The Use of Indices in Surveys

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Abstract. The paper deals with some new indices for ordinal data that arise from sample surveys. Their aim is to measure the degree of concentration to the "positive" or "negative" answers in a given question. The properties of these indices are examined. Moreover, methods for constructing confidence limits for the indices are discussed and their performance is evaluated through an extensive simulation study. Finally, the values of the indices defined and their confidence intervals are calculated for an example with real data.

Key words: multinomial proportions, ordinal data, indices, confidence intervals, sample surveys

1. Introduction

Various types of indices are widely used in real world applications. Some fields where the use of indices is widespread are index numbers (see e.g., Mudgett (1951)), statistical quality control (see e.g., Kotz and Lovelace (1998) and Montgomery (1997)), economics (see e.g., Cowell (1995)), fundamental analysis (see e.g., Ritchie (1996)) and sample surveys (see e.g., Bnerjee et al. (1999)).

In the area of sample surveys, questions requiring answers that have a somewhat natural ordering are frequently included. A common example of such type of answers is "Very Good", "Good", "Moderate", "Bad" and "Very Bad". In practice, the presentation of the observed proportions of the possible answers of such questions is restricted to frequency tables, graphs (bar and pie charts) and some coefficients such as Cohen's (1960) Kappa and its modifications (see e.g., Bnerjee et al. (1999) and Doner (1999)). A detailed presentation of categorical data analysis can be found in Agresti (1990). However, no measure of the potential concentration of the positive or negative answers is used.

In this paper we introduce some indices that can be used to measure this concentration, based on the observed proportions of the answers. In Section 2 we define three alternative indices, we examine the properties of these indices and compare their behavior. The third section deals with the construction of confidence intervals for the true values of the indices. In particular, some methods for assessing simultaneous confidence intervals for multinomial proportions are reviewed briefly. These methods can be implemented for constructing confidence intervals for one of the indices defined in Section 2. Furthermore, three bootstrap methods applied to these indices (standard, percentile and bias corrected percentile) are illustrated and their use for obtaining confidence intervals for the indices is also described. The results of a simulation study aimed at testing the performance of the bootstrap confidence limits for the indices are shown in detail in Section 4. From these results it is observed that the coverage of these bootstrap confidence limits is very satisfactory, since it is quite close to the nominal, in most of the cases. An illustrative example that clarifies the assessment of the indices and their corresponding confidence intervals is given in Section 5. Further topics on the indices are presented in the last section.

2. Definition and Properties of Indices

Consider a question in a study where the person who answers has to choose one out of k possible answers. These answers have a natural ordering and thus can be ordered from "positive" to "negative" ones. We assume that the number of "positive" answers is equal to the number of "negative" ones. Let p_i, $i = 1, 2, \ldots, k$ denote the observed percentage (%) of answers in each of the k categories, where p_1 refers to the "best" available answer, and p_k to the "worst" one. Obviously, the "neutral" answer, if such an answer exists (i.e. if k is odd), is located at point $\left[k/2\right] + 1$. We should remark that among the k possible answers we include the "neutral" answer (if it exists), but we do not take into consideration answers of the kind "No opinion/No answer". If such a type of answer exists, we recalculate the observed proportions excluding this answer and we proceed using the theory developed in the following sections. In what follows, we define three alternative indices.

2.1. INDEX I_1

Let p_0 denote the quantity $\left\{\left[k/2\right] \cdot (1/k)\right\} 100$. We define an index I_1 as

$$I_1 = \frac{\sum_{i=1}^{\left[\frac{k}{2}\right]} p_i}{p_0} = \frac{p_+}{p_0}$$

where in the numerator we have the sum of the percentages (%) of "positive" answers, represented by p_+, and in the denominator the value of p_0 is equal to the expected percentage (%) of the "positive" answers assuming that all the answers are uniformly distributed (i.e. each answer is chosen with the same frequency). The use of the integer part for the computation of the "positive" answers ensures that regardless of whether the number of available answers is odd or even we include all the "good" answers in the computation of I_1.

For illustration let us assume that we have a question with five possible answers, which are "very good" (25%), "good" (20%), "moderate" (30%), "bad" (10%) and "very bad" (15%) (in the parenthesis we have the observed percentages of each answer). Then $k = 5$, $p_0 = 40$, $p_+ = 45$ and therefore $I_1 = 1.125$.

Index I_1 can take values between 0 and $100/p_0$. When the index takes the value 0 it means that none of the given answers are among the $[k/2]$ "positive" answers. On the other hand, when I_1 takes the value $100/p_0$, all of the given answers are among the $[k/2]$ "positive" answers. A value close to unity is an indication that the number of positive answers is close to what we would expect if the answers are uniformly distributed. It is obvious that I_1 has always a finite value. In addition, it is easy to compute confidence intervals for this index, not only via bootstrap, but also by using some methods for simultaneous confidence intervals for multinomial proportions as well (see Section 3.1). On the other hand, index I_1 ignores "negative" and "neutral" answers neglecting the information of these answers.

2.2. INDEX I_2

We define an index I_2 as follows

$$I_2 = \frac{p_+}{p_-}$$

where p_+ is defined as in I_1 and p_- is the sum of the percentages of the $\left[k/2\right]$ "negative" answers. For the previous example and for the index I_2 we have that $k = 5$, $p_+ = 45$, $p_- = 25$ and $I_2 = 1.8$.

Index I_2 takes values between 0 and infinity. A value 0 means that nobody has answered one of the $[k/2]$ "positive" answers, whereas an infinite value means that everyone has selected one of the $[k/2]$ "positive" answers. A value close to unity is an indication that the number of positive answers is similar to that of the negative ones. Values greater than unity show a tendency towards the positive answers, whereas values smaller than unity show a negative concentration. The fact that I_2 can become infinite is a disadvantage, even though this is an extreme case. Another drawback of this index is that it excludes the "neutral" answer. Also, as explained in Section 3, the construction of confidence intervals for I_2, without resorting to bootstrap, is a difficult task, since it requires knowledge of the distribution of ratios of multinomial proportions. However, I_2 is superior to I_1 because it takes into account "negative" answers and, at the same time, its calculation is fairly easy.

2.3. INDEX I_3

The third index that we consider is the index I_3, defined as

$$I_3 = \frac{p_+ + p_n}{p_- + p_n}$$

where p_+, p_- are defined as previously and p_n is the percentage (%) of the "neutral" answers. In the example of Section 2.1 $I_3 = 1.36$, since $k = 5$, $p_+ = 45$, $p_- = 25$ and $p_n = 30$.

The values that I_3 can take, lie between 0 and infinity. I_3 takes the value 0 when everyone has answered one of the ”negative” answers and is equal to infinity when everyone has chosen one of the “positive” answers. The interpretation of this index is similar to that of the index I_2. A disadvantage of I_3 is the difficulty in constructing confidence intervals for it. The only way to overcome this problem with I_3 is to use the method of bootstrap. Also, its value is not finite in some cases. However, this is not a very probable scenario. The advantage of I_3 is that it takes into account every category in its calculation, a property that makes this index preferable to the previous two.

2.4. INTERRELATION OF THE THREE INDICES

The following relations hold for the three indices defined:

- I_1 is greater (smaller) than I_2 if p_- is greater (smaller) than p_0.
- I_2 exceeds I_3 if $p_+ > p_-$ (or equivalently if $I_2 > 1$) and vice versa.

3. Confidence Intervals

This section is devoted to the construction of confidence intervals for the indices defined. These indices are functions of multinomial proportions. Therefore, the construction of confidence intervals for these indices can be based on the construction of simultaneous confidence limits for multinomial proportions. This is a problem that many authors have dealt with and is described briefly in Section 3.1. However, such confidence intervals can be used only in connection to I_1. Confidence intervals for I_1 can also be obtained using the binomial distribution since regardless of the number of categories considered, we end up with two categories - the “positive” and the “rest” (see Section 3.1.). The construction of parametric confidence intervals for indices I_2 and I_3, which are ratios of sums of multinomial proportions, is much more complicated and cannot be based on the existing theory. For this reason we resort to the well-known method of bootstrap for obtaining such limits for them.

3.1. PARAMETRIC CONFIDENCE INTERVALS FOR INDEX I_1

A first attempt for constructing simultaneous confidence limits for multinomial proportions was made by Quesenberry and Hurst (1964). They concluded that one can obtain simultaneous confidence intervals for the actual proportions (probabilities) (π_i, $i = 1, \ldots, k$) using the formula

$$\frac{1}{2\left(n + \chi^2_{k-1,1-\alpha}\right)} \times$$

$$\left(\chi^2_{k-1,1-\alpha} + 2X_i \pm \sqrt{\chi^2_{k-1,1-\alpha}}\sqrt{\chi^2_{k-1,1-\alpha} + \frac{4}{n}X_i\,(n - X_i)}\right), \tag{1}$$

where n is the total number of answers corresponding the specific category, $\chi^2_{k-1,1-\alpha}$ denotes the $(1-\alpha)100\%$ percentile of the chi-square distribution with $k-1$ degrees of freedom and X_i is the observed number of answers in category i (i.e. $X_i = p_i \cdot n/100$).

Goodman (1965), proposed a modification of the previous interval. More specifically, he found that the confidence interval (1) becomes shorter if one substitutes $\chi^2_{1,1-\alpha/k}$ for $\chi^2_{k-1,1-\alpha}$. Hence, according to Goodman (1965), simultaneous confidence intervals for π_i's can be obtained through the formula

$$\frac{1}{2\left(n + \chi^2_{1,1-\alpha/k}\right)} \times \left(\chi^2_{1,1-\alpha/k} + 2X_i \pm \sqrt{\chi^2_{1,1-\alpha/k}}\sqrt{\chi^2_{1,1-\alpha/k} + \frac{4}{n}X_i\,(n - X_i)}\right). \tag{2}$$

Fitzpatrick and Scott (1987) suggested the use of the interval

$$\frac{p_i}{100} \pm \frac{d}{\sqrt{n}}. \tag{3}$$

The value of d depends on the desired coverage and it has to be 1 for 90% coverage, 1.13 for 95% coverage and 1.4 for 99% coverage.

Sison and Glaz (1995) proposed another method for constructing simultaneous confidence intervals for multinomial proportions. This method is much more complicated than the three methods described so far. However, as Sison and Glaz (1995) point out their method achieves coverage closer to the nominal in comparison to the coverage that the intervals (1), (2) and (3) achieve. A short description of this method is given here. The method of Sison and Glaz (1995) (see also Glaz and Sison (1999)) leads to confidence intervals of the form

$$\left(\frac{p_i}{100} - \frac{c}{n},\ \frac{p_i}{100} + \frac{c + 2\gamma}{n}\right), \tag{4}$$

where

$$\gamma = \frac{(1-\alpha) - v\,(c)}{v\,(c+1) - v\,(c)}$$

and c is an integer such that

$$v\,(c) < 1 - \alpha < v\,(c+1)$$

and finally

$$v(c) = P\left(X_i - c \leq X_i^* \leq X_i + c; i = 1, \ldots, k\right). \quad (5)$$

Here, $\left(X_1^*, \ldots, X_k^*\right)$ follows a multinomial distribution with parameters n, $\frac{p_1}{100}, \ldots, \frac{p_k}{100}$. In order to find the value of c, Sison and Glaz (1995) showed that (5) can be rewritten as

$$\frac{n!}{n^n e^{-n}} \left\{ \prod_{i=1}^{k} P\left(X_i - c \leq V_i \leq X_i + c\right) \right\} f_e \left(\frac{n - \sum_{i=1}^{k} \mu_i}{\sqrt{\sum_{i=1}^{k} \sigma_i^2}} \right) \frac{1}{\sqrt{\sum_{i=1}^{k} \sigma_i^2}},$$

where $V_i, i = 1, \ldots, k$ are independent Poisson random variables with parameters np_i and the function $f_e(x)$ is defined as

$$\begin{aligned} f_e(x) \;=\; & \left(\frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}} \right) \\ & \times \left[1 + \frac{\gamma_1}{6}\left(x^3 - 3x\right) + \frac{\gamma_2}{24}\left(x^4 - 6x^2 + 3\right) \right. \\ & \left. + \frac{\gamma_1^2}{72}\left(x^6 - 15x^4 + 45x^2 - 15\right) \right], \end{aligned}$$

where

$$\gamma_1 = \frac{\sum_{i=1}^{k} \mu_{3,i}}{\left(\sum_{i=1}^{k} \sigma_i^2\right)^{3/2}},$$

and

$$\gamma_2 = \frac{\sum_{i=1}^{k} \left(\mu_{4,i} - 3\sigma_i^4\right)}{\left(\sum_{i=1}^{k} \sigma_i^2\right)^2}$$

μ_i, σ_i^2 and $\mu_{3,i}$, $\mu_{4,i}$ are the expected values, the variances and the central moments of the truncated Poisson distribution with mean $np_i/100$, to the interval $[X_i - c, X_i + c]$. These central moments can be assessed using a formula for the factorial moments of the truncated Poisson distribution provided by Sison and Glaz (1995). May and Johnson (1997) studied the performance of various methods for simultaneous confidence intervals for multinomial proportions and concluded that the methods of Goodman (1965) and Sison and Glaz (1995) are superior.

The $100(1 - a)\%$ confidence interval of index I_1 based on any of the preceding methods is given by

$$\left(\frac{100 \cdot \sum_{i=1}^{[k/2]} p_L^{(i)}}{p_0}, \frac{100 \cdot \sum_{i=1}^{[k/2]} p_U^{(i)}}{p_0} \right), \quad (6)$$

where $p_L^{(i)}$, $p_U^{(i)}$ are the lower and the upper simultaneous confidence limits for category i, using any of the previous methods.

Since, in index I_1, the k categories are separated into only two groups - the first group consisting of the positive answers and the second of the remaining answers - we may compute exact confidence intervals for it using the existing theory for binomial proportions (see e.g., Johnson et al. (1993)). Using the property that

$$\sum_{x=r}^{n} \binom{n}{x} \left(\frac{p_+}{100}\right)^x \left(1 - \frac{p_+}{100}\right)^{n-x} = P\left[F \leq \frac{\nu_2 p_+}{\nu_1 (100 - p_+)}\right],$$

where F follows the F distribution with $\nu_1 = 2r$ and $\nu_2 = 2(n - r + 1)$ degrees of freedom, a confidence interval for $\sum_{i=1}^{[k/2]} \pi_i$ is given by

$$\left(\frac{\delta_1 F_{\delta_1,\delta_2,a/2}}{\delta_2 + \delta_1 F_{\delta_1,\delta_2,a/2}}, \frac{\delta_3 F_{\delta_3,\delta_4,1-a/2}}{\delta_4 + \delta_3 F_{\delta_3,\delta_4,1-a/2}}\right), \tag{7}$$

where $\delta_1 = 2X_+$, $\delta_2 = 2(n - X_+ + 1)$, $\delta_3 = 2(X_+ + 1)$, $\delta_4 = 2(n - X_+)$ and $X_+ = \sum_{i=1}^{[k/2]} X_i$. If we denote the limits of (7) by p_L^+ and p_U^+ we conclude that the $100(1 - a)\%$ confidence interval for I_1 is given by

$$\left(\frac{100 \cdot p_L^+}{p_0}, \frac{100 \cdot p_U^+}{p_0}\right). \tag{8}$$

3.2. BOOTSTRAP CONFIDENCE INTERVALS

As it is well-known bootstrap is a non-parametric technique that can be used whenever it is troublesome to create confidence intervals for a parameter using standard statistical techniques. The method was introduced by Efron (1979) and a detailed description of it and its implementation for the construction of confidence intervals can be found in Efron and Tibshirani (1993). In this section we illustrate how the bootstrap method is used for constructing confidence intervals for the indices that were defined previously. For simplicity we adopt the general notation I for all indices defined.

Let us assume that we have a sample with k categories, n observations and observed proportions $p_1/100$, $p_2/100, \ldots, p_k/100$. From this initial sample we generate a large number of multinomial samples, say B, by sampling with replacement. The choice of B is arbitrary, but its value must be sufficiently large. In practice, the number of B that is preferred, is 1000. The B samples are called bootstrap samples. For each bootstrap sample the value of the index I is calculated. After the assessment of all B index values, we order them in a non-descending order and we denote the $i - th$ of these values by

$$I_{(i)}, i = 1, \ldots, B.$$

In the sequel, we describe three alternative methods that one can apply in order to create bootstrap confidence intervals. These methods, are the standard bootstrap, the percentile bootstrap and the bias-corrected percentile bootstrap (see e.g., Efron and Tibshirani (1993)).

3.2.1. *The Standard Bootstrap*

According to this method, a $100(1-\alpha)\%$ confidence interval for the index I is given by

$$\left(\widehat{I} - z_{1-\alpha/2} S_I, \widehat{I} + z_{1-\alpha/2} S_I\right),$$

where z_α denotes the $100\alpha\%$ percentile of the standard normal distribution,

$$S_I = \sqrt{\frac{1}{B-1}\sum_{i=1}^{B}\left(I_{(i)} - \overline{I}\right)^2}$$

is the standard deviation of the B index values,

$$\overline{I} = \frac{1}{B}\sum_{i=1}^{B} I_{(i)}$$

is the mean of the B index values and $\widehat{I}$ is the index value that was assessed from the initial sample.

3.2.2. *The Percentile Bootstrap*

According to this approach, the $100(1-\alpha)\%$ confidence limits for the index I are the $100\,(a/2)\,\%$ and $100(1-\alpha/2)\%$ percentile points of the bootstrap distribution of I. Consequently, the interval is

$$\left(I_{(B\alpha/2)}, I_{(B(1-\alpha/2))}\right).$$

It has to be remarked that, sometimes, $B\alpha/2$ or $(1-\alpha/2)B$ are not integers and so we cannot find the exact $100\,(a/2)\,\%$ and $100(1-\alpha/2)\%$ percentiles. In such cases, we take the nearest integers to $B\alpha/2$ and $(1-\alpha/2)B$.

3.2.3. *The Bias-corrected Percentile Bootstrap*

This third approach is similar to the second but involves a slight correction. The reason why this correction is made is the potential bias. This method, despite the fact that it is more complicated than the two previously described, performs usually better than they do. According to this method, we firstly find the two successive values $I_{(i)}$ and $I_{(i+1)}$ between which the value of the index that was assessed from the initial sample ($\widehat{I}$) lies. Then, we assess the value for which the cumulative distribution function of the standard normal distribution Φ takes the value i/B. If we denote this value by z_0, then $z_0 = \Phi^{-1}(i/B)$. Finally, we calculate the probabilities p_l and p_u, which are defined as

$$p_l = \Phi(2z_0 + z_{\alpha/2})$$

and

$$p_u = \Phi(2z_0 + z_{1-\alpha/2}).$$

Using these probabilities we end up with a $100(1-\alpha)\%$ confidence interval of the form

$$\left(I_{(B\cdot p_l)}, I_{(B\cdot p_u)}\right).$$

4. A Simulation Study

The performance of the three bootstrap methods that were described in the previous section is examined through a simulation study, whose results are presented in the current section. In this study 10000 random samples from the multinomial distribution with parameters $n = 250$ and $n = 500$ and various combinations of $\pi_1, \pi_2, \ldots, \pi_7$ were generated. We selected 9 combinations of proportions so as to include cases where the values of the indices are small, moderate or large. The number of the selected categories is $k = 7$. Other choices of k are not considered since the values of the three indices depend only on the percentages of the positive, the negative and the neutral answers no matter how many positive and negative answers exist. The selected combinations of the proportions are these presented in Tables *AI–AIV* in the Appendix.

From each of the samples we generated $B = 1000$ samples. In each case we found the observed coverage (OC), which must be as close as possible to the nominal coverage. The nominal coverage is 0.90 (Tables AI and $AIII$) and 0.95 (Tables *AII* and *AIV*). The first entry of each cell corresponds to the standard bootstrap (SB) method, the second to the percentile bootstrap (PB) and the third to the bias corrected percentile bootstrap (BB). Moreover the tables present the mean range (MR) of the confidence intervals that each method gives. Similarly, the first value corresponds to the SB, the second to PB and the third to BB.

From the tables we observe that:

– The observed coverage is not affected by the value of n (250 or 500). Thus, we may construct confidence intervals for the indices even when we have a relatively small number of available observations.
– For the index I_1, method BB does not give satisfactory results in many cases. On the other hand, method SB appears to be the one with the best results. The mean range of the confidence intervals produced from the three methods is nearly the same.

– For index I_2, in almost all cases, PB and BB provide confidence intervals with very good coverage. However, using the method SB seems to be ill-conditioned and it also gives the largest mean range. The mean range of the other two methods is quite close even though BB method results generally in shorter intervals.
– For index I_3 methods PB and BB result in coverage close to the nominal in all the examined cases. On the contrary, SB performs quite well in most of the parameters combinations, but leads to unsatisfactory results when the proportion of positive answers is very large. Generally, method SB gives wider intervals while BB gives shortest ones.

Note that the mean range of I_2 and I_3 can not be computed in any case as these indices may equal infinity. Finally, it should be remarked that the previously described procedure was also implemented for $B = 500$ and the obtained results were similar. However, we suggest the use of $B = 1000$, since it is the standard practice in most related papers.

5. An Illustrative Example

In order to illustrate the assessment of the indices defined in this paper and the construction of confidence intervals for their true values we used the data analyzed by Jensen (1986). These data were collected between 1973 and 1976 from the only Catholic high school and its two neighboring public high schools of a southeastern city of the United States. Questionnaires were given to about 60% of the students of each school. (More details on the survey design and the data collection are given in Jensen (1986)). The questionnaires that were given to the students include some questions with ordinal answers for which one can implement the theory developed in the preceding sections.

In Table 5 of Jensen (1986) we have answers on some questions related to the attitudes of the students toward school. These questions are

1. *The things we learn in school are important to me*
2. *Going to school is making me a better person*
3. *Getting good grades is important to me*
4. *I wish I could drop out of school*

and the possible answers were “strongly agree” (SA), “agree” (A), “uncertain” (U), “disagree” (D), “strongly disagree” (SD).

Jensen (1986) gives the observed proportions of the answers for public-school and catholic-school students separately. The observed proportions (%) of public and catholic schools are displayed on Tables *I* and *II*, respectively.

In Table III, we present the values of the three indices for the two types of schools and for all the four questions. For the first three questions, we see that the students seem to prefer the positive answers (SA and A) since the values of all the indices are greater than one. On the other hand, for the fourth question the values

Table I. Public schools

Question	SA	A	U	D	SD	n
1	25.2	49.1	14.6	7.5	3.7	1463
2	24.0	41.3	21.9	8.4	4.5	1481
3	37.3	47.0	8.0	5.7	2.0	1481
4	3.7	4.7	9.0	25.4	57.2	1478

Table II. Catholic schools

Question	SA	A	U	D	SD	n
1	27.9	48.5	11.7	8.9	3.0	437
2	27.2	39.0	20.9	10.0	2.9	441
3	45.6	41.7	7.0	3.6	2.0	441
4	2.3	5.7	12.9	26.5	52.6	441

of all the indices are very small, which means that the students avoid selecting positive answers.

In Tables IV–IX we present confidence intervals for the indices using the techniques described in Section 3. In particular, Tables IV and V refer to bootstrap confidence intervals for the three indices for public and catholic schools, respectively. Tables VI–IX correspond to the parametric methods of Section 3.1 for confidence intervals of index I_1.

From all the confidence intervals we conclude that in the first three questions the students seem to prefer the positive answers (SA and A) since all the values contained in the intervals are greater than one. In the fourth question the range of values of all the intervals is restricted to values less than one, which indicates that the students do not tend to select positive answers. Furthermore, according to

Table III. Values of the three indices

Question	Public schools			Catholic schools		
	I_1	I_2	I_3	I_1	I_2	I_3
1	1.857	6.634	3.446	1.910	6.420	3.733
2	1.632	5.062	2.506	1.655	5.132	2.577
3	2.108	10.948	5.879	2.182	15.589	7.484
4	0.210	0.102	0.190	0.200	0.101	0.227

Table IV. Bootstrap confidence intervals

Question		Public schools I_1	I_2	I_3
	SB	(1.8007, 1.9143)	(5.5403, 7.7276)	(3.1001, 3.7914)
1	PB	(1.8011, 1.9122)	(5.7676, 7.9643)	(3.1401, 3.8134)
	BB	(1.8011, 1.9122)	(5.6774, 7.8085)	(3.1157, 3.7886)
	SB	(1.5719, 1.6931)	(4.2574, 5.8666)	(2.2987, 2.7128)
2	PB	(1.5699, 1.6965)	(4.3807, 6.0488)	(2.3175, 2.7421)
	BB	(1.5716, 1.6965)	(4.3670, 5.9814)	(2.3110, 2.7220)
	SB	(2.0615, 2.1535)	(8.7435, 13.1527)	(5.1139, 6.6441)
3	PB	(2.0645, 2.1556)	(9.2313, 13.5376)	(5.2326, 6.7246)
	BB	(2.0679, 2.1590)	(9.2481, 13.5914)	(5.2852, 6.8209)
	SB	(0.1756, 0.2444)	(0.0834, 0.1199)	(0.1677, 0.2122)
4	PB	(0.1776, 0.2453)	(0.0842, 0.1209)	(0.1681, 0.2117)
	BB	(0.1810, 0.2503)	(0.0852, 0.1226)	(0.1704, 0.2151)

Table V. Bootstrap confidence intervals

Question		Catholic schools I_1	I_2	I_3
	SB	(1.8091, 2.0109)	(4.4006, 8.4397)	(2.9849, 4.4812)
1	PB	(1.8078, 2.0080)	(4.8060, 8.8974)	(3.1074, 4.5814)
	BB	(1.8078, 2.0080)	(4.8333, 8.9487)	(3.1176, 4.5862)
	SB	(1.5434, 1.7666)	(3.5798, 6.6837)	(2.1727, 2.9811)
2	PB	(1.5420, 1.7630)	(3.9286, 7.1860)	(2.2256, 3.0226)
	BB	(1.5420, 1.7687)	(3.9437, 7.1905)	(2.2470, 3.0630)
	SB	(2.1038, 2.2612)	(7.9414, 23.2371)	(5.3908, 9.5774)
3	PB	(2.1032, 2.2619)	(10.8000, 26.1333)	(5.8429, 10.2381)
	BB	(2.0918, 2.2562)	(10.7429, 25.1875)	(5.8857, 10.2927)
	SB	(0.1378, 0.2622)	(0.0668, 0.1355)	(0.1831, 0.2712)
4	PB	(0.1417, 0.2664)	(0.0708, 0.1377)	(0.1827, 0.2720)
	BB	(0.1474, 0.2721)	(0.0718, 0.1399)	(0.1849, 0.2744)

the index values and confidence intervals we do not observe significant differences in the degree of concentration to the positive answers in all the questions for the two types of schools (the corresponding confidence intervals have common values). However, Jensen (1986) implemented chi-square test in order to capture differences

Table VI. Confidence intervals (8)

Question	Public schools	Catholic schools
1	(1.7995, 1.9131)	(1.8041, 2.0083)
2	(1.5702, 1.6930)	(1.5397, 1.7655)
3	(2.0578, 2.1522)	(2.0957, 2.2566)
4	(0.1757, 0.2480)	(0.1398, 0.2716)

Table VII. Confidence intervals (1)

Question	Public schools	Catholic schools
1	(1.6741, 2.0492)	(1.5772, 2.2693)
2	(1.4544, 1.8213)	(1.3341, 2.0121)
3	(1.9132, 2.3052)	(1.8296, 2.5489)
4	(0.1431, 0.3056)	(0.1002, 0.3935)

in the way that the students of public and catholic schools answered and concluded that there exist significant differences in the third question.

Finally, we have to remark that the method of Sison and Glaz (1995) has not been implemented as it would have been extremely cumbersome due to the large sample size of our example.

6. Discussion-Conclusions

In the previous sections we introduced some new indices for ordered answers in questionnaires. Various methods for constructing confidence intervals for these indices are outlined. Finally, the performance of some of these methods was investigated.

Table VIII. Confidence intervals (2)

Question	Public schools	Catholic schools
1	(1.8342, 1.8866)	(1.8547, 1.9844)
2	(1.6104, 1.6620)	(1.6039, 1.7318)
3	(2.0818, 2.1351)	(2.1214, 2.2531)
4	(0.2029, 0.2371)	(0.1910, 0.2740)

Table IX. Confidence intervals (3)

Question	Public schools	Catholic schools
1	(1.7098, 2.0052)	(1.6405, 2.1810)
2	(1.4855, 1.7792)	(1.3863, 1.9244)
3	(1.9599, 2.2535)	(1.9135, 2.4516)
4	(0.0628, 0.3567)	(0.0072, 0.4675)

It would be useful to compare the observed coverage of the confidence intervals for index I_1 based on formulae (1), (2), (3) and (4), even though the coverage of the bootstrap confidence intervals seems to be quite satisfactory in most of the cases. Nevertheless, we believe that bootstrap confidence limits should be preferred since the method of Sison and Glaz (1995), which performs better than the other three parametric methods (see Sison and Glaz (1995)), is extremely complicated and time consuming. Finally, it should be noted that we currently work on possible modifications of the indices introduced, that overcome some of their drawbacks.

Appendix

See tables on following pages

Table AI. $B = 1000$, $n = 250$, (90%)

Proportions								I_1		I_2		I_3	
								OC	MR	OC	MR	OC	MR
							SB	0.8831	0.1450	0.9015	0.1655	0.9004	0.1344
0.03	0.02	0.05	0.45	0.05	0.2	0.2	PB	0.8892	0.1446	0.8978	0.1646	0.8994	0.1343
							BB	0.8711	0.1435	0.8992	0.1653	0.9004	0.1343
							SB	0.8888	0.1934	0.9005	0.2926	0.8999	0.1753
0.06	0.06	0.08	0.4	0.1	0.15	0.15	PB	0.8897	0.1931	0.8979	0.2906	0.8976	0.1750
							BB	0.8813	0.1925	0.9013	0.2910	0.8979	0.1751
							SB	0.9005	0.2218	0.9100	0.4611	0.9044	0.2332
0.1	0.1	0.1	0.35	0.15	0.10	0.10	PB	0.8954	0.2216	0.9015	0.4574	0.9008	0.2327
							BB	0.8913	0.2213	0.9024	0.4570	0.9008	0.2327
							SB	0.8962	0.2373	0.9131	0.7043	0.9056	0.3184
0.15	0.15	0.1	0.3	0.1	0.1	0.1	PB	0.8942	0.2372	0.9059	0.6967	0.9021	0.3173
							BB	0.8917	0.2371	0.9052	0.6951	0.9028	0.3171
							SB	0.9054	0.2422	0.9121	0.8438	0.9100	0.4513
0.2	0.15	0.15	0.2	0.10	0.10	0.10	PB	0.9170	0.2419	0.9011	0.8342	0.9031	0.4489
							BB	0.9134	0.2422	0.9027	0.8312	0.9048	0.4484
							SB	0.9015	0.2373	0.9129	0.9823	0.9077	0.6768
0.2	0.2	0.2	0.1	0.1	0.1	0.1	PB	0.9000	0.2372	0.9024	0.9710	0.9005	0.6718
							BB	0.8955	0.2377	0.9029	0.9667	0.9021	0.6699
							SB	0.8994	0.2219	0.9188	2.0187	0.9099	1.1332
0.25	0.25	0.2	0.1	0.1	0.05	0.05	PB	0.9104	0.2217	0.8953	1.9790	0.8960	1.1206
							BB	0.9062	0.2226	0.8970	1.9600	0.8941	1.1146
							SB	0.8942	0.1935	0.9396	6.9855	0.9216	2.2949
0.25	0.25	0.3	0.1	0.05	0.03	0.02	PB	0.8960	0.1933	0.8968	6.6062	0.8964	2.2509
							BB	0.8937	0.1947	0.9015	6.4264	0.9013	2.2249
							SB	0.8844	0.1449	0.8668	–	0.9367	7.7877
0.3	0.3	0.3	0.05	0.02	0.02	0.01	PB	0.9135	0.1446	0.8979	–	0.8931	7.3692
							BB	0.9178	0.1471	0.8947	–	0.8973	7.1222

Table AII. $B = 1000$, $n = 250$, (95%)

Proportions								I_1		I_2		I_3	
								OC	MR	OC	MR	OC	MR
							SB	0.9388	0.1728	0.9451	0.1972	0.9482	0.1601
0.03	0.02	0.05	0.45	0.05	0.2	0.2	PB	0.9396	0.1720	0.9450	0.1967	0.9476	0.1600
							BB	0.9286	0.1708	0.9466	0.1975	0.9470	0.1601
							SB	0.9433	0.2304	0.9456	0.3487	0.9492	0.2089
0.06	0.06	0.08	0.4	0.1	0.15	0.15	PB	0.9388	0.2299	0.9473	0.3479	0.9483	0.2088
							BB	0.9307	0.2293	0.9499	0.3483	0.9487	0.2088
							SB	0.9494	0.2643	0.9542	0.5495	0.9534	0.2779
0.1	0.1	0.1	0.35	0.15	0.10	0.10	PB	0.9483	0.2639	0.9503	0.5481	0.9510	0.2777
							BB	0.9402	0.2636	0.9507	0.5477	0.9501	0.2777
							SB	0.9477	0.2828	0.9569	0.8393	0.9526	0.3794
0.15	0.15	0.1	0.3	0.1	0.1	0.1	PB	0.9464	0.2824	0.9511	0.8363	0.9507	0.3790
							BB	0.9428	0.2824	0.9513	0.8343	0.9498	0.3788
							SB	0.9530	0.2886	0.9571	1.0054	0.9549	0.5378
0.2	0.15	0.15	0.2	0.10	0.10	0.10	PB	0.9588	0.2883	0.9458	1.0016	0.9512	0.5368
							BB	0.9576	0.2886	0.9476	0.9978	0.9500	0.5360
							SB	0.9489	0.2827	0.9566	1.1704	0.9555	0.8064
0.2	0.2	0.2	0.1	0.1	0.1	0.1	PB	0.9496	0.2824	0.9495	1.1662	0.9484	0.8045
							BB	0.9458	0.2832	0.9509	1.1610	0.9488	0.8021
							SB	0.9482	0.2644	0.9575	2.4054	0.9556	1.3503
0.25	0.25	0.2	0.1	0.1	0.05	0.05	PB	0.9559	0.2641	0.9472	2.3900	0.9484	1.3455
							BB	0.9529	0.2654	0.9471	2.3662	0.9474	1.3385
							SB	0.9477	0.2306	0.9642	8.3238	0.9589	2.7346
0.25	0.25	0.3	0.1	0.05	0.03	0.02	PB	0.9473	0.2303	0.9442	8.1424	0.9484	2.7173
							BB	0.9448	0.2321	0.9471	7.9110	0.9504	2.6863
							SB	0.9355	0.1726	0.8864	–	0.9601	9.2796
0.3	0.3	0.3	0.05	0.02	0.02	0.01	PB	0.9558	0.1722	0.9411	–	0.9431	9.0766
							BB	0.9581	0.1752	0.9486	–	0.9479	8.7654

Table AIII. $B = 1000$, $n = 500$, (90%)

Proportions								I_1		I_2		I_3	
								OC	MR	OC	MR	OC	MR
							SB	0.9015	0.1027	0.9016	0.1156	0.8999	0.0949
0.03	0.02	0.05	0.45	0.05	0.2	0.2	PB	0.8984	0.1025	0.8995	0.1152	0.8992	0.0948
							BB	0.8902	0.1022	0.9013	0.1156	0.8983	0.0949
							SB	0.9003	0.1371	0.9064	0.2045	0.9068	0.1238
0.06	0.06	0.08	0.4	0.1	0.15	0.15	PB	0.8992	0.1371	0.9085	0.2038	0.9044	0.1237
							BB	0.8934	0.1368	0.9075	0.2040	0.9039	0.1237
							SB	0.8988	0.1570	0.9040	0.3190	0.9018	0.1640
0.1	0.1	0.1	0.35	0.15	0.10	0.10	PB	0.8971	0.1570	0.8998	0.3176	0.8988	0.1637
							BB	0.8947	0.1570	0.8986	0.3177	0.8981	0.1638
							SB	0.9003	0.1680	0.9043	0.4853	0.9006	0.2235
0.15	0.15	0.1	0.3	0.1	0.1	0.1	PB	0.8984	0.1680	0.8984	0.4828	0.8956	0.2230
							BB	0.8957	0.1680	0.8984	0.4825	0.8950	0.2231
							SB	0.9000	0.1714	0.9056	0.5806	0.9035	0.3151
0.2	0.15	0.15	0.2	0.10	0.10	0.10	PB	0.9093	0.1713	0.8970	0.5772	0.9006	0.3142
							BB	0.9086	0.1715	0.8963	0.5767	0.9025	0.3142
							SB	0.8922	0.1679	0.9062	0.6762	0.9037	0.4701
0.2	0.2	0.2	0.1	0.1	0.1	0.1	PB	0.8949	0.1679	0.9005	0.6723	0.8963	0.4684
							BB	0.8928	0.1681	0.9009	0.6713	0.8976	0.4680
							SB	0.8983	0.1571	0.9068	1.3638	0.9027	0.7803
0.25	0.25	0.2	0.1	0.1	0.05	0.05	PB	0.9087	0.1570	0.8965	1.3518	0.8964	0.7761
							BB	0.9058	0.1573	0.8948	1.3466	0.8953	0.7747
							SB	0.8988	0.1371	0.9239	4.3475	0.9122	1.5451
0.25	0.25	0.3	0.1	0.05	0.03	0.02	PB	0.9006	0.1370	0.9003	4.2565	0.8998	1.5309
							BB	0.8977	0.1375	0.9025	4.2109	0.9006	1.5238
							SB	0.8938	0.1027	0.9348	15.385	0.9202	4.8820
0.3	0.3	0.3	0.05	0.02	0.02	0.01	PB	0.9096	0.1025	0.8907	14.505	0.8960	4.7789
							BB	0.9093	0.1035	0.8947	14.078	0.8956	4.7180

Table AIV. $B = 1000$, $n = 500$, (95%)

Proportions								I_1		I_2		I_3	
								OC	MR	OC	MR	OC	MR
							SB	0.9482	0.1224	0.9497	0.1377	0.9512	0.1131
0.03	0.02	0.05	0.45	0.05	0.2	0.2	PB	0.9467	0.1221	0.9485	0.1375	0.9514	0.1130
							BB	0.9401	0.1217	0.9490	0.1378	0.9498	0.1131
							SB	0.9481	0.1634	0.9529	0.2436	0.9542	0.1475
0.06	0.06	0.08	0.4	0.1	0.15	0.15	PB	0.9463	0.1633	0.9548	0.2433	0.9520	0.1474
							BB	0.9434	0.1631	0.9554	0.2436	0.9518	0.1475
							SB	0.9467	0.1871	0.9489	0.3801	0.9492	0.1954
0.1	0.1	0.1	0.35	0.15	0.10	0.10	PB	0.9450	0.1870	0.9483	0.3796	0.9481	0.1953
							BB	0.9428	0.1869	0.9496	0.3796	0.9475	0.1953
							SB	0.9513	0.2002	0.9511	0.5783	0.9499	0.2663
0.15	0.15	0.1	0.3	0.1	0.1	0.1	PB	0.9511	0.2001	0.9478	0.5774	0.9479	0.2662
							BB	0.9478	0.2002	0.9489	0.5771	0.9482	0.2663
							SB	0.9479	0.2043	0.9528	0.6918	0.9509	0.3754
0.2	0.15	0.15	0.2	0.10	0.10	0.10	PB	0.9527	0.2041	0.9505	0.6903	0.9502	0.3750
							BB	0.9522	0.2044	0.9504	0.6896	0.9490	0.3750
							SB	0.9472	0.2001	0.9514	0.8057	0.9507	0.5602
0.2	0.2	0.2	0.1	0.1	0.1	0.1	PB	0.9471	0.1999	0.9473	0.8042	0.9475	0.5594
							BB	0.9457	0.2003	0.9476	0.8031	0.9470	0.5591
							SB	0.9488	0.1871	0.9537	1.6251	0.9537	0.9298
0.25	0.25	0.2	0.1	0.1	0.05	0.05	PB	0.9563	0.1871	0.9491	1.6200	0.9502	0.9283
							BB	0.9567	0.1876	0.9485	1.6141	0.9498	0.9265
							SB	0.9472	0.1634	0.9577	5.1804	0.9533	1.8411
0.25	0.25	0.3	0.1	0.05	0.03	0.02	PB	0.9495	0.1632	0.9471	5.1430	0.9469	1.8355
							BB	0.9462	0.1640	0.9485	5.0884	0.9474	1.8275
							SB	0.9447	0.1223	0.9589	18.332	0.9589	5.8172
0.3	0.3	0.3	0.05	0.02	0.02	0.01	PB	0.9537	0.1221	0.9423	17.902	0.9456	5.7755
							BB	0.9560	0.1233	0.9460	17.367	0.9486	5.7009

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