A Numerical Method for Pricing Electricity Derivatives for Jump-Diffusion Processes Based on Continuous Time Lattices

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ABSTRACT

We present a numerical method for pricing derivatives on electricity prices. The method is based on approximating the generator of the underlying process and can be applied for stochastic processes that are combinations of diffusions and jump processes. The method is accurate even in the case of processes with fast mean-reversion and jumps of large magnitude. We illustrate the speed and accuracy of the method by pricing European and Bermudan options and calculating the hedge ratios of European options for the Geman-Roncoroni model for electricity prices.

Introduction

Jump-diffusion models are widely used in energy finance to describe the behavior of spot electricity prices. While the jumps are due to limited generation supply and inelastic and volatile demand which result in electricity prices that fluctuate violently over short periods of time, the difficulty of calibrating models based on supply and demand has led researchers to try to match characteristics of electricity prices using reduced form models, allowing for large jumps.[1]

A problem that arises in using models with jumps is the relative lack of fast and accurate numerical methods for pricing derivative contracts and for determining the optimal exercise policy of American-type claims. A straightforward method based on Monte-Carlo simulation is inefficient, because of slow convergence due to the large magnitude of the jumps, and to inherent difficulties in identifying the optimal exercise policy. In this paper we present an alternative numerical method that tries to overcome this difficulty. Instead of approximating the process directly, we approximate the generator of process, under the risk-neutral measure, by the generator of a discrete Markov chain.[2]

The main idea of the method is to choose the generator of the Markov chain so that it converges, in the limit of an infinite number of states, to the generator of the underlying continuous process. Approximating the generator of the process has the advantage that jumps are explicitly accounted for in the transition probabilities between the states of the Markov chain. Another advantage is that the time between the steps of the Markov chain can be chosen arbitrarily without influencing the accuracy of the approximation. The flexibility of choosing the time steps allows us to consider the values of the Markov chain only at times of interest based on the specifics of the problem; e.g., possible exercise dates, settlement dates, etc.

We illustrate our method by applying it to the case of pricing electricity derivatives using the model proposed by Geman and Roncoroni (2006) applied to average daily, on-peak, electricity prices. Due to lack of closed-form formulæfor European option prices and hedge-ratios in the Geman-Roncoroni model, it is difficult to measure the accuracy of our method. Monte Carlo method is not suitable to calculate the hedge-ratios in this case. To address this issue, we apply our method to the Merton's jump-diffusion model where it posses semi-analytic formula for price and hedge-ratios for European call option and compare the results by applying our method against them.

[1] Such reduced form models models have been proposed, for example, by Kholodnyi (2004) and Geman and Roncoroni (2006) — see also Eydeland and Wolyniec (2003) for an overview of both reduced form and structural models, and the difficulties involved in their calibration.

[2] Our method is an extension of the method described in Albanese and Kuznetsov (2003), adapted to the case of electricity processes (other methods for pricing derivatives in the presence of jumps include Hirsa and Madan (2004), Cont and Tankov (2004), and Levendorskii, Kudryavtsev, and Zherder (2006)).

The paper is organized as follows: We describe the numerical method in the context of the Merton jump-diffusion model in Section 1. Section 2 discusses the modifications necessary to apply the method to the Geman-Roncoroni model of electricity prices. Section 3 provides a calibration and numerical simulations of option prices and hedge-ratios of European options and the optimal exercise threshold of daily-exercisable Bermudan options for the Geman-Roncoroni model. Section 4 concludes.

1. Description of the Numerical Method

We illustrate the numerical method for the case of the Merton jump-diffusion model. This model is an ideal candidate to demonstrate the accuracy of the method since the underlying stochastic process exhibits jumps and allows for closed-form formulas for the price and hedge-ratios of European call options.

Merton's jump-diffusion model under the risk-neutral measure can be specified through the stochastic differential equation

$$\frac{dS_t}{S_{t-}} = (r - \lambda m)dt + \sigma dW_t + (J_t - 1)dN_t \tag{1}$$

where S_{t-} stands for the left limit of S at time t, r is a constant risk-free interest rate, σ is a constant diffusion volatility, W is a standard Brownian motion, and N_t is a Poisson process with constant intensity λ which is independent of W. J_t is the amplitude of the multiplicative jump size which is lognormally distributed with expectation $\mathbf{E}[J_t] = \overline{J}$, variance $\mathbf{Var}[\log J_t] = b^2$ and $m = \overline{J} - 1$.

The price, delta, and gamma of a European call option are given by

$$\text{Price} = \sum_{n=0}^{\infty} e^{-\lambda' t} \frac{(-\lambda' t)^n}{n!} \text{BS}(S_0, \sigma_n, T, r_n, K) \tag{2}$$

$$\text{Delta} = \sum_{n=0}^{\infty} e^{-\lambda' t} \frac{(-\lambda' t)^n}{n!} \text{BSD}(S_0, \sigma_n, T, r_n, K) \tag{3}$$

$$\text{Gamma} = \sum_{n=0}^{\infty} e^{-\lambda' t} \frac{(-\lambda' t)^n}{n!} \text{BSG}(S_0, \sigma_n, T, r_n, K) \tag{4}$$

where $\lambda' = \lambda(1 + m)$, $\sigma_n^2 = \sigma^2 + b^2 n/T$, $r_n = r - \lambda m + n \log(1 + m)/T$, and BS(.), BSD(.), BSG(.) denotes the Black-Scholes call option price, delta and gamma formula respectively. We assume that the jump component is independent of the diffusion component and that the parameters, including the risk-free interest rate r are constant.

We consider equation (1) in logarithmic scale. By Ito's lemma,

$$d \log S_t = (r - \lambda m - \frac{1}{2}\sigma^2)dt + \sigma dW_t + \log J_t dN_t \tag{5}$$

Let $\xi_t = \log S_t$.

To price options under the stochastic process described in Equation (5) we approximate the evolution of the stochastic differential equation on a discrete lattice of prices. Let Ω be a finite set $\{0, \cdots, N\}$ containing the first N integers together with 0 and let $\xi : \Omega \to \mathbb{R}$ be a non-negative function which satisfies the following two conditions: $\xi(0) \geq 0$ and $\xi(x) > \xi(x-1)$ for all $x = 1, \ldots, N$. Given such a function ξ, the discretized price process ξ_t^Ω can take any of the values $\xi(x)$, where x is an element in Ω and time $t \in \mathbb{R}_+$.

The dynamics of the price are specified by the Markov generator $\mathcal{L}_\Omega$ which is given by

$$\mathcal{L}_\Omega = \mathcal{L}_\Omega^d + \mathcal{L}_\Omega^j \tag{6}$$

where $\mathcal{L}_\Omega^d$ is the discretization of the generator of the diffusion component and $\mathcal{L}_\Omega^j$ is the discretization of the generator of the jump component.

We first consider the diffusion part of the model. The Markov generator of a diffusion process given by the stochastic differential Equation (5) acts in the following way on any twice differentiable real function ϕ:

$$(\mathcal{L}^d \phi)(\xi^d) = (r - \lambda m - \frac{1}{2}\sigma^2)\frac{\partial \phi}{\partial \xi^d}(\xi^d) + \frac{\sigma^2}{2}\frac{\partial^2 \phi}{\partial {\xi^d}^2}(\xi^d).$$

To insure that the dynamics of the discretized energy price process corresponds to the dynamics specified by the Equation (5), elements of the discretized Markov generator for diffusion $\mathcal{L}_\Omega^d(x, y)$ are obtained by solving the following linear systems for all $x, y \in \Omega$, which guarantee that the mean and variance of the diffusion part of the process are matched locally:

$$\sum_{y\in\Omega} \mathcal{L}^d_\Omega(x,y) = 0, \tag{7}$$

$$\sum_{y\in\Omega} \mathcal{L}^d_\Omega(x,y)(\xi(y)-\xi(x)) = r-\lambda m-\frac{1}{2}\sigma^2, \tag{8}$$

$$\sum_{y\in\Omega} \mathcal{L}^d_\Omega(x,y)(\xi(y)-\xi(x))^2 = \sigma^2. \tag{9}$$

Equation (7) ensures probability conservation over the infinitesimal time interval. Equations (8) and (9) are the instantaneous first and second moment matching conditions for the discretized log-price process ξ^Ω_t respectively.

At each end of the domain Ω, we impose absorbing boundary conditions by setting $\mathcal{L}^d_\Omega(x,y) = 0$ for all $y \in \Omega, x = 0, N$. This is a reasonable requirement of the underlying process for two reasons. First, since the size of the set Ω is a parameter of our model, we can make sure that we choose it large enough so that the process does not reach the boundary. Second, this choice of boundary conditions makes it easy to detect if the domain Ω is not large enough, which would not necessarily be the case had we used reflecting boundary conditions. The resulting matrix $\mathcal{L}^d_\Omega(x,y)$ is tri-diagonal and in the continuous state-space limit reduces to the generator of a diffusion $\mathcal{L}^d$.

After obtaining the Markov generator for the diffusion component, we consider the jump component of the Merton model. The jumps in the model are controlled by a Poisson process with constant intensity. In the time interval $(t, t+\delta t)$ the probability of a jump is roughly proportional to δt, while the probability of two or more jumps is negligible.

The Markov generator for the jump component is given by:

$$\mathcal{L}^j_\Omega(x,y) = \begin{cases} \lambda \int_A^B \phi(z,\mu,b^2)dz = \lambda[\Phi(B,\mu,b^2)-\Phi(A,\mu,b^2)], \text{if } x \neq y \\ -\sum_{z\neq x} \mathcal{L}^j_\Omega(x,z), \text{if } x = y \end{cases}$$

for all $x, y \in \Omega$, where

$$\begin{aligned}
\phi(z,\mu,b^2) &= \frac{1}{\sqrt{2\pi b}} e^{-\frac{1}{2}\left(\frac{z-\mu}{b}\right)^2} \\
\Phi(X,\mu,b^2) &= \int_{-\infty}^{X} \phi(z,\mu,b^2)dz \\
\mu &= \log(\overline{J}) - \frac{1}{2}b^2 \\
A &= \frac{\xi(y)+\xi(y-1)}{2} - \xi(x) \\
B &= \frac{\xi(y)+\xi(y+1)}{2} - \xi(x)
\end{aligned}$$

A. Computing the probability kernel

To obtain the probability kernel of the process ξ_t^Ω from its Markov generator $\mathcal{L}_\Omega$, we consider the following eigenvalue problem

$$\mathcal{L}_\Omega u_n = \lambda_n u_n$$
$$\mathcal{L}_\Omega^T v_n = \lambda_n v_n$$

where λ_n are the eigenvalues of the operator $\mathcal{L}_\Omega$ and u_n and v_n are its right and left eigenvectors respectively and the superscript T denotes matrix transposition. In general, the Markov generator is not symmetric, hence u_n and v_n are different and the eigenvalues are not real. We can only guarantee that the real part of the eigenvalues is non-positive and that the complex eigenvalues appear in pairs, in the sense that if λ is an eigenvalue then its complex conjugate, $\overline{\lambda}$, is also an eigenvalue.

We assume that the operator $\mathcal{L}_\Omega$ admits a complete set of eigenvectors. If U is a matrix whose columns are given by the eigenvectors u_n we have

$$\mathcal{L}_\Omega = U\Lambda U^{-1}$$

where Λ is a diagonal matrix having eigenvalues $\{\lambda_i\}_{i=0}^N$ as elements and $U^{-1} = V$ with V a matrix whose rows are given by the vectors v_n.

Key to our construction is the remark that, if the Markov generator is diagonalizable, we can apply to it an arbitrary function ϕ, defined on the spectrum of the generator, by means of the following formula:

$$\phi(\mathcal{L}_\Omega) = U\phi(\Lambda)U^{-1}. \tag{10}$$

Formula (10) is useful because the task of calculating $\phi(\Lambda)$ is a very simple one indeed:

$$\phi(\Lambda) = \begin{pmatrix} \phi(\lambda_0) & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & \phi(\lambda_N) \end{pmatrix}.$$

Formula (10) is the basis of "functional calculus". As Itô's formula regarding functions of stochastic processes is central in the stochastic analysis for diffusion processes, functional calculus for Markov generators plays a pivotal role in our framework.[3] Formula (10) allows us to express $p(x,t;y,T)$ by

$$p(x,t;y,T) = (e^{(T-t)\mathcal{L}_\Omega})(x,y) = \sum_{n=0}^{N} e^{\lambda_n(T-t)} u_n(x) v_n(y).$$

B. Regularization of the Markov Chain Generator

A potential problem in the representation of the transition density $p(x,t;y,T)$ as

$$p(x,t;y,T) = \left(Ue^{(T-t)\Lambda}U^{-1}\right)(x,y) \tag{11}$$

arises in the case of underlying stochastic processes with large drift or large asymmetric jumps, common for processes describing electricity prices. In such cases the Markov generator $\mathcal{L}_\Omega$ is highly asymmetric, leading to large condition numbers and to numerical instabilities when inverting the matrix U.[4]

We propose a regularization algorithm to account for numerical instabilities. The method avoids diagonalization of the Markov generator $\mathcal{L}_\Omega$, by estimating the transition densities by means of *binary-exponentiation*. Our objective is to find the transition density $p(x,t;y,T)$ induced by $\mathcal{L}_\Omega$ and it must satisfy the following four criteria:

1. Conservation of probability: The sum of transition densities should equal one

$$\sum_y p(x,t;y,T) = 1$$

[3] For more details see Albanese and Kuznetsov (2003).
[4] The condition number of a matrix is defined as the ratio of its largest to its smallest eigenvalue.

2. Positivity: Transition densities should be between zero and one

$$0 \leq p(x,t;y,T) \leq 1$$

3. Real values: Transition densities should be real numbers.

4. Chapman-Kolmogorov conditions:

$$\sum_z p(x,t_1;z,t_2)p(z,t_2;y,T) = p(x,t;y,T), \quad \text{where } t_1+t_2=T.$$

We will use the four criteria to measure the accuracy of the approximation of the transition densities calculated by using the regularization method.

The numerical instability in equation (11) is due to the inversion of the right eigenvector matrix U. Our aim in the regularization method is to find the transition density matrix $\mathbf{P}_T$ from the Markov generator $\mathcal{L}_\Omega$ without diagonalization.

The transition density matrix $\mathbf{P}_{dt}$ induced by the Markov generator $\mathcal{L}_\Omega$ is given as:

$$\mathbf{P}_{dt} = e^{dt\mathcal{L}_\Omega}.$$

If dt is small, $\mathbf{P}_{dt}$ can be approximated by

$$\mathbf{P}_{dt} \approx \mathbf{I} + dt\mathcal{L}_\Omega,$$

where $\mathbf{I}$ is the identity matrix, and the error is of order $(dt)^2$.

Since $\mathcal{L}_\Omega$ is time homogenous, $\mathbf{P}_T$ can be obtained by:

$$\mathbf{P}_T = (\mathbf{P}_{dt})^{T/dt}, \tag{12}$$

where T/dt is an integer.

While dt is small, T/dt can be quite large and to compute $(\mathbf{P}_{dt})^{T/dt}$ is computationally expensive. A common trick is to use an algorithm so-called binary-exponentiation to overcome this problem. First we find a constant τ_1 such that $\mathbf{P}_{\tau_1}(x,y)$ satisfy the four criteria that are described earlier in this section for all $x, y \in \Omega$. By the Perron-Frobenius theorem[5], the spectral radius for any finite transition density matrix equals one. This will guarantee the power of the matrix $\mathbf{P}_{\tau_1}$ would not explode and also satisfy

[5]For more details see (Horn and Johnson 1990)

the four criteria for transition density. Second, we find an integer ν, which is the smallest integer such that

$$2^{\nu} > \frac{T}{\tau_1}.$$

Finally, we set $\tau_2 = T/(2^{\nu})$. We can now rewrite equation (12) as:

$$\begin{aligned}
\mathbf{P}_{2\tau_2} &= \mathbf{P}_{\tau_2} \times \mathbf{P}_{\tau_2} \\
\mathbf{P}_{4\tau_2} &= \mathbf{P}_{2\tau_2} \times \mathbf{P}_{2\tau_2} \\
&\dots \\
\mathbf{P}_T = \mathbf{P}_{2^{\nu}\tau_2} &= \mathbf{P}_{2^{(\nu-1)}\tau_2} \times \mathbf{P}_{2^{(\nu-1)}\tau_2}
\end{aligned} \tag{13}$$

where only $\nu = \log_2(T/\tau_2)$ matrix multiplications are performed. Clearly, as we decrease τ_2, the approximation to equation (11) becomes more accurate but it will slow down the computation.

C. Hedge-ratios

Let $C_0(x)$ be the price of an European option on the spot energy price given the current spot energy price is $S(x)$, where $S(x) = \exp(\xi(x))$ and $x \in \Omega$. The delta and gamma of $C_0(x)$ can be computed using symmetric differences

$$\begin{aligned}
\Delta(x) &:= \frac{C_0(x+1) - C_0(x-1)}{S(x+1) - S(x-1)} \\
\Gamma(x) &:= 4\frac{C_0(x+1) + C_0(x-1) - 2C_0(x)}{(S(x+1) - S(x-1))^2},
\end{aligned}$$

Notice that calculating $C_0(x+1)$, or any other value of the option $C_0(y)$ with a starting point $y \in \Omega$ different from x, requires no further calculations because the probability density function corresponding to a value other than x is given by a different row of the matrix $\mathbf{P}_t$, which has already been calculated during the pricing of the original contract $C_0(x)$.

D. Numerical examples

We illustrate the accuracy of the numerical method under the following set of parameters: spot price $S_0 = 100$, time to maturity $T = 2$, risk free rate $r = 0$, diffusion volatility $\sigma = 50\%$, jump intensity

$\lambda = 75\%$, mean of jump size $\overline{J} = 80\%$ and jump size volatility $b = 75\%$. The lattice $\xi(.)$ is evenly spaced with $\xi(0) = 0, \xi(N) = 11.9184$ and $N = 500$. Tables 1, 2 and 3 show the European call option prices, deltas and gammas respectively. The second row of each table shows the time to compute all the prices, deltas and gammas for that particular column (i.e. for the results in all three tables). The second column is the strike of the European call options, the third column is the value calculated based on the closed-form formula for Merton's jump-diffusion model price, delta and gamma. The forth column is the result obtained using the representation $e^{T\mathcal{L}_\Omega}$ without regularization. Notice that the Merton's jump-diffusion model with the above parameters does not exhibit numerical instabilities when using the representation $e^{T\mathcal{L}_\Omega}$ to compute the transition density. The fifth to ninth column are the results obtained by using the binary-exponentiation with different τ_2, where τ_2. We can see from all three tables that as τ_2 decreases, the result by the binary-exponentiation is closer to the closed-form solution. Since there are no numerical instabilities introduced by the above set of parameters, the representation $e^{T\mathcal{L}_\Omega}$ can be regarded as the binary-exponentiation with $\tau_2 \to 0$. Therefore, one would expect the representation $e^{T\mathcal{L}_\Omega}$ approximates the closed-form solution better than binary-exponentiation. Furthermore the smallest diagonal entry of $\mathcal{L}_\Omega$ is -72.45082 (i.e. min $\mathcal{L}_\Omega(x,x)$, for all $x \in \Omega$.), this implies if $\tau_2 < \frac{1}{72.45082} = 0.0138$, $\mathbf{P}_{\tau_2}(x,y)$ is guaranteed to satisfy the four criteria that are described in Section 2.B for all $x, y \in \Omega$.

We show that if there are no numerical instabilities, one would want to use the representation $e^{T\mathcal{L}_\Omega}$ to compute the transition density because is it more accurate and faster than binary-exponentiation. There are two sources of discretization error introduced by binary-exponentiation. We have illustrated the time discretization error in the tables above; we showed that as τ_2 decreases, the result converges to the closed-form solution. The representation $e^{T\mathcal{L}_\Omega}$ is immune from the time discretization error as one can consider it is taking $\tau_2 \to 0$. The other source of discretization error is the state-space discretization error which appears in both methods. In Tables 1, 2 and 3 the number of lattice points N is 500. Table 4, 5 and 6 shows the European call option prices, deltas and gammas using the representation $e^{T\mathcal{L}_\Omega}$ with different values of N. All three tables show that as the number of lattice points increases, the closed-form solution is more closely approximated by $e^{T\mathcal{L}_\Omega}$. All the calculations in this paper were performed using Pentium M 1.60GHz processor with 480MB RAM.

Table 1
European Call option prices for Merton's jump-diffusion model.

	Strike	closed-form formula (2)	$e^{T\mathcal{L}_\Omega}$	$\tau_2 = 0.00138$	0.00414	0.00690	0.00966	0.01242
time(s)			41	78	67	62	60	58
	85	46.94320	46.94709	46.94777	46.94912	46.95048	46.95183	46.95319
	90	45.15809	45.16293	45.16365	45.16508	45.16651	45.16795	45.16938
	95	43.46686	43.47072	43.47147	43.47297	43.47447	43.47596	43.47746
	100	41.86367	41.86625	41.86703	41.86858	41.87013	41.87168	41.87323
	105	40.34305	40.34596	40.34676	40.34835	40.34994	40.35153	40.35313
	110	38.89991	38.90250	38.90331	38.90493	38.90656	38.90818	38.90980
	115	37.52949	37.53325	37.53407	37.53571	37.53735	37.53900	37.54064
	120	36.22735	36.23221	36.23304	36.23469	36.23634	36.23799	36.23965
	125	34.98936	34.99414	34.99496	34.99662	34.99827	34.99992	35.00157
	130	33.81166	33.81425	33.81507	33.81671	33.81836	33.82001	33.82165

Table 2
European Call option deltas for Merton's jump-diffusion model.

	Strike	closed-form formula (3)	$e^{T\mathcal{L}_\Omega}$	$\tau_2 = 0.00138$	0.00414	0.00690	0.00966	0.01242
time(s)			41	78	67	62	60	58
	85	0.7812294	0.7812700	0.7812697	0.7812692	0.7812686	0.7812680	0.7812674
	90	0.7642699	0.7643129	0.7643136	0.7643152	0.7643168	0.7643183	0.7643199
	95	0.7474597	0.7475021	0.7475040	0.7475078	0.7475116	0.7475154	0.7475192
	100	0.7308420	0.7308820	0.7308851	0.7308912	0.7308973	0.7309034	0.7309095
	105	0.7144529	0.7144945	0.7144987	0.7145072	0.7145156	0.7145240	0.7145325
	110	0.6983223	0.6983633	0.6983686	0.6983794	0.6983902	0.6984009	0.6984117
	115	0.6824744	0.6825210	0.6825275	0.6825406	0.6825536	0.6825666	0.6825797
	120	0.6669287	0.6669819	0.6669895	0.6670048	0.6670200	0.6670353	0.6670505
	125	0.6517001	0.6517548	0.6517635	0.6517809	0.6517982	0.6518156	0.6518330
	130	0.6368002	0.6368435	0.6368532	0.6368726	0.6368920	0.6369114	0.6369308

Table 3
European Call option gammas for Merton's jump-diffusion model.

	Strike	closed-form formula (4)	$e^{T\mathcal{L}_\Omega}$	$\tau_2 = 0.00138$	0.00414	0.00690	0.00966	0.01242
time(s)			41	78	67	62	60	58
	85	0.0028931	0.0028918	0.0028916	0.0028913	0.0028909	0.0028906	0.0028902
	90	0.0030407	0.0030393	0.0030391	0.0030387	0.0030383	0.0030379	0.0030375
	95	0.0031769	0.0031756	0.0031754	0.0031749	0.0031745	0.0031741	0.0031736
	100	0.0033018	0.0033006	0.0033004	0.0032999	0.0032994	0.0032990	0.0032985
	105	0.0034155	0.0034143	0.0034140	0.0034135	0.0034130	0.0034126	0.0034121
	110	0.0035184	0.0035172	0.0035170	0.0035165	0.0035159	0.0035154	0.0035149
	115	0.0036109	0.0036095	0.0036093	0.0036087	0.0036082	0.0036077	0.0036072
	120	0.0036934	0.0036918	0.0036916	0.0036911	0.0036905	0.0036900	0.0036895
	125	0.0037665	0.0037648	0.0037646	0.0037641	0.0037635	0.0037630	0.0037625
	130	0.0038305	0.0038292	0.0038289	0.0038284	0.0038279	0.0038274	0.0038269

Table 4

European Call option prices for Merton's jump-diffusion model using $e^{T\mathcal{L}_\Omega}$ with different number of lattice points N.

	Strike					
N		300	500	700	900	closed-form formula(2)
time(s)		11	41	102	231	
	85	46.95314	46.94709	46.94560	46.94448	46.94320
	90	45.17129	45.16293	45.16011	45.15893	45.15809
	95	43.47958	43.47072	43.46822	43.46788	43.46686
	100	41.87103	41.86625	41.86491	41.86436	41.86367
	105	40.35500	40.34596	40.34485	40.34437	40.34305
	110	38.91363	38.90250	38.90232	38.90104	38.89991
	115	37.54356	37.53325	37.53148	37.53091	37.52949
	120	36.24128	36.23221	36.22963	36.22857	36.22735
	125	35.00320	34.99414	34.99093	34.99042	34.98936
	130	33.82565	33.81425	33.81410	33.81280	33.81166

Table 5

European Call option deltas for Merton's jump-diffusion model using $e^{T\mathcal{L}_\Omega}$ with different number of lattice points N.

	Strike					
N		300	500	700	900	closed-form formula(3)
time(s)		11	41	102	231	
	85	0.7813222	0.7812700	0.7812454	0.7812352	0.7812294
	90	0.7643687	0.7643129	0.7642859	0.7642752	0.7642699
	95	0.7475596	0.7475021	0.7474748	0.7474655	0.7474597
	100	0.7309301	0.7308820	0.7308569	0.7308472	0.7308420
	105	0.7145567	0.7144945	0.7144699	0.7144603	0.7144529
	110	0.6984356	0.6983633	0.6983422	0.6983294	0.6983223
	115	0.6825929	0.6825210	0.6824934	0.6824831	0.6824744
	120	0.6670505	0.6669819	0.6669496	0.6669368	0.6669287
	125	0.6518257	0.6517548	0.6517180	0.6517078	0.6517001
	130	0.6369311	0.6368435	0.6368238	0.6368088	0.6368002

Table 6

European Call option gammas for Merton's jump-diffusion model using $e^{T\mathcal{L}_\Omega}$ with different number of lattice points N.

	Strike					
N		300	500	700	900	closed-form formula(4)
time(s)		11	41	102	231	
	85	0.0028921	0.0028918	0.0028925	0.0028928	0.0028931
	90	0.0030393	0.0030393	0.0030401	0.0030404	0.0030407
	95	0.0031755	0.0031756	0.0031764	0.0031766	0.0031769
	100	0.0033010	0.0033006	0.0033013	0.0033015	0.0033018
	105	0.0034140	0.0034143	0.0034149	0.0034152	0.0034155
	110	0.0035167	0.0035172	0.0035177	0.0035181	0.0035184
	115	0.0036090	0.0036095	0.0036103	0.0036106	0.0036109
	120	0.0036915	0.0036918	0.0036928	0.0036931	0.0036934

2. The Geman-Roncoroni model of electricity prices

In the Geman-Roncoroni model the evolution of the spot price of electricity S_t includes a mean-reverting component and a jump component. The direction of the jumps depends on the level of the spot price: if the price is below a threshold all jumps increase the spot price, while if the spot price is above the threshold the jumps decrease the spot price. The existence of the threshold aims to reproduce the observed spikes in electricity prices. We assume that the spot price describes day-ahead, average on-peak price per megawatt-hour of electricity.

The evolution of the spot price under the statistical measure P is given by

$$S_t = S_0 e^{\xi_t + \mu_P(t)}$$

where ξ_t includes a mean-reverting and a jump component and solves the following stochastic differential equation:

$$d\xi_t = -\theta_1 \xi_t dt + \sigma dW_t + h(t, S_{t^-}) dJ_t \tag{14}$$

where S_{t^-} stands for the left limit of S at time t. The term μ_P is a deterministic function of time and represents the predictable seasonal trend of electricity prices[6]

$$\mu_P(t) = \alpha + \beta t + \gamma \cos[\epsilon + 2\pi t] + \delta \cos[\zeta + 4\pi t].$$

Jumps are characterized by their occurrence, magnitude and direction. The function J_t has the form

$$J_t = \sum_{i=1}^{N(t)} Y_i,$$

where $N(t)$ is a Poisson process with time-varying intensity $\iota(t)$ and the jumps Y_i are independent and identically distributed random variables with truncated exponential density $q(z; \theta_3, \psi)$ given by

$$q(z; \theta_3, \psi) = \frac{\theta_3 \exp(-\theta_3 z)}{1 - \exp(-\theta_3 \psi)}, \qquad 0 \le z \le \psi. \tag{15}$$

and

$$\iota(t) = \theta_2 \times s(t),$$

where $s(t)$ represents a normalized (and possibly periodic) jump intensity shape and the constant θ_2

[6]Since we are interested in pricing derivatives, it is the form of μ_P under the risk neutral measure that is important. We discuss the transformation between the real and risk neutral measures in Section 3.

can be interpreted as the maximum expected number of jumps per time unit.

To account for spikes in electricity prices, jumps occur in a positive direction when prices are below a threshold $\Upsilon(t)$, and in a negative direction otherwise. The switching function, h, is given by

$$h(t, S) = \begin{cases} +1, & \ln S < \Upsilon(t) \\ -1, & \ln S \geq \Upsilon(t). \end{cases}$$

The threshold is defined by a constant spread Δ over the average trend:

$$\Upsilon(t) = \mu_P(t) + \Delta.$$

We consider the case where the mean-reverting level of the diffusion process is *constant* (i.e. $\mu_P(t) = \alpha$) and the compound Poisson process controlling jumps has *constant* intensity $\iota(t) = \iota$ and *constant* jump reversion threshold $\Upsilon = \alpha + \Delta$. For simplicity we also assume that the risk-free interest rate r is constant.

In the Geman-Roncoroni model, the diffusion component and the jump component are independent. The dynamics of the energy price are specified by the Markov generator $\mathcal{L}_\Omega$ which is given by

$$\mathcal{L}_\Omega = \mathcal{L}_\Omega^d + \mathcal{L}_\Omega^j \tag{16}$$

where $\mathcal{L}_\Omega^d$ is the discretization of the generator of the diffusion component and $\mathcal{L}_\Omega^j$ is the discretization of the generator of the jump component.

The mean-reverting process is related to the solution ξ_t^d of the following stochastic differential equation:

$$d\xi_t^d = -\theta_1 \xi_t^d dt + \sigma dW_t. \tag{17}$$

The Markov generator of a diffusion process given by the stochastic differential Equation (17) acts in the following way on any twice differentiable real function ϕ:

$$(\mathcal{L}^d \phi)(\xi^d) = -\theta_1 \xi_t^d \frac{\partial \phi}{\partial \xi^d}(\xi^d) + \frac{\sigma^2}{2} \frac{\partial^2 \phi}{\partial {\xi^d}^2}(\xi^d).$$

Elements of the discretized Markov generator for diffusion $\mathcal{L}_\Omega^d(x, y)$ are obtained by solving the following linear systems for all $x, y \in \Omega$, which guarantee that the mean and variance of the diffusion part of the process are matched locally:

$$\sum_{y\in\Omega}\mathcal{L}^d_\Omega(x,y) = 0, \quad (18)$$

$$\sum_{y\in\Omega}\mathcal{L}^d_\Omega(x,y)(\xi(y)-\xi(x)) = -\theta_1\xi(x), \quad (19)$$

$$\sum_{y\in\Omega}\mathcal{L}^d_\Omega(x,y)(\xi(y)-\xi(x))^2 = \sigma^2. \quad (20)$$

After obtaining the Markov generator for the diffusion component, we consider the jump component of the Geman-Roncoroni model. The jumps in the model are controlled by the compound Poisson process with constant intensity ι and jump size z with truncated exponential density $q(z;\theta_3,\psi)$.

In the Geman-Roncoroni model, Y_i is distributed according to the density given by Equation (15). The transition density for the continuous-time process to jump from state x to state y between times t and $t+dt$ is given by

$$p(x,t;y,t+\delta t) = \iota q(|\xi(y)-\xi(x)|;\theta_3,\psi)dt = \iota\frac{\theta_3 e^{-\theta_3|\xi(y)-\xi(x)|}}{1-e^{-\theta_3\psi}}dt,$$

where $|\xi(y)-\xi(x)|\le\psi$.

To discretize this formula on the lattice $\xi(a), a\in\Omega$, we replace it with

$$\begin{aligned} p(x,t;y,t+\delta t) &= \iota dt\int_{|\frac{\xi(y)+\xi(y-1)}{2}-\xi(x)|}^{|\frac{\xi(y)+\xi(y+1)}{2}-\xi(x)|} q(z,\theta_3,\psi)dz \\ &= \iota dt\frac{e^{-\theta_3|\frac{\xi(y)+\xi(y-1)}{2}-\xi(x)|}-e^{-\theta_3|\frac{\xi(y)+\xi(y+1)}{2}-\xi(x)|}}{1-e^{-\theta_3\psi}} \end{aligned}$$

The Markov generator $\mathcal{L}^j_\Omega$ for the compound Poisson process is given by

$$p(x,t;y,t+\delta t) = e^{dt\mathcal{L}^j_\Omega}(x,y)\approx\delta_{xy}+\mathcal{L}^j_\Omega(x,y)dt$$

where $\delta_{xy}=1$ if $x=y$, and zero otherwise. $\mathcal{L}^j_\Omega$ can be defined as follows:

$$\mathcal{L}^j_\Omega(x,y) = \begin{cases} \iota\frac{e^{-\theta_3|\frac{\xi(y)+\xi(y-1)}{2}-\xi(x)|}-e^{-\theta_3|\frac{\xi(y)+\xi(y+1)}{2}-\xi(x)|}}{1-e^{-\theta_3\psi}}, & \text{if } x\neq y \\ -\sum_{z\neq x}\mathcal{L}^j_\Omega(x,z), & \text{if } x=y \end{cases}$$

for all $x,y\in\Omega$.

To obtain the Markov generator for the Geman-Roncoroni model with constant mean-reversion level, constant jump intensity and constant jump reversion threshold, we combine $\mathcal{L}_\Omega^j, \mathcal{L}_\Omega^d$ according to Equation (16). Since $\mathcal{L}_\Omega^d$ and $\mathcal{L}_\Omega^j$ are Markov generators, $\mathcal{L}_\Omega$ is a Markov generator as well.

3. Numerical Examples for the Geman-Roncoroni model

In the Geman-Roncoroni model, due to the high mean-reversion rate and the large jump amplitude of the spot electricity price, the corresponding Markov generator is highly asymmetric. This will cause numerical instabilities when computing the transition density using $e^{T\mathcal{L}_\Omega}$. The Geman-Roncoroni model does not allow for closed-form formulas for European option prices. In order to test the accuracy of the numerical method, we compare the price of European call options on spot energy price to results computed using Monte Carlo simulation. We also compute the hedge-ratios for European call options on spot which are presented in Figures 4 and 5. Furthermore we compute the transition densities by using the regularization method and analyse using the four criteria. The results are presented in Table 11.

A. Calibration

Geman and Roncoroni (2006) estimate the parameters of their model under the statistical measure. To incorporate the price of risk we replace the drift $\mu_P(t)$ with the drift under the risk neutral measure $\mu_Q(t) = \alpha + a(t)$, estimated to reproduce futures prices, where $a(t)$ is a deterministic function of time t.[7]

The specification of the futures contract in electricity markets involves delivery over several days of the delivery month. For example, for the PJM Interconnection Monthly Peak Electricity Futures contract, the trading unit is 40 megawatt hours (MWhs) per peak day.[8] Depending on the number of peak days in the month, the number of MWhs will vary (between 760 MWhs and 920 MWhs). The futures price is given by:

$$F(0,t) = \frac{1}{M} \sum_{i=1}^{M} \mathbf{E}_0^Q \left[S_{t_i} e^{(t_i - t)r} \right] \tag{21}$$

where $F(0,t)$ is the futures price at time 0 with settlement date t, r is the constant risk-free rate, t_i is

[7] This change of measure effectively assumes that the price of risk is associated with the diffusion part of the process. If derivative prices are available, additional changes could be made to the frequency and magnitude of the jumps, as well as the position of the threshold. Since we are interested in presenting the numerical method for pricing electricity derivatives, we do not focus on the calibration of the risk-neutral measure.

[8] PJM stands for the Pennsylvania, New Jersey, Maryland electricity market.

a delivery day, S_{t_i} is the electricity price at time t_i, M is the number of peak days in a month and $\mathbf{E}_0^Q$ denotes expectation taken at time 0 under the pricing measure Q. We calibrate the model, under the measure Q, by finding a factor $a(t)$ to satisfy Equation (21).

We illustrate the calibration of our discrete state-space approximation in Figure 1. For simplicity, we assume there are only four delivery days and $t_{i+1} - t_i = t_1 - t$, for all i. We want to calibrate the lattice to the current futures prices $F(0, t)$. In this example, we only have seven lattice points. Therefore, the Markov generator will be a 7×7 matrix. We need to compute two transition probability matrices: one from time 0 to time t and the other from time t_i to time t_{i+1}.

$$\mathbf{P}_t = Ue^{t\Lambda}U^{-1}$$

$$\mathbf{P}_{t_1-t} = \mathbf{P}_{t_{i+1}-t_i} = Ue^{(t_{i+1}-t_i)\Lambda}U^{-1}$$

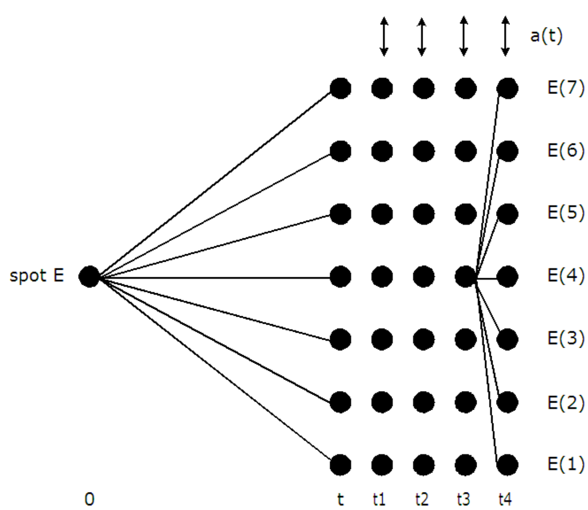

Figure 1. Lattice calibration.

In general, for traded futures contracts, the transition matrices that need to be estimated are: from the current time to the settlement date; from the settlement date to the first delivery date; for the time between delivery dates during the week; and for the time between delivery dates on Fridays and Mondays.

The parameters estimated by Geman and Roncoroni (2006) under the statistical measure for COB are $\theta_1 = 13.3815, \theta_2 = 13.2269, \theta_3 = 1.0038, \alpha = 2.8928, \beta = 0.1382, \gamma = 0.1979, \delta = 0.0618, \epsilon = 1.7303, \zeta = 1.7926, \sigma = 1.3631, \psi = 1.0169, \Delta = 1$. For simplicity, we illustrate our numerical method in the case where the mean-reverting level $\mu_p(t)$, the jump intensity $\iota(t)$ and the jump reversion threshold $\Upsilon(t)$ are all constant. We approximate all three time dependent paramters by setting $\mu_p(t) = \alpha$,

$\Upsilon(t) = \alpha + \Delta$ and , $\iota(t) = \theta_2 \times 0.275$ (where 0.275 is the average of $s(t)$ for COB in Geman and Roncoroni (2006)). The numerical method can be easily extended to the case of piece-wise constant time dependent functions.

We can match the futures prices $F(0,t)$ by finding $a(t)$ and the number of peak days in a month, $M = 22$. We have used 700 evenly spaced states to discretize the electricity price with $\xi(0) = 0$ and $\xi(N) = 6$. Constant risk-free rate $r = 5\%$. The results are given in Table 7. The parameters we used in our numerical method imply the mean-reversion level is approximately \$18/MWh and the jump reversion threshold is \$49/MWh under the statistical measure. The initial spot electricity price S_0 is \$40/MWh.

Table 7
Calibration

This table presents Futures prices (in $/MWh) and the values of the function $a(t)$, that accomplishes the transformation of the drift between the real and the risk neutral measures.

t	$F(0,t)$	$a(t)$
1 month	42	0.55531
2 months	44	0.68736
3 months	45	0.73701
6 months	46	0.77161
1 year	48	0.81462
2 years	50	0.85544

B. Pricing European Options on futures price

The price of a European call option on futures price with expiration date s on the futures contract with settlement date t, with strike price K is given by

$$e^{-rs}\mathbf{E}_0^Q[F(s,t)-K]^+$$

To price a European option on futures prices on a lattice, we need to compute the following transition probability matrices: from the current time to the option expiration date, from the option expiration date to the settlement date of the futures contract and for the time between the delivery dates of the futures contract. Table 8 shows the prices of European call option on futures prices for the Geman-Roncoroni model with the parameters described in Section 3.A.

C. Results

Table 9 to 10 shows the European call option prices in the Geman-Roncoroni model with the parameters described above. The results in Table 9 are obtained using the binary-exponentiation algorithm with different numbers of lattice points. The second row of Table 9 is the τ_2's that are used to compute the transition densities. The third row is the maximum τ_2's that can be used to compute the transition densities. Notice that as the number of lattice points increases, the minimum diagonal entry of $\mathcal{L}_\Omega$ and the maximum τ_2 also decreases. The τ_2's in the second row are ten times smaller than their corresponding maximum τ_2 in the third row. As the number of lattice points increases, the absolute difference between the prices against the prices on its left-hand column decreases.

Table 8
Prices of European Call option on futures price for the Geman-Roncoroni model

	Strike	binary-exponentiation $N = 700$ $\tau_2 = 0.00000394$
time(s)		960
expiration = 0.5m settlement = 1m	35	6.992765
	40	2.519862
	45	0.462843
	50	0.067240
	55	0.006477
expiration = 1m settlement = 2m	35	8.962578
	40	4.006070
	45	0.555799
	50	0.019986
	55	0.000085
expiration = 1.5m settlement = 3m	35	9.937695
	40	4.968854
	45	0.572591
	50	0.001116
	55	0.000000
expiration = 3m settlement = 6m	35	10.863356
	40	5.925467
	45	0.987587
	50	0.000000
	55	0.000000
expiration = 6m settlement = 1y	35	12.679029
	40	7.802479
	45	2.925930
	50	0.000000
	55	0.000000
expiration = 1y settlement = 2y	35	14.268441
	40	9.512294
	45	4.756147
	50	0.010922
	55	0.000000

Table 9
Prices of European Call option on spot for the Geman-Roncoroni model with varying number of lattice points N, using the binary-exponentiation algorithm.

	Strike	N = 200	300	400	500	600	700
τ_2		0.00004833	0.00002138	0.00001210	0.00000772	0.00000538	0.00000394
max τ_2		0.00048326	0.00021382	0.00012097	0.00007718	0.00005378	0.00003942
min $\mathcal{L}_\Omega(x,x)$		-2069.27	-4676.73	-8266.57	-12956.53	-18595.31	-25367.81
time(s)		25	55	140	312	550	960
1m	20	26.03813	26.03770	26.03119	26.03078	26.02881	26.02886
	40	8.59357	8.60463	8.59732	8.60015	8.60033	8.60105
	60	1.99321	2.01762	2.00965	2.01502	2.01793	2.01957
	80	0.55297	0.57173	0.56670	0.56968	0.57157	0.57284
	100	0.17291	0.18228	0.17935	0.18087	0.18186	0.18252
2m	20	25.25277	25.25603	25.25139	25.25160	25.25085	25.25107
	40	8.37763	8.39744	8.39240	8.39530	8.39666	8.39780
	60	2.09861	2.12328	2.11918	2.12386	2.12707	2.12845
	80	0.62260	0.64102	0.63805	0.64077	0.64286	0.64396
	100	0.21131	0.22113	0.21891	0.22043	0.22155	0.22219
3m	20	25.25698	25.25886	25.25671	25.25693	25.25660	25.25675
	40	8.46933	8.47910	8.47980	8.48233	8.48409	8.48486
	60	2.16885	2.18695	2.18583	2.19043	2.19368	2.19488
	80	0.65821	0.67446	0.67252	0.67533	0.67742	0.67858
	100	0.22955	0.23866	0.23683	0.23843	0.23957	0.24026
6m	20	25.50501	25.50524	25.50512	25.50515	25.50514	25.50515
	40	8.77002	8.78098	8.78236	8.78406	8.78590	8.78666
	60	2.29306	2.31161	2.31146	2.31540	2.31875	2.31994
	80	0.70655	0.72314	0.72189	0.72447	0.72667	0.72789
	100	0.25102	0.26050	0.25894	0.26048	0.26171	0.26245
1y	20	26.76664	26.76669	26.76671	26.76671	26.76672	26.76672
	40	9.94342	9.95151	9.95375	9.95459	9.95714	9.95758
	60	2.73935	2.75705	2.75735	2.76118	2.76518	2.76605
	80	0.86273	0.88070	0.87961	0.88252	0.88505	0.88624
	100	0.31452	0.32526	0.32365	0.32548	0.32692	0.32771
2y	20	27.27492	27.27495	27.27496	27.27496	27.27497	27.27497
	40	10.87110	10.87616	10.87773	10.87977	10.88132	10.88152
	60	3.15961	3.17480	3.17452	3.17947	3.18312	3.18418
	80	1.01642	1.03463	1.03335	1.03674	1.03953	1.04085
	100	0.37905	0.39048	0.38871	0.39084	0.39255	0.39344

Table 10
Prices of European Call option on spot for the Geman-Roncoroni model using the regularization method and Monte Carlo simulation.

maturity	Strike	binary-exponentiation $N = 700$ $\tau_2 = 0.00000394$	MC 3 million runs	stderr	MC 1 million runs	stderr	number of time steps for MC
	20	26.02886	26.04722	0.13417	26.04811	0.23105	
	40	8.60105	8.62039	0.09921	8.62357	0.17056	
1m	60	2.01957	2.01976	0.03531	2.03141	0.06018	500
	80	0.57284	0.57628	0.01136	0.58606	0.01910	
	100	0.18252	0.18301	0.00368	0.18730	0.00611	
	20	25.25107	25.29484	0.14781	25.31287	0.25715	
	40	8.39780	8.43917	0.10625	8.46312	0.18486	
2m	60	2.12845	2.14900	0.03995	2.16031	0.06968	1000
	80	0.64396	0.65269	0.01405	0.65717	0.02457	
	100	0.22219	0.22625	0.00512	0.22734	0.00899	
	20	25.25675	25.28812	0.15083	25.29397	0.26033	
	40	8.48486	8.51468	0.10904	8.51871	0.18794	
3m	60	2.19488	2.20630	0.04198	2.21403	0.07197	1500
	80	0.67858	0.68213	0.01523	0.68825	0.02593	
	100	0.24026	0.24193	0.00575	0.24505	0.00967	
	20	25.50515	25.52919	0.16148	25.53177	0.26194	
	40	8.78666	8.79671	0.11521	8.80657	0.19286	
6m	60	2.31994	2.32005	0.04345	2.32759	0.07586	3000
	80	0.72789	0.72832	0.01688	0.73020	0.02785	
	100	0.26245	0.26462	0.00694	0.26968	0.01060	
	20	26.76672	26.80746	0.17119	26.82140	0.27284	
	40	9.95758	9.99812	0.12130	10.00884	0.21274	
1y	60	2.76605	2.78785	0.05738	2.79466	0.08991	6000
	80	0.88624	0.89090	0.01982	0.89603	0.03438	
	100	0.32771	0.32894	0.00843	0.33113	0.01364	
	20	27.27497	27.29581	0.18660	27.30452	0.28090	
	40	10.88152	10.89948	0.13638	10.90463	0.24058	
2y	60	3.18418	3.19141	0.06644	3.19403	0.10440	12000
	80	1.04085	1.04822	0.02338	1.05077	0.03969	
	100	0.39344	0.39447	0.00960	0.39660	0.01581	

Table 11
Regularization of the Markov Generator

This table presents the extent to which the four criteria described in the text are violated by the original transition probability matrix and the regularized matrix. Criterion 1 measures the conservation of probability, $\sum_y p(x,t;y,T) = 1$. Criterion 2 measures whether the entries of the transition probability matrix satisfy $0 \geq p(x,t;y,T) \geq 1$. Criterion 3 measures whether the entries in the transition probability matrix are real numbers. Criterion 4 measures whether the Chapman-Kolmogorov conditions are satisfied. Time to maturity $(T-t)=1$, spot price = \$40/MWh, machine precision 10^{-9}. The number of discretization levels for the electricity price is 700.

Criteria		original matrix	matrix by binary-exponentiation
1		0.999986	1
2	# violations	85	0
	maximum absolute error	0.01	0
	average absolute error	0.00085	0
3	# violations	199	0
	maximum absolute error	0.05	0
	average absolute error	0.006	0
4		violated	satisfied

D. Bermudan Options: Optimal Exercise Boundary

A significant advantage of approximating the stochastic process with a Markov chain is the ease of pricing options with more than one possible exercise dates, as well as determining the optimal exercise boundary. In the case of options on electricity spot prices this boundary is not trivial, due to both the mean-reversion of the diffusion part of the process and to the reversal of jumps around a threshold. In Figure 2 we demonstrate the optimal exercise boundary for two options with 1 month maturity. The first option is written on the on-peak average daily electricity price, which can be exercised once, with possible exercise dates any weekday during the month (notice of exercise is delivered on the day previous to exercise, i.e. for delivery on Tuesday, one needs to provide notice on Monday). The second option is written on the same underlying but assume that it can be exercised everyday in the month (assuming there are price fluctuations everyday).

One interesting feature uncovered by the numerical results is the non-monotonicity of the exercise boundary for the weekday-exercise-only option. From Figure 2, it is clear that in the early days of the month the option is exercised at lower prices on Thursday compared to the preceding Wednesday or the following Sunday. The intuition for this behavior follows if one considers mean-reversion in electricity prices as a state-dependent "dividend": on every possible exercise day, other than Thursday, the "dividend" is calculated until the next possible exercise day, which is one day ahead. On Thursdays, the "dividend" is larger, since it is calculated over 3 days; i.e., until Sunday, the next possible exercise day. Since calls on assets that pay dividends are exercised at lower prices when the dividend increases, the fact that on-peak prices only apply to delivery during weekdays leads to a non-monotonous exercise boundary.

4. Conclusions

We have described a fast and accurate numerical method for pricing electricity options in a model with mean-reverting diffusion and with jumps whose direction depends on the price level. The method constructs an approximating Markov Chain for the stochastic process, discretizing the state-space, and allowing for arbitrary timesteps. We calibrated the model using futures prices and used the method to compute the price and hedge-ratios of European options, as well as determine the optimal exercise strategy for Bermudan options.

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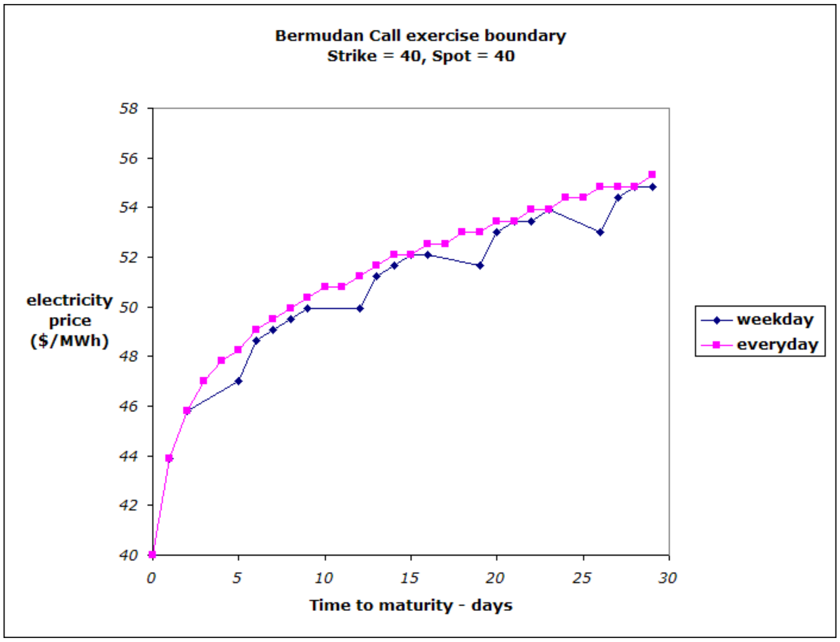

Figure 2. Exercise Boundaries for the weekday-exercise-only and everyday-exercise Bermudan call option on spot.

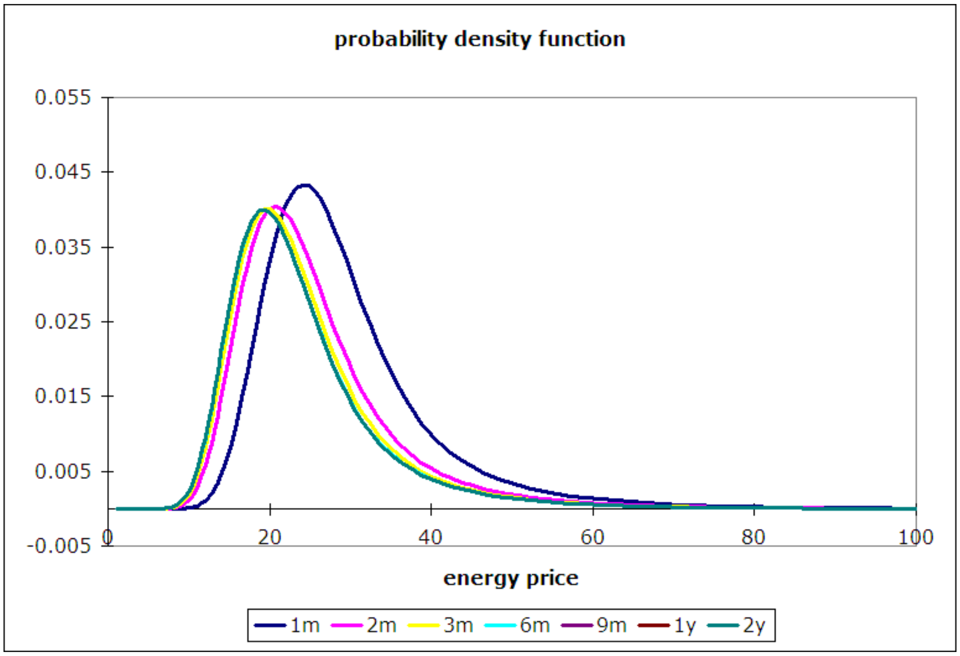

Figure 3. probability density function between 1 month to 2 years by applying the binary-exponentiation algorithm.

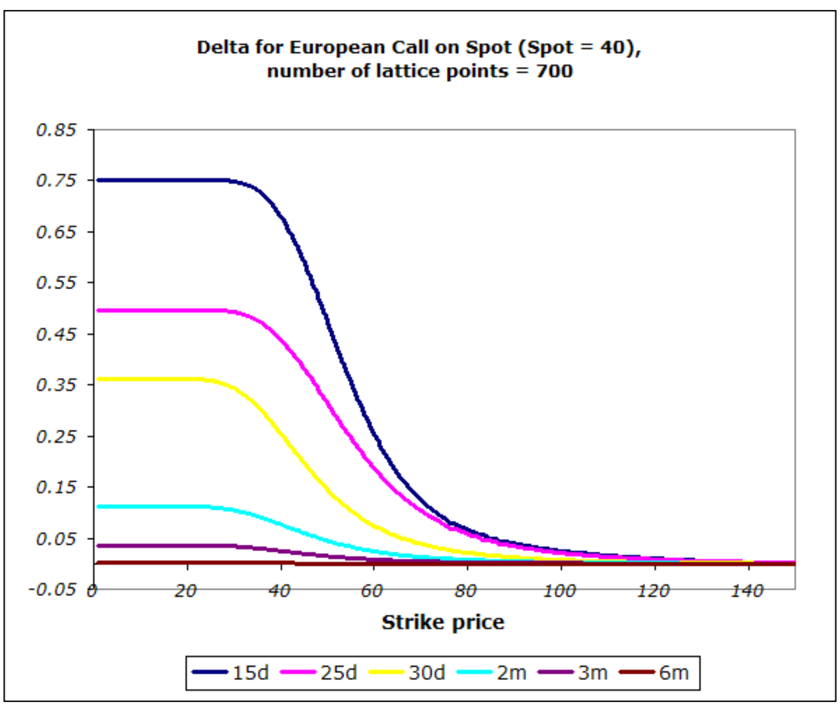

Figure 4. Delta profiles of European call options on spot price with maturities between 15 days and 6 months, spot price equals 40.

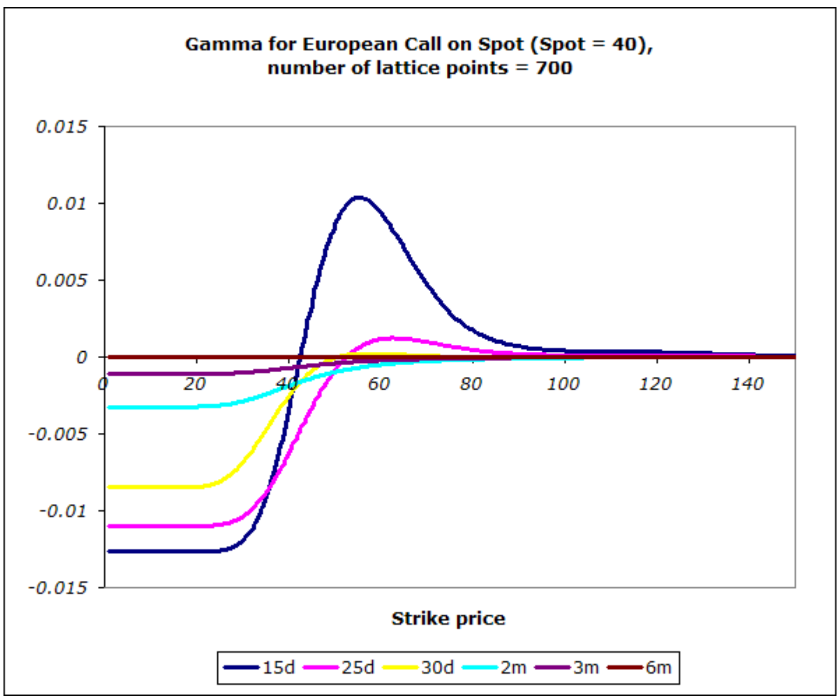

Figure 5. Gamma profiles of European call options on spot price with maturities between 15 days and 6 months, spot price equals 40.