared to an economy without credit frictions? The authors analyse the response of macroeconomic variables in a model with credit frictions and nominal rigidities (based on the mechanisms of Bernanke, Gertler and Gilchrist (1999)), and compare this with two alternatives. First, the response of a frictionless economy, which provides the benchmark of what the optimal response of all variables should be. Second, a New Keynesian economy with nominal rigidities but no credit frictions. They conclude that in the case of a gradual productivity increase (which is akin to a demand shock, as the bulk of the productivity increase occurs in the future), it is sufficient for

<sup>&</sup>lt;sup>1</sup>By fully optimal monetary policy I mean the Ramsey solution to a planning problem. See e.g. Ljunqvist and Sargent (2004), chapter 30 for a very general formulation

monetary policy simply to respond to inflation. A stronger response to inflation will bring the economy closer to a frictionless economy. But in the case of shocks to net worth, responding more strongly to inflation causes output to deviate further from its optimal path, so there is a short-run policy trade-off between inflation and output variability. A rule that responds to net worth as well as inflation can achieve lower output variability at the expense of higher inflation variability. They conclude, as many others have done, that there is little benefit from monetary policy responding to asset prices, but they speculate that it may well pay to respond to net worth or the spread between risky and risk-free interest rates, although they do not develop this idea any further. This paper takes the next logical step in the literature, which is to carry out a full quantitative analysis of what paths of macroeconomic variables monetary policy should try to achieve in order to maximise welfare, if there are both nominal rigidities and credit frictions in the economy.

## 4 The environment

The model features a basic credit frictions mechanism due to Kiyotaki (1998), which is extended to allow for endogenous labour supply, monopolistic competition and a role for monetary policy.

There is a continuum of entrepreneurs. They are identical in terms of preferences. Their production technology is also identical, up to a productivity factor, which randomly switches between high ( $\alpha$ ) and low ( $\gamma$ ). Denote those who currently have high productivity 'producers', and those who currently have low productivity 'investors'. The productivity factor follows an exogenous Markov process with transition probability matrix

$$P = \begin{bmatrix} 1 - \delta & \delta \\ n\delta & 1 - n\delta \end{bmatrix}$$
(1)

so the probability of switching from high productivity to low productivity is  $\delta$ , and the probability of switching from low productivity to high productivity is  $n\delta$ . This probability matrix implies that from any initial distribution, the distribution will converge to a stationary distribution with a ratio of productive to unproductive agents of n.

Producers maximise life-time utility given by

$$\max_{c_t} E_0 \sum_{t=0}^{\infty} \beta^t \ln c_t \tag{2}$$

s.t. budget constraint,

$$c_t + x_t + q_t(k_t - k_{t-1}) + z_t l_t = \frac{y_t}{\varphi_t} + \frac{b_t}{r_t} - b_{t-1}$$
(3)

production technology,

$$y_t = \alpha \left(\frac{k_{t-1}}{\sigma}\right)^{\sigma} \left(\frac{x_{t-1}}{\eta}\right)^{\eta} \left(\frac{l_t}{1-\eta-\sigma}\right)^{1-\eta-\sigma} \tag{4}$$

and borrowing constraint

$$b_t \leqslant E_t q_{t+1} k_t \tag{5}$$

The variable  $c_t$  denotes consumption,  $x_t$  denotes a non-durable input (eg inventories),  $k_t$  denotes durable capital,  $z_t$  denotes the wage paid,  $l_t$  denotes the quantity of labour employed,  $b_t$  denotes the amount of real borrowing taken out at time t and repayable at time t + 1,  $q_t$  is the price of capital, and  $r_t$  is the real interest rate payable on borrowing  $b_t$ .

It is assumed that producers do not consume their output directly, but sell it to a monopolistically competitive retailer, who then offers the diversified goods back to producers, investors and workers with a mark-up of  $\varphi_t$ . That means that one unit of output produced can be sold to retailers for  $1/\varphi_t$ . All variables are denominated in terms of a consumption index. Define a Dixit-Stiglitz (1977) aggregate of a continuum of differentiated goods of type  $z \in [0, 1]$  each with price p(z)

$$c_t = \left[\int_0^1 c_t(z)^{\frac{\theta-1}{\theta}} dz\right]^{\frac{\theta}{\theta-1}}$$
(6)

The corresponding price index, defined as the minimum cost of a unit of the consumption aggregate, is defined as

$$p_t = \left[\int_0^1 p_t(z)^{1-\theta}\right]^{\frac{1}{1-\theta}} \tag{7}$$

For simplicity, it is assumed that inventories are costlessly created from the consumption aggregate, so that their relative price in terms of consumption is 1.

Following Kiyotaki and Moore (1997), borrowing constraints are interpreted as follows: it is assumed that when an entrepreneur has installed some capital, he invests some specific skill into that capital to generate output. The total value of his project is therefore the next period resale value of the installed capital plus the value of the output that can be generated using his specific skill. But he cannot commit to investing his specific skill: once the capital is in place, he can always choose to walk away. Because of this inability to commit to full repayment, the investor will never lend more than the resale value of capital. It is assumed that, should the value of collateral fall short of what was expected at the time the loan was taken out, the entrepreneur still repays the borrowing in full, because by the time he finds out about the realisation of the aggregate shock, he has already produced, and no longer has the opportunity to walk away.<sup>2</sup> Also following Kiyotaki and Moore (1997), it is assumed that, after the initial uncertainty about aggregate productivity is resolved, agents assume that future aggregate productivity is constant. In other words, their decisions are assumed to be unaffected by aggregate uncertainty.

It is useful to define  $u_t \equiv q_t - E_t \frac{q_{t+1}}{r_t}$ , the user cost of a unit of capital.

If we assume the borrowing constraint is binding, which will be verified later, we can rewrite the budget constraint as

$$c_t + x_t + u_t k_t + z_t l_t = \frac{\alpha}{\varphi_t} \left(\frac{k_{t-1}}{\sigma}\right)^{\sigma} \left(\frac{x_{t-1}}{\eta}\right)^{\eta} \left(\frac{l_t}{1 - \eta - \sigma}\right)^{1 - \eta - \sigma} + q_t k_{t-1} - b_{t-1}$$
(8)

To solve this, we break up the problem into two steps. First, given last period's capital and intermediate goods, what is the optimal demand for labour?

$$\pi_t = \max_{l_t} \left\{ \frac{\alpha}{\varphi_t} \left( \frac{k_{t-1}}{\sigma} \right)^{\sigma} \left( \frac{x_{t-1}}{\eta} \right)^{\eta} \left( \frac{l_t}{1 - \eta - \sigma} \right)^{1 - \eta - \sigma} - z_t l_t \right\}$$
(9)

It can be shown that the maximised profit after paying for labour input is

$$\pi_t = (\eta + \sigma) \frac{y_t}{\varphi_t} \tag{10}$$

For the second step of the producer's problem, we analyse what combination of capital and inventories he should buy to minimise expenditure, given a desired level of profits.<sup>3</sup>

$$m_t = \min_{k_t, x_t} \{ u_t k_t + x_t \}$$
(11)

$$s.t.E_t \pi_{t+1} \geqslant \overline{\pi} \tag{12}$$

Let  $\lambda_t$  denote the Lagrangean multiplier on the profit constraint. Substituting the optimal level of labour demanded into the production function, the first-order conditions become

 $<sup>^{2}</sup>$ He could still have an incentive to walk away if the debt burden exceeds not only the value of his collateral, but exceeds the value of his collateral plus current output. It is assumed that shocks are never that large.

 $<sup>^{3}</sup>$ The actual level of profits is irrelevant to the optimisation problem given the constant returns to scale technology.

$$u_t = E_t \left\{ \lambda_t \left( \frac{\alpha}{\varphi_{t+1}} \right)^{\frac{1}{\eta+\sigma}} z_{t+1}^{-\frac{1-\eta-\sigma}{\eta+\sigma}} \left( \frac{\sigma}{\eta} \right)^{\frac{\eta}{\eta+\sigma}} \left( \frac{x_t}{k_t} \right)^{\frac{\eta}{\eta+\sigma}} \right\}$$
(13)

$$1 = E_t \left\{ \lambda_t \left( \frac{\alpha}{\varphi_{t+1}} \right)^{\frac{1}{\eta+\sigma}} z_{t+1}^{-\frac{1-\eta-\sigma}{\eta+\sigma}} \left( \frac{\sigma}{\eta} \right)^{-\frac{\sigma}{\eta+\sigma}} \left( \frac{x_t}{k_t} \right)^{-\frac{\sigma}{\eta+\sigma}} \right\}$$
(14)

This can be simplified to

$$u_t = \frac{\sigma x_t}{\eta k_t} \tag{15}$$

Note that  $\lambda_t$  is the resource cost of another unit of profit, or, in other words,  $1/\lambda_t$  is the return on investment. For convenience we define this as a new variable:

$$r_t^p \equiv E_t \left\{ \left(\frac{\alpha}{\varphi_{t+1}}\right)^{\frac{1}{\eta+\sigma}} z_{t+1}^{-\frac{1-\eta-\sigma}{\eta+\sigma}} u_t^{-\frac{\sigma}{\eta+\sigma}} \right\}$$
(16)

In a similar way, we can also calculate the expost return from having used resources  $x_{t-1}$ ,  $k_{t-1}$  and  $l_t$  in the optimal combination given  $u_{t-1}$ ,  $z_t$  and  $\varphi_t$ . This return is equal to:

$$r_{t-1}^{p} \equiv \left\{ \left(\frac{j}{\varphi_{t}}\right)^{\frac{1}{\eta+\sigma}} z_{t}^{-\frac{1-\eta-\sigma}{\eta+\sigma}} u_{t-1}^{-\frac{\sigma}{\eta+\sigma}} \right\}$$
(17)

In this equation,  $j = \alpha, \gamma$  depending on whether the entrepreneur had high or low productivity in the previous period.

Substituting the optimal labour demand and factor demand conditions into the production function, we can now write the budget constraint as

$$c_t + m_t = r_{t-1}^j m_{t-1} + q_t k_{t-1} - b_t$$
(18)

This can be interpreted as a savings problem with uncertain returns (eg Sargent (1987)). The optimal decision rules for consumption and investment are linear in wealth:

$$c_t = (1 - \beta)(r_{t-1}^j m_{t-1} + q_t k_{t-1} - b_t)$$
(19)

$$m_t = \beta(r_{t-1}^j m_{t-1} + q_t k_{t-1} - b_t)$$
(20)

#### 4.1 Investors

Let lower-case variables with a prime denote the choices of an individual investor. The labour demand conditions facing the agents with low productivity, i.e. the investors, are the same as those for the producers, so the maximised profits after paying the wage bill are

$$\pi'_t = (\eta + \sigma) \frac{y'_t}{\varphi_t} \tag{21}$$

The second step of the problem, minimising the expenditure on  $x'_t$ and  $k'_t$ , is solved by maximising

$$\min_{x'_t,k'_t} \left( q_t - E_t \frac{q_{t+1}}{r_t} \right) k'_t + x'_t \tag{22}$$

$$s.t.\pi_{t+1}' \geqslant \overline{\pi} \tag{23}$$

Using our earlier definition of  $u_t$ , this problem is again parallel to that faced by producers, except that the rate of return for investors is

$$r_t^i \equiv E_t \left\{ \left(\frac{\gamma}{\varphi_{t+1}}\right)^{\frac{1}{\eta+\sigma}} z_{t+1}^{-\frac{1-\eta-\sigma}{\eta+\sigma}} u_t^{-\frac{\sigma}{\eta+\sigma}} \right\}$$
(24)

Just as for producers, the decision rule for consumption and investment of investors is therefore also linear in wealth with the same coefficients.

#### 4.2 Retailers

Retailers buy output and use a costless technology to turn output goods into differentiated consumption or input goods, which they sell onwards. The separation of producers and retailers is a modelling choice similar to Bernanke, Gertler and Gilchrist (1999) and is chosen to introduce monopolistic competition while maintaining tractable aggregation of producers. If producers operate directly in monopolistically competitive markets, they no longer face constant returns to scale at the firm level, and their optimisation problem will no longer yield the linear decision rules that are needed for tractable aggregation. Per period real profits for the retailers are given by

$$\Pi_t(p_t(z)) = \frac{(p_t(z) - p_t^p)}{p_t} y_t^R(z)$$
(25)

where  $p_t^p$  is the nominal price of output goods, so that  $\frac{p_t^p}{p_t} = \frac{1}{\varphi_t}$ . In other words,  $\varphi_t$  is the retail sector's average mark-up. Retailer output is denoted  $y_t^R(z)$ .

Demand for each retailer's output is given by

$$y_t^R(z) = \left(\frac{p_t(z)}{p_t}\right)^{-\theta} Y_t^R \tag{26}$$

where  $Y_t^R$  is aggregate demand for retail goods, which is given by

$$Y_t^R = \left[\int_0^1 y_t^R(z)^{\frac{\theta-1}{\theta}} dz\right]^{\frac{\theta}{\theta-1}}$$
(27)

In the baseline model, it is assumed that some fraction  $\kappa$  of retailers must set their price,  $p_{2,t}(z)$ , one period in advance, while the remainder can change their price,  $p_{1,t}(z)$  each period. Each type of retailer maximises profits, leading to the following first order conditions:

$$\frac{p_{1,t}(z)}{p_t} = \frac{\theta}{\theta - 1} \frac{1}{\varphi_t}$$
(28)

$$E_{t-1}\left\{\Lambda_{t-1,t}\frac{Y_t^R}{p_t^{-\theta}}\left[\frac{p_{2,t}(z)}{p_t} - \frac{\theta}{\theta - 1}\frac{1}{\varphi_t}\right]\right\} = 0$$
(29)

The term  $\Lambda_{t-1,t}$  is a discount factor applied at time t-1 to profits earned at time t. It is assumed that retailers are owned by workers, so it is the workers' discount factor that is relevant here. The aggregate price level evolves according to:

$$p_t = \left[ (1 - \kappa) p_{1,t}^{1-\theta} + \kappa p_{2,t}^{1-\theta} \right]^{\frac{1}{1-\theta}}$$
(30)

I will end up working with a log-linearised model, and it is convenient to note already that the first-order conditions for retailer profit maximisation, combined with the evolution of the aggregate price level, once linearised, will give the following pricing equation:

$$\widehat{\pi}_t = E_{t-1}\widehat{\pi}_t - \frac{1-\kappa}{\kappa}\widehat{\varphi}_t \tag{31}$$

where  $\hat{x}_t$  denotes log deviations from the steady-state.

In an extension of the model, I consider an environment where retailers face a quadratic cost of changing their price, following Rotemberg (1982). This is implemented by adding a cost term to the per period profit function so that it becomes:

$$\Pi_t(p_t(z)) = \frac{(p_t(z) - p_t^p)}{p_t} y_t^R(z) - \frac{\psi}{2} \left(\frac{p_t(z)}{p_{t-1}(z)} - 1\right)^2$$
(32)

Adjustment cost in prices is convenient to work with in welfare analysis because we can consider equilibria where all agents set the same price. This stands in contrast to the Calvo (1983) staggered price formulation, in which different producers charge different prices, which significantly complicates aggregation. Schmitt-Grohe and Uribe (2004a) use the quadratic cost formulation for welfare analysis.

This leads to the following aggregate pricing equation, or Phillips curve:

$$\pi_{t} (\pi_{t} - 1) = \beta E_{t} \left[ \frac{u_{w,t+1}}{u_{w,t}} \pi_{t+1} (\pi_{t+1} - 1) \right] + \frac{\theta - 1}{\psi} (y_{t} + y_{t}') \left( \frac{\theta}{\theta - 1} \frac{1}{\varphi_{t}} - 1 \right)$$
(33)

Retailers are owned by workers, so it is their marginal utility  $u_{w,t}$  that determines how future profits are to be discounted. I consider only symmetric equilibria where all retailers set the same price.

#### 4.3 Workers

4

There is a set of agents in the economy who have no access to productive technology, but who can work for the producers and investors. They derive utility from consumption and leisure, and their objective is to maximise

$$\max_{c_t, l_t} \sum_{t=0}^{\infty} \beta^t \ln \left( c_t - \frac{\chi}{1+\tau} l_t^{1+\tau} \right)$$
(34)

$$s.t.c_t^w + \frac{b_t^w}{r_t} = z_t l_t + b_{t-1}^w + \Pi_t$$
(35)

where  $l_t$  is the fraction of time spent on work, and  $\Pi_t$  are the profits from the retail sector, which is owned by the workers.<sup>4</sup>

Setting the workers' marginal utility of leisure equal to their marginal utility of consumption, the labour supply decision is

$$z_t = \chi l_t^{\tau} \tag{36}$$

Paying profits to the workers makes the model very tractable, but strictly speaking the workers would not want to own the retailers in equilibrium, because they do not want to save, as will be shown later. They are prevented from selling the retailers by assumption. An alternative would be to consider retailers as consuming agents in their own right, i.e. give the retailers a utility function, so that they themselves could consume the profits from their technology of diversifying goods. Just like the workers, retailers would not want to save in equilibrium due to the low interest rate, and they would not be able to borrow against future profits because there is no collateral. So they would simply consume the profits each period. The model results would therefore be identical, but come at the expenses of more complexity.

It is to be verified later that the interest rate on bonds is below the rate of time preference  $1/\beta$ . This implies that, near the stationary state, the workers will choose not to hold any bonds, and simply consume their wage and profit income. Their consumption therefore becomes:

$$c_t^w = z_t l_t + \Pi_t \tag{37}$$

#### 4.4 Monetary authorities

Prices in the economy are set in money terms. I assume such a 'cashless limit' (Woodford (2003)) economy here, so that money balances, and therefore the central bank's balance sheet, approach zero. Given this assumption, it is a reasonable approximation to omit money from the agents' utility function and budget constraint. A similar approach is used, for example, by Aoki (2001) who also omits money balances from a model that allows the central bank to set nominal interest rates. The central bank simply announces the one-period nominal interest rate  $R_t$ , which means that it stands ready to deposit or lend any amount<sup>5</sup> the private sector desires at this rate, subject to a (infinitely small) spread. The spread ensures that the private sector will attempt to clear the loan market first without resorting to the central bank. No private agent would be willing to borrow at a rate higher than that offered by the central bank, and no private agents would deposit funds that receive a lower return than that offered by the central bank. This arbitrage mechanism is similar to the way actual monetary policy operates in countries such as New Zealand, Canada, the United Kingdom and Scandinavian countries, although in practice the spreads are of course not infinitely small. This environment gives rise to an arbitrage condition between real and nominal rates of return, evaluated using the marginal utility of the investors.

$$E_t \left\{ \beta R_t \frac{P_t}{P_{t+1}} \frac{1}{c'_{t+1}} \right\} = E_t \left\{ \beta r_t^i \frac{1}{c'_{t+1}} \right\}$$
(38)

The central bank is assumed to follow a simple rule for setting monetary policy, <sup>6</sup> by responding to current inflation. There are also random deviations from the rule, which we will interpret as monetary policy shocks.

<sup>&</sup>lt;sup>5</sup>The central bank does not have better enforcement mechanisms for the collection of loan repayments than does the private sector. It will therefore not lend any funds to a producer who is already at the binding borrowing constraint.

<sup>&</sup>lt;sup>6</sup>Sargent and Wallace (1975) showed that if the interest rate follows an exogenous path, the price level is indeterminate. However, McCallum (1981) showed that the price level can be determinate under an interest rate rule if interest rates respond to a nominal variable, such as the price level in his paper, or inflation in my case.

$$\frac{R_t}{r^i} = \pi_t^\lambda \exp(\varepsilon_t^R) \tag{39}$$

## 5 Competitive Equilibrium

We now look for a competitive rational expectations equilibrium for this model economy. This will consist of aggregate decision rules for consumption, investment, labour supply and asset holdings, and aggregate laws of motion so that market clearing and individual optimality conditions hold. As will be shown, the distribution of wealth can be summarised by the share of wealth owned by producers.<sup>7</sup> Let capital letters denote aggregate variables. The market clearing conditions are that

$$B_t + B'_t + B^W_t = 0 (40)$$

$$K_t + K'_t = \overline{K} \tag{41}$$

and that labour supply equals labour demand. The market clearing condition goods is then  $^8\colon$ 

$$Y_t^R = Y_t + Y_t' \tag{42}$$

$$C_t + C'_t + X_t + X'_t + C^w_t = \frac{Y_t + Y'_t}{\varphi_t} + \Pi_t$$
(43)

Aggregate retailers' profits will be equal to:

$$\Pi_{t} = \left(1 - \frac{1}{\varphi_{t}}\right) (Y_{t} + Y_{t}') - \frac{\psi}{2} (\pi_{t} - 1)^{2}$$
(44)

where the quadratic term is omitted in the case of staggered pricing. Note that the individual decision rules for consumption and investment

<sup>&</sup>lt;sup>7</sup>In model simulations I will consider a stochastic process for aggregate productivity. Because each entrepreneur's problem collapses to a linear savings problem with log consumption, the fact that future returns are uncertain does not affect the consumption and savings decision. Where uncertainty might affect decision rules is that borrowers may not want to borrow up to the borrowing limit if uncertainty about future asset prices is large. In other words, they might not leverage to the maximum permitted, to reduce the risk of leveraged loss under an adverse aggregate shock. Similar to Iacoviello (2005) and Kiyotaki (1998), I only consider an approximation of the model where the borrowing constraint binds at all times.

<sup>&</sup>lt;sup>8</sup>This clearing conditions holds only in a neighbourhood of the steady state for the staggered pricing model, due to the different aggregators for consumers and retailers. But for the quadratic adjustment cost model, all retailers choose the same output level so the aggregation is exact

are all linear, so that we can simply sum them to obtain aggregate decision rules and laws of motion. Each agent consumes a fraction  $1 - \beta$  of their wealth and reinvests a fraction  $\beta$  of their wealth.

The following is asserted, to be verified later: we consider equilibria near a steady state where the investors hold some capital for their own production. This has two implications. First, investors must then be indifferent between holding capital for production and bonds, so that they equalise the expected return to each

$$r_t^i = r_t \tag{45}$$

Second, because we have shown that

$$r_t^p = \left(\frac{\alpha}{\gamma}\right)^{\frac{1}{\eta+\sigma}} r_t^i > r_t^i \tag{46}$$

it follows that the borrowing constraint is indeed binding near the steady state, since producers achieve a larger return on their own productive investment than the interest rate they have to pay on the bonds they issue.

Next, it is useful to define aggregate wealth as the quantity of output available for consumption or reinvestment, i.e. after paying the wage bill.

$$W_t = (\eta + \sigma) \frac{Y_t + Y'_t}{\varphi_t} + q_t \overline{K}$$
(47)

We also define the share of wealth held by producers as  $s_t$ . We can now write a law of motion for aggregate wealth as

$$W_{t+1} = \left[ r_t^p s_t + r_t^i (1 - s_t) \right] \beta W_t \tag{48}$$

Using the Markov-process for the way agents switch between having high and low productivity, the law of motion for the share of wealth can be written as

$$s_{t+1} = \frac{(1-\delta)\widetilde{\alpha}s_t + n\delta\widetilde{\gamma}(1-s_t)}{\widetilde{\alpha}s_t + \widetilde{\gamma}(1-s_t)}$$
(49)

where  $\tilde{\alpha} = \alpha^{\frac{1}{\eta + \sigma}}$  and similarly for  $\tilde{\gamma}$ .

I now want to consider an aggregate disturbance to productivity. I achieve this by multiplying  $\alpha$  and  $\gamma$  by a productivity disturbance  $\varepsilon_P$ . The assumed stochastic process for the productivity disturbance is that its log follows an autoregressive process with a normally distributed shock:

$$\widehat{\varepsilon}_{P,t+1} = \rho \widehat{\varepsilon}_{P,t} + \nu_{t+1} \tag{50}$$

The full list of model equations is listed in the appendix for ease of reference.

## 6 Model solution

#### 6.1 Dynamics

The system of difference equations that constitute the full model is log-linearised around the steady state, and solved using the Schur decomposition as described in Soderlind (1999), to write the non-predetermined variables as a linear function of the predetermined variables and the shocks. The steady state is the level that aggregate variables tend to when there are no aggregate shocks. Associated with these levels for aggregate variables is a stationary wealth distribution summarised by the share of wealth owned by producers,  $s_t = \overline{s}$ .

We consider only non-explosive, determinate solutions. For a solution to be determinate (following Blanchard and Kahn (1980)), it is necessary for the number of eigenvalues outside the unit circle to correspond to the number of non-predetermind variables. In the calibration that I use this is indeed the case, for a monetary policy rule that responds to inflation with a coefficient greater than 1.

#### 6.2 Steady state

It is instructive to consider the expression for the steady-state interest rate:

$$r = \frac{1}{\beta} \left( \frac{\widetilde{\gamma}}{\widetilde{\alpha}s + \widetilde{\gamma}(1-s)} \right) < \frac{1}{\beta}$$
 (51)

Since s < 1, the real interest rate is strictly lower than the (inverse of) the rate of time preference. At these low interest rates, workers will not wish to save, so workers choose not to participate in asset markets. This proves the earlier assertion that workers simply consume their wage and profit income in each period.

#### 6.3 Frictionless model

Before turning to the properties of the full model, I show what the properties of the model would be without binding borrowing constraints. In that case, the efficient allocation would always be reached, in the sense that the most productive agents would always hold the entire capital stock. It can be shown that the law of motion for aggregate output is:

$$Y_{t+1} = \varepsilon_{P,t+1}^{\frac{\tau+1}{\tau+\eta+\sigma}} \left(Y_t\right)^{\frac{\eta(\tau+1)}{\tau+\eta+\sigma}} c \tag{52}$$

where c denotes a constant term that is a function of the model parameters. This implies that output dynamics are entirely driven by the exogenous process for aggregate productivity and lagged output.

parameter	assigned value
$\beta$	0.99
$\eta$	0.1
$\sigma$	0.3
au	0.5
$\chi$	0.29
$\gamma$	0.12
$\kappa$	0.5
heta	11
$\lambda$	1.5
$\alpha/\gamma$	1.034
n	0.0073
δ	0.5

Table 1: Calibrated parameter values for the baseline model

There is no feedback from any net worth or asset price variable in the model. The equations for the asset price and wealth are

$$q_t = \frac{\sigma\beta}{\varphi \overline{K}(1-\beta)} Y_t \tag{53}$$

and

$$W_t = \frac{\eta + \sigma - \eta\beta}{\varphi \left(1 - \beta\right)} Y_t \tag{54}$$

So asset prices and entrepreneurial wealth are simply proportional to output.

#### 6.4 Calibration

The model contains 13 parameters. Some of the parameters are standard, in the sense that they can be chosen to match key steady-state ratios in the economy. Other parameters, in particular those specific to the credit mechanism, are more difficult to assign values to. The calibration I have chosen is designed to show how the mechanism might work, not how it most likely does work, as there is little guidance from actual observation in choosing plausible values for these parameters. Table 1 shows the parameter values chosen for the baseline model.

The model is calibrated so that each period can be interpreted as one quarter of a year. The discount factor  $\beta = 0.99$  is a standard choice in many general equilibrium macromodels (see e.g. Cooley and Prescott (1995)). While in this model such a discount factor will lead to

a lower real interest rate compared with models where there is perfect enforcement or commitment, the difference is small under the baseline parameterisation: the steady-state annual real interest rate is just under 4%. The values for  $\eta, \sigma, \tau, \chi, \gamma$  were chosen to achieve a capital to output ratio of 10, a labour share in output of 0.6, hours worked of 0.31 as a fraction of total available time, and a wage elasticity of labour supply of 2, values very close again to those in Cooley and Prescott (1995) and subsequent literature. The monetary policy reaction function parameter  $\lambda$  is set at the value used by Taylor (1993), although the reaction function does not have exactly the same form. The rule used in this paper is certainly too simplistic to be realistic, and is used to illustrate the basic mechanisms of the model. The elasticity parameter  $\theta$  determines a steady-state net mark-up for consumption goods of 0.10, corresponding to the empirical findings by Basu and Fernald (1997). The share of prices that are set one period in advance,  $\kappa$ , is set at 0.5. In the extended model, with a cost of price adjustment, there is a cost parameter  $\psi$  which is set as follows. Because the linearised Rotemberg pricing equation is identical to a linearised pricing equation with Calvo (1983) probabilities of price changes, the cost of price adjustment can be calibrated to be quantitatively equivalent to a particular Calvo adjustment frequency.<sup>9</sup> In this model the equivalent of a Calvo probability of keeping prices fixed of 2/3 is to set the cost parameter of price changes  $\psi = 5.4$ . This calibration implies that firms change their price on average every 3 quarters, in line with the estimates in Sbordone (2002). The extended model also features a more realistic monetary policy rule, which helps generate plausible inflation dynamics. The form of the rule in the extended model is

$$\widehat{R}_{t} = (1 - \rho_{R}) \lambda_{\pi} \widehat{\pi}_{t} + (1 - \rho_{R}) \lambda_{\varphi} \widehat{\varphi}_{t} + \rho_{R} \widehat{R}_{t-1} + \varepsilon_{R,t}$$
(55)

In other words, monetary policy now responds gradually to inflation, and also responds to the mark-up, which is a proxy for the deviation of output from the level of output that would prevail under flexible prices (when the mark-up is constant). The calibrated values for  $\{\lambda_{\pi}, \lambda_{\varphi}, \rho_R\}$ are  $\{1.5, -2, 0.9\}$ .

The crucial parameters for the strength of the credit mechanism are the productivity difference between producers and investors  $\alpha/\gamma$ , the steady-state ratio of productive to unproductive agents n, and the probability of a highly productive agent becoming less productive,  $\delta$ . The parameters n and  $\alpha/\gamma$  were chosen so that productive agents hold about 2/3 of the capital stock in steady state, the same value as that in Kiyotaki and Moore (1997). But other combinations of these

<sup>&</sup>lt;sup>9</sup>Although the calibration can be set so that the linearised pricing equations are identical, the welfare effects, and therefore the optimal monetary policy, are not necessarily identical because they are based on the non-linearised versions of the pricing equation.

parameters could achieve the same ratio, and generate either more or less persistence. The parameter  $\delta$  was chosen to be low enough so that the credit mechanism generates substantial persistence, while still producing model responses that appear well behaved.

#### 6.5 Response to aggregate productivity shock

In this section I consider the response of the model economy to aggregate productivity shocks. I compare these responses with the responses of a 'flexible price' version of the model (with  $\kappa = 0$ , so that all prices can be changed in each period), and also with the response of the fully efficient model, outlined in section (6.3). Figure 1 shows the response of output, the price of capital, and aggregate entrepreneurial wealth. The units on the vertical axes are percentage deviations from steady state. The units on the horizontal axes are quarters, with the shock taking place in quarter 1. The productivity shock is a 0.25 per cent fall in aggregate productivity, which lasts only for a single period. In other words, aggregate productivity follows a white noise process. Output in the efficient model falls by about 1.7 times the fall in productivity, which is the combined effect of lower productivity and lower labour inputs. After the shock, output returns fairly quickly to its steadystate value. We know from equation (52) that, if productivity follows a white noise process, then the persistence of output, as measured by the autocorrelation coefficient, is equal to  $\frac{\eta(\tau+1)}{\tau+\eta+\sigma}$ . Using the baseline calibration, this is equal to 0.17. Asset prices and aggregate wealth respond with the same proportional magnitudes as output. For the flexible price model with credit frictions, the initial output response is the same as the efficient response, because all determinants of output other than labour (i.e. last period's borrowing decision, the share of capital held by productive agents, and investment in inventories) are predetermined. But note that the asset price falls more than twice as much. This amplification is due to the following mechanism. In period 1. producers and investors experience an unanticipated loss of output, as well as an unanticipated reduction in the value of producers' collateral. This means that in period 1, producers cannot maintain their share of the capital stock: they can now afford less than the steady-state share, because they buy capital with the reinvested share of output and with collateralised borrowing. This means that capital will be less efficiently used for production from period 2 onwards. Because today's capital price is the present discounted value of all future marginal returns to capital, the price of capital falls by more than in the efficient model, and this fall further exacerbates the reduction in producers' net worth. Output in period 2, rather than rising back towards the steady-state, falls further due to the shift in capital from highly productive to less productive entrepreneurs. After period 2,

it takes time for the most productive agents to rebuild their share of wealth, and it therefore takes time for asset prices and output to return to their steady-state values. It is interesting to note that the high degree of amplification is achieved with a plausible parameter value for the capital share and a plausible parameter value for the intertemporal elasticity of substitution (log utility implies a value of 1). Cordoba and Ripoll (2004) find that, in a model where the agents' productivity level is fixed permanently, no substantial amplification can be achieved unless either of these two parameters take on values that are well outside the range usually thought to be plausible, such as capital shares in excess of 0.5, and elasticities of substitution below 0.1. However, their results are the consequence of a set of modelling choices that imply a very close link between capital shares, productivity differences and the rate of time preferences. It is therefore not possible to vary one without getting implausible results for another. This is not the case for our model, where productivity levels are distinct from the capital share in production, allowing greater productivity gaps between investors and entrepreneurs while still preserving plausible steady state distributions of output and capital.

In the full model, with sticky prices as well, the initial fall in aggregate output is slightly muted relative to the efficient and flexible price models. As output falls, the nominal price level needs to rise for any given monetary policy stance that does not fully accommodate the output fall. But prices are sticky, so they do not rise enough. This causes the real marginal cost of the retail sector to rise, as not all retailers are able to charge their desired mark-up. For the entrepreneurs, however, paying a lower mark-up is beneficial: it increases the value of their output in consumption terms, which in turn increases the amount of labour they want to hire, relative to the amount of labour they would want to hire with constant mark-ups. This mechanism, while appearing perhaps non-standard when described this way, is simply the New Keynesian channel whereby those who cannot change prices change output to meet demand. Ouput is therefore higher than it would have been under flexible prices. So aggregate output falls by less in the period of the shock. This has important consequences for output dynamics in future periods. Because output falls by less, there is a smaller redistribution of wealth from producers to investors. There is therefore a smaller response of asset prices and aggregate wealth, because less of the capital stock shifts from producers to investors during the transmission of the shock. The entire credit - asset price effect has been dampened by the stickiness of prices. The response of inflation, nominal interest rates and the mark-up in the sticky price model are also shown in figure 1.

The key difference, relative to standard sticky-price monetary models, is that under flexible prices the output fall from period 2 onwards

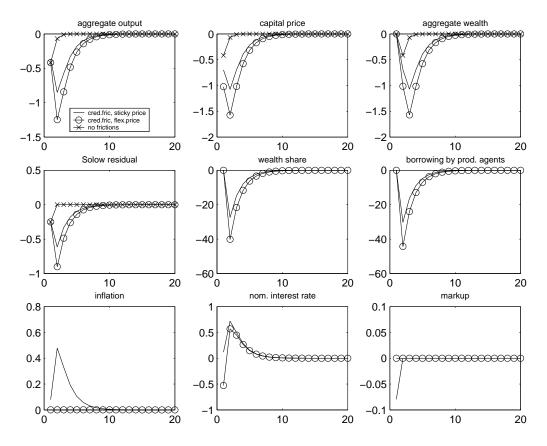


Figure 1: Response to productivity shock (baseline model)

is no longer fully efficient. This can be seen from the fact that the no-frictions level of output, which also corresponds to a social planner solution in the absence of all frictions, lies strictly above the flexibleprice level of output from period 2 onwards.<sup>10</sup>. In standard sticky-price monetary models, it is considered desirable for monetary policy to respond aggressively to inflation following a productivity shock, as this will simultaneously reduce inflation and ensure that output follows the same path as a model without price stickiness. In those models, as soon as productivity has returned to its steady-state level, so does the flexible price level of output. But in the credit frictions model considered in this paper, only the initial fall in output is an efficient response to a change in aggregate productivity. The subsequent further fall, and the slow return to steady state are the result of inefficiencies in the credit market.

How large the dampening effect of sticky prices will be depends on how aggressively monetary policy responds to inflation. As the adverse productivity shock puts upward pressure on inflation, the monetary policy reaction function dictates that the nominal interest rate should rise. The more aggressive the rise in interest rates, the smaller the resulting increase in inflation, and the smaller the reduction in markups. As monetary policy becomes sufficiently aggressive in its response to inflation, the economy's response to productivity shocks approaches that of the flexible price economy, where mark-ups are constant. As monetary policy becomes less aggressive, by responding less strongly to inflation, output fluctuations become smaller. However, in order to ensure determinacy of the equilibrium, monetary policy must react to inflation with a coefficient of at least 1, so aggressiveness cannot be toned down too far. Ensuring determinacy of the equilibrium is one interpretation of what central banks refer to as anchoring inflation expectations.

One further aspect of the model that is worth mentioning is that, even though the level of productivity of each firm is only perturbed for a single period, the measured aggregate level of productivity falls persistently. Panel 4 of figure 1 shows the response of the Solow residual,  $A_t$ . This is calculated as the total factor productivity in the economy under the assumption that there is no heterogeneity in productivity. When log-linearised, it is equal to

$$\hat{A}_{t} = \frac{y}{y+y'}\hat{y}_{t} + \frac{y'}{y+y'}\hat{y'}_{t} - \eta\hat{X}_{t-1} - (1-\sigma-\eta)\hat{l}_{t}$$
(56)

<sup>&</sup>lt;sup>10</sup>It is important to emphasise that to achieve the first best it is necessary for the path of all variables to match the social planner path, not just output. I am using output deviations here as an indication of whether we are moving further from or closer to an optimal path. A full welfare analysis is carried out in the next section.

The shift in capital from producers to investors causes measured aggregate productivity to fall further in the period following the shock, and given that the shift in capital is persistent, the fall in aggregate productivity is persistent too. Furthermore, the extent of the fall depends on how monetary policy reacts to the shock. If monetary policy keeps inflation strictly constant, aggregate productivity falls further, relative to the case where monetary policy allows inflation to rise temporarily. The model therefore gives an interesting perspective on the interaction between aggregate productivity, heterogeneity and monetary policy. This is discussed in more detail in the next section.

#### 6.6 Response to monetary policy shock

Figure 2 shows the model economy's response to a temporary white noise shock to the monetary policy rule, where the model now features price adjustment costs and the monetary policy rule  $(55)^{11}$ . The shock is calibrated to cause a 0.25 per cent rise in the annualised nominal interest rate. The discussion here is brief, because most of the mechanism is similar to that in the case of a productivity shock. Only the initial phase of the transmission of the disturbance differs. Nominal interest rates rise in response to the shock. Because retailers are unable to lower their prices sufficiently in response to the monetary contraction, their mark-ups rise. Entrepreneurs therefore face a fall in the consumption value of their output, which reduces net worth both via a direct effect of the mark-up and via the consequent reduction in labour inputs. The fall in output is only 10bp, but total wealth is around 70bp. Because of the leverage effect, producers suffer a larger fall in net worth than investors. Their share of total wealth falls by nearly 30 per cent, so the wealth distribution is shifted from those with high productivity to those with low productivity. This lowers return on capital in future periods, which causes a fall in the price of capital today, resulting in a reduction of net worth that is much larger than the reduction of the initial period's output alone. Output in the following period is lower still, because capital is now being used less efficiently. The return to the steady-state happens gradually, as producers rebuild their share of wealth, so that the wealth distribution returns to its stationary distribution. Note that in this case the efficient path of output, as well as the path of output under flexible prices, remains constant, because monetary policy would have no effect in this model absent sticky prices.

It is also interesting to note that aggregate productivity, as measured by the Solow residual, falls in response to a monetary contraction,

<sup>&</sup>lt;sup>11</sup>The response of this price adjustment cost version of the model to productivity shocks is omitted, but is quantitatively and qualitatively very close to the baseline model.

as capital shifts from high to low productivity agents, and is therefore less efficiently used even for a given level of inputs. This puts an interesting perspective on the Real Business Cycle and monetary policy literature. The RBC tradition is to claim that monetary policy does not explain much of the variation in output, because a large share of the fluctuation can be explained as an endogenous response to exogenous productivity or technology shocks (see e.g. Prescott (1986) and Plosser (1989). But if measured aggregate productivity is not exogenous, but instead is affected by monetary policy shocks, as well as by the systematic response of monetary policy to other shocks, this conclusion in unwarranted. More recently, several authors of the real business cycle tradition have questioned the interpretation of aggregate productivity as strictly determined by technology alone (see e.g. Prescott (1998) and Kehoe and Prescott (2002)). Chari, Kehoe and McGrattan (2004) have suggested that aggregate productivity, rather than being taken as given, is something that needs to be formally explained by a model.<sup>12</sup> They call it the "efficiency wedge". The model I present here is one possible formalisation of a process that makes the efficiency wedge endogenous, and sensitive to monetary policy. It also counters Chari, Kehoe and McGrattan's (2004) claim that credit frictions are unlikely to explain a significant share of business cycle fluctuations, by showing that credit frictions can be the cause of variations in the efficiency wedge.

## 7 The optimal policy problem

Having analysed the model properties under a simple ad hoc monetary policy rule and with monetary policy that stabilises inflation instantly and perfectly, I now turn to the question of what optimal monetary policy is.

#### 7.1 Objective of the policy-maker

The policy-maker maximises the weighted sum of the welfare of entrepreneurs and of workers. The one-period welfare function is therefore the sum of the utility of all the agents. There is no unique way to sum utilities, but one candidate is

$$U_t = \ln \left( c_t + c_t' \right) + \mu \ln \left( c_t^w - \frac{\chi l_t^{\tau+1}}{\tau+1} \right)$$
(57)

 $<sup>^{12}</sup>$ In the vintage capital version of RBC models, aggregate productivity is largely endogenous, as technology shocks only affect the newest vintage of capital, and the remaining dynamics of aggregate productivity are driven by the optimal switch to new capital.

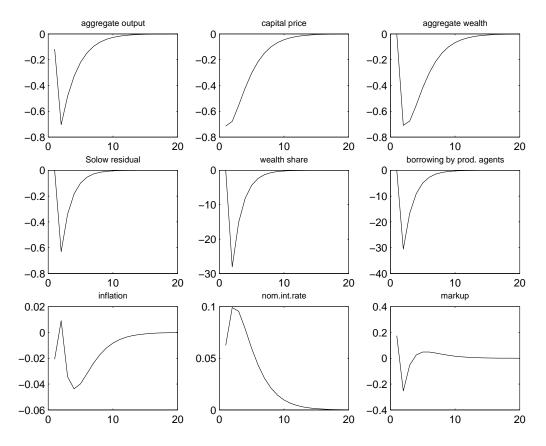


Figure 2: Response to monetary policy shock (price adjustment cost model)

This formulation uses total consumption across entrepreneurs, who are ex-ante identical. Workers are not identical to entrepreneurs: they face different constraints and have a different utility function, so they are treated separately, and added to the aggregate welfare function using  $\mu$ , the Pareto weight on workers.<sup>13</sup>

The policy-maker then solves the dynamic problem of maximising welfare, conditional on being in some given initial state, subject to the private sector model equations outlined in the appendix. This problem takes the form

$$\max \sum_{t=0}^{\infty} \beta^t \left\{ U_t - \lambda_t f(x_{1,t-1}, x_{1,t}, x_{2,t}, x_{2,t+1}) \right\}$$
(58)

where f(.) is a vector of the equations describing the behaviour of the private sector,  $x_1$  is a vector of the natural state variables of the private sector model,  $x_{2,t}$  is a vector of non-predetermined private sector variables and  $\lambda_t$  is a vector of Lagrange multipliers. The maximisation is subject to initial conditions  $x_{1,-1}$ , which are the initial conditions of the natural state variables. The natural state variables of the private sector model are the level of borrowing  $b_{t-1}$ , the lagged user cost  $u_{t-1}$ , and the level of capital held by productive agents  $k_{t-1}$ .<sup>14</sup> As discussed in, e.g. Ljungqvist and Sargent (2004), we must be careful how to treat the Lagrange multipliers on the various constraints. The multipliers on equations with a forward-looking element must be treated as additional state variables. This is because these Lagrange multipliers capture the policy-maker's earlier promises upon which private sector expectations were formed. It is this particular treatment of past promises that makes the policy a 'commitment' policy. It is assumed that the policy-maker acts as a Stackelberg leader, and does not re-optimise after the private sector has formed expectations. The remaining Lagrange multipliers are treated as non-predetermined, i.e. they can jump freely at period t. The predetermined Lagrange multipliers in this particular system are the multipliers on the borrowing constraint, the Phillips curve, the expected return on investment and the asset-pricing condition for capital, which are the equations of the private sector model that involve expectations of future variables.

<sup>&</sup>lt;sup>13</sup>This particular welfare function does not give any importance to the distribution of consumption across entrepreneurs, as only total entrepreneurial consumption matters. The distribution of consumption across entrepreneurs matters indirectly, of course, because a reallocation of resources away from highly productive producers lowers total output, and hence total consumption. Using a welfare function that takes into account the distribution of consumption among entrepreneurs explicitly would make credit-driven fluctuations more costly in welfare terms, and therefore likely lead to higher optimal inflation variability.

<sup>&</sup>lt;sup>14</sup>There is no unique way to choose state variables. One could also work with wealth and the share of wealth held by producers as states.

The system of first-order conditions is solved by log-linearising it around its steady-state, and then solving the resulting system of linear difference equations using the Schur decomposition as described in Soderlind (1999).

#### 7.2 Optimal response to productivity shock

To understand what optimal monetary policy is trying to achieve, it is useful to consider, in addition to the optimal policy solution, two other solutions for the model, also considered in the section 6.5. First, I consider the solution of the model when there are no credit frictions and prices are fully flexible. Recall from section 6.3 that his implies that output dynamics are driven entirely driven by the exogenous process for aggregate productivity and lagged output. There is no feedback from any net worth or asset price variable in the model. And asset prices and entrepreneurial wealth are simply proportional to output.

A second version of the model that is useful for comparison is the model with credit frictions but flexible prices. This can be interpreted either as an economy where there are no impediments or costs to changing prices, or as an economy where the monetary policy maker is concerned only with stabilising inflation, which can be achieved perfectly in this model.

Let us now consider the optimal monetary policy or Ramsey solution. This is the model economy with credit frictions and sticky prices, and with a monetary policy maker who maximises the welfare of the private sector agents as outlined in detail in section (7.1). As shown in figure (3), the initial output fall is smaller than in the frictionless model and the flex-price credit model, and inflation is allowed to rise initially. The formal solution to a full optimisation problem confirms the intuition gained in the previous section: it is optimal for the policymaker to dampen the initial output fall, because of the consequences it has in future periods. The policy-maker is therefore trading off the efficiency loss of dampening the initial output and asset price fall (and the temporary rise in inflation) against the efficiency gain from limiting the damaging effect of the credit propagation mechanism in subsequent periods.

In effect, the combination of both credit frictions and sticky prices has resulted in a traditional short-run trade-off between the deviation of output from its efficient level and inflation. A trade-off between the output gap and inflation in the short run is largely absent from the New Keynesian literature unless one considers shocks that hit the price level directly.<sup>15</sup> . This absence of a fundamental trade-off has been dubbed

<sup>&</sup>lt;sup>15</sup>In models such as those discussed by Clarida, Gali and Gertler (1999) and Woodford (2003), the level of output that prevails under flexible prices is the appropriate target for

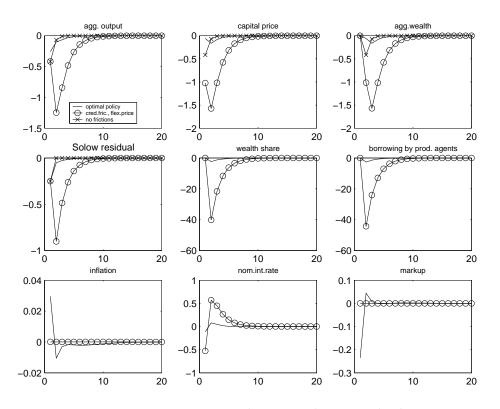


Figure 3: Response to adverse productivity shock

	Ramsey	Frictionless	Flex-price credit	
s.d.(y+y')	1.157	2.000	6.658	
s.d.(q)	0.798	2.000	8.974	
$s.d.(\pi)$	0.128	0	0	
a.r.(y+y')	0.431	0.167	0.779	
a.r.(q)	0.698	0.167	0.806	
$a.r.(\pi)$	-0.246	0	0	
$s.d.(\varepsilon_{P,t})$	1.000	1.000	1.000	

Table 2: Theoretical moments of selected variables

Notes: s.d. denotes standard deviation, expressed in per cent, and a.r denotes first-order autocorrelation.

Moments were calculated for log-linear deviations of aggregate output (y + y'), the price of capital (q), and inflation $(\pi)$ .

Each column represents a different version of the model.

the "divine coincidence" by Blanchard (2005), in reference to the fact that closing the welfare-relevant measure of the output gap coincides with stabilising inflation. Angeletos (2003) also discussed this problem with the New Keynesian models. In my approach, there is no longer any divine coincidence, because stabilising prices does not stabilise output around its efficient level, or even its constrained efficient level. And as shown in figure 3, the optimal policy involves allowing inflation to rise briefly following an adverse productivity shock. The nature of the propagation mechanism due to credit frictions implies that in this model the trade-off is not between current inflation and the *current* gap between output and its efficient level. Instead, there is a dynamic trade-off between current inflation and the *future* gap between output and its efficient level. This dynamic nature of the trade-off has important consequences for the concept of the output gap. It means that, even if we could measure it accurately, the distance between output and its efficient level at any point in time is not a useful summary of the objective of monetary policy, in the way that the New Keynesian gap between output and its flexible price level summarises the monetary policy objective.

Table 2 illustrates the trade-off and the desirability of smoothing

monetary policy, and this level can theoretically be achieved as long as there are no direct shocks to the price level. For the case of productivity shocks, there is therefore no trade-off between output fluctuations from their flex-price level and inflation deviations from target. This is not the case if other frictions are added. For example, Erceg, Henderson and Levin (2000) show that a trade-off also exists if both wages and prices are sticky.

output and asset price fluctuations. Under optimal or Ramsey policy, inflation variability<sup>16</sup> is non-zero. It is of the same order of magnitude as the variability of actual inflation in low-inflation industrialised countries such as the US<sup>17</sup>. Output variability under optimal policy is much smaller than in the flex-price credit model. The reduction in output variability also implies a reduction in asset price variability. Quantitatively, the ability of the monetary policy-maker to affect the real economy in the short run allows most of the adverse effects of credit frictions to be off-set. In the illustrative calibration used here, the standard deviation of output under optimal policy is around one sixth of the standard deviation of output under price stability. In other words, a little inflation variability buys a large reduction in output variability.

Comparing the Ramsey outcome with the frictionless model, we see that aggregate output is more persistent, but less variable, under the Ramsey policy than in the frictionless model. The increased persistence of Ramsey output arises because it is not efficient to off-set the initial output fall entirely, so there is still some persistence from the credit mechanism that prevents output from rising back to its steady-state level as quickly as the frictionless model. This is illustrated in figure (3) by the fact that the wealth share of producers still falls under optimal policy.

## 8 Robustness Checks

In this section I want to explore the extent to which the quantitative conclusions are sensitive to the particular choice of parameters. I will vary 4 key parameters. I explore the consequences of (a) putting a smaller Pareto weight on workers ( $\mu = 0.1$ ), (b) making labour supply less elastic ( $\tau = 1$ ), (c) making goods prices less sticky ( $\psi = 2$ ), and (d) weakening the credit channel by lowering the productivity gap between producers and investors ( $\frac{\alpha}{\gamma} = 1.01$ ).

The results appear to be robust to even these large parameter changes, with the crucial parameters being the strength of the credit channel and the extent of nominal rigidities, as can be expected, since these are the two frictions the policy-maker is trading off against each other. The changes in the model properties help to firm up the underlying intuition, so I will describe them case by case.

Lowering the welfare weight on workers makes output less variable,

<sup>&</sup>lt;sup>16</sup>The theoretical standard deviations and autocorrelations of the model variables were calculated using the method described in Hamilton (1994), p. 265-266.

 $<sup>^{17}</sup>$  The standard deviation of US quarterly inflation, on the GDP deflator measure, is 0.25% for the sample period 1983:1-2005:1. (Source: US BEA). The standard deviation of US quarterly output, measured as deviations from a Hodrick-Prescott filtered trend, was 1.11% over the same period.

	baseline	workers	lab.elast.	nom.rigid.	prod.gap
s.d.(y+y')	1.157	1.1337	1.105	1.270	1.427
s.d.(q)	0.798	0.597	0.764	1.133	1.139
$s.d.(\pi)$	0.128	0.196	0.160	0.313	0.094
a.r.(y+y')	0.431	0.373	0.359	0.558	0.300
a.r.(q)	0.698	0.665	0.660	0.676	0.371
$a.r.(\pi)$	-0.246	0.245	-0.347	-0.372	0.220
$s.d.(\varepsilon_{P,t})$	1.000	1.000	1.000	1.000	1.000

Table 3: Robustness of optimal policy results to parameter changes

Notes: s.d. denotes standard deviation, expressed in per cent, and a.r. denotes first-order autocorrelation.

Moments were calculated for log-linear deviations of aggregate output (y + y'), the price of capital (q), and inflation $(\pi)$ .

Each column represents a different version of the model.

but inflation more so. That is because, in order to dampen the effect of the shock on initial output, the expansionary monetary policy dampens output partly by its effect on labour. Since only workers supply labour, if policy is less concerned with worker welfare, it can tolerate greater deviations from the optimal path of labour, meaning it will dampen the output fall more strongly and tolerate higher inflation variability.

Less elastic labour supply implies that output falls by less following an adverse productivity shock, even in the frictionless model. That automatically weakens the credit channel, leading to less output variability. But it also means that monetary policy has to generate more inflation to dampen output by a given amount. In other words, the slope of the short-run Phillips curve has become steeper. So inflation variability is higher.

Lowering price stickiness gives monetary policy less traction, but leaves the strength of the credit channel unchanged. Monetary policy is therefore less able to dampen output responses, and a stronger burst of inflation is needed to dampen output by a given amount. The result is that both output variability and inflation variability under optimal policy are larger.

Finally, weakening the credit channel brings the model closer to the frictionless model. Higher variability of output can be tolerated, because it no longer has strong effects on the efficiency with which capital is used. And inflation variability is smaller, because there is no longer the need to use inflation to dampen the output response to a productivity shock as strongly.

## 9 Conclusion

I have outlined a macroeconomic model where credit markets operate less than perfectly due to enforcement problems, and I have used this model to discuss the implications of imperfect credit markets for monetary policy. The analysis has shown that the credit mechanism can amplify shocks and make them highly persistent, so that small, temporary disturbances to productivity or monetary policy have large and persistent effects on output. The basic mechanism is that, because highly productive agents find it optimal to borrow from less productive agents, they are leveraged. Any aggregate disturbance will affect borrowers' net worth more than lenders' net worth due to leverage, and so will affect the wealth distribution. The most productive agents will end up holding less of the economy's productive resources, which will lower future aggregate output and further depresses the current price of capital, exacerbating the shift in the wealth distribution. It takes time for the most productive agents to rebuild their share of wealth, and output therefore deviates from its steady-state level for many periods. Aggregate productivity in this model is endogenous, and is affected by the systematic response of monetary policy to non-policy shocks, as well as by monetary policy shocks. In the presence of sticky prices, monetary policy has the ability to dampen the output effect of productivity shocks by allowing temporary fluctuations in inflation. Such initial dampening will mitigate the reallocation of resources away from the most productive agents in the future.

I then conduct a quantitative analysis of optimal monetary policy. It is shown that, in the case of productivity shocks, the presence of both types of rigidities creates a trade-off between inflation variability and output variability. Because the initial output fall leads to a reallocation of resources toward less productive agents, it will result in large *future* deviations of output from its efficient level. So there is a trade-off between the rise in inflation immediately following the shock, and the fall in *future* output relative to its efficient level. The dynamic nature of this trade-off implies that neither the gap between output and its flexible-price level, nor the gap between output and its unconstrained efficient level are adequate descriptions of monetary policy objectives. Allowing a small temporary rise in inflation following an adverse productivity shock is optimal, because it results in output being much closer to its efficient level in future periods. A large reduction in output variability can be achieved by allowing only a small amount of inflation variability. Conversely, the cost of stabilising inflation too aggressively can be large.

Of course, real life policy decisions must be made without detailed knowledge of the state of the economy and of the laws of motion of the economy, so even if credit frictions are quantitatively important, the large reduction in output variability suggested by this model may not be achievable. Nevertheless, a realistic policy prescription might be not to stabilise inflation too aggressively following a shock that pushes output and inflation in opposite directions, especially at times when the financial system is fragile or borrowers' balance sheets are weak.

## A Model equations

The full model is described by the following equations. The timing convention is that all variables that are decided at date t after the realisation of the period t shock will have the subscript t. Predetermined variables therefore have a subscript t - 1. The wage of workers is denoted  $z_t$ .

Total wealth of entrepreneurs  $(W_t)$ 

$$W_t = (\eta + \sigma) \frac{y_t + y'_t}{\varphi_t} + q_t \tag{59}$$

the share of wealth held by producers  $(s_t)$ 

$$s_{t}W_{t} = (1 - \delta) \left[ (\eta + \sigma) \frac{y_{t}}{\varphi_{t}} + q_{t}k_{t-1} - b_{t-1} \right] + n\delta \left[ (\eta + \sigma) \frac{y'_{t}}{\varphi_{t}} + q_{t}(1 - k_{t-1}) + b_{t-1} \right]$$
(60)

the user cost of capital  $(u_t)$ 

$$u_t = \frac{\sigma}{\eta} \left(\beta W_t - q_t\right) \tag{61}$$

capital held by producers  $(k_t)$ 

$$k_t = \beta \frac{\sigma}{\eta + \sigma} \frac{s_t W_t}{u_t} \tag{62}$$

borrowing constraint  $(b_t)$ 

$$b_t = E_t q_{t+1} k_t \tag{63}$$

Phillips curve  $(\pi_t)$ 

$$\pi_{t} (\pi_{t} - 1) = \beta E_{t} \left[ \frac{u_{w,t+1}}{u_{w,t}} \pi_{t+1} (\pi_{t+1} - 1) \right] + \frac{\theta - 1}{\psi} (y_{t} + y'_{t}) \left( \frac{\theta}{\theta - 1} \frac{1}{\varphi_{t}} - 1 \right)$$
(64)

return on investment  $(r_t)$ 

$$r_t = E_t \left( \varepsilon_{P,t+1}^{\frac{1}{\eta+\sigma}} \gamma_{t+1}^{\frac{1}{\eta+\sigma}} \varphi_{t+1}^{-\frac{1}{\eta+\sigma}} z_{t+1}^{-\frac{1-\eta-\sigma}{\eta+\sigma}} u_t^{-\frac{\sigma}{\eta+\sigma}} \right)$$
(65)

pricing equation for capital  $(q_t)$ 

$$u_t = q_t - E_t \frac{q_{t+1}}{r_t}$$
(66)

labour market equilibrium in terms of workers' wage  $(z_t)$ 

$$z_t^{\frac{1+\tau}{\tau}} \left(\frac{1}{\chi}\right)^{1/\tau} = \frac{1-\eta-\sigma}{\eta+\sigma} \left(W_t - q_t\right) \tag{67}$$

Fisher equation<sup>18</sup> determining the nominal interest rate  $(R_t)$ 

$$E_t \frac{R_t}{\pi_{t+1}} \frac{1}{(\eta + \sigma) \frac{y'_{t+1}}{\varphi_{t+1}} \varphi + q_{t+1}} = E_t r_t \frac{1}{(\eta + \sigma) \frac{y'_{t+1}}{\varphi_{t+1}} \varphi + q_{t+1}}$$
(69)

producers' output  $(y_t)$ 

$$y_t = \alpha^{\frac{1}{\eta+\sigma}} \varepsilon_{P,t}^{\frac{1}{\eta+\sigma}} u_{t-1}^{\frac{\eta}{\eta+\sigma}} \left( z_t \varphi_t \right)^{-\frac{1-\eta-\sigma}{\eta+\sigma}} \frac{k_{t-1}}{\sigma}$$
(70)

investors' output  $(y'_t)$ 

$$y_t' = \gamma^{\frac{1}{\eta+\sigma}} \varepsilon_{P,t}^{\frac{1}{\eta+\sigma}} u_{t-1}^{\frac{\eta}{\eta+\sigma}} \left( z_t \varphi_t \right)^{-\frac{1-\eta-\sigma}{\eta+\sigma}} \frac{\left(1-k_{t-1}\right)}{\sigma} \tag{71}$$

definition of  $u_w$ , which is the marginal utility of consumption of workers

$$\frac{1}{u_{w,t}} = \frac{\tau}{\tau+1} \left(\frac{1}{\chi}\right)^{\frac{1}{\tau}} z_t^{\frac{1+\tau}{\tau}} + (y_t + y_t') \left(1 - \frac{1}{\varphi_t}\right) - \frac{\psi}{2} (\pi_t - 1)^2 \quad (72)$$

the aggregate productivity process  $(\varepsilon_{P,t})$ 

$$\log \varepsilon_{P,t} = \rho \log \varepsilon_{P,t-1} + v_t \tag{73}$$

When the model is solved for an ad hoc monetary policy, as opposed to optimal monetary policy, the final equation to close the model is the monetary policy rule.

$$E_t c'_{t+1} = (1 - \beta) \left[ (\eta + \sigma) \frac{y'_{t+1}}{\varphi_{t+1}} + q_{t+1} \right]$$
(68)

<sup>&</sup>lt;sup>18</sup>This is the standard asset pricing arbitrage condition, based on the marginal utility of consumption of the investors. I have made the following substitution:

## References

- ALVAREZ, F., AND JERMANN, U. J. Efficiency, equilibrium, and asset pricing with risk of default. *Econometrica* 68, 4 (2000), 775–798.
- [2] ANGELETOS, G.-M. Optimal monetary and fiscal policy: A linear-quadratic approach (comment). NBER Macroeconomics Annual 2003 (2003), 350–361.
- [3] AOKI, K. Optimal monetary policy responses to relative-price changes. Journal of Monetary Economics 48, 1 (2001), 55–80.
- [4] BARLEVY, G. Credit market frictions and the allocation of resources over the business cycle. *Journal of Monetary Economics* 50, 8 (2003), 1795–1818.
- [5] BASU, S., AND FERNALD, J. G. Returns to scale in u.s. production: Estimates and implications. *Journal of Political Economy* 105, 2 (1997), 249–283.
- [6] BERNANKE, B., AND GERTLER, M. Agency costs, net worth, and business fluctuations. *American Economic Review* 79, 1 (1989), 14–31.
- [7] BERNANKE, B., GERTLER, M., AND GILCHRIST, S. The financial accelerator in a quantitative business cycle framework. *Handbook* of Macroeconomics 1C (1999), 1341–1393.
- [8] BERNANKE, B. S., AND GERTLER, M. Should central banks respond to movements in asset prices? *American Economic Review* 91, 2 (2001), 253–257.
- [9] BLANCHARD, O. Inflation targeting in transition economies: A comment. In *The Inflation-Targeting Debate*, B. Bernanke and M. Woodford, Eds. University of Chicago Press, Chicago, 2005.
- [10] BLANCHARD, O., AND KAHN, C. M. The solution of linear difference models under rational expectations. *Econometrica* 48, 5 (1980), 1305–1311.
- [11] BROCK, W. A., AND MIRMAN, L. Optimal economic growth and uncertainty: The discounted case. *Journal of Economic Theory* 4 (1972), 479–513.
- [12] CALVO, G. Staggered prices in a utility-maximizing framework. Journal of Monetary Economics 12, 3 (1983), 383–398.
- [13] CARLSTROM, C. T., AND FUERST, T. S. Agency costs, net worth, and business fluctuations: A computable general equilibrium analysis. *American Economic Review* 87, 5 (1997), 893–910.
- [14] CARLSTROM, C. T., AND FUERST, T. S. Agency costs and business cycles. *Economic Theory* 12, 3 (1998), 583–597.

- [15] CHARI, V. V., KEHOE, P. J., AND MCGRATTAN, E. R. Business cycle accounting. NBER Working Paper, 10351 (2004).
- [16] COOLEY, T. F., MARIMON, R., AND QUADRINI, V. Aggregate consequences of limited contract enforceability. *Journal of Political Economy* 112 (2004), 817–847.
- [17] COOLEY, T. F., AND PRESCOTT, E. C. Economic growth and business cycles. In *Frontiers of Business Cycle Research*, T. F. Cooley, Ed. Princeton University Press, Princeton, 1995, pp. 1– 39.
- [18] CORDOBA, J.-C., AND RIPOLL, M. Credit cycles redux. International Economic Review 45, 4 (2004), 1011–1046.
- [19] DIXIT, A. K., AND STIGLITZ, J. E. Monopolistic competition and optimum product diversity. *American Economic Review 67*, 3 (1977), 297–308.
- [20] EISFELDT, A. L., AND RAMPINI, A. A. Capital reallocation and liquidity. Northwestern University Working Paper (2005).
- [21] ERCEG, C. J., HENDERSON, D. W., AND LEVIN, A. T. Optimal monetary policy with staggered wage and price contracts. *Journal* of Monetary Economics 46, 2 (2000), 281–313.
- [22] FAIA, E., AND MONACELLI, T. Optimal interest rate rules, asset prices and credit frictions. Universitat Pompeu Frabra Working Paper (2004).
- [23] FISHER, I. The debt-deflation theory of great depressions. *Econo*metrica 1, 4 (1933), 337–357.
- [24] GERTLER, M. Financial structure and aggregate economic activity: An overview. Journal of Money, Credit and Banking 20, 3 (1988), 559–588.
- [25] GILCHRIST, S., AND LEAHY, J. V. Monetary policy and asset prices. Journal of Monetary Economics 49, 1 (2002), 75–97.
- [26] HAMILTON, J. D. Time Series Analysis. Princeton University Press, Princeton, NJ, 1994.
- [27] HUBBARD, R. G. Capital-market imperfections and investment. Journal of Economic Literature 36, 1 (1998), 193–225.
- [28] IACOVIELLO, M. House prices, borrowing constraints and monetary policy in the business cycle. American Economic Review 95, 3 (2005), 739–764.
- [29] JAFFEE, D. M., AND RUSSELL, T. Imperfect information, uncertainty, and credit rationing. *Quarterly Journal of Economics* 90, 4 (1976), 651–666.

- [30] JERMANN, U., AND QUADRINI, V. Stock market boom and the productivity gains of the 1990s. NBER Working Paper, 9034 (2002).
- [31] KEHOE, T. J., AND LEVINE, D. K. Debt-constrained asset markets. *Review of Economic Studies* 60, 4 (1993), 865–888.
- [32] KEHOE, T. J., AND PRESCOTT, E. C. Great depressions of the twentieth century. *Review of Economic Dynamics* 5, 1 (2002), 1–18.
- [33] KIYOTAKI, N. Credit and business cycles. Japanese Economic Review 49, 1 (1998), 18–35.
- [34] KIYOTAKI, N., AND MOORE, J. Credit cycles. Journal of Political Economy 105, 2 (1997), 211–248.
- [35] KOCHERLAKOTA, N. R. Implications of efficient risk sharing without commitment. *Review of Economic Studies* 63, 4 (1996), 595– 609.
- [36] KOCHERLAKOTA, N. R. Creating business cycles through credit constraints. *Federal Reserve Bank of Minneapolis Quarterly Re*view 24, 3 (2000), 2–10.
- [37] LJUNGQVIST, L., AND SARGENT, T. J. Recursive Macroeconomic Theory, second edition ed. MIT Press, Cambridge, MA, 2004.
- [38] MCCALLUM, B. T. Price level determinacy with an interest rate policy rule and rational expectations. *Journal of Monetary Economics* 8, 3 (1981), 319–329.
- [39] PLOSSER, C. I. Understanding real business cycles. Journal of Economic Perspectives 3, 3 (1989), 51–77.
- [40] PRESCOTT, E. C. Theory ahead of business cycle measurement. Federal Reserve Bank of Minneapolis Quarterly Review, Fall (1986), 9–22.
- [41] PRESCOTT, E. C. Needed: A theory of total factor productivity. International Economic Review 39, 3 (1998), 525–51.
- [42] ROTEMBERG, J. J. Sticky prices in the united states. Journal of Political Economy 90 (1982), 1187–1211.
- [43] SARGENT, T. J. Dynamic Macroeconomic Theory. Harvard University Press, Cambridge, MA, 1987.
- [44] SARGENT, T. J., AND WALLACE, N. "rational" expectations, the optimal monetary instrument, and the optimal money supply rule. *Journal of Political Economy* 83, 2 (1975), 241–254.
- [45] SBORDONE, A. M. Prices and unit labor costs: a new test of price stickiness. *Journal of Monetary Economics* 49, 2 (2002), 265–292.
- [46] SCHEINKMAN, J. A., AND WEISS, L. Borrowing constraints and aggregate economic activity. *Econometrica* 54, 1 (1986), 23–45.

- [47] SCHIANTARELLI, F. Financial constraints and investment: Methodological issues and international evidence. Oxford Review of Economic Policy 12, 2 (1996), 70–89.
- [48] SCHMITT-GROHE, S., AND URIBE, M. Optimal fiscal and monetary policy under sticky prices. *Journal of Economic Theory* 114, 2 (2004), 198–230.
- [49] SCHMITT-GROHE, S., AND URIBE, M. Solving dynamic general equilibrium models using a second-order approximation to the policy function. *Journal of Economic Dynamics and Control 28*, 4 (2004), 755–775.
- [50] SODERLIND, P. Solution and estimation of re macromodels with optimal policy. *European Economic Review* 43, 4-6 (1999), 813– 823.
- [51] STIGLITZ, J. E., AND WEISS, A. Credit rationing in markets with imperfect information. *American Economic Review* 71, 3 (1981), 393–410.
- [52] TAYLOR, J. B. Discretion versus policy rules in practice. Carnegie-Rochester Conference Series on Public Policy 39 (1993), 195–214.
- [53] WOODFORD, M. Interest and Prices. Princeton University Press, Princeton, NJ, 2003.