Measurement Error in Monetary Aggregates: A Markov Switching Factor Approach

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Abstract

This paper compares the different dynamics of simple sum monetary aggregates and the Divisia indexes over time, over the business cycle, and across high and low inflation and interest rate phases. Although the traditional comparison of the series may suggest that they share similar dynamics, there are important differences during certain times and around turning points that can not be evaluated by their average behavior. We use a factor model with regime switching that offers several ways in which these differences can be analyzed. The model separates out the common movements underlying the monetary aggregate indexes, summarized in the dynamic factor, from individual variations in each one series, captured by the idiosyncratic terms. The idiosyncratic terms and the measurement errors represent exactly where the monetary indexes differ. We find several new results. In general, the idiosyncratic terms for both the simple sum aggregates and the Divisia indexes display a business cycle pattern, especially since 1980. They generally rise around the end of high interest rate phases – a couple of quarters before the beginning of recessions – and fall during recessions to subsequently converge to their average in the beginning of expansions. We also find that the major differences between the simple sum aggregates and Divisia indexes occur around the beginning and end of economic recessions, and during some high interest rate phases.

KEY WORDS: Measurement Error, Divisia Index, Aggregation, State Space, Markov Switching, Monetary Policy.

JEL Classification Code: E40, E52, E58

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1. Introduction

There is a vast literature on the appropriateness of aggregating monetary components using simple sum. Although some may justify it theoretically based on Hickisian aggregation (Hicks 1946), this theory only holds under the assumption that user costs of money services do not change over time. Simple sum also requires that the relative prices between two monetary assets be equal to unity. This condition implies that each asset is a perfect substitute for the others in the set. Since financial assets provide different services, each yields a particular rate of return, which can be time-varying. The empirical literature finds that the relative prices of U.S. monetary assets fluctuate considerably, posing serious concerns on the reliability of the simple sum aggregation method. In addition, an increasing numbers of imperfect substitute short term financial assets have emerged in the last two decades. Finally, since monetary aggregates from simple sum do not accurately measure the quantities of monetary services chosen by optimizing agents, shifts in the series can be spurious as they do not necessarily reflect a change in the utility derived from money holdings.

Microeconomic aggregation theory offers an appealing alternative to the definition of money compared to the simple-sum method. The quantity index under this approach measures income effects of changes in relative prices separately from substitution effects, which should be invariant for constant utility. The simple sum index, on the other hand, does not distinguish between income and substitution effects if its components are not perfect substitutes. A theoretical-based definition of money that internalize substitution effects is the superlative Divisia index, which is constructed by computing expenditure shares as the index weights. Barnett (1978) constructs theoretical user cost of each monetary asset, which allows computation of Divisia indexes. The weights resulting from this approach are different across assets depending on their rate of returns, and they can be time-varying at each point in time. For a detailed description of the theory underlying this construction, see Barnet (1982).

Several authors have studied the empirical properties of the Divisia index compared to the simple sum index. Some examples are Jones and Nesmith (1997), Belongia (1996), Schunk (2001), and the comprehensive survey found in Barnett and Sertelis (2000). In particular, Jones and Nesmith (1997) compare the statistical properties of the indexes obtained from both methods. Belongia (1996) replicates some studies on the impact of money on economic activity and compares the results obtained from using a Divisia index instead of the original used simple sum index, while Schunk (2001) investigates the forecasting performance of the Divisia index compared to the simple sum aggregates.

In this paper we compare the different dynamics of simple sum monetary aggregates and the Divisia indexes not only over time, but also over the business cycle, and across high and low inflation and interest rate phases. The potential differences between the series can be economically very important. If one of the indexes corresponds to a better measure of money, their differences increase the already considerable uncertainty regarding the effectiveness and appropriateness of monetary policy. We aim to find the nature of the differences and whether they occur during particular periods. This information about the state of monetary growth is premium especially around times in which policymakers may wish to change monetary policy, such as when inflation enters a high growth phase or the economy starts to weaken.

Although the traditional comparison of the series may suggest that they share similar dynamics, there are important differences during certain times and around turning points that can not be evaluated by their average behavior. The proposed model offers several ways in which these differences can be analyzed. The model separates out the common movements underlying the monetary aggregate indexes – summarized in the dynamic factor – from individual variations in each one the indexes – captured by the idiosyncratic terms. The idiosyncratic terms and the measurement errors represent exactly where the monetary indexes differ.[1] The idiosyncratic terms show the movements that are peculiar to each series, whereas the measurement error captures the remaining noise inherent in the data. That is, the factor represents simultaneous downturns and upturns movements in money growth indexes. On the other hand, if only one of the indexes declines, this would not characterize a monetary contraction in the model, and it would be captured by its idiosyncratic term.

We model both the common factor as well as the idiosyncratic terms for each index as following a different Markov process each. Given that the idiosyncratic movements are peculiar to each index, the Markov processes are assumed to be independent on each other. In addition, we allow the idiosyncratic terms to follow autoregressive processes. These assumptions entail a very flexible framework that can capture the dynamics of the differences across the indexes without imposing dependence on them.

Factor models with regime switching have been widely used to represent business cycle (see e.g., Chauvet 1998, 2001, Kim and Nelson 1998, among several others). The proposed model

[1] In aggregation theory measurement error refers to the tracking error in a nonparametric index number's approximation to the aggregator function of microeconomic theory, where the aggregator function is the subutility or subproduction function that is weakly separable within tastes or technology in an economic agent's complete utility or production function, which implies that the aggregator function is increasing and concave and needs to be estimated econometrically. On the other hand, state space model use the term measurement error to mean unmodeled noise in the data, which is not captured by the state variable or idiosyncratic terms. In this paper, measurement error refers to this latter definition.

differs from the literature in its complexity as it includes estimation of the parameters of three independent Markov processes. In addition, the focus is not only on the estimated common factor, but on the idiosyncratic terms that reflect the divergences between the monetary aggregate indexes.

To our knowledge, there is no parallel work in the literature that formally compare simple sum aggregate and the Divisia index directly using a multivariate framework to *estimate* the differences between these series. Our contribution goes beyond the simple comparison over time, as we also focus on major measurement errors that might have occurred during some periods, such as around the beginning or end of recessions or in transition times, such as from low (high) to high (low) inflation or interest rate phases.

We estimate three models, one for each pair of the monetary indexes: simple sum M_1 and Divisia MSI_1 (Model 1), simple sum M_2 and Divisia MSI_2 (Model 2), and simple sum M_3 and DivisiaMSI_3 (Model 3). Our findings confirm some of the previous literature in addition to several new results. In general, the idiosyncratic terms for both the simple sum aggregates and the Divisia indexes display a business cycle pattern, especially since 1980. They generally rise around the end of high interest rate phases – a couple of quarters before the beginning of recessions – and fall during recessions to subsequently converge to their average in the beginning of expansions. We find that the major differences between the simple sum aggregates and Divisia indexes occur around the beginning and end of economic recessions, and during some high interest rate phases. This is particularly the case for the period between 1977 and 1983, which includes a slowdown, two recessions, two recoveries and the change in the Fed's operating procedure. Notice that this period also corresponds to a high interest rate phase, which took place from 1977:2 to 1981:2. Another time in which we find that the indexes diverge substantially is around the 1990 recession. A more detailed summary of findings is found in section 4.

The structure of the paper is as follows: section 2 describes the model, section 3 reports the empirical findings; section 4 summarizes the main results; section 5 concludes.

2. The Model

Let $\mathbf{Y}_t$ be the nx1 vector of monetary indexes for $n = 1,\ldots, N$:

(1) $$\Delta\mathbf{Y}_t = \lambda\, \Delta F_t + \gamma\tau_t + \mathbf{v}_t,$$

where $\Delta = 1 - L$ and L is the lag operator. Changes in the monetary aggregates, $\Delta\mathbf{Y}_t$, are modeled as a function of a scalar unobservable factor that summarizes their commonalities, ΔF_t, an idiosyncratic component nx1 vector that captures the movements peculiar to each index, $\mathbf{v}_t$, and a potential time trend τ_t. The factor loadings, λ, measure the sensitivity of the series to the dynamic factor, ΔF_t.[2] Both the factor and the idiosyncratic terms follow autoregressive processes:

(2) $\Delta F_t = \alpha_{S_t} + \phi(L)\,\Delta F_{t-1} + \eta_t \qquad \eta_t \sim N(0, \sigma^2),$

(3) $\mathbf{v}_t = \mathbf{\Gamma}_{S_t^h} + \mathbf{d}(L)\mathbf{v}_{t-1} + \boldsymbol{\varepsilon}_t, \qquad \boldsymbol{\varepsilon}_t \sim \text{i.i.d. } N(0, \mathbf{\Sigma}).$

where η_t is the common shock to the latent dynamic factor, and $\boldsymbol{\varepsilon}_t$ are the measurement errors. In order to capture potential nonlinearities across different monetary regimes, the intercept of the monetary factor switches regimes according to a Markov variable, S_t, where $\alpha_{S_t} = \alpha_0 + \alpha_1 S_t^{\alpha}$, and $S_t^{\alpha} = 0, 1$. That is, monetary indexes can either be in an expansionary regime, where the mean growth rate of money is positive ($S_t^{\alpha} = 1$), or in a contractionary phase with a lower or negative mean growth rate ($S_t^{\alpha} = 0$).

We also assume that the idiosyncratic terms for each index n=1,…, N follow *distinct* two-state Markov processes by allowing their drift terms, $\mathbf{\Gamma}_{S_t^h}$, to switch between regimes. For example, in the case of two monetary indexes, n=2, there will be two idiosyncratic terms, each one following an independent Markov process S_t^{β} and S_t^{δ}, where $S_t^{\beta} = 0, 1$ and $S_t^{\delta} = 0, 1$. Notice that we do not constraint the Markov variables S_t^{α}, S_t^{β}, and S_t^{δ} to be dependent on each other, but allow them instead to move according to their own dynamics. In fact, there is no reason to expect that the idiosyncratic terms would move in a similar manner to each other or to the dynamic factor, since by construction they represent movements peculiar to each index not captured by the common factor.

The switches from one state to another is determined by the transition probabilities of the first-order two-state Markov processes, $p_{ij}^k = P(S_t^k = j \mid S_{t-1}^k = i)$, where $\sum_{j=0}^{1} p_{ij}^k = 1,\ i,j = 0,1,$ for $k = \alpha, \beta, \delta$, representing the Markov processes for the dynamic factor, and the idiosyncratic terms, respectively.

The model separates out common signal underlying the monetary aggregates from individual variations in each one of the indexes. The dynamic factor captures simultaneous downturns and upturns movements in money growth indexes. On the other hand, if only one of the variables declines, e.g. M1, this would not characterize a monetary contraction in the model, and it would be captured by the M1 idiosyncratic term. A monetary contraction (expansion) will occur when

[2] The factor loading for the Divisia monetary index series is set equal to one to provide a scale for the latent dynamic factor. This normalization is a necessary condition for identification of the factor and the choice of parameter scale does not affect any of the time series properties of the dynamic factor or the correlation with its components.

all n variables decrease (increase) at about the same time. That is, η_t and $\mathbf{v}_t$ are assumed to be mutually independent at all leads and lags, for all n variables, and $\mathbf{d}$(L) is diagonal. The dynamic factor is the outcome of averaging out the discrete states. Although the n monetary indexes represent different measurements of money, the estimated dynamic factor is a nonlinear combination of them, representing broader movements in monetary aggregates in the U.S.

Dynamic factor models with regime switching have been widely used to represent business cycle (see e.g., Chauvet 1998, 2001, Kim and Nelson (1998), among several others). The proposed model differs from the literature in its complexity as it includes estimation of the parameters of three independent Markov processes.

The model is cast in state space form, where (4) and (5) are the measurement and transition equations, respectively:

(4) $\Delta\mathbf{Y}_t = \mathbf{Z}\,\xi_t + \mathbf{G}\tau_t$

(5) $\xi_t = \boldsymbol{\mu}_{\xi_{st}} + \mathbf{T}\,\xi_{t-1} + \mathbf{u}_t,$

A particular state space representation for the estimated indicator using two variables is:

$$\Delta\mathbf{Y}_t = \begin{vmatrix} \Delta Y_{1t} \\ \Delta Y_{2t} \end{vmatrix}, \quad \mathbf{Z} = \begin{bmatrix} \lambda_1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 \end{bmatrix}, \quad \xi_t = \begin{vmatrix} \Delta F_t \\ v_{1t} \\ v_{2t} \\ F_{t-1} \end{vmatrix} \quad \boldsymbol{\mu}_{\xi_{st}} = \begin{vmatrix} \alpha_{st} \\ \beta_{st} \\ \delta_{st} \\ 0 \end{vmatrix},$$

$$\mathbf{T} = \begin{bmatrix} \phi_1 & 0 & 0 & 0 \\ 0 & d_1 & 0 & 0 \\ 0 & 0 & d_2 & 0 \\ 1 & 0 & 0 & 1 \end{bmatrix}, \quad \mathbf{G} = \begin{vmatrix} \gamma_1 \\ \gamma_2 \end{vmatrix} \text{ and } \quad \mathbf{u}_t = \begin{vmatrix} \eta_t \\ \varepsilon_{1t} \\ \varepsilon_{2t} \\ 0 \end{vmatrix}.$$

The term F_{t-1} is included in the state vector to allow estimation of the dynamic factor in levels from the identity $\Delta F_{t-1} = F_{t-1} - F_{t-2}$.

The model is estimated using an extended version of the nonlinear Kalman filter to compute the latent dynamic factor and each one of three Markov processes. The nonlinear filter forms forecasts of the unobserved state vector, $\xi^{(i,j)}_{t|t-1}$, and the associated mean squared error matrices, $\theta^{(i,j)}_{t|t-1}$, based on information available up to time t-1, $I_{t-1} \equiv [\Delta\mathbf{Y}'_{t-1}, \Delta\mathbf{Y}'_{t-2},..., \Delta\mathbf{Y}'_1]'$, on the Markov state S_t for each $S_t = S^{\alpha}_t, S^{\beta}_t, S^{\delta}_t$ taking on the value j, and on S_{t-1} taking on the value i, for $i, j = 0, 1$:

(6) $\xi^{(i,j)}_{t|t-1} = E(\xi_t \mid I_{t-1}, S_t = j, S_{t-1} = i)$

(7) $\theta^{(i,j)}_{t|t-1} = E[(\xi_t - \xi_{t|t-1})(\xi_t - \xi_{t|t-1})' \mid I_{t-1}, S_t = j, S_{t-1} = i)].$

The filter uses as inputs the joint probability of the Markov-switching states at time t-1 and t conditional on information up to t-1, $P(S_{t-1} = i,\ S_t = j\ |I_{t-1});$ an inference about the state vector using information up to t-1, given $S_{t-1} = i$ and $S_t = j$, that is, $\xi^{(i,j)}_{t-1|t-1}$; and the mean squared error matrices, $\{\boldsymbol{\theta}^{(i,j)}_{t-1|t-1}\}$. The outputs are their one-step updated values. The nonlinear Kalman filter is:

$$\xi^{(i,j)}_{t|t-1} = \boldsymbol{\mu}_{\xi_{st}} + \mathbf{T}\xi^{i}_{t-1|t-1} \tag{8}$$
$$\boldsymbol{\theta}^{(i,j)}_{t|t-1} = \mathbf{T}\boldsymbol{\theta}^{i}_{t-1|t-1}\mathbf{T}' + \mathbf{H} \tag{9}$$
(prediction equations)

$$\xi^{(i,j)}_{t|t} = \xi^{(i,j)}_{t|t-1} + \mathbf{K}^{(i,j)}_{t}\mathbf{N}^{(i,j)}_{t|t-1} \tag{10}$$
$$\boldsymbol{\theta}^{(i,j)}_{t|t} = (\mathbf{I}_n - \mathbf{K}^{(i,j)}_{t}\mathbf{Z})\boldsymbol{\theta}^{(i,j)}_{t|t-1} \tag{11}$$
(updating equations)

where $\mathbf{H}$ is the variance-covariance matrix of the vector of disturbances $\mathbf{u}_t$, $\mathbf{I}_n$ is the identity matrix, $\mathbf{K}^{(i,j)}_{t} = \boldsymbol{\theta}^{(i,j)}_{t|t-1}\mathbf{Z}'[\mathbf{Q}^{(i,j)}_{t}]^{-1}$, $\mathbf{N}^{(i,j)}_{t|t-1} = \Delta\mathbf{Y}_t - \mathbf{Z}\xi^{(i,j)}_{t|t-1}$ is the conditional forecast error of $\Delta\mathbf{Y}_t$, and $\mathbf{Q}^{(i,j)}_{t} = \mathbf{Z}\boldsymbol{\theta}^{(i,j)}_{t|t-1}\mathbf{Z}'$ is its conditional variance.

The probability terms are computed using Hamilton's filter, for each $S_t = S^{\alpha}_t, S^{\beta}_t, S^{\delta}_t$ as:

$$P(S_{t-1} = i,\ S_t = j\ |I_{t-1}) = p^{ij}\sum_{h=0}^{1} P(S_{t-2} = h,\ S_{t-1} = i\ |\ I_{t-1}). \tag{12}$$

From these joint conditional probabilities, the density of $\Delta\mathbf{Y}_t$ conditional on S_{t-1}, S_t, and I_{t-1} is:

$$f(\Delta\mathbf{Y}_t\,|S_{t-1} = i,\ S_t = j,\ I_{t-1}) = [(2\pi)^{-\mathbf{n}/2}\,|\mathbf{Q}^{(i,j)}_{t}|^{-1/2}\ exp(-\frac{1}{2}\mathbf{N}^{(i,j)'}_{t|t-1}\mathbf{Q}^{(i,j)^{-1}}_{t}\mathbf{N}^{(i,j)}_{t|t-1}). \tag{13}$$

The joint probability density of states and observations is then calculated by multiplying each element of (12) by the corresponding element of (13):

$$f(\Delta\mathbf{Y}_t,\ S_{t-1} = \mathrm{i},\ S_t = \mathrm{j}|\ \mathrm{I}_{t-1}) = f(\Delta\mathbf{Y}_t|\ S_{t-1} = \mathrm{i},\ S_t = \mathrm{j},\ I_{t-1})\ P(S_{t-1} = \mathrm{i},\ S_t = \mathrm{j}|\ I_{t-1}). \tag{14}$$

The probability density of $\Delta\mathbf{Y}_t$ given I_{t-1} is:

$$f(\Delta\mathbf{Y}_t\,|I_{t-1}) = \sum_{j=0}^{1}\sum_{i=0}^{1} f(\Delta\mathbf{Y}_t,\ S_{t-1} = \mathrm{i},\ S_t = \mathrm{j}\ |I_{t-1}). \tag{15}$$

The joint probability density of states is calculated by dividing each element of (14) by the corresponding element of (15):

$$P(S_{t-1} = i,\ S_t = j\ |\ I_t) = f(\Delta\mathbf{Y}_t,\ S_{t-1} = i,\ S_t = j\ |\ I_{t-1})\ /\ f(\Delta\mathbf{Y}_t\ |\ I_{t-1}) \tag{16}$$

Finally, summing over the states in (16), we obtain the filtered probabilities of expansions or recessions:

$$P(S_t = j\ |\ I_t) = \sum_{i=0}^{1} P(S_{t-1} = i,\ S_t = j\ |\ I_t). \tag{17}$$

As in the linear Kalman filter, the algorithm calculates recursively one-step-ahead predictions and updating equations of the dynamic factor and the mean squared error matrices, given the parameters of the model and starting values for $\xi^{j}_{t|t}$, $\theta^{j}_{t|t}$, and the probabilities of the Markov states. However, for each date t the nonlinear filter computes 2^k forecasts, where k is the number of states, and at each iteration the number of cases is multiplied by *k*. This implies that the algorithm would be computationally unfeasible even for the simplest cases. Kim (1994), based on Harrison and Stevens (1976), proposes an approximation introduced through $\xi^{j}_{t|t}$ and $\theta^{j}_{t|t}$ for $t>1$. This approximation consists of truncating the updating equations into averages weighted by the probabilities of the Markov states.

The conditional likelihood of the observable variables is obtained as a by-product of the algorithm at each *t*, from equation (13), which is used to estimate the unknown model parameters. The filter evaluates this likelihood function, which is then maximized with respect to the model parameters using a nonlinear optimization algorithm. The maximum likelihood estimators and the sample data are then used in a final application of the filter to draw inferences about the dynamic factor and probabilities, based on information available at time *t*. The final estimated state vector is calculated as:

$$\xi_{t|t} = \sum_{i=0}^{1} P(S_t = j|I_t)\xi^{j}_{t|t}.$$

The estimation is implemented through a numerical procedure. The nonlinear discrete filter produces two outputs: the state vector containing the dynamic factor and the idiosyncratic terms, $\xi_{t|t}$, and the associated probabilities of the Markov states. The filtered probabilities give at time *t* the probability of the Markov state using only information available at *t*, $P(S_t = 0, 1\ |I_t)$. On the other hand, the smoothing probabilities are obtained through backward recursion using the information in the full sample, $P(S_t = 0, 1|\ I_T)$.

3. Empirical Results

Data

We use the Federal Funds Rate as the interest rate and the log first difference of Consumer Price Index as inflation. The series and the simple monetary aggregates M1, M2, and M3 as well as their corresponding Monetary Service indexes (Divisia) MSI1, MSI2, MSI3 were all obtained from the Federal Reserve Bank of Saint Louis. The Research Division of the Saint Louis Fed produces the MSI indexes on a regular basis. The MSI or Divisia indexes are a measurement of the flow of monetary services obtained by households and firms from holding monetary assets. For the theory and methodology utilized in the construction of these indexes, and for details of the construction of these indexes see Anderson, Jones, and Nesmith (1997a and b). For a survey of the theory of monetary aggregation theory, empirical comparisons, and important papers on the subject see Barnett and Serletis (2000). We use data at the quarterly frequency from 1960:2 to 2005:4, which corresponds to the period in which the data on Divisia indexes are available.

Specification Tests

The dynamic factor structure captures cyclical comovements underlying the observable variables. The resulting dynamic factor is highly correlated with all the monetary aggregates used in its construction, indicating that the structure was not simply imposed on the data by assuming large idiosyncratic errors.

In addition, tests for the number of states strongly support the single factor specification. This is tested in different ways. First, the eigenvalues of the correlation matrix of the common factor indicate adequacy of the single factor specification.[3] Second, the model assumes that the factor summarizes the common dynamic correlation underlying the observable variables, which implies that the idiosyncratic terms $\mathbf{v}_{m,t}$, for $m = 1,..,M,$ are uncorrelated with the observed variables $\Delta\mathbf{Y}_{n,t}$, $n = 1,\ldots N$, for $n \neq m.$[4] In order to test this assumption, the idiosyncratic terms $\mathbf{v}_{m,t}$ are regressed on six lags of the observable variables $\Delta\mathbf{Y}_{n\neq m,t}$, and the parameters of the equations are found to be insignificantly different from zero. In addition, the one-step-ahead conditional forecast errors, $\mathbf{N}_{t|t-1}$ – obtained from the filter described in section 2 – are not predictable by lags of the observable variables. These results support the single factor specification, since these error terms are not capturing common information underlying the observable variables.

With respect to the measurement errors $\boldsymbol{\varepsilon}_t$ the i.i.d. assumption is tested using Ljung-Box statistics on their sample autocorrelation, and Brock, Dechert, and Scheinkman's (1996) diagnostic test.[5] Both tests fail to reject the i.i.d. assumption at any level.

High and Low Inflation and Interest Rate Phases

We study changes in monetary growth across business cycle phases and high and low inflation and interest rate periods. We use economic recessions and expansions as dated by the NBER to analyze changes across business cycle states. Regarding inflation, we are mostly interested in identifying times in which there is a persistent change in this series. We classify a

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[3] The magnitude of the n eigenvalues for each factor reflects how much of the correlation among the observable variables is explained by $k \leq n$ potential factors. For each of the three composite indicators, there is only one eigenvalue greater than one, while the others are close to zero.

[4] The model was estimated allowing either AR(1) or AR(0) processes for the disturbances $\Delta\mathbf{v}_t$. The likelihood ratio test favors the AR(1) specification at the 1% level.

[5] Leads of 2, 3, 4, 5, and 6 months are used for the residuals and the distance between the two vectors of residuals is set to be equal to their standard deviation.

high inflation phase as one in which inflation increases persistently for several quarters until it reaches a peak. By the same token, low inflation phases start when inflation falls for several quarters until it reaches a trough. A high (low) inflation phase may include periods in which the level of inflation is still relatively low (high) but is increasing (decreasing) persistently. That is, the level of inflation is not as relevant as its rate of change. For example, inflation was historically low in the early 2000s, but since its derivative turned positive in 2002:1 and remained so for a couple of quarters, this date indicates the beginning of a high inflation phase.

The metric proposed to determine inflation phases is as follows: a high inflation phase starts in quarter *t* if inflation π_{t-1} was in a low phase in quarter t-1 and $\pi_{t+2} \geq \pi_{t+1} \geq \pi_t \geq \pi_{t-1}$. That is, inflation grows for three consecutive quarters. A low inflation phase starts in quarter *t* if inflation π_{t-1} was in a high phase in quarter t-1 and $\pi_{t+1} < \pi_t < \pi_{t-1}$. That is, inflation falls for two consecutive quarters. This is similar to the rule of thumb of two quarters decrease (increase) in GDP to determine beginning of recessions (expansions), although we use an asymmetric number of quarters for high and low phases based on inflation persistence. However, the results do not change if we use instead two quarters decrease or increase.

We also use Bry and Boschan (1971) routine to determine inflation phases. Bry and Boschan (B-B) formalizes turning point dating rules into a computer routine, which has been refined by Haywood (1973) to include an amplitude criterion.[6] The turning points obtained coincide with our proposed criterion described above. In fact, both methods select turning points that would be easily picked simply by visual inspection of the smoothed series.

The resulting inflation phases are plotted in figure 1a together with smoothed inflation, inflation, and NBER recessions. As it can be seen, when inflation starts increasing it does so slowly and steadily. However, when inflation falls, it drops abruptly, which makes it easier to identify the beginning of a low inflation phase than the start of a high inflation phase. Notice that inflation phases are associated with NBER recessions. In particular, all recessions begin around the end of high inflation phases. In addition, there were only two high inflation phases, in 1983-1984 and 2002, in which a recession did not follow. However, the economy entered a slowdown in 1984-1986.

With respect to interest rate, the determination of peaks and troughs is made simpler by the fact that this series is smoother than inflation. We use a similar metric than the one used for

[6] The main steps of the B-B routine are: 1) the data are smoothed after outliers are discarded; 2) preliminary turning points are selected and compared with the ones in the original series; 3) duration of the phases is checked and if it is below 6 months the turning points are disregarded; 4) Amplitude criterion is applied, based on a moving standard deviation of the series. In the end, the program selects turning points that would be easily picked simply by visual inspection.

inflation. However, using two or three quarters of change as the cut off for dating the phases results in exactly the same dating. Thus, we use the following metric: a high interest rate phase starts in quarter *t* if interest rate $i_{t-1}$was in a low phase in quarter t-1 and $i_{t+1} \geq i_t \geq i_{t-1}$ and a low interest rate phase starts in quarter *t* if interest rate i_{t-1} was in a high phase in quarter *t*-1 and $i_{t+1} < i_t < i_{t-1}$. That is, the turning point of interest rate phases takes place when it falls or rises for two consecutive quarters. Once again, we use Bry and Boschan (1971) routine to determine interest rate phases and find the same turning points as the two-consecutive-quarter rule of thumb.

The interest rate phases are shown in figure 1b as well as smoothed interest rate, interest rate, and NBER recessions. Interest rate phases are also associated with the NBER recessions and expansions – the peak generally is at or right before economic recessions whereas the trough is roughly in the middle of expansions. One exception is for this last expansion in which the high interest phase started a lot earlier, at the trough of the 2001 recession.

3.1 Results

Table 1 shows the maximum likelihood estimates of the Markov switching dynamic factor model applied to the monetary aggregates. Three models were estimated, one for each pair of the monetary indexes: M_1 and MSI_1 (Model 1), M_2 and MSI_2 (Model 2), and M_3 and MSI_3 (Model 3).

The Markov states for the factors are statistically significant across the specifications. State 1 has a positive mean growth rate, α_1, while state 0 has a negative mean growth rate, α_0, for models 1 and 3. For model 2, the mean growth rates in both states are positive, although the one in state 0 is smaller than the one in state 1, and they both are statistically significant at the 1% level.

The autoregressive coefficient for the factor, ϕ, is positive and around 0.5 across all specifications. The factor loadings measure how changes in the dynamic factor affect changes in the observable variables. The loadings for the Divisia monetary indexes are set equal to one to provide a scale for the latent dynamic factors. This normalization is a necessary condition for identification of the factors. The choice of parameter scale does not affect any of the time series properties of the dynamic factor or the correlation with its components. We find that the estimated factor loading for the simple monetary aggregate is positive and close to one across all models, indicating that the Divisia index and the simple sum aggregate have a similar and proportional impact on the factor for each model.

All other parameters of the model are statistically significant as well. We discuss their dynamics for each model below.

Simple M1 Aggregate and Divisia M1

The factor extracted from the growth rates of the simple sum aggregate M1 and from the Divisia M1 (MSI1) is plotted in Figure 2a together with the probabilities of low monetary growth and NBER recessions (DF1). During the 1960s and 1970s, the factor is mostly positive with an average quarterly growth of 1.2%. In the second half of the sample there are times in which money growth decreases substantially, reaching negative values. The smoothed probabilities identify four phases of negative monetary growth during this period: 1989:1-1989:4, 1994:4-1997:2, 2000:2-2000:4, and 2005:1-20005:3; and a pulse change in 1980:2.

The factor is highly correlated with its components, with values of 0.988 for M1 and 0.998 for MSI1, respectively (Table 2). Notice that M1 and MSI1 are more correlated with the factor than with each other. Figure 2b plots these series and NBER recessions. Although the comparison of the series suggests that they share very similar dynamics, there are important differences during certain times and around turning points that can not be evaluated by their average behavior. The proposed model offers several ways in which these differences can be analyzed. The model separates out the common movements of these series, which is summarized in the dynamic factor. However, the idiosyncratic terms and the measurement errors represent exactly where the monetary indexes differ. The idiosyncratic terms show the movements that are peculiar to each series, whereas the measurement error captures the remaining noise inherent in the data.

The idiosyncratic term for MSI1 is highly autocorrelated (0.98) and smooth whereas the one for M1 is a lot less persistent (0.48) and more jagged (Table 1 and Figure 2c). Both idiosyncratic terms display a business cycle pattern from 1980 on. In particular, they rise before the beginning of recessions and fall during recessions, but subsequently converge to their average in the beginning of expansions. During the 1980s and 1990s expansions, the idiosyncratic terms increased steadily until reaching a peak in the middle of these expansions.

Figure 2d plots the squared difference between the idiosyncratic terms for M1 and MSI1, NBER recessions, and phases of high inflation and interest rates. From 1960 until 1976 the difference between them was almost zero. However, analysis of the second part of the sample reveals some interesting divergent patterns. The major differences took place right around the beginning or end of recessions. Notice that the beginning of recessions is also the end of high interest rate and inflation phases. The largest difference occurred at the end of the 1981-82 recession and in 2005:3, followed by divergences before the 1980-81 and 1981-82 recessions and at the trough of the 1990-91 recession. In addition, persistent differences took place during times in which inflation and interest rates were in a high phase. It can be observed that differences also occur when there are some major changes in the magnitude of monetary growth. This is especially the case between 1994:4-1997:2 when both the rate of growth of M1 and of the Divisia index MSI1 decreased substantially to negative values.

Figure 2e shows the measurement error from simple sum aggregate M1 growth, from Divisia M1 growth (MSI1), and NBER recessions. As it can be seen, the measurement error from Divisia is a lot smaller throughout the sample compared to the measurement error from M1 growth. As discussed in the previous session, linear and nonlinear tests fail to reject the hypothesis of i.i.d. for the measurement errors. However, some interesting patterns can be observed in their *squared differences*. Since 1984, the measurement error of M1 growth is greater than Divisia growth in the middle of expansions and smaller from the second half of expansions until around the beginning of recessions. The difference becomes positive during recessions but reverts to negative at their end. The major difference between the two took place in the first quarter of 1983, when the measurement error for M1 growth reached its maximum value.

Figure 2f shows the squared difference between the measurement errors. As for the idiosyncratic terms, the difference between the measurement errors is almost zero before 1976. However, its highest levels occurred during the high inflation phase between 1977 and 1983. It also increased at the peak and trough of the 1990-1991 recession and between 1999 and 2000 – during the high inflation and interest rate phase that preceded the 2001 recession. As for the idiosyncratic terms, the only time that the difference between the two measurement errors was large but not associated with a high inflation or interest rate phase or a recession was between 1995-1996. This period corresponds to a shift of monetary growth from historically positive to large negative.

This analysis confirms previous results (see e.g. Belongia 1996), which find large differences between M1 and Divisia MSI1 between 1984 and 1987 and between 1995 and 1997, with the former being greater than the latter.

Simple M2 Aggregate and Divisia M2

The factor obtained from the growth rates of the simple sum aggregate M2 and from the Divisia M2 (MSI2) is highly correlated with these series – 0.95 and 0.96, respectively (Table 2). Figure 3a shows this factor (DF2) and probabilities of high monetary growth. The most noticeable feature of the factor (and of its components) is its rise between 1970-73 and between 1975-78. In fact, these periods are captured by the smoothed probabilities, as well as the fast monetary growth phases right after the 1980-81 and 1981-82 recessions, and during the 2001 recession. There were other times in which money growth was well above its average also in 1985-86 and 1998, as depicted by the probabilities.

The dynamics of the factor DM1 differ substantially from the factor DM2, especially after 1990 (Figures 1c and 3b) and the overall correlation between them is only 0.34. First, the DM1 factor does not increase substantially as the DM2 factor in the 1970s. Second, the DM2 factor moves in the opposite direction as the DM1 factor from 1991 to 1994, with DM2 reaching its highest level of growth during this period. A divergent movement also takes place in 1995-1996, when the DM1 grows and the DM2 falls. This same pattern is found by comparing the growth rate of M1 and MSI1 with M2 and MSI2.

The idiosyncratic terms for M2 and MSI2 are shown in Figure 3c. There are marked differences between them. Although they generally move in the same direction in the first part of the sample, they differ substantially around turning points and in the second period. For example, the idiosyncratic term for M2 increased during the 1970 and 1974-75 recessions, even when interest rate was already in a low phase. The idiosyncratic term for the MSI2, on the other hand, decreased during these periods. From 1982 there are several instances in which these series display divergent movements.

Figure 3d shows the squared difference between these two series, NBER recessions, and phases of high inflation and interest rates. For the most part the discrepancies between the idiosyncratic terms take place in transition times, such as around business cycle turning points or the beginning and end of interest rate or inflation phases. The largest differences were from the middle to the trough of the 1980-81 and 1981-82 recessions, at the end of the high interest rate phase in 1989 (and the beginning of an economic slowdown), and between 1991 and 1996. In this last period the differences were not only large, they were also the longest in the sample, corresponding to cyclical movements of DM1 and DM2 to opposite direction as explained above. There were other important divergences as the ones during the 1970 and 1990 recessions, and during transition from tight to loose monetary policies.

These differences are economically very important. If one of the aggregates correspond to a better measure of monetary aggregate in the economy, their differences add to the uncertainty of the economy and of the effectiveness and appropriateness of monetary policy exactly at times in which there information about the state of monetary growth is premium, such as around business cycle turning points and changes in inflation phases.

Figure 3f plots the difference between the measurement errors for M2 and MSI2 growth. The main discrepancies between these two series occur between 1979 and 1982. This period includes a slowdown, two recessions and a small recovery, and coincides with the time in which the Federal Reserve changed its operating procedures.

The other times in which these series differ is in the transition between two phases in 1989. In particular, a larger difference takes place at the peak of interest rates cycle. While interest rate started decreasing in 1989:2 inflation remained in a high phase until 1990:2.

Simple M3 Aggregate and Divisia M3

Figure 4a shows the factor (DF3) resulting from the growth rates of the simple sum aggregate M3 and from the Divisia M3 (MSI3) while Figure 1c compares the three dynamic factors, DF1, DF2, and DF3. As it can be observed, the factor DF1 moves in opposite direction as the factors DF2 and DF3 during some periods, whereas in general DF2 and DF3 display very similar dynamics (Figure 1c). However, DF3 (as well as M3 and MSI3 growth) did not present a high growth in the 1970s as did DF2. In fact, the Markov probabilities for DF3 capture instead a large drop in the underlying series M3 and MSI3 growth between 1989:2 and 1995:1 as the most salient variation in the series. Other important low growth phases captured by the probabilities are in 1966, between 1969-70, 2002, and in 2004-05.

The factor DF3 is highly correlated with M3 and MSI3 growth, but more so with the former (0.98) than with the latter (0.90) (Table 2). However, the correlation between the factor and the growth of MSI3 is a lot higher if the period between 1978 and 1982 is excluded. During this time MSI3 growth oscillated substantially (Figure 4b).

The idiosyncratic terms for M3 and MSI3 growth are shown in Figure 4c. The term corresponding to M3 is smoother and has smaller fluctuations. Although they have general similar dynamics, the two idiosyncratic terms differ substantially during some important times. Figure 4d plots their squared difference. The major divergences between M3 and MSI3 growth coincide in time and amplitude with the differences between M2 and MSI2 growth. The largest discrepancies took place during the high inflation phase between 1978 and 1981, and during the 1981-82 recession. Times of high uncertainty are associated with larger asynchronous movements between M3 and MSI3 growth, such as during recessions or when interest rate has a turning point. This is the case, for example, between 1989 and 1990, when the high interest rate phase ended, but inflation remained in a high phase until right before the beginning of the 1990 recession. This is also the case in 1965-67, during the 1969-70 and 1990-91 recessions, and during 1972-74, which corresponds to a high inflation phase and recession.

Another way of gauging the differences between M3 and MSI3 growth is through the measurement errors. Figure 4e shows the squared difference between their measurement errors.

Analysis of these series indicate that the major differences took place in 1979:4, 1982:1, and in the middle of the 1969-73 recession, in addition to the dissimilarities captured by the idiosyncratic terms.

4. Summary of Findings

In general, the idiosyncratic terms for both the simple sum aggregates and the Divisia indexes display a business cycle pattern, especially since 1980. They generally rise around the end of high interest rate phases – a couple of quarters before the beginning of recessions – and fall during recessions to subsequently converge to their average in the beginning of expansions.

We find that the major differences between the simple sum aggregates and Divisia indexes occur around the beginning and end of economic recessions, and during some high interest rate phases. This is particularly the case for the period between 1977 and 1983, which includes a slowdown, two recessions, two recoveries and the change in the Fed's operating procedure. Notice that this period also corresponds to a high interest rate phase, which took place from 1977:2 to 1981:2. Another time in which the indexes diverge substantially is around the 1990 recession.

In the case of M1 and MSI1, the main divergence between the two indexes is in 1983:1. The idiosyncratic term for M1 counter intuitively increased to its highest level in a quarter that marked the beginning of a high interest rate phase. The MSI1, on the other hand, had only a minor rise. At that time, Milton Friedman, based on the movements of M1, warned in newspapers that this 'monetary explosion' was bounded to cause a contractionary policy by the Fed, which would lead to another period of stagflation. William Barnett, on the other hand, correctly predicted that there was no reason for panic, since monetary growth was at its average rate based on the Divisia index MSI1. In fact, Barnett correctly reckoned *in real time* that the large increase in M1 was a 'statistical blip'.

The differences and similarities between the pairs M2-MSI2 (model 2) and M3-MSI3 (model 3) are closer than the ones for M1 and MSI1 (model 1). First, the Divisia indexes MSI2 and MSI3 decrease a lot more before recessions (at the peak of inflation phases) and increase substantially more during recessions and recoveries (low interest rate phases) than the simple sum aggregates M2 and M3, respectively. That is, the dynamics of these Divisia indexes correspond more closely to the expected movements related to interest rates and inflation.

A noticeable difference between the Divisia MSI2 and the simple sum aggregate M2 is their movement to opposite directions between 1991 and 1995. During the recovery after the 1990 recession, M2 increased more than MSI2, while interest rates were falling. However, M2 continued to increase even during the high interest rate phase that started in 1993:3 and ended in 1995:1. On the other hand, MSI2 showed a movement more consistent with changes in interest rates, decreasing during this period.

Another difference that is observable in both pairs M2-MSI2 and M2-MSI3 is their behavior at the end of the 1981 recession, when there was a large increase in the idiosyncratic terms from the Divisia indexes, and only a minor rise for the simple sum aggregates. Accordingly, the Divisia indexes display a business cycle pattern more consistent with monetary policy.

With respect to the idiosyncratic terms for MSI3 and for the simple sum aggregate M3, the idiosyncratic terms for these series move in opposite directions in several occasions. In particular, the Divisia index increases during the expansion in the early 1970s while M3 counter intuitively decreases. In addition, M3 shows a steadily increase since the end of the 1981-82 recession until 1989, showing no link with the high interest rate phase that took place during 1986:4-1989:1. On the other hand, MSI3 increased during the low inflation phase following the 1981-82 recession, but fell during this high interest rate phase. More recently, the idiosyncratic term from the M3 has been counter intuitively high during the latest high interest rate phase that started in 2004, whereas the Divisia MSI3 shows the expected decrease.

5. Conclusions

Microeconomic aggregation theory offers an appealing alternative to the definition of money compared to the simple-sum method. The quantity index under this approach measures income effects of changes in relative prices separately from substitution effects, which should be invariant for constant utility. The simple sum index, on the other hand, does not distinguish between income and substitution effects if its components are not perfect substitutes. In this paper we compare the empirical differences between a theoretical-based definition of money that internalize substitution effects – the Divisia index, with the simple sum aggregate indexes as used by the statistical agencies to measure money.

Our focus is not only on differences in their average behavior but also during some important periods of time, such as around business cycle turning points and across high and low inflation and interest rate phases. We propose a factor model with regime switching to evaluate the common dynamics of the indexes as well as their idiosyncratic movements.

We find some interesting new results. The idiosyncratic terms for both indexes display a business cycle pattern, especially since 1980. We also find that the major differences between the simple sum aggregates and Divisia indexes occur around the beginning and end of economic recessions, and during some high interest rate phases. The period between 1977 and 1983 is the one where the most notable differences take places. This period not only includes a slowdown, two recessions, two recoveries and the change in the Fed's operating procedure, but it also corresponds to a high interest rate phase, which took place from 1977:2 to 1981:2.

These results suggest that further investigation on the differences of these series is warranted. In particular, we have as on-going projects the examination of the relationship between money, output, and prices using the framework proposed in this paper.

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Table 1: Maximum Likelihood Estimates

Parameters	M1 and MSI1	M2 and MSI2	M3 and MSI3
α_0	-0.226 (0.022)	0.621 (0.115)	-0.767 (0.137)
α_1	0.636 (0.226)	0.731 (0.195)	0.949 (0.141)
Φ	0.556 (0.070)	0.518 (0.082)	0.497 (0.071)
d_M	0.431 (0.084)	0.976 (0.020)	0.962 (0.039)
d_{MSI}	0.979 (0.010)	0.589 (0.095)	0.603 (0.075)
σ^2	0.511 (0.056)	0.254 (0.038)	0.157 (0.026)
σ^2_M	0.030 (0.003)	0.006 (0.003)	0.005 (0.002)
σ^2_{MSI}	1.099 (0.018)	0.047 (0.007)	0.093 (0.011)
$\lambda_{\mathbf{M}}$	1.099 (0.018)	0.977 (0.034)	1.172 (0.054)
p^{α}_{00}	0.987 (0.016)	0.970 (0.031)	0.857 (0.076)
p^{α}_{11}	0.941 (0.059)	0.795 (0.150)	0.967 (0.022)
p^{β}_{00}	0.560 (0.209)	0.633 (0.144)	0.992 (0.009)
p^{β}_{11}	0.967 (0.019)	0.977 (0.011)	0.976 (0.021)
p^{δ}_{00}	0.954 (0.019)	0.681 (0.138)	0.679 (0.136)
p^{δ}_{11}	0.701 (0.137)	0.971 (0.014)	0.972 (0.014)
β_0	-0.322 (0.063)	-0.549 (0.059)	-0.040 (0.010)
β_1	0.024 (0.012)	0.009 (0.002)	0.262 (0.015)
δ_0	-0.018 (0.010)	-0.703 (0.433)	-0.857 (0.086)
δ_1	0.096 (0.020)	0.008 (0.003)	0.051 (0.020)
τ	0.002 (0.001)	0.002 (0.001)	0.004 (0.0007)
LogL(θ)	-88.404	-68.893	-77.295

Asymptotic standard errors in parentheses.

Table 2: Correlation Between Monetary Indexes and Dynamic Factors

Parameters	M_1	MSI_1	M_2	MSI_2	M_3	MSI_3
DFM_1	**0.988**	**0.998**	0.337	0.423	0.150	0.265
DFM_2	0.354	0.339	**0.947**	**0.963**	0.767	0.883
DFM_3	0.120	0.128	0.793	0.732	**0.987**	**0.902**
M_1	1	0.984	0.354	0.429	0.139	0.260
MSI_1	0.984	1	0.332	0.418	0.151	0.261
M_2	0.354	0.332	1	0.894	0.802	0.806
MSI_2	0.429	0.418	0.894	1	0.693	0.904
M_3	0.139	0.151	0.802	0.693	1	0.858
MSI_3	0.260	0.261	0.806	0.904	0.858	1

Figure 1a – Smoothed Inflation (—), Inflation (—),High Inflation Phases (—), and NBER Recessions (Shaded Area)

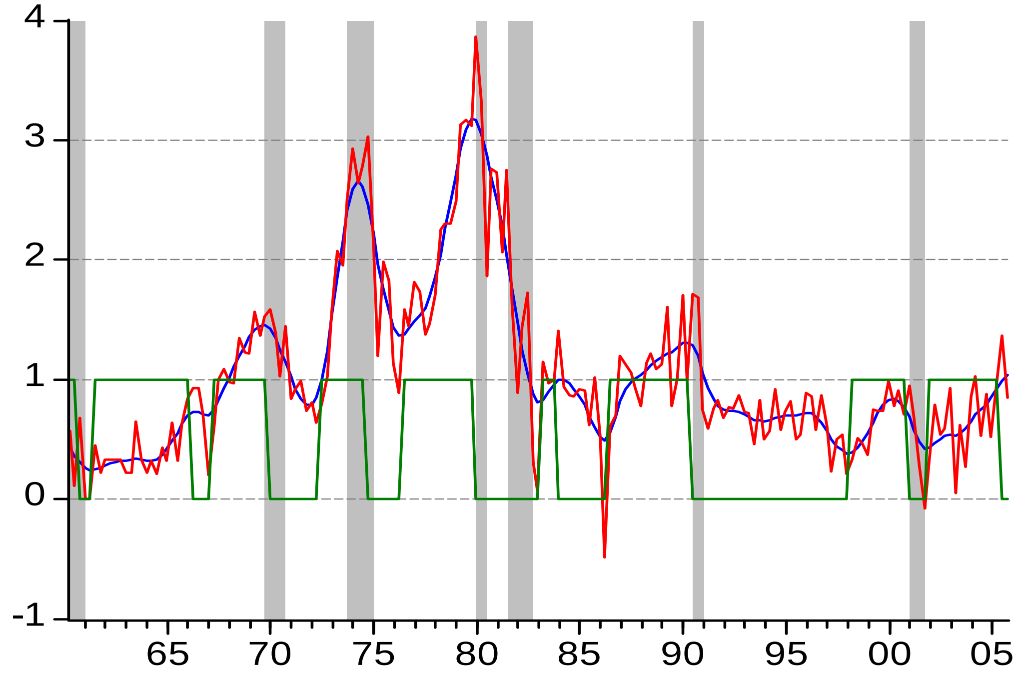

Figure 1b – Interest Rates (—), High Interest Rates Phases (—), and NBER Recessions (Shaded Area)

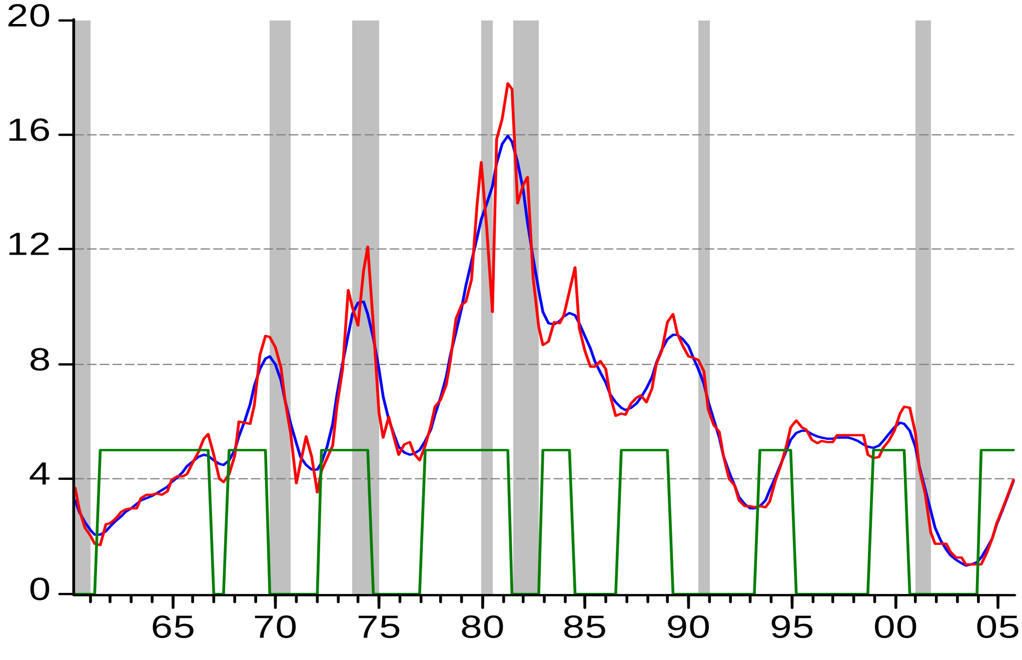

Figure 1c – Dynamic Factors from the Pairs M1-MSI1 Growth (—), M2-MSI2 Growth (—) and M3-MSI3 (—), High Interest Rate Phases (—), High Inflation Phases (—), and NBER Recessions (Shaded Area)

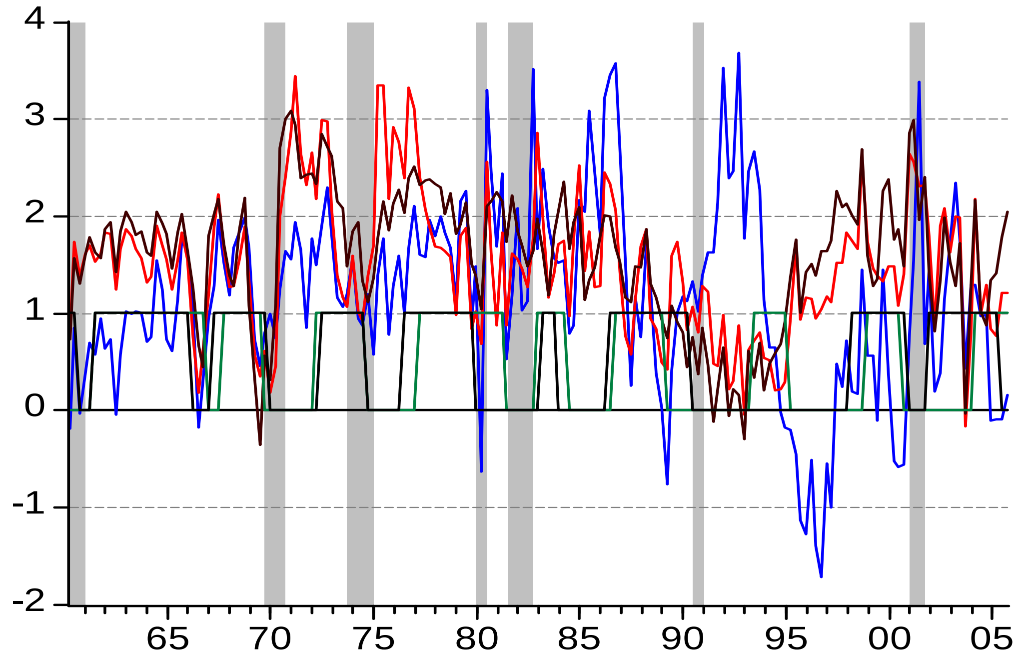

Figure 2a – Dynamic Factor (—) and Probabilities of High Monetary Growth Based on M1 and MSIa (—), and NBER Recessions (Shaded Area)

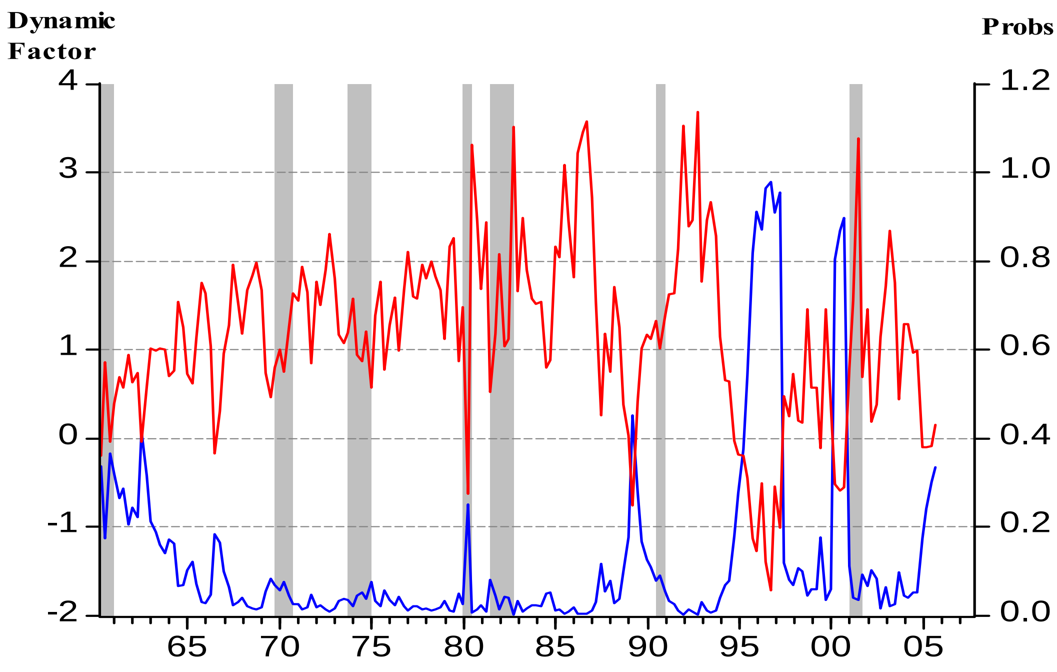

Figure 2b – Dynamic Factor (—), Rate of Growth of M1 (—) and MSI1 (—), and NBER Recessions (Shaded Area)

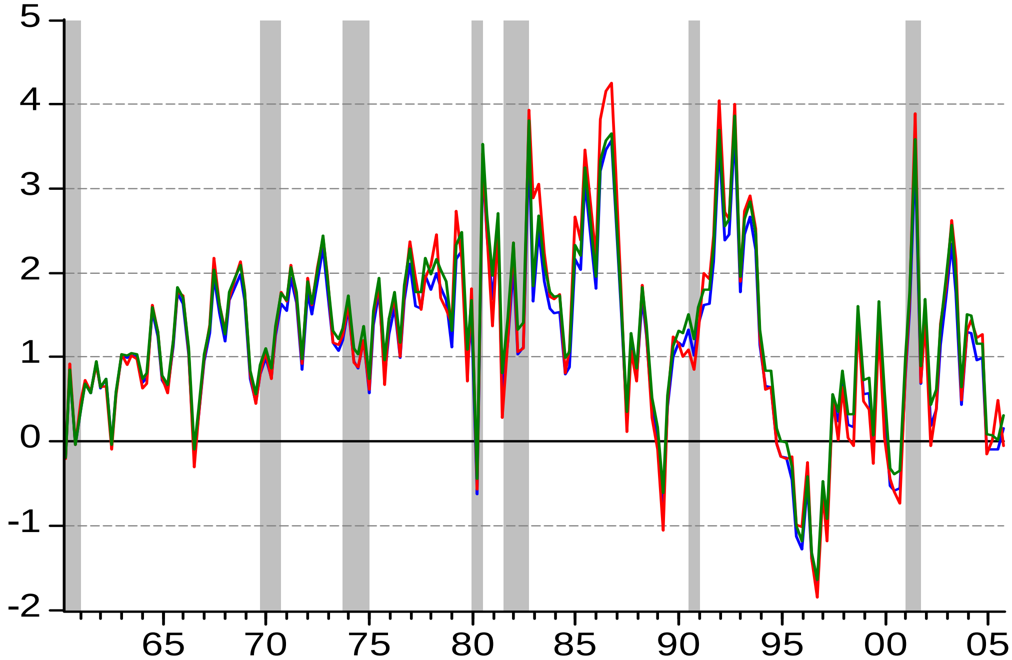

Figure 2c – Idiosyncratic Terms for M1 (—) and MSI1 Growth (—),High Interest Rate Phases (—), High Inflation Phases (—), and NBER Recessions (Shaded Area)

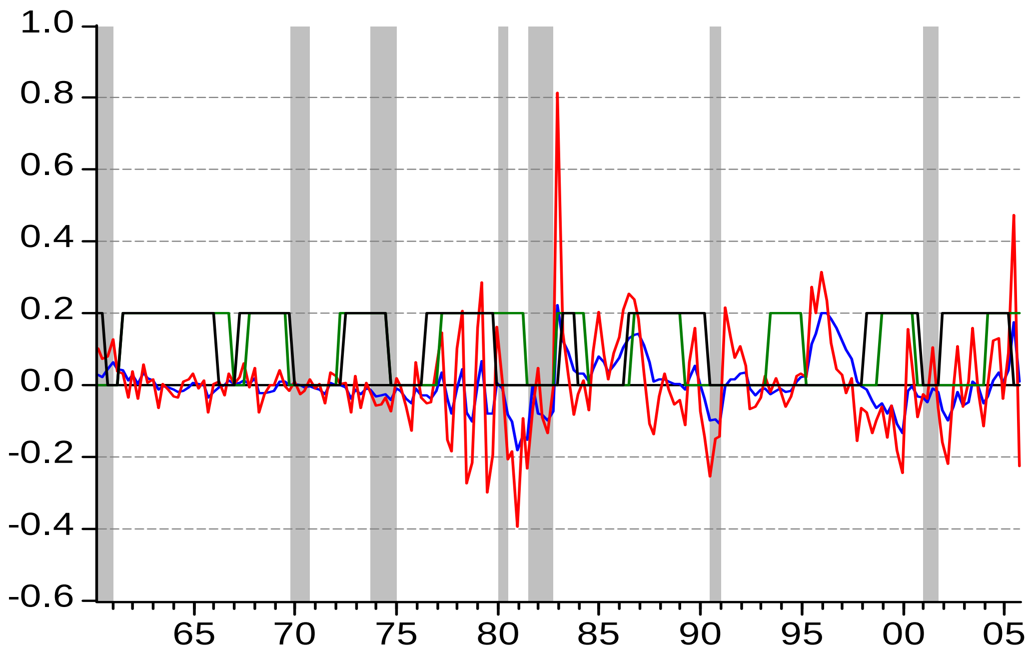

Figure 2d – Difference between Idiosyncratic Terms for M1 and MSI1 Growth Without (—), and With Dummy (—),High Interest Rate Phases (—), High Inflation Phases (—), and NBER Recessions (Shaded Area)

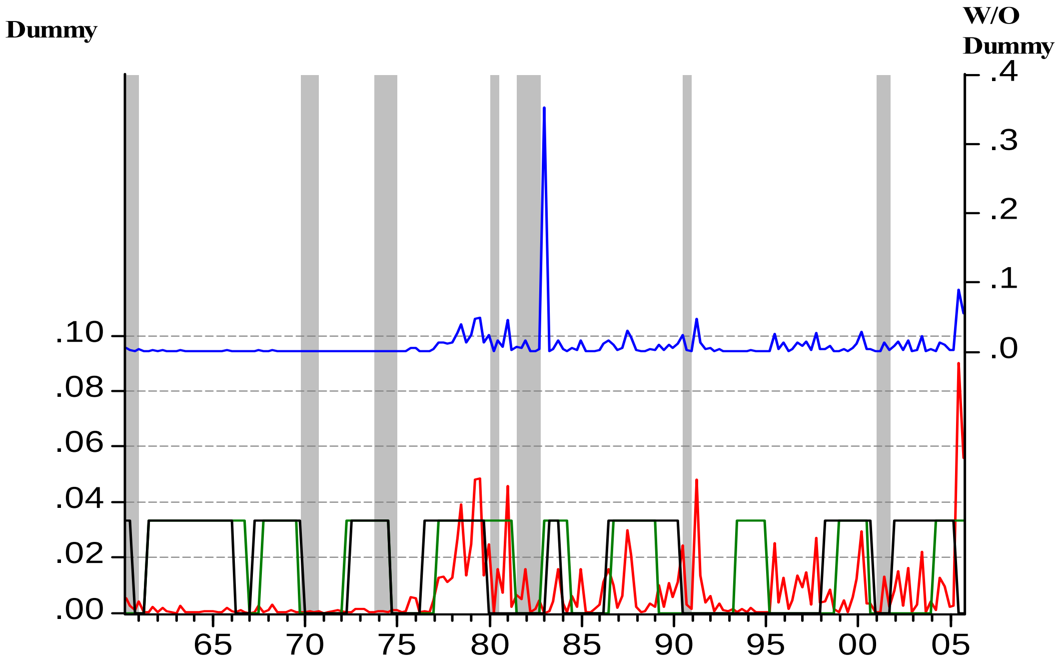

Figure 2e – Measurement Errors for M1 (—) and MSI1 Growth (—), High Interest Rate Phases (—), High Inflation Phases (—), and NBER Recessions (Shaded Area)

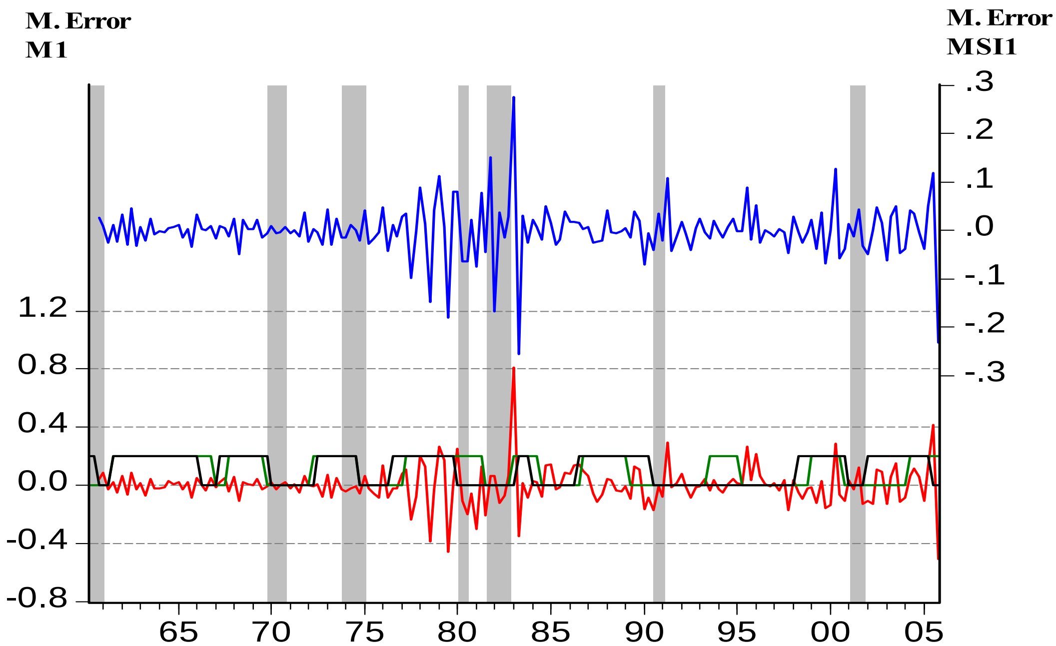

Figure 2f – Difference between Measurement Errors for M1 and MSI1 Growth Without (—), and With Dummy (—), High Interest Rate Phases (—), High Inflation Phases (—), and NBER Recessions (Shaded Area)

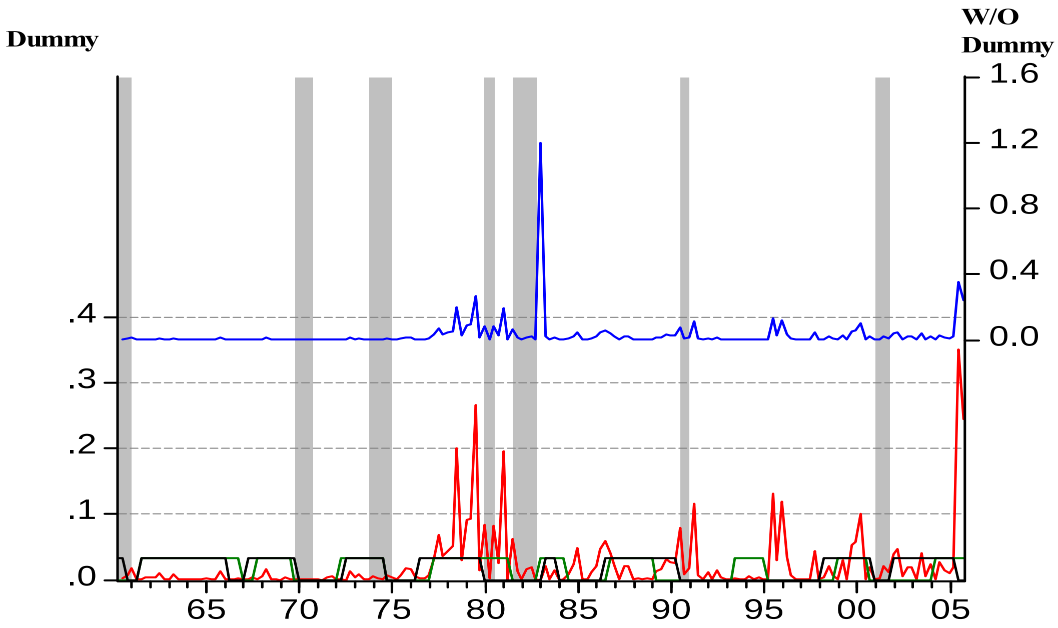

Figure 3a – Dynamic Factor (—) and Probabilities of High Monetary Growth Based on M2 and MSI2 (—), and NBER Recessions (Shaded Area)

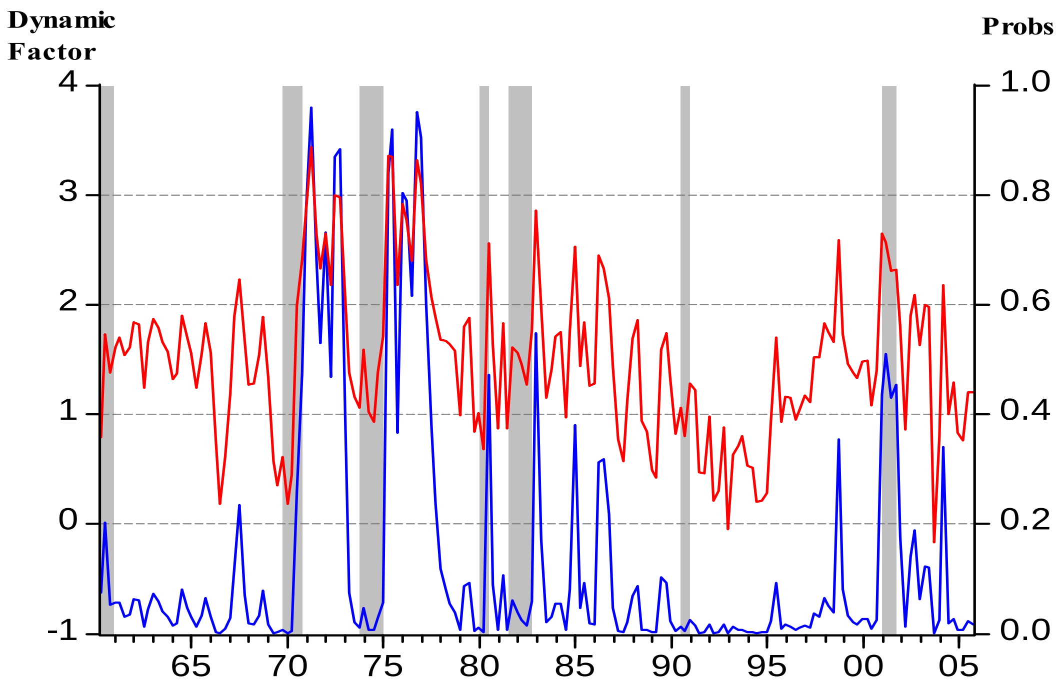

Figure 3b – Dynamic Factor (—), Rate of Growth of M2 (—) and MSI2 (—), and NBER Recessions (Shaded Area)

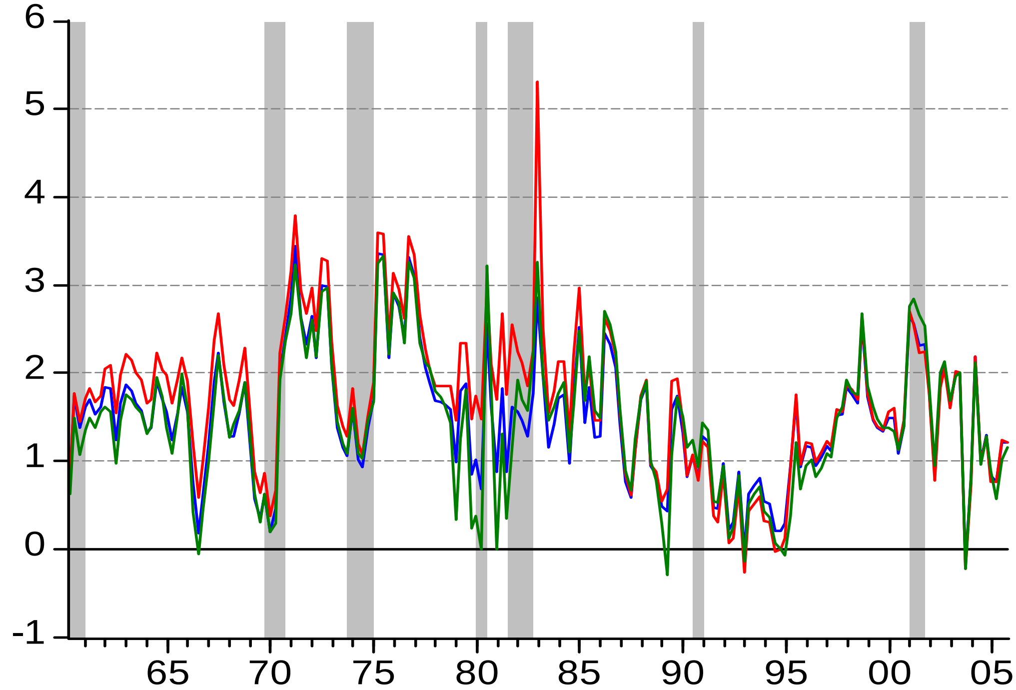

Figure 3c – Idiosyncratic Terms for M2 (—) and MSI2 Growth (—),High Interest Rate Phases (—), High Inflation Phases (—), and NBER Recessions (Shaded Area)

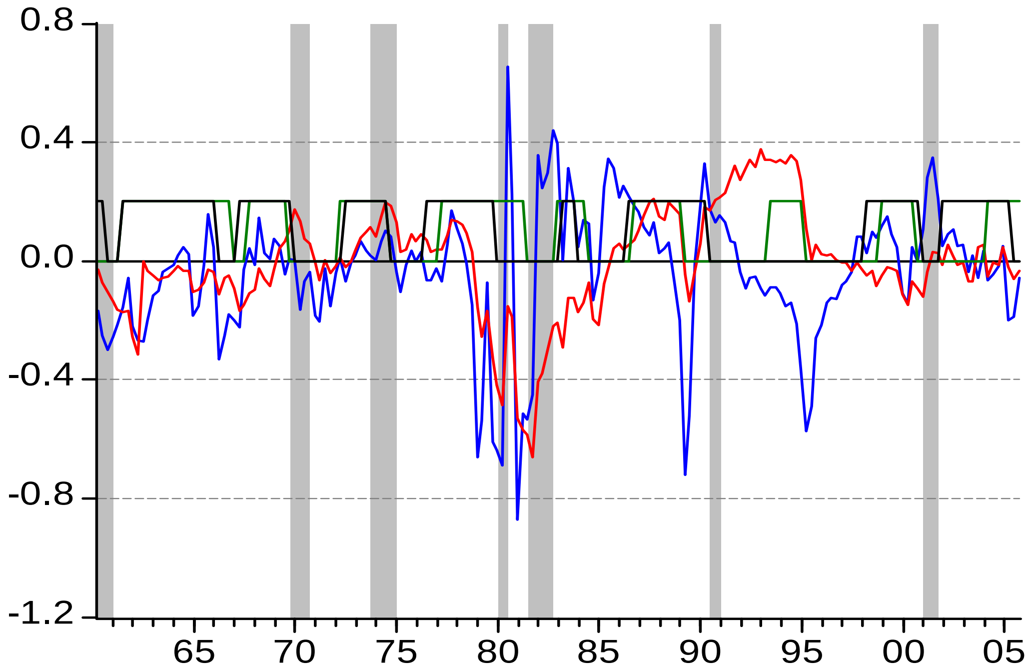

Figure 3d – Difference between Idiosyncratic Terms for M2 and MSI2 Growth (—), High Interest Rate Phases (—), High Inflation Phases (—), and NBER Recessions (Shaded Area)

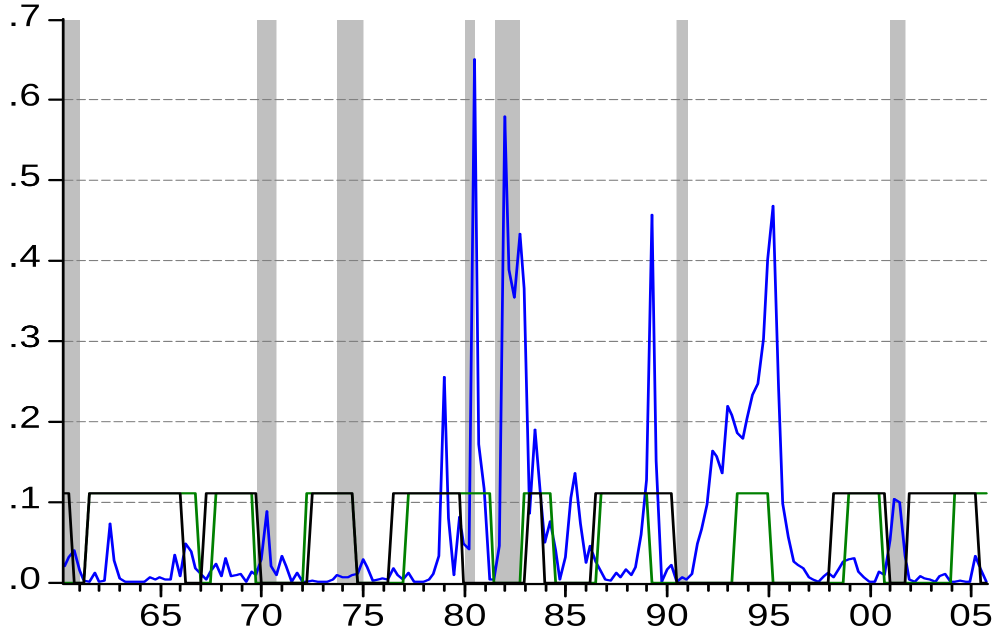

Figure 3f – Difference between Measurement Errors for M2 and MSI2 Growth (—), High Interest Rate Phases (—), High Inflation Phases (—), and NBER Recessions (Shaded Area)

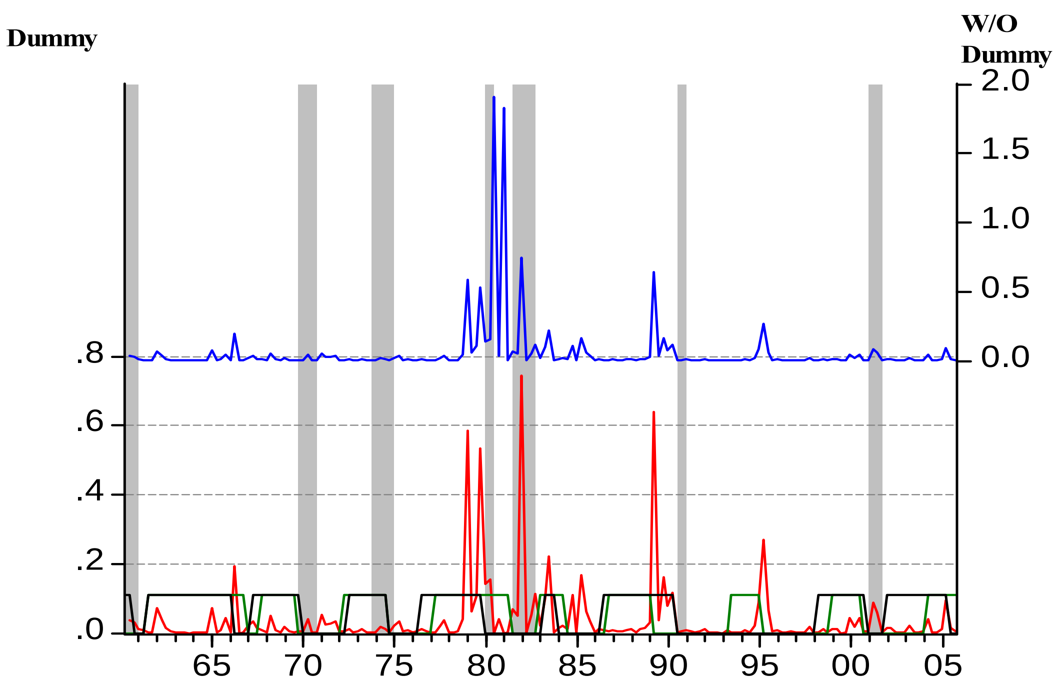

Figure 4a – Dynamic Factor (—) and Probabilities of Low Monetary Growth Based on M3 and MSI3 (—), and NBER Recessions (Shaded Area)

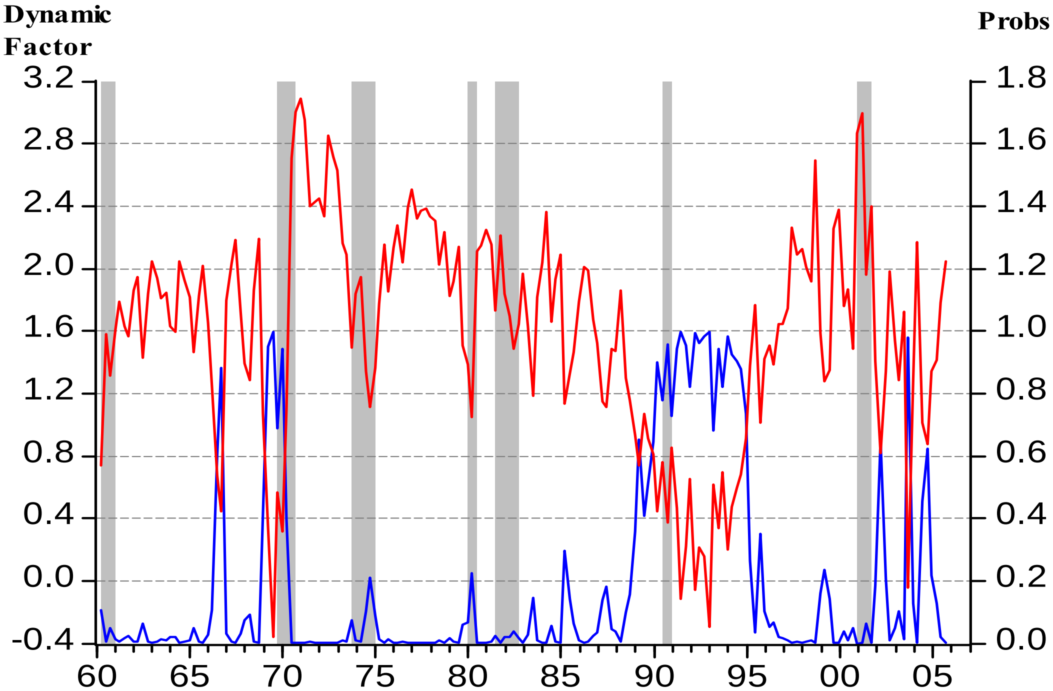

Figure 4b – Dynamic Factor (—), Rate of Growth of M3 (—) and MSI3 (—), and NBER Recessions (Shaded Area)

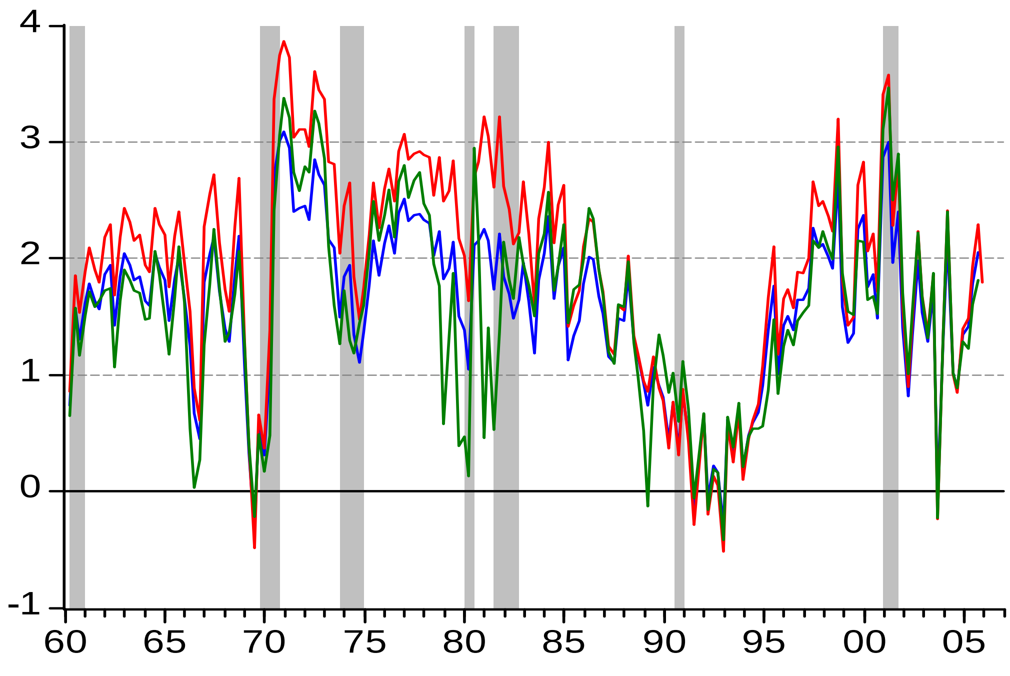

Figure 4c – Idiosyncratic Terms for M3 (—) and MSI3 Growth (—), High Interest Rate Phases (—), High Inflation Phases (—), and NBER Recessions (Shaded Area)

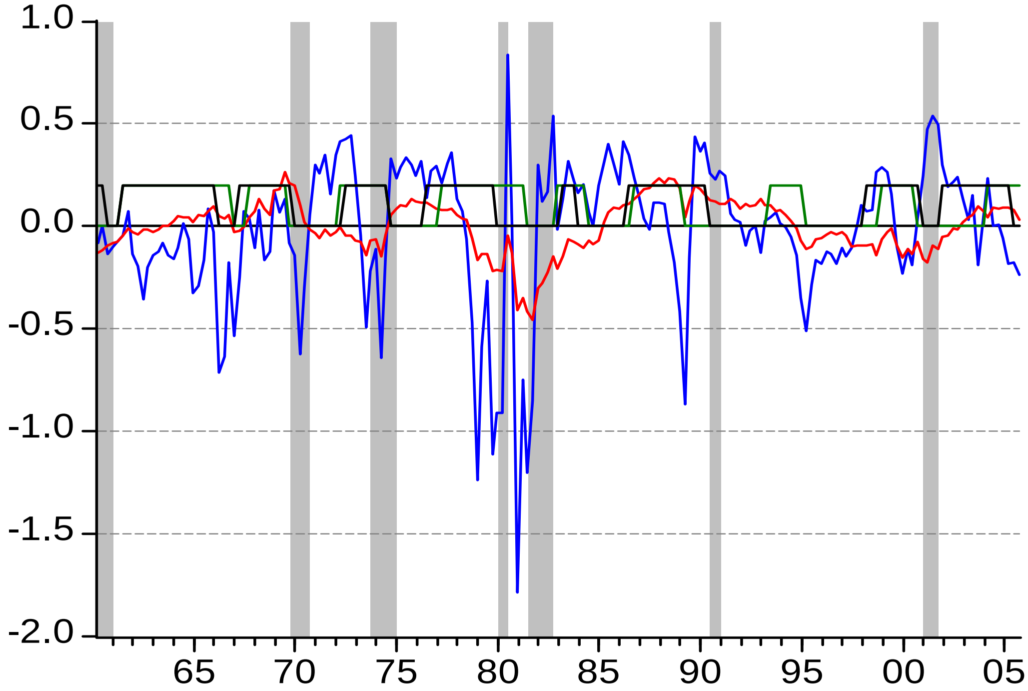

Figure 4d – Difference between Idiosyncratic Terms for M3 and MSI3 Growth Without (—), and With Dummy (—),High Interest Rate Phases (—), High Inflation Phases (—), and NBER Recessions (Shaded Area)

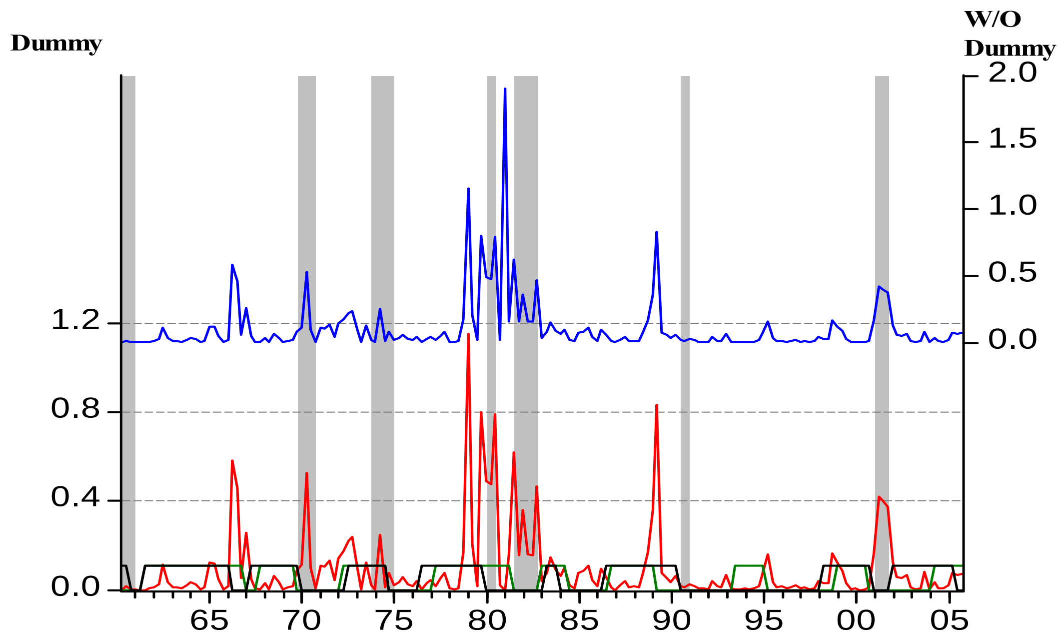

Figure 4e – Difference between Measurement Errors for M3 and MSI3 Growth Without (—), and With Dummy (—), High Interest Rate Phases (—), High Inflation Phases (—), and NBER Recessions (Shaded Area)