Endogenous Network Formation In the Laboratory

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Abstract

This paper provides an experimental test of a theory of endogenous network formation. A group of subjects face a decision problem under uncertainty. The subjects are endowed with a private information about the fundamentals of the problem, and they are supposed to make a decision one after the other. The key feature of the experiment is that a subject can observe the decisions of the preceding subjects by forming links. A link is costly, yet it enables a subject to observe previous decisions of those to whom he is linked. We show that subjects respond to changes in the information structure and the cost of link formation in the expected manner. However, we also show that behavior systematically deviates from the Bayesian benchmark as subjects form more links than theory predicts. Subjects also exhibit a tendency to conform rather than follow their own information. In order to explain this pattern, we provide an econometric model that posits that subjects care about their relative standing in the group. We show that the modified model provides a better fit than a standard QRE.

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1 Introduction

Evidence that social networks are important in disseminating information has been found in many contexts. Kelly and Ó Gráda [24] demonstrate the importance of information networks in market panics using data from two panics in 1850s' New York. Foster and Rosenzweig [16], Conley and Udry [11], and Munshi [30] show that technology diffusion is driven by an underlying social learning process. In particular, Conley and Udry [11] report the results of field work in rural areas of Ghana indicating that the main channel for the adoption of new technology in an agricultural sector is social learning through networks. Also, it is well-documented in the labor economics literature (see Ioannides and Loury [21]) that social networks are the main source of information about jobs. These are just a few examples that demonstrate the significance of such questions as:

(1) How does the network structure affect the dissemination of information?

(2) How do social networks form, as far as information aggregation is concerned?

The answers to these questions are sought via the theory of the economics of social networks.[1] There are a number of studies concerning the first question (viz. Bala and Goyal [1], and Gale and Kariv [18]). These papers address such questions as: how does information propagates? and, how do agents learn from each other in exogenously given network structures? Çelen, Choi and Kariv [10] attempts to answer the second question. Çelen et al. [10] extends the canonical *social learning* model in order to analyze the role of information externality on the formation of networks, as well as the impact of the dynamic evolution of networks on learning dynamics. The present paper carefully modifies Çelen et al. [10] to design an experiment that contributes to the study of endogenous network formation in a pure information externality environment.

In the canonical social learning model, agents receive private signals regarding the state of the world and then make decisions sequentially, after having observed the action choices of all or some of their predecessors. If agents are Bayesian, they infer valuable information from their observations, often leading to herd behavior or informational cascades. A critical assumption of these models is that the interaction between agents is exogenously determined. This particular assumption is in contrast with many real-life situations where agents form their own networks and choose whom to observe. Put differently, in order to acquire more information, an agent can decide to form *links* to other agents to observe their decisions and thereafter make his decision. This is what we mean by saying that the network evolves endogenously.

Our experiment consists of a group of four subjects who sequentially make decisions on the same problem. Each subject is (potentially) endowed with a piece of valuable information

[1] For recent and comprehensive surveys of the literature see Jackson [22, 23].

regarding the fundamentals of the problem. In addition to having this private information, each subject, before making a decision, is allowed to form links to his predecessors. Forming a link is costly, yet it entitles a subject to observe the actions of those with whom he linked. While the subjects cannot observe the actions without a link, they can observe the structure of the network that evolved until it is their decision turn. This is an important feature of our design, since the structure itself divulges the degree of the network's informativeness, should the subject decide to form any links. We have two different treatments: the full information treatment (FI) and the partial information treatment (PI). The two treatments differ in that in the former treatment, subjects always receive an informative signal, while in the latter they receive a signal with probability less than one. This allows us to analyze how subjects react to changes in the quality of information in this environment.

In a nutshell, each subject decides first whether or not to form any links and then, upon observing the actions through his links, makes an action decision. This two-step procedure gives us the leverage to pose many interesting questions: how do subjects search for information, how do subjects respond to the cost of link formation? how much information is transmitted by the network? how do subjects aggregate the information obtained from any link decisions?

Section 2 lays out the model that we implement in the laboratory and provides the predictions of the theory. We first describe what we call the network formation game. Then we elaborate on the specifics of link formation. In particular we explain the details of link formation and the way agents can collect information through what we call the Bayesian Sequential Link Procedure. Finally, we state the theoretical predictions of the model concerning the behavior of the Bayesian agents.

Section 3 explains *what* happens in the laboratory, as well as *how* it happens. We also explain some of our choices concerning the experimental design. In Section 4, we provide a descriptive analysis of the observed link and action decisions of the subjects in our experiment. This is where we show that there are systematic deviations from the Bayes-rational benchmark of Section 2. For example, in both information treatments, informed subjects in the second position link to the first subject approximately 30% of the time, although it is not optimal for any positive cost. Remarkably, though, when the observed action and the signal of the second subject disagree, there is a strong tendency in the FI treatment to conform to the decision of the first subject, while there is no such tendency for the partial information treatment. We also show that subjects display herd behavior in link formation: that is, the third subject is much more likely to link if he observes a link between the subjects in the first and second positions; the fourth subject almost always links when the third subject has a link to the second, who is also linked to the first. Finally, we show that subjects do not necessarily form links to the most informative node; instead, they prefer to link to the *larger* sub-network. All of these deviations should not be construed to mean that subjects' behavior is completely erratic. Indeed, subjects respond to changes in the cost of link formation just as the theory predicts. Moreover, by looking across information treatments, we are able to see that subjects do respond to information. For example, herding in link formation is less pronounced in the partial information treatment than in the full information treatment.

The observations gleaned from our descriptive analysis in Section 4 lead us to a number of hypotheses that we attempt to rigourously test in Section 5. To explain the linking behavior that we observe, we reformulate our model to incorporate the relative income hypothesis à la Duesenberry [14]. In particular, we argue that a subject is motivated to form a link above and beyond the standard reasons found in the Bayesian benchmark because he wishes to compare his expected earnings to those of his predecessor(s). We parameterize this kind of behavior with the parameter α: when $\alpha > 0$, subjects care about their relative standing, while for $\alpha = 0$, the model reduces to a more standard Quantal Response Equilibrium model, which is also discussed in Section 5. Finally, in Section 6 we offer some concluding remarks and directions for future research.

2 Theoretical Background

The design of our experiment is based on a theory of network formation through information transmission. In this section, we introduce the model and carefully discuss its theoretical predictions. These predictions offer the rational benchmark for the analysis of subjects' behavior in the laboratory.

2.1 Network Formation Game

The basic structure of the problem—which we call the **network formation game**—is as follows. There are two equally likely states of the world $\theta \in \{-1, 1\}$. The game consists of four agents who are randomly assigned to a position in a decision line indexed by $i = 1, 2, 3, 4$. Agents act sequentially in a predetermined order. The agents' problem involves correctly identifying the true state of the world. Precisely, each agent i is supposed to take an action $a_i \in \{-1, 1\}$, which we call the **action decision**. If an agent's action matches the true state, then he receives a payoff $m > 0$; otherwise his payoff is zero. Thus, we can represent the preferences of agent i by the utility function

$$u_i(a_i; \theta) = \begin{cases} m & \text{if } a_i = \theta, \\ 0 & \text{otherwise.} \end{cases}$$

Before he makes a decision, agent i receives a private signal $\sigma_i \in \{-1, 0, 1\}$. We say that the agent is **informed** when the signal he receives is either -1 or 1. The signals $\sigma_i \in \{-1, 1\}$ are informative about the true state because, conditional on the true state, the probability that the signal matches the state is $p = 2/3$. In contrast, the signal $\sigma_i = 0$ is uninformative because, given $\sigma_i = 0$, the probability of state $\theta = -1$ and $\theta = 1$ are both 1/2. Hence, the agent cannot distinguish the states of the world based on his signal. Therefore, we say that agent i is **uninformed** if he receives the signal $\sigma_i = 0$. Finally, we assume that an agent is informed with probability $q \in (0, 1]$, and uninformed with probability $1 - q$.

The signals that agents receive are independently and identically distributed conditional on the true state. Table 1 summarizes the probabilities with which an agent receives each signal conditional on the state of the world.

TABLE 1: INFORMATION STRUCTURE

σ	θ	
	-1	1
1	$q/3$	$2q/3$
0	$1-q$	$1-q$
-1	$2q/3$	$q/3$

After receiving a private signal but before making the action decision, an agent has the option to observe action decisions made by the preceding agents in the decision line. The decision on whether to observe any of the preceding agents' actions—and, if so, whose actions to observe—is called the **link decision**. We make two assumptions on the process of information gathering through link formation.

(1) First, link decisions are assumed to be public information: that is, each agent observes all the link decisions made by the preceding agents but not their action decisions.

(2) Second, by forming a link to one of the preceding agents, an agent not only observes the action decision of this agent, but also all the actions that this agent observed through his link decision(s). The cost of each link is assumed to be $c \geq 0$.

The next section provides a detailed description of the process of link formation.

2.2 BAYESIAN SEQUENTIAL LINK PROCEDURE

For each agent the process of link formation is sequential. In other words, if an agent can form more than one link, he forms the links one after the other by comparing the potential benefit and cost of an additional link at each stage.

Figure 1: Bayesian Sequential Link Procedure: An Example

Let us be more specific and explain what we call the **Bayesian Sequential Link Procedure** (BSLP) through an example, illustrated in Figure 1. It is the fourth agent's turn to move and he observes the following: the second agent did not form a link to the first, and took his action based on his private information. The third agent formed a link to the second; yet, after observing the action of the second, he did not form a link to the first agent. Therefore, he took his action based on the information deduced from the second agent's action and his private information. There are two links that the fourth can form: a link to the third agent, through which he can observe the action of the third and the second, and a link to the first agent, through which he can observe the action of the first agent.[2]

According to BSLP, the fourth agent evaluates the problem in the following way. Based on his own information, he decides whether it is optimal to incur c and form a link to the third. But in doing so he keeps in mind that he could continue and form a link to the first by incurring the cost c again. More precisely, he considers all possible action profiles that he can observe through his link to the third agent. Also, conditional on his private information, he knows the probability with which he can observe each action profile. Furthermore, for each one of these contingencies, he considers the action profile he can observe by a second link and decides whether he would form the second link or not as if he is at that situation. Finally, with this continuation value in mind, he decides whether to form his first link or not. In what follows, we will explain this procedure more formally. All the results that we will report in the following sections are based on the use of the formula we derive here.

As is the case in the example, forming a link is equivalent to saying that the agent observes the outcome (the action profile) of an experiment (forming a link). For the purposes of the present paper, it is enough to look at the case where there are two random variables, X_1, X_2, that are independent conditional on θ, but not necessarily identical. The realizations of the random variables are denoted by x_1 and x_2 respectively. Therefore, an agent facing X_1 and X_2 first decides whether to take an optimal action simply based on his private information, or to experiment X_1. If he decides to experiment X_1, for any realization x_1, he specifies whether to take an optimal action, or to further experiment X_2. If he decides to experiment X_2, for all realizations x_2, he specifies the optimal action he should take.

[2] By Blackwell's [6] celebrated theorem, it is clear that it is not optimal to form a link to the second, rather than the third. Similarly, it is not optimal to form a link to the first and then to the third agent.

Let $s_0 = (\sigma), s_1 = (\sigma, x_1), s_2 = (\sigma, x_1, x_2)$ denote the information nodes where the agent observes only his private information, his private information and the realization of X_1, and his private information, the realization of X_1 and of X_2, respectively. We denote the maximum expected utility an agent can get at a node $s \in \{s_0, s_1, s_2\}$ without further experimentation by

$$\underline{v}(s) := \max\left\{ \Pr(\theta = 1|s), \Pr(\theta = -1|s)\right\}m. \tag{1}$$

Also, at node s_j, where $j \in \{0, 1\}$ the ex ante maximum expected utility from further experimentation is

$$\bar{v}(s_j) \;:=\; \textstyle\sum_{s_{j+1}} \Pr(s_{j+1}|s_j) \max\left\{\underline{v}(s_{j+1}), \bar{v}(s_{j+1}) - c\right\}, \tag{2}$$

since at each s_j an agent compares $\underline{v}(s_j)$ against $\bar{v}(s_j) - c$ to decide whether to experiment X_{j+1} or to take the optimal action at s_j. To capture this, we define what we refer to as the **value of information** from further experimentation by

$$\begin{aligned} &v(s_j) := \bar{v}(s_j) - \underline{v}(s_j) - c \;\text{ for } j \in \{0, 1\}, \\ &v(s_2) := \bar{v}(s_2). \end{aligned} \tag{3}$$

Therefore at s_j, an agent decides to further experiment if $v(s_j) > 0$, otherwise he takes the optimal action at s_j without experimenting X_{j+1}.

By using the equations (1), (2), (3) and backward induction, we can fully describe the optimal strategy of an agent. The following proposition formally states the complete characterization of BSLP that we illustrated.

PROPOSITION 1. *The optimal policy of Bayesian sequential link procedure for an agent is characterized by a pair* (τ, a^*) *such that*

$$\tau \;= \min\left\{j \in \{0, 1, 2\} : v(s_j) \le 0\right\}, \tag{4}$$

$$a^* \;\in \arg\max_a \left\{ \textstyle\sum_\theta \Pr(\theta|s_\tau)u(a, \theta)\right\}. \tag{5}$$

Proposition 1, and the value of information, as defined by (3), provide us with the full characterization of the decision problem. In other words, an agent stops at the first node at which the value of information is negative; otherwise, he keeps forming links. If the agent stops at the node s_τ, then he takes the action that maximizes his expected utility given his information. The equilibrium results that we present in the following section are derived by use of this characterization.

2.3 Theoretical Predictions

In this section we state the equilibrium behavior of agents $i \in \{1, 2, 3, 4\}$ as corollaries to Proposition 1. Furthermore, we discuss the equilibrium networks that can be observed in the network game. The proofs of the corollaries are in Appendix A.

First Agent. The decision problem of the first agent is easy because there is no preceding agent; thus, he takes an action based only on his private signal. If the first agent is informed, then he follows his signal. If he is uninformed, he randomizes between the two possible actions. Therefore, unless $q = 1$, the second agent cannot determine the status of the first agent as either informed or uninformed. Figure 2 depicts this situation. We reserve the diamond to refer to an agent whose status cannot be determined.

Figure 2: After the First Agent

(1.a) 

Second Agent. The second agent faces a more interesting problem because he has the option of observing the first agent's action. Here, optimal behavior depends on whether or not the agent is informed. If the second agent is informed, then by applying Proposition 1 we find that he does not form a link for any positive cost. Intuitively, this is easy to see: if the second agent incurs the cost c and forms a link to the first, either he will observe an action that is the same as his own private information or he will observe the opposite action. In the former case, he will take the action that he would have taken without a link. In the latter, for any $q < 1$, he will still favor the action that is in line with his information. At best, when $q = 1$, he will become indifferent between the two actions. This suggests that forming a link does not provide any value to an informed second agent.

On the other hand, if he is uninformed, then it is optimal to form a link to the first agent if the cost is low enough, because there is a positive probability that the first is informed. If the cost is high, an uninformed second agent does not form a link.

Corollary 1 summarizes the optimal behavior of the second agent.

Corollary 1. *The optimal decision rule of the second agent is characterized as follows:*

1. *Let $\sigma_2 \in \{-1, 1\}$ and $q \in (0, 1]$. For any $c > 0$, the second agent does not form a link and takes action $a_2 = \sigma_2$.*

2. *Let* $\sigma_2 = 0$ *and* $q \in (0, 1]$*. There exists a threshold* $c^* = qm/6$ *such that for any* $c < c^*$*, the second agent links to the first and takes action* $a_2 = a_1$*; if* $c \geq c^*$*, the second agent does not form a link and randomizes between the two actions.*

Notice that the threshold value c^* increases linearly in the probability of being informed, q, and in the payoff from a correct action, m. Intuitively, when q is higher, the first agent is more likely to be informed and thus, from the perspective of an uninformed second agent, it becomes more valuable to form a link to the first agent. In a similar vein, when the payoff from a correct decision is larger, the same amount of information increases the potential benefit from that information. Figure 3 depicts the networks that can emerge in the equilibrium for $0 < c < c^*$, and $c \geq c^*$. The square indicates the event in which it can be deduced that the agent is uninformed, while the circle indicates that he is informed.

FIGURE 3: AFTER THE SECOND AGENT

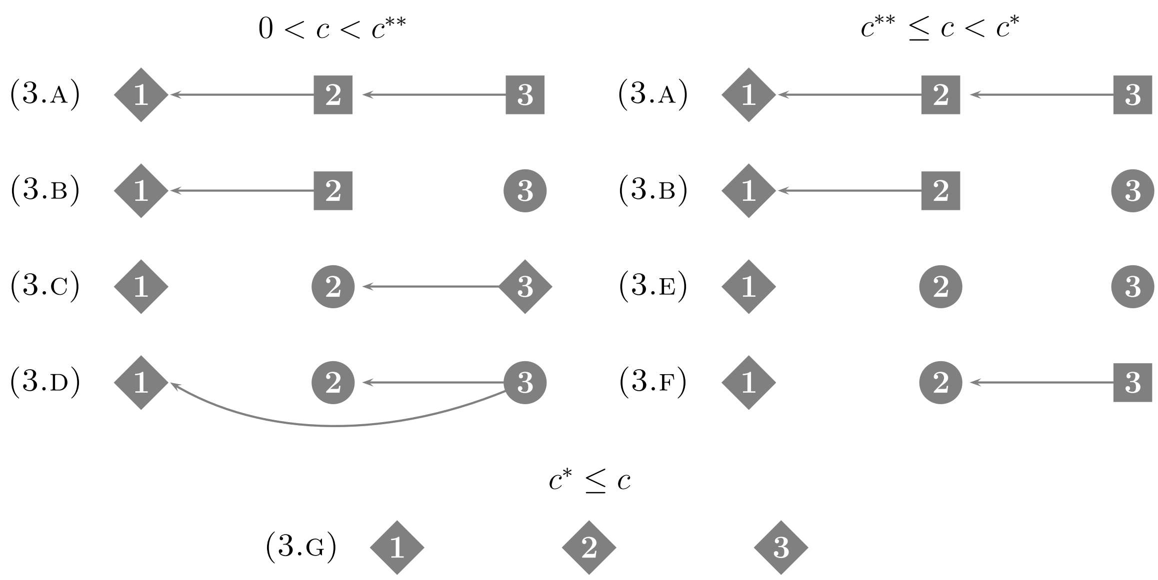
THIRD AGENT. Note that when $c < c^*$, the third (and fourth) agent(s) can discern whether the second agent is uninformed ((2.A) in Figure 3) or informed ((2.B) in Figure 3) simply by observing the network structure emerging from his link decision. As such, the network structure itself contains valuable information and affects significantly the behavior of all agents coming after the second agent.

The decision problem for the third agent is interesting because the network structure becomes more significant in determining the optimal behavior. When the third agent observes a link between the second and the first agents, he rationally infers that the second agent is uninformed and his action conveys no information. In this case, his problem is equivalent to the second agent's problem: if he is informed, he does not form a link for any positive cost, whereas if he is uninformed, he forms a link to the second agent for $c < c^*$.[3] The networks (3.A) and (3.B) in Figure 4 exhibit these situations.

Suppose that $0 < c < c^*$ and the third agent faces the network (2.B) in Figure 3. When the third agent observes that there is no link between the first and the second agents, he

[3] Note that it is also optimal to form a link to the first agent and the third observes only the first agent's action. In order to get around an unnecessary multiplicity of equilibria, we assume that an agent starts to form a link to the closest agent in a line.

Figure 4: After the Third Agent



knows that the second agent is informed. However, he knows that the first is informed only with probability $q \in (0, 1]$. Therefore, an uninformed third agent forms a link to the second and imitates his action ((3.C) and (3.F) in Figure 4). Note that after observing the second agent, the uninformed third agent is exactly the same as an informed second agent. Hence, a link to the first agent is worthless.

On the other hand, if the third agent is informed, we find that the value of information is positive only when the cost is low enough (i.e. $c < c^{**} = 2qm/39$.) Suppose that $c < c^{**}$. An informed third agent starts forming a link to the second. If he finds out that the signal of the second agent is the same as his, he imitates the second's action without forming a link to the first ((3.C) in Figure 4.) Otherwise, he proceeds with a link to the first agent ((3.D) in Figure 4.) Since there is a chance an informed third agent does not form a link to the third, the fourth agent facing the network (3.C) in Figure 4 cannot tell whether the third is informed or not. On the other hand, when $c^{**} \leq c < c^{*}$, an informed third agent does not form a link to the second agent ((3.E) shown in Figure 4.)

Finally, when the cost is high enough (i.e., $c > c^{*}$), the third agent observes the empty network where the second did not form a link. Because the third agent cannot distinguish whether the second agent is informed, he faces two opportunities for linking with equal amounts of information ((3.G) in Figure 4.) However, due to the high cost, it is never optimal for him to form any link, regardless of whether he is informed.

We summarize this discussion in the following Corollary.

Corollary 2. *The optimal decision rule of the third agent is characterized as follows:*

1. *Suppose there is a link between the first and the second agents.*

   (a) *Let* $\sigma_3 \in \{-1, 1\}$ *and* $q \in (0, 1]$. *Then, for any* $c > 0$, *the third agent does not form a link and takes action* $a_3 = \sigma_3$.

   (b) *Let* $\sigma_3 = 0$ *and* $q \in (0, 1]$. *Then, for any* $c < c^*$, *the third agent links to the second and takes action* $a_3 = a_2$; *if* $c \geq c^*$, *the third agent does not form a link and randomizes between the two actions.*

2. *Suppose there is no link between the first and the second agents.*

   (a) *Let* $\sigma_3 \in \{-1, 1\}$ *and* $q \in (0, 1]$.

      i. *There exists a threshold* $c^{**} = 2qm/39$ *such that for any* $c < c^{**}$, *the third agent links to the second agent. If* $a_2 = \sigma_3$, *then he does not form a link to the first and takes action* $a_3 = a_2$; *if* $a_2 \neq \sigma_3$, *then he links to the first and takes action* $a_3 = a_1$.

      ii. *For any* $c \geq c^{**}$, *the third agent does not form a link and takes action* $a_3 = \sigma_3$.

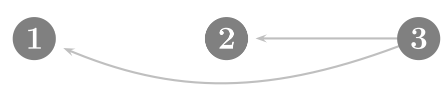
   (b) *Let* $\sigma_3 = 0$ *and* $q \in (0, 1]$.

      i. *For any* $c < c^*$, *the third agent links to the second and takes action* $a_3 = a_2$.

      ii. *For any* $c \geq c^*$, *the third agent does not form a link and randomizes between the two actions.*

FOURTH AGENT. Since most of our attention will be focused on the first three agents in the analysis of the experimental data, we will not provide a full characterization of optimal behavior for agents in the fourth position. We can point out, however, that often the problem of the fourth agent will be strategically equivalent to either the second or the third agent. For example, consider the case where the cost of link formation is low enough, and the fourth agent observes the network displayed in Figure 5.

FIGURE 5: THE FOURTH AGENT'S PROBLEM:
AN EXAMPLE



Given this network, the fourth agent can infer that the second agent was informed. He also can infer that the signals of the second and third must have disagreed (since the third agent also formed a link to the first). Therefore, the signals of the second and third essentially cancel each other out, leaving the only relevant signal that of the first agent—exactly as it is for the second agent. Therefore, if the fourth agent is informed, he should not link for any positive cost, while if he is uninformed, he should link provided that c is small enough. To be sure, there are situations in which the fourth agent's problem is not equivalent to one of his predecessors. However, since this is not our main focus, and the intuition is relatively straightforward, we will omit such details.

3 Experimental Design

The experiment was run at the Experimental Economics Laboratory of the Center for Experimental Social Sciences (C.E.S.S.) at New York University. The 72 subjects were recruited from undergraduate classes at New York University and had no previous experience in social learning experiments. In each session, after subjects read the instructions they were also read aloud by an experimental administrator.[4] Each session lasted for about one hour and fifteen minutes and each subject participated in only one session. An \$8.00 participation fee and subsequent earnings, which averaged about \$13.60, were paid in private at the end of the session. Throughout the experiment, we ensured anonymity and effective isolation of subjects in order to minimize any interpersonal influences that could stimulate uniformity of behavior.[5]

In each session, subjects played the network formation game described in section 2.1, for 40 independent rounds. At the beginning of each session, subjects' positions were randomly assigned to either 1, 2, 3 or 4. Moreover, their positions were held fixed for the duration of the experiment. In each round, the cost of link formation, in experimental points, was randomly drawn by the computer uniformly from the set $\{0, 2, \ldots, 18, 20\}$.

In our experiment, we conducted two treatments, which we call the full information treatment (FI) and the partial information treatment (PI). In treatment FI, all the subjects received an informative signal (i.e., $q = 1$), while in treatment PI we set $q = 2/3$ so there was a chance of a subject receiving an uninformative signal. In all treatments, the informativeness of the signal was held fixed at $p = 2/3$, and the cost of each link was determined as described above. Table 2 summarizes the details of our experiment.

In each round, subjects earned $m = 100$ points for correctly guessing the state and 0 points otherwise. Subjects' net point total was determined by subtracting the appropriate number of points for each link that a subject made from the points collected in that round

[4] At the end of the first round, subjects were asked if there were any misunderstandings. No subject reported any problems with understanding the procedures or using the computer program.

[5] Participants' work stations were isolated by cubicles making it impossible for participants to observe others' screens or to communicate. We also made sure that all remained silent throughout the session. At the end of a session, participants were paid in private according to the number of their working-stations.

Table 2: Summary of Experiments

	Number of Subjects	c	q	p	Number of Rounds
Session 1 (FI)	20	Random	1	2/3	40
Session 2 (FI)	16	Random	1	2/3	40
Session 3 (PI)	20	Random	2/3	2/3	40
Session 4 (PI)	16	Random	2/3	2/3	40

for guessing the state. At the end of the experiment, the computer randomly selected three rounds for which subjects would be paid. The total number of points was then converted back to dollars at the rate of $1.00 = 15 points.

3.1 Some Remarks on The Design

Subjects' Positions. In the experiment, the subjects engage in a two-step decision process. Our decision to fix the subjects' position throughout the session aims to allow them to develop a *strategy* and to play accordingly. Therefore, for fixed p and q, subjects decide whom to link at different levels of cost. It is critically important to note that in a given session, there are either 16 or 20 subjects, and the groups (of four subjects) are reshuffled at each round. Therefore, a subject knows that at each round he is a member of a (possibly) different group. This is how we avoid any collusive behavior in the experiment.

Random Cost. As section 2.3 discussed in detail, the cost of link formation is one of the critical parameters that affects rational behavior according to BSLP. Therefore, having a subject play at different cost levels for 40 rounds allows us to observe how his behavior responds to the cost of link formation. The costs were chosen so that subjects in each (non-trivial) position would experience link formation costs such that, according to BSLP, the optimal action is to form a link (if c is low) or not form a link (if c is high).

Treatments FI & PI. Like the level of cost, different values of q generate different behaviors, according to BSLP.[6] The two treatments FI and PI give us the opportunity to compare, across treatments, the effect of varying the probability with which subjects receive a private signal and determining whether the predicted comparative statics hold true.

[6] The exact behavioral differences are evidenced in the threshold costs of link formation. In general, the higher the probability that subjects receive signals, the higher the threshold cost.

PAYOFFS. We employ a random lottery incentive system in our experiment as we randomly choose three rounds to determine the payoffs of the subjects. Cubitt et al. [12] demonstrates that this incentive scheme generates reliable data. Furthermore, it has many advantages. First, it helps us to generate a large data set while economizing on cost. Second, and more importantly, it assures homogenous behavior during the experiment by mitigating any wealth effects. For instance, had a subject earned many points in earlier decision rounds, his or her behavior might be distorted in the final rounds of the session. Instead, the random lottery incentive scheme makes each round equally important, so behavior should be less history dependent.

4 DESCRIPTIVE ANALYSIS

We start our analysis with a basic description of the experimental data. We organize our presentation by focusing separately on the behavior of the subjects at each decision turn. Our goal is to have a first look at the subjects' behavior and to develop the hypotheses that we test thoroughly in Section 5. In particular, we question how subjects use the information available to them and how/why they deviate from the predictions of the theory.

4.1 THE FIRST SUBJECT

The decision problem of the subjects in the first position is simple: if a subject is informed with signal $\sigma_1 \in \{-1, 1\}$, he should optimally follow his own signal and choose $a_1 = \sigma_1$; if his signal is uninformative, i.e., $\sigma = 0$ (in the PI treatment), then randomization seems to be a natural way to determine the action.

TABLE 3: THE BEHAVIOR OF THE INFORMED FIRST SUBJECT

	FI	PI $\sigma_1 \in \{-1, 1\}$
Number of errors	9	51
Number of decisions	360	236
Frequency	2.5%	21.6%

Table 3 summarizes the behavior of the informed subjects in the first round. One striking observation is the difference in the number of errors between the FI and PI treatments. In the FI treatment there are nine cases in which the subjects take an action that conflicts with their signals. In the PI treatment, this number goes up to 51 out of 236 cases in which subjects were informative. It is worth noting that the relatively high frequency of errors is spread across seven subjects in the first position—but two subjects are responsible for 25 of the 51 errors.

4.2 The Second Subject

A subject in the second position must make two decisions: a link decision and an action decision. We summarize our data in three tables: Tables 4, 5, and 7 fully describe the decomposition of link decisions and action decisions in the FI treatment, the PI treatment when subjects are informed, and the PI treatment when subjects are uninformed, respectively. In Table 6 we also compare different aspects of subjects' behavior across treatments.

4.2.1 FI Treatment

In the FI treatment, the theory predicts that a subject should not form a link unless the cost is zero. Out of 360 decisions, we observe 239 cases in which subjects do not form a link and thus make their decisions based on their signals. Among these 239 cases, subjects take the action that is the same as their signal in 214 instances. In 25 cases, however, they take the action that is opposite to their signal. Table 4 displays this data.

There are a total of 121 cases in which subjects decide to form a link to the first subject. However, in 25 of these cases the cost is zero, hence the link decisions are (weakly) optimal.[7] In order to distinguish the link decisions that are taken at zero cost, we use the notation $x : y$, meaning that x links are formed when the cost is positive and y are formed when the cost is zero. When there exists a link between the first and the second subject, there are four different possibilities:

(1) In 61 cases, the subjects observe the same action as their signal, and take the same action. Thus, even though the link decision is a deviation from the theory (for 48 of the cases), the resulting action decision is backed up with two matching pieces of information.
(2) In 21 cases, the subjects observe an action that is different from their signal. Facing two conflicting pieces of information, they decide to follow their signal. This is a curious case, because the action choice of the subject makes it even harder to justify the link decision.
(3) In 35 instances the subjects observe an action different from their signals, but prefer to follow the first subject's decision. This is the opposite of the second

[7] While it is formally not an error to form a link when the cost is zero, that is not to say that the information *should not* be ignored—this is especially true if subjects believe that their predecessors are prone to make mistakes.

situation: there they disregard the action of the first subject; here they disregard their private signal.
(4) Finally, in 4 cases the subjects observe the same action as their signals but they ignore both the action they observe and their signals by choosing an opposite action. Such behavior is extremely irrational but, fortunately, it is also quite rare.

Table 4: The Behavior of the Second Subject in fi Treatment

		Number of Subjects	
		No Link	Link
$a_2 = \sigma_2$	$a_2 = a_1$	214	48:13
	$a_2 \neq a_1$		15:6
$a_2 \neq \sigma_2$	$a_2 = a_1$	25	30:5
	$a_2 \neq a_1$		3:1

4.2.2 pi Treatment—Informed Subjects

In the pi treatment, there are 244 cases in which the subjects are informed. Out of these 244 cases, subjects in 169 of them do not form a link. This behavior is consistent with the predictions of the theory. However, in 61 instances subjects decide to form a link even though the cost is strictly positive.

Table 5: The Behavior of the Second Subject in pit Treatment: Informed Subjects

		Number of Subjects	
		No Link	Link
$a_2 = \sigma_2$	$a_2 = a_1$	157	31:4
	$a_2 \neq a_1$		24:9
$a_2 \neq \sigma_2$	$a_2 = a_1$	12	5:0
	$a_2 \neq a_1$		1:1

Table 5 is the same as Table 4, except for informed subjects in pi treatment. The second column shows the number of subjects in each of the four possibilities that we discussed earlier. We elaborate more on this in the next subsection as we compare the behavior of informed subjects in the pi and fi treatments.

4.2.3 Comparison of Informed Subjects in PI and FI Treatments

To compare the two treatments, we provide in Table 6 the frequency of the decisions in each. Although subjects tend to form links slightly less often in the PI treatment, the difference is not statistically distinguishable.

However, there is one notable compositional difference between the two treatments that is statistically significant. Specifically, we observe that subjects tend to give more weight to their own signal in the PI treatment than in the FI treatment. In the FI treatment, subjects form a link, observe an action different from their signal, and take an action consistent with their signal, 5.83 percent of the time. This goes up to 13.52 percent in the PI treatment. Similarly, in the FI treatment subjects form a link, observe an action different from their signal, but take an action consistent with the action they observe 9.72 percent of the time; that percentage goes down to 2.05 in the PI treatment.

Table 6: The Frequency of Links by Informed Subjects in the FI and PI Treatments.

		FI Treatment		PI Treatment	
		No Link	Link	No Link	Link
$a_2 = \sigma_2$	$a_2 = a_1$	59.44%	16.94%	64.34%	14.34%
	$a_2 \neq a_1$		5.83%		13.52%
$a_2 \neq \sigma_2$	$a_2 = a_1$	6.94%	9.72%	4.92%	2.05%
	$a_2 \neq a_1$		1.11%		.08%
	Total	66.39%	33.61%	69.26%	30.24%

Observation 1. *Linking behavior is not substantially different across treatments. However, it deviates from the rational benchmark. In contrast, action decisions differ across treatments. Whereas in the* PI *treatment, subjects who observe an action that conflicts with their own signal almost always follow their own signal, in the* FI *treatment there is a noticeable tendency to conform to the observed action.*

4.2.4 PI Treatment—Uninformed Subjects

Recall from Corollary 1 (and Figure 3) that the optimal behavior of the second subject depends on the level of cost. It turns out that with the parameters of the experiment, an uninformed subject should always form a link when the cost is less than 11, and should avoid forming a link when it is above 11. This is how Table 7 organizes the data. From the Table, we observe that the majority of the decisions are in this line: the number of links when the cost is less than the threshold is more than the number of links when the cost is greater than the threshold.

Table 7: The Behavior of the Second Subject in pi Treatment: Uninformed Subjects

$c < 11$			$c \geq 11$		
	Number of Subjects			*Number of Subjects*	
	No Link	Link		No Link	Link
$a_2 = a_1$	17	33	$a_2 = a_1$	34	9
$a_2 \neq a_1$		16	$a_2 \neq a_1$		7

The exact threshold cost is $c^* \simeq 11.11$.

4.2.5 Responsiveness to Cost

While it is theoretically a mistake for the informed second subject to form a link to the first at any positive cost, it is a more costly mistake when the cost is higher. Therefore, one would expect that there is a decrease in the number of links as the cost of forming a link increases. This is exactly what we find. Figure 6 depicts the observed empirical frequency of a link forming at each possible cost of link formation. There is strong evidence that the frequency of link formation decreases as the cost increases.[8]

Observation 2. *While subjects' linking behavior deviates from the Bayesian benchmark, the frequency with which links are formed is decreasing with the cost of link formation, as would be predicted by a relaxed model which allows for stochastic best response.*

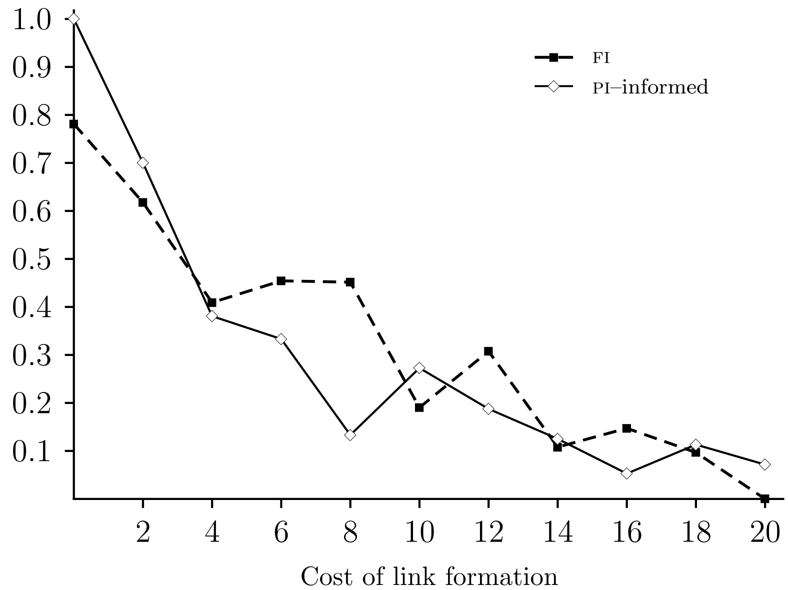
4.3 The Third Subject

The subjects in the third position face a more complicated problem. In fact, there are two possible networks that they can observe. In the first network there is a link between the first and the second subjects, while in the second there is no link between the first and the second subjects.

We begin our analysis with the link formation behavior of the subjects in the fi treatment. Table 8 provides these data. Note that when $q = 1$, as in the fi treatment, the network of the first type (the second is linked to the first subject) is not an equilibrium network for any $c > 0$. Nevertheless, we observe this off-the-equilibrium path network in the experiment. The first two rows in Table 8 show the behavior of the third subject facing such networks. There

[8] We will elaborate on this point further in section 4.4.

Figure 6: Frequency of Link Formation:
Informed Second Subjects



are 121 networks of this type. Note that in 24 of them, the cost is zero, which rationally justifies the existence of the link. In all of the cases in which $c > 0$, the third decides to form a link 62.5 percent of the time. It is worth analyzing the action behavior of the third in these situations. Table 9 summarizes the decomposition of the action decisions of the third subject.

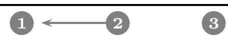
Table 8: The Distribution of Networks in the FI Treatment

Networks	$c < 5$	$c > 5$
(3.A) 1 ← 2 ← 3	52^{24}	32
(3.B) 1 ← 2 3	12^{1}	25
(3.G) 1 2 3	19^{2}	174
(3.C) 1 2 ← 3	13^{1}	12
(3.D) 1 ← 3, 2 ← 3	14^{4}	7

The exact threshold cost is $c^{**} \simeq 5.13$.
The highlighted cells indicate the equilibrium path decisions and networks.

Let us look first at the case where the third forms a link to the second. If the two actions that the third observes are the same as his signal, then he always chooses the action that matches this information (39 instances versus 0.) If, however, the actions of the second and third subjects are the same but differ from his signal, then the third subject tends to follow the actions that he observes more often than his signal (20 instances versus 6 instances.) Finally, if the actions of the first and the second are different, then his private signal becomes more decisive. In fact, in 16 cases out of 19, the third subject observes two different actions and finally takes the same action as his signal.

TABLE 9: THE DECOMPOSITION OF ACTION DECISIONS IN THE FI TREATMENT

Networks		$a_3 = \sigma_3$	$a_3 \neq \sigma_3$
(3.A) 	$a_1 = a_2 = \sigma_3$	39	0
	$a_1 = a_2 \neq \sigma_3$	6	20
	$a_1 \neq a_2$	16	3
(3.B) 		29	8
(3.G) 		167	26
(3.C) 	$a_1 = \sigma_3$	20	1
	$a_1 \neq \sigma_3$	2	2
(3.D) 	$a_1 = a_2 = \sigma_3$	4	4
	$a_1 = a_2 \neq \sigma_3$	0	0
	$a_1 \neq a_2$	11	2

The other network that the third subject observes is when the second subject does not form a link to the first (the last three rows in Table 8). In this case, there are three possibilities for the third subject: either he does not form any links, he forms only one link, or he forms two links. There are 239 such instances. Among those cases, he does not form any links 81 percent of the time, he forms a link to the second and stops 10 percent of the time, and he forms two links 9 percent of the time. Note that, according to the predictions of the theory, if the cost is greater than 5 the third subject should not form any links. This indeed happens in 90 percent of those situations. In terms of action decisions, as Table 9 indicates, we do not observe a substantial number of mistakes.

We also undertake the analysis for the PI treatment. Tables 10 and 11 provide a summary of the data for the PI treatment. The left panel of Table 10 details the networks formed by informed subjects, whereas the right hand side panel details the networks formed by uninformed subjects. Table 11 decomposes the informed subjects' action decisions in the PI treatment.

OBSERVATION 3. *There is a substantial difference in linking behavior when the third subject faces network (2.A) or (2.B). In particular, when the third subject observes a link between*

Table 10: The Distribution of Networks in pi Treatment

Networks	$\sigma_3 \in \{-1, 1\}$			$\sigma_3 = 0$		
	$c < 3$	$3 < c < 11$	$c > 11$	$c < 3$	$3 < c < 11$	$c > 11$
(3.A)	25^{15}	15	2	21^{11}	12	5
(3.B)	9^{2}	17	17	3	9	5
(3.G)	7	35	68	5	12	26
(3.F)	0	4	10	2	5	6
(3.D)	1	15	11	1	6	6

$c^* \simeq 11.11$ and $c^{**} \simeq 3.41$.
The highlighted cells indicate the equilibrium path decisions and networks.

Table 11: The Decomposition of Action Decisions in pi Treatment—Informed Subjects

Networks		$a_3 = \sigma_3$	$a_3 \neq \sigma_3$
(3.A)	$a_1 = a_2 = \sigma_3$	17	0
	$a_1 = a_2 \neq \sigma_3$	5	9
	$a_1 \neq a_2$	11	0
(3.B)		41	2
(3.G)		103	7
(3.C)	$a_1 = \sigma_3$	12	1
	$a_1 \neq \sigma_3$	0	1
(3.D)	$a_1 = a_2 = \sigma_3$	4	0
	$a_1 = a_2 \neq \sigma_3$	1	6
	$a_1 \neq a_2$	14	2

the first and the second (network (2.A)), he is very likely to form a link, whereas when the third subject observes that no link has been formed (network (2.B)), he is very unlikely to form a link.

Observation 3 is remarkable because, at least in the PI treatment, the latter network signals that the second subject was *informed*, while the former network signals that he was *uninformed*. Therefore, the Bayesian model would predict a higher frequency of link formation in the latter network than in the former network.

4.4 Observed Network & Link Decisions

Finally, we want to understand the relationship between the observed network and the link decisions of the third and fourth subjects. Specifically, we look at the binary decision of forming at least one link and how it depends upon the observed network. Tables 12 and 13 report the results of a series of estimations. The variable `informed` is a dummy variable which takes the value 1 if the subject was informed and zero otherwise. Similarly, variables depicted as networks are dummy variables which take the value 1 if the subject observes that particular network and zero otherwise. The variable `link cost` is simply the cost of link formation. For each information treatment, we first report the results of a logit estimation (coded as .1) and then the results of a fixed-effect logit estimation (coded as .2) which allows for subject heterogeneity. Clearly, allowing for subject heterogeneity does not change the results qualitatively, but it does change the magnitude of the effects; in particular, by including the fixed-effects terms, we derive estimated coefficients that are substantially larger in absolute value.

These tables reinforce some points that we have already made in our descriptive analysis. First, as was apparent in Figure 6 for the second subject, the negative coefficient on `link cost` for the third and fourth subjects shows that the greater is the cost of forming links, the lower is the probability that they will be formed. Second, informed subjects are less likely to form links than their uninformed counterparts, although this is only marginally significant for the fourth subject. Notice, too that the estimated coefficients on the network dummies are almost always smaller in the PI treatment than in the FI treatment. Thus our subjects do have a sense that information matters, even if they tend to form links more often than the theory predicts.

However, there is information in these tables. First, notice that Observation 3 carries through to the fourth subject. In both treatments and for all specifications, the estimated coefficient on the dummy for network 3.A is substantially larger than that of network 2.A. Therefore, there is a strong tendency towards what we call **herding in link formation**. Finally, there is further evidence that subjects understand the informativeness of networks, at

Table 12: Logit Regressions: Third Subject

	FI.1	FI.2	PI.1	PI.2
`informed`	NA	NA	-0.607	-1.964
	NA	NA	(0.238)	(0.453)
(2.A) 	1.562	2.559	0.703	2.217
	(0.298)	(0.497)	(0.252)	(0.478)
`link cost`	-0.247	-0.404	-0.076	-0.226
	(0.031)	(0.053)	(0.021)	(0.045)
`constant`	0.867	NA	0.447	NA
	(0.298)	NA	(0.319)	NA
`LL`	-146.88	-67.87	-220.61	-71.86
`N`	360	360	360	360

Estimated standard errors are in parentheses.

Table 13: Logit Regressions: Fourth Subject

	FI.1	FI.2	PI.1	PI.2
`informed`	NA	NA	-0.453	-0.535
	NA	NA	(0.296)	(0.361)
(3.B) 	0.160	0.866	1.211	1.930
	(0.435)	(0.537)	(0.371)	(0.485)
(3.A) 	5.069	6.589	1.930	2.922
	(1.034)	(1.305)	(0.378)	(0.506)
(3.D) 	3.472	5.432	3.086	3.823
	(1.065)	(1.393)	(0.467)	(0.669)
(3.C) 	2.246	3.708	1.034	1.307
	(0.624)	(0.906)	(0.518)	(0.672)
`link cost`	-0.160	-0.162	-0.158	-0.242
	(0.033)	(0.037)	(0.027)	(0.038)
`constant`	0.376	NA	0.238	NA
	(0.402)	NA	(0.397)	NA
`LL`	-122.05	-84.48	-170.09	-103.46
`N`	360	360	360	360

Estimated standard errors are in parentheses.

least across treatments. For the FI treatment compare the estimated coefficients on the (non-equilibrium) network 3.B and the (equilibrium) network 3.F. The coefficient is substantially and significantly larger in the latter case, indicating a greater propensity to form a link when faced with this network than the out-of-equilibrium network 3.B.[9]

5 Econometric Analysis

In the previous sections, we described behavior of the second, third, and fourth subjects in various situations. Although the behavior clearly deviates from the fully rational theory as espoused by the BSLP, we argue that it can be *rationalized* in the sense that there are systematic patterns of behavior that can be modeled. In particular, as the Bayesian theory predicts, the cost of link formation matters: the higher the cost, the less likely links are to be formed. Information matters: often behavior is substantially different between the FI and PI treatments. Finally, the structure of links (i.e., the network) matters—even if it matters differently than the Bayesian theory predicts.

Our goal in this section is to derive and estimate a parsimonious model of behavior that explains the key patterns of behavior described in our descriptive analysis. Our discussion will focus almost exclusively on the second and third subjects. Clearly, there is very little to be said about the first subject. The analysis of behavior for the fourth subject is exceedingly complex and, moreover, we have very few observations for each network. We begin with the standard Quantal Response Equilibrium (QRE) model of behavior and show that we can reject the rational model. Then, we extend the model by modifying the preferences of agents for incorporating the effects of local interactions via the relative income hypothesis à la Duesenberry [14]. In this way, we are able to describe subjects' linking and action decisions better than the QRE model.

5.1 A Model of Stochastic Best Response

5.1.1 The Second Agent

In the standard model of stochastic best response, agents experience a random shock to each of their possible decisions. For simplicity, consider the second agent receiving signal $\sigma_2 = 1$. Let $\ell_{i,j} = 0, 1$ denote the existence of a link between the i^{th} and the j^{th} agents when $j > i$. There exists a link between them if and only if $\ell_{i,j} = 1$. The expected utility of forming a

[9] Interestingly, for both networks and across both treatments, the fourth subject almost always forms a link to the *larger* sub-network, rather than to the most informative node. Thus, subjects would seem to prefer to observe more action decisions, even if, *from a strictly informative* point of view, doing so is suboptimal.

link is given by:

$$\varphi(\sigma_2 = 1, \ell_{1,2} = 1) := r \max\big\{sm, (1-s)m\big\} + (1-r)\max\big\{tm, (1-t)m\big\} - c + \epsilon_1 \quad (6)$$

where $r = \Pr(a_1 = 1|\sigma_2 = 1)$, $s = \Pr(\theta = 1|\sigma_2 = a_1 = 1)$ and $t = \Pr(\theta = 1|\sigma_2 = 1, a_1 = -1)$. Whereas the expected utility of not forming a link is given by:

$$\varphi(\sigma_2 = 1, \ell_{1,2} = 1) = \max\big\{pm, (1-p)m\big\} + \epsilon_0 \quad (7)$$

where $p = \Pr(\theta = 1|\sigma_2 = 1)$. Under standard assumptions on ϵ_1 and ϵ_0, the probability that a link is formed is given by:

$$\Pr(\ell_{1,2} = 1|\sigma_2, c) = \Big[1 + \exp\big\{\lambda_l\big(\varphi(\sigma_2, \ell_{1,2} = 0) - \varphi(\sigma_2, \ell_{1,2} = 1)\big)\big\}\Big]^{-1} \quad (8)$$

where λ_l is a parameter to estimate and captures the subject's ability to best-respond in his link decision.

From these equations, one can easily write the likelihood function given a set of observations and obtain the maximum likelihood estimates of the model. This is precisely what the left panel of Table 14 depicts. Before discussing these results, however, recall that subjects make an action decision as well as a link decision and, as we have seen, this is another source of errors in decision making. In this case, the probability that the second subject will take an action depends upon his signal and the first subject's action choice whenever there is a link. For example,

$$\Pr(a_2 = 1|\sigma_2 = 1, a_1 = 1) = \Big[1 + \exp\big\{\lambda_a(1-2s)m\big\}\Big]^{-1} \quad (9)$$

where λ_a is a parameter to be estimated and captures the subject's ability to best respond in his action decision. By taking the action decision into account, we can write the *full* likelihood function for the subject's decision problem and obtain maximum likelihood estimates of λ_l, λ_a. The results of this exercise are reported in the right panel of Table 14.

There are a number of points worth making. First, likelihood ratio tests easily allow us to reject the hypothesis that behavior is purely random (i.e., that the λ's are all zero).[10] Second, the Bayesian model also is easily rejected (i.e., that the λ's are infinite).[11] Third, $\lambda_l > \lambda_a$, implying that subjects are more likely to make an erroneous action decision than link decision. This effect is particularly pronounced in the FI treatment and much less so

[10] The calculated LR statistics are, respectively, 101.3, 96.5, 328.3 and 351.5, while the 5% critical values are $\chi^2(1) = 3.841$ and $\chi^2(2) = 5.991$.

[11] We approximated the Bayesian model by taking $\lambda_l = \lambda_a = 8$. The calculated LR statistics are, respectively, 250.0, 203.9, 928.5 and 513.3.

TABLE 14: ESTIMATION RESULTS: SECOND SUBJECT
PURE QRE

	Link Only		Link & Actions	
	FI	PI	FI	PI
λ_l	2.160	2.362	2.160	2.362
λ_a	NA	NA	1.185	1.805
LL	-198.9	-201.3	-334.900	-323.30
N	360	360	360	360

in the PI treatment. We have two explanations for this observation. First, we assumed that the second subject *correctly* anticipates that the first will make errors.[12] Therefore, in the event that $\sigma_2 \neq a_1$, it is strictly optimal to follow one's signal. However, as we saw in the descriptive analysis, over half the time $\sigma_2 \neq a_1$ and the second subjects actually chose $a_2 = a_1$. We assert that this is not so much an error but a behavioral pattern that deserves further exploration.

REMARK 1. *Note that λ_l's are the same in the* Link Only *and the* Link & Actions *specifications. This is because of the independence of errors in the link and action decisions. Alternatively, one may argue that, for example,*

$$\varphi(\sigma_2 = 1, \ell_{1,2} = 1) = r \max\{sm + \epsilon_1, (1-s)m + \epsilon_2\} + (1-r)\max\{tm + \epsilon_1, (1-t)m + \epsilon_2\} - c$$

so that, before his link decision, the second agent receives a preference shock in each of his possible action decisions. The realization of these preference shocks not only determine his eventual action decision, but also determine his link decision. This substantially complicates the estimation, since it is no longer possible to obtain closed form solutions for the probability of any given action. Nevertheless, we were able to estimate by drawing vectors of random numbers, ϵ_i, from a type I extreme-value distribution with parameter λ. The results we obtained are roughly similar to what we report here. Also, perhaps more importantly, it is hard to justify that this is the correct approach. As Table 14 shows, and from our descriptive analysis, there are clear differences between the link and action decisions.

5.1.2 The Third Agent

The empirical analysis of the third agent is in the same spirit as that of the second agent. However, matters are substantially more complicated. In particular, the third agent may

[12] That is, the anticipated error rate, $1 - \delta_1$, is the empirical error rate from our data set. For example, $\Pr(a_1 = 1|\theta = 1) = \frac{2q}{3}\delta_1 + \frac{q}{3}(1 - \delta_1) + \frac{1-q}{2}$, where, for the FI treatment, $q = 1$ and $\delta_1 = 0.975$.

face one of two different networks and has the option of forming 0, 1, or 2 links to his predecessors. Instead of giving a full derivation of the empirical choice model, we simply note the key steps required.

In addition to the required calculations from the previous section, we need more tedious calculations to be able to solve for the value of information. The task is even more complicated, since we assume that the third agent anticipates the errors made by the first and second agents. For example, consider the case in which the third agent observes that the second linked to the first. For ease of exposition, let us introduce the following notation:

$\Pr(a_1, a_2 \mid \sigma_3, \ell_{1,2} = 1)$			
(a_1, a_2)	σ_3		
	-1	0	1
$(1, 1)$	a_{11}	a_{12}	a_{13}
$(1, -1)$	a_{21}	a_{22}	a_{23}
$(-1, 1)$	a_{31}	a_{32}	a_{33}
$(-1, -1)$	a_{41}	a_{42}	a_{43}

$\Pr(\theta = 1 \mid a_1, a_2, \sigma_3, \ell_{1,2} = 1)$			
(a_1, a_2)	σ_3		
	-1	0	1
$(1, 1)$	c_{11}	c_{12}	c_{13}
$(1, -1)$	c_{21}	c_{22}	c_{23}
$(-1, 1)$	c_{31}	c_{32}	c_{33}
$(-1, -1)$	c_{41}	c_{42}	c_{43}

Then, the ex ante expected utility of the third, who observes a link between the first and the second, is

$$\begin{aligned}\varphi_j &= a_{1j} \max\left\{c_{1j}m, (1-c_{1j})m\right\} + a_{2j} \max\left\{c_{2j}m, (1-c_{2j})m\right\} \\ &\quad + a_{3j} \max\left\{c_{3j}m, (1-c_{3j})m\right\} + a_{4j} \max\left\{c_{4j}m, (1-c_{4j})m\right\},\end{aligned}$$

where $j = 1$ for $\sigma_3 = -1$, $j = 2$ for $\sigma_3 = 0$, and $j = 3$ for $\sigma_3 = 1$, and an informed third agent will form a link if and only if:

$$\max\left\{pm, (1-p)m\right\} + \epsilon_1 < \varphi_j - c + \epsilon_0.$$

If no link is observed, then the decision is even more complicated, because now the agent must decide whether to link to the first agent, second agent, or not at all; then, if he decides to link, he must decide whether to form another link, and only then to make his action decision.

In Table 15 we report the results of two different estimations for the third subjects's link decision. The first column considers the simpler problem in which the third subject observes a link between the first and the second subjects. The second column considers only the third agent's *first* link decision, whether or not the second subject linked to the first. We do not want to go into too much detail here. However, there are a couple of points that we would like to make. First, it is again the case that we can easily reject, with one exception, both the random behavior, $\lambda_l = 0$, and the fully rational behavior, $\lambda_l = \infty$, in favor of the model of stochastic best response.[13] The second point, which is particularly striking in the FI treatment, is the difference in λ_l in the first and the second columns. As we saw in the descriptive analysis, when the second agent linked to the first, the third agent was *very* likely to form a link—very often this is a mistake. Table 8 shows that λ_l is substantially higher when we consider the more general linking problem. When there was no link between the first and second agents, it was rare that the third formed a link if the cost was above the threshold. Therefore, we add to the mix a great number of instances in which the third agent took the correct action. Why does this result not appear to hold in the PI treatment? Probably because the decision problem is generally more difficult and, moreover, the aversion to forming links when the second did not link to the first appears to persist, even though linking would more often be optimal. If no link is observed, then the decision is still more complicated because the agent must decide whether to link to the first agent, second agent, or not at all. Then, if he decides to link, he must decide whether to form another link, and only then to make his action decision.

Table 15: Estimation Results: Third Subject
Pure QRE

	FI		PI	
	$\ell_{1,2}=1$	$\ell_{1,2}\in\{0,1\}$	$\ell_{1,2}=1$	$\ell_{1,2}\in\{0,1\}$
λ_l	1.184	4.361	1.983	2.249
LL	-82.330	-207.98	-87.54	-285.2
N	121	360	140	360

Remark 2. *We also could consider the full linking problem; however, we choose not to for two reasons. First, it was very rare that subjects actually formed two links. Second, of the subjects that did form two links, the first link was always to the first subject. Moreover, in the* PI *treatment, two subjects were responsible for all but two of these instances, while in the* FI *treatment, two subjects were responsible for the vast majority of cases.*[14]

5.2 A Model of Local Interactions

The QRE model in Section 5.1 formally demonstrates the existence of deviations from rational behavior; however, any deviation is simply termed a mistake and manifests itself as a

[13] The calculated LR statistics, going from left to right, for the $\lambda_l = 0$ hypothesis are: 3.1, 276.9, 19.0 and 107.1, while for the $\lambda_l = \infty$ hypotheses they are: 69.3, 61.8, 79.9 and 308.7. The one exception noted above is that we cannot reject the random behavior hypothesis for the case in which the third agent observed a link between the first and second in the FI treatment.

[14] Estimation results for the full link decision problem are available upon request.

lower estimate of λ. But why do subjects in the second position link over 30% of the time, while at the same time showing a tendency to conform in the FI treatment but not in the PI treatment? The baseline model of stochastic best response is not able to capture this. The purpose of this section therefore is to generalize our model in order to better capture behavior.

5.2.1 The Second Agent

We reformulate the utility function of the subjects in order to account for possible preferences for *relative* well-being. In a way, we modify the preferences to incorporate the relative income hypothesis, à la Duesenberry [14], which simply states that people often care more about their relative well being than their absolute well being.[15] In an axiomatic study, Ok and Koçkesen [31] proposes the following representation under some plausible axioms on the relation $\succsim$:

$$(a, y) \succsim (b, x) \quad \textit{if and only if} \quad \frac{a}{\mu(a, y)} \geq \frac{b}{\mu(a, x)}.$$

This representation states that an individual who satisfies the axioms prefers a wealth level a when the wealth distribution of the peer groups is given by a vector y, to a wealth level b when the wealth distribution of the peer groups is x, if and only if the ratio of the individual's wealth a to average income $\mu(a, y)$ is larger than the ratio of the individual's wealth b to average income $\mu(b, x)$. The representation is silent in the existence of risk. Nevertheless, having been inspired by this representation, we propose the following utility function for the second agent:

$$U(a_2, a_1; \theta, \sigma) := u(a_2; \theta) + v(a_1, a_2; \theta, \sigma),$$

where u is given by

$$u(a_2; \theta) := \begin{cases} m & \text{if } a_2 = \theta \\ 0 & \text{otherwise} \end{cases}$$

and v is the *relative* utility specified as

$$v(a_2, a_1; \theta, \sigma) := \begin{cases} \epsilon_0 & \text{if } \ell_{1,2} = 0, \\ \alpha \frac{\mathbf{E}(u(a_2;\theta)|\sigma)}{\mathbf{E}(u(a_1;\theta)|\sigma)} + \epsilon_1 & \text{if } \ell_{1,2} = 1. \end{cases}$$

That is, if the second agent forms a link to the first, he gets a utility kick of α times his relative standing vis-à-vis the first agent. Notice that this model generalizes the QRE model of the previous subsection—if $\alpha = 0$, second subject's relative standing does not matter and we are back to the standard case. In the specification above, ϵ_1, ϵ_0 are random preference shocks. We assume that they are independently distributed according to the Type I extreme-value

[15] For a literature review of the literature see Ok and Koçkesen [31] and the references therein.

distributions, with parameter λ_1.

Suppose, for example, $\sigma_2 = 1$. If no link is formed, then his expected utility is $pm + \epsilon_0$, where $p = \Pr(\theta = 1|\sigma_2 = 1)$. On the other hand, if he forms a link, then his ex ante expected utility is:

$$\begin{aligned} \varphi &= r \max\left\{sm + \alpha, (1-s)m + \alpha\frac{(1-s)m}{sm}\right\} \\ &\quad +(1-r)\max\left\{tm + \alpha\frac{tm}{(1-t)m}, (1-t)m + \alpha\right\} + \epsilon_1 \\ &= \varphi' + \epsilon_1 \end{aligned}$$

where r, s, and t are as in section 5.1.1. It is clear that the second agent will link if the following inequality is satisfied:

$$pm + \epsilon_0 \leq \varphi' + \epsilon_1 - c.$$

Again, one can easily write the likelihood function for the entire data set and obtain the Maximum Likelihood estimates of α and λ_l.

As with our QRE specification above, we also can make use of choice data. Specifically, for all those who have formed a link to the first subject, we can calculate the probability that he will take any particular action. Consider the case in which $\sigma_2 = a_1 = 1$. The expected utility from conforming, i.e., $a_2 = 1$, is $sm + \alpha + \epsilon_c$. On the other hand, the expected utility of not conforming is: $(1-s)(m + \frac{\alpha}{s}) + \epsilon_n$. The probability of conforming is then given by:

$$\begin{aligned} \Pr(a_2 = 1|\sigma_2 = a_1 = 1) &= \Pr\left(sm + \alpha + \epsilon_c - \left((1-s)\left(m + \frac{\alpha}{s}\right) + \epsilon_n\right) \geq 0\right) \\ &= \Pr\left(\tilde{\epsilon} \geq (1-2s)\left(m + \frac{\alpha}{s}\right)\right). \end{aligned}$$

Now consider the case in which $\sigma_2 \neq a_1 = 1$. The expected utility from conforming is $(1-t)m + \alpha + \epsilon_c$. On the other hand, the expected utility of not conforming is: $t(m + \frac{\alpha}{1-t}) + \epsilon_n$. The probability of conforming is then given by:

$$\begin{aligned} \Pr(a_2 = 1|\sigma_2 \neq a_1 = 1) &= \Pr\left((1-t)m + \alpha + \epsilon_c - \left(t\left(m + \frac{\alpha}{1-t}\right) + \epsilon_n\right) \geq 0\right) \\ &= \Pr\left(\tilde{\epsilon} \geq -(1-2t)\left(m + \frac{\alpha}{1-t}\right)\right). \end{aligned}$$

As earlier, we assume that ϵ_c and ϵ_n are independently distributed with a Type I extreme-value distribution with parameter λ_a, where $\lambda_l \neq \lambda_a$ to reflect potentially different best-response frequencies for link and action decisions. Given this, one can write the full log-likelihood as: Pr(choice|link decision) · Pr(link decision) and estimate the parameters of the model. For both the FI and PI treatments, the results are presented in Table 16.

TABLE 16: ESTIMATION RESULTS: SECOND SUBJECT

	Link Only		Link & Actions	
	FI	PI	FI	PI
α	0.236	0.192	0.232	0.194
λ_l	3.643	3.894	3.623	3.905
λ_a	NA	NA	1.168	1.763
LL	-190.22	-190.1	-326.77	-311.45
N	360	360	360	360

There are a number of interesting observations here. First, notice that likelihood ratio tests easily allow us to reject the null hypothesis that $\alpha = 0$ in all cases.[16] A more important observation is the fact that the α's are estimated to be the *same* whether we use only linking data or linking and choice data.[17] This provides our model with an extra degree of external validity, for it is able to explain: 1) why subjects form links, and 2) why subjects have a tendency to conform in the FI treatment but not in the PI treatment. For example, one may be tempted to say that subjects have a preference for conformity. In the FI treatment, this could accurately rationalize the data. However, in the PI treatment, one would need an ex ante preference for conformity (to explain the linking behavior) and an unanticipated ex post Bayes rationality (to explain choice behavior). Our model succeeds in explaining both facts with one parameter.

Notice that λ_a is not significantly different in this specification than in the QRE specification, even though more action decisions are being labeled correct. However, since $\alpha > 0$, the payoff difference between the *correct* and *incorrect* actions is widened and, therefore, the same λ_a generates greater accuracy. On the other hand, notice that λ_l is significantly higher in the present specification than in the QRE specification. This may seem to conflict with the explanation offered for why λ_a is constant across the two specifications; however, there is a relatively straightforward explanation. For action decisions, from a strictly Bayesian perspective, the payoff difference from choosing $a_2 = 1$ or $a_2 = -1$ is not that great if the second agent's signal differs from the observed action; therefore, systematic errors in favor

[16] For the "Link & Actions" estimation, we obtain LR statistics of 16.2 and 23.64 for the FI and PI treatments, respectively. For the "Link Only" estimation the LR statistics are 17.28 and 22.44.

[17] The LR statistic for the Link Only case is 0.79, while for the Link & Actions case is 0.59. Also, in neither case can we reject the null hypothesis that $\alpha_{FI} = \alpha_{PI}$.

of one action will not bias λ_a downwards much in the QRE specification. However, again from a strictly Bayesian perspective, because linking requires the payment of a positive (and sometimes large cost), systematic deviations toward linking *necessarily* bias λ_l downwards, unless there is something else to compensate by raising the payoff to linking. That something else is $\alpha > 0$.

5.2.2 The Third Agent

Just as it was possible to generalize the QRE specification to the third agent, we can do the same with our model of local interactions. Consider the case in which the third agent observes that the second linked to the first.[18] It can be shown that the ex ante expected utility of the third who observes a link between the first and the second is:[19]

$$\begin{aligned}\varphi_j &= a_{1j} \max\left\{c_{1j}m + \alpha, (1-c_{1j})m + \alpha\frac{1-c_{1j}}{c_{1j}}\right\} \\ &\quad + a_{2j} \max\left\{c_{2j}m + 2\alpha c_{2j}, (1-c_{2j})m + 2\alpha(1-c_{2j})\right\} \\ &\quad + a_{3j} \max\left\{c_{3j}m + 2\alpha c_{3j}, (1-c_{3j})m + 2\alpha(1-c_{3j})\right\} \\ &\quad + a_{4j} \max\left\{c_{4j}m + \alpha\frac{c_{4j}}{1-c_{4j}}, (1-c_{4j})m + \alpha\right\},\end{aligned}$$

where $j = 1$ for $\sigma_3 = -1$, $j = 2$ for $\sigma_3 = 0$, and $j = 3$ for $\sigma_3 = 1$. Here, a_{ij} and c_{ij} represent the same probabilities as in the QRE specification; however, because we assume that agents anticipate the errors of their predecessors, they will indirectly depend on λ_l, λ_a, and α, which we estimated earlier for the second subject. An informed third agent will form a link if and only if:

$$\max\{pm, (1-p)m\} + \epsilon_1 < \varphi_j - c + \epsilon_0$$

As in Table 15, Table 17 reports the results of two different estimations: first, considering only those instances in which the third subject observes a link between the first and second; then, considering the third subject's first link decision problem, regardless of whether there is a link between the first two subjects. Three points are of immediate interest. First, for both information treatments, the estimated α declines substantially in the second (more general) estimation. Second, while α always remains positive for the FI treatment, it becomes negative (but insignificant) in the PI treatment. Finally, the model appears to fit the data

[18] In this case we write the *relative* utility for the third agent as

$$v(a_3, a_2, a_1; \theta, \sigma) := \begin{cases} \epsilon_0 & \text{if } \ell_{2,3} = 0, \\ \alpha\frac{\mathbf{E}(u(a_3;\theta)|\sigma)}{\mathbf{E}(u(a_1;\theta)|\sigma) + \mathbf{E}(u(a_2;\theta)|\sigma)} + \epsilon_1 & \text{if } \ell_{2,3} = 1. \end{cases}$$

[19] Details of this and other calculations are available upon request.

generated from the FI treatment substantially better than that of the PI treatment.

TABLE 17: ESTIMATION RESULTS:
THIRD SUBJECT'S LINK DECISION

	FI		PI	
	$\ell_{1,2}=1$	$\ell_{1,2}\in\{0,1\}$	$\ell_{1,2}=1$	$\ell_{1,2}\in\{0,1\}$
α	0.369*	0.091*	0.227*	-0.083
λ_l	3.415*	5.033*	2.607*	1.974*
LL	-65.668	-204.131	-81.956	-285.190
n	121	360	140	360

* Significant at 5% level

Table 17 lends further support to what was apparent in the descriptive analysis: subjects in the third position view their decision problem *substantially* differently depending on whether they observe a link between the first and the second subjects. When they observe a link, they are very likely to form a link; conversely when they do not observe a link, they are very unlikely to form a link. To be sure, because the network evolves endogenously, and subjects in the second position are more likely to form a link for low costs, the third subject is more likely to observe a link when the cost is low and not to observe a link when the cost is high (in which case, the Bayesian theory would predict that no link should be formed). However, our estimation procedure controls for the cost of link formation—yet the apparent difference remains!

We are at somewhat of a loss to explain this feature of the data. According to BSLP, the reason that subjects in the third position do not form a link upon observing the empty network is that they realize that by forming one link, they may observe that their linked subject took the opposite action from his signal, therefore necessitating another link, and hence the costs of two links. If c is high enough, this tradeoff is simply not worth it. However, our subjects appear to go beyond even what would be predicted by BSLP: upon observing the empty network, they only link infrequently.

5.3 DISCUSSION

Our empirical results suggest that local interactions between subjects are important in driving the evolution of the network. For the second agent, it appears, the motivating factor is a desire to compare his standing with that of the first agent. This shows up as a positive estimate for α and can explain the inflation of links in both treatments, a tendency to conform in the FI treatment, and the strong incentive to follow one's own signal despite a contradictory observation. Eliaz and Schotter [15] experimentally study a problem similar to the second agent's: A prize is in Urn A with probability h in the high state and with probability $l \in (\frac{1}{2}, h)$ in the low state; in either state, the prize is in Urn B with complementary probability. Subjects can pay a cost, $c > 0$, in order to learn whether the state is high or low. As in our setting, the subjects should never pay the cost because choosing Urn A is always better than choosing Urn B. Similar to us, they find a substantial number of subjects who are willing to pay for this information, and argue that subjects have a preference over beliefs. While this could explain some of our results, it cannot explain why many subjects actually conformed to the first subject, even though it went against their own signal. Thus, in our experiment, the information gained by forming a link *was* often decisive.

Eliaz and Schotter [15] discuss two alternative explanations: the "disjunction effect" of Tversky and Shafir [34] and Kreps and Porteus' [25] preference for early resolution of uncertainty. It is difficult to see how the latter explanation could rationalize our data, because the act of forming a link does not actually resolve any uncertainty about the state of the world. The disjunction effect, which posits that subjects do not fully anticipate their final decision when making their initial link decision, also has some merit; however, we believe it fails on two grounds: first, like Eliaz and Schotter [15], it cannot explain why the information gained from linking is sometimes decisive. Second, since subjects played the network formation game forty times, we would expect any disjunction effect to dampen over time as subjects learn. However, we found no such tendency in our data. Therefore, we believe that our explanation remains the leading one.[20]

For the third subject, matters are more complicated because now subjects can condition their behavior on one of two networks; indeed, when faced with the empty network they must consider the possibility that if they form one link, they may form subsequent links. Here we saw the importance of the network: when the second linked to the first, the third was very likely to link (hence $\alpha > 0$), but when the second did not link to the first, the third was much less likely to link (hence the dramatic drop in α). It is of interest here to examine subject heterogeneity in order to determine the reasons for the decline in α. In Table 18, we re-estimated our empirical model for the third agent but allowed for a subject-specific α.

Three things are apparent. First, there is a great deal of variation in the estimated α's across subjects. Second, going from the network with a link between the first and second subject to the one without a link, the estimated α diminishes in all but one case, sometimes becoming negative. Why this uniform drop in α? Two explanations come to mind: either subjects in the third position like to mimic the link decision of the second, or the prospect of having to form two links *scares* them away from forming *any* links. Finally, it is apparent that allowing for subject heterogeneity dramatically improves the model's fit; however, it

[20] From the discussion in Remark 1, one could argue that our empirical model already contains a disjunction effect. While this is a valid point, it was done largely for empirical convenience. As was discussed above, the empirical results are qualitatively similar if we estimate the more complicated model discussed in Remark 1; i.e., we still obtain $\alpha > 0$.

Table 18: Estimation Results: Subject-Specific α's

	FI		PI	
	$\ell_{1,2} = 1$	$\ell_{1,2} \in \{0, 1\}$	$\ell_{1,2} = 1$	$\ell_{1,2} \in \{0, 1\}$
λ	6.701	7.445	3.807	4.114
α_1	0.935	0.458	0.680	1.884
α_2	0.319	0.156	0.811	0.678
α_3	0.019	-0.068	-0.246	-0.518
α_4	0.725	0.505	0.138	-0.281
α_5	0.281	0.188	-0.382	-0.523
α_6	0.466	0.242	0.276	-0.138
α_7	0.120	0.053	0.933	0.430
α_8	-0.239	-0.250	0.126	-0.067
α_9	-1.276	-1.833	-0.384	-0.516
LL	-35.293	-137.134	-49.806	-162.994
N	121	360	140	360

does not alter the qualitative results, with one exception: it is not true that all subjects are averse to forming links when faced with the empty network (as one might conclude from Table 17), but rather there appears to be a mixture of behavioral *types*.

6 Concluding Remarks

In this paper we present the results of an experiment on endogenous network formation. Not surprisingly, there were substantial deviations from the Bayesian theory. In particular, subjects in the second position tended to form many links; in some instances there was a tendency to conform to the decision of one's predecessor; the observed network was *extremely* important in driving linking behavior; and, rather than linking to the most informative node, subjects tended to prefer linking to larger networks. There was also a noticeable pattern of herding in link formation. However, behavior did conform to many of our expectations: the frequency of link formation was negatively related to the cost, and there were important differences in behavior between the FI and PI treatments, suggesting that subjects do grasp certain aspects of information transmission through the networks.

All of our results suggest that *local interactions* matter: whether in their link or action decisions, people emulate their predecessors and are heavily influenced by what they observe. Indeed, the empirical model in Section 5.2, in which subjects are motivated to form links because they care about their relative standing, does an excellent job of rationalizing the data. This suggests that there is value in seeking out further connections between the social learning/network formation literature and the literature on local interactions—both in terms of theory and experimental design.

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Appendices

Appendix A: Omitted Proofs

Corollary 1. *The optimal decision rule of the second agent is characterized as follows:*

1. *Let $\sigma_2 \in \{-1, 1\}$ and $q \in (0, 1]$. For any $c > 0$, the second agent does not form a link and takes action $a_2 = \sigma_2$.*
2. *Let $\sigma_2 = 0$ and $q \in (0, 1]$. There exists a threshold $c^* = \frac{qm}{6}$ such that for any $c < c^*$, the second agent links to the first and takes action $a_2 = a_1$; if $c \geq c^*$, the second agent does not form a link and randomizes between the two actions.*

Proof.

1. Let $q \in (0, 1]$, and without loss of generality, suppose that $\sigma_2 = 1$. Then, the value of information from linking to the first agent when the unit cost of a link is $c > 0$ is

$$v(\sigma_2) = \sum_{a_1} \Pr(a_1|\sigma_2) \max_{a_2} \left\{ \sum_{\theta} \Pr(\theta|\sigma_2, a_1) u(a_2, \theta) \right\} - \max_{a_2} \left\{ \sum_{\theta} \Pr(\theta|\sigma_2) u(a_2, \theta) \right\} - c$$
$$= \sum_{a_1} \Pr(a_1|\sigma_2) \sum_{\theta} \Pr(\theta|\sigma_2, a_1) u(1, \theta) - \sum_{\theta} \Pr(\theta|\sigma_2) u(1, \theta) - c < 0$$

The second equality comes from the fact that choosing $a_2 = 1$ is always optimal, regardless of a_1 and hence the first two terms are canceled out. Therefore, it is never optimal to form a link to the first agent for any positive unit cost $c > 0$. The case of $\sigma_2 = -1$ is similar.

2. Let $q \in (0, 1]$, and suppose that $\sigma_2 = 0$. Then a simple computation leads the value of information from linking to the first agent to be

$$v(\sigma_2) = \frac{qm}{6} - c.$$

Therefore, when $c < \frac{qm}{6}$ the second agent forms a link to the first. However, when $c \geq \frac{qm}{6}$ the second agent does not form a link.

□

Corollary 2. *The optimal decision rule of the third agent is characterized as follows:*

1. *Suppose there is a link between the first and the second agents.*
   (a) *Let $\sigma_3 \in \{-1, 1\}$ and $q \in (0, 1]$. Then, for any $c > 0$ the third agent does not form a link and takes action $a_3 = \sigma_3$.*
   (b) *Let $\sigma_3 = 0$ and $q \in (0, 1]$. Then, for any $c < c^*$ the third agent links to the second and takes action $a_3 = a_2$; if $c \geq c^*$, the third agent does not form a link and randomizes between the two actions.*
2. *Suppose there is no link between the first and the second agents.*
   (a) *Let $\sigma_3 \in \{-1, 1\}$ and $q \in (0, 1]$.*

i. *There exists a threshold* $c^{**} = \frac{2qm}{39}$ *such that for any* $c < c^{**}$*, the third agent links to the second agent. If* $a_2 = \sigma_3$*, then he does not form a link to the first and takes action* $a_3 = a_2$*; if* $a_2 \neq \sigma_3$*, then he links to the first and takes action* $a_3 = a_1$*.*

ii. *For any* $c \geq c^{**}(q, m)$ *the third agent does not form a link and takes action* $a_3 = \sigma_3$*.*

(b) *Let* $\sigma_3 = 0$ *and* $q \in (0, 1]$*.*

i. *For any* $c < c^*$ *the third agent links to the second and takes action* $a_3 = a_2$*.*

ii. *For any* $c \geq c^*$*, the third agent does not form a link and randomizes between the two actions.*

Proof.

1. Suppose there is a link between the first and the second agents. When there is a link between the first and the second agents, the decision problem of the third agent is equivalent to that of the second agent (see the proof of Corollary 1.)

2. Suppose that there is no link the first and the second agents.

(a) Let $\sigma_3 \in \{-1, 1\}$ and $q \in (0, 1]$.

i. The value of information is

$$\begin{aligned} v(\sigma_3) &= -c + \sum_{a_2} \Pr(a_2|\sigma_3) \max\{v(\sigma_3, a_2), 0\} \\ &= -c + \frac{4}{9}\left(\frac{qm}{6} - c\right) = \frac{2qm}{27} - \frac{13}{9}c. \end{aligned}$$

Therefore, when $c < \frac{2qm}{39}$ we have $v(\sigma_3) > 0$, hence the third agent links to the second agent. If $a_2 = \sigma_3$, the value of information is $v(\sigma_2, \sigma_3) = v(\sigma_2)$; hence the third does not form a link. If $a_2 \neq \sigma_3$, then the value of information is equivalent to $v(\sigma_3 = 0)$. We already know that in this case the third forms a link.

ii. If $c \geq \frac{2qm}{39}$ we have $v(\sigma_3) < 0$, hence the third agent does not form a link to the second agent.

(b) Let $\sigma_3 = 0$ and $q \in (0, 1]$.

i. The value of information is

$$\begin{aligned} v(\sigma_2) = &\sum_{a_2} \Pr(a_2|\sigma_3) \max_{a_3}\left\{\sum_{\theta} \Pr(\theta|\sigma_3, a_2) u(a_3, \theta)\right\} - \max_{a_3}\left\{\sum_{\theta} \Pr(\theta|\sigma_3) u(a_3, \theta)\right\} \\ &- c + \sum_{a_2} \Pr(a_2|\sigma_3) \max\{v(\sigma_3, a_2), 0\} = \frac{m}{6} - c, \end{aligned}$$

where $v(\sigma_3, a_2) < 0$ for any (σ_3, a_2). Thus, for $0 < c < \frac{qm}{6}$, it is optimal to form a link to the second agent. After forming a link, for any action he observes, the value of information for the third agent is identical to that of an informed second agent. Therefore, the third does not form a link to the first, and takes action $a_3 = a_2$ (see the proof of Corollary 1.) and follow his action decision.

ii. When $c \geq \frac{qm}{6}$, the value is $v(\sigma_3) < 0$. Therefore, the third does not form a link to the second agent.

□

Appendix B: Instructions

General Instructions

This is an experiment in the economics of decision-making. Your earnings will depend partly on your decisions and partly on chance. By following the instructions and making careful decisions you will earn varying amounts of money, which will be paid at the end of the experiment. Details of how you will make decisions and earn money will be provided below.

In this experiment, you will participate in 40 independent rounds, each of which contains four decision positions in a decision queue. In each round you will be asked which of two urns has been randomly chosen (called *action decision*); however, before making your action decision, some subjects will be able to observe the actions of those who have gone before them (called *link decision(s)*) by paying a cost that will be determined by the computer at the beginning of each round.

Before the first round, you will be randomly assigned to a position in the decision queue labeled **1**, **2**, **3** or **4**. One-fourth of the participants will be randomly assigned to each of the four positions. Your position depends solely on chance and will remain constant in all rounds throughout the experiment. When you are called to make decisions, in the center of the computer screen you will be informed of your position and any link decisions made by those in preceding decision positions; however, you will not observe their action decisions.

A Decision Round

Each round starts by having the computer randomly form groups of four participants by selecting one participant from each of the four positions. The groups formed in each round depend only on chance and are independent of the groups formed in any of the other rounds.

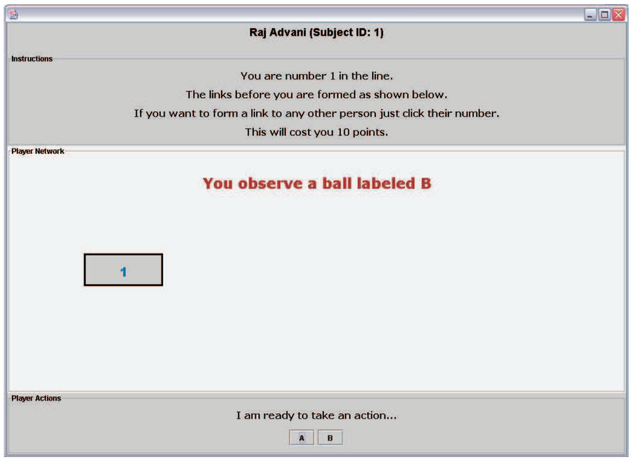
In each round you will be asked to predict which of two urns, labeled **A** and **B**, has been chosen. For each group of four, it is equally likely that urn **A** or urn **B** will be chosen. **Urn *A* contains *2 balls* labeled *A* and *1 ball* labeled *B*. Urn *B* contains *2 balls* labeled *B* and *1 ball* labeled *A*.**

To help you determine which urn has been selected, you will be allowed to observe one ball, drawn at random, from the urn *at no cost.* In addition, if you are in position **2, 3 or 4**, you will be given a chance to see action decisions in preceding positions at a cost determined by the experimental software.

Your private draw in each round is independent of the draw received by any other participant. The result of your draw will be your private information and should not be shared with any of the other participants. You will see your private draw in the middle portion of the computer screen.

After each draw, the ball will be returned to the urn before making a private draw for the next participant. This is done by the experimental software.

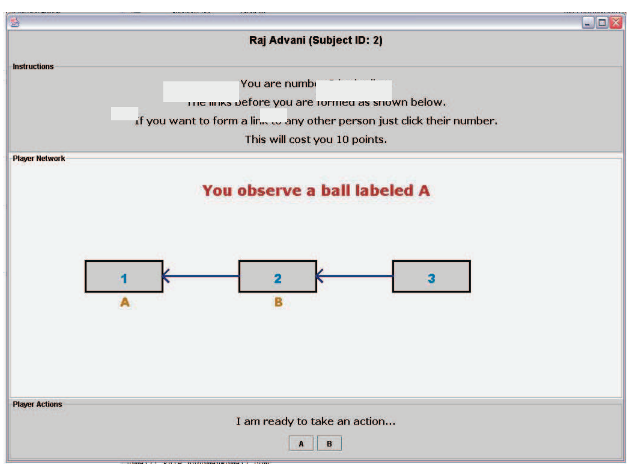
Participants assigned to position **1** may see the following screen on your computer screen:



In this case, since you are in the first position in a decision queue, all you need to do is make your action decision based on your private information. This is done at the bottom of the screen by simply clicking on either **A** or **B**.

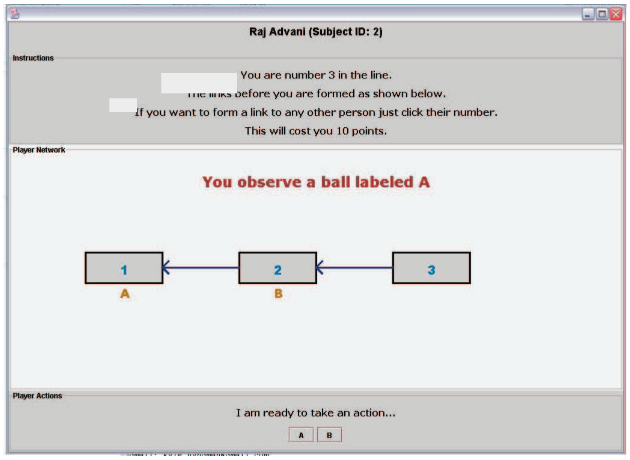
For participants assigned to positions **2**, **3** and **4**, there will be other participants in the same group who have already made their action and link decisions. In addition to your private draw, you will have the opportunity to observe the action decisions of those in preceding positions at a cost determined by the experimental software at the beginning of each round. When it is your turn to move, you will see a graphical representation of all the link decisions made by those who precede you in the decision queue.

For example, suppose that you are assigned to position **3** in the queue. You may see the following screen:



In this example, your private draw was a ball labeled **A**. In addition, you observe that the participant in the second position chose **not** to form a link to the first position. That is, the participant in the second position predicted which urn was more likely to be chosen, while having chosen not to observe the action decision by the participant in the first position.

Continue with the example above and suppose that you wish to form a link to the second position. To do this, simply click on the box labeled **2**. Then you will observe the action decision made by the participant in position **2** while incurring the cost of forming a link. This is depicted below:



Note that once you form a link to one preceding participant, you see not only his/her action choice *but also* the action choices of all those with whom that person linked.

For example, suppose that the second person *had* actually formed a link to the first person in the queue. In this situation, by forming a link to the second position in the queue you would see the action decisions of **both** participants in the first and the second positions in the queue.

In this example, if you wish to observe more information, you may form a link to the first position, **at an additional cost**, and observe his or her action decision. If not, and you are ready to make your decision, simply click on the box labeled **A** or **B** at the bottom of your screen, corresponding to which urn you think was more likely to have been chosen.

Once you have made your decision for that round, you will be informed which urn was actually used and what your potential payoffs are for that round. By clicking on the **OK** button you will be taken to a waiting screen and then the next person in the line will be able to make his or her decisions.

This concludes one decision round. All of the participants will then be randomly placed into a new group of four people. In total, you will repeat 40 independent rounds with various levels of costs.

Remember: In each round, the same urn applies to all members of a group. That is, the experimental software picks **one** urn for each group in each round.

Cost of Forming Links

Now, we will describe in detail how the cost of forming a link will be determined in each of the 40, independent, rounds. In all rounds throughout the experiment, the cost of forming a link can be any **even** number between 0 and 20, inclusive; that is, the cost will be one of the following numbers, 0, 2, 4, …, 16, 18, 20.

In each round, the computer will randomly assign a cost to each group of four. The chance that the computer selects any even number between 0 and 20 points is exactly the same. That is, the chance that a cost of 2 is selected is the same as the chance that a cost of 14 is selected and so on. Moreover, the cost assigned in one decision round is independent of the cost in any other decision round.

Remember: The cost of forming a link in each round is the same for all members of a group.

Moreover, the cost for each link is the same (*e.g.,* If you form one link at a cost of 10 points, you are free to form another link by paying an additional cost of 10 points.).

Payoffs

Your potential earnings for each round are determined as follows. If you made the correct action decision regarding which urn was used, you will be awarded 100 points for that round; otherwise, you will be awarded nothing. From this amount, either 100 or 0, we will subtract the appropriate cost for **each** link decision that you made. For example, if, in round 10, the cost of link formation was 18 points, then in determining your potential earnings for round 10, 18 points will be subtracted, from either 100 or 0, for every link decision that was made. For example, if, after having made one link decision, you correctly guessed which urn was chosen, your potential earnings would be 100 - 18 = 82 points.

At the end of the 40 rounds, the experimental software we will randomly select three rounds from which you will be paid. The total number of points earned will be summed up for each of these three rounds — 100 points for each correct decision, from which we will subtract the appropriate number of points for each link decision. This will be converted to a dollar amount according to the rule:

$$\$1 = 15 \text{ points}$$

This amount will then be added to the $8.00 participation fee to give your payment for this experiment. Payments will be made in private via petty cash vouchers at the conclusion of the session.

Rules

Please do not talk with anyone during the experiment. We ask everyone to remain silent until the end of the last decision problem.

Your participation in the experiment and any information about your earnings will be kept strictly confidential. Your receipt of payment and consent form are the only places on which your name will appear. This information will be kept confidential in the manner described in the consent form.

If you have any questions please ask them now. If not, we will proceed to the experiment.