# MPRA

Munich Personal RePEc Archive

# Putting Free-Riding to Work: A Partnership Solution to the Common-Property Problem

Martin Heintzelman and Stephen W. Salant and Stephan Schott

Clarkson University, University of Michigan (Ann Arbor), Carleton University

29. July 2008

# Putting Free-Riding to Work: A Partnership Solution to the Common-Property Problem

Martin D. Heintzelman, Stephen W. Salant, and Stephan Schott[*]

July 28, 2008

**Abstract** The common-property problem results in excessive mining, hunting, and extraction of oil and water. The same phenomenon is also responsible for excessive investment in R&D and excessive outlays in rent-seeking contests. We propose a "Partnership Solution" to eliminate or at least mitigate these excesses. Each of $N$ players joins a partnership in the first stage and chooses his effort in the second stage. Under the rules of a partnership, each member must pay his own cost of effort but receives an equal share of the partnership's revenue. The incentive to free-ride created by such partnerships turns out to be beneficial since it naturally offsets the excessive effort inherent in such problems. In our two-stage game, this institutional arrangement can, under specified circumstances, induce the social optimum in a subgame-perfect equilibrium: no one has a unilateral incentive (1) to switch to another partnership (or create a new partnership) in the first stage or (2) to

deviate from socially optimal actions in the second stage. The game may have other subgame-perfect equilibria, but the one associated with the "Partnership Solution" is strictly preferred by every player. We also propose a modification of the first stage which generates a unique subgame-perfect equilibrium. Antitrust authorities should recognize that partnerships can have a less benign use. By organizing as competing partnerships, an industry can reduce the "excessive" output of Cournot oligopoly to the monopoly level. Since no partner has any incentive to overproduce in the current period, there is no need to deter cheating with threats of future punishments.

# 1  Introduction

In some fisheries in Japan, fishermen from several vessels share their catch. Their pooled output is sold through a common outlet and members of each partnership divide equally the resulting gross revenue, no matter how little someone has contributed. Such egalitarian catch-sharing among vessels is a prescription for free-riding. Multiple partnerships compete for the catch, or the induced free-riding would be even greater.

Received economic theory cannot account for such partnerships. They do not appear to be a response to uncertainty or asymmetric information. Catch-sharing partnerships must have *some* advantage, however, since as of the census of 1988, 147 different fishing groups in Japan were engaging in such income pooling. To understand why such partnerships arise, Platteau and Seki (2000) interviewed skippers in the glass-shrimp industry in Japan and, when feasible, used more objective measures to validate their responses. They concluded that "The most prominent result emerging from this exercise is certainly the fact that stabilization of incomes was not mentioned a single time." Instead, the main motive appeared to be the reduction of congestion: "The desire to avoid the various costs of crowding while operating in attractive fishing spots appears as the main reason stated by Japanese fishermen for adopting pooling arrangements."[1]

---

[1]A similar arrangement seems to have been adopted by some groups of New Jersey fishermen as well, as discussed in Feeny et al. (1990). A more traditional society in Tonga, as described by Bender, Kägi, and Mohr (2002) also has features of the partnership arrangement we explore here.

These Japanese fishermen appear to have rediscovered an ancient solution to the common-property problem. According to anthropologists, hunter-gatherer cultures that have survived to the modern era typically share their kill and work short hours; moreover, they consume enough quantity and variety to be characterized by one distinguished anthropologist as the "Original Affluent Society" (Sahlins, 1972). Kägi (2001) was the first to point out that these phenomena, which have been studied extensively but separately, may be connected: those hunter-gatherer cultures surviving to modern times owe their success to their practice of sharing the fish and game caught by groups of hunters since extensive sharing dulls hunting effort sufficiently to protect common property from over-exploitation.[2]

At the opposite end of the technological spectrum, individuals who form research joint ventures to share revenue from their discoveries may have hit upon the same solution. Without joint ventures, individuals vying for a patent awarded to the best innovation will invest too much (Baye and Hoppe, 2003) even taking account of the fact that the expected value of the winning patent grows with aggregate investment. Baye and Hoppe also demonstrate that such innovation tournaments are strategically equivalent to rent-seeking contests where the value of the prize increases with the

---

[2] "The literature on traditional hunter-gatherers provides ample evidence that work effort is extremely low in traditional societies and that natural resources are not overexploited but rather under-exploited" (Kägi, 2001, p.45). "We do not know whether traditional societies have introduced sharing consciously. . . Once introduced (or chosen by accident), however, it appears to be a stable means to regulate resource use." (Kägi, 2001, p.67).

4

total outlay. So, in the absence of prize-sharing within interest groups, rent-seeking outlays are also excessive (Chung, 1996).[3]

---

[3]In an important contribution, Nitzan (1991) analyzed a contest for a fixed prize among rent-seeking individuals exogenously allocated to partnerships and found that aggregate rent-seeking outlays decline because of free-riding. Each partnership was assumed to use the same exogenous sharing rule of which our egalitarian rule is a special case. In other cases, his rule partially rewards an individual for making larger effort relative to other members of his group, which implicitly requires that group members costlessly monitor each other's efforts. We extend this original insight and show its implications for the common property and cartel problems. The case we examine is isomorphic to a rent-seeking contest where the prize is a strictly increasing function of aggregate outlays and where groups form endogenously (see footnote 6). Since the prize in Nitzan's contest is fixed, the partition generating the highest social welfare occurs when every player is in a single group. In our variable-prize case, putting every player in one group induces too little extraction effort and the social optimum instead occurs at an "interior" solution with the correct number groups. In Section 5, we show how our Partnership Solution easily generalizes when groups share according to Nitzan's rule provided the weight on relative effort is not excessive. Baik and Lee (2001) extend Nitzan's original model of group rent-seeking with a fixed prize by endogenizing group formation and the choice of the sharing rule. However, our analyses are quite different because there is no variable prize (the counterpart to our production) in their application. Consequently, it is efficient for everyone to join a single group, make no rent-seeking outlay, and share the fixed prize; in our context,

Common property extraction provides a particularly relevant illustration of the same strategic considerations. In the absence of sharing, fishing effort is excessive (Gordon, 1954) even when account is taken of the fact that aggregate catch grows with aggregate effort. Sharing arrangements promote free-riding and, when effort would otherwise be excessive, this constitutes a social improvement. We investigate the circumstances when sharing agreements can be used to restore the social optimum in extraction from common properties. As we make clear, however, our conclusions apply equally to (1) innovation tournaments and (2) rent-seeking contests.[4]

Besides these beneficial uses, partnerships can also be used for a more sinister purpose. In the absence of partnership agreements, service providers in an industry can be expected to reap at most oligopoly profits. But if they can organize themselves into a collection of competing revenue-sharing partnerships (a common organizational form in some service industries), they can potentially reap monopoly profits.[5] There is no need for interactions to be ongoing so that the prospect of a price war in the future deters the current temptation to expand output. The revenue-sharing inherent in a partnership structure *eliminates* the current temptation to expand output. Antitrust authorities should be aware that in industries where partnerships predominate, prices may approach monopoly levels even though competition among these partnerships is

the "prize" (production) would completely vanish in the absence of outlays (effort). Another key difference arises because we consider the role of team production.

[4]We clarify the strategic equivalence of these three applications in footnote 8 after introducing notation.

[5]Law firms, medical firms, and consultancies are some examples.

vigorous.

We consider a two-stage game where individuals choose their partnerships at the first stage and their effort levels at the second stage. If every individual chooses in the first stage to work in a different partnership as its sole "partner," aggregate effort in the second-stage equilibrium will exceed the social optimum. At the other extreme, if every player joins the same grand partnership, aggregate effort in the second stage will be inadequate due to rampant free-riding. As we show, aggregate effort is a strictly increasing function of the number of partnerships formed at the first stage.[6] Socially optimal effort can, therefore, be induced (or approximated if there are integer problems) if the $N$ players partition themselves into an intermediate number of partnerships in such a way that each agent's tendency to work too hard is exactly offset by his tendency to free ride. We refer to this as the "Partnership Solution."

In reality, of course, the Partnership Solution is viable if and only if each person in a given partnership has no incentive to switch to some other partnership (pre-existing or new). We refer to such partnerships as "stable." We refer to each partition of players into partnerships in the first stage of our game as a "partnership structure." Whether a partnership structure is stable (formally, whether it is part of a subgame-perfect Nash equilibrium) turns out to depend on the disadvantages of solo production relative to team production. This follows since the principal source of first-stage instability is going into business for oneself (deviating unilaterally to a *new* partnership).

---

[6]For experimental confirmation of this prediction, see Schott et. al., 2007 .

There are typically multiple subgame-perfect equilibria in the game. In the equilibrium associated with the Partnership Solution, however, every individual receives the largest expected payoff. This suggests two ways the game can be modified to select this "payoff-dominant" equilibrium. Before the agents select the partnerships, an outside mediator, with the power neither to monitor nor enforce compliance with his suggestion, can recommend this number of partnerships. His only function is to focus the expectations of the players on the payoff-dominant equilibrium. Alternatively, each agent can record the number of partnerships he prefers and the proposal of one agent, selected randomly, would be implemented. Either of these modified game forms should result in the implementation of the Partnership Solution.

It is natural to ask what advantages the Partnership Solution has over Pigouvian taxes or auctioned quotas. While taxes or quotas could induce the same extraction effort as the Partnership Solution, resource users would receive larger aggregate surplus under the Partnership Solution as long as *any* of the revenue collected under these two alternative policies was diverted to general coffers rather than being redistributed to the $N$ players.

We proceed as follows. In the next section, we introduce our notation, define socially optimal effort, and show that this level of aggregate effort arises in the second-stage subgames if and only if a requisite number of partnerships forms at the first stage. In Section 3, we determine conditions sufficient for this Partnership Solution to be stable and discuss two mechanisms which can be used to select this equilibrium. Section 4 generalizes the analysis while maintaining the assumption of egalitarian sharing within partnerships. Section 5 discusses justifications for our assumption of

equal sharing and then generalizes the Partnership Solution when sharing ceases to be purely egalitarian but instead takes some account of relative effort. Section 6 concludes the paper.

## 2    Decentralization in a Two-Stage Partnership Game

Suppose $N$ individuals expend effort to extract a common resource and consider two extremes.[7] On the one hand, suppose everyone pays his own effort cost and acts independently; then, as is well known, aggregate effort will be excessive because of congestion externalities. On the other hand, suppose everyone must share the fruits of his labor equally with the other $N - 1$ individuals while paying his own cost of effort; then aggregate effort will be insufficient because of free-riding. Each of these two extremes is a special case of the following arrangement: players partitioned into competing partnerships simultaneously choose effort levels, with each partnership's share of aggregate revenue equal to its share of aggregate effort and every member of each partnership required to pay his own effort cost but to share equally with his colleagues the gross revenue he brings in. In the first of the extremes above, there are $N$ "solo" partnerships while in the second, there is 1 "grand" partnership to which all $N$ individuals belong. We show below that aggregate effort is a strictly

---

[7]In assuming that $N$ is exogenous, we are abstracting from the entry of outsiders. In reality, partnerships have also been effective in protecting their own territories from outsiders. For an interesting analysis of how this has been accomplished see Acheson and Gardner's (2005) discussion of the Maine lobster fishery.

increasing function of the number of partnerships and hence socially optimal effort can be achieved as a Nash equilibrium if a particular number of partnerships forms at the first stage. To begin, we define the following notation that will be used throughout the paper.

$m_i$ = number of members of group $i$

$x_{ik}$ = effort level of agent $k$ in group $i$

$Y_i^{-k}$ = aggregate effort level of members of group $i$ other than agent $k$

$X_{-i}$ = aggregate effort of other groups

$X$ = total effort level (sum of all agents' efforts)

$F(X)$ = aggregate production function

$c$ = constant marginal cost of effort

$n$ = number of groups

$N$ = total number of agents

$A(\cdot) = \frac{F(X)}{X}$ = average product

$\bar{x}_i = \frac{\left(x_{ik} + Y_i^{-k}\right)}{m_i}$ = mean effort level in group $i$

$\beta$ = effort advantage of team production compared to solo production

Until Section 4, we make the assumption standard in the common-property literature that the price of output, $\bar{p}$, is a constant (normalized to unity). In addition, we assume that (1) $A(X)$ is bounded, strictly positive, strictly decreasing, and twice continuously differentiable; (2) $A(0) - c > 0$; and (3) that $A'(X) + XA''(X) < 0$, holds for all $X \geq 0$ (the Novshek condition).

Socially optimal effort maximizes total net benefit ($X^* = \text{argmax } \bar{p}X(A(X) - c)$).

10

Since $\bar{p}$ is normalized to one, socially optimal effort must therefore satisfy the following first-order condition:

$$A(X^*) + X^* A'(X^*) - c = 0. \tag{1}$$

Since the Novshek condition holds, $X^*$ is unique. This aggregate effort level is the goal we seek to achieve by decentralization through our Partnership Solution.

Let $n \leq N$ denote the number of distinct partnerships formed at the first stage and index these groups $i = 1, \ldots, n$. Then, in the second stage, agents simultaneously choose their effort after observing each agent's choice of group. To verify that the partnership solution is subgame-perfect, we must show that it forms a Nash equilibrium in every subgame. We demonstrate this through backwards induction, considering the problem of effort choice first.

## 2.1   Equilibrium Effort Choice in Second-Stage Subgames

Consider second-period subgames in which individuals grouped into partnerships simultaneously choose their effort levels.

An individual in group $i$ would choose his own effort level $(x_{ik})$ taking as given the aggregate effort level of his colleagues in partnership $i$ $(Y_i^{-k} = \sum_{l \neq k} x_{il})$ as well as the aggregate effort levels of the other partnerships $(X_{-i})$. Hence, he would maximize

$$\pi_{ik} = \underset{x_{ik}}{Max} \left\{ \frac{1}{m_i} \left[ \frac{x_{ik} + Y_i^{-k}}{X} \right] \cdot F(X) - c x_{ik} \right\},$$

where $X = x_{ik} + Y_i^{-k} + X_{-i}$ and $m_i$ is the number of partners in his group.[8] This is

---

[8]We can now formally show the connection between the common-property problem and the two other applications discussed in the introduction. Let $x_{ik}$ denote the rent-seeking outlay of player $k$ in group $i$, $Y_i^{-k}$ denote the outlays of his partners, $X$ denote the total outlay of all contestants, and $F(X)$ denote the prize. Then, assuming the probability that a group wins the prize is proportional to its rent-seeking outlays and the prize is divided evenly among members of the winning group, the foregoing objective function of player $k$ is simply his expected payoff from an outlay of $x_{ik}$. Similarly, consider an innovation tournament in which player $k$ in group $i$ makes research effort $x_{ik}$. Following Baye and Hoppe (2003), conceive of each researcher's discovery effort as his drawing ideas worth \$y if awarded a patent ($y \in [0, U]$, where $U$ is an exogenous highest conceivable value) from a differentiable probability distribution $G(y)$. Let $x_{ik}$ denote the integer number of times ideas are drawn with replacement from the distribution. If each draw costs the researcher $c$, then researcher $k$'s cost would be $cx_{ik}$. Moreover, if the prize goes to the partnership with the highest realized draw, then the probability that partnership $i$ wins equals its aggregate number of draws as a percentage of the total number of draws of all players in the tournament (the factor in square brackets). It remains to show that—whoever wins—the expected value of the prize is a strictly increasing, strictly concave function of the total number of draws. This is both intuitive and straightforward to show (see Theorem 1 of Baye and Hoppe, 2003): $\int_{y=0}^{U} yX G^{X-1}(y) G'(y) dy = 1 - \int_0^U G^X(y) dy = F(X)$, where the first integral is the expected value of the prize when there are $X$ draws from distribution $G(\cdot)$ and the largest of them is designated $y$. It is straightforward to verify by differentiating

equivalent to maximizing:

$$m_i \pi_{ik} = \left(x_{ik} + Y_i^{-k}\right) \cdot A \left(x_{ik} + Y_i^{-k} + X_{-i}\right) - m_i c x_{ik}. \tag{2}$$

To find the best response of member $k$ in partnership $i$, we differentiate the objective function (2) with respect to $x_{ik}$ and substitute $X = x_{ik} + Y_i^{-k} + X_{-i}$ to arrive at the following $N$ first-order conditions:

$$A\left(X\right) + \left(x_{ik} + Y_i^{-k}\right) \cdot A'\left(X\right) = m_i c \text{ for } i = 1, \ldots, n \text{ and } k = 1, \ldots, m_i. \tag{3}$$

Each of the $N$ first-order conditions in (3) clarifies why player $i$ reduces his effort in a multiperson partnership compared to his effort operating solo, for unchanged effort of the other $N - 1$ players. There are two effects, each of which leads him to reduce his effort: the "internalization effect" and the "diversion-of-benefits effect." First, since in a multiperson partnership, player $i$ *receives* a share of the receipts generated by his partners, he would refrain from imposing as large a negative externality on them as he would if he operated solo. That is, the first factor in the second term is larger by $Y_i^{-k}$ than it would be if he operated solo. This "internalization effect" would induce him to reduce his effort in a multiperson partnership even if $c = 0$ but the effect would disappear if under the rules of the partnership he received nothing

that the resulting expected value of the winning prize ($F(X)$) is a strictly increasing and concave function of the aggregate research effort as our analysis assumes.

from his partners. Second, since in a multiperson partnership, player $i$ *relinquishes* a share of the benefits of his effort but pays the full cost of generating them, he would reduce his effort. That is, the right-hand side of the equation is $m_i > 1$ times as large as it would be if he were operating solo. This "diversion-of-benefits effect" would persist even if he received nothing from his partners but nonetheless had to give them a portion of his catch. This effect would disappear if $c = 0$.

Consider any solution to the $N$ equations in (3) and the 2 equations defining $X$ and $Y_i^{-k}$. Notice that if the efforts within any partnership are rearranged without altering their sum than each of these $N + 2$ equations still holds. Formally, therefore, there are multiple Nash equilibria in the final stage of our game, with the payoff to player $k$ lower in those equilibria in which he undertakes a larger effort. We regard the multiplicity of equilibria in the second stage of our game as an artifact of our assumptions since introducing even the slightest convexity in the effort cost functions or the slightest weight on relative effort in the sharing rule would eliminate all of the asymmetric equilibria, leaving only the symmetric one. We therefore focus on the symmetric equilibrium by assuming as a "refinement" that agents anticipate at the first stage that efforts will be shared equally in any partnership they join.[9]

---

[9]In exactly these circumstances, Benoit and Krishna (1985) have shown that many first-stage configurations can be supported as subgame perfect equilibria. To clarify what we are ignoring, suppose anyone deviating to a different partnership at the first stage anticipated that, as "hazing" reserved for new members, his new colleagues would undertake no effort and would make him carry the entire effort burden of the partnership by himself. The anticipated situation would deter his deviation and is

## 2.2 Partnership Effects on Effort Choice

Since the first-order conditions in (3) depend only on the aggregate effort of partnership $i$ and not on the efforts of its individual members, we can re-write the conditions in terms of the mean effort in partnership $i$ $(\bar{x}_i)$. Rewriting the first factor of the second term gives us:

$$A(X) + m_i \bar{x}_i \cdot A'(X) - cm_i = 0, \quad \text{for } i = 1, \ldots, n. \tag{4}$$

These $n$ equations plus the equation $X = \sum_{i=1}^{n} m_i \bar{x}_i$ uniquely determine the $n$ mean effort levels $\{\bar{x}_i\}_{i=1}^{n}$ and $X$. We can solve (4) for $\bar{x}_i$, the mean effort level in group $i$:

$$\bar{x}_i = \left(\frac{1}{-A'(X)}\right)\left(\frac{A(X)}{m_i} - c\right). \tag{5}$$

If partnerships of different sizes form at the first stage, then their mean effort levels will differ at the second stage. In particular,

**Proposition 1** *In any equilibrium, strictly larger groups have strictly smaller mean effort levels.*

Proof: As (5) reflects, the strictly positive mean effort level at the $i^{th}$ partnership can be represented as the product of two positive factors. The second factor will be smaller at a partnership with a larger number of members $(m_i)$ while the first factor

───────────────────────

technically "credible" since it is a Nash equilibrium. While exploiting this aspect of our problem would allow us to show that the Partnership Solution is stable in a wider set of circumstances, we refrain from doing so.

15

will be the same for all the partnerships. Hence, the larger the partnership the smaller the mean effort.∎

Intuitively, the larger the group, the more free-riding occurs within it.

Next we verify that *aggregate* effort in the second-stage depends only on the *number* $(n)$ of groups formed at the first stage and not on the distribution of agents among the different groups:

**Proposition 2** *Aggregate effort (X) in the second stage depends only on the number of groups formed in the first stage and not on the size of those groups.*

Proof: Adding together the $n$ first-order conditions in (4), we obtain the following condition:[10]

$$nA(X) + XA'(X) - cN = 0. \tag{6}$$

Thus aggregate effort $(X)$ induced in the Nash equilibria of second-stage subgames depends only on the number of groups formed at the first stage and not on the specific partition. ∎

A monotonic relationship exists between the number of partnerships formed at the first stage and the aggregate effort expended at the second stage.

**Proposition 3** *If the number of groups formed at the first stage is strictly larger, the aggregate effort level at the second stage is strictly larger.*

———————————————

[10]Our proposition reinterprets the result in Bergstrom and Varian (1985) that, in an interior equilibrium of a Cournot oligopoly model with constant marginal costs, aggregate output depends only on the sum of the marginal costs.

Proof: Differentiating (6) implicitly, we obtain:

$$\frac{dX}{dn} = \frac{A(X)}{-[(n+1)A'(X) + XA''(X)]} > 0,$$

where the inequality follows from $A(X) > 0$, $A'(X) < 0$, and the Novshek condition.∎

Since aggregate effort in our game is a continuous, strictly increasing function of the number of groups formed at the first stage and since $n = 1$ induces too little aggregate effort and $n = N$ generates too much, some unique intermediate number of groups will (if we provisionally ignore integer constraints) induce the socially optimal level of effort at the second stage. We can find this number by plugging $X^*$ into (6) and then solving for $n^*$.

**Proposition 4** *If* $n^* = \frac{c(N-1)}{A(X^*)} + 1$ *groups form at the first stage, then the aggregate effort chosen in the Nash equilibrium of the second stage will be socially optimal.*

Proof: Substitute $n^* = \frac{c(N-1)}{A(X^*)} + 1$ and $X^*$ into (6). This gives us:

$$\left(\frac{c(N-1)}{A(X^*)} + 1\right)A(X^*) + X^*A'(X^*) - cN = 0.$$

Simplifying, we obtain:

$$A(X^*) + X^*A'(X^*) - c = 0$$

which is the same as (1), the condition defining $X^*$.∎

Proposition 4 implies that whenever $c = 0$, the social optimum is achieved by putting everyone into a single partnership ($n = 1$), while for $c > 0$, the optimum is achieved by dividing them among several partnerships. To understand these results, suppose each of the $N$ players joins a single partnership and everyone's effort equals $1/N^{th}$ of the aggregate profit-maximizing level. If costs are zero, aggregate revenue

17

would also be maximized so that if someone marginally reduced his effort unilaterally, total revenue would not change nor would his share of it. Since deviating is unprofitable, the social optimum is achieved as a Nash equilibrium. On the other hand, if everyone is assembled into a single partnership and asked to undertake $1/N^{th}$ of the socially optimal effort when $c > 0$, there is an incentive to reduce effort. In this case, such a reduction does not affect aggregate profit because aggregate gross revenue and aggregate costs fall by equal amounts. However, since all of the cost savings accrue to the deviator while only $1/N^{th}$ of the aggregate revenue loss is borne by him, he has a strict incentive to reduce his effort and it is no longer possible to achieve the social optimum with a single partnership.

When $c > 0$, therefore, the Partnership Solution requires dividing the $N$ players among *several* partnerships. Suppose again, each player undertakes $1/N^{th}$ of the socially optimal effort. If someone marginally reduced his effort unilaterally, he would save the same costs as before but now would incur a larger loss in revenue both because his partnership's losses are larger and because he bears a larger share of those losses.

In the Partnership Solution ($n = n^*$), the social optimum is achieved as a Nash equilibrium because each person fully anticipates the *social* consequences of his deviation. If anyone increased his effort, he would incur the entire cost increase. Moreover, since the gross revenue of his colleagues would expand by exactly as much as the gross revenue of noncolleagues would contract, the deviator's share of his partnership's increased revenue would just equal the aggregate gain in revenue from his deviation.[11]

---

[11]More precisely, since at the social optimum the planner equates to marginal cost the marginal revenue from additional effort while in the Partnership Solution each

## 2.3 Partnerships and Pigouvian Taxes

We conclude this section by clarifying the relationship between partnerships and Pigouvian taxes. Partition the $N$ players in any way into $n$ partnerships and compute the Nash equilibrium in the simultaneous-move effort game. We could dissolve one or more of these partnerships and instead tax the effort of each of its members at the group-specific, constant rate $\tau_i = (1 - \frac{1}{m_i})A(X)$. In response to the taxes, each member of a group would choose $\hat{x}_i$ to maximize $\hat{x}_i A(\hat{x}_i + X_{-i}) - (c + \tau_i)\hat{x}_i$ by solving $A(X) + \hat{x}_i A'(X) = c + (1 - \frac{1}{m_i})A(X)$ or $A(X) + m_i \hat{x}_i A'(X) = m_i c$ where $\sum_{i=1}^{n} m_i \hat{x}_i = X$. But this is precisely the same first-order condition as before and hence the effort $\hat{x}_i = \bar{x}_i$: taxing the groups in this way induces the same effort as before—when every group was a partnership. Notice that whereas before effort declined with partnership size because of free-riding, now it declines with tax-group size because larger groups are subjected to higher tax rates.

Although, dissolving a partnership and instead subjecting each of its former members to such taxes would not alter equilibrium effort levels, it would alter equilibrium

---

player equates to that same marginal cost the increase in gross revenue he would receive through the partnership agreement if he marginally increased his effort, the following equality must hold:$[A + XA'] = \frac{1}{m_i}[A + (X - X_{-i})A']$. But this is equivalent to the following: $\frac{m_i - 1}{m_i}[A + (X - X_{-i})A'] + X_{-i}A' = 0$. The first term can be interpreted as the aggregate increase in gross revenue of the deviator's colleagues and the second term can be interpreted as the exactly offsetting aggregate decrease in the gross revenue of players outside the partnership.

payoffs. In particular, every member of group $i$ would earn $\tau_i \bar{x}_i$ less when taxed than as a member of a partnership. Hence, unless all of the tax revenues are rebated, participants would strictly prefer the Partnership Solution to the Pigouvian tax scheme.

# 3 Partnership Choice in the First Stage: Equilibria and Selection

In the first stage, $N$ players simultaneously choose partnerships, resulting in the formation of between 1 and $N$ groups. We now investigate whether any agent would have an incentive to deviate from the chosen partnership structure. When no such incentive exists, the partnership structure will be part of a subgame-perfect Nash equilibrium. We first discuss the requirements for this equilibrium. We derive conditions when the Partnership Solution will be stable and show that under these circumstances other equilibria may exist as well. We conclude the section by discussing selection of the Partnership Solution from among the equilibria.

## 3.1 Equilibrium in the First Stage

Deviations at the first stage fall into two categories: (1) an agent can abandon the colleagues in his prescribed group for the members of some other group or (2) he can abandon his prescribed group to go into business for himself. The following proposition allows us to eliminate as potential subgame-perfect equilibria most partnership structures which could form at the first stage.

**Proposition 5** *A necessary condition for a partnership structure to be part of a subgame-perfect Nash equilibrium is that no partnerships differ in size by more than one member.*

Proof: First note that, from Proposition 2, a deviation which maintains the number of groups formed at the first stage will not alter aggregate effort ($X^*$) exerted at the second stage. Second, each player in group $i$ anticipates a payoff of ($\pi_i$):[12]

$$\pi_i = \bar{x}_i(A(X^*) - c). \tag{7}$$

This is strictly increasing in $\bar{x}_i$ since $A(X^*) - c > 0$. Each member's payoff is larger in groups with a larger mean level of effort. Proposition 1 tells us that a group with a smaller number of members will have a larger mean effort since its smaller size will discourage free-riding. Hence, any player can strictly improve his payoff if he can switch to a partnership which will be smaller (even after he joins it) than his existing partnership. Hence, profitable defections are always possible when two partnerships differ in size by two or more members but never possible when every partnership differs in size by at most one member.∎

Proposition 5 rules out as part of a subgame-perfect equilibrium any partnership structure formed at the first stage in which two partnerships differ in size by two or more members.

There remain $N$ partnerships structures to consider as potential subgame-perfect Nash equilibria. In particular, for any $n$ ($= 1, \ldots, N$) there will be a *unique* partnership structure where sizes differ by at most one member. To visualize this, imagine that the $N$ players are being dealt sequentially like cards in a card game to the $n$ partnerships arranged around a table. When all of the players have been dealt out

---

[12]To see this, begin with the objective function (2) and see that, if player $k$ makes effort $x_{ik}$ in a group with mean effort $\bar{x}_i$ when aggregate effort is $X$, then his payoff is: $\pi_{ik} = \bar{x}_i A(X) - cx_{ik}$.

(assigned to partnerships) some of the partnerships may have one fewer player than rest. The Partnership Solution with its $n^*$ groups is among these potential equilibria.

Any of these partnership structures will be subgame-perfect if, in addition, no agent can strictly improve his expected payoff by forming a new, singleton, group. Whether this is profitable or not depends upon the disadvantage of solo production compared to team production. The literature on the theory of the firm emphasizes that multi-agent firms are rife with incentive problems to which single-agent firms are immune. But, since multi-agent firms abound, there must be a countervailing advantage to such arrangements—individuals working in teams must be able to produce more output per man-hour than those working alone.[13] Following the literature on team production, therefore, we consider the possibility that a team can produce more than an individual working by himself the same number of man-hours; in extreme cases, a team may be necessary in order to produce at all.[14]

---

[13]Alchian and Demsetz (1972) were the first to emphasize the importance of team production in the theory of the multi-person firm and their insights have now percolated down to undergraduate treatments of that theory. For an extensive discussion, consult the textbooks by Eaton et. al (Chapter 19, 2002) and Campbell (Chapter 2.5, 1995).

[14]An alternative interpretation could be that there are subjective "social benefits" to remaining in a team. This is the approach of Osés-Eraso and Viladrich-Grau in a standard common-property setting. They, however, fail to recognize the further benefit of sharing in reducing aggregate effort.

Suppose that to duplicate the efforts of 1 man-hour of team effort, a single individual must work $1/\beta$ hours, for $\beta \in [0,1]$. Then, if we continue to express effort in man-hours of team effort, the marginal cost of effort for an individual working alone would be $\frac{1}{\beta}c$.

Partition the $N$ players into $n$ groups in such a way that no two groups differ in size by more than 1 member. Recall that for any $n$ there is a unique partition that satisfies this restriction. In the case where some partnerships are one member larger than others, these larger partnerships will generate more free-riding in the equilibrium of the second stage (Proposition 1). Anticipating lower payoffs in the second stage, every member of a larger partnership would have a stronger incentive to deviate to a solo partnership at the first stage. Let $g(n,\beta)$ denote the gain a member of a larger partnership would achieve by setting up his own partnership. If $g(n,\beta) \leq 0$ then he has no incentive to deviate and *a fortiori* neither does any member of a smaller partnership; hence the partition under consideration is stable. If, however, $g(n,\beta) > 0$ then he has an incentive to deviate and the partition under consideration is unstable. By analyzing properties of the $g(\cdot,\cdot)$ function, we show below that for any $n$, including $n^*$, there is a unique $\beta(n) \in (0,1]$ such that the Partnership Solution is stable for all $\beta \leq \beta(n)$.

### 3.1.1 Team Production is Essential ($\beta = 0$)

In many applications, "it takes two workers to perform a given task" (Holmstrom and Tirole, p. 67). That is, solo production is infeasible. For example, no matter how hard a person works he/she cannot catch a whale by himself; nor can he/she stay

awake every day and night of his medical career to help patients with their medical emergencies. In other applications deviating to solo groups may be illegal since many partnership agreements contain 'non-compete' clauses which prevent an individual, when leaving a partnership, from competing in the same market as the group he is leaving.[15]

Whenever solo production is infeasible, $g(n, 0) < 0$ and we can conclude:

**Proposition 6** *When solo production is infeasible, the Partnership Solution is subgame perfect and solves the common-property problem.*

Proof: As we have verified, no unilateral deviation to an existing partnership is strictly advantageous to any agent. Moreover, since $g(n, 0) < 0$, no deviation to a solo partnership is profitable for any $n$, including $n^*$. ∎

If solo production is infeasible, however, there will be other subgame-perfect equilibria as well. At the end of the section, we will discuss two mechanisms for selecting from the set of subgame-perfect equilibria the equilibrium inducing socially optimal effort.

### 3.1.2 Solo Production Is Feasible ($\beta \in (0, 1]$)

If solo partnerships are legal and feasible, we must investigate further. If $\beta < 1$, then social welfare can never be maximized as long as any solo partnership is involved. For, if there are any solo partnerships, then even if in equilibrium optimal effort ($X^*$) results, the cost of achieving it will strictly exceed $cX^*$, which a planner could achieve

---

[15]Various courts have upheld such clauses, including the Georgia Supreme Court in Rash v. Toccoa Clinic Med. Assoc., 253 Ga. 322, 320 S.E.2d 170 (1984).

just by assembling a team of all $N$ players and commanding that level of effort.

So we assume that $n = 1, 2, \ldots, \lfloor N/2 \rfloor$ partnerships, where $\lfloor Z \rfloor$ denotes the greatest integer less than or equal to Z. For example, if $N = 15$, there are at most $\lfloor 15/2 \rfloor = 7$ partnerships: six with two members and one with three members.

Equation (6) implicitly defines the aggregate effort which would result from $n$ partnerships, each of which has two or more members. Denote the aggregate effort implicitly defined by this equation as $X(n)$. If $X(\lfloor N/2 \rfloor) \geq X^*$, then the Partnership Solution can potentially achieve the first best by generating more free riding and thereby bringing effort down toward $X^*$.

Denote the payoff of a potential deviator, prior to his deviation, as $\pi^C$ and his payoff after going solo as $\pi^D$. $\pi^C$ is independent of $\beta$. A partner who deviates, therefore, gains $g(n, \beta) = \pi^D - \pi^C$. His gain from going solo, his effort, everyone else's effort, and aggregate effort, will depend on the parameter $\beta$. Define $\underline{\beta}$ such that for any $\beta > \underline{\beta}$, the deviator going solo would make strictly positive effort while for any smaller $\beta$ he would make zero effort. When $\beta \in [0, \underline{\beta}]$, the deviator would receive a zero payoff ($\pi^D = 0$) following his deviation. Hence, $g(n, \beta) = g(n, 0) = -\pi^C < 0$ for any $\beta \in [0, \underline{\beta}]$. When $\beta \in (\underline{\beta}, 1]$ the consequences of one agent's going solo are described by the four variables $\pi^D$, $X$, $X_{-1}$, and $\bar{x}_1$ which are defined by equations (8)-(11) below, where for simplicity we assign the index "1" to the deviator's solo partnership (and therefore denote his effort as $\bar{x}_1$ and the aggregate effort of all others after observing his first-stage deviation as $X_{-1}$):

$$\pi^D = \bar{x}_1(A(X) - \frac{c}{\beta}) \qquad (8)$$

$$A(X) + \bar{x}_1 A'(X) - \frac{c}{\beta} = 0 \qquad (9)$$

$$nA(X) + X_{-1}A'(X) - (N-1)c = 0 \qquad (10)$$

$$\bar{x}_1 + X_{-1} = X. \qquad (11)$$

Equation (10) is obtained by adding up the first-order conditions of the $n$ original partnerships (that is, excluding from the sum the solo partnership) after effort levels in the following stage have adjusted in response to the deviation.

**Proposition 7** $g(n, \beta)$ *is a differentiable function of $\beta$ in the neighborhood of any $\beta$ in $(\underline{\beta}, 1]$.*

Proof: Since $\pi^C$ is independent of $\beta$, it is sufficient to show that $\pi^D$ is differentiable in $\beta$. Use (11) to eliminate $X$ from (8)-(10). Equation (10) does not involve $\beta$. Given the Novshek condition, $(n+1)A' + X_{-1}A'' \neq 0$; therefore, the implicit function theorem insures that, in a neighborhood of any solution $(\bar{x}_1, X_{-1}, X)$ induced by $\beta \in (\underline{\beta}, 1]$ we can write (10) as $X_{-1} = f(\bar{x}_1)$ where $f(\cdot)$ is a continuous function with derivative $f' = -\frac{nA' + X_{-1}A''}{(n+1)A' + X_{-1}A''} \in (-1, 0)$. Equation (9) does involve $\beta$. Replace $X_{-1}$ in this equation by $f(\bar{x}_1)$. Given the Novshek condition and $A' < 0$, $(1 + f')(A' + \bar{x}_1 A'') + A' \neq 0$; therefore, the implicit function theorem insures that we can write (9) locally as $\bar{x}_1 = h(\beta)$ for some continuous function $h(\cdot)$ with derivative $h' = -\frac{c/\beta^2}{A' + (1+f')(A' + x_1 A'')} > 0$. Substituting both of these differentiable functions into (8), we obtain:

$$\pi^D(\beta) = h(\beta)\left[A\left(h(\beta) + f(h(\beta))\right) - c/\beta\right].$$

Since $A(\cdot)$ is differentiable and since sums, products, and compositions of differentiable functions are differentiable, $\pi^D$ is a differentiable function of $\beta$ in a neighborhood of any solution $(\pi_D, \bar{x}_1, X_{-1}, X, \beta)$. Given this conclusion, there can be no $\beta \in (\underline{\beta}, 1]$ where $\pi^D$ is non-differentiable. It follows that $g(n, \beta)$ is differentiable in the neighborhood of any $\beta$ in the interval $(\underline{\beta}, 1]$. ∎

Since $g(n, \beta) = \pi^D(\beta) - \pi^C$, we can differentiate to obtain the partial derivative, $g_\beta(n, \beta)$.

**Proposition 8** $g_\beta(n, \beta) > 0$ *for any* $\beta$ *in* $(\underline{\beta}, 1]$.

Proof: Since $\bar{x}_1 > 0$ for any $\beta$ in $(\underline{\beta}, 1]$, $h(\beta) > 0$. Recall that $A' < 0$. Differentiating our expression for $g(n, \beta)$ and using (9) to simplify (an application of the envelope theorem) we conclude that:

$$g_\beta(n, \beta) = h'[A + hA' - c/\beta] + h[A'f'h' + c/\beta^2] = h[A'f'h' + c/\beta^2] > 0$$

for any $\beta$ in $(\underline{\beta}, 1]$.∎

We have shown that $g(n, \beta)$ is differentiable and strictly increasing in $\beta$ in the interval $(\underline{\beta}, 1]$ and $g(n, \beta) = -\pi^C$ for $\beta \in [0, \underline{\beta}]$. The following lemma establishes that there is no discontinuity at the boundary $\beta = \underline{\beta}$.

**Lemma 1** *The function* $g(n, \beta)$ *is continuous in* $\beta$ *at the point* $\underline{\beta}$.

Proof: Since $g(n, \beta) = -\pi^C$ for $\beta \in [0, \underline{\beta}]$, it suffices to verify that $\lim_{\beta \to \underline{\beta}} g(n, \beta) = \lim_{\beta \to \underline{\beta}} (\pi^D - \pi^C) = -\pi^C$. But this follows from (8) since $\lim_{\beta \to \underline{\beta}} \bar{x}_1 = 0$ and $A$ is bounded. ∎

We can therefore, conclude:

**Proposition 9** *If the partition indexed by* $n$ *is stable for some* $\beta$*, then it is stable for all smaller* $\beta$.

Proof: This follows from Proposition 8. ∎

We now use the results above to prove the existence and uniqueness of a 'threshold' $\beta(n)$ which separates stable from unstable partitions.

**Proposition 10** *For any $n \leq \lfloor N/2 \rfloor$, there exists a unique $\beta(n) \in (\underline{\beta}, 1]$ such that for any $\beta < \beta(n)$, the partition indexed by $n$ can be supported as a subgame-perfect equilibrium, while for $\beta > \beta(n)$ the partition can never be supported.*

Proof: For any given $n$, suppose that at $\beta = 1$, $g \leq 0$. Then that partition can be supported as an subgame-perfect equilibrium for any $\beta \in (0, 1]$ and we can define $\beta(n) = 1$. Now suppose that at $\beta = 1$, $g > 0$. Then by continuity (Proposition 7), there will exist one or more roots, $\beta \in (0, 1)$, such that $g(n, \beta) = 0$. Denote any root as $\beta(n)$. Uniqueness of $\beta(n)$ then follows since $g$ is strictly increasing (Proposition 8).∎

This makes precise the intuitive notion that the socially optimal partnership partition is stable if solo production is "sufficiently disadvantageous": solo production need not be infeasible ($\beta = 0$) but $\beta$ can not be strictly larger than $\beta(n^*)$. Alternatively, for any given $\beta$, $\beta(n)$ also defines partnership partitions which are stable:$\{n : \beta(n) > \beta\}$.

Is the Partnership Solution *always* stable even when team production confers no advantage whatsoever ($\beta = 1$)? A single counterexample suffices to eliminate this possibility. Recall the example introduced at the outset where $N = 12$ producers in an industry, each with constant marginal cost of $c = 3$, face an inverse demand curve of $P = 19 - X$ and attempt to achieve monopoly profits by dividing into $n = 4$ partnerships of equal size. It is easily verified that for any $\beta \leq .39$, full monopoly profits (\$64) can be achieved, but for $\beta > .39$ the configuration of four partnerships is unstable.

## 3.2 Selection of the Partnership Solution from the Set of Equilibria

When the Partnership Solution is among the subgame-perfect equilibria, several alternative methods can be used to select it, of which we focus on two. Both methods designate the formation of a specific number of partnerships and implicitly assume that players will be assigned to these partnerships at random and in such a way that sizes differ by at most one member (recall the card-dealing analogy).

The first method of selecting the Partnership Solution from the set of equilibria is a coordination device first proposed by Schelling (1960, p. 63 and p. 302). An outside "mediator" could recommend publicly to all the players the establishment of $n^*$ partnerships. By assumption, this mediator could neither monitor nor enforce compliance with his suggestion. His only function would be to focus the expectations of the players on the payoff-dominant equilibrium.[16] In two-player experiments, Van Huyck et. al. (1992) have shown that subjects disregard such recommendations if they are not Nash or are Nash but not payoff-dominant; however, subjects follow the recommendations which are Nash and payoff-dominant 98% of the time. Brandts and MacLeod (1995) extend this experimental work to two-stage games and find that the mediator is equally influential in the selection of subgame-perfect equilibria.[17]

---

[16]In certain settings, one could imagine a regulatory authority having the further ability to enforce the Partnership Solution, say in a fishery. In this case, there is no need for the partnership solution to be 'stable' in the sense described above.

[17]A useful discussion of these two papers is contained in Camerer (2003, 362-5).

Alternatively, each agent could simply record the number of partnerships he prefers and the proposal of one agent, selected randomly, would be implemented. Since the socially optimal number of partnerships is also privately optimal for each agent, and the selection mechanism makes each agent pivotal with positive probability, each agent should choose to record $n*$.

# 4    Generalizations Using the Same Sharing Rule

Until now, we have assumed that (1) no costs were *shared* within a partnership and (2) no individual or partnership had the power to change the price of output. Even in the case of the Japanese fishermen, however, neither of these simplifications is entirely realistic. According to Platteau and Seki, some costs are shared among the members of each partnership. Moreover, although the primary reason why Japanese fisherman partition themselves into partnerships is to reduce congestion, a secondary reason is to raise prices: "Fishermen believe that by limiting effort they can cause fish prices to rise." Platteau and Seki's statistical analysis of price data confirmed this effect.

To relax these two maintained assumptions, we need only reinterpret our previous analysis. In addition, we show how partnerships can be valuable as a way to increase

Further evidence that pre-play communication can secure the payoff-dominant profile is provided by "cheap talk" experiments (Cooper et. al., 1992) where the players themselves communicate prior to play without benefit of a mediator. These cheap-talk experiments suggest that a cartel could coordinate on the payoff-dominant Partnership Solution even without a mediator.

payoffs even when the first-best is unattainable.

Suppose we partition $N$ homogeneous agents into $n$ payoff-sharing groups indexed by $i$, each playing a simultaneous-move game. Assume agent $k$ in group $i$ chooses $x_{ik}$ to maximize $\frac{1}{m_i}\left[x_{ik}+Y_i^{-k}\right]\cdot G(x_{ik}+Y_i^{-k}+X_{-i})-cx_{ik}$. If we make the same assumptions about $G(X)$ that we made about $A(X)$ then we will get the corresponding results. So assume that (1) $G(X)$ is strictly positive, strictly decreasing, and twice continuously differentiable; (2) $G(0)-c>0$; and (3) the Novshek (1985) condition, $G'(X)+XG''(X)<0$, holds for all $X\geq 0$. These assumptions are sufficient to insure the existence of a pure-strategy Nash equilibrium in the simultaneous-move game. Because $G(\cdot)$ is downward-sloping, there is a negative externality: agent $k$ is adversely affected by increases in $X_{-i}$. We have derived conditions sufficient for the aggregate payoff, $X(G(X)-c)$, to be maximized: provided $n^*\leq\lfloor N/2\rfloor$ and $\beta<\beta(n^*)$, the optimum can be achieved by setting up $n^*$ partnerships differing in size by at most one member.

Suppose $G(X)=A(X)-K$, where $K$ denotes cost per unit effort for those costs shared within the partnership. Then the Partnership Solution maximizes producer surplus. Since price is constant, this maximizes social welfare as well.

Next suppose $G(X)=P(F(X))A(X)-K$, where $P(\cdot)$ is the industry price when aggregate output $F(X)$ is put on the market. This generalization fits the case of the Japanese fishermen, who share some but not all costs and who use their partnerships not merely to curb congestion but to raise price. Again, the Partnership Solution maximizes producer surplus.

Finally, suppose $G(X)=P(X)-K$, where $X$ is now interpreted as *output* and

31

$K$ (respectively, $c$) as the cost per unit *output* rather than effort, which is shared (respectively, not shared) within the partnership. In this case, there is no congestion externality and hence no common property problem. The Partnership Solution can be used to curb excessive output and permits a cartel to maximize profits without any need for supergame strategies. An outside observer would simply see an industry with a collection of firms organized as partnerships in competition with one another.

In cases where $\beta > \beta(n^*)$, the advantages of team production are insufficient to achieve the first-best using the Partnership Solution. In such cases, a generalization of the Partnership Solution can nonetheless lead to a *second-best* equilibrium with a large increase in the aggregate payoff. To illustrate, recall the example where $N = 12, c = 3$, and $G(X) = 19 - X$. In that case $n^* = 4$ and $\underline{\beta} = .39$. Suppose that $\beta = .56 > .39$. Then dividing the agents into four partnerships of equal size is not feasible since each member would have an incentive to go solo. However, if the 12 agents are divided into six partnerships of equal size, then industry profit is $54.12—not the first-best level of $64 but approximately *triple* the result in the oligopoly (or common property) solution.

## 5 Generalizing the Sharing Rule

In any game, one must specify the feasible strategies of each player and the payoff each receives for any strategy profile. In each second-stage subgame, we assume that each member of a partnership anticipated that he would receive an equal share of the gross revenue accruing to the partnership, and that there is no opportunity for players to deviate from this sharing rule *ex post*. This can, most easily, be justified

by assuming that the sharing rule is contracted at the time of group formation, in which case agents are legally obligated to comply.[18] We now generalize our analysis to other sharing rules.

In the rent-seeking game of Nitzan (1991), the group winning the fixed prize divides it using a sharing function $S_{ik}$ which is an exogenous weighted average of our egalitarian rule and a rule rewarding relative effort:

$$S_{ik} = (1 - a)\frac{1}{m_i} + a\frac{x_{ik}}{(x_{ik} + Y_i^{-k})}, \tag{12}$$

where $a \in [0, 1]$.[19] We have so far restricted attention to the egalitarian case ($a = 0$) because it is simpler and does not require that members perfectly and costlessly monitor each other's efforts. When such monitoring is possible, however, we can show that our results generalize. For any $a \in (0, 1]$, it is straightforward to verify that any partition of players into groups generates a Nash equilibrium in the second stage in which every member of a larger group makes smaller effort and receives a smaller payoff. Therefore, as in our model, the groups formed at the first stage will differ in size by at most one member.[20]

---

[18]Another possible justification is that even if one of the partners were able to seize the entire allocation of the group, each partner would be equally likely to be the thief, and each would, in expectation, get an equal share of the total.

[19]Sen (1966) previously used this weighted-average rule when discussing sharing within cooperatives.

[20]As with the egalitarian rule, there will also be an incentive to deviate by going

If member $k$ of group $i$ makes effort $x_{ik}$, his payoff is $S_{ik}(x_{ik} + Y_i^{-k})A(X) - cx_{ik}$. Summing each player's best reply,[21] we obtain an equation linking effort to the number of groups and the weight on the relative effort component of the sharing rule:

$$KA(X) + XA'(X) - Nc = 0, \tag{13}$$

where $K = aN + (1-a)n$. If sharing is purely egalitarian ($a = 0$), this reduces to equation (6). As before, aggregate effort is strictly increasing in the number of groups and independent of the distribution of players among these groups. However, with this more general sharing rule, decreasing the weight ($a$) on relative effort provides a second channel through which to stimulate free riding and reduce aggregate effort.

Using our egalitarian sharing rule ($a = 0$), we previously found that aggregate effort could be reduced to the socially optimal level ($X^*$) by reducing the number of groups from $N$ to $n^*$ defined in Proposition 4. Equation (13) can be used to generalize the Partnership Solution when the sharing rule puts more weight on relative effort. Since $n^*$ groups would generate socially optimal effort ($X^*$) when $a = 0$, the number

solo unless solo production is either impossible or sufficiently costly compared to production with teams of two or more players.

[21]The best reply ($x_{ik}$) solves the following equation: $[(1-a)\frac{1}{m_i} + a]A(X) + [(1-a)\frac{1}{m_i} + a\frac{x_{ik}}{x_i}]x_i A'(X) - c = 0$, where $x_i$ denotes aggregate effort in partnership $i$. For any $a > 0$, this equation implies that in equilibrium every individual in the same group makes the same effort. This explains why we *assumed* equal efforts when $a = 0$ even though in that polar case individual effort levels in a group are indeterminate.

of groups $(n(a))$ which would achieve the social optimum when more weight is put on relative effort is:

$$n(a) = \frac{n^* - aN}{1 - a}.$$ (14)

As $a$ increases, $n(a)$ decreases until, at $a^* = \frac{n^*-1}{N-1}$, a single group is required to achieve $X^*$. Partnerships can solve the common property problem only if $a \in [0, a^*]$.

# 6  Conclusion

In this paper, we showed how the free-riding induced in partnerships can be harnessed to increase payoffs when aggregate effort or output would otherwise be excessive. We showed how this idea can be applied to curb excessive extraction from common properties or excessive production from cartels. The same mechanism can achieve the social optimum in innovation tournaments and rent-seeking contests with variable prizes. Inducing limited free-riding may be beneficial in other contexts as well. For example, tips in some restaurants are *separately* solicited by various team members whose combined effort makes a dining experience pleasurable: the maître d', the sommelier, the waiter, the musician, the coat-room provider, the parking valet, etc. In such situations, aggregate effort per customer may be excessive and pooling tips among subsets of these service providers (an increasingly common practice in restaurants) can be used to raise their net payoffs. Japanese fishermen who have formed partnerships report that pooling revenues reduces congestion and raises price. As we have seen, these are consequences to be expected from such partnerships.

As we have shown there is a temptation in the Partnership Solution to flee one's

free-riding partners by going solo. However, other forces often act as a counterbalance. Going solo is sometimes infeasible for technological or legal reasons and, when it is feasible, ceases to be attractive when there are sufficient benefits from team production, fixed costs of setting up a solo practice, or social benefits to remaining in existing groups. In such circumstances, the Partnership Solution can be used to maximize or at least to raise significantly the aggregate payoff.

Throughout, we assumed that a partnership had to admit every applicant. It might have been more realistic to assume that members of an existing partnership could deny admission to anyone if opposition to him within the partnership was "sufficiently widespread." This change in assumption would in fact have *increased* the scope of the Partnership Solution. For, every solution we identified as stable would continue to be stable since no one in such solutions has any incentive to join an existing partnership even when assured of admission. But partitions we identified as unstable under our old assumption would become stable under this new assumption. To illustrate, suppose going solo was infeasible and we set up $n^*$ non-solo partnerships some of which differed by two or more members. Such an arrangement could not achieve the first-best under our old assumption because every member of the largest partnership would deviate unilaterally to a smaller partnership with less free-riding. But this same arrangement *would* achieve the first-best under the new assumption since admitting him would be blocked unanimously by existing members who anticipated that expanding the number of partners would stimulate free-riding and would lower each of their payoffs.[22] In assuming that no applicant could be rejected by

---

[22]Either the anticipation of rejection by one's new colleagues or of having to carry

existing members, therefore, we *understated* the usefulness of partnerships in solving the common-property and cartel problems.

It is natural to ask if partnerships can ever solve or ameliorate the common property problem when agents differ in their constant marginal costs. The answer, surprisingly, is yes as the following example illustrates. Suppose N agents have marginal cost $c$ and are grouped into the socially optimal number of partnerships $(n^*)$. Assume these partnerships are of equal size. Suppose we now add $\sigma n^*$ more agents (where $\sigma$ is a positive integer) and assume they have a very high common marginal cost $(c_h >> c)$. Put $\sigma$ of them in each of the $n^*$ partnerships so that each group has the same composition. The social optimum is to have none of the new workers active and to generate the unchanged aggregate effort, $X^*$, from the more efficient workers. Suppose, for simplicity, that the common marginal cost of the additional agents is so high that they never have an incentive to work in the decentralized solution. Even so, their presence still has an effect. Because each more efficient worker now receives a smaller share of the gross revenue of his partnership, he has an incentive to work less hard. To enlarge aggregate effort, however, we could group individuals into a *larger* number of partnerships, each with fewer people. In this way, it may be possible to *approximate* the social optimum and, upon occasion, to duplicate it.[23] Note that

---

a disproportionate share of the new partnership's workload (see footnote 9) could explain why the pools of Japanese fishermen persist even with sizes differing by more than one member.

[23]It is straightforward to produce an example of a partnership equilibrium where agents are heterogeneous, *every agent* is active, and the social optimum is approxi-

since every partnership has the same composition, no agent would have any incentive to switch unilaterally to another partnership.

The issue of defections to solo partnerships, which we examined in the homogeneous case, continues to arise unless the technology makes solo partnerships untenable. As before, there may be more than one subgame-perfect equilibrium and the same issue of selecting the best one arises. For simplicity, suppose one of these equilibria replicates the social optimum. In the heterogeneous case, the two types of agents will disagree. The free-loaders will strictly prefer to have more than the socially optimal number of partnerships because that would generate more output; after all, someone else is expending the effort to produce that output. But since social welfare weakly declines when output expands, the low-cost workers would be made worse off with more than the socially optimal number of partnerships; after all, it is they who must work harder to expand the output. Hence, low-cost workers want fewer partnerships than high-cost workers. This conflict between heterogeneous members of the partnerships is reminiscent of conflicts identified in the literature on heterogeneous cartels and common properties although in contexts without sharing it is the higher cost agents which prefer reduced output.[24] A more complete analysis of partnerships with

mated. Simply reduce the higher marginal cost in our example until each freeloader makes a slight effort. Admittedly, it is impossible to *achieve* the social optimum when the high cost agents are active.

[24]In their classic paper, Johnson and Libecap (1982) argue that because of heterogeneity "limits on individual effort are extremely costly to agree to and enforce" in common property environments, such as fisheries. Cave and Salant (Section III,B,

heterogeneous agents must be left to future research.

———————————————————

1995), discuss how such cost differentials create similar tensions within cartels which vote on their quotas. Finally, Tarui (2007) finds, in an overlapping-generations framework relying upon punishment strategies, that heterogeneity in agent productivity reduces the ability of agents to achieve a sustainable harvest when harvests are shared equally amongst agents. Sherstyuk (1998), in a special case of partnerships with heterogeneous agents and no effort choice, and thus no shirking, finds that partnerships can be stable and efficient, but only when they are segregated by 'ability,' in contrast to the results suggested here.

# References

[1] James M. Acheson and Roy J. Gardner. Spatial strategies and territoriality in the maine lobster industry. *Rationality and Society*, 17(3):309–341, 2005.

[2] Armen A. Alchian and Harold Demsetz. Production, information costs, and economic organization. *American Economic Review*, 62(5):777–795, December 1972.

[3] Kyung Hwan Baik and Sanghack Lee. Strategic groups and rent dissipation. *Economic Inquiry*, 39(4):672–684, October 2001.

[4] Michael R. Baye and Heidrun C. Hoppe. The strategic equivalence of rent-seeking, innovation, and patent-race games. *Games and Economic Behavior*, 44:217–226, 2003.

[5] Andrea Bender, Wolfram Kägi, and Ernst Mohr. Informal insurance and sustainable management of common-pool marine resources in ha'apai, tonga. *Economic Development and Cultural Change*, 50(2), January 2002.

[6] Jean-Pierre Benoit and Vijay Krishna. Finitely repeated games. *Econometrica*, 53(4), 1985.

[7] Theodore C. Bergstrom and Hal R. Varian. Two remarks on cournot equilibrium. *Economics Letters*, 19(1):5–8, 1985.

[8] Jordi Brandts and W. Bentley MacLeod. Equilibrium selection in experimental games with recommended play. *Games and Economic Behavior*, 11:36–63, 1995.

[9] Colin F. Camerer. *Behavioral Game Theory : Experiments in Strategic Interaction.* The Roundtable Series in Behavioral Economics. Princeton University Press, Princeton, NJ, 2003.

[10] Donald Campbell. *Incentives: Motivation and the Economics of Information.* Cambridge University Press, Cambridge, 1995.

[11] Jonathan Cave and Stephen W. Salant. Cartel quotas under majority rule. *The American Economic Review*, 85(1):82–102, March 1995.

[12] Tai-Yeong Chung. Rent-seeking contest when the prize increases with aggregate efforts. *Public Choice*, 87(1-2):55–66, April 1996.

[13] Russell Cooper, Douglas DeJong, Robert Forsythe, and Thomas Ross. Communication in coordination games. *The Quarterly Journal of Economics*, 107:79–771, 1992.

[14] B. Curtis Eaton, Diane F. Eaton, and Douglas W. Allen. *Microeconomics, 5th Edition.* Prentice Hall, Toronto, 2002.

[15] David Feeny, Fikret Berkes, Bonnie J. McCay, and James M. Acheson. The tragedy of the commons: Twenty-two years later. *Human Ecology*, 18(1), 1990.

[16] H. Scott Gordon. The economic theory of a common-property resource: The fishery. *Journal of Political Economy*, 62:124–42, April 1954.

[17] Bengt Holmstrom and Jean Tirole. Theory of the firm. *Handbook for Industrial Organization, Volume 1*, 1989. Edited by Richard Schmalensee and Robert D. Willig, Elsevier Press.

[18] John B. Van Huyck, Ann B. Gilette, and Raymond C. Battalio. Credible assignments in coordination games. *Games and Economic Behavior*, 4:606–626, 1992.

[19] Ronald N. Johnson and Gary D. Libecap. Contracting problems and regulation: The case of the fishery. *The American Economic Review*, 72(5):1005–1022, December 1982.

[20] Wolfram Kägi. The tragedy of the commons revisited: Sharing as a means to avoid environmental ruin. IWOE Discussion Paper 91, Institute for Economy and the Environment, University of St. Gallen, August 2001.

[21] Shmuel Nitzan. Collective rent dissipation. *The Economic Journal*, 101(409):1522–1534, November 1991.

[22] W. Novshek. On the existence of cournot equilibrium. *Review of Economic Studies*, 52:85–98, 1985.

[23] Nuria Osés-Eraso and Montserrat Viladrich-Grau. On the sustainability of common property resources. *Journal of Environmental Economics and Management*, 53(3):393–410, May 2007.

[24] Jean-Phillippe Platteau and Erika Seki. Community arrangements to overcome market failures: Pooling groups in japanese fisheries. In M. Aoki and Y. Hayami, editors, *Market, Community, and Economic Development*. Clarendon Press, Oxford, 2000.

[25] Marshall Sahlins. *Stone Age Economics.* Aldine de Gruyter, New York, 1972.

[26] Thomas C. Schelling. *The Strategy of Conflict*. Harvard University Press, 1960.

[27] Stephan Schott, Neil Buckley, Stuart Mestelman, and R. Andrew Muller. Output sharing in partnerships as a common pool resource management instrument. *Environmental and Resource Economics*, 37(4):697–711, August 2007.

[28] Amartya K. Sen. Labour allocation in a cooperative enterprise. *Review of Economic Studies*, 33(4):361–371, 1966.

[29] Katerina Sherstyuk. Efficiency in partnership structures. *Journal of Economic Behavior and Organization*, 36:331–346, 1998.

[30] Nori Tarui. Inequality and outside options in common-property resource use. *Journal of Development Economics*, 83:214–239, 2007.