

Yield curve, time varying term premia, and business cycle fluctuations

Matteo Modena

University of Glasgow

May 2008

Online at <http://mpra.ub.uni-muenchen.de/8873/> MPRA Paper No. 8873, posted 26. May 2008 18:17 UTC

Yield Curve, Time-Varying Term Premia, and Business Cycle Fluctuations

Matteo Modena

University of Glasgow

First Draft: December 2007 **This Draft: April 2008**

Abstract

Using data for U.S. and Canada, we find evidence of the time-varying nature of risk premia, which are obtained as difference between long term interest rates and their expected values. We then apply Kalman filtering to extract the conditional variance of term premia prediction errors; our results highlight that this variable is informative beyond term premia and spreads, and it significantly improves upon prediction capability of standard models. In particular, the conditional variance of term premia, reflecting the high volatility of financial markets, anticipates movements in the output growth. Empirical evidence supports the inverse correlation between term premia and business cycle fluctuations. Data suggest that a deterioration of financial markets conditions, as captured by the increased volatility of term premia, anticipates a decline in the output growth. Therefore, term premia conditional volatility has an adverse effect on the economy.

 JEL classification: C01, C22, E32, E44, G12. *Keywords: Term Structure, Term Premia, Kalman Filtering, Industrial Production Growth*

1 Introduction

Forecasting future time series is a recurring theme in empirical economics. A large portion of the recent *macro-finance* literature suggests that financial variables have information content in predicting future economic activity. Financial market prices are appealing since financial markets are useful at distilling economic information. In particular, it has been argued that the yield spread, the difference between long term and short term interest rates, is a significant indicator of the future level of output (Stock and Watson, 1989; Harvey 1989; Estrella and Hardouvelis, 1991). Hence, if financial markets collect and process efficiently available information, as it is normally assumed, the yield curve summarizes quite accurately agents' expectations regarding the future stance of monetary policy, as reflected in bond prices. Therefore, the slope of the yield curve is regarded to influence output fluctuations because it mirrors agents' expectations about the incoming stance of monetary policy.

The expectations hypothesis of the term structure of interest rates states that the slope of the yield curve is capable of predicting future changes in interest rates. However, there is weak empirical support for the expectations theory and this has been attributed to time-varying term premia (Mankiw and Miron, 1986; Fama, 1986; Cook and Hahn, 1989; Lee, 1995; Tzavalis and Wickens, 1997; Hejazi and Li, 2000). Campbell and Shiller (1991) have shown that the yield spread can be seen as the sum between the expected changes in short term rates and a term premium, which is a function of maturities.

Recently, term premia have been shown to be relevant for predicting business cycle movements, where the term premium represents the difference between the yield spread and the *theoretical*, or *perfect foresight*, spread (see *inter alias*, Hamilton and Kim, 2002; Favero, Kaminska, and Soderstrom, 2005; Ang, Piazzesi, and Wei, 2006). In principle, large values of the yield spread are due either to the expected stance of monetary policy or to the effect of a risk premium, as long as investors do not like bearing risk in bad times. The decomposition of the yield spread into an expectations-based component and a risk premium allows examining separately the effect exerted on output by the expected stance of monetary policy and by risk aversion respectively. We believe that the aforementioned decomposition of the spread, although useful and appealing, can be further improved upon to obtain better forecast of the future level of economic activity.

In this work we suggest that financial distress, associated to the augmented volatility of term premia, rather than simply risk aversion, provides useful information to predict future output movements. Most of the existing literature has improved the forecasting model by incorporating macro variables in a reduced form empirical model (Evans and Marshall 2001; Favero, Kaminska, and Soderstrom 2005; Rudebusch, Sack and Swanson, 2007).

We find robust empirical evidence that the conditional variance¹ of term premia prediction errors, rather than term premia, is what matters to make effective inference regarding the future level of economic activity. We thus suggest that financial distress, incorporated in the conditional variance of term premia prediction errors, rather than risk aversion, as merely captured by the level of term premia, reflects agents' expectations regarding the future level of output.

From the left to the right the diagrams of Figure 1 show the yield spread, the term premium, and the conditional variance of the term premium prediction errors. The yield spreads decrease, and eventually become negative, before recessions (shaded areas). The term premium appears to anticipate a decline in real activity as well; it rises substantially before recessions. Finally, the conditional variance of term premia appears to be quite informative about business cycle fluctuations; in particular, the conditional variance rockets immediately before recessions.

In this paper we differ from previous works in many respects. Firstly, we provide evidence to support the existence of time-varying term premia². Second, we find evidence that the conditional volatility of term premia has information content for predicting of economic fluctuations. Hamilton and Kim (2002) argue that interest rate variability is an important determinant of both the yield spread and the term premium, but not of GDP cyclical movements. We augment their model by considering the conditional variability of term premia, which we find to provide useful information for predicting industrial production growth.

We believe that term premia volatility, rather than interest rate volatility, can explain future output fluctuations. We thus emphasize the role of risk aversion and financial distress as opposed to the

¹ We thus follow Engle's (1982) suggestion that the conditional variance, i.e. the variance conditional upon the information available at the time of forecasting, rather than the unconditional variance which instead is based on the whole sample, is what really matters for the behaviour of economic agents.

² Consistent with Pesando (1975), Engle, Lilien and Robins (1987), Tzavalis and Wickens (1997).

unpredictability of monetary policy as reflected by the variability of interest rates. Finally, in this work we use monthly data, rather than quarterly, because at higher frequency financial data provide a more accurate picture of markets' sentiment.

The aim of this work is to show that the conditional variance of term premia forecast errors is informative about the future level of real economic activity. For U.S. and Canada we find robust empirical evidence of inverse correlation between term premia conditional variance and future business cycle fluctuations. In particular, high conditional variance tends to predict lower growth of real output. The increasing volatility of term premia on financial markets is symptomatic of financial distress and anticipates a worsening of the economic conjuncture. A deterioration of financial markets conditions predicts a decline of the output growth. As a further robustness check, we show that including term premia conditional variance in a probit model augments significantly the probability of forecasting recessions.

This paper is organized as follows. In the next Section we present a survey of the empirical macrofinance literature. In Section 3 a preliminary battery of stability test is performed in order to show that term premia are time-varying. In Section 4 the Hansen stability test is presented in details. In Section 5, we describe the Kalman filtering approach. In Section 6 empirical results are discussed. In Section 7 we present results from a probit model. Section 8 concludes. All data are presented in *Appendix I*.

2 Literature Review

In a seminal work in the *macro-finance* literature Stock and Watson (1989, 1996) found that the interest rate spread can be regarded among leading economic indicators to predict output change. The usefulness of the yield spread for forecasting future economic activity has found extensive support afterwards. Estrella and Hardouvelis (1991), using U.S. average quarterly data, find that the slope of the term structure, as measured by the spread between the 10-year Treasury bond (T-bond) and the 3-month T-bill rates, is a good predictor of future real GDP growth. Moreover, they claim that the predictive accuracy of the spread for cumulative changes from 5 to 7 quarters ahead is quite impressive; in that horizon the spread explains more than one-third of the variations of future output changes. Estrella and Mishkin (1997) extended the analysis to some European countries. They gauge the effect of the spread on both output and inflation, concluding that the spread is a powerful tool in forecasting future inflation; in addition, using a probit model they provide evidence that the spread is capable of anticipating recessions with a significant positive probability. There is robust empirical evidence that the effect of the spread on future output growth is positive, so that a lower yield spread tends to predict slower GDP growth. The rationale works as follows. When the central bank tightens, the spread decreases and eventually becomes negative, as long as short term interest rates raise more than long term rates; the level of aggregate demand diminishes through the channel of monetary transmission, and future output falls. Dueker (1997) has shown that the yield spread, among leading indicators, is a relatively good predictor of recessions.

However, as pointed out by Feroli (2004), the predictive ability of the spread to forecast output fluctuations is contingent on the monetary authority's reaction function; the predictive power of the spread depends on the accuracy of the expectations about the future stance of monetary policy. Feroli thus proposes a small macro model that ties the predictive power of the term structure to the parameters of the monetary policy reaction function; simulation results show that, depending on the parameters' values, the model can account for the diminished predictive power of the spread after 1979.

Recently, researchers have also highlighted the role of term premia as predictors of future output growth. Hamilton and Kim (2002) provide a decomposition of the spread using ex-post observed short-term rates data series instead of ex-ante expected rates. The spread is thus split into two components: the expected future changes in short-term rates and a term premium. They show that both components help predict real GDP growth. The estimated effect of both components on future output growth is significantly positive. This empirical result is also confirmed by Ang, Piazzesi and Wei (2006). The distinction of these two components is important to obtain a clear understanding of the forecasting model. The spread is assumed to describe the expected stance of monetary policy; while term premia are related to economic agents' risk aversion. Hamilton and Kim suggest that interest rate volatility is not informative regarding the future GPD growth; however, interest rate volatility is said to be an important empirical determinant of both the spread and the term premium. Along the same line, Favero, Kaminska, and Soderstrom (2005) decompose the spread into an expectational component and a pure term premium, claiming that it allows a better understanding of the forecasting model. In addition, they show that adding some macroeconomic variables in a reduced form empirical model improves the forecasting ability of the spread. Using quarterly data, they find that the spread between 5-year and 3-month interest rates, and the term premia associated to those maturities, are reliable predictors of the GDP quarterly change. Consistently with previous findings, they provide evidence that a lower term premium predicts slower GDP growth. Kim and Wright (2005) employ a standard arbitrage-free dynamic latent factor term structure model to obtain a measure of risk premia. They ascribe the so-called conundrum, i.e. the decline in long term rates in response to a policy tightening action in 2004, to a fall in term premia. Wright (2006) investigates whether the yield spread and a measure of the term premium are useful predictors of recessions; he finds that the risk premium is able to predict recessions over a six-quarter horizon, but not from two to four quarters. Consistently with previous research, he remarks that a lower term premium raises the probability of a recession in the future. The probit model suggests that also the spread is a reliable instrument for predicting recessions; moreover, Wright claims that the inclusion of the policy rate in the model improves the forecasting power both in- and out-of-sample. Hejazi (2000) exploits the aforementioned decomposition and reconsiders the information content of the term structure predict fluctuations in real monthly industrial production. He argues that term premia are linearly related to the conditional variance of excess returns; therefore, he adopts a GARCH-M (GARCH-in-mean) model to analyse the role of conditional variances. Results suggest that interest rate variability is a significant empirical determinant of future level of the industrial production index; in particular, high interest rate variability can account for future contraction in the industrial production. In addition, he finds strong evidence that the spread between the 10-year T-bond and the 1-month T-bill is informative to gauge future movements in industrial production.

In our paper we show that the decomposition of the spread into an expectational factor and a term premium can be further improved upon to obtain better predictions of the output growth. In particular, we focus on the dynamic properties of risk premia. We propose a time-varying multifactor model for term premia; then we analyse whether the conditional variance of term premia forecast error enriches significantly the information set to predict business cycle fluctuations. We show that term premia conditional variance is an important empirical determinant of business cycle movements.

Our approach is based upon the variability of term premia. There exists substantial evidence that term premia are time-varying (Pesando, 1975; Fama, 1984; Campbell, 1987; Lee, 1995; Tzavalis and Wickens, 1997; Hejazi and Li, 2000). In particular, term premia variability over time has been suggested to justify the empirical failure of the expectations hypothesis (Mankiw and Miron, 1986; Fama, 1986; Cook and Hahn, 1989). The expectations theory has always found little empirical support indeed. According to the so-called Campbell and Shiller paradox (1991), the slope of the term structure does not return an accurate forecast of future changes in short-term rates, and gives a forecast in the wrong direction for the short-term change of long-term rates. It has been argued that the failure of the expectations hypothesis is due to the presence of a time-varying term premium. From this we find the rationale to examine term premia in a time-varying parameter model. As shown in the next Sections, results indicate a considerable instability of the parameters in the term premia equation over time. The inherent instability of the term premia equation is thus interpreted as a sign of financial distress. We find strong empirical evidence that factors related to term premia

time-instability, in particular the conditional variance, are quite informative regarding the future level of economic activity.

3 Time Variation in Term Premia and Stability Tests

Time variation in term premia is mentioned as a possible cause of the failure of the expectations hypothesis. Cook and Hahn (1986) remark this view; their explanation for the poor performance of the expectations hypothesis assumes small changes through time in the term premium. Term premia are defined in the following way. For any couple of maturities (*n*, *m*) term premia (*tp*) are simply the difference between the actual long term yield $(i_iⁿ)$ and its value implied by the expectations hypothesis, i.e. the average of expected short term yields (i_i^m) :

$$
i_t^n = \left(\frac{m}{n}\right) \sum_{q=0}^{n-m} E_t i_{t+mq}^m + t p_t^{n,m}
$$
 (1)

Throughout the paper, we have considered 120 and 60 months as long tem maturities (*n*); while 3, 6 and 12 as short term maturities. The empirical analysis is performed with data from January 1987 and June 2007 in two countries: United States and Canada. Samples are automatically adjusted as imposed by equation (1); as a consequence, the most recent observations are lost due to the effect of expected future observations of the short term yield. Yields data are presented in *Appendix 1*.

In Figure 2 we plot the time series of term premia $(tp_i^{n,m})$ with short term maturity $m = 3$ as obtained by (1). According to Hamilton and Kim (2002), term premia can be thought as the sum of a liquidity premium and a risk premium. Term premia are proxies for excess bond returns³. A mere visual inspection suggests the time-varying nature of term premia; term premia do not exhibit any stochastic or deterministic trend though.

 3 After adjusting for a scaling factor, term premia implied by (1) are identical to bond risk premia, or excess log returns, as in Cochrane and Piazzesi (2005).

Some descriptive statistics about term premia are reported in Table 1. Keeping the maturity of the short term yield constant, the mean is increasing with the maturity of the long term yield; similarly, given the maturity of the long term yield, the mean diminishes with the increase of the short term maturity. The standard deviation of term premia is higher at shorter horizons. With few exceptions, the highest standard deviation is displayed by term premia whose longer maturity is 36 months. These descriptive statistics are consistent with some stylized facts in bond pricing. Firstly, at long horizons investors require a positive liquidity premium, which is increasing with maturity. Secondly, the medium-short end of the yield curve is more volatile than the long end. Yields are quite volatile at short maturities; whereas, long term rates tend to be smooth and persistent.

Table 1

In Table 2 we report the results of the augmented Dickey-Fuller test to check for the presence of unit root in term premia time series. For any pair of maturities (*n*, *m*) the null hypothesis of unit root is rejected. Therefore, term premia turn out to be integrated of order zero, i.e. stationary. The KPSS test confirms these results: the null hypothesis of stationarity cannot be rejected.

Table 2

We introduce an important relation that links term premia, risk aversion, and the intertemporal rate of substitution between savings and consumption. Dynamic asset pricing theory is helpful to unveil the time-varying nature of term premia. The fundamental equation in asset pricing asserts that the price of a security is simply the discounted value of its expected future payoffs. Equation (2) states that the price, at time *t*, of *n*-period bond is simply the expected discounted value of its price oneperiod ahead. The superscript indicates the maturity of a bond. The stochastic discount factor (sdf,) is used to assign values to all the possible future state-contingent asset payoffs.

$$
p_t^n = E_t \left(s df_{t+1} \ p_{t+1}^{n-1} \right) \tag{2}
$$

Using the above formula we can express the price of the bond at time $t+1$ as the present value of its future payoffs $p_{t+1}^{n-1} = E_{t+1} \left[sdf_{t+2} p_{t+2}^{n-2} \right]$. Substituting in (2) yields:

$$
p_t^n = E_t \left[sdf_{t+1} E_{t+1} \left(sdf_{t+2} p_{t+2}^{n-2} \right) \right]
$$
 (3)

The law of iterated expectations, otherwise known as the tower property, allows iterating the process recursively forward, leading to the following expression:

$$
p_t^n = E_t \left(\prod_{q=1}^n s df_{t+q} \right) \tag{3.4}
$$

To obtain the above expression we employ the trivial relation $p_{t+q}^0 = 1$ ($\forall q$), i.e. the price of one dollar delivered at time *t* is merely one dollar. According to (4), the price of a bond thus depends upon the sequence of future stochastic discount factors along its entire life, i.e. till its maturity *n*. The stochastic discount factor is tied to the real economy through the marginal utility of consumption:

$$
sdf_{t+1} = \delta E_t \left(\frac{u^t(c_{t+1})}{u^t(c_t)} \right) \tag{5}
$$

where δ is the subjective discount factor; a parameter that describes the temporal preferences of the representative consumer. The lower δ , the lower the weight given to future consumption, and the more impatient the consumer. The utility function is increasing, so that it reflects the desire for more consumption. The concave shape of the utility function $u(\bullet)$ indicates aversion to both risk and intertemporal substitution; therefore, consumers usually prefer a smooth stream of consumption which is steady over time and across states of nature. The stochastic discount factor is also called the marginal rate of substitution, i.e. the rate at which the representative agent is willing to shift consumption from present to future, or the other way around. The demand for assets is thus determined by the relative convenience of saving to consuming. Using the inverse relation between bond prices and returns $(i^n_i = -(1/n) \log p^n_i)$, term premia $(tp^{n,m}_i)$ can be expressed as the summation of future stochastic discount factors (*^t sdf*):

$$
tp_t^{n,m} = i_t^n - \left(\frac{m}{n}\right) \sum_{q=0}^{\frac{n-m}{m}} E_t i_{t+mq}^m =
$$

= $-\frac{1}{n} \log p_t^n - \left(\frac{m}{n}\right) \sum_{q=0}^{\frac{n-m}{m}} E_t \left[\left(-\frac{1}{m}\right) \log p_{t+mq}^m \right] =$
= $-\frac{1}{n} \log E_t \left(\prod_{q=1}^n s df_{t+q} \right) + \left(\frac{1}{n}\right) E_t \left(\sum_{q=1}^{\frac{n-m}{m}} \log E_{t+mq-m} s df_{t+mq} \right)$ (6)

In general the stochastic discount factor is responsive to a great variety of shocks that hit the economy, such as monetary and fiscal shocks, but also technological and institutional changes. These different types of shocks have implications for the determination of output and other economic variables. Therefore, the derivation of the term premium in (6) provides the theoretical justification for modelling term premia by means of a multifactor model. Rudebusch et al. (2007) point out that the relation between term premia and output growth depends on the nature of shocks that drives the change in term premia. In the light of these considerations, our time-varying approach to term premia, developed in Section 5, appears to be particularly effective because it deals with the unpredictable nature of disturbances that affect macroeconomic and financial variables. The term premia conditional variance suggests that there are two sources of uncertainty in our model, one is due to future idiosyncratic disturbances; one arises from the evolutionary behaviour of regressing coefficients in (7).

In this analysis we assume that term premia to be a function of some macroeconomic variables that have been recognized to be important determinants⁴ in the macro-finance literature. The multifactor model for term premia is:

$$
tp_t^{n,m} = \beta_0 + \beta_1 \, rate_t + \beta_2 \, unemp_t + \beta_3 \, inf_t^{epi} + \beta_4 \, spread_t^{120,3} + e_t^{n,m} \tag{7}
$$

The above equation has been estimated for any combination of maturities (*n*, *m*) and for all countries (U.S. and Canada). Different statistical and econometric tests have been performed on the linear term premia equation in order to demonstrate its instability. Firstly we performed a test against the alternative hypothesis of unstable regression coefficients. Brown, Durbin, and Evans (1975) proposed two tests; one based on the cumulative sum of recursive residuals, one based on

⁴ According to (7) term premia depend on a constant, the policy interest rate, unemployment, the inflation rate, and the slope of the term structure. In Section 5 we justify the specific functional form of equation (7); in addition, we describe how term premia are modelled using a time-varying approach.

the cumulative sum of the square of recursive residuals. In the former, the test statistics is the ratio between the sum of recursive residuals and the residuals sum of squares from the full-sample regression. In the latter, by splitting the entire sample into some arbitrary nonoverlapping subsamples, the statistics is calculated as the ratio of the between-group over within-group mean squared residuals. Under the null hypothesis of stable coefficients, the tests statistics is distributed like an *F* with $(kp - k, T - kp)$ degrees of freedom, where *k* is the number of parameters, *p* is the number of nonoverlapping sub-samples, ad *T* is the whole sample size. Test results reject the null hypothesis in favour of the alternative of unstable coefficients in the term premia equations⁵.

The first row reports tests results for U.S., while the second row refers to Canada. These results are obtained for the selected pair of maturity ($n = 60$, $m = 3$). The panels on the left in each row of Figure 3 show the plot of residuals; at a first sight they seem to be serially correlated and heteroscedastic. In the second chart the recursive residuals are shown to break the standard errors bands. In the third and fourth panels the cumulative sum and cumulative sum of squared residuals are plotted respectively. All these tests reveal coefficients instability in the above linear equation (7). A visual inspection of the correlogram confirms the presence of autocorrelation in the residuals; in addition, the test performed using the Ljung-Box Q-statistics leads to the rejection of the null hypothesis of absence of serial correlation. Moreover, in any linear regression the Durbin-Watson statistics falls by far below 2, denoting positive serial correlation in the residuals.

 $⁵$ In Figure 3.2 we report the results for some combinations of maturities in different countries. For all the remaining</sup> couples of maturities results are similar (available upon request).

The particular time-varying nature of the regression coefficients in (7) deserves further investigation. Engle and Watson (1985) suggested the presence of a unit root for the coefficients in case of structural change when agent adjusts their estimation of the state after that new information becomes available. Therefore a statistical test can be performed to test whether or not coefficients follow a random walk stochastic process. Under the alternative hypothesis of random walk coefficients, the residuals from an ordinary least squares regression have a particular heteroscedastic form. Breusch and Pagan (1979) and Godfrey (1978) propose a method to check whether residuals from an ordinary least squares regression are heteroscedastc. The Breusch and Pagan (1979) test is used also to check whether coefficients are subject to random variation, i.e. follow a random walk stochastic process. Under the null hypothesis of stable coefficients, one half times the explained sum of squares from an OLS regression of $(\hat{e}_i^2 / \hat{\sigma}_e^2)$ onto $(t \cdot x_i^2)$ $(t \cdot x_t^2)$ is distributed like a *chi-square* with *k* degrees of freedom, where *k* is the number of explanatory variables⁶ in x_t . The higher the value of the regression (explained) sum of squares, the more highly correlated the independent variable with the error variance, and the less likely the null hypothesis of homoscedasticity (stable coefficients) to hold. The null hypothesis of stable coefficient is decisively rejected for equation (7), for any pairs of maturities and for any country. The calculated Breusch and Pagan test statistics is generally very large. As pointed out in Section 5, on the basis of these results the transition matrix of the state equation in the state-space form model will be a diagonal matrix. In addition, the White test reinforces the above conclusion: the null hypothesis of homoscedasticity is rejected in all cases.

Finally, the ARCH test is performed on the residuals obtained from equation (7); for all combinations of maturities very strong evidence of the ARCH effect was found. To determine the source of the ARCH effect we checked whether the serial correlation still remains after the timevarying parameter estimations. After Kalman filtering⁷ the term premia in the macro-finance setting, both the forecast errors and the squared forecast errors turn out to be serially uncorrelated. Table 3 reports the OLS estimates of the first-order autoregressive model of the squares of forecast errors. Estimates suggest that the autocorrelation coefficient is not significantly different from zero 8 .

⁶ The explanatory variables in the auxiliary regression may be a function of the independent variables in the main regression from which residuals are taken.

⁷ Details are discussed in Section 5.

⁸ In Table 3 regression results refer only to few couples of maturities. For all the remaining pairs of maturities results are similar and available upon request.

Table 3

If we had found any serial correlation after running the time-varying parameter estimation, we might have suspected that the ARCH effect was due to reasons other than evolutionary behaviour of the coefficient in the term premia model. The null hypothesis of absence of serial correlation cannot be rejected, thus the existence of the ARCH effect in the linear term premium function is mainly due to the evolutionary pattern of the regressing coefficients. In this Section stability tests for the regression coefficients in (7) provide a clear rationale for time-varying parameter modelling the term premia. In the next Section we perform an additional test to show that term premia at different maturities are a time-variant function of the state of the economy, as captured by some fundamental macroeconomic variables.

4 The Hansen Stability Test

Hansen has proposed a test to check for parameter constancy in linear models. Differently from the Chow test, the main advantage of the Hansen test is that no prior knowledge about the structural break is required. Again, eventual parameter instability in equation (7) provides a rational to model term premia using a time-varying model. The Hansen test overcomes some drawbacks of the CUSUM and CUSUM of squares proposed by Brown, Durbin, and Evans (1975). In particular, the former has been criticized for being a trivial test to detect instability in the intercept of a model; the latter suffers from poor asymptotic power. Hansen has proposed a test which has locally optimal power. The variables in the linear equation must be weakly dependent process, i.e. they cannot contain any deterministic or stochastic trend. The residuals from the ordinary least squares regressions must be stationary as well. In our context we rewrite equation (7) in the following way:

$$
tp_i^{n,m} = \beta^l x_i + e_i^{n,m} \tag{8}
$$

where x_t is the matrix of regressors in (7). Usual conditions must hold. The disturbance term has zero mean $E(e_t | x_t) = 0$. The second moment is $E(e_t^2) = \sigma^2$. Zero covariance between noise and the explanatory variables $E(x_i^I e_i) = 0$. Equation (8) is estimated by ordinary least squares. The vector of parameter estimates is $(\hat{\beta}^{\prime}, \hat{\sigma}^2)$. Residuals from (8) are

$$
\hat{e}_t^{n,m} = \hat{e}_t = tp_t^{n,m} - x_t^I \hat{\beta}
$$
\n
$$
\tag{9}
$$

Rewriting the first-order conditions in a slightly different way yields:

$$
\sum_{t=1}^{T} x_{it} \hat{e}_t = 0 \tag{10}
$$

$$
\sum_{t=1}^{T} \left(\hat{e}_t^2 - \hat{\sigma}^2 \right) = 0
$$

Defining a new variable f_i :

$$
f_{it} = \begin{cases} x_{it}\hat{e}_t \\ \hat{e}_t^2 - \hat{\sigma}^2 \end{cases}
$$
 (11)

Expressions (10) becomes equivalent to:

$$
\sum_{t=1}^{T} f_{it} = 0 \tag{12}
$$

the variables f_i are the first-order conditions, and are akin to the *score* in the maximum likelihood estimation. The Hansen test statistics are based on the cumulative sums of the f_i , namely:

$$
S_{it} = \sum_{t=1}^{T} f_{it} \tag{13}
$$

Two versions of the tests are available. To check for individual parameter stability the test is based on the following statistics:

$$
Li = \frac{1}{TV_i} \sum_{t=1}^{T} S_{it}^2
$$
 (14)

where V_i is the cumulative sum of f_i^2 . Asymptotic critical values for the individual parameter stability test are given by Hansen (1992). At 5% significance level the critical value is 0.47; the 10% critical value is 0.353. Large values of the test statistics (*Li*) implies a violation of the firstorder conditions, and thus suggest rejection of the null hypothesis of parameter stability. The *Li* test by proposed by Hansen is similar to the *t*-test to assess significance of individual parameter of an OLS regression.

The test statistics to assess joint parameter stability is:

$$
Lc = \frac{1}{T} \sum_{t=1}^{T} s_t^I V^{-1} s_t \tag{15}
$$

where $s_t = (S_{1t}, S_{2t},..., S_{k+1,t})$, $f_t = (f_{1t}, f_{2t},..., f_{k+1,t})$, and $V = \sum_{t=1}^{T}$ *t* $V = \sum_{t=1}^{T} f_t f_t^T$. Under the null hypothesis of parameter constancy, the first-order conditions are mean zero, thus the cumulative sum tend to be distributed around zero. Under the alternative hypothesis of parameter instability, the cumulative sum does not have zero mean and the test statistics tends to assume large values. Therefore, the distribution is not standard and is tabulated by Hansen (1992). There are six explanatory variables in model (7) including both the constant and the errors variance. At 5% significance level the critical value is 1.68, while the 10% critical value is 1.49. The null hypothesis of joint parameter stability is rejected if the test statistics exceeds the critical values. The Hansen joint test for parameter stability reminds of the *F*-test to assess the joint significance of parameters in an ordinary least squares regression. Hansen reveals "*if a large number of parameters are estimated,…, the joint significance test is a more reliable guide*".

In Table 4 we report the results of the Hansen test for the combinations of maturities we are going to deal with later on⁹. The top part of the table refers to U.S., while the bottom row refers to Canada. Test results suggest clear parameter instability. Therefore the Hansen test reinforces results we found in the previous Section, and provides a strong argument to model term premia in a timevarying framework as we are going to do in the next Section.

5 A Time-Varying Parameter Model

In this Section we present the Kalman filter model for term premia. We recall that our main concern is to determine whether term premia and their dynamic properties are informative about future business cycle fluctuations. As pointed out in Section 3, term premia are obtained from an application of the expectations theory as implied by the Campbell and Shiller equation (1). In particular, the term premium is the difference between the long term rate implied by the expectations hypothesis and the effective long term rate $(i_iⁿ)$; where *m* denotes the short term maturity, and *n* the long term maturity. According to the expectations hypothesis *tp* in (1) should be a constant term premium, which is simply a function of maturities (*n*, *m*), but not a function of time. Unfortunately, the empirical investigation of the expectations theory has been unsuccessful, and the hypothesis has almost always been rejected. One possible explanation for the empirical failure of the EH is the presence a time-varying term premium (Mankiw and Miron, 1986; Fama, 1986; Cook

⁹ The results for other pairs of maturities are similar (available upon request).

and Hahn, 1989). In this work we take into account this possibility, and assume a time-varying macroeconomic structure for term premia. It is interesting to show the decomposition the yield spread into an expectational component and a term premium:

$$
i_t^n - i_t^m = \left(\frac{m}{n} \sum_{q=0}^{m-m} E_i i_{t+q}^m - i_t^m \right) + \left(i_t^n - \frac{m}{n} \sum_{q=0}^{n-m} E_i i_{t+q}^m \right)
$$
(16)

The second part on the RHS of the above decomposition is the term premium. Term premia in (16), can be viewed, for instance, as the sum of a liquidity premium and a risk premium. An extensive analysis of term premia is carried out in Section 3, where we have shown that the term premium is a function of the stochastic discount factor.

There is substantial evidence of the time-varying behaviour of term premia (Pesando, 1975; Fama, 1984; Campbell, 1987; Lee, 1995; Hejazi and Li, 2000); moreover, financial economists use to attribute the lack of empirical support for the expectations hypothesis to time variation in term premia.

Therefore, we adopt a time-varying model to analyse the macroeconomic foundations of term premia. The Kalman filter has been largely used in economics and finance because it is a convenient and practical way to describe how agents process information as new pieces of it become available; this filter uses all available information and takes into account how agent form their expectations, updating continuously their knowledge in a Bayesian fashion. Kalman filtering is suitable to depict how rational economic agents would revise their estimates of the coefficients when new information becomes available.

We thus propose a time-varying multifactor model for risk premia. Term premia are assumed to be a time-varying function of the policy interest rate¹⁰, unemployment, inflation, as measured by the annual change in the CPI index, and the slope of the term structure, i.e. the spread between the 10 year and the 3-month yields¹¹. If follows a brief explanation. As long as risk premia are a component of the yield spread, they are believed to depend on the stance of monetary policy, as captured by the policy rate¹² which exerts an important effect on the short end of the term structure. In addition, Hamilton and Kim (2002) show that the interest rate variability is a determinant of term

 10 The Canadian interest rate is the overnight rate. The policy rate for U.S. is the effective federal funds rate.

 11 U.S. term premia seem not to be sensitive to the CPI inflation and to the spread (120, 3), so that these variables have been replaced by the PPI inflation and the effective exchange rate.

 12 In this framework we do not allow explicitly for either the credibility of the monetary regime or the reputation of the monetary authority; both aspects are regarded to influence the level of risk premia required by investors. We believe that the time-varying pattern of coefficients captures implicitly these two effects.

premia. Unemployment affects term premia through risk aversion. Backus and Wright (2007) provide evidence of the cyclical behaviour of term premia. Their findings are in line with the thesis that high levels of unemployment are associated to high premia required by economic agents. According to Cochrane (2005) "*an asset that does badly in states of nature like a recession, in which the investor feels poor and is consuming little, is less desirable than an asset that performs badly in states of nature like a boom in which the investor feels wealthy and is consuming a great deal*". Inflation is another important determinant of term premia as long as economic agents aim at preserving the real value of their financial investments. Ang and Bekaert (2002, 2006) show that the positive slope of the term structure is due to a inflation risk premium indeed. Finally, Lee (1995) emphasizes the role of the yield spread in explaining the magnitude and the variability of risk premia. It follows the empirical specification of equation (7) for Canada and U.S.

$$
tp_{t,CAN}^{n,m} = f_{CAN}(x_{t,CAN}) = f_{CAN}(const, rate_t, \text{ unemp}_t, \text{ in } fl_t^{cpi}, \text{spread}_t^{120,3})
$$
\n
$$
(7. a)
$$

$$
tp_{t,US}^{n,m} = f_{US}\big(x_{t,US}\big) = f_{US}\big(const, ffr_t, \text{ unemp}_t, \text{ in } fl_t^{ppi}, \text{ eer}_t\big) \tag{7.b}
$$

The state space form of the Kalman filter is represented by two basic equations. The observation equation, or the measurement equation, is:

$$
tp_{t,j}^{n,m} = d_j + x_{t,j} \beta_{t,j} + e_{t,j}^{n,m}
$$
 (17)

The observation equation relates the dependent variable to the explanatory variables; the subscript *j* indicates the country, U.S. and Canada respectively. $e_{t,j}$ is a country specific stochastic disturbance *i.i.d.* $(0, \sigma_{\varepsilon})$. The state, or transition, equation captures the evolution of coefficients over time:

$$
\beta_{t,j} = \mu_j + F_j \beta_{t-1,j} + v_{t,j} \tag{18}
$$

 $v_{t,j}$ is an idiosyncratic noise *i.i.d.* $(0, \sigma_{v})$. On the basis of results obtained in Section 3, we assume that each of the regression coefficients follows a random walk; matrix F in equation (18) is thus the identity matrix. The Kalman filter is an iterative algorithm which we summarize here by means of by the following expressions:

$$
P_{t|t-1} = E[(\beta_t - \beta_{t|t-1})(\beta_t - \beta_{t|t-1})^t]
$$
\n(19)

Equation (19) represents the variance-covariance matrix of the coefficients conditional on information up to $t-1$; equation (20) is the forecast of the term premium based on information available up to time $t-1$; equation (21) represents the prediction error, while equation (22) is its conditional variance.

$$
tp_{t|t-1} = x_t \beta_{t|t-1} \tag{20}
$$

$$
\eta_{t|t-1} = tp_t - x_t \beta_{t|t-1} = tp_t - tp_{t|t-1}
$$
\n(21)

$$
h_{t|t-1} = E[\eta_{t|t-1}^2] = x_t P_{t|t-1} x_t^1 + \sigma_{\varepsilon}^2
$$
\n(22)

One of the major features of the Kalman filter is that two sources of uncertainty characterize the conditional variance of the forecast error $(h_{t|t-1})$: one form of uncertainty is due to the evolutionary behaviour of estimated coefficients, the other is a random noise associated to future unpredictable disturbances, such as political, institutional, or technological shocks. As shown in Section 3, risk premia are a function of the expected path of the stochastic discount factor, which is regarded to respond to a variety of shocks. Hence, the assumption of a constant variance of nominal shocks to term premia within a country over time does not seem realistic; the variance conditional upon available information at the time of forecasting is assumed to be time-varying due, for instance, to a continuously changing regime, as captured by evolutionary behaviour of β coefficients, or to some unpredictable shocks that hit the economy, as captured by the stochastic noise. One of the major features of Kalman filtering is that expectations are adjusted continuously, since they are changeable over time depending on the state of the economy. A quick look at Figure 4 shows the improvement of adopting a time-varying parameter model (bottom) rather than a fixed coefficient regression (top)¹³. We plot U.S. and Canadian term premia on the left panel and on the right panel respectively.

¹³ We report the actual and fitted values of term premia for all countries but only for some pairs of maturities. U.S. top panel (120, 12) and bottom panel (60, 3). Canada top panel (120,6) and bottom (60,3). For all the remaining couples of maturities results are similar.

Estimates are run for the period from January 1987 to June 2007; samples are adjusted automatically as indicated in the last column of Table 1. In the above figures the actual and fitted values of term premia are plotted. In the top panel we plot results from a fixed coefficient regression, while in the bottom figures results from the tvp model are plotted. It is immediate to notice that a time-varying parameter (henceforth tvp) model returns by large a better fit. In addition, residuals obtained with the fixed coefficients regression are serially correlated, while the errors from the tvp model are not. The goodness of fit of the tvp model is remarkably higher than the one returned by model (7). Results of the tvp Kalman filter coefficients estimates for term premia are reported in Figure 5. Estimation results indicate that the variability of all macro-finance variables in the multifactor models (7.a) and (7.b) affect significantly the term premia dynamics¹⁴. Only in few cases macroeconomic variables are not statistically significant, as expected the plot of coefficients over time, and the respective standard error bands, fluctuates around zero.

¹⁴ We report here estimations and plots of the time-varying parameter estimates only for few pairs of maturities. For the remaining combinations of maturities results are similar.

Table 5

U.S. tvp estimates in Figure 5.a indicate that the influence of the federal funds rate on term premia has clearly decreased over time (left chart). As expected the effect of the unemployment change on risk premia is significantly positive though decreasing over time, as shown in the second panel. PPI inflation does not seem to be a significant determinant of term premia; finally, the effect of the effective exchange rate has been extremely variable over time. The shaded areas indicate two recessions: from quarter two of 1990 to quarter one of 1991, and from quarter one of 2001 to quarter one of 2002.

In Figure 5.b, starting from the left we show the effect over time on the term premium (60, 3) of the Canadian interest rate, the change in unemployment, the CPI inflation and the spread (120, 3). Canadian estimates show a dramatic reduction of the inflation component over time, with a sharp drop after the introduction of the inflation targeting regime in 1991. Differently from U.S. estimates the effect of unemployment on term premia is negative. The tvp estimates of the spread reveals a structural break in 1992, which is captured by the dynamics of the conditional variance as shown in the central panel of Figure 6 below.

In Section 3 we proved that the tvp model is correctly specified for any country; as a consequence the forecast errors obtained by Kalman filtering are serially uncorrelated for any pair of maturities. We recall that the ARCH test was performed on the fixed coefficient version (7) of the multifactor model for term premia; the test revealed the presence a strong ARCH effect. This effect could be due to reasons other than the mere evolutionary behaviour of the regressing coefficients in the equation (17). We thus show that the absence of serial correlation in the (squares of) forecast errors from the tvp multifactor model (17) rules out this possibility; the tvp model (17) does seem to be correctly specified. For the pair of maturities ($n = 60$, $m = 3$) we plot both the forecast errors and the associated conditional variances of term premia obtained from Kalman filtering (U.S. and Canada).

6 Empirical Results

In the previous Section we have examined the time-varying features of term premia. In this Section we investigate whether they are informative about future business cycle fluctuations. The U.S. seasonally adjusted industrial production series is from the FRED database of the Federal Reserve; while the Canadian seasonally adjusted industrial production series are from the IMF database, available from *Datastream¹⁵* .

We adopt a standard model to examine whether some financial indicators are informative about future movements of real economic activity. Let $IP_{t,j}$ denote the level of the seasonally adjusted industrial production index in country *j* at time *t*, and $y_{t+T,j}$ the average annualized growth over the period *t* to *t+T*

$$
y_{t+T,j} = \frac{12}{T} \log \left(\frac{IP_{t+T,j}}{IP_{t,j}} \right) * 100
$$
 (23)

The time index *T* indicates the forecast horizon ($T = 3, 6, 12, 24, 36$ months); in this study we have considered forecast horizons from one quarter to three years ahead. The basic models are the following:

$$
y_{t+T,j} = \alpha_0 + \alpha_1 h_{t|t-1,j} + \alpha_2 \eta_{t|t-1,j}^{n,m} + \alpha_{3a} t p_{t,j}^{n,m} + \alpha_{3b} t h s p_{t,j}^{n,m} + v_{t,j}
$$
(24)

$$
y_{t+T,j} = \alpha_0 + \alpha_1 h_{t+1,j} + \alpha_2 \eta_{t+1,j}^{n,m} + \alpha_3 \text{spread}_{t,j}^{n,m} + u_{t,j}
$$
 (25)

where *j* is the subscript indicating the country (U.S. and Canada). $h_{t|t-1}$ is the conditional variance of term premia prediction errors discussed in the previous Section; $\eta_{t|t-1}$ are term premia forecast errors; $tp_t^{n,m}$ is the term premium, and $thsp_t^{n,m}$ is the expectational component, i.e. the theoretical, or *perfect foresight*, spread, according to the Campbell and Shiller (1991) terminology. We recall the actual spread between long term (*n*) and short term (*m*) interest rates can be decomposed into the sum of a term premium ($tp_t^{n,m}$) and an expectations-based factor ($thsp_t^{n,m}$).

Quite a few issues deserve attention when estimating regressions (24) and (25) by ordinary least squares. Firstly, the Newey and West (1987) correction must be imposed to deal with overlapping

¹⁵ Data are described in *Appendix* 1.

nonspherical disturbances. Secondly, Mishkin (1982) and Pagan (1984) pointed out that generated regressors in the above equation might influence the distribution of test statistics, and, consequently, invalidate the inference procedure to verify parameters' significance. In order to prove our results are robust we have estimated different specifications of the above regressions. The augmented equations include more explanatory variables such as the policy interest rate, the effective exchange rate, and the nominal bilateral exchange rates between the two considered economies¹⁶. Results are definitely robust to different model specifications. In addition we can count on a sufficiently large number of available observations¹⁷. Third, the functional form of coefficient α_2 has also been chosen to avoid any potential multicollinearity problem in equations (24) and (25). Following Kim and Nelson (1989) coefficient α_2 has been set to be a function of the term premium conditional variance: $\alpha_{2,j} = \phi_{0,j} + \phi_{1,j} \ln(h_{t_{t+1,j}})$. Both the two-step estimation procedure and the joint estimation confirm results are robust¹⁸. Finally, as shown in the following equations, the actual value of the industrial production growth has been included in order to show that the financial indicators are robust also to the inclusion of a real variable.

$$
y_{t+T,j} = \alpha_0 + \alpha_1 h_{t+1,j} + \alpha_2 \eta_{t+1,j}^{n,m} + \alpha_{3a} t p_{t,j}^{n,m} + \alpha_{3b} t h s p_{t,j}^{n,m} + \alpha_4 y_{t,j} + v_{t,j}
$$
(26)

$$
y_{t+T,j} = \alpha_0 + \alpha_1 h_{t|t-1,j} + \alpha_2 \eta_{t|t-1,j}^{n,m} + \alpha_3 \text{spread}_{t,j}^{n,m} + \alpha_4 y_{t,j} + u_{t,j}
$$
(27)

In the following tables we report empirical results for the U.S. economy. Our main result is that the conditional variance of term premia appears to be a powerful predictor of the industrial production growth. The negative sign of coefficient α_1 reveals that high values of the conditional variance of term premia forecast errors are associated to low expected growth in economic activity. Therefore, financial distress, as reflected in an excessive variability of term premia dynamics, tends to anticipate a future slowdown in real activity. Moreover, results show that it is a deeper analysis of the term premium that allows a better understanding of the forecasting model, and not the mere decomposition of the spread into a term premium and an expectational component, as claimed by Favero et al. (2005). In particular, if we compare the goodness of fit from regressions on the left

¹⁶ The exchange rates are U.S. – U.K. and Canada – U.K. Details are given in Appendix I.

¹⁷ Depending on the pair of maturities considered (n, m) the lowest number of observation is N = 117, so that statistical inference is based on distributions with 112 degrees of freedom. When the long term maturity is 60-month inference is based on statistics with 177 degrees of freedom.

 18 This functional form has been chosen only to prove our results are robust. Our analysis emphasizes the role of term premia conditional variance; so that, if we drop the level of forecast errors from the equations, we do not loose any significant information and we obtain similar results.

column with the one from regressions on the right we do not notice any significant difference. The adjusted- R^2 increases substantially when we add the variables that capture the volatility of term premia. We thus claim that the inclusion of a new financial variable, i.e. the conditional variance, leads to a considerable improving of the forecasting model. The conditional variance of term premia might be interpreted as a sign of financial fragility; it measures how current uncertainty affects future output growth. In addition, also coefficient α_2 turns out to be negative and statistically significant; so that business cycle movements are inversely related not only to the volatility of term premia, as captured by the conditional variance, but also to the magnitude of prediction errors.

Table 6

Term premia forecast errors and the associated conditional variance can anticipate movements in real activity up to three years ahead. However, if we consider the pair of maturities ($n = 120$, $m =$ 12), the magnitude of the estimated coefficients α_1 and α_2 progressively diminishes with the extension of the forecasting horizon.

Many authors have documented that lower term premia tend to predict slower GDP growth, since the estimated α_{3a} coefficient turn out to be positive¹⁹. We claim that this is contrary to common

¹⁹ Hamilton and Kim (2002); Favero, Kaminska, and Soderstrom (2005); Ang, Piazzesi, and Wei (2006).

wisdom, as long as risk aversion should affect output negatively. In accordance with Rudebusch, Sack, and Swanson (2007), our results show that over a 24- and 36-month horizons high term premia tend to anticipate slowdown in economic activity. Term premia thus seems to be inversely correlated with the business cycle. Finally, the coefficient of the yield spread α_3 is positive over horizons of six to twelve months, but turns to negative when the forecasting horizon enlarges. This empirical fact has an important macroeconomic interpretation. Large values of the spread are typically associated to accommodative stance of monetary policy and stimulus to real economic activity. This effect fades away within one year though. Over longer horizons agents expect an inversion in the conduct of monetary policy and thus a subsequent decline in real activity. In Table 7 we reports results for U.S. when the term premium is computed using the pair of maturities ($n =$ 60, $m = 3$). Results are similar.

Table 7

The variability of financial market sentiment displays a significant negative effect on the economic conjuncture ($\alpha_1 < 0$; $\alpha_2 < 0$). Again we point out that term premia are inversely related to the business cycle; therefore, a decline in term premia tends to stimulate economic activity. The effect

of conditional variance seems robust to different specifications; for instance, it does not vanish after the inclusion of the current level of output growth²⁰.

So far we have examined the effect on output exerted by term premia obtained from the entire length of maturity spectrum of the term structure ($n = 120, 60$; $m = 6, 3$). If we focus on the medium and short end of the yield curve $(n = 36, 24)$ results are not so encouraging. In particular, when the long term rate is $n = 36$, the estimated coefficient α_1 , which describes the effect on output by the conditional variance of term premia, is informative about business cycle only over short forecasting horizon, i.e. from one to two quarters. The magnitude of the coefficient is quite high though; $\hat{\alpha}_1 = 0.84$ when *T* is 3 months, and $\hat{\alpha}_1 = 0.67$ when *T* is 6 months.

Estimates for the Canadian economy return similar results as shown in the Tables below. The predictive ability of term premia conditional variance is significantly negative. Differently from U.S., Canadian estimates suggest that the effect of conditional variance on the output growth becomes more intense at longer forecasting horizons. Prediction errors are not statistically significant. In Canada the coefficients of term premia is positive; term premia thus tend to anticipate faster growth in industrial production.

Equations (24) and (25) have also been estimated on the sample between the two recessions: from April 1991 to December 2000. The coefficients of the conditional variance remain statistically significant but decrease in magnitude. This result may reflect the slowdown in industrial production in the mid 1990s.

Our results do not prove any direct influence running from financial markets to the real economy; however, empirical evidence suggests that financial markets do anticipate future movements in real activity. In this paper we emphasize the signalling role played by uncertainty without insinuating any clear causality implication for the real economy. We can only speculate that, when increased uncertainty is reflected by bond markets, agents heavily discount expected future events in current prices through the stochastic discount factor. In addition, the changing conditions on financial markets due to greater uncertainty might be accompanied by further perverse behaviours, such as adverse selection and moral hazard, which, in turn, may contribute to worsening the expectations about future economic conjuncture.

 20 Our results are comparable with those obtained by Schwert (1989), who analyses the effect of stock market volatility on economic growth. He shows that stock market volatility is particularly high during recessions, and thus it is an important cyclical indicator. Focusing on bonds market we find evidence that term premia volatility contains valuable information to predict business cycle fluctuations.

Table 9

In this paper we have developed an innovative method to extract valuable information from financial markets; and we show its usefulness to make inference about the future level of economic growth. Our approach highlights the role of term premia unpredictability in forecasting future industrial production growth. Data suggest that term premia conditional volatility signal incoming future adverse effect on the economy; this might happen when rational agents think to bear an unnecessary high risk. We have provided evidence that the conditional variance of term premia is an important cyclical indicator; in particular it helps in predicting the future evolution of industrial production.

7 Forecasting Recessions: a Probit Model

 \overline{a}

In this Section we consider two alternative probit models to forecast recessions. The dependent variable is a dummy assuming value one during NBER recessions (shaded areas in the figures below). The notation has been introduced in previous sections. $\Psi(\bullet)$ is the standard Normal cumulative distribution function.

$$
p(recession_t = 1) = \Psi(\alpha_0 + \alpha_3 spread_{t,j}^{n,m})
$$
\n(28)

$$
p(recession_t = 1) = \Psi(\alpha_0 + \alpha_1 h_{t|t-1,j}^{n,m} + \alpha_2 \eta_{t|t-1,j}^{n,m} + \alpha_3 spread_{t,j}^{n,m})
$$
\n(29)

We believe that the forecasting performance of model (29), which includes the conditional variance of term premia prediction errors $(h_{t|t-1}^{n,m})$ $\binom{n,m}{|t-1|}$, is superior to the predictive performance of model (28). Therefore we show that uncertainty and financial distress, as reflected by the term premia conditional volatility, are informative to forecast imminent recessions. In both equations the actual value of the industrial production growth has been included. We report results for both U.S. and Canada for the pairs of maturities²¹ ($n = 120$, $m = 6$) –top- and ($n = 60$, $m = 3$) –bottom-.

²¹ The U.S. combination of maturities when the long term rate is 120 months is $(m = 120, n = 12)$, as explained in previous Sections.

Table 10.b

Results show a clear dominance of model (29) over model (28). The McFadden measure of goodness of fit is definitely larger for model (29). In addition, the Bayesian information criteria (Akaike, Schwarz, and Hannan-Quinn) for model (29) are lower compared to those associated to model (28). Finally, model (29) returns a much higher probability of forecasting a recession (shaded area) before it takes place, as shown in the figures below²². The red line represents the probability of recession associated to model (29), while the blue line the one associated to model (28). The green line represents the industrial production rate of growth. The probability model (29) is also able to anticipate the sharp drop in industrial production in mid 1990s, which is not predicted by model (28). Both the Theil inequality coefficient²³ and the root mean squared error support the better forecasting performance of model (29). Results are particularly significant for U.S.

 22 Figure 7 reports probability for the pair of maturities (120,6); while Figure 8 reports the probability from the model estimated on maturities (60,3).

 23 The Theil inequality coefficient, by construction, lies between zero and one, where zero indicates perfect fit.

Figure 7

In addition, to prove our results are robust, we have compared the predictive ability of models (29) with model (28) augmented with the federal funds rate, as suggested by Wright (2006). Our results show that the model specification we propose (29) is definitely more informative to predict future recessions.

8 Conclusion

In this paper we have confirmed that the information content of the yield curve is relevant for predicting future output growth. After decomposing the yield spread into an expectational component and a term premium, we have analysed the time-varying behaviour of term premia. We proposed a dynamic multifactor model for term premia in order to give a deeper interpretation of the term premium effect on output growth.

We have showed that adding the conditional variance of the term premium to the traditional equation for predicting real economic activity leads to a considerable improvement in the forecasting model. In particular, we have gone beyond the mere analysis of risk aversion by

providing evidence that uncertainty and financial distress are important elements to anticipate future business cycle movements.

We have provided evidence that term premia conditional variance is inversely correlated with future growth in the industrial production index; therefore, high values of term premia conditional variance tend to predict slower output growth. Results are robust for both U.S. and Canada; the predictive ability of term premia conditional variance is significant over horizons from six months to three years.

Finally, in contrast with previous studies, we find evidence of inverse correlation between term premia and future output growth; we find that a rise in the term premium is thus associated with a decline in future real economic activity.

References

.

Ang A., Bekaert G., 2003, *The Term Structure of Real Rates and Expected Inflation*, Columbia University and NBER Working Papers.

Ang A., Piazzesi M., 2003, *A No-Arbitrage Vector Autoregression of Term Structure Dynamics with Macroeconomic and Latent Variables*, Journal of Monetary Economics, Vol. 50, n. 4, 745-787.

Ang A., Piazzesi M., **Wei M.**, 2006, *What Does the Yield Curve Tell Us about GDP Growth?*, Journal of Econometrics, Vol. 131.

Backus K. D., Wright J. H., 2007, *Cracking the Conundrum*, Brooking Papers on Economic Activity, 1:2007.

Bagliano F., Favero C.A., 1997, *Measuring Monetary Policy with VAR Models: an Evaluation*, European Economic Review, Vol. 42, 1069-1112.

Bernanke B., 1990, *On the Predictive Power of Interest Rates and Interest Rate Spreads*, NBER Working Papers.

Breusch T.S., Pagan A.R., 1979, *A Simple Test for Heteroscedasticity and Random Coefficients Variation*, Econometrica, Vol. 47, 1287 - 1294.

Campbell J.Y., 1987, *Stock Returns and the Term Structure*, Journal of Financial Economics, Vol.18, 373 – 399.

Campbell J.Y., Shiller R.J., 1991, *Yield Spreads and Interest Rate Movements: A Bird's Eye View,* The Review of Economic Studies, Vol. 58, n.3, 495-514.

Caporale G.M., Pittis N., 1998, *Term Structure and Interest Differentials as Predictors of Future Inflation Changes and Inflation Differentials*, Applied Financial Economics, Vol. 8, 615-625.

Cochrane J.H., 1999, *New Facts in Finance*, Economic Perspecives XXIII, Fed of Chicago.

Cochrane J.H., 2005, *Asset Pricing*, Princeton University Press.

Cochrane J.H. Piazzesi M., 2005, *Bond Risk Premia*, The American Economic Review, Vol. 95, n. 1, 138- 160.

Cook T., Hahn T., 1989, *Interest Rate Expectations and the Slope of the Money Market Yield Curve*, Federal Reserve Bank of Richmond.

Cook T., Hahn T., 1989, *The Effect of Changes in the Federal Funds Rate Target on Market Interest Rates in the 1970s*, Journal of Monetary Economics, Vol. 24, 331-351.

Dueker M.J., 1997, *Strengthening the Case for the Yield Curve as a Predictor of U.S. Recessions*, Fed of Saint Louis Review, March/April.

Engle R.F., 1982, *Autoregressive Conditional Heteroscedasticity with Estimates of the Variance of United Kingdom Inflation*, Econometrica, Vol. 50, 987 - 1007.

Engle R.F., Lilien D.M., Robins R.P., 1987, *Estimating Time-Varying Risk Premia in the Term Structure: the ARCH-M Model,* Econometrica, Vol. 55, 391 - 407.

Engle R.F., Watson M.W., 1985, *Application of Kalman Filtering in Econometrics*, unpublished report, World Congress of the Econometric Society. Cambridge, MA.

Estrella A., Hardouvelis G.A., 1991, *The Term Structure as a Predictor of Real Economic Activity*, The Journal of Finance, Vol. 46, n. 2, 555-576.

Estrella A., Mishkin F.S., 1997, *The Predictive Power of the Term Structure of Interest Rates in Europe and the United States: Implications for the European Central Bank*, European Economic Review, Vol. 41, 1375-1401.

Estrella A., Mishkin F.S., 1999, *Predicting U.S. Recessions: Financial Variables as Leading Indicators,* NBER Working Papers.

Evans C.L., Marshall D.A., 2007, *Economic Determinants of the Nominal Treasury Yield Curve*, Journal of Monetary Economics, Vol. 54, n. 7, 1986 – 2003.

Fama E.F., 1984, *The Information in the Term Structure,* Journal of Financial Economics, Vol. 13, 509-528.

Fama E.F., 1984, *Term Premium in Bonds Returns,* Journal of Financial Economics, Vol. 13, 529-546.

Fama E.F., 1986, *Term Premiums and Default Premiums in Money Markets,* Journal of Financial Economics, Vol. 17, 175-196.

Fama E.F., Bliss R.R., 1987*, The Information in Long Maturity Forward Rates*, The American Economic Review, Vol. 77, n. 4, 680-692.

Favero C.A., 2001, *Does Macroeconomics Help Understand the Term Structure of Interest Rates?*, Paper prepared for the Conference "EMU Macroeconomic Institutions and Policies".

Favero C.A., 2006, *Taylor Rules and the Term Structure*, Journal of Monetary Economics, Vol. 53, 1377 - 1393.

Favero C.A., Kaminska I., Soderstrom U., 2005, *The Predictive Power of the Yield Spread: Further Evidence and a Structural Interpretation*, CEPR Discussion Paper.

Feroli M., 2004, *Monetary Policy and the Information Content of the Yield Spread*, Federal Reserve Board, Washington D.C., Finance and Economics Discussion Series.

Hamilton J.D., Kim D.H., 2002, A *Reexamination of the Predictability of Economic Activity Using the Yield Spread*, Journal of Money, Credit, and Banking, Vol. 34, 340 – 360.

Hansen B.E., 1992, *Testing for Parameter Instability in Linear Models, Journal of Policy Modeling*, Vol. 14, n. 4, 517 – 533.

Hardouvelis G.A., 1988, *The Predictive Power of the Term Structure During Recent Monetary Regimes*, The Journal of Finance, Vol. 43, n. 2, 339-356.

Hardouvelis G.A., 1994, *The Term Structure Spread and Future Changes in Long and Short Rates in G-7 Countries: Is There a Puzzle?*, Journal of Monetary Economics, Vol. 33, 255-283.

Hejazi W., Li Z., 2000, *Are Forward Premia Mean Reverting?*, Applied Financial Economics, Vol. 10, 343 – 350.

Hejazi W., 2000, *Yield Spreads as Predictors of Industrial Production: Expectations on Short Rates or Term Premia?*, Applied Economics, Vol. 32, 945 - 951.

Hejazi W., Lai H., Yang X., 2000, *The Expectations Hypothesis, Term Premia, and the Canadian Term Structure of Interest Rates*, Canadian Journal of Economics, Vol. 33, 133 – 148.

Kim C.J., Nelson C.R., 1989, *The Time-Varying-Parameter Model for Modeling Changing Conditional Variance: the Case of the Lucas Hypothesis*, Journal of Business & Economic Statistics, Vol. 7, 433 – 440.

Kim D., Wright J.H., 2005, *An Arbitrage-Free Three-Factor Term Structure Model and the recent Behaviour of Long-Term Yields and Distant-Horizon Forward Rates*, Fed Working Paper.

Kozicki S., Sellon G., 2005, *Longer-Term Perspectives on the Yield Curve and Monetary Policy*, Fed of Kansas City Working Paper.

Lee S.S., 1995, *Macroeconomic Sources of Time-Varying Risk Premia in the Term Structure of Interest Rates,* Journal of Money, Credit and Banking, Vol. 27, n.2, 549-569.

Mankiw N.J., Miron J.A., 1986*, The Changing Behaviour of the Term Structure of Interest Rates*, The Quarterly Journal of Economics, Vol. 101, n. 2, 211-228.

Miskin F.S., 1982, *Does Anticipated Monetary Policy Matters? An Econometric Investigation*, The Journal of Political Economy, Vol. 90, 22 – 51.

Miskin F.S., 1988, *The Information in the Term Structure: Some Further Results*, Journal of Applied Econometrics, Vol. 3, n. 4, 307-314.

Miskin F.S., 1988, *What Does the Term Structure Tell Us About Future Inflation?*, NBER Working Paper.

Pagan A., 1984, *Econometric Issues in the Analysis of Regressions with Generated Regressors*, International Economic Review, Vol. 25, 221 – 247.

Pesando J.E., *Determinants of Term Premiums in the Market for United States Treasury Bills*, The Journal of Finance, Vol. 30, 1317 - 1327.

Rudebusch G.D., Wu T., 2004, *A Macro-Finance Model of the Term Structure, Monetary Policy, and the Economy*, Fed of San Francisco Working Paper.

Rudebusch G.D., Sack B.P., Swanson E.T., 2007, *Macroeconomic Implications of Changes in the Term Premia*, Fed of St Louis Review.

Schwert G. W., 1989, *Business Cycles, Financial Crises, and Stock Volatility*, Carnegie Rochester Conference Series on Public Policy, Vol. 31, 83-126.

Stock, J.H., Watson M.W., 1989, *New Indexes of Coincident and Leading Indicators*, In Blanchard O., Fischer S., NBER Macroeconomics Annual, Vol. 4, MIT Press.

Thornton D.L., 2003, *Tests of the Expectations Hypothesis: Resolving the Campbell-Shiller Paradox*, Fed of St. Louis Working Paper.

Tzavalis E., Wickens M. R., 1997, *Explaining the Failures of the Term Spread Models of Rational Expectations Hypothesis of the Term Structure*, Journal of Money, Credit and Banking, Vol. 29, n. 3, 364- 380.

Wright J.H., 2006, *The Yield Curve and Predicting Recessions*, Fed Working Paper.

Appendix I

All data have monthly frequency; the sample starts in January 1987. The core econometric analysis, after Kalman filtering, is thus performed from January 1988 in order to rule out the first 12 observations.

Industrial Production and Unemployment. The U.S. series of seasonally adjusted industrial production is from the FRED database (Federal Reserve Economic Data). The Canadian series of seasonally adjusted industrial production is from the IMF database (available from *Datastream*). The U.S. seasonally adjusted unemployment rate series (civilian unemployment) is from the FRED database; the source is the U.S. Department of Labour (Bureau of Labour Statistics). The Canadian unemployment rate series (seasonally adjusted percentage of civilian labour force) is from the OECD database (available from *Datastream*).

Blue lines are used for U.S. data, while red lines for Canadian data; shaded areas indicate periods of (NBER) recession. Plots from the left to the right in the above figure refer respectively to: the (log) industrial production index; the annual change in the log-IP index; the unemployment rate and, finally, the annual change in the unemployment rate. The above diagrams clearly show that during recessions there is both a dramatic reduction of the industrial production index and a sharp increase of the unemployment rate. The log-industrial production growth and the unemployment rate are covariance stationary as suggested by both the augmented Dickey-Fuller test and the Kwiatkowski-Phillips-Schmidt-Shin test.

The ADF test rejects the null hypothesis of unit root; while the null hypothesis of stationarity cannot be rejected by the KPSS test. To match the monthly frequency of data, the rule of thumb selected number of lags in the auxiliary regression is either 11 or 12. The automatic lag selection based on different criteria (Akaike, Schwarz, Hannan-Quinn) is consistent with our choice. Unit root test results obtained with the automatic lag selections are similar. The critical values of the KPSS test are 0.739 (1%), 0.463 (5%), and 0.347 (10%) when the intercept is included in the auxiliary model. The compute KPSS statistics never falls in the critical region.

Interest Rates. U.S. yields data are from different sources. Before January 1999 the 3-and 6-month, and the 10-year yields are from the McCulluch database, while the yields associated to the remaining maturities (1-, 2-, 3-, 5-year) are from the Fama and Bliss CRSP database, as reported by Cochrane and Piazzesi (2005). After 1999 U.S. data series are the ZCB yield from *Datastream*. The U.S. effective federal funds rate is from the FRED database. Yields data for Canada are from the central Bank of Canada. The Fibor (before January 1999) and the Euribor (afterwards) is from *Datastream*. Precisely, Fibor is the Germany Interbank 3-month offered rate; while Euribor is the 3 month offered rate. In the Figure below we plot the series of the U.S. federal funds, the Canadian overnight rate, and the Fi-Euribor.

The KPSS statistics critical values are 0.216 (1%), 0.146 (5%) and 0.119 (10%) when both the intercept and a trend are included in the test equation. The statistics reported in the table above indicate that the null hypothesis of stationarity cannot be rejected, since the empirical statistics are lower than the critical values.

Spreads. According to both the ADF and the KPSS tests he yield spreads are stationary.

Bold values refer to the specific spreads used in the analysis of business cycle (Sections 6 and 7).

Exchange Rates. The nominal bilateral exchange rates series between U.S. Dollar and both the Canadian and U.K. currencies are from the FRED database. The nominal bilateral exchange rate between the Canadian Dollar and the U.K. Sterling has been derived from the two above series. The plots below show the annual change in the nominal bilateral exchange rates between the three considered economies. The ADF test confirms that these series are stationary.

The KPSS statistics critical values are 0.739 (1%), 0.463 (5%) and 0.347 (10%) when only the intercept is included in the auxiliary equation. The statistics reported in the table above indicate that the null hypothesis of stationarity cannot be rejected, since the empirical statistics are lower than the critical values.