DOUBLE BASS INTONATION: A SYSTEMATIC APPROACH TO SOLO AND ENSEMBLE PLAYING

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Abstract

This study uses an interdisciplinary approach to analyze double bass intonation as it occurs in a solo (i.e., without playing with any additional instruments) and ensemble contexts, develops a systematic approach to double bass intonation (subsequently referred to as "the system"), and applies that system to double bass literature to theoretically test its applicability. While the examples used come mostly from the orchestral literature, the material presents passages often heard in an orchestral audition context in which the bass is played by itself. Intonation generally is examined according to acoustics, psychoacoustics, cognition, historical and modern performance practice, and pedagogy. The salient principle extracted is that acceptable intonation is generated from the satisfaction of several factors, including clear categorical assignment of an interval's size in a tonal context, highly rated timbral characteristics of the sound produced, and tone placement conforming to emotional schematic expectations, and a general model of acceptable intonation is thereby proposed. With this background, the particular intonation difficulties of the double bass are analyzed, including acoustic roughness, psychoacoustic roughness, and part-specific intonation expectations. The resonance system of double bass intonation proposed is intended to minimize acoustic and psychoacoustic roughness while staying within the categorical bounds of intonation and maximizing conformity to schematic expectations. The system's efficacy is theoretically tested against examples from the double bass literature from various time periods, keys, and modes. It is found to conform in most cases to intonation expectations, and where not, alternative readings and tone placements are suggested. A possible course of study to implement the system is then suggested, as are extensions and ideas for further related research.

Keywords: Double Bass
Intonation
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Temperament
Acoustics
Psychoacoustics
Cognition
Performance Practice
Pedagogy

N.B. All words marked with a $^+$ are further defined in Appendix C: Glossary, p. 255

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Chapter 1: Introduction

"Intonation is the cross we all have to bear as string players."

-Anonymous audition committee member

The search for "good" intonation is one that beguiles many a string player. It is shrouded in myth and legend, conflicted in artistry and physics. Many proceed through their education without a clear understanding of what it is they are measuring and the manner in which they are measuring it. Intonation forms part of the foundation of the musical art, being one of the three components, along with timbre and timing, which a musician is able to expressively manipulate. For string players, we have the most flexibility of where to place our tones[+] but also the most burden, having only four fixed tones which to reference. Attaining artist-level intonation was elusive for me since my first days as an undergraduate, and not until my senior year was I given a method of approaching how to learn to play in tune in a systematic way. It was a 10 minute section of a lesson which has informed how I approach intonation and has formed the impetus for the research for this paper.

A bit more abstractly presented here than was originally, the principle is that each note[+] on a stringed instrument resonates in a particular way relative to the overtone series[+] of the other strings and that the best acceptable level of intonation is perceived when the instrument is ringing sympathetically. This principle, which has been in extant in the violin family pedagogical literature for generations, has yet to penetrate to the double bass literature. In fact, intonation is a topic which has only recently begun to be discussed and debated seriously in the double bass literature. Double bass pedagogy has tended to focus strictly on finger placement and hand-frame approaches, and has focused less on developing an idealized intonation. Many rely on equal temperament-based electronic tuners to tell them whether or not they are in tune, which has been equated to playing video games as opposed to actually learning to play in tune.[1] There is even been a mythology developed that, in contrast to the other string instruments, the double bass is not able to be played in tune.

The purpose of this study is to begin to address the issues which concern double bass intonation perception, build a systematic approach to an acceptable root intonation for the bass (subsequently referred to as a "system" for ease of reading), to apply this system to examples of the performance, and to outline some aspects for which it would benefit the pedagogy of the double bass. I say "begin to address" because be it good, bad, preferred, or acceptable, the issues relating to intonation are those which are not fully described in the literature at large, and in addition, intonation issues of the contra register are not generally included. Many issues of how exactly to dissect and examine intonation have yet to be resolved. So many codependent factors influence the perception of intonation that many questions are still to be studied in detail. That intonation is a problem which can be viewed and interpreted in so many ways is a reason it remains fascinating to me.

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[1] Lambert 2006

The null hypothesis, if one may allow me to term it that, is that the bass cannot be played in tune; the alternate hypothesis is that, because of the bass's historical development, (psycho)acoustics, and musical function, the bass is perceived to be in tune in a different manner than the rest of the string family in particular, and much of the rest of the instrumental world in general. Many of the principles which underlie the perception of intonation acceptable at an artistic level of the other symphonic string instruments will apply to the bass, but because the bass is a historically differently derived instrument, the outcome will be slightly different.

I use the term "acceptable" in particular, because while large variability in each musician's preference for a given intonation in a given musical context, a general consensus does exist on what falls within acceptability in most contexts. Once a player has developed a sense of a grounded intonation framework, expressive variants can be made from that framework, leading toward an artist-level sense of intonation. This is very similar to the idea of rubato, which must be grounded strongly in a player's sense of rhythm. Just as rubato generates its expressive meaning from its intentional resistance to the entrained[+2] rhythmic structure, so do expressive variants of intonation get their meaning from an intentional deviation from the established intonation framework.

Once the system of double bass intonation is drawn, I will show its application to and durability with the double bass literature. This will show that the system will provide an acceptable starting point, if not a full solution, to intonation quandaries which occur in

[2] This is a somewhat peculiar usage of the word fairly unique to the psychological field describing the alignment of a subject's internal rhythmic sense or pattern expectation to an external rhythm or other pattern. In this case, the external rhythm is the piece being performed and the subject is the auditor.

the literature. As I alluded to earlier, I have no illusions that this system will work in every instance. It is a system that does not necessarily take into account contextual functionality or expressive variation, but it will provide a starting point for a more detailed exploration of the structure and possible meaning of a given passage. Conversely, given the function and structure of basslines and audience expectations of string intonation, the system will fall within perceptual acceptability far more often than not.

The larger purpose of the system will be to further the pedagogy of the instrument. In speaking with my colleagues, bassists seem to have the least clear conception of what acceptable intonation should be and that conception develops the more slowly than for other instrumentalists. There are a number of legitimate reasons for this, not least of which are the acoustic and psychoacoustic limitations inherent with the register, as well as intonation being not nearly as well discussed as in the literature of the other instruments. Bassists, as a community, have tended to develop a sense of intonation largely by focusing on hand-frame approaches, by immersion with players executing superior intonation, and trial-and-error under the guidance of a teacher, who learned in a similar manner. We tend to not talk about how one achieves acceptable intonation, but are quick to point out when someone is not in tune! Rarely, one finds a teacher who will discuss the theoretical underpinnings of an intonation system for the double bass. One of my specific goals when I set out to conduct this study was to bring the underlying factors determining the acceptability of intonation into the discussion and suggest a solution. By proposing a method of approaching double bass intonation, I hope to aid students by increasing the speed of their mastery of basic manipulation of one of the elements of the instrument. In this way, intonation and its expressive variants can be quickly integrated into a bassist's playing and become part of the whole artistic concept of a passage, not just a utilitarian requirement of the job of playing bass.

Because intonation is a multidimensional phenomenon, I have tried to focus my approach as much as possible. The first concession is that intonation is a perceptual problem, not primarily one of physical execution. While I admit that physical acoustics play a large role in intonation, it is how those physical parameters are received and interpreted which concern me more. Much ink has been spilt over absolute frequencies and ratios which in the end can have very little to do with the perceived acceptability of adjacent pitches[+], for example. That said, I will make a few allowances for the sake of brevity. Firstly, any references to a fundamental frequency are assumed to refer to the perceived pitch, unless otherwise indicated. This is not a safe assumption when one is dealing with real[+3], stretched, dense strings, but since we won't be directly measuring the frequencies of any tones, it is an acceptable conceit in this narrow case. The corollary is that each fundamental frequency referenced is assumed to have a regular harmonic[+] overtone series above it. Again, these stipulations are for ease of the general discussion; the primary focus is on the perception of the tones involved.

[3] Acousticians use "real" to describe the behavior of actual materials or instruments, as opposed to "theoretical" materials or instruments. While theoretical systems behave in very predictable and easily calculated fashions, real systems rarely do so, having numerous anomalies which are not easily accounted for.

The pedagogical literature has been very good about addressing manners of execution which lead to proper intonation. But, the assumption taken in the literature is that the student is correcting poor intonation which has already been discriminated. At its worst, some philosophies teach that proper movement alone leads to proper intonation. What I want to address is the underpinnings of acceptable double bass intonation, not how it is physically achieved. There is theory available to explain an approach to intonation, but most of it is not written to explicitly explain intonation in the contra register. However, examining the theoretical literature and pedagogical material should provide enough information for extrapolation of a working system. Putting this system into musical context will not only show its perceptual functionality, but also how the approach might better educate a student on the reasons behind what is or is not acceptably in tune.

Specifically, I have used a cross-disciplinary approach in studying this problem so as to not forsake one aspect of intonation for another. As with any academic discipline, the closer one looks at one aspect of intonation, the less one is able to say about other aspects. For example, if one were to run a controlled experiment on the preference of perfect fifths, the controls on the experiment would often be such that the fifth would be abstracted from a musical context and would lead to one preference. However, in a musical context a range of intervals loosely called a perfect fifth might be acceptable to an educated listener. Conversely, a study on the performance practice of various fifths may give far too much latitude, beyond general acceptability, for perfect fifths. This is in part why I've insisted on "acceptable" intonation. The literature is littered with studies[4] trying to determine a population's "preferred" intonation or background temperament[+]. There are as many different results as there are studies and individual preferences. However, the literature supports the idea that while each individual has an ideal intonation of an interval in a given context, there is a range of acceptable intonation of an interval in a given context which does not vary significantly within a population, e.g. musically educated persons as one population or expert musicians as another.[5] To avoid these pitfalls, I have chosen to examine intonation from the standpoints of acoustics and psychoacoustics[+], musical cognition[+], temperament and historical performance practice pedagogy, and current performance practice pedagogy in order to demonstrate the bounds of what "acceptable intonation" might be.

Of course, any good study needs a list of assumptions and exclusions. A central posit of this study is that we are using the Western Tonal System (WTS[+]). While there is good psychoacoustic support for the "naturalness" of some intervals, at the level of nuance of intonation,[6] we have moved beyond simple, harmonically formed intervals and into their assemblage into a categorically-learned cognitive construct.[7] Given the literature and context which I will examine, the WTS seems to be the most logical choice. The other central posit is that we're dealing with a double bass played in isolation. This is firstly because this study is intended as a way for individuals to learn to play acceptably in tune more expeditiously and solve the intonation difficulties inherent to the

[4] See for example Rakowski 1990 for a summary
[5] See Chapter 3 for more detail on this point.
[6] as termed by Snyder 2000, 85
[7] c.f. Krumhansl 1990

instrument itself. The second, quickly following reason is that inclusion of other instruments would add problems in blending the intonation of separate instruments in addition to the other instruments' internal intonation hazards, though application of this system will aid in ensemble intonation. Thirdly, an acoustic environment will not be considered. I'm sure we've all heard how changes in an acoustic environment can change the perception of an instrument, and Benade has noted the similar findings,[8] showing the influence of reference cues and tone overlap generating intonation changes in an actual hall. Instead, here I'll assume that the environment does not play a significant role in perception so as to only address the inherent problems and imitations of the double bass itself. One further limit regarding the bass in particular is to exclude considerations of the C-extension. While many of the principles discussed below would apply to the resonances of a C-extension, the first step in developing this system is to apply it to the standard four-string double bass.

I'll assume that a given performer has a basically good tone, with a prominent first partial[+] and well-defined overtone series. As will be discussed at length later, tone and intonation are strongly interrelated. But, in the initial stages of the argument, this has as much to do with pitch salience[+] as with a theory of intonation. Later, it will be shown how tone and intonation can reinforce one another acoustically. An acoustic/psychoacoustic consideration to be expressly excluded will be vibrato. This largely rests on the conceit that intonation is primarily a perceptual phenomenon. While debate has raged over which portion of vibrated tone is considered the center of the pitch, both the literature and common experience agree that there is a center of pitch in a

[8] Benade 1976, 286

vibrated tone. The assumption then is that intonation is judged pitch center to pitch center, whatever may define that pitch center.[9]

[9] Brown 1996

Chapter 2: What is this "acceptable intonation" thing anyway?

I can't tell you what it is, but I know it when I hear it.

-after Justice Potter Stewart's comment in *Jacobellis v. Ohio*

One of the major difficulties encountered when writing about intonation is that no consistent definition of "good" intonation exists. What is particularly vexing is that there are such wildly divergent opinions on the subject. The divergence is rooted in the conflict of acoustic principles, psychoacoustic realities, cognitive interpretation, and notational eccentricities of the Western tonal system.

The physical canvas

The acoustic difficulties of assessing intonation stem from the particular psychoacoustic and tonally structural emphasis placed upon the octave and the (so-called) perfect fifth, which have frequency ratios of 2:1 and 3:2 respectively. If one were to stack 7 octaves and twelve perfect fifths from the same bass tone, and the top tone came out to be the same frequency on both stacks as conventional modern enharmonic equivalency and notation hold, we would have many fewer conversations about the nature of the musical universe. Seeing as one can never multiply 3 by itself enough times to ever have it come out to an even number, we find an odd ratio of 129.746:128 and are left to ponder how to deal with what has become known as the ditonic (Pythagorean) comma, which is roughly 23.5 cents large. If one were to count using half steps, 7 octaves and 12 perfect fifths have the same number of half steps; however in mixing acoustically perfect and imperfect measures, one may find many discrepancies.[10]

Many, especially those in the non-fixed pitch realm, regard these discussions as purely theoretical or merely in the purview of keyboard instruments that have no control over their pitch and must develop a way to have a closed twelve-tone system. Hasn't equal temperament solved all of these issues? In some ways, yes, but in many, no. I would offer two examples to illustrate. Firstly is that with equal temperament, the ditonic comma is divided evenly between the twelve fifths comprising the circle of fifths, yielding a 1/12 comma meantone temperament,[11] making each fifth slightly flat by roughly two cents and out of tune as compared to a pure fifth. This by itself is not particularly bothersome to most listeners, expert or otherwise,[12] but the tempering of the fifth has repercussions throughout the rest of the tonal system. Most notable is the gross enlargement of the major third, which is roughly 14 cents wide of a purely tuned major third. Because of the string inharmonicity[+] of the piano in particular, this is accepted harmonically on the piano; the overtones beat[+] much more slowly and acceptably than theory would predict.[13] But, in an instrument with a harmonic overtone series, this is generally considered far too wide to be considered in tune.[14]

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[10] See Hall 2002 and Duffin 2007 for excellent comparisons of the various theoretical and acoustic discrepancies in addition to the ditonic comma.
[11] Duffin 2007
[12] See Vos 1984.
[13] Benade 1976, 322
[14] c.f. Vos 1984 and Hall 2002

The second illustration of why the commas in their various shapes and sizes matter in particular to string players is Casals' comment about always starting the day by finding where to put an E.[15] If a cellist (or a double bassist tuning in fifths) tunes his/her strings anywhere from pure fifths to slightly tight fifths, the E played on the D-string can be tuned to be beatless and resonate beautifully with the E overtones on the A-string or it can be tuned to be beatless and resonate beautifully with the E overtones on the C-string. The catch is that they will not be the same E, but will be about a ditonic comma to four cents apart! This is a direct result of the math of acoustics, and much of the literature concerning intonation is ultimately directed at solving this inequity. From the acoustics standpoint, the problem of the E can start to be solved by tempering the open strings, which brings the E overtones of the open strings closer to alignment and narrows the range of placement of the E on the D-string, and brings it closer to the rest of the quartet.[16]

Much of the preceding would have very little value were it not for a biomechanical receiver collecting these sound waves and beginning to give some meaning to them. One of the most significant bridges between what physically happens and what we perceptually hear is the relationship between the frequency of a sound wave and the pitch at which we perceive it. As Benade observed through binaural trials, subjects could tune two tones to be beatless in one ear, and tune two tones presented one

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[15] Blum 1977

[16] See Duffin 2007 for additional illustrations of this point.

to each ear but the two tones played in one ear would have horrendous beating.[17] This illustrates the perceptual difference between tuning[+] and pitch matching.

Notions of pitch

As Snyder notes, the perception of a pitch is actually a process of a fairly high order, transferring the frequency as encoded by the cochlea into a pitch code usable by the auditory cortex and higher brain areas.[18] The cochlea responds to frequencies from about 20 Hz[+] to about 20 kHz, though the region of best pitch identification is generally held to be from 200 Hz to 5000 Hz[19], which is due to the internal structure of the cochlea. If the length of the cochlea were envisioned as a rolled, conical tube, it would be divided roughly in half by the basilar membrane, which holds a single row of hair cells along its entire length, and each hair cell is connected to about 30,000 nerve cells.[20] When the fluid of the cochlea is set in motion by a sound wave, a corresponding standing wave is encouraged at the specific frequency location in the cochlea, and that standing wave sets in motion the hair cells in the surrounding area. More formally, this is basilar membrane activation. This in turn starts a train of nerve impulses along the auditory (VIII) nerve, which we will continue to follow shortly.

As we move from the opening of the cochlea, which responds to the highest frequencies, to the coiled center, which responds to the lower frequencies, we find that the basilar membrane at around the 5000 Hz response region starts growing three

[17] Benade 1976, 268
[18] Snyder 2000
[19] Incidentally, 5000 Hz is around the equal temperament frequency of C_8.
[20] Stainley & Cross 2008, 48

additional rows of hair cells along the one which runs the entire length of the cochlea. While the single row of hair cells is particularly sensitive to the place of the standing wave, the secondary rows of hair cells are particularly tuned to matching the periodicity of the standing wave, meaning the nerve impulses imparted by the secondary hair cells is tuned to the stimulus of the crests and troughs of the standing wave. It is interesting to note that, though there has not been much formal study on the effects, recursive looping[+] in the auditory neurology has been shown to actively move the second row of hair cells, which has the effect of shaping and sharpening the standing wave in the cochlea to focus basilar membrane activation.[21]

These two similar functionings of the hair cells of the cochlea are known as place coding and temporal (phase) locking coding and for many years, debate raged as to which predominated as the primary source coding for what would become a pitch sensation in the auditory cortex. The contemporary two-component view is that at any given time, each is of roughly equal perceptual weight. The groundwork for the modern combined theory of pitch perception was laid by Terhardt in his research developing the concepts of spectral pitch and virtual pitch.[22] Spectral pitch is basically the raw data of each mechanical sound wave being transduced into neural impulse train. It's the basic representation of each sine component of a stimulus being sent up the auditory (VIII) nerve for further processing. As the train of nerve impulses from each hair cell group travels up to the auditory cortex, processes turns the spectral pitch (raw data) into a categorized percept, the virtual pitch, which is extracted and synthesized from the raw

[21] See Hall 2002, 393; Roederer 1995, 65; and Stainley & Cross 2008, 49.
[22] Terhardt 1974

data presented to the auditory system. Listeners tend to have varying degrees of synthetic and analytic listening. Synthetic listening is when the whole stimulus is heard as a single percept, which is most common for musically untrained listeners and the general utilitarian mode of listening for trained listeners. Analytic listening is the ability to hear each individual sine component of a stimulus, and is less common.[23]

The pitch is generated from the interaction of the upper partials of a complex tone.[24] The clearest example of this is the sensation of a pitch from a tone with a missing fundamental. Earphone speakers are almost physically incapable of producing sound waves in the neighborhood of 55 Hz, for example. By spectral coding of a complex sound, we would not have any sensation of a pitch in the region around A_1,[25] but we clearly hear a double bass playing an open A-string when listening to our favorite .mp3 player. The trick used by audio compression (and organ builders for generations) is to use the third and fourth harmonics (165 and 220 Hz in this example) to cause the *sensation* of a fundamental pitch at 55 Hz, which really isn't there. In a very real sense, the brain supplies the fundamental and second harmonic as if it were present in the spectral pitch. While not specifically related, most musicians would tend to be familiar with this phenomenon being analogous to Tartini or difference tones. Modern magnetoencephalographic (MEG)[26] studies have supported Terhardt's theory by showing

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[23] See Terhardt 1974, Burns & Houtsma 1977, and Stainley & Cross 2008, 47.
[24] Stainley & Cross 2008
[25] The system used to reference a specific note/tone conforms to the Acoustical Society of America notation system, where the letter references the standard note name and the numeral specifies the octave. The referential note middle C in ASA notation is C_4.
[26] An MEG study is similar to electroencephalographic (EEG) studies, but using changes in magnetic fields to measure brain activity rather than changes in electric fields. This form of study offers better spatial resolution in near real-time than an EEG study, but is not of as high of resolution or as slow as a functional MRI scan.

that the tonotopic arrangement by frequency of the cochlea does not always match the tonotopic arrangement by pitch in the auditory cortex.[27] To put this in a more neurologic context, this system is most similar to a vector processing and pattern-matching model analogous to the other senses.[28] The two lower level inputs activate a categorical representation, perceptually changing the raw data into a higher-order percept of a pitch.[29]

The degree to which a stimulus activates the pitch category is called pitch salience, more loosely how much a stimulus gives the sense of a discreet pitch, and is influenced by several factors, mainly by the spacing of the stimulus's overtone series. For example, a string instrument has regularly spaced overtones, yielding a relatively clear sensation of pitch. Conversely, a tam-tam has overtones which are irregularly spaced, yielding a poor or no sensation of a central pitch. Overly abstract thinking might lead one into thinking that the strongest sensation of pitch would be yielded by a sine tone, as it would activate one section of the basilar membrane strongly and easily phase lock with the parallel hairs. However, the most salient pitch is one which has the greatest number of clear partials on a focused area of the basilar membrane firing a close group of place-coding neurons and synchronously-firing temporally-locking neurons of the outer hair cells.[30] To draw an analogy with string playing, most players have the most secure sense of left hand position when there are multiple points of contact between the player's body and the instrument, and the more points of contact, the more salient the sense of

[27] Weinberger 1999
[28] Churchland 1996 and Hall 2002, 389-91
[29] See also Roederer 1995, 160; Burns 1999, 241; and Howard & Angus 2001, 133.
[30] c.f. Houtsma & Smurzynski 1990; Shepard 1999b, 189-191; and Stainley & Cross 2008, 49-50

place. Pitch works in a similar manner: the more strictly harmonic overtones one has, the greater the sense of a specific pitch will be. Thus, a stimulus with a frequency of 400 Hz and strong harmonic overtones (i.e., integer multiples of 400) will generally have among the strongest sensations of pitch. In this case, the patterns of basilar membrane activation and the phase locking of the inner hair cells would have a very high correlation leading to a very clear signal[+] which is easy for the higher processing centers to interpret a pitch. There are multiple points which the brain can use to interpolate a pitch position.

This is born out in the just-noticeable-difference (JND[+]) statistics[31] of frequency perception. Hall lists the JND for sine tones below 1000 Hz to be a 1 Hz,[32] and Butler lists a similar figure at 3 Hz.[33] However, for complex tones, Houtsma & Smurzynski showed that JND decreases as the tone complexity increases, that is as the tone increases in the number of clear partials.[34] Hall lists the JND as low as 0.1 Hz, with 5 cents being easily discriminated.[35] Burns cites a study by Lynch, et al. which reported a JND of 10 cents in a tonal context for experienced musicians.[36]

Most real sounds do not enjoy the benefits of perfectly harmonic overtone series. However, most musical instruments do have overtone series which are approximately harmonic, and the brain tends to treat nearly harmonic overtone series as harmonic.[37]

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31 JND statistics, as its name implies, are experimentally derived figures for how far apart two stimuli must be to sound as two different entities. For example, two tones very close in frequency will sound the same until a certain point when the brain can distinguish each as separate tones.
32 Hall 2002, 96-97
33 Butler 1992, 40
34 Houtsma & Smurzynski 1990
35 Hall 2002, 411
36 Burns 1999, 233 citing Lynch, et al. 1990
37 Benade 1976, 265-266

Just as an optometrist might ask if lens one or two were better in reading the large letter E on the chart, a pitch with harmonic overtones might be clearer, more salient, than one with slightly inharmonic overtones, but the distinction might be slight. As the overtones get progressively farther from harmonic, the pitch salience declines. The border between a stimulus having a pitch and not having a pitch is by no means a clear one, and ultimately relies in the stimulus having the sufficient center of gravity to be assigned a pitch. This is how many pitched percussion instruments, which often do not have a clear harmonic series[+], are able to be understood as having a pitch, that in their sound spectra there is a strong enough pattern conveyed to activate the pitch category. Stringed instruments generally enjoy overtone series which are nearly perfectly harmonic, especially from a perceptual standpoint.[38] There is some deviation because they have real stretched strings, but the effects are minor, especially as compared to piano strings which have very stretched strings and very stretched overtone series, where the overtone series occurs in multiples greater than the harmonic integer ratios.[39]

Given that the stimuli with the strongest sensations of pitch associated with them are strongly harmonic, it is very tempting to assume that the fundamental can represent the pitch of a stimulus, and this is not an unreasonable supposition when one is working with broad divisions of the octave, such as half-steps. However, when one is doing research on intonation, which involve divisions of the octave on the order of hundredths or thousandths,[40] one must consider that the perception of pitch is not generated by the fundamental pitch, which is nearly completely ignored in fact. The perception of pitch is

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[38] Benade 1976, 57
[39] See Benade 1976, 313-315 for the mathematics of resonance for real strings under tension.
[40] See Blackwood 1985.

more governed by the overtone series above it and which overtones are most in use is highly register-dependent.

The band of frequencies which are mainly responsible for generating a sense of pitch actually remains fairly constant and independent of the register of the fundamental,[41] and especially for the lower-registered tones, the upper partials bear the weight of the pitch perception.[42] Butler cites Plomp's table of which partials are responsible for pitch above their associated fundamentals (f_1):[43]

above 1400 Hz:	all partials f_1 and above
700-1400 Hz:	all partials f_2 and above
350-700 Hz:	all partials f_3 and above
below 350 Hz:	all partials f_4 and above.

The example from Rasch & Plomp is that a tone with a harmonic series of 204, 408, 612, 800, 1000, and 1200 Hz would have the pitch sensation of a fundamental at 200 Hz, even though the lower partials would suggest 204 Hz.[44] A complication with this is that a low-frequency signal in the range of 100-400 Hz and at a loudness of 50 dB$^+$ would have perceived harmonic strength of only a little above 10 dB at partials 4, 5, and 6,[45] so there is less signal at these bands to transmit to the auditory cortex.

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[41] Plomp 1967b
[42] Rasch & Plomp 1999
[43] Butler 1992, 42
The notation of f_n in this case does indicate a particular pitch in a particular octave, but the order number of a partial of a tone. So, f_1 is the fundamental frequency of the tone, f_2 the second partial, f_3 the third, etc.
[44] Rasch & Plomp 1999, 97
[45] Ritsma 1967

This creates particular problems for contra register tones, as there tends not to be enough information physically present for accurate pitch assessments. Pierce in slight contrast to Plomp, emphasizes that at low frequencies, the gestalt waveform, partials 1-6, or partials 7-12 can be analyzed by the ear to generate a pitch sensation, but not in great detail.[46] Both Pierce and Stainley & Cross emphasize that the best pitch resolution for low tones is temporal locking of the upper partials leading to synchronous neural firings.[47] As the example from Rasch & Plomp cited above suggests, this can cause situations in which the fundamental can drift far from theoretically correct values, but the pitch would remain constant and acceptable.[48] Butler shows the opposite example, where the tones in a scale had fundamentals consistent with equal temperament values, but the upper partials were modified to cause pitch drift in excess of acceptable limits.[49] It has been shown experimentally that a fundamental can be up to a half-step from the expected frequency without disturbing the perceived pitch.[50]

The interaction of pitch and timbre is an area which is only relatively recently receiving empirical consideration. At its most basic level, timbre is the quality of a sound as experienced based on the specific placement and strength of components of a sound's overtone series[+], and it is very onset time-dependent. The main confound in attempting to study these two processes is that both use the same raw data, place coding of basilar membrane activation, to derive their perceptions. A study by Robinson & Patterson has shown that pitch and timbre are in fact two separate processes by comparing the number

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[46] Pierce 1999a, 62
[47] Pierce 1999a and Stainley & Cross 2008
[48] c.f. Chambers & Feth 1984
[49] Butler 1992, 44-45
[50] Burns 1999, 250

of cycles of a sound needed to extract each feature.[51] The musically trained subjects were able to detect above the level of chance the timbre of a tone in two cycles, whereas it took between 8 and 16 cycles, depending on register, to extract the pitch of the tone, the lower registers taking longer.

The initial transient also has a large effect on the perception of an instrument's pitch. Benade noted that the imposed frequency of the transient can be much different from the frequency of the steady state tone.[52] Balzano was able to show that each third of an oboe tone had a distinct pitch and each contributed independently to the total percept of the overall pitch.[53] Fyk reached similar conclusions in her study of violin tones, though the deviation was much less pronounced.[54]

Most practical experience would tell us that pitch and timbre are strongly interrelated, though two separate phenomenon. Research has tended to support the notion that more brilliant tones are considered sharper than dark tones with the same computed fundamental frequency.[55] In terms of spectral content, a bright tone has a broad spectrum of overtones characteristic of the instrument, with decibel power shifted towards the mid- to high range of the overtone series.[56] A dark tone will have its power centered in the lower harmonics and have few, if any higher harmonics.

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[51] Robinson & Patterson 1995
[52] Benade 1976, 160
[53] Balzano 1986, 308
[54] Fyk 1995
[55] c.f. Adey 1998 and Brooks 2007
[56] Rasch & Plomp 1999

Comparison of pitches: where intonation begins

When two tones containing pitch information are brought into close proximity, it is very difficult for a trained musician to not have a clear sense, either implicitly or explicitly, of their relative distance and the musical meaning of that distance. The process of the categorization of an interval is largely a bottom-up cognitive one which happens nearly automatically and is a step in the abstraction of pitch information for ease of use by higher cognitive functions. The direct comparison of high-level, high-resolution raw data, such as is sent from the cochlea, would demand a great deal of processing power; thus its abstraction brings it to a more manageable chunk.+[57] In a visual comparison, imagine how manic one would feel having to identify the specific color of each object one saw. Isn't it enough to know that the sky is blue or gray and that the grass is green and not red?

In melodic or harmonic coding, a fully nuanced understanding of the exact distance of each interval is not only unnecessary from a functional standpoint, but would be detrimental given the limits of primarily the Short-Term Memory (STM+) systems. As I mentioned previously, in order to quickly process the vast amount of raw data provided by the sensory systems, the higher order sensory centers code the raw data into categorized abstractions, which are generalized forms of the raw input.[58] These can either be perceptual categories, such as colors which are more or less "hard-wired" into our nervous system, or conceptual categories, which are the result of exposure to certain patterns of stimuli over a prolonged period. After much debate, intervals have been

[57] For chunk as a technical term, see Snyder 2000, 53.
[58] Snyder 2000, 84

classified as an example of a conceptual category.[59] While the twelve interval classes of the WTS approximate many of the so-called natural intervals (e.g. perfect fourths and fifths), which would indicate a perceptual category, two pieces of evidence indicate otherwise. Other cultures have developed consistent intervallic categories which do not conform to natural intervals, such as the intervals of the Javanese gamelan, which do have wide pitch categories not corresponding to either WTS or "natural" divisions of the octave.[60] One's perceptual categories are also fairly fixed in their acuity, and any perceived increase in acuity is usually the result of improvements in the language assignment (labeling), which is an even higher level task than discrimination.[61] Conversely, learned categories are highly refinable, else it would make no sense to have aural skills classes.[62]

That intervals are part of a categorical encoding process brings pitch information into a usable format for the STM system and subject to STM limitations, primarily the 7±2 rule of units of information held by STM, which is consistent across sensory systems,[63] and the time limit of nominally 5 seconds to a maximum of 12 seconds non-rehearsed retention for each element for hearing.[64] STM is able to hold larger amounts of data in memory if individual units are able to be associated into chunks; for example 1776149220011984 is very easily remembered as 1776 1492 2001 1984.[65] Music is able to be chunked into groups of notes and into phrases in a similar manner, and is an

[59] c.f. Burns & Ward 1974; Burns & Ward 1978; Perlman & Krumhansl 1996; and Burns 1999
[60] Perlman & Krumhansl 1996
[61] See Wapnick, Bourassa, & Sampson 1982 and Morrison & Fyk 2002.
[62] See Fyk 1995, 115 citing Houtsma; Burns 1999, 230; Burns & Houtsma 1999; Weinberger 1999, 55; and Snyder 2000, 81.
[63] Burns 1999, 218-219
[64] Stevens & Byron 2009 and Snyder 2000, 12
[65] Snyder 2000, 54

example of hierarchical grouping to reduce memory loading. Notes can be grouped into intervals, intervals into melodic patterns, and melodic patterns into phrases. Also, the more prototypical the patterns, the less memory loading is required to retain and process the elements.[66] Repetition of a note or pattern serves as a rehearsal, encouraging the prolongation of those elements in STM and interaction with Long-Term Memory (LTM+) and activating a tonal schema+, for example.[67] An example of all of the above would be the common major scale, in any given key. It is a highly rehearsed collection of notes, which easily fits the 7±2 rule, is easily groupable, is highly predictable, and imbues a strong tonal implication with very low memory loading.

The comparison of two or more pitches is highly dependent on memory loading and the interaction of echoic memory+, which is a low-level memory system retaining a high-resolution image of the raw perceptual data, or trace+, for a very brief time on the order of one to two seconds,[68] STM, and LTM, which provides tonal and other contexts for the percepts involved, and will be discussed at length later. Larson's trace model[69] and Zartorre's experiments with speed-sorting of intervals[70] strongly support a two-component theory of pitch comparison for consecutively presented tones.[71] When a tone is presented, it leaves a trace in echoic memory while being categorized for comparison in STM. Concurrent with the categorization process, a copy of the trace of the tone bypasses the categorization process and is brought directly into focused awareness, more

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[66] Snyder 2000, 81
[67] See Chapter 2: Categories in Context for a more full discussion of schemas.
[68] Snyder 2000, 16 and 127
[69] Larson 1997
[70] Zatorre 1983
[71] See also Cohen 1984; Burns 1999; and Burns & Houtsma 1999.

colloquially "conscious mind," which provides a precise sense of where that pitch is in pitch-space. With the second incoming tone, the two categorized pitches are compared for their absolute size difference in the form of an interval category, and precise heights of each trace copy are immediately compared against the idealized interval categories, but not against each other directly in focused awareness.

The second trace can either overlap with the first tone's trace or displace it in echoic memory, which is below the level of focused awareness. If two tones are presented simultaneously, both leave overlapping traces in echoic memory, which allow direct comparison of the raw data, while the two tones are simultaneously categorized and compared categorically. This offers the highest resolution for tone comparison. If the two tones are presented sequentially and the second tone is a minor third or more from the first tone, the trace of the first tone will persist in echoic memory, allowing for direct, though deteriorating, comparison of the raw data, while the second tone is categorized and compared to the categorical representation of the second tone, the categorical representation giving the sense of distance.[72] If the second tone is a step or half-step above the original tone, the trace of the original tone is displaced by the trace of the second with almost no overlap, but the information is still available for a short time of direct comparison, though to much less of an extent than if the trace had not been displaced. If melodically or harmonically unrelated tones are inserted between the two test tones, comparison performance further degrades, as one is almost totally reliant on

[72] Burns & Campbell 1994

abstracted, categorized data which will be losing salience in STM without rehearsal or reinforcement.[73]

A fact which seems quite odd, given how sensitive musicians seem to be about intonation, is that humans are generally described as having very good *inter*category discrimination but very poor *intra*category discrimination. In other words, once we have decided an interval is a perfect fifth, we either don't notice or it doesn't matter if the two tones are slightly narrow or wide of our ideal perfect fifth. Before we scrap this notion of intonation sensitivity, let us consider two parallel ideas. The first is that the processing we've been discussing is to a very large extent at the subconscious level in STM and is highly contextualized, aside from the trace copies sent directly into focused awareness. When that perfect fifth is played, one might not identify it as a perfect fifth by name and maybe not as any interval in particular at all. But, the subconscious processing of its size and meaning is occurring none the less. The labeling of the interval is a different process than for observing the procession of pitches occurring in focused awareness. The second idea is that intonation is a nuance[74] particularly of high-resolution pitch information in focused awareness interacting with the categorized interval. Especially with trained musicians, intonation is the sense of precisely where within an interval category two pitches fall, but because a nuance is not in itself categorized so as to protect the necessary data resolution, it is not processed directly by the memory systems and is not retained.[75] Thus, it is perfectly reasonable that musicians in particular are able to discriminate what an interval "is," i.e., assign it a categorical interval size, to a broad extent, yet be able to

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[73] See Larson 1994 and Larson 1997.
[74] For nuance as a technical term; see Snyder 2000, 86.
[75] Burns & Houtsma 1999

judge how closely a particular example fits the prototype interval size of the category with a fair degree of precision.[76]

That said, I don't want to totally divorce absolute pitch distance, categorical assignment, and intonation, for it is only against the categorical assignment of an interval that intonation has meaning. If two pitches fall on a listener's categorical distance for a given interval, then the memory load incurred attempting to assign category is very small, as it meets all expectations for that interval category. The intonation of those two particular tones would go unnoticed as the categorical distance and the nuance distance match without any higher-order mediation. If two pitches begin to vary from the prototypical interval size, more memory resources are required to assess the interval size and assign it a category, incurring a higher memory load. Depending on the size of the variation, this may be treated as normal and ignored, and the intonation in focused awareness also ignored or perhaps noted as atypical if mediation between the perceived intonation and categorical backdrop were needed. However, as the difference in the two pitches reaches and crosses the categorical bounds of the interval, the memory load required to assign the interval to a category distracts from other analytic tasks and forces the incongruence of the intonation and interval categorical assignment into focused awareness, leading to a violation of expectation and an out-of-tune judgment.[77]

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[76] c.f. Burns & Campbell, 1994; Acker, Pastore, & Hall, 1995; and Snyder 2000, 84
[77] c.f. Snyder 2000, 137 and 141; and Burns 1999, 231

Categories in context: getting to the key

Categories assemble in larger scale orders around a mental framework called a schema, which form the basic structure of LTM storage and the basis for expectation. The most familiar type of schema is of the temporal-order type, such as the procedure for rising in the morning and preparing for a weekday, though any learned behavior is processed through a schema. Abstract concepts, such as tonality, are processed by schemas, and even emotional states can be accessed through schematic processes. Each unit of a schema is arranged hierarchically, and schemas can be grouped into larger schemas, which are also hierarchically arranged, such as modulatory key centers around a home key. The individual units of a schema have learned default values but are subject to modification if incoming information strongly warrants such a change. Schemas also have the potential to occasionally over-ride valid incoming data and replacing it with the default value should the incoming information be too contrary to the established schema![78]

LTM and STM have a significant interaction in that elements in STM activate the schema found in LTM, and the schema in turn provides context to the elements in STM. The elements in STM suggest a set of expectations based on a schema, and LTM replies with what the specific schematic expectations should be. The key difference between the two systems is the STM actively processes the incoming elements, while LTM provides the backdrop against which those elements are processed. That tonality was a schematic process was subjected to intense debate for a number of years, but finally came to rest on the intervallic rivalry theory[+] of Butler, Brown, & Jones, where the sensation of a tonal

[78] See Snyder 2000, 95-96 and Huron 2006, 216.

center is formed based on the succession of key-defining intervals such as the major second and tritone,[79] and the key profile theory[+] of Krumhansl and Shepherd, where the sensation of a tonal center is formed by the number and distribution of the tones presented.[80] After much wrangling between the two camps, both were found to empirically support tonality as a schematic process. Intervallic rivalry accesses the expectations established by exposure to tonality as shown by the key profile.[81] Huron does make a valid point that the key profiles are models of completion expectation and that his model of statistical distribution of tones in a key more accurately models the moment-to-moment expectations of a given piece.[82] However, the key profiles are extremely useful in the broader evaluation of expectation, especially in the context of intonation discrimination. The reason key activation is so important is that it changes the context and meaning of intervals, which influences the acceptability of intonation. The work of Lerdahl & Jackendoff and Schenker are not often brought into the same context,[83] but if a combined view of Butler, Brown, & Jones and Krumhansl is appropriate, as I think the evidence suggests,[84] then for the purposes of intonation, the combination of event and tonal hierarchies would lead to the stability context, at least on the local level (i.e., within 3-4 chord or pitch events), by which intonation is judged.[85]

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[79] Butler, Brown, & Jones 1994
[80] Krumhansl 1990
[81] Also observed by Huron & Parncutt 1993 and Schmuckler 2008.
[82] Huron 2006, 150
[83] cf. Larson 1997 and Lerdahl 1997
[84] c.f. Huron & Parncutt 1993 and Snyder 2008
[85] See Cuddy 1997, Larson 1997 and Bigand & Poulin-Charronnat 2008.

The intervallic rivalry model argues that a sense of key center is generated from the surface level[86] of music, that each note is taken as a tonic until it can be supplanted by a better candidate. Thus, a lone G_1 will be perceived as tonic until it is followed by C_2. Because the rising perfect fourth in a tonal context is a highly specific motion, an event hierarchy will be established that poses C in a hierarchically superior position to G. Because C has been established in a superior position to G, the tonal schema is activated, centered around the perceived pitch of C, establishing a tonal hierarchy. Granted, this would be a weaker assertion of the key of C major: based on only two events, there are still many directions in which we might be led, but given that there are no alternates presented it serves for the time being.

If we were to insert a B into the sequence, yielding G_1, B_1, and C_2, a tonal hierarchy centered around C major would be even more highly suggested. Not only do we have the motion of the G to the C, and the traces of each pitch, in STM[87] to suggest a tonal schema, but also the ascending half-step motion from B to C, which is arguably more specialized in the tonal context and more suggestive of a tonal schema centered around C. Looked at from an event hierarchy perspective, the G presents the first and most stable context for a tonal center, and it is then supported by the inclusion of the B. The rising M3 is a non-specialized, and thus very stable, tonal motion, which offers no challenge to the previous event for key centrality and is therefore of a lower hierarchical position. From a memory processing point of view, these two events are so closely related as to chunk, which in combination with their traces, implying a chord, which also

[86] The surface level represents the basic "notes on the page" reading of a passage and is opposed to deeper structural levels, such as the middle- and background levels.
[87] See Larson 1997, Snyder 2000, and Schmuckler 2008 for melodic encoding and processing.

encourages the activation of a tonal schema.[88] With the arrival of the C, we have both the ascending P4 and m2, which offers a strong challenge to the established hierarchy of G major, and C takes over as the center of the tonal schema, even though a G major chord is more firmly established, because the intervallic pattern more highly suggests that C is in a higher hierarchical position than either the G, the B, or a combination of both.

That said, this may still not be the strongest activation of the tonal schema. While there is support for literal activation of a C-centered chord from hearing a lone C, for example,[89] there is still much G-centered chord activation. Adding E_2 would further strengthen the activation of the tonal schema in C. Just as the inclusion of B with the G implies and literally traces a G major chord, so too would the inclusion of E with the C. The rising M3 would closely associate the E with the C, putting it in a lower hierarchical position than the C but higher than the G, as the E is elaborating or prolonging the implied C major chord. This would be a strong activation of a tonal hierarchy, as there are two chunks equally activated, but in a clear hierarchical relationship.

To take this to its logical conclusion, perhaps the strongest activation of a tonal hierarchy would be G major-minor-seventh chord leading to a C major chord. All elements of each chord would be strongly active, as each would be physically present. Because each element is physically present and encoded in STM, seven elements would be active which is near the limit of STM capacity, even if chunked into two elements. In isolation, these would consume the entire "focus" of STM. In addition to the hierarchal

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[88] See also Parncutt 2011.
[89] c.f. Bharucha 1994 and Parncutt 2011

elements already established, the added elements only serve to provide more points of reference. Primary among these is the contraction of the tritone between B and F to C and E. The tritone is an extremely rare interval in a tonal context and its resolution is highly suggestive of a tonal center. In this case, the tritone has a very low hierarchical position and is moving to tones which have a very high hierarchical position. Given the context of an ascending P4, the descending M2 between D and C highly encourages the hierarchical superiority of C major. While the melodic motion of a descending M2 is not particularly rare, that particular motion in that particular context of surrounding motions is. Thus, between the high memory loading and combination and succession of rare intervals, a tonal schema centered on C would tend to be suggested to the exclusion of other possibilities.[90]

The activation of a stable tonal center in turn activates a stable tonal schema. When a schema is activated, the categories or prototypes within the schema are also activated.[91] The strength of the prototype activation relates directly to the strength of the total schematic activation,[92] often times with a default value if a category is not directly activated.[93] As I've noted, Huron has established a tonal schema quite different from Krumhansl, saying that his better conforms to actual pitch-to-pitch expectation while Krumhansl's better predicts closure or stability of a particular pitch in a sequence. While I don't dispute Huron's line of thinking in a general way, I would submit a working assumption that Krumhansl's key profile best represents a pitch's stability and that the

[90] Credit should also be given to Cook 2009 for the notion of stability arising from instability.
[91] See Acker, Pastore, & Hall 1995.
[92] Snyder 2008
[93] Snyder 2000, 98

expectation of stability of the next pitch or group of pitches is more important to intonation than which specific pitch is next. This is somewhat born out in a recent MEG studies reported by Tan, Pfordresher, & Harre,[94] though this is inferential as the study was not on this direct question. In the studies, an out-of-tune pitch was detected more quickly than an out-of-context pitch, based on speed of changes in the event-related potential/field (ERP/F) readings generated from the MEG study. This was also observed by Bharucha using a slightly different experimental technique.[95] When in a tonal context with highly activated prototypes, the expectation is that each tone will be within the range of the prototype and that this is processed at a fairly low level.[96] Another way of phrasing this is that intonation is best discriminated when the tonal plan is clear and the tonal schema is fully activated.[97] If one is required to shift schemas to account for a pitch more appropriate to a different context, this both requires more time and processing of intonation only in retrospect. In a highly stable context, intonation judgments can be rendered very quickly and tones of a higher hierarchical position have a tighter range of acceptability.[98]

The bearing that this has on intonation is that the more stable a pitch is in the tonal schema, the closer it has to be to the prototype, especially if that prototype is particularly activated by the event hierarchy context. It may be no surprise that the key profile, tonal pitch space model,[99] and observed cognitive interval strength are all very

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[94] Tan, Pfordresher, & Harre 2010, 91
[95] Bharucha 1994
[96] See Burns 1999, 233 and citing Umemoto 1990.
[97] c.f. Acker, Pastore, & Hall 1995 and Fyk 1995, 175
[98] Burns 1999
[99] Lerdahl 1988

closely correlated.[100] For instance, $\hat{1}$, $\hat{4}$, and $\hat{5}$ are very closely related acoustically and functionally, and exhibit tighter tolerances for intonation variation than the relation between $\hat{6}$ and $\hat{1}$, which can have multiple meanings and functionalities. This tendency was particularly demonstrated by Shackford in a study in which he designed a set of compositions to test the robustness of various intervals in a tonal string quartet context.[101] The tonally structural intervals tended to be held to a very low dispersion, particularly the tonic, and even in excerpts designed to have an ambiguous tonic, the intended implied tonic had drift only on the order of a few cents. Sundberg with singers[102] and Sachaltueva with cellists[103] were able to show experimentally that in performance, $\hat{1}$, $\hat{4}$, and $\hat{5}$ were held to very stable standards close to their acoustically pure sizes. That tonic in particular is actively held for comparison of intonation has also been documented by Matthews & Sinus, Shepard & Jordan, Fyk, Brown, and Shepard.[104]

To say that the establishment of a tonic and a tonal schema is a strict, rules-based process based solely on the sequence of surface events would be an error. There is a long tradition on the performance side which holds that an interpreter has a great deal of control over the choices about how a piece is conveyed and heard by selection of how surface elements are grouped in a performance. There is evidence of high correlations between intended affect and perceived affect, and intended harmonic motion and perceived harmonic motion.[105] Part of the basis of this lies in the structure of the major

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[100] See Rakowski 1990.
[101] Shackford 1961 and 1962
[102] Sundberg 1999
[103] Sachaltueva 1960 cited in Fyk 1995
[104] Matthews & Sinus 1981, Shepard & Jordan 1984, Fyk 1995, Brown 1996, and Shepard 1999b
[105] c.f. Shackford 1962, Fyk 1995, Cuddy 1997, and Bigand & Poulin-Charronnat 2008

scale system, which is asymmetrical in its construction. This asymmetry is a relatively efficient way of establishing and promoting position-finding within the implied tonal hierarchy.[106] The demonstration by exclusion would be to take the examples of the Second Viennese School and the French Symbolists, who by philosophical and aesthetic percept sought to negate any sense of a tonal center by using symmetrical constructions.[107]

The rub about the asymmetrical construction of the major scale providing swift position-finding is that it is still not terribly efficient, especially as compared to chordal constructions. As with most melodic constructions, a scale contains non-structural notes which interfere with the establishment of tonic if presented in a purely sequential, non-grouped way. In moving through a collection of four tones, with intervals between each being two whole-tones followed by a semi-tone, the brain has to wait through four elements, and only three vaguely structural ones, before being fairly certain what key is really being implied and assigning the pitches scale degree values of $\hat{1}$ to $\hat{4}$. Even then, the key is implied only weakly, based on the premise of the first note holding hierarchical superiority until it can be sufficiently challenged. Performed "correctly", it is just as likely that the motion was not $\hat{1}$ to $\hat{4}$ but $\hat{5}$ to $\hat{1}$! If we were to follow the scale up to $\hat{5}$, we would have a fairly clear sense of the key, but at a high memory loading: we are forced to retain a relatively large number of elements in STM for as long as it took to progress through $\hat{1}$ to $\hat{5}$ (remembering STM time limit is about 5-8 seconds and 7±2 elements). Only after all five elements are in place can the gestalt be brought together,

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[106] c.f. Shepard 1999b and Stevens & Byron 2009
[107] Shepard 1999b

the key implied, and the melodic motion chunked for further processing.[108] This is opposed to a chordal presentation, which has all structural elements of a key presented at once at a lower memory loading. Even an arpeggiated presentation of $\hat{1}$, $\hat{3}$ and $\hat{5}$, while incurring a higher memory load, would have a quick, low-load/high-key-activation processing: $\hat{1}$ is presented, $\hat{3}$ reinforces and is grouped to $\hat{1}$, $\hat{5}$ does the same, and the chunk is able to be processed much more easily as one element.

One theory as to why we are sensitive to intonation at all is that it helps with reducing memory loading required establishing tonal groupings in melodic contexts.[109] In particular, the compression of the minor second, i.e. making it smaller than the "typical" size of 100 cents, serves as a pointer to the structurally stable elements of $\hat{1}$, $\hat{4}$, and $\hat{5}$.[110] To help put this is context, it is best to wipe the image of the scale from our minds and think of the tonal schema's structural order of $\hat{5}$, $\hat{1}$, and $\hat{4}$. If $\hat{1}$ is presented first, then $\hat{4}$ and $\hat{5}$ are going to be activated schematically in firm relative distances from $\hat{1}$, for as long as that particular tonal context lasts. Any notes in between $\hat{1}$ and $\hat{4}$ are of a lower hierarchical order, and thus have a greater dispersion of acceptability. If, when moving linearly through pitch space, the distance from $\hat{1}$ to $\hat{2}$ and $\hat{2}$ to $\hat{3}$ is enlarged just slightly, but noticeably from the expected prototype value, it will serve as a pointer and melodic-grouping shorthand to stand for the tonal melodic figure of motion from $\hat{1}$ to $\hat{4}$, provided that $\hat{4}$ remains in its expected distance from $\hat{1}$. STM is able to

[108] The STM loading limitations are also one theory of why cross-culturally the divisions of the octave generally fall within the 5-8 unit range (see Snyder 2008 and Burns 1999).
[109] Bigand & Poulin-Charronnat, 2008
[110] See also Shepard & Jordan 1984 and Burns 1999.

chunk the entire configuration and activate the appropriate tonal schema in only three elements, as opposed to at least four. This has been shown somewhat in reverse by Shepard & Jordan,[111] in which each step was modified to an extent below the practical note-to-note tolerance, but by the time tonic was reached, all subjects were aware that the "tonic" was out of tune versus the original internalized tonic tone.

The compression of small intervals may also serve as a low-level perceptual shortcut.[112] In processing fast passages, intervals of roughly the same size, such as whole- and half-steps tend to get homogenized into units of roughly equal perceptual size; there just isn't time or bandwidth to process the relatively slight differences, especially considering the rate at which traces are continuously being supplanted. By enlarging whole-steps and compressing half-steps in actual distance, the perceptual homogenization is offset to a large degree.

Much of what I have presented to this point has implicitly regarded sequentially presented tones. While most, if not all, of what I touched on still applies to simultaneously presented tones, especially complex tones, they present an additional crinkle in that the physical congruency between tones must be considered. When two slightly mistuned tones are played together, the interference patterns of any of the partials results in a measureable change in the amplitude of the resultant combined sound wave, which are known as beats. This beating, because it is a physical property of the sound wave is detectable to a greater or lesser degree by the ear, depending on the decibel level

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[111] Shepard & Jordan 1984
[112] See particularly Shepard & Jordan 1984 and Shepard 1999b.

and extent of mistuning.[113] The sensation caused by physical beating is either perceived as roughness[+114] or beating. There are two causes for this, the first of which being the basilar membrane faithfully transducing the physical rise and fall of the combined tone's amplitude.

The second is more subtle and begins with the recollection that the basilar membrane responds to an area of activation, not the specific spot of the standing wave in the cochlea. This area is known as the critical band and forms the basis of psychoacoustic consonance and dissonance, work pioneered by Plomp & Levelt.[115] If two sine tones are the same frequency, then they will activate the same portion of basilar membrane and the same critical band, causing the sensation of a clear partial at that frequency, and in fact generally reinforcing the sensation of the particular partial. However, as the frequencies of the two partials change, each will try to active its own critical band, causing a conflict if the two critical bands overlap. If the difference between the two partials is 0% to 15% of the width of a critical band, this is perceived most often as simple roughness, but one partial. Between 15% and 25% of the critical band difference, the sensation of two separate but competing partials emerges, with the psychoacoustic sensation of beats beginning at about 15% and the maximum dissonance between the two tones at 25% of the critical band. Between 25% and 100% of the critical band difference, beating is still a present percept, but the dissonance decreases logistically, and nearing 100% of the critical band the perceived dissonance drops back to

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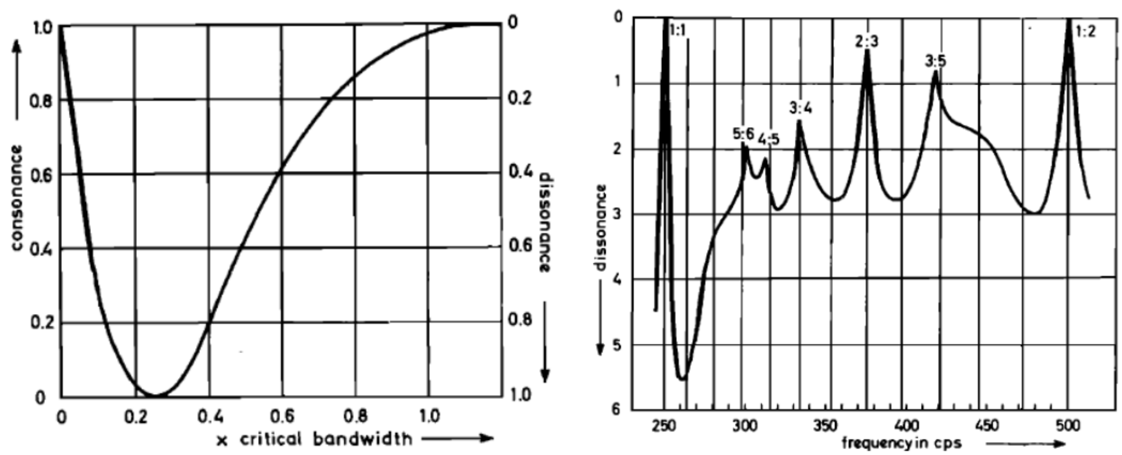
[113] See particularly Plomp 1967, Vos 1982 and 1984 and Vos & van Vianen 1985.
[114] This is a usage particular to psychoacoustics which describes the physiological auditory sensation of dissonance.
[115] Plomp & Levelt 1965

0. Though not quite as precisely described, this is the source of Helmholtz's annoyance due to dissonant complex tones.[116] When judging the psychoacoustic consonance or dissonance of two simultaneous complex tones, each partial's dissonance is summed to generate the total dissonance of the interval.[117] In looking at the consonance curves generated by Plomp & Levelt,[118] two were drawn: one representing responses from sine tone, described above and one for complex tones. What is interesting about the complex tone curve is that it has local peaks of high consonance corresponding to natural intervals tuned to the overtone series.



a) b)

Fig. 1: The consonance ratings for (a) sine tones and (b) complex tones as measured by Plomp & Levelt (1965).

The trouble with psychoacoustic dissonance is that, left unresolved through harmonic motion, it tends to be judged as much less attractive in tonal music than consonance, though each has its place, of course. If beats and roughness were as acceptable in this modern age of equal temperament as some would have us believe, then

[116] Helmholtz 1954
[117] See Rasch & Plomp 1999 and Pierce 1999b.
[118] Plomp & Levelt 1965

this intonation notion would be much less of a battle. However, Vos in particular has spent much of his career researching the acceptability of various sizes of simultaneous intervals, particularly the major third and the perfect fifth, and found that harmoniousness peaked at acoustically pure versions of each, and fell off exponentially as the intervals were tempered away from pure. Incidentally, the results did not change if "harmonious" was replaced with "euphonious," "pleasant," "stable," or "in-tune."[119] Granted for the P5, this represents a relative non-event, as in most tuning/tempering systems, the deviation from pure is below or only slightly above the JND level, and does not violate the categorical boundary of the P5. The tempered P5 perceptually resembles the pure P5 to such a degree as to almost go unnoticed, even with the roughness caused by the superposition of two slightly mistuned complex tones.

In contrast, the tempering of the major third presents a significant problem. The brain tends to respond to perceived distance between two notes in a similar fashion, despite their being presented sequentially or simultaneously, and treats them categorically relative to the center of the tonal schema. To keep things in general terms for the moment, let's assume that the real size of a purely tuned M3 is smaller than the average categorical (tempered) size of a M3, and it is so small that it is outside the normal range of variation allowed for automatic, no-load processing, but not so small as to automatically be judged out of tune. What is now set up is a conflict between the low-level psychoacoustic processing, which desires low levels of roughness and beating, and the high-level categorical processing which desires activation of the category center. The determining difference usually lies with the method of presentation. When presented

[119] Vos 1986 and Sundberg 1982

simultaneously, all elements of each tone are available to the various memory systems, but with tuned signals, the sense of congruence of the tones is particularly strong due to actual direct signal comparison. Even though the STM many return a verdict of "slightly too small," the predominating factor will be the sense of congruency of the signal and the low psychoacoustic load and represents the best compromise between the two factors.[120] With a tempered signal, the actual beating of the tones causes neural beating, which greatly increases physical neural load and echoic/STM load.[121] Mistuned partials which generate beats on the basilar membrane generate aphasic nerve firings,[122] which in turn are perceived as dissonance.[123] When neural beating is present, not only must the brain separate and measure the two tones, but must also factor in the resultants of the beating and the increased signal noise associated with incongruent signals. Because the purely tuned M3 does not explicitly violate the boundary of an acceptable interval and has a very low psychoacoustic load, it tends to be preferred in a simultaneous presentation, as the brain prefers synchronous nerve firings.[124]

If the tones are presented sequentially, the first signal exists only as an echoic trace, so the direct interaction between the two tones is not as strong, though still present, especially in arpeggiated presentations. In such cases, the categorical representation will have a greater perceptual weight, often a dominating one. However, if the trace of the first tone is strong, as it is in an arpeggiated figure, then the perceptual weight of the

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[120] Pierce 1999a and Cohen 1984
[121] See Burns 1999 and Weinberger 1999.
[122] Plomp 1967a
[123] Weinberger 1999
[124] Bharucha 1994 and Katy 1995; c.f. Burns 1999

psychoacoustic consonance or dissonance will be factored in the acceptable/unacceptable judgment of interval size.

As with many other processes we've discussed, intonation discrimination of simultaneous tones is generally much more acute than sequential tones. As with pitch, key finding, etc., simultaneous tones afford the sensory and memory systems two very strong points of reference, whereas sequential tones have one strong point of reference tied to an imprecise measuring tool (the ear) and a second which may or may not be present or relevant, no matter how precise the tool. It may not be surprising then that simultaneous and cognitively overlapping tones tend to have a preferred size trending toward ones with beatless tuning, with the psychoacoustic expectation of smoothness fulfilled. The question of how practically useful this fact is has been asked many times by such persons as Hindemith, Schoenberg, and Schenker.[125] Vos has catalogued the strength and deviation from how beatlessly-tuned an interval needed to be for beats to be detected, and both are high numbers, apparently supporting the idea that tuning by beat elimination is not practical in the performance context. However, I posit a question: if, given the amount of subconscious processing that occurs, is it not possible that even if the level of neural disturbance caused by beating is not significant enough to be brought into focused awareness, might there be enough, even in rapid passages, to cause a slight discomfort with an interval, a certain perception of roughness, and to influence an acceptable/unacceptable judgment?

[125] Hindemith 1945, Schoenberg 1978, and Schenker 1954

As Snyder points out, much of our cognition and understanding of music is highly dependent on the different time-scales of presentation: the formation of rhythm and pitch on the millisecond level in echoic memory, metric and melodic grouping and coding in STM and under 10 seconds working retention, and notions of form and expectation involving LTM for categorized data in units over 16 seconds.[126] Intonation is equally as influenced by time, if not more so, as it is a nuance and inherently not coded or well retained and influenced by the preceding pitches and expected following pitches. Because intonation is highly perishable, its strongest comparison is going to be the echoic and categorized versions of the hierarchically highest preceding pitch, followed less strongly by the categorized versions of the previous pitch events more generally. This is also subject to the limits of STM, 7±2 events or event groups and 5-8 seconds. Events outside of the limits of STM would tend to not directly influence intonation judgments, as the information with which to compare the present stimulus in focused awareness is simply not retained, is so weak as to be of little relevance, or processed through LTM as part of the tonal schematic information.

Part of the difficulty of tonal schema activation is that it is an ongoing process, not necessarily defined by a specific event. Therefore, events in STM are at the same time judged by the tonal schema and are in the process of confirming that tonal schema or activating a new one with a different center, based on the events in STM. Intonation plays a part in this process, as a performer can use interval manipulation (read: intonation), such as half-step compression, to focus the attention of the audience on a new tonal center. Conversely, unintentional interval malformation (read: poor intonation)

[126] Snyder 2000, 32

would tend to destabilize an active tonal schema and be a major thwarting of expectation! Even given a strong and stable activation of a tonal schema, the normal course of music does not typically stay in one key, depending on the particular style of the piece of course, and the sense of a "home" key is also a perishable idea, given the correct series of intervening events.[127]

Assuming a relatively stable tonal schema, intonation judgments would be bounded by the limits of STM. While theoretically this provides wide latitude of comparison pitches, in practice Fyk has found that the practical limit is two to three pitches for active comparison, and defined this as the "intonation horizon," as part of her dynamic intonation model.[128] While this seems to be a very short area of focus, it is still based on the assumption of stability of certain intervals within a tonal context. This is supported in a study by Brown, which found that while performers tend to base intonation from an average tonic (prototype activation model), the average is based on short cells involving different iterations of tonic.[129] Because the tonal schema is active, there is expectation generated by its activation, which implies a view slightly over the intonation horizon. A pedagogical technique known as audiation, in which the next expected pitch is heard internally by the performer, has been used in aiding the development of proper intonation. It might be the case that more or less strong instance of this might occur with the audience, especially with highly learned examples.

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[127] Butler, Browne, & Jones 1994
[128] Fyk 1995
[129] Brown 1996

Modeling categories: the general case

The many studies probing the categorical boundaries of interval centers and widths has all lead to pretty much the same conclusion: interval categories tend to center around equally tempered intervals and tend to be about ±30 cents for all intervals. This may strike many as very improbable, especially considering that some intervals seem to be less subject to the variability tolerated by the above statement, that intracategory discrimination is generally thought to be quite poor, that musicians tend to think they have exceptional intonation discrimination, and that the statement suggests that equal temperament indeed does rule the day.

The ±30 cents rule (which can be as low as ±25 cents depending on how the study is run) is a figure which occurs very often in the literature[130] and seems to be the categorical boundary of *identification* of, or being able to assign a label to an interval, which is a separate cognitive process from discrimination. Within ±30 cents, a musically trained person can identify the interval presented, or extrapolating for our purposes, what the interval is *supposed* to be. Between each interval category lies a region, about 40 cents wide, in which the subjects tend to get confused about what size of interval is being presented and cannot identify it. For example, if we were to assume that a subject's ideal interval size were 400 cents for a M3 and 500 cents for a P4 and we presented an interval 425 cents wide, under this model, the subject would be able to identify it as a M3 and tell a researcher that it is a M3. The previously ignored portion might have been that the subject might think that it was an absurdly large M3, but that was less the point of

[130] c.f. Siegal & Siegal 1977; Wapnick, Bourassa, & Sampson; Hall & Hess 1984; Rakowski 1990; and Fyk 1995

previous studies. A subject presented with an interval 455 cents wide would probably be confused and not able to identify the interval. Thus, we can see that, as with pitch, this is a categorical exercise of: is this a recognizable interval or not, and which interval is it supposed to be?

Rakowski spent a good deal of time refining that general model and developed the idea of interval center and strength as it deviates from equal temperament, the inference being that more highly learned categories would have a stronger center and a lower dispersion.[131] These begin to reflect the spectrum between identification and discrimination, based on the experimental design. There is more of an emphasis on how well a particular interval represents the category, and where that category is centered, than previous studies. Siegal & Siegal ran a series of similar experiments, and while the experimental design was quite poor for the conclusions they ultimately drew, the raw data reported is quite telling as well.[132] Their subjects had a categorical width of about ±30 to ±20 cents, depending on the interval, but inherent in the experimental design was gathering data on sharp v. flat v. in-tune judgments of the subjects. The detail misinterpreted by Siegal and Siegal was that within roughly ±10-15 cents of the equally tempered center of a given interval, the subjects almost totally lost the ability to make sharp/flat discrimination judgments. Another observation was that subjects exhibited a region of acceptability of an interval's size roughly 80% the total width of the identification category. From the standpoint of studying intonation, the main difficulty with both studies is that they conflate the ideas of the width of an intervallic identification

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[131] Rakowski 1990
[132] Siegal & Siegal 1977

category (the range of sizes which can be identified as a given interval) with the width of an intervallic preference category (the range of sizes preferred for a given interval). Thus, the two need to be differentiated for further evaluate intonation acceptability.

Hall & Hess, in a series of experiments which asked more appropriate questions of interval preference size within the identification category, demonstrated that each interval has its own regions of identifiability with poor preference, acceptability with fair or conditional preference, and a central region of about ±10 cents around the idealized center in which the ability to judge the interval size falls off remarkably.[133] The center of each zone versus equal temperament and its width varied with each specific interval, and a key point is that there is a zone of very poor discrimination around the idealized center of the interval size category. Shackford took this as an assumption in his performance analysis studies, and set this region as ±10 cents, and his results indeed tended to fall well-within or well outside of this range.[134] The data of Brown also confirmed this number as a practical average of the range of indistinct size discrimination.[135] Fyk combined her data with that of other Eastern European researchers, coming to very similar conclusions.[136] While citing Garbuzov's10 intonation zones as he coined them, she reduced the number to 3 acceptability ranges, depending on the context: small, centered, and large.[137] However, Fyk continually ran up against the "operational limit" of contextual intonation discrimination of about ±10 cents, which did differ from the more

[133] Hall & Hess 1984
[134] Shackford 1961 and 1962
[135] Brown 1996
[136] Fyk 1982 and 1995
[137] Fyk, 1995, 31

clinical JND of intonation discrimination of ±3 cents.[138] A large factor for this is informal masking, where the performance of the auditory system degrades as the stimulus "varies across multiple dimensions" and the auditory system doesn't have enough channel capacity to track pitch changes while it is tracking all other qualities of a sound simultaneously.[139] Finally, van Bestow, Brereton, & Howard found a similar ±10-15 cent region of subjects not being able to discriminate between changes of pitch.[140]

Part of what I am illustrating here is that intonation judgments are primarily built around the perceptual anchors described above. A perceptual magnet helps create very clear distinctions between categories in a schema. One visual example is the perceptual puzzle of the old/young woman (see Fig. 2).

Fig. 2: The old/young woman illusion

[138] Fyk 1995, 30
[139] Brown 1996, 86 citing Watson 1987
[140] van Bestow, Brereton, & Howard 2008

One tends to see one or the other, and almost certainly not at the same time. The way the visual cortex and associated identification systems assimilate the cues within the picture assign it to the “old woman” category or “young woman” category, without much, if any, sensation of being “in between” the two images. Each category prototype acts as a magnet, pulling and retaining the highest cued perception. An example with a little more gray area would be the two faces/cup puzzle (see Fig. 3).

Fig. 3: The two faces/goblet illusion.

Most people tend to see two faces or the cup, but it is possible to hold both constructs in mind at the same time with some effort. Typically, the easiest perception (read: least memory loading) is to let the perceptual magnets identify the image as either two faces or a cup. An auditory example would be pitch. While there are many natural sounds which don’t have anything close to an identifiable pitch, there are some which generate some sense of pitch centrality, activating the pitch prototype and categorizing the sound as pitched. Once one begins to hear a sound as having a pitch, it tends to be very hard to unhear that sensation! A sound, once drawn into the pitched category, tends to stay in that category.

The anchoring prototype allows relatively clear discrimination within the identification category. The clearest visual comparison (though not strictly analogous) is with color discrimination, where most of us have a clear idea of what our ideal (read: prototypical) version of each color is. These prototypes anchor our perceptions of categorical identification of what color an object is, but also serve to give a point of reference for the finer distinctions of hue, brightness, and saturation. To look at the categorical extremes in no-context situations, one probably isn't able to discriminate in what direction and to what extent the colors on the sign of the local McDonald's restaurant are from one's prototypical versions of red and yellow. Those are typically so close to a person's prototype that without an exemplar comparison, he/she won't be able to describe the difference, even if one is somewhat perceived. At the other extreme, if one were to ask a subject to identify mauve as pink or purple, the subject might very well reply "yes?," indicating the utter confusion caused by mauve often being such a poor representation of either category as to cause it to be not assigned to any.

The position of stimulus relative to the anchor of a category is often perceived as an emotional response to the intonation of that stimulus, the visceral "ick" of a mistuned tone. Huron's book, *Sweet Anticipation*, is a general examination of the development of musical expectation and the consequences of violating that expectation. Central to his thesis is the ITPRA model relating the interaction of expectation and emotional responses.[141] ITPRA stands for the emotional responses of an individual before, during, and after an event: **Imagination** – anticipation of what one's feelings about an event will be; **Tension** – emotional preparation for an event; **Prediction** – feelings about the

[141] Huron 2006

anticipated quality of an event; **Reaction** – direct response to an event; **Assessment** – evaluation of an event after thought and consideration has been given to the context of that event.[142] While Huron mainly focuses on illustration of the model for one single event, it is clear that one could have a cascading series of ITPRA responses based on multiple, succeeding (pitch) events, and that multiple ITPRA models could be active per event or event series. Though described in active voice, much of this processing occurs far below the conscious level. This model is useful in this study because it offers a way to understand intonation not just in terms of defining the "correctness" of an interval based on its absolute size, but to understand that good/poor judgements of intonation are based on an interval's filling an *expectation* of how large the interval should be and why poor intonation is so disruptive to the musical experience.

If we term our "single event" as the paired-pitch event of an isolated melodic interval, which also neatly avoids most of the psychoacoustic problems associated with bottom-up neural disturbance, we can begin to form a basic application of the ITPRA model. The initial pitch would cue the imagination of a generic schema of all possible intervals in the WTS, the expectations generated by that schema of what specific pitch may follow the initial one, and the possible emotional responses to each of those possibilities. The imagination of the next pitch event then cues a tension response, drawing focus in anticipation of the next pitch event and "pre-loading" a series of possible responses to that event.

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[142] Huron 2006, 15-18

According to Huron, once the pitch event occurs, the Reaction and Prediction responses occur simultaneously. The reaction response is probably the most important response for the valuation of intonation especially in isolation, as it is the response which most directly interacts with the schema and is by its nature a "fast-track" response.[143] The Reaction response is what gives us the direct like or dislike of an interval, governed by the principle that if an interval falls within the schema for an interval, it yields a positively valenced[+144] emotional response, and if it falls outside of the schema category, it yields a negatively valenced emotional response. The Prediction response is the evaluation of how well the schema performed, and how well the Imagination and Tension responses prepared for the interval event. For example, if, for some reason the Imagination and Tension responses cued for the possibility of a failure of the schema in predicting the succeeding pitch and the pitch failed to meet the schema expectations, a couple of emotional states would be generated by the Prediction response. First would be a negatively valenced emotion due to the failure of the schema itself. This is a complement to the Reaction response's negative emotion due to the stimulus's failure to meet the schematic expectation. The Reaction response and the negative emotion generated by the Reaction response then aids in schema evaluation and modification. If a schema fails too many times, an alternate schema is suggested in its place. Secondly, the Imagination and Tension responses did correctly predict the possibility of the stimulus's failure to satisfy the expectations generated by the tonal schema. So, even though the stimulus failed to satisfy the schema and the schema failed to predict the stimulus's actual position, the Prediction response still returns another, positively valenced emotional

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[143] Huron 2006, 131

[144] This is a somewhat peculiar usage of the word fairly unique to the psychological field describing a subject's attraction or aversion to a stimulus.

response to reinforce the ultimately correct failure prediction. This partly, but not completely, offsets the negatively valenced emotions generated by the other two responses.[145]

The last response, Assessment, is the evaluation of the stimulus in context, and begins immediately after the Reaction and Prediction responses. Because this evaluation involves the hippocampus and cerebral cortex, it is a slower response, but has the power to totally reverse the emotional responses generated by the previous functions. Huron's illustration of this is surprise (as in a type of birthday party), where physiologically, surprise is closely related to terror in preparation for flight and is the initial emotional reaction, the ITPR responses generating negatively valenced emotions.[146] However, when the full context of the party portion of the surprise is known and with the full knowledge that one's life in not going to end (predictably) in the next few seconds, the Assessment response is able to generate a positively valenced response which can totally supplant the negatively valenced states engendered by the ITPR portions. This contrastive valence is often more pleasurable because not only is one getting pleasure from a party being thrown for the occasion of one's birth, but added to that is the positive emotion of just getting back to the status quo! The total pleasure felt represents not only the warm feelings of being at a party with friends and loved ones, but added to that is the emotional change required to pass from the negative emotional state to the positive. This is Huron's contrastive valence, representing the total change in emotional state.[147]

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[145] Huron 2006, 143
[146] Huron 2006, 19
[147] Huron 2006, 21

Taking a musical example to illustrate the impact of Assessment, let's consider the melodic sequence $\hat{5}$ - $\hat{7}$ - $\hat{1}$ in a case where we hear each pitch functioning as the scale degree listed of a major tonal collection, i.e., that the tonal schema is previously fully activated. Let us further assume that a performer for some reason decides to enlarge the interval between $\hat{5}$ and $\hat{7}$ to the point of being at or beyond the categorical boundary for a major third. If the example were to stop on $\hat{7}$, the ITPRA model would predict quite a bit of unrest because the key-centered tonal schema is activated! Primarily, for our purposes, a listener has a very specific set of interval expectations associated with the $\hat{5}$ - $\hat{7}$ interval, which have not been met, as the stimulus failed to meet the requirements of the schema (the interval being too large), yielding a negatively valenced response. Additionally, this sequence of stimuli would cue melodic, closure, and emotional schemas inherent with the scale degrees, all of which would remain unsatisfied.

However, if continued to $\hat{1}$, with the underlying $\hat{5}$ - $\hat{1}$ interval sized to be within the categorical bounds, we can see the impact of Assessment with the completion of the underlying motion. Taken as a unit, the open question left by the first two events is resolved, satisfying the melodic and closure schemas with a relatively high contrastive valence. The negatively valenced emotional state caused by the categorical violation of $\hat{5}$ - $\hat{7}$ can then be assessed in the context of these two schemas and the more proximal response of the $\hat{7}$ - $\hat{1}$ motion, which by definition would be sized within the m2's categorical bounds and positively valenced. The highly affective portrayal of the notated $\hat{5}$ - $\hat{7}$ interval resolving to $\hat{1}$ would have an extremely high contrastive valence when the full context of the motion is unfolded and considered, and even though the specific motion of $\hat{5}$ - $\hat{7}$ would have been judged out of tune, in context of the whole unit, the interval may be judged in tune, even as an effective manipulation of affect for expressive purposes.

Huron makes extensive use of cognitive information theory relating statistical expectation and memory loading to emotional valence throughout much of his book. The general explanation of the theory is that high-probability, low-information events, such as cadences,[148] have generally positively valenced responses, but low-probability, high-information events, such as the musical surprise when the nursery rhyme weasel goes "POP!" have at least an initial negatively valenced response.[149] This is in part due to the high-probability, low-information events being very conformal to the schematic constraints defining the expectation of the event and incurring a very low memory load to process the stimulus, both of which tend to generate positively valenced emotional responses. Conversely, low-probability, high-information events require high amounts of memory to process the occurrence of a schema failure, and to search and switch to a new schema, which generally gives a negatively valenced response.

This view impacts this study in several ways, the first being analysis of a random out of tune pitch. If a performer has been playing in a manner which consistently conforms to a listener's schematic expectations of pitch, then that constancy tends to generate positively valenced responses and allows more "background" processing of intonation, as the intonation processing has turned into a series of high-probability, low-

[148] Huron 2006, 154
[149] Huron 2006, 114

information set of events in a similar manner to rhythmic entrainment. However, should a pitch become errant, the schematic failure immediately draws attention to the error, starts the process of schematic evaluation, and increases the Tension response until a new or reaffirmed schema is established.[150] The poorly intoned pitch has become in context a low-probability, high-information event, which generates higher memory loading and negatively valenced responses. Further, the failure represents a high contrastive valence because the general emotional experience before the failure is positively valenced, but the swing from a positively to negatively valenced affect draws even more attention to the failure and creates the impression of a far more dramatic change.

If the previous example is a post-event destabilization, a performer not seeming to be able to find a stable intonation pattern would be an example of a pre-event destabilization, again of the low-probability, high-information variety, because each new event in the intonation chain represents a new distinct piece of information in the search for a stable schema. However, instead of acting mainly on the PRA portion, because the schema continually fails, the IT portion is destabilized as no referential schema from the preceding pitch events suggests a viable prediction. The Imagination and Tension responses left unable to generate a prediction, which cause heightened attention and memory loading to the incoming pitch stimuli. This in turn generates negatively valenced responses, but these are not as strong as the random out of tune pitch. Because the listener is essentially starting from a neutral emotional position, there isn't the attention drawn to a particular pitch in the sequence and the contrastive valence is not present as it occurs with the schematic failure associated with the random pitch. Also,

[150] Huron 2006, 108

somewhat perversely, a certain expectation of the unexpected occurs, which mediates the negative valence. As long as IT are knowingly sending forward a null prediction, even if the reaction response is negative, the prediction response is at least able to send a positively valenced report to the Assessment response that the IT were right about not being able to predict anything! However, this would still create the unrest from which out-of-tune performances seem to suffer.

As I have pointed out previously, conceptual categories and schemas can be changed, overwritten, or sharpened. This is generally how experience is translated into expertise, and the process is not fundamentally different for interval categories and intonation discrimination and tolerance. Given this and that the consistently occurring number for identification category width for musically trained subjects is ±30 cents, let's say for the sake of argument that expert musicians sharpen that sense of categorical identification to ±20 cents. Let's say for the sake of argument that an acceptability range for a given interval would be 80% of the identification width, or ±16 cents, based on Siegal & Siegal.[151] Finally, we have a poor-discrimination region of ±10 cents. Combining the above points, I propose a general model of acceptable intonation, where i is the generic interval being evaluated, measured in cents:

[151] Siegal & Siegal 1977

For i > ±30 cents of prototype:	not categorically assigned, judged out-of-tune
For ±30 cents > i > ±20 cents:	poor categorical assignment the sense of "this interval should be *x*," judged out-of-tune
For ±20 cents > i > ±16 cents:	categorical assignment boundary sensation judged marginally in-tune judged in-tune contextually
For ±16 cents > i > ±10 cents:	categorical assignment size relative to prototype discriminated zone of acceptability judged in-tune, especially if serving an expressive function
For ±10 > i:	categorical assignment size relative to prototype not discriminated judged unconditionally in-tune.

Does this model cover all situations? Of course not, as intonation is a cognitive process with a wide variety of influences, but I think it does offer a reasonable approach to average situations. As discussed earlier, the tolerances for perfect intervals would undoubtedly be tighter, especially when functioning structurally. But even intervals subject to expressive expansion or contraction still conform to these standards, falling in the region around ±16 cents or less.[152] These ranges for the interval models also conform to the interval strength model of Rakowski.[153]

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[152] See Rakowski 1994, Rosner 1999, and Burns 1999.
[153] Rakowski 1990

Some Influences of Instrument Acoustics

If pitch is essentially a psychological experience of the sum of the frequencies of a stimulus, then the timbre of a stimulus is the psychological experience of the spectral content of the stimulus. What makes this interesting for the present study is that mechanistically, both processes are making use of the exact same signal, and this begins to explain much of the interdependence of pitch and tone quality in the performance and pedagogical literature.[154] For example, singers are acutely aware of this trait of tone interpretation, knowing that a change in the vowel form of a sound can affect its pitch.[155] This has even been demonstrated experimentally, with the perceived pitch of a sung sound changing with a change in the vowel, but the measured fundamental frequency remaining constant.[156]

This interdependence has begun to be explored experimentally and tends to confirm the anecdotal assessment of the relationship between pitch and timbre. For example, it is being shown experimentally that of two stimuli of the same fundamental frequency, the one with more power in the higher end of the spectrum (a higher spectral centroid, or center of gravity of the signal spectrum) is perceived as having a brighter timbre than the one with a lower spectral centroid, which is perceived as darker. Further, given the same stimuli, the brighter tone is perceived to have a higher pitch than the darker tone, even though both tones have the same fundamental frequency![157]

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[154] c.f. Flesch 1939, Stubbins 1965, Rayner 1975, Mozart 1951, Leuba 1984, Smith 1993, Kanno 2003, Ross 2003, and Kimber 2005

[155] See Howard 2007.

[156] Vurma & Ross 2006

[157] See particularly Fyk 1995, 148 citing Sachaltueva 1964, and Geringer, Madsen & Dunnigan 2001.

Madsen & Geringer, and later with Dunnigan, took the different tack by questioning the relationship between timbre and intonation in a series of experiments.[158] Again, the results generally support the pedagogical tradition, affirming that judgments of good timbre and good intonation are mutually supporting in the optimum case. However, what is interesting to note is how strongly correlated results are from the pairings of good timbre/poor intonation and poor timbre/ (mathematically) good intonation. The subjects in these trials were far more accepting of wide deviations, and the intonation rated much more highly, in calculated pitch provided that the timbre of the tone was considered good. Conversely, even if the tone is mathematically close to an ideal model, if the timbre is judged to be poor, the rating of the intonation is far below that of the good tone/good intonation rating. The suggestion is that judgments of intonation are not limited to the pure categorical assignment of pitch and interval, but are interdependent, even confused, with assessments of timbre. The implication for intonation pedagogy is that one of the mainstays of achieving acceptable intonation must not only be correct tone placement, but also achieving an excellent timbral quality as well. The confusion is further supported practically by a leading international principal double bassist, who advocates tuning in fifths an octave below the cello, to match the intonation of the rest of the string section. In a clarification of earlier comments,[159] the issue was not so much that the intonation of the string section was easier to match, but it was primarily easier to match and blend the timbre of the double bass in fifths with the rest of the string section and

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[158] Madsen & Geringer 1976 and Geringer, Madsen, & Dunnigan 2001
[159] c.f. Brun 2000, 154 and Masuzzo 2002

also allowed better intonation given the improved alignment of the overtone spectrum, as evidenced by lower acoustic dissonance.[160]

This raises the question as to if an instrument tuned in fourths+ fundamentally functions differently from an instrument tuned in fifths. Without bogging down with too many numbers at this point, I would point out that the harmonic series of the open strings of the instrument tuned in fourths line up differently amongst themselves than the harmonic series of the open strings of an instrument tuned in fifths. This has given rise to the observation that double basses fundamentally sound different from their violin family counterparts, particularly noted by 1835 Paris Opera director Habeneck,[161] Rodion Azarkhin,[162] and Bertram Turetzky.[163] Given a pair of instruments, such as the violin and the double bass, which have four strings of the same pitch classes but one is tuned in fifths and the other is in fourths in different registral positions. They will share the same overtone series of each string, as may be obvious. The more subtle difference is how the displacement in octaves of the overtone series affects the sympathetic resonance of the instruments. For example, a properly tuned open D-string will sympathetically resonate the G-string on the lowest common D harmonic. For a violin, this would be the third partial of the G-string, which would add a measureable resultant resonance to the second partial of the D-string,[164] while on the bass, the third partial of the G-string would resonate, but this would add a resultant resonance to the fourth harmonic of the D-string. While this may not seem to be a significant difference, if one remembers that timbre is

[160] Quarrington 2013
[161] Duffin 2007, 96
[162] Applebaum 1983, 166
[163] Turetzky 1989, 10 and Applebaum 1975, 290
[164] c.f. Fyk 1995, 100, Ross 2003, and Meyer 2009

dependent on the placement and strength of partials within the overtone series, the impact may become clearer. The overall resonance of an instrument determines its timbral characteristics, which can be directly affected by the choice of tuning used.

As an example of how this choice affects stopped tones, the pitch classes of E and G share a common partial of B. Let's assume two four-stringed instruments had the pitch classes E and G as their outer strings, one tuned in fourths the other in fifths, leading to the fourths instrument having its low string be the E and the high string the G, and the instrument tuned in fifths has a bottom string of G and top of E. The common partial B between the low E-string and high G-string is the sixth harmonic of the E-string and the fifth of G-string for the instrument tuned in fourths. For the one tuned in fifths, the common partial is the fifth harmonic of the low G-string and the third of the high E-string (we'll ignore the octave separation for now). Given standard tuning practices for both fourths and fifths, the B's of the G-strings will always be lower than the B's of E-strings by about a ditonic comma. If players were to tune a stopped B to the lowest strings of each instrument, then each tone would be internally in tune with each instrument, but each stopped B would not be in tune with the other!

The question of the effect of fourths versus fifths tuning is not quite as academic as it might first appear, but is one of the important differences between the da braccio and the da gamba families, not only from a classification standpoint, but from an acoustic one, as was previously demonstrated. Many of the differences in construction between the two families are of concern only from a strict taxonomic point of view, and they share many acoustically significant features. For instance, the overlapping or flush edges of an instrument have a negligible impact on the sound of an instrument, but are extremely important for the classification. Comparing the post-Renaissance viols to da braccio instruments, there are almost more acoustic similarities than differences. Both groups have upper and lower bouts separated by narrow waists, both have sound holes near the center waist region, both have similar bass bar/soundpost internal structure, both have purfled tops if not also backs, and both have similar top graduations. Aside from the number and tuning of the strings, the more important construction differences affecting the acoustics of the instruments lie in the proportions of each instrument, as discussed with Thomas Sparks, Senior Lecturer of Music (String Technology) at Indiana University.[165] The primary difference is that the da gamba family is proportioned so that the bridge ideally divides the body approximately in half, whereas the da braccio family is built around an approximate Golden Section division (618:1000), which results in differences in how the top of each family resonates. The bridge itself is different between the two families, the gamba bridge being thinner and more widely waisted (even given the increased width necessitated by the increased number of strings!) than its da braccio counterparts, changing its modes of vibration. Perhaps nearly important as the division of the top is the shape of the sound holes, which partly control the shape of the modes of vibration of the top and the propagation of the sound wave.[166] The narrowing of the top plate between the top of f-holes of the violins tend to bound the major movement of the top, while the generally more widely spaced c-holes of the gambas tend to allow greater

[165] Personal communication, April 2011
[166] Peterlongo 1979, 51

movement of the top at the fundamental frequency of the note played. This is confirmed in harmonic spectrograms of each group of instruments.[167]

The measurable results of the acoustically significant differences in construction are mainly what's known as a "bridge hill" in the spectrographic plots of violin-family response, which is absent in the gamba family, and a tighter and more harmonic response pattern for the viols, i.e., narrower bandwidth per harmonic, than the violin family.[168] The bridge hill is an area of increased resonance around the resonance frequency of the bridge, which contributes vowel definition to the resultant sound, and total resonance falls off sharply above the bridge hill. Because the vibrating area of the top plate of a gamba functions more like a theoretical hinged plate,[169] the harmonic vibration modes are much more sharply defined, leading to little extra sound energy around the center of each harmonic spike, and the harmonic spectrum falls away from the fundamental in a much more regular manner than in the violins. The timbral experience given these qualities is a comparatively full, complex sound to the violins and a pure, open sound to the viols.

The attribution of "resonance" is somewhat more difficult, since both instruments are capable of having strings vibrate sympathetically, though the gambas have technique specifically built around this principle, have more opportunities to create resonances on other open strings, and are historically known for being resonant instruments. The mechanics of sympathetic resonance, more formally known as "coupled resonance," has

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[167] McClure 2010, 320-321
[168] See Benade 1976, 540; Meyer 2009, 88 and 100; McClure 2010, 322; and Tan, Pfordresher, & Harre 2010, 10.
[169] See Benade 1976, 131.

been understood for quite some time,[170] but there has been a lack of direct observation and comment about how sympathetic vibration impacts the sound of a violin-family instrument. While it is clear that it is a real, perceptible phenomenon, its extent depends on how close the frequency of the stimulus is to the center of the frequency of the target,[171] and its effect is large enough to be a confound in experiments in which spectral analysis takes place.[172] An interesting effect about sympathetically driven strings is that, in a narrow band around each harmonic, the driving string will impose its specific vibrating frequency on the sympathetically driven string, even if those frequencies do not exactly match, but outside that band, the driven string will still respond, but at its natural harmonic frequency and induce beating in the resultant sound.[173] For a hypothetical example, if the driving string is vibrating at 443 Hz, and an adjacent string has a harmonic at 440 Hz with a 5 Hz bandwidth, the adjacent string will actually sympathetically vibrate at 443 Hz, despite its natural harmonic position at 440 Hz. However, if the driving string is vibrating at 446 Hz, the adjacent string may still vibrate sympathetically, but at 440 Hz, and there will be a 6 Hz beat in the resultant sound. While each individual instrument has its own bandwidths for "pulled" resonances[174] and driven resonances, the above is at least an illustration of how this phenomenon behaves.

The performance literature would suggest that sympathetic vibration contributes to the sensation of free, ringing, and brilliant tone,[175] and the pedagogical literature would

[170] See particularly Weinreich 1977 for coupled unison piano strings and Gough 1981 for violins with strings coupled through the bridge.
[171] See Benade 1976, 159 and 513; and Gough 1981.
[172] c.f. Fyk 1995, 100; Meyer 2009, 88; and Schoonderwaldt 2009
[173] See Gough 1981.
[174] As Gough 1981 puts it.
[175] See for example Stanton 1965; Pandos 1981, 169; Fyk 1995, 149; Adey 1998, 698-700; and Ross 2003.

tend to support this.[176] However, the acoustic literature has less directly examined the influence of sympathetic vibration on the resultant spectrum and the differentiation of a spectrum with and without coupled components. Gough demonstrated that sympathetic resonance at a given harmonic sharpens the response of, or narrows the bandwidth at, a given harmonic and also increases the output of the coupled system.[177] On violins, this has been almost parenthetically mentioned by Meyer, where A_5 was 15 dB higher when played on the E-string versus being played on the A-string, though the conclusion that this increase in output was not outwardly drawn.[178] In analyzing the spectrographs of Culver and especially comparing the spectrum of the damped viola to the rest of the undamped violin family, it can be inferred that the sympathetic resonances of the open strings contribute significantly to the development and propagation of the upper harmonics.[179] In the readily available literature, the only direct report of the influence of string-string coupling has come through Yoshikawa and Peterson.[180] Yoshikawa, in studying the shamisen, a Japanese plucked instrument, measured a significant increase in the power output and perceived quality of the sound when the strings were allowed to resonate sympathetically. Peterson, in analyzing hammered dulcimers, found that sympathetic resonance contributes significantly to the tone quality and sound duration. Another factor influencing the spectrum of a sound is that a sympathetically stimulated string will continue to ring after the driving string has ceased vibrating, to the point that it does actively interfere with successive tones as was observed experimentally by Fyk,[181]

[176] e.g. Geringer, Madsen, & Dunnigan 2001
[177] Gough 1981
[178] Meyer 2009, 88
[179] Culver 1941, 94-103
[180] Yoshikawa 2010 and Peterson 2010
[181] Fyk 1995, 153

Peterson,[182] and particularly by Karr in the double bass performance realm.[183] These residual vibrations cause interference patterns in the resultant spectrum, which are usually judged to contribute to a poor sound quality. Mismatched stimulus/sympathetic frequencies will tend to cause an interference pattern, an extreme example being wolf tones.[184] Properly matched, however, a sympathetic vibration can add an open string-like brilliance to a stimulus tone.[185]

The example of the wolf tone is also an example of how normal and heterodyne interactions of the instrument can be perceived as vibro-tactile feedback. Askenfelt & Jansson measured more normal vibrations of the double bass to be of the proper rate and extent to be easily perceived through the left hand on the neck and the right hand on the bow.[186] It was also highly suggested that vibrations of the bass could be directly perceived through the air.[187] These results lend credence to the anecdotal notion that an instrument whose strings are out of tune feels out of tune, as the mismatched overtones would be either creating interference vibrations with each other or dampening the free vibration of the top, both of which could be perceived through the right hand. Conversely, well-matched overtones should provide a smooth feeling in the bow, as no interference patterns would exist. It also supports the pedagogical stances taken by some

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[182] Peterson 2010
[183] Karr 1996
[184] A wolf tone is more properly described as an interference pattern resulting from the string-body-coupled system where the string resonance approaches the main body resonance, but the phenomenon is analogous.
[185] See Rakowski 1990 and Fyk 1995, 88.
[186] Askenfelt & Jansson 1992
[187] See also Karr 1979.

teachers, that for the bass, the feeling of a smooth sound is more important that trying to listen for and compare pitches.[188]

General conclusions

If we didn't already know this, I think I've demonstrated that the processes underpinning intonation are as diverse as they are complex. The central problem of intonation is that, like music in general, it is a neurologically global phenomenon,[189] incorporating interactions between multiple memory systems, sensory consonance (raw data), categorical data, tonal schema, and emotional schema, to name but a few factors. Further, intonation suffers from excellent intrajudge reliability, but extremely poor interjudge reliability,[190] meaning that each person's concept of intonation and the underlying tonal schema are highly variable between individuals even if they are able to repeat performances to a high degree of accuracy.

I think what best characterizes good intonation from a cognitive point of view is that intonation which incurs the least memory loading from all schemas: psychoacoustic, tonal, emotional, timbral, etc. However, this again begets the most central question: how does one balance the acoustic constraints with the mathematical approximations which we call notation and the cognitive constraints we place upon ourselves? I think there is groundwork for approaching a system of intonation using a low psychoacoustic load approach which satisfies the schematic requirements of key discovery and tonal schema activation. What we need is a system which creates the best pitch relations with the most

[188] c.f. Karr 1979, Tirado 2002, and Beckendorf 2007
[189] See especially Tan, Pfordresher, & Harre 2010.
[190] Brown 1996, 1

satisfying emotional content, the least beating, and the most tonal stability, where appropriate.

Chapter 3: How does this play out in practice?

"That's what the book says; now I'm going to tell you how things really work."

-Dr. James VanNess, Dean, McCormick School of Engineering and Applied Science, Northwestern University

Temperament

Given that fretless stringed instruments (where the double bass most clearly shows the influence of the violin family) have a theoretically infinite flexibility of intonation, temperament might strike one as a curious place to start looking at performance practice issues, but I do so for a number of practical reasons. The first is that intonation is almost always discussed in terms of its relation to a given tuning or temperament. Usually one of the just, meantone, equal, or Pythagorean temperament systems is used for such a comparison. Even if one is trying to steer away from such interference by using the cent as a basis for comparing interval sizes, it is still a unit of interval measurement based on the equal-temperament scale. Similarly, once one ventures from pure tuning by ratios, one has modified, literally tempered, the intervals producing an ad hoc temperament system, even if it is not fully closed or circulating, such as with well-temperament or equal-temperament systems.[191] If nothing else, reference to a temperament system provides a stable point of comparison which is useful

[191] c.f. Lindley & Turner-Smith 1993, 48, and Duffin 2007, 124

when one is comparing the psychological experience of vibrating air molecules, for which we have few other means of measuring.

Perhaps most importantly, the study of temperament as a whole can inform us as to the practical bounds of acceptability of the size of a given interval. As Hall and others have noted, particularly Lindley, there is no one perfect temperament for all music, each piece having been conceived in a given temperament in a given time period.[192] Blackwood, in his book outlining his mathematical models of temperament construction, puts the matter into the perspective that one can create diatonic temperaments, even circulating ones, which are unrecognizable as cohesive temperament systems, others which are recognizable as diatonic, but are not acceptable, and others which are acceptable.[193]

While it is tempting to treat fretless instruments as having an infinitely variable intonation, as has been done so often before, this is not practically true of instruments with multiple strings. In some manner, one must deal with the fact that there are four fairly fixed tones on the instruments of the violin family (of which I will include the double bass in this discussion for practicality's sake). We know this is an issue simply because of how much has been written in discussing how the violin family should be tuned, whether it be pure fifths, tight fifths, two pure fifths with the third fifth tightened by a ditonic comma.[194] The question of the specific tuning of the strings is as much a mechanical question of how an instrument is to be tuned as how the different tunings

[192] Hall 2002 and Lindley 1977
[193] Blackwood 1985
[194] c.f. Halfpenny 1974; Barbieri & Mangsen 1991; Watkins 2004; Duffin 2007, 129; and Borup 2008

implies differing resultant intonation and temperament frameworks and how the relative sizes of intervals are perceived by the performer.[195] While there are extant fingering charts showing the differing positions of enharmonically equivalent sharps and flats,[196] there is also debate as to whether a performer, especially a modern performer of the violin, could be reasonably asked to modify fingerings with such precision.[197] Finally, if intonation is at least partly mediated or informed by the sympathetic resonance of the instrument,[198] then the tuning of the instrument would affect the location of the overtones in pitchspace. Where the performer would need place fingered tones to encourage the activation of the overtones and the specific ratios relating two pitches would also be similarly modified. This results in an ad hoc intonation system which would vary with each tuning.[199]

I will admit that taking temperaments as a group loses much of the resolution that characterizes each individual temperament, the keys generated by a temperaments, specific interval sizes, and their uses. However, taking the historically used temperaments in amalgamation, certain informative patterns tend to emerge. As Benade and Blackwood have noted,[200] and a perusal of Jorgensen and Barbour will confirm,[201] most tempered interval sizes tend to cluster at the equally tempered interval sizes and in two groups, ± 10 cents of their equal temperament counterparts. Barbour goes further by referencing period sources citing temperaments with a mean deviation from equal

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[195] c.f. Barbieri & Mangsen 1991; Fyk 1995, 33 and 211; Loosen 1995; and Duffin 2007, 122
[196] c.f. Haynes 1991 and Barbieri & Mangsen 1991
[197] e.g. Flesch 1939, 22
[198] as Raab 1978, among others, strongly suggest
[199] c.f. Barbieri & Mangsen 1991, Lindley & Turner-Smith 1993, and Borup 2008
[200] Benade 1967, 295 and Blackwood 1985
[201] Jorgensen 1977 and Barbour 1951

temperament of ±10 cents as preferred temperaments, ±16 cents as good temperaments, and anything beyond ±20 cents as unusable.[202] This equates with the experiments of Rayner, which measured equal-tempered piano tunings, modified to suite each piano's particular resonance strengths.[203] No resultant temperament was beyond ±8 cents. While there is plenty of reason to question Barbour's motivation of seeing temperament as the continual drive to equal temperament and the statistical method he used from a performance practice standpoint, I think the process still gives valid observations for generalized comparisons, given the support of period documentation as to what have been historically acceptable interval sizes and yields practical data when pooled across all occurrences, especially given its coincidence with the psychoacoustic data.

As I've noted earlier, the current theory is that the tonal schema is largely a learned one, and I would argue that regardless of the specific temperament used at a particular time, the intervallic categorical centers making up the tonal schema will approximate an equally tempered schema[204]. This would most likely still occur in period practice because, with exposure to enough of the keys and chords within a given temperament, various sizes of intervals would be experienced, which would tend to average to equal-tempered sizes. The notion that keys have a particular color or function at the visceral level is one almost completely lost on a musical culture immersed in equal

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[202] Barbour 1951
[203] Rayner 1975
[204] There is a great deal of debate on this issue, which mainly rests on how approximate "approximate" is. Loosen 1994 and 1995, Leukel & Stoffer 2004, and Vurma & Ross 2006 tend to support Burn's 1999 assertion of the tonal schema's categorical centers roughly aligning with equal temperament. Even though Rakowski's 1990 measurements of categorical centers were within a few cents of equal temperament, he seemed unwilling to come to the same conclusion. Wapnick, Bourassa, & Sampson 1982, Hall & Hess 1984, and Acker, Pastore, & Hall 1995 found varying ranges, some intervals centering on equal tempered sizes, others centering towards just interval sizes.

temperament, but one which was quite present in the minds of composers and performers before the prevalence of equal temperament and relies on the sense of an "average" interval size for each interval class. For example Galeazzi, expounding on the qualities of various keys, reports that Bb major is tender, soft, and sweet, while E major is piercing, shrill, and youthful.[205] Mattheson cycled through all 24 major and minor keys, relating the various qualities of each.[206] This supports two notions: one, that deviation from the tonal schema causes an emotional reaction, which when used purposefully and strategically could be considered "expressive",[207] and two, that the amalgamation of interval sizes of a prevalent temperament or set of temperaments forms the outline for a generalized tonal schema approximating equal temperament.[208] As Barbour, Sundberg, and Hall noted, equal temperament represents an approximate average of the usable non-equal temperaments.[209] If the tonal schema is the sum of all previous tonal experiences, then the only logical outcome is that the internalized schema is going to approximate an equally tempered system to a greater or lesser degree. The historical record indicates that, in a harmonic context, tonally stable intervals were sized more towards pure than present, only due to the stylistic expectation of smoother (more acoustically pure) harmonic intervals.[210] Conversely, when not mediated by any other psychoacoustic or harmonic factor, we have evidence much more strongly indicative of contemporary schemas being closer to equal temperament in the survey of Rakowski, Loosen, and Leukel & Stoffer, more directly measuring internalized interval size versus the equally

[205] cited by Duffin 2007, 44
[206] Mattheson 2002
[207] See particularly Huron 2006.
[208] See also Burns 1999.
[209] Barbour 1951, Sundberg 1982, and Hall 2002
[210] c.f. Quantz 1985 and Norrington 2004

tempered scale, and the results of Krumhansl, which indicated that when abstracted, musicians find each key to be psychologically equivalent.[211]

The differentiation I am trying to draw is that while our historical musical counterparts would have drawn a specific rhetorical and affectual implication from a key's deviation from the equally tempered schema, modern listeners often find such divisions, such as in a ¼ comma meantone temperament, intriguing or simply strange, at worst. The modern expectation is that a piece played in Eb major will sound just as convincing, in fact "the same," in E major and that there is no inherent qualitative difference between the two keys, whereas the historic view would see that due to tempering affecting the relative sizes of intervals in the two keys, the piece played in each key would generate massively different affects, a difference which might or might not be sought by the composer. However, this does not preclude that modern listeners would have some "inherent" understanding as to the affect achieved by the change of tempered keys. The deviations from the tonal schema are thought to be universal enough to thwart the schematic expectation and elicit an emotional reaction across stylistic boundaries.[212]

This puts us at a curious crossroads: while modern listeners may have the intellectual notion that all keys are created equal, many retain the feeling that some keys have a different affect than others, especially for non-fixed pitched instruments. Barbieri & Mangsen illustrated this point by asking a violinist to play a melody in F# major and

[211] Rakowski 1990, Loosen 1994 and 1995, Leukel & Stoffer 2004, and Krumhansl 1990
[212] e.g. Fyk 1995, 23

then in Gb major.[213] When the performer was asked why he chose to modify his intonation choices given that F# and Gb are enharmonically the same notes, he replied that F# major is a much brighter key than Gb major and he adjusted accordingly, demonstrating that there remains the performance practice expectation that key choice dictates some affective influence. To separate Gb major from F# major, a performer could select tone placements which would have less chance of ringing clearly against an open string, yielding a duller sound, and smaller interval sizes, such as a smaller M3 between $\hat{1}$ and $\hat{3}$, which is schematically less brilliant, being more harmonically stable and less directional. Similarly, especially for the violin and the double bass, D major remains known as a very bright key, whereas Ab major is a dull one.[214] In practical terms, much of this difference revolves around the availability and desirability of resonance on the instrument.[215] D major as a bright key is one which benefits most from a pure fifths tuning: in order to have the greatest opportunity for low-level sympathetic resonance, the tone placements necessary to achieve that resonance widens the M2, M3, M6, and M7, while compressing the m2 and m3, which would perceptually render a brightening effect. Conversely, Ab major has few notes with low-level sympathetic resonance, and if a performer chooses to favor what little resonance is available, the resulting interval sizes will be more of the stable, harmonically smooth size, which is effectively less brilliant, even if the tone is more so. This illustrates how changes in desired affect can create a temporary ad hoc temperament which will change with the demands of the music and how the performer wishes to portray it.

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[213] Barbieri & Mangsen 1991
[214] See also Adey 1998, 698-700.
[215] See Fyk 1995, 149.

Much as many would like us to believe otherwise, the mythos of purely equal temperament still haunts fixed pitch instruments and the resonance issue affects their tuning. As equal temperament began to take root near the mid-1800's, tuning practices had yet to fully catch the precise mathematical gymnastics required for dividing the octave by basing a tuning system on the ratio of the twelfth root of two. Piano tuners, still tuning by mainly ear, tended to shade the tunings of the instruments toward favoring more harmonically smooth tunings for the keys around C, even when the tuners thought that they were tuning a precise equal temperament.[216] Even with White's description of how to tune without the use of computerized electro-acoustic feedback systems, this practice persisted into the early twentieth century.[217]

The use of the stretched octave is another phenomenon which pulls a piano from being truly equally tempered and is in reaction to the non-linear pitch response. While the mechanism underlying the response is still poorly understood, the consistent observation is that for an octave to be heard as pure, it often needs to be stretched slightly by about 2-5 cents, which greatly varies across the best-pitch range, with the extremes effectively stretched more than the central region.[218] While this might be of little consequence to instruments of relatively narrow compass, such as the violin, it becomes extremely relevant to instruments of larger compass, such as keyed percussion instruments, which need to keep perceptual consistency throughout their range.[219]

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[216] Duffin 2007, 114
[217] Rayner 1975
[218] c.f. Terhardt 1974 and 1984, Fyk 1982, Rakowski 1990, and Burns 1999
[219] See also Gillessen 2000.

Lastly, to shatter the notion that we live in a purely egalitarian equally-tempered world, since the 1960's when precise frequency measurement was possible on a manageable scale, piano tuners have intentionally made the decision to deviate from equal temperament to favor the resonance of the particular piano which they are tuning. While the capability existed for pianos to be tuned fairly precisely to an equally tempered system, it was found that such tuning produced either a dull sounding instrument or one which was not perceived as being in tune, even though it was mathematically perfect![220] By modifying the tuning slightly from equal temperament but favoring the resonance of instrument, the acceptability of both the intonation and the tone of the instrument improved greatly. This performance practice result supports Madsen & Geringer, Fyk, and Geringer, Madsen, & Dunnigan in the rigorous clinical realm.[221]

Performance practice

The history of intonation performance practice and pedagogy can generally be observed as a tale of the relative position of sharps and flats or of the move from static intonation to dynamic intonation, as described by Fyk.[222] To describe the phenomenon in terms of temperament would be to arc from a fixed or extended meantone system to an equi-tempered/Pythagorean one.

As I alluded, I think there is some validity to the notion that a period's intonation can be loosely described by the dominant temperament of the period.[223] This is most

[220] Rayner 1975
[221] Madsen & Geringer 1976; Fyk 1995, 149; and Geringer, Madsen, & Dunnigan, 2001
[222] Fyk 1995
[223] See Barbieri & Mangsen 1991; Haynes 1991; Loosen 1995; Fyk 1995, 18; and Ornoy 2006.

clearly seen beginning in the Baroque, where the documentation of keyboard and temperament development closely parallels the descriptions of where a violinist was to put his fingers.[224] If, for the sake of brevity I may describe the period in one large swath, it would be to say that it was dominated by shades of a meantone temperament, as even at the end of the Baroque, the irregular and well-temperaments were basically derivatives of the meantone system.[225] The implication for intonation is that there was a clear sense of pitches having a fixed position, especially in that flats were higher than sharps, which is the by-product of extending the meantone tempering system beyond the basic circle of fifths.[226] This represents Fyk's static model of intonation, emphasizing sensory consonance of more purely tuned intervals.

The main factor influencing the decision of using the meantone system, even for solo instruments, was the compositional practice of the time of a much tighter integration of the vertical and linear elements as a unit and the rhetorical implication of each shade of acoustic consonance or dissonance. Thus, to keep the acoustic alignment of the M3, the resultant placement of flats is above the enharmonically equivalent sharp. Leopold Mozart specifically notes that flats and sharps should be differentiated by "the correct amount" of a comma,[227] but did limit the sharp/flat differentiation, specifically saying that, though theoretically different, F-double-sharp was to be the same pitch as G.[228] Galeazzi laid down fingering placements with the flats in a higher position to the

[224] See Geminiani 1951, Quantz 1985, and Muffat 2001.
[225] See Benade 1976, 312-313; Lindley 1977; and Hall 2002, 429.
[226] See Geminiani 1951, Mozart 1951, Barbieri & Mangsen 1991, and Duffin 2007, 80.
[227] Mozart 1951, 70; Mozart was using the more general notion of a comma being a small unit of difference, rather than a technical comma, such as the ditonic comma
[228] Mozart 1951, 49

sharps.[229] Quantz also appears to have preferred a meantone system, though it is never mentioned by name, and takes acoustic purity as much more essential than today, saying that the refined ear finds antipathy with beating.[230] He lauded the strings for being able to make adjustments to be in agreement with the basso continuo and advocated for a differentiation of the D# and Eb by use of a separate key on the flute, with the Eb being higher than the D#.

As the Baroque transitioned to the Classical, the extended meantone approach was maintained and evidenced by all three composers of the First Viennese School. Mozart was the most direct, noting specifically of the differentiation of sharps and flats that "these tones the Harpsichord [sic] has not, but all other Instruments [sic] have" referring almost by name to the existence of major (diatonic) and minor (chromatic) semitones.[231] Based on Mozart's corrections of his student, Thomas Atwood, Chestnut extrapolated that Mozart's conception of non-fixed intonation was essentially an extended meantone intonation, with a 19-part division of the octave.[232]

Beethoven and Haydn were a bit more circumspect, however. In the Heiligenstadt testament, Beethoven described how keyboard instruments he had heard all had definite key colors, whereas orchestras had very muted key colors. Because the winds fit very well into a specific set of keys, or tuned to do so, and the strings were trained to adjust for sharps and flats to create a more consonant sound, this would tend to

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[229] cited by Barbieri & Mangsen 1991
[230] Quantz 1985
[231] Chestnut 1977, 265
[232] Chestnut 1977, 269

eliminate the specific orientation of major and minor semitones and is characteristic of a meantone approach to intonation.[233] Because the testament is so famously dated, it also informs us that this was the state of affairs of intonation and temperament through 1802. Haydn also left us a dated piece in his Quartet Op. 77/2 of 1799, in which he noted that the pitch of the D# was to be maintained as the notation changed to Eb. To a modern reader, this may seem redundant and be quickly overlooked. But it begs the question: if Eb is the same as D# are the same pitch, why the note? The contextual answer is that, in order to maintain a consonant ratio with the other parts, the Eb would need to be raised to a level approaching the meantone Eb level. However, it can be conjectured based on the contemporary practice, that the acoustic impurity of the D# was the desired effect at that particular event.[234] The preference for a meantone approach in ensembles was carried through the Baroque[235] to the Classical,[236] and even was a theme maintained in the Romantic specifically for ensemble playing, with Berlioz specifically pointing out that the position of sharps *above* flats was a technique reserved for soloists, not orchestral players.[237]

A compositional change between the Baroque and the Classical and into the Romantic was the shift from contrapuntally integrated linear sonic events to a more melody/accompaniment construction. In the solo literature, this change began a movement towards a more dynamic view of intonation which was less tied to the more rigid constructs of the meantone system. Campangnoli left us the first surviving

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[233] Duffin 2007, 86-87
[234] Duffin 2007, 79-80
[235] See Haynes 1991 and Duffin 2007.
[236] See also Chestnut 1977.
[237] Duffin 2007, 103

documentation of the practice of placing sharps above flats in1797,[238] and if one were to look at this as a change of temperament, it would look very much like a shift towards Pythagorean intonation, which has sharps being higher than flats as part of the larger purely tuned fifth extending out along the cycle of fifths. However, allusions to the expansion of the M3 and M7 above tonic, what we might now call "functional intonation," were recorded by Delusse in 1760, Roussier in 1770, and Eximeno in 1775.[239]

Concurrent with superposition of sharps over flats was the rise of equal temperament in the keyboard world from about 1810 to about 1850.[240] As the idea of an equally tempered system began to dominate the keyboard and compositional landscape in the second decade of the nineteenth century, indications of its permeation into the pedagogical realm began to surface as well. Spohr is perhaps the most famous "advocate" of equal temperament in string playing, and Geminiani, in explaining his fingering chart, states that the chromatic notes are to split the diatonic whole steps evenly.[241] However, a couple caveats apply, the first being that Spohr explicitly emphasizes that his advocation for the use of equal temperament was a pedagogical tool for the beginner only,[242] and likewise, Geminiani admonished that his static positioning of the chromatic tones on his fingerboard chart was only for the beginner.[243] The implication is that equal temperament was not the standard practice for advanced string

[238] Duffin 2007, 80
[239] Barbieri & Mangsen 1991
[240] Tittle 1978, 13
[241] Geminiani 1951
[242] See Duffin 2007, 94 and Borup 2008.
[243] Geminiani 1951

players of the time and that the differentiation between major and minor semitones was still in place in the string world, but considered an advanced concept not suitable for the beginning player.[244] The second caveat is that compared to other temperaments in use previously, equal temperament and Pythagorean intonation look very similar, with important intervals, such as the M3 and P5, being within the practical just noticeable difference, if not the technical perceptual just noticeable difference. So, perceptually and pedagogically, there is very little difference between equally tempered and Pythagorean intonation. A side effect of a more Pythagorean approach to intonation is that, when the circle of fifths is extended beyond the initial circle of fifths, the position of sharps and flats reverse, so that sharps are in their more familiar position of being higher than flats.

The coming of the twentieth century saw the full development of the philosophy of dynamic intonation, based on the aesthetics of linear playing and categorical perception as it applies to solo playing.[245] To build on the intense chromatic harmonies and longer melodic lines, performers began to explicitly use intonation inflection as a tool of emotional illustration. Casals famously coined this "expressive intonation," but Flesch was one of the first pedagogues to detail this technique in print.[246] While he is perhaps more known for his description of the best intonation as the one which corrects a faulty pitch before the audience notices, his description of tone placement has been noted favorably with its encouragement of pitch alteration based on its context and affect desired. However, what is often overlooked is the full context of the statement in which Flesch is fairly clear that affective pitch alterations are a sparingly used tool and that the

[244] Borup 2008
[245] Fyk 1995
[246] Flesch 1939, 20ff

normative intonation is more towards a meantone temperament/just tuning approach.[247] If one also reads Casals fully in context, he actually had much the same approach, only carried to its fully-wrought conclusion. His approach uses pitch alteration to a much greater extent, essentially any time its inclusion would yield an emotional impact without the listener becoming saturated by the experience.[248] Casals is one of the first to further advocate the compression of tendency intervals, such as a minor second derived from a chromatic alteration pointing up towards its resolution note, and that $\hat{1}$, $\hat{4}$, and $\hat{5}$, and their respective chords, should be held as inviolate "pillars" of the key and not subject to alteration, thereby totally grounding the key and preventing pitch drift.[249] We can also trace the popularization of the notion that "sharps go up and flats go down" to Casals.[250] A point of emphasis I would iterate is that Casals does not indicate that every M3 or every leading tone should always be sharpened. However, as succeeding generations have read and interpreted Casals' writing, some of his suggestions have taken on a dogmatic nature, and the cautions Casals included have on occasion been forgotten.[251]

Current practice seems to have taken the best lessons of the past by trading vertical consonance with linear expression when necessary to convey the affect implied. The expectation is that a performer will know his or her role in the music being played and adjust the intonation accordingly, bending towards purer intonation for chordal playing and towards Pythagorean when playing melodically. Violinists, as might be expected, seem to have the most inherent tendency towards playing with Pythagorean

[247] Flesch 1939, 22
[248] c.f. Applebaum & Applebaum 1972, 272, and Blum 1977, 102
[249] Blum 1977, 103
[250] Blum 1977, 105
[251] c.f. Eisenberg 1966 and Blum 1977

intonation,[252] though most artists are keenly aware of needing to modify their pitch when playing chordally so as to be in acoustic agreement with the other tones.[253] Conversely, violists are keenly aware of needing to brighten their pitches when playing solos, as the nature of their ensemble music trains them in a more pure tuning than tempered.[254] As would seem to follow the train of logic, cellists are almost fully schizophrenic on the issue, as acting as a bass voice to the upper strings and a melodic voice of the lower strings compels them to be able to change between the two modes of thinking. A commonality between each is the position of I, IV, and V remaining fixed within the given harmonic context with the ability to vary intonation between the extremes of pure and Pythagorean as the music and ensemble requires. The more ensemble-oriented cellists tend towards a basic harmonically tuned intonation, whereas the more soloistic artists trend towards an intonation expressly based on expressive playing.[255] In non-tonal contexts, there is some division as to how intonation is to be treated. Some advocate training in equal temperament for just such an eventuality, while others advise determining a pitch center and playing to that pitch center.[256]

Intonation pedagogy

For as much as the technology and technique of instrumental playing has improved, it is somewhat amazing how stable the methods of training and improving intonation have been. The methods mostly fall into four categories: teacher guidance, ear

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[252] See particularly Loosen 1993, 1994, and 1995.

[253] See Kanno 2003; Duffin 2007, quoting Steinhardt, 69 and 122; Ornoy 2006; and Brooks 2007, quoting DeLay; see also Shackford 1961 and 1962.

[254] See Applebaum & Applebaum 1972, interviewing Katims, 236; Wallace 1996; and Kimber 2005; see also Shackford 1961 and 1962.

[255] c.f. Applebaum 1973, interviewing Soyer, 302; and Blum 1977

[256] c.f. Eisenberg 1966; Applebaum 1973, interviewing Soyer, 302; Blum 1977; Smith 1993; Young 1995, interviewing Rowell; and Whitcomb 2010

training, play-alongs, and mechanistic approaches. Perhaps the oldest, most consistently used, and most reliable is teacher criticism and correction, where a teacher, presumed to have an expert's level of intonation discrimination and production, provides feedback to a student to correct deficient intonation. Over time, with proper exposure to other appropriate models, the student is able to develop and refine the pitch categories necessary to play in a manner which could be considered in tune by an audience. The overall goal is to help the student develop his or her own internal standard to be able to self-discriminate and self-correct. While this may seem primitive or overbearing to some, this technique has been shown to be among the fastest and most effective methods of teaching and improving intonation.[257] A refinement of the relatively recent past has been to focus on developing a sense of intonation more methodically by first anchoring $\hat{1}$, then $\hat{5}$, and $\hat{3}$, then $\hat{6}$ and $\hat{4}$, and so on.[258] The intent is to focus the student's attention to returning to the same pitch repeatedly, maintaining a consistent point of reference until all of the categories have been established within a tonal context.

Ear training, especially at the collegiate level, is often assumed to be concurrent with the teacher-correction model. There is some debate as to how productive ear training is for intonation improvement as it is based on the assumption that ear training will automatically improve intonation performance. However, ear training primarily focuses on categorical discrimination, not reproduction, which is a separate cognitive task. Morrison & Fyk argue that many neural and physical problems may impede a student's reproduction skills, even though that student has excellent discrimination skills,

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[257] c.f. Auer 1921, Corey 1965, Salzberg 1980, and Morrison & Fyk 2002
[258] See particularly Pondos 1981, Powell 1991, and Watkins 2004.

or a student may be able to perform far in excess of his or her demonstrated discrimination abilities.[259] While the former may be very true, the counter to the latter is that many of the ear training tasks focus on establishing categorical bounds of pitch as implicit knowledge and also develop the skill to link that implicit knowledge with the explicit knowledge of labeling that given category, such as a minor sixth. In addressing the student with excellent discrimination but poor performance, Gordon might argue that the student is not as poorly off as it might seem, as the student has developed the ability to at least know that her/his intonation is poor and can imagine, or audiate[+], a better pitch.[260] His aural-oral feedback model relies on pitch discrimination and pitch matching tasks as a precursor to guide establishing the internal, implicit schema, mapping it to a conceptualized, audiated, and expected pitch, and using that to map the neural motor responses to achieve the expected pitch in performance.[261] The ability to audiate, or the ability to hear internally a pitch not physically present in a tonally functional context, and then making the instrumentally-produced pitch match the audiated pitch has become a staple of intonation pedagogy.[262]

Play-alongs include exemplar models, duets with like or dissimilar instruments, and drones. Playing along with an exemplar model can take forms ranging from the traditional teacher playing with the student to playing with a computerized rendition of the piece being worked on. This technique serves to both refine categorical boundaries, develop pitch matching abilities, and to offer direct feedback to the motor-neural system

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[259] Morrison & Fyk 2002
[260] Gordon 1989
[261] See also Hiatt & Cross 2006.
[262] See particularly Stubbins 1965, 295-296; Green 1985; Applebaum 1986; Phillips & Winkle 1992; Galamian 1995; Conway 2003; Lambert 2006; and Grodner 2011.

as to the execution precision of the intonation.[263] Duets with similar instruments, or dissimilar such as a piano, offer some of the same benefits as playing with an exemplar, but also develop vertical awareness between the two instruments. The object is often to provide a stable frame of reference so that the student is given indirect feedback as to his intonation performance, but this method also promotes beat elimination. As the referential pitch is further removed from the line being played by the student, he or she must have (or quickly develop!) a stable internalized idea of the line being played. Drones have been employed as a pedagogical tool for longer than most people would probably imagine, with references for using pipe organs instead of pianos for their greater pitch stability,[264] though with modern chromatic tuners, an electronic tone is often used now. Set to the tonic, dominant or other reference pitch, the drone provides a stable reference point from which to assess intonation.[265] Also, while not often thought of this way, comparison of a pitch to or playing in tandem with an open string is essentially using a drone.[266] While able to employ any temperament system derived from the reference pitch, the curious detail is that a drone will often encourage beatless tuning with it so as to avoid Helmholtz's annoyances. This would mean a raised m3, lowered M3, and lowered M6 above the reference, and a lowered leading tone against the dominant, if that is the reference point.

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[263] See Morrison & Fyk 2002.
[264] See Martens 1929, interviewing Kuzdo, 84.
[265] See particularly Stubbins 1965, Leuba 1984, Barrett 2005, and Lambert 2006.
[266] See Flesch 1939, 21; Ross 2003; Watkins 2004; and Lambert 2006.

Mechanistic approaches to improving intonation have been pooh-poohed by some in the pedagogical realm, such as Flesch,[267] as being un-artistic, but another argument is that, if one is in trouble, a mechanical reference is a good place to get one's bearings, especially if a student is working independently and not under the watchful eye of his or her teacher. Techniques include using resultant pitches, visual tuning aids, and resonance training. The use of resultant pitches in tuning chords and linear passages has been advocated most strongly by Tartini, to the point that a resultant pitch is a Tartini tone by any other name, but the technique's advocates have included Mozart,[268] Rostropovich,[269] Fournier,[270] Green,[271] Leuba,[272] Applebaum,[273] and Hindemith[274] to name but a few advocates. The idea is to tune two tones so precisely that the beats between the two tones disappear and the resultant pitch becomes apparent. This can be done against two stopped tones, a stopped tone and an open string, or a stopped tone against an external drone. The exercise is primarily to develop a sense of the position of each tone independent of the reference tone, if not have the resultant tone appear in performance. The time necessary for two tones to come into alignment and a resultant tone to be perceived has been the chief criticism leveled against this technique, by Schoenberg in particular.[275] However, the technique is routinely employed by barbershop quartets for tuning purposes and pipe organs and double bass sections supplying the 32' or 64' octave

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267 Flesch 1939, 23
268 Mozart 1951, 163
269 Applebaum 1973, 282
270 Applebaum 1973, 284
271 Applebaum 1980, 168
272 Leuba 1984
273 Applebaum 1986
274 Hindemith 1945
275 Schoenberg 1978

by difference tone. If one is measuring linear playing by this technique, then the resulting tone arrangement will be of the just variety.

Electronic tuners and other computerized systems are becoming pervasive as the technology becomes smaller and lighter. The chief advantage to these systems is that they are able to quickly analyze a tone and give its position relative to the equi-tempered scale. Its chief disadvantage, however, is that because it is so quick and precise, a less mindful student could seek only to satisfy the needle, but not any musical consideration and not be developing the correct neurological link between the auditory perception and motor response.[276] Under study, electronic tuners have mixed results, with initial training progressing well, but plateauing over time.[277]

Aside from double basses, playing an instrument for maximal resonance as a method for learning to play in tune has been a staple of wind, brass, and string training. Pedagogically, this capitalizes on the ear's confusion of tone quality and intonation. The acoustic principle behind this for string players is that if the overtones of a fingered tone are matched with the harmonics of an open string, the open string will resonate, causing a net increase in the total decibel output of the system and sharpening the overtone response at that harmonic. From a psychoacoustic standpoint, this has a further effect (read: benefit) of sharpening the sensation of pitch, and generally improves the subjective rating of the tone (see also Chapter 2: Some Influences of Instrument Acoustics). If the goal is to align the overtones in the sound, then the tools available to teach and refine that

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[276] For a philosophical underpinning, see Gordon 1989, and for a more recent examination, see Morrison & Fyk 2002.

[277] c.f. Corey 1965, Salzberg 1980, and Smith 1995

practice are playing with an external drone, playing in doublestops, using an adjacent open string as a drone, checking a finger position against an open string or harmonic, and finally directly observing the sympathetic ringing of the instrument, either visually or aurally through the change in tone color.

In order to fully appreciate this approach to intonation, one has to link the ideas of resonance, tone color, and intonation in a hopefully virtuous cycle. Stubbins, Leuba, and Cousins, each wind players of some stripe, are fairly emphatic that the first step in developing good solo intonation is to play in tune with the natural resonances of one's instrument, which on a wind instrument means that the frequency imposed at the embouchure matches the natural frequency of the bore.[278] Each allows for refinement from there to fit the musical requirements of a specific situation, but the basis of intonation starts with encouraging the maximal resonance of the instrument. For string players, the technique is modified so as to allow the open strings to resonate, which allows for the greatest resonance of the instrument. Raab specifically correlated the presence of sympathetic vibration with the evaluation of a student being in tune,[279] Sariti advocated using the lack of beating in the resultant sound as an indicator of being in tune,[280] and Flesch was implicitly advocating this approach by encouraging the use of an open string drone and listening for a "locked in" sound.[281] Pondos even advocated keeping a finger in place to add that string's sympathetic vibration at the stopped length,

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[278] Stubbins 1965, Leuba 1984, and Cousins 1992
[279] Raab 1978
[280] Sariti 2009
[281] Flesch 1939, 23

a technique prevalent in viol pedagogy.[282] Ross noted that tone quality was significantly improved by the inclusions of sympathetic resonance,[283] Adey and Kanno cited players' tendency to choose finger placements in accordance with the instrument's resonance versus the absolute distance between tones,[284] and Benade noted the improved instrument response and quality of tone when played in accordance with its resonance characteristics.[285] In the bass realm, Lambert essentially says the same thing as his counterparts, noting "good tone is the foundation of good intonation,"[286] while Turetzky comes from the other direction blaming poor intonation mostly on poor tone.[287]

While most acousticians seem reticent to comment on how a sound's spectral characteristics are perceived, the psychoacousticians are fortunately less so. Much work has been done to more formally link descriptors such as "bright," "brilliant," or "resonant" with sharp intonation and descriptors such as "dark," "dull," or "covered" with flat intonation.[288] What can be noted in that same breath is that bright/brilliant/resonant/sharp tones are associated with spectra with numerous clearly defined overtones, particularly above partial 8 (higher centroid), while dark/dull/covered/flat tones have fewer or less well defined overtones (lower centroid). Often in the pedagogical and performance literature, there is a reported preference for bright tones which is also linked with resonant tones.[289]

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[282] Pondos 1981
[283] Ross 2003
[284] Adey 1998, 700; and Kanno 2003
[285] Benade 1976, 299-300
[286] Lambert 2006, 55
[287] Turetzky 1960
[288] See particularly Fyk 1995, 149 and 217; and also Benade 1976; Rasch & Plomp 1999, citing Terhardt 1971; Rayner 1975; Geringer, Madsen & Dunnigan 2001; and Howard 2007.
[289] e.g. Pondos 1981, Leuba 1984, and Sariti 2009; see also Chapter 2: Some Influences of Instrument

From the performance and pedagogical perspective, this approach is significant for several reasons, the first of which is that is offers auditory, visual, and tactile feedback.[290] So even if the ear is unsure of the placement of a tone, there is still an internal check of the intonation which conforms to intervallic and timbral expectations. A student will know a tone is not in tune because the tone quality is not as it should be. This also plays to the strength of the psychoacoustic associations of timbre and relative sharpness or flatness. Of course, if the tone acoustically agrees with the harmonic series of its neighbors, there will be a visual cue as well in the string ringing sympathetically, which is a technique in use at least since Mozart.[291] And if the tone quality isn't correct, there will most likely be a corresponding feeling in the hands of the tone not being correct, as interference patterns will be imposed on the strings being played, which as discussed above, are perceptible to a performer.

Closely related to the first point is that playing for sympathetic resonance provides inherent anchor points, both in pitch and tone.[292] Benade noted that it is far easier and more repeatable to practice for maximal resonance and tone quality, which has the benefit of mechanical feedback, than to practice spot-to-spot using only the blunt instrument of the training ear.[293] Most obviously, keeping a piece's intonation anchored to the sympathetic vibration of a string would prevent pitch drift. More subtly, encouraging a student to aim for the same resonance repeatedly would help further refine

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[290] Raab 1978

[291] Mozart 1951, 163

[292] Garam 1990, Ross 2003, and Irvine 2005

[293] Benade 1976, 300

the categorical bounds by using an external check to ensure the intervals are of the same size and establishing that interval size as the prototype for that interval category, assuming he or she is actively listening at all times!

Lastly, in performance, knowledge of this approach gives a performer not only security when needed, but also interpretative freedom. Whitecomb noted that at the very least, playing in this manner provides "good default locations," which could be subsequently modified to suit the expressive requirements at any given time.[294] Mozart noted that the key of a piece must be particularly noted so that it could be appropriately fitted to the instrument to have the proper affect;[295] similarly, as noted before, Barbieri & Mangsen related how a performer played a piece in F# major differently from the same piece notated in Gb major.[296] The interrelation of tone color and intonation can provide flexibility to satisfy both an emotional schema and an intervallic one which might otherwise be in conflict.

What we really do, really

There are two facets to how most of this plays out in practice: how we hear intonation (error detection) and how we perform intonation (detectable reproduction error). The difficulties for the former are that error detection is highly context-dependent, not only in the tonal sense, but also as to whether the auditor is in an "error detection" mode of listening, tracking intonation specifically, or the auditor is tracking the progress of the gestalt of the presentation, which results in a large range of results of intonation

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[294] Whitecomb 2010, 78
[295] Mozart 1951, 154
[296] Barbieri & Mangsen 1991

acceptability. The latter is best summed by an anecdote related by Duffin: George Bernard Shaw is reported to have preferred the intonation (or "temperament," as he put it) of Sarasate's playing over that of Joachim's in their respective solo performances.[297] Part of the subtext was not that either played out of tune, per se, but the supposition is that Sarasate, as a trained soloist, would have tended towards a Pythagorean-like intonation, and Shaw had a preference for this intonation in a solo context. Joachim, at heart a great quartet leader, would have practiced an intonation closer to just, which in the solo realm for Shaw seems to have been less satisfying but not unacceptable.

We've seen how much influence the hierarchical position of a tone can influence how strongly it is perceived relative to its ideal categorical position.[298] However, to take a step or two back and look more generally at intonation tolerance, Hall reports that when not in detection mode, intonation variation can be up to ±50 cents and that in professional performances, the reproduction error (from whatever idealized metric) is in the ±10-20 cents range.[299] Even octaves, perfect fifths and major thirds in isolation were deemed unacceptable only outside of a ±10-15 cent window. Burns reported a ±14-22 cent acceptability window for non-octave tones and ±10 cents for octaves.[300] Lynch, et al. found a ±10 cent tolerance in a tonal context with experienced musicians.[301] Sundberg found a slightly smaller tolerance at ±7 cents for performances by singers whose performances were judged to be in tune.[302] Meyer puts the tolerance width for bass

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[297] Duffin 2007, 124-125
[298] e.g. Rakowski 1990 and Burns 1999
[299] Hall 2002, 411; the assumption is, as with most analyses of professional performances, that the performance is judged to be "in tune" by the auditors.
[300] Burns 1999
[301] Lynch, et al. 1989, cited by Burns 1999, 233
[302] Sundberg 1999, 205

voices (as in persons who sing bass!) at ±9-14 cents, with a preference width of ±3-5 cents.[303] Though given all of the above, it is interesting to note that Fyk was able to report that in a performance context, musically trained and untrained subjects have similar intonation tolerances.[304]

While these tolerances may seem to be absurdly large for professional musicians, one must remember the large roles of context and informal masking. While listening to a performance and even allowing for an auditor's focused attention on intonation, we have many, many streams of information to process simultaneously and there simply isn't enough bandwidth to accurately measure each variable, what Burns refers to as information transfer limits.[305] Vurma & Ross confirmed this from the performers' side, revealing a standard deviation of 22 cents and showing that we indeed can't accurately hear what we are doing while we do it![306] Butler sees intonation as very "time-evaluation" dependent, in that one can only evaluate so many data points in a given time,[307] and Fyk determined that accurate intonation judgments occur at the (generically called) phrase level, not note-to-note evaluations.[308] The result is that when looking at note-to-note judgments in a musical context, error detection is much lower in a musical context than in isolation,[309] as a tone which lies within the categorical bounds may be far from the idealized center but may not be noticed. But, the tones with a higher hierarchical position retain their tighter tolerances and tend to remain stable throughout a

[303] Meyer 2009
[304] Fyk 1982
[305] Burns 1999
[306] Vurma & Ross 2006
[307] Butler 1992, 55
[308] Fyk 1995, 166
[309] Burns 1999

reference passage.[310] Particularly in a tonal context, the contextual identity and relative stability of a tone is much more important to the judgment of its intonation quality than the absolute size of the intervals formed by adjacent tones.[311]

Looking from our current vantage point, the importance of the stability of $\hat{1}$, $\hat{4}$, and $\hat{5}$ could hardly seem to be overemphasized. Their role in establishing the tonal schema in turn helps stabilize the framework by which intonation of a given passage will be judged. Shackford was able to show their stability in a performance context, and that the tonic of an excerpt is retained throughout the passage, even with significant modulatory intervention or a passages' ambiguous tonic orientation.[312] Fyk was able to concur with that conclusion with different melodic material.[313] Brown connected these findings with the establishment and retention of an internal tonic standard through analysis of the intonation patterns of recordings of artist-level violinists.[314] This, in turn, supported the pedagogical literature which holds the retention of an internal pitch standard as one of the keys of successful intonation.[315] The other two hierarchically important pitches, $\hat{4}$ and $\hat{5}$, are similarly shown to remain stable by Shackford,[316] Brown,[317] and Sundberg,[318] when they are functioning as IV and V of the home key.

[310] See particularly Shackford 1961 and 1962; Fyk 1995, 204; and Brown 1996.
[311] See Butler 1992 and Umenton 1990, cited by Burns 1999; c.f. Gabrielsson 1999.
[312] Shackford 1961 and 1962
[313] Fyk 1982 and 1995, 204
[314] Brown 1996
[315] e.g. Galamian 1995 and Grodner 2013, 22ff
[316] Shackford 1961 and 1962
[317] Brown 1996
[318] Sundberg 1999

Again, this supports the emphasis placed on these tones by the performance and pedagogical literature.[319]

The idea of a scale degree having a particular function has received a good bit of attention, most notably by Larson,[320] in the cognitive and theoretical world, but for intonation, this has the characteristic of establishing the tonic schema. As has been noted previously, one explanation of expressive intonation in linear playing is that it points out the stable pitches of the tonic schema,[321] to the point that the sense of a key center can be wholly modified by these variants.[322] This is reflected in the performance literature, most notably by Casals[323] and Eisenberg,[324] who both expressly used intonation to outline and emphasize tonic. However, this emphasizes that intonation is judged at a larger level than note-to-note and that the intonation of that larger unit is judged as a gestalt. At the level of the individual interval, there is a good deal of "wiggle room," depending on the context and function of that interval. Taking the M3 as an example, Shackford, Fyk, Gabrielsson, and Leukel & Stoffer all found distinct species of M3 which occurred depending on the melodic or harmonic function of the M3 at a particular instance.[325] M3's in the beginning of a passage or generated by expanding melodic motion were generally in the range of 400-408 cents, while concluding M3's or other harmonically stable M3's tended to be closer to the purely tuned 386 cent-variety. The emphasis in the performance practice of advanced intonation is not so much of the size of a given

[319] e.g. Blum 1977, interviewing Casals; Fyk 1995, citing Sachaltueva 1960; Conway 2003; and Duffin 2007, 69, citing Blum 1986 interviewing Steinhardt
[320] Larson 1994 and 1997
[321] e.g. Fyk 1995, 211
[322] Cuddy 1997
[323] Blum 1977, 103
[324] Applebaum 1972, 301
[325] Shackford 1961 and 1962, Fyk 1995, Gabrielsson 1999, and Leukel & Stoffer 2004

interval, but on the perception relative to an internal standard, the control to correct one's intonation relative to that standard, and the ability to deviate from that standard for expressive purposes.[326] In a performance situation, the preference is not for a specific interval reference prototype or schema, such as the equal temperament scale, but for the stability of the tonal schema and the repeatability of the reference pitches therein.

As definitive as that might sound, there are still a couple of loose ends to tie. While it's probably clear by now that I don't particularly subscribe to the dichotomy of linear versus chordal playing due to the terms' prevalence, it is a bit easier to reference them on occasion. To take an example, the expansion of the M3 and M7 in linear playing in particular can be explained as an expressive variant in order to point out the structural positions of $\hat{4}$ and $\hat{1}$, which has long been advocated in the pedagogical literature.[327] This has led some to try to assign violin intonation as being Pythagorean in nature, most notably Seashore and Loosen.[328] If we accept the priority of the stability of $\hat{1}$, $\hat{4}$ and $\hat{5}$ to the point of using intonation variants to accentuate their perception, then as Fyk points out, the resultant intonation is going to look very Pythagorean, even though performing to that theoretical standard was not the goal.[329] To take a parallel tack and looking more closely at the materials used in Loosen, he used a C major scale on a violin but did not report the absolute distance between each open string.[330] Given that violinists at least partly base their intonation choices on the resonances of the instrument and the tuning of an instrument has an effect on the resultant intonation, the tuning of the violins in the

[326] See Morrison & Fyk 2002.
[327] e.g. Auer 1921, Eisenberg 1966, and Applebaum 1986
[328] Seashore 1937a, 1937b and 1938, and Loosen 1993 and 1994
[329] Fyk 1995, 211
[330] Loosen 1993

experiment would be an interesting fact to know, because tuning in perfect fifths generally yields Pythagorean-esque results, and this would be particularly true for a C major scale! It may not be that violinists are particularly looking to play in Pythagorean tuning or equal temperament, but the resultant intonation may look very similar to those tunings under analysis for reasons quite external to playing in accordance to a theoretical model. When looking at violins tuned with tight fifths, the resultant intonation is closer to a meantone background temperament.[331] However, when taking the sum of intonation research, what is most clear is that it is a misnomer to say that a person performs anywhere near "in" a temperament, as in Gabrielsson reported only 11% of measured tones corresponding to any theoretical system.[332]

When in a tonal context, for linear playing which is nearly chordal such as in arpeggiated sections or sections over pedal points, the intonation strongly resembles the more purely tuned varieties of intervals seen in choral-type playing.[333] Here, we can see the influence of the trace model of hearing on the performance. Because the previous pitch is still strongly cognitively present (and probably physically present also, as will be discussed in Chapter 4), the sensory consonance between the two tones is still a prime factor in their placement. As with two tones physically juxtaposed, sensory consonance is dependent on the alignment of the upper partials of each tone. In addition to the direct coupled vibration of an instrument, this may additionally explain the phenomenon of fitting the key of the piece into the tuning of the instrument or the tuning of the

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[331] See particularly Barbieri & Mangsen 1991 and Duffin 2007.

[332] Gabrielsson 1999, citing Fyk 1994; see also Schneider 1994, Loosen 1993, 1994 and 1995, and Burns 1999.

[333] See Garam 1990 and Leukel & Stoffer 2004.

instrument to the key of the piece, as often happened with viols and period double basses.[334] The "after-ring" on the physical and cognitive sides may influence the choice of key placement on the instrument just as much as the direct sympathetically driven motion.

The tendency of chord-like passages towards just tuning also parallels the results of Cohen, which sought to test the effects of inharmonic partials on the tuning of simultaneous intervals.[335] The study suggests that while winds, brass, and vocalists respond categorically to mistuned partials until they become nearly harmonic, string players respond by almost exclusively tuning the upper partials and not categorically according to the perceived fundamental. It is also interesting to relate the above to a study by Rasch, which using two-part musical fragments, tested the interrelation of melodic intonation of the melody, melodic intonation of the bass, and the harmonic intonation between the two lines.[336] What was particularly revealing was that while the melody line was judged as most in tune when the intonation resembled an equal-tempered, Pythagorean, or expressive framework, the ratings of the bassline intonation were robust to changes in intonation ranging from just to Pythagorean tunings. When analyzed further, the primary factor for bassline intonation judgments was its harmonic agreement with the melody, and secondly, that the highest ratings for bassline melodic intonation corresponded to patterns which closely resembled intervals in just tuning to meantone temperament, though equal-tempered and Pythagorean variations were accepted. Putting this in the context of this chapter, the subjects of Rasch responded

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334 See Chapman 2003.
335 Cohen 1984
336 Rasch 1985

most highly to basslines with high levels of overtone agreement, both vertically and linearly, in contrast to very linearly tuned examples in the melody.[337] An inference suggested by this is that the more arpeggiated, less melodically driven basslines are generally more subject to partial overlap and a tendency towards natural intervals affecting the over-all intonation ratings.

337 Rasch 1985

Chapter 4: Detailed Problems of Double Bass Intonation and Some Solutions

"If we're going to be damned, let's be damned for what we really are."

-Capt. Jean-Luc Picard, *Star Trek: The Next Generation*

Such a long preamble is necessary to provide at least a cursory background on the issues that bassists confront when analyzing intonation and the resulting difficulties it poses for the instrument. As I mentioned, the impetus for this study was the relative lack of pedagogical information regarding intonation in the double bass literature. No small part of the problem is that the double bass has been playing catch-up to the rest of the string world: when the violins had a fully fleshed, impactful treatise, the basses barely had a pamphlet.[338] Even into the early 20th century, Torello, one of the grandfathers of double bass teaching in the United States, observed that the double bass methodology was in "chaos."[339] Part of this is understandable, since the bass in its modern form is a relative newcomer and was still struggling to find its place in the world even by 1800, when the "standard" of the modern double bass began to coalesce.

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338 c.f. Mozart 1951; Planyavsky 1998, citing Corrette 1781; Brun 2001, citing Corrette 1781
339 Martens 1923, 284

For most of its history, double bass pedagogy has focused on fingering patterns rather than developing a cohesive theory of intonation. Stanton,[340] among others, hypothesizes this is mainly because the traditional method books, e.g. Simandl and Bille,[341] were originally published barely a generation or two after frets were removed from the double bass and the teachers of that generation had a mindset of not really needing to worry about intonation as long as one was on the fret.[342] This has carried through to later method books such as Montag and even into the jazz realm with Berryman, being a jazz version of a Simandl-type method.[343] Even more recent developments, such as Rabbath and Karr,[344] while not totally abandoning a position-oriented approach, use fewer positions and anchor these positions by reference to harmonics along the string length, rather than by a half-step position progression up the string length. What has been written subsequently about implementing these methods has focused on correct motion leading to correct intonation,[345] which parallels the more traditional focus of a good hand frame leading to correct intonation.[346]

Only recently has any attention been paid specifically to double bass intonation. Goilav and Bradetitch both wrote relatively substantial sections on intonation.[347] Both are excellent starting points of an overview of how intonation functions practically and

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[340] Stanton 1965

[341] Simandl 1984 and Bille 1922

[342] Of course, even with over-wound strings, with gut frets there is a bit of play in intonation depending on exactly where the finger is placed relative to the fret.

[343] Montag 1955 and Berryman 1997

[344] Rabbath 1977 and Karr 1996

[345] e.g. Dennis 1981, Cross 1989, Aaron & Gannett 1994, Cameron 1996, and Heath 2008 interviewing Sturm

[346] e.g. Martens 1923 interviewing Maloney, Turetzky 1960, and Applebaum 1981, 194 interviewing Walter

[347] Goilav 2003 and Bradetitch 2009

offer traditional “quick and dirty” solutions to a broad range of intonation problems. Both books are written from the performance perspective, are broad in their scope, and need to offer a similarly broad overview of intonation in multiple contexts. While not in the scope of their respective works, that neither develop a “first principles” theory of intonation is still a fair observation, and that theory is still lacking in the double bass literature.

Acoustic, psychoacoustic, and cognitive problems

Bass players spend their lives in the low frequencies, and the brain, being the wonderfully plastic organ that it is, is thought to be able to be trained to hear those frequencies better. This may be why bass players are able to interpret tones which others have legitimate trouble processing. The question then becomes if that ability becomes a handicap when other instrument types are listening to a double bass performance. While bassists may be listening for pitch and interval information as low as possible, others might be listening for pitch information in frequency bands much closer to their nominal performance range. So, a violinist may only be processing information above 440 Hz for pitch information, whereas a bassist may not look any higher than 880 Hz to find pitch information. That only leaves one octave of overlap where both instruments are using the same information for pitch determination. Because no acoustic instrument is precisely harmonic, this may lead to differing perceptions of what the pitch actually is, because of two different conceptions of the pitch, each equally valid within the appropriate overtone band. Also, a bassist might ignore higher-band overtone interference which might trouble treble instrumentalists.

The real problems begin at the intersection of acoustics, psychoacoustics, and pitch salience. By the numbers, the “bread and butter” performance range of the bass, E_1 to A_3 (41-220 Hz), is below the range of best pitch recognition, but a more detailed look into why is in order. Two factors which particularly rob the double bass of its pitch recognition is induced roughness of the double bass sound and the number of cycles needed to extract a pitch. The double bass responds with an essentially harmonic spectrum and a power declination of 6dB per octave across that spectrum, which is a similar response to other members of the violin and viol families. The challenge basses face is that because their fundamental tones are so low, their audience is at the outer edge of physiologically and psychologically being able to process the tone information and extract usable timbre and pitch information. Recalling our earlier discussion on pitch extraction,[348] the fundamental is almost never used to determine pitch, but the overtones are. As the frequency of a tone is reduced, the lowest harmonic used in determining the pitch of that tone actually *increases* as the octave decreases, meaning that the frequency range used for pitch extraction remains relatively stable as the fundamental changes. So, while on a violin, partials at or above 2 or 3 may be the highest used to determine pitch, by the time the basses play the same note, only partials 4 or 5 and above may be available for use in pitch extraction for the general population of expert listeners. So even at the slighter difference between A_3 (220 Hz) played on the cello and A_2 (110 Hz) played on double bass, an auditor would have significantly more information available to extract a pitch from the cello tone than the bass tone.

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[348] See Chapter 2: Notions of Pitch.

When looking at spectral plots of string instruments, this represents a significant loss of raw data with which a listener of the double bass is able to work, not only in numbers of partials available to draw upon but also the quality, as instruments tend to get "noisier" in the higher part of their spectrum.[349] At this low range, the critical band and a psychoacoustic detail known as the phon[+] scale become important as well. The vernacular "loudness" of a sound is often measured by sound pressure level in decibel (dB) units, which is a logarithmic transformation of the measurement of sound pressure, which for most purposes, mimics the perceived loudness and loudness changes of sounds. However, the dB scale does not take into account the perceived loudness of a sound as it varies over the frequency spectrum. This phenomenon is mapped by the phon scale, also known as an equal-loudness curve, which is a more precise unit of measurement of perceived loudness, and is covariant with frequency and sound pressure levels (see Fig. 4). The ear tends to be most sensitive around the center of the region of best pitch recognition and falls off towards the ends of the spectrum. For instance, if a trombone and trumpet were played at the same dB level, the trumpet would be perceived as louder because its frequency range is in a more sensitive part of the hearing range, and thus would be higher on the phon scale.

[349] c.f. Kohut, Matthews, & Miller 1973, Vos, 1986, and Askenfelt 2010, 264

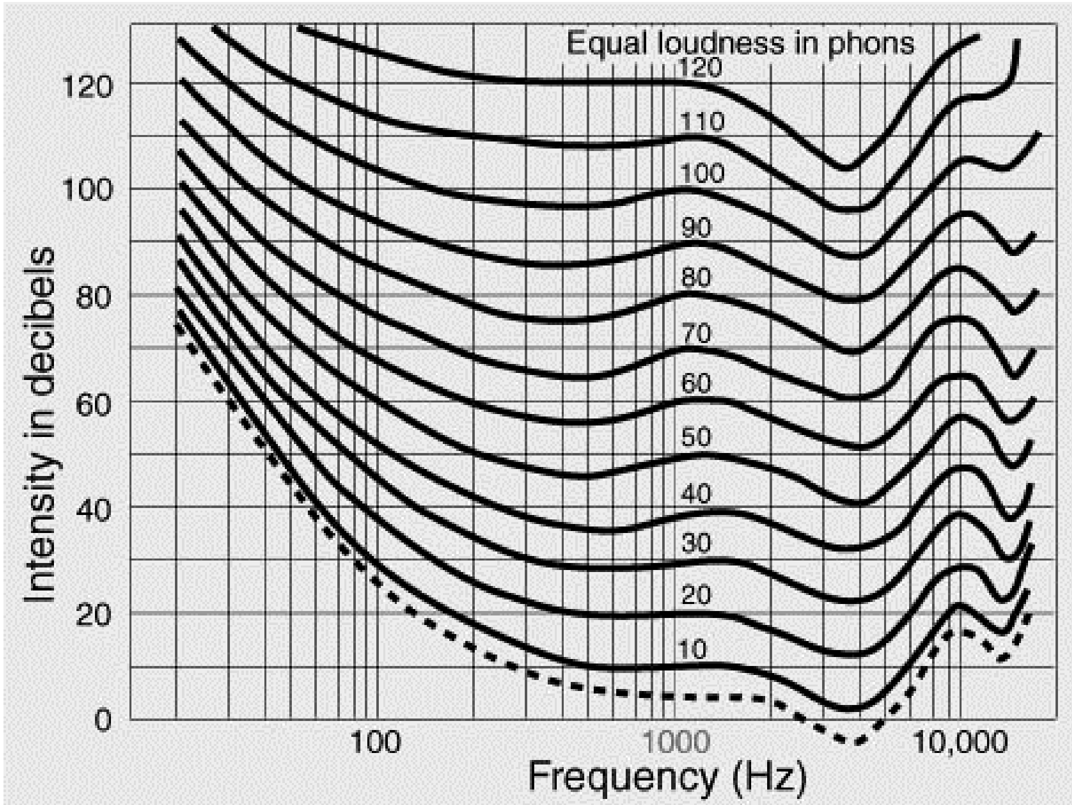

Fig. 4: Phon scale chart of sine tones

As the critical band width is not strongly frequency dependent and the double bass's fundamental frequencies start at such a low level, an interesting intersection occurs with the phon scale. For an idealized double bass tone, the region at which the overtones begin to be unresolved in separate critical bands approaches or is in the same region as the most sensitive part of the phon scale and remains within the place and temporal locking region of the basilar membrane. For example, for a bass playing a tone at 110 Hz (A_2), the partials will begin to be unresolved at about the 8^{th} partial (880 Hz), which is directly in the path of the dip of the phon curve. So, even though the power of the overtone series is declining at 6 dB per octave, the ear is becoming more perceptually sensitive to the frequencies of the higher parts of the overtone series as those same parts of the overtones series of the double bass begin to psychoacoustically roughen. To compare a violin tone at 440 Hz, the violin partials begin to be unresolved at about 3520 Hz, and while that is about the region of maximum sensitivity on the phon curve, power level of the violin spectrum is very low at this point and the phon curve quickly falls off, leaving the rest of the partials declining in pressure level and more quickly in perceived loudness. Thus, the portion of the tone which would cause the perceptual roughness will be much less present, leaving the pitch-defining section of the tone very clear in the perception. One theory of why some bows[350] and some instruments[351] sound better than others is that each acts as a comb filter, suppressing alternate harmonics in the spectrum. By suppressing adjacent partials within the same critical band, the sound spectrum becomes less noisy, and the perceived quality of the sound increases.

One of the acoustic distinguishing differences between the violin and viol families is that the violins have an added region of resonance called the bridge hill, corresponding to the interaction of the body resonance and the resonance frequency of the bridge.[352] This is believed to add the brilliance, power, and to some extent clarity to the violins which the viols lack. The viols tend to have a resonance profile much closer to the ideal 6 dB/octave power declination. In comparing a high quality violin and cello, both have bridge hill resonances which equal or surpass the main body resonance.[353] However, even high quality double basses have much less distinct bridge hills, which are not nearly as present as those of the violins. Lower quality basses, such as student models, have

[350] Matthews, et al. 1965
[351] Askenfelt 2010
[352] Askenfelt 2010 and Campbell & Campbell 2010; see also Chapter 2: Some Influences of Instrument Acoustics.
[353] Curtin & Rossing 2010

hardly any bridge hill to speak of whatsoever.[354] In this way, the double bass is very similar to its historical heritage, but while the gambas tend to be of a high enough tessitura to prevent muddiness, the lack of a distinct bridge hill in most double basses leads to a darker and more diffuse sound than the rest of the violin family. Part of the gamba technique is to hold the instrument and strings in such a way as to encourage the maximum resonance. When the viol's strings (either open or closed) are allowed to ring sympathetically, added high-end resonance is allowed to evolve in the sound profile, adding clarity to and brightening the sound.[355]

When two partials interact in the same critical band, or close to the same critical band, masking of one or the other partial may occur. This will occur for components of a complex sound as well as for individual sine tones.[356] If the louder component is 6-8 dB above the less loud component, it will mask the less loud component.[357] The absolute loudness of a component can also affect the area of the basilar membrane it masks, with louder components masking a larger area of the membrane. In the case of two components in adjacent critical bands, if the louder component is of a lower frequency than the less loud component, upward masking may occur if the lower component is sufficiently loud.[358] Downward masking tends not to occur unless the component is very loud. Masking has been observed to interfere with the perception of violin tones in performance contexts.[359] If sympathetic resonance were properly chosen and

[354] See particularly Askenfelt 2010.
[355] c.f. Tan, Pfordresher, & Harre 2010, 40
[356] Howard & Angus 2001, 233
[357] Hall 2002, 400
[358] See also Pierce 1983, 130-135.
[359] Fyk 1995, 153

encouraged, it could act effectively in a similar manner to the comb filter action of the bow by masking nearby components, clarifying the sound not only by sharpening the instrument response, but by psychoacoustically masking noisy upper partials. Beating in the sound, however, has the effect of decreasing any masking effect.[360]

An acoustic factor related to the size of the double bass is the decay time of a resonance after power has ceased being put into the instrument. While the decay time of a violin sound is on the order of .5 second at the fundamental,[361] the double bass resonates for upwards of 10 seconds for an open string and 3 seconds for a closed tone.[362] This makes the decaying resonance even more of a factor in double bass performance than the violin; in practice, we are never playing one note at a time! Overlapping resonances of adjacent tones will cause interference and psychoacoustic roughness, often above the threshold of perception. Tones in the same overtone series will dampen less quickly as they share power in the same specific overtone frequencies. However, if these overtones aren't matched, they will beat, and because they are close in frequency, the overtones will not be damped quickly. This will cause roughness in the over-all sound, leading to bad tone quality, which was observed experimentally in the violin by Fyk.[363] Again, because the effects of the ear's sensitivity versus frequency (i.e., the phon curve), this affects the double bass more than the other string instruments, as the lower overtones are disproportionally psychologically represented in the sound. This is one reason

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[360] Pierce 1983, 133
[361] Benade 1976, 510
[362] Meyer 2009, 102
[363] Fyk 1995, 153

gambas in particular matched the tuning of the instrument to the key of the piece they were playing.[364]

That timbre and pitch are separate phenomenon was shown in Robinson & Patterson's study which demonstrated that timbre was able to be identified within one to two fundamental cycles, while pitch was identified in 10 to 15 fundamental cycles, which did not tend to vary across the frequency range.[365] While the violin's pitch is identified in about .04 second, the double bass's pitch can only be identified after about .1 to .2 second, which is equivalent to a sixteenth-note at mm=144. The double bass's tone is identified in about .01 second. Clearly, at any given time in a playing condition, there is far more timbre information than pitch information for the double bass! In a similar experiment using sine tones, Butler found that the pitch of a tone at 100 Hz is identified at 45 msec, while a tone at 500 Hz is identified at 26 msec.[366] To put these timings in context, the initial attack transient decay time for a violin is on the order of 60 msec, while the double bass's transient decay time is on the order of 120 msec.

Basslines have a peculiar function in music, in that at their core, they carry not only melodic elements, but are primarily responsible for generating or supporting the harmonic underpinning more explicitly than any other voice. Because basslines are centered on articulating harmony, the tonal schema will be highly activated which both creates the expectation of more purely tuned intervals and tightens the tolerance on

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[364] See Chapman 2003 and Lindley 1984.
[365] Robinson & Patterson 1995
[366] Butler 1992, 84

interval size.[367] However, because the intervals have melodic function at least in that they are sequentially presented and form a line if not an actual melody, there still can be an expectation of directional alteration, i.e. an expressive variant of the interval size, which may run counter to the harmonic expectations implied by the line, bringing the tension between melodic and harmonic intonation expectations to the fore in one musical unit. Some in an audience may look for melodic direction while others may look for tonal stability. More broadly, while there is some notion that each instrument has intonation which is expected of it,[368] each listener will still have an expectation of a segment of music that is informed by that listener's background. However, because the differentiation of linear and harmonic ideas is a learned process, an approach using bottom-up consonance limiting psychoacoustic noise may yield a more consistent solution.

Being part of the solution

At this point, I could hardly blame anyone if they felt playing the bass in tune was a hopeless venture, labeling it impossible for all of the reasons above. However, experience with artist-level bassists tells us that it is indeed possible, though some may still quibble! I think most artist-level players explicitly or intuitively play in tune with the overtone series and sympathetic resonances of the open strings, a supposition concurred with by a principal bassist of a major United States symphony.[369] They may couch it in terms or phrases like "ring," "resonance," "brightness," "clarity," "transparency," or "the bass liking it when the pitch is in tune," all of which imply an

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[367] c.f. Rakowski 1990 and Butler 1992, 115
[368] c.f. Stubbins 1965, Leuba 1984, and Garam 1990
[369] Personal communication, J. Turner, March 2011.

absence of interference patterns in the waveform of the instrument. This is a cue given to us by almost every other instrument, which the bass community as a whole has not discussed openly. However, for a number of reasons to be detailed, I think that playing the bass so that it is in tune with the overtone series of the open strings is a solution to the problem of teaching and performing on the double bass which needs to be discussed as a means of speeding the teaching and improving the performance of the double bass. Primarily, practicing to repeat the resonance of a tone returns the most consistent, repeatable tones and can form the basis from which to help define a student's intervallic categories, and for a basic intonation structure upon which a more refined sense and execution of intonation can be built.

Real basses have sound spectra which are somewhat removed from the nice tidy 6 dB/octave declination of the theoretical string instrument. It can be quite messy in fact, with partials looking like hills in the spectrograph instead of tidy spikes, but still exhibit a declination roughly correlated to the ideal.[370] The "spikes" at each overtone can vary in bandwidth, which has the sonic consequence of variation in the clarity or tightness of the sound. A bass with wide overtone spreads will sound fuzzy or raspy, while a bass with narrow-banded overtones will sound clear and defined. One advantage of high quality basses is that they have relatively sharply defined overtone series. Another is that a high-quality bass and bow combination acts as a comb filter, suppressing or encouraging at various intervals, which is probably a result of the bridge hill and has several effects.[371] It helps sharpen the overtone series and cuts out close overtones. It also has the

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[370] See Tan, Pfordresher, & Harre 2010.
[371] See particularly Askenfelt 2010.

psychoacoustic effect of clarifying the sound, as it reduces the number of overtones per critical band, thereby reducing psychoacoustic roughness and increasing pitch resolution downstream. Additionally, because the bandwidth of each partial is relatively narrow, the range at which it will resonate is comparably narrow. This means that an artist training for maximal resonance must repeatedly match frequency very precisely.

Unfortunately, students do not usually have basses with these advantages, so the question is how to mimic this effect on their instruments. In principle, playing with partials tuned to those of the open strings will have a similar effect. When looked at graphically, the power of a partial is spread across its width and height (frequency range and dB output, respectively). So, a partial with a wide frequency spread, but low dB height would have the same total power as a narrow partial with a high dB output. A short-hand approximation of power output of a partial would be to describe this as the height-to-width ratio. The narrow partial (higher ratio) would be perceived as clearer and louder than the more defuse wider partial (lower ratio). If we assume that sympathetic vibrations are essentially additive, then the effect of adding two relatively widely spread partials would be to significantly increase the height of the partial without increasing its width, which would increase the ratio and begin to make it look more like the spike model. This is essentially what Gough's model and observations show.[372] Over the entire spectrum range, this would not actively suppress unwanted resonances as on a finer instrument, but it would increase the "signal-to-noise" ratio, putting far more output into aligned partials, which would draw more attention and lessen the impact of the roughness caused by the remnant partials, or would in fact mask unwanted partials.

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[372] Gough 1981

At both ends of the bass quality scale, the effect is the perception of better tone quality. Because of increased partial definition, the tone will be clearer, have a better pitch definition, and have better resonance. I'm not claiming that a student model instrument will sound like a fine, artist-quality Italian instrument, but a bass played in this manner will at least reach its full timbral potential. Conversely, if the overtones are not aligned, the effect will be a perception of a less desirable tone quality, either by direct beating and roughness of the sound or a dull, non-resonant sound. In the first instance, if a stimulus partial is within the percent bandwidth of an open string partial to vibrate it sympathetically but not closely coinciding with the partial, acoustic beating will occur within the sound and more than one partial will be active per critical band. The most extreme example of this is a wolf tone.[373] Both are deleterious to tone ratings, and occur more in the mid- to upper overtone ranges, which happen to fall in the low- to mid-listening ranges of most auditors. The range which would cause the most grief to most listeners is the range tending to be most ignored by bassists! If however there is enough frequency spread between the stimulus partial and the open partial so as to not activate the open partial at all, the open string will instead act to dampen the resonance of the instrument as a whole, the full mass and pressure of the dead string acting upon the top plate of the instrument.[374] At best, this yields a muffled, dull tone, otherwise known as a flat tone, both in timbre and often in pitch.

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[373] See Chapter 2: Some Influences of Instrument Acoustics, note 180.

[374] Gough 1981

The interplay of overtone series and resonance is an important consideration because, by pedagogy, empiric study, and anecdote, correct placement of pitch is often mistaken for, or interchangeable with, excellent tone quality. What would otherwise be borderline errors in pitch, when viewed solely by the tonal schema, are mollified or overridden by an increase in the tone quality judgment. Conversely, theoretically correct frequencies tend to be judged as less well in tune if the tone quality is rated as poor. All of that said, gradations of placement and acoustical purity can also be part of the artistic pallet, recalling the differentiations between a piece in F# major and Gb major.

When we begin to consider the interaction of performance practice, acoustics, and real basslines, I tend to take the approach that composers knew what they were doing, wrote to take advantage of the prevailing temperament, and wrote to the best advantage of the instruments at their disposal. For instance, Mozart's Symphony 35 in D major would have been sparklingly resonant on a Viennese violone, which was nominally tuned F'A'DF#A, but probably would have employed a lower F# to comply with the prevailing extended meantone temperament ideas and the D major harmonic series. However, as we'll explore in the next chapter, the bass lines are constructed in a way that this is not deleterious to the linear aspects of the bass part. Even while pushing towards equal temperament, in Schubert's Symphony 9 in C, third movement at rehearsal B, we still see evidence of thought put into key color, and a resonance system of intonation can still help anchor the intonation and contribute to the evolution of the soundscape. Even as equal temperament has taken hold and composers pushed to the edge of chromatic tonality, the resonance system still is useful in anchoring the performance intonation of such excerpts as Strauss's *Don Juan*, rehearsal F, a passage notoriously difficult to play in tune due to its chromatic design, and the first bars of Bartok's *Concerto for Orchestra*, which shows the resilience of the system even in non-tonal contexts, without violating the directionality of the bassline, the "harmonic" implications, or temperament expectations. I would even argue that the solo works of composers such as Koussevitzky, working in the heyday of expressive intonation, wrote their pieces to take advantage of the resonance and expressive implications of tones in certain positions. The confluence of performance practice, tonal expectation, and the resonance system should imply lowered harmonies, not violating a linear bassline, fulfilling temperament expectations, optimizing tone quality, and increasing repeatability of pitches.

Chapter 5: The System and Its Application

"Measure it to the micrometer, mark it with chalk, and cut it with an ax."

-Col. James D. Hooppaw, USAF (ret.), *Where the BUFF Fellows Roamed*

An acoustic solution: the illusion of good intonation

As everyone may have gathered, the goal is not to play "in tune," which hopefully will have become a bit of a defunct concept by now, but to make an audience *think* one is playing in tune. Even if the exact frequency ratios are rather approximate versus the theoretical values of any given system, the tones which have the best tone quality will most likely be preferred over the "correct" tone. The trick is to get this phenomenon to work in conjunction with the tonal schema, instead of against it. The approach I've taken is to fit each tone to the overtone series of one or more of the strings of the bass, which will take maximum advantage of the bass's resonance and closely follow the tonal schema. Again, I'm not arguing that this is THE answer of how to play in tune, but offer it as a root intonation, providing basic, consistent reference points from which artistic choices can be made. I would also iterate that while I may reference notes by their fundamental frequency, I am NOT advocating frequency counting as a means of playing "in tune;" it is simply an artifact of easy reference by the written word. The purpose here is to fill in the gap left by authors who said "...but the ear should be the guide" by illustrating a method for approaching what a player should be listening for with specific tools to do so.

Objectives for the system

This systematic approach to double bass intonation, to which I refer as the resonance system of intonation for readability, was developed out of my trying to interpret and apply intonation pedagogy of other instruments to the double bass. By using the overtone series of stopped tones tuned to the overtones of the open strings, the aural placement of the chromatic collection of tones used on the bass can be generated (predicted) as a system of intonation, analogous to how extended meantone intonation is systemized, but these placements are mediated by acoustic, stylistic, and melodic considerations. While I will reference specific positions of tones and interval sizes, I will reiterate that this is an artifact of the written word and that the system is designed around using the ear to tune the overtones of the stopped tone to the overtones of the open strings. The focus on using the ear to tune the overtones is also due to the variations of real instruments. Depending on bow placement or variations in how the string is depressed, the same position on the fingerboard can yield two different tones; however, if the focus is on the aural sensation of overtone alignment, the system becomes repeatable.

Of primary consideration in developing the system was to encourage the maximal resonance of the instrument by aligning the overtones of fingered tones with those of remaining open strings. This would tend to encourage the maximum pitch salience, maximum tone quality, and maximum repeatability. The resultant intonation would generally be in accordance with interval sizes more towards natural tunings, which are advantageous in bassline tuning. Further, the resultant intonation will reduce psychoacoustic roughness and beating, and reduce adverse waveforms imposed on the strings, leading to a smooth feel under the bow. In the long-term, by consistently returning to the same tonic and structural intervals, the ear training of the student can be further enhanced.

Developing the system: tuning

For so simple a thing as tuning the open strings, there are a multitude of opinions: by harmonics, by open strings, perfect fourths, tight fourths, wide fourths, with a piano, with a tuner, with a drone, etc. In approaching the subject, I will apply a universal musical caveat: almost anything can be appropriate in a given context. In the solo bass context, my approach is to use perfect fourths as given by matching the unison harmonics, so that, for example, the D harmonics of the G- and D-strings will be beatlessly tuned to each other, resulting in a theoretically purely tuned fourth between the G- and D-strings. The primary consideration in this choice is that it keeps the harmonic series generated by sequences of fourths and fifths in the greatest amount of agreement. Thus, the bass is set up for the open strings to resonate sympathetically to each other by the third and fourth partial pairs of the adjacent strings. Beating will be readily apparent here, or the open strings will generate their greatest resonance if the harmonics are tuned. This approach also provides the most consistency of finger placement across the strings, without, for example, having to choose between tuning an A on the G-string with the overtone series of the D- or A-strings and having it be a different pitch from an A played on the D-string. Perfect fourths tuning also has other advantages when tuning other tones to the open strings, which will become apparent as the system develops.

I have little trouble with an equi-tempered or wide fourths tuning, especially in an ensemble context, as this matches the rest of the strings which tend to tune with “tight” fifths. In practicality, because the bass is a real system with real strings under tension, the overtone series will be stretched slightly, which result in slightly wide fourths even when tuning by harmonics. The irony is that wide fourths produces a resultant intonation of a rough meantone system, as shown by Duffin, if a static model of intonation is applied.[375] The tight fifths, especially in quartet literature, help match the E partial of the viola and cello C-strings to the open E-string of the violin. Using normal finger patterns, sequential thirds tend to be lowered towards pure tuning by the nature of the lowered fifths, which are perceived as better in tune in an ensemble context. This tuning also tends to pull down thirds when tuned sympathetically.

The only tuning which will get a player into trouble is the tight fourths, which seems to result from a player wanting to tune in a similar manner to the violins, but not thinking the matter through! When looked at as complimentary intervals, if one tightens the fifth, the fourth must be expanded to maintain a stable octave. If one tightens the fourth, it expands the fifth, which I have never read as being advocated by the violin community! The resultant intonation would be beyond Pythagorean in its exaggeration of interval sizes such as the major third and perfect fifth, both of which suffer severely when stretched beyond that boundary. Blackwood’s calculations of closed, recognizable

[375] Duffin 2007

temperaments also broke down after the expansion of the perfect fifth much beyond pure.[376] The studies of Vos also showed that the acceptability of fifths stretched beyond purely tuned fell off dramatically faster than when the fifth is tightened.[377] The resultant sympathetic intonation will similarly follow suite and expand beyond the categorical bounds established.

Developing the system: setting unisons, octaves, and fifths

To put it simply, any octave of E, A, D, and G must sympathetically vibrate their associated open strings: E_1, A_1, D_2, or G_2, respectively. These are the only "fixed" sets of pitches, as the open strings are the only fixed pitches on the instrument. There is the maximum partial overlap with the strongest of the partials, offering the greatest chance for resonance and conversely offer the greatest chance to deaden the sound if not aligned. Similarly, the pitch classes a fifth above the open strings (B_1 in particular and the pitch classes E and A in general) should have an octave partial tuned to the third, sixth, twelfth, etc. partial of the corresponding open string. So, the fundamental of A_3 will match up directly with the third partial of the open string, D_2, and will also activate the harmonic series of the open A_1 string. The second partial of B_1 should align with the third of the open E-string.

The fifths below the open strings can also be set in this manner, though much of the work is taken care of by the open strings. If tuned in perfect fourths and depending on placement and octave, a closed G, such as G_1, could vibrate both the open D-string via

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376 Blackwood 1985
377 Vos 1982, 1984, and 1986

the G's third partial (pitch class D and its octave equivalents) and the open G_2 string at once. The main new beneficiary is C_2, which can have its third partial matched with the second of G_2, and the other octaves of C can then follow suit. Conveniently, the finger placement for C and G can be carried horizontally across the bass with no or minimal vertical movement, setting the position of F and Bb.

B_2 and the pitch class members octaves above would seem to benefit from the same sympathetic resonance as B_1 enjoys with the E-string. However, because of its proximal relation to the G- and D-strings in physical distance and harmonic series and the distant to the one in distance with the E-string, this does not appear to be so on most basses I have observed. While I will describe this in more detail below, it seemed appropriate to mention now, given the fifth relation of the B/E pitch classes.

Developing the system: filling in the half-position tones with thirds and fifths

When deriving the positions of F_1, Bb_1, and Eb_2, the fifth partial of each will match the fourth of the A-, D-, and G-strings, respectively. While the sympathetic vibration encouraged here is high in the spectrum, resulting in the perception of a relatively high ring, it seems to be quite effective on most instruments. Mathematically speaking, perfect octaves of F_2 and Bb_2 will be in the acoustically pure fourth series above the C_2 derived earlier, as well as all pitch class pairs F/C, Bb/F, Eb/Bb enjoying acoustically pure fifth relations.

G$\#_2$ in half position is harder to neatly derive, as F, Bb, and Eb are, because it does not enjoy a harmonically and physically proximal distance to a string sharing an overtone. The E-string does have an overtone approximating a noted G#, which would be consistent with the approach I am taking with this system of intonation. However, two problems executing this approach exist: lack of actual added resonance achieved and mismatch of placement relative to the other finger-placements in half position.

Because of the asymmetrical internal construction of the bass, when the bass-side of the bridge is activated, the more vertical motion of the bass side is converted to side-to-side motion on the treble side of the bridge, as the top "swings" around the soundpost.[378] The leverage afforded by the bridge very effectively transfers the energy of the activated bass side to the treble side. However, the reverse is not true, as energy input into the treble side is working directly against the soundpost and not encouraging the swinging action to as great an extent, radiating energy only locally.[379] Thus, G$\#_2$ is not able to effectively able to convert its second partial's energy into sympathetic vibration of the fifth partial of the E-string. Much of this depends on where exactly the G-string groove is cut relative to axes of the treble side leg and the waist. As G$\#_2$ played on the D-string seems perfectly capable of resonating the fifth partial of the E-string on most basses, it is conceivable that a G-string cut sufficiently inside the treble leg and outside the waist to provide enough leverage to activate the E-string.

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378 See particularly Benade 1976, 529.

379 Here again the phon curve comes into play: the treble side of the bridge does not need to encourage as much vertical movement in the top, which translates into sound pressure output. Because the treble tones are of higher frequency, the ear is more sensitive to them and less sound pressure is needed for a higher tone to be perceived as equally loud to a lower tone.

To tune the partials of F, Bb, and Eb to their respective open strings, the finger will in most cases be set higher than the mathematical mean dividing the whole-tone between the open string and the step above. To tune G# to the E-string would require the finger be placed below the mean. While this is not a large difference in practice, it might vex some who wish to carry "perfect fourths" directly across the fingerboard. A mathematically and pedagogically simple approach would be to simply use an "Ab," the tone a perfect fourth above the Eb, which would carry the finger straight across the strings with no vertical movement. This may not be the perfect solution for every musical case, but it retains the consistency established by the other three tones in half position. Also, there is no "good" resonance solution to this particular division. Lastly, if one is practicing in a passage which $G\#_2$ is necessarily on the G-string, the surrounding musical context could provide the answer, whether a preceding E_2 suggests a lowered G# for more harmonic consonance or a running motion suggests the need for a raised G# to point towards A_2 melodically.

Developing the system: filling in first position with the switch hitters

B_2, $F\#_2$, $C\#_2$, and $G\#_1$are surprisingly hard to place: B_2 has multiple acceptable positions from a resonance point of view, $F\#_2$ and $C\#_2$ have limited resonance options, and $G\#_1$ has almost none.

B_2 has a theoretically strong resonance of its fundamental with the third partial of the E-string, B_2, and the octave B's above; however in practice, while fairly strong, it is not as strong as might be expected, probably for the reasons outlined above regarding the

$G\#_2$ on the G-string. The second option is to tune the third partial of B_2 to the fifth of the D-string, $F\#_4$. Both options are of about equal resonance in practice, but a peculiarity of bandwidth comes into play as well. Because the higher power of the lower partials, they tend to have greater resonating bandwidth. Even when B_2 is in the lower position, and depending on the instrument, it can sympathetically resonate the B partials of the E-string, which are higher than the partials of the fingered B_2, and the partials of the D- and G- strings. In practice, this tends to yield much less roughness than might be expected due to the tendency for a sympathetically driven string to respond at the driver's frequency within a certain range,[380] and any stopping of B_2 between the lower and higher positions would seem to be acceptable.

For the first time in this approach to intonation, we have two equally and independently valid approaches to a note, which by the wonderful math of acoustics are a comma (22 cents) apart. I can't advise about picking one over the other, as each is valid in its own context and each has its uses. The lower B_2 is an acoustically pure major third against G_2 and a pure major sixth against D_2, which is very useful for harmonic passages with stability built around these tones. However, this is theoretically low in pitch, even by the general model of acceptable intonation, for a perfect fifth against E_2, which would be best served by the higher position of B_2, an acoustically pure fifth, and will also incidentally create a perfect octave against B_1. The higher position also yields a Pythagorean third against G_2, which is useful melodically.

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[380] See Gough 1981.

I think the solution lies in what context an interval is found and in what the system has offered thus far about listening to the harmonic series above the fundamental. I don't think the question is truly about the high/low position of B_2, but rather given the two options, which position will sound in tune in a particular context. To illustrate, as we've discussed, E_2 is positioned in a relatively fixed position versus A_1 and E_1. When the perfect fifth is formed by E_2 and B_2, the performer must choose between the two positions of B_2 using the criteria of the size requirement given the interval's function in context and the requirements for acoustic purity of the interval itself, since both tones will resonate well independently. The most clear, or restrictive, example is where the fifth forms the structural fifth of E major/minor, requiring both a strong (read: stable) categorical assignment and great acoustic purity. This context requires the higher position of B_2.

Moving towards the ambiguous, if we have the same fifth in G major, the question becomes how the B is functioning in the context: is it $\hat{3}$ of G or the fifth of vi? Even if acting as the fifth of vi, is G more acoustically and/or psychologically active than E? Because the E minor chord is relatively distant from the G major tonal center in terms of chordspace,[381] less categorical precision is required for the fifth between E and B. Further, the fifth between E and B may be totally incidental: the B could belong harmonically to I in G major and the E could belong to IV. With the categorical latitude afforded by its weak functional position, the resonance requirements of the surrounding harmony would dominate. If G was active as the local as well as broader tonal center and

[381] See Lerdahl 1988.

was physically active on the instrument, then the lower position for B might be appropriate, as it agrees acoustically and affords the best overall tone, which would override a momentary acoustic (but not psychological!) clash with a following E. Conversely, even if G was the tone center, but E minor was active locally, the B may need to be in the higher position to acoustically agree with the E. The principle carried through here is that the performer should look towards the agreement (or even disagreement, as aesthetics dictate) of the overtones to determine the position of tones among several options.

In order to have any resonance in first position, $F\#_2$ and $C\#_2$ are basically limited to matching their third partials with the fifth partials $C\#_4$ of the A_2 and $G\#_3$ of the E_1, respectively. These are strong and resonant partials, especially as there are no other competing partials. This position is an acoustically pure fourth relation to the lower B_2 position. However, the position congruous with the higher B_2 is also useful, depending on the harmonic or melodic context.

$G\#_1$ represents one of the greatest difficulties, in that it has no strong natural resonance with any of the remaining open strings, and this is partly why G# typically has such a poor tone on the bass. I think the best advice is to let the fingers follow the perfect fourths down to the E-string, matching the B, F#, and C# above. Often the G#, acting as Ab, will be in the higher position to match the more timbrally stable C and Eb.

By hearing how acoustically pure intervals can resonate, musicians can choose the size of a fingered interval. Often, in sequentially presented tones, the improvement in tone by tuning the notes to the open strings as well as to each other will "cover" for a theoretically poorly executed interval. However, if for example the shortened fifth between the B and the F# is bothersome, there is nothing here which prevents a musician from moving the F# or the B, particularly in the case of a doublestop. Sometimes the resonances of a particular series of tones could override position of tones by tuning with the open strings.

In specifically addressing the "short" intervals, one benefit of the bright ring when the upper tone is played sympathetically is that it sounds sharper, with the benefit of splitting the perceptual difference between the acoustically pure and melodically satisfying Pythagorean sizes. The behavior of real strings also helps in this area. As we are dealing with a real system with real strings under tension, the harmonics of the open strings and fingered tones are going to be stretched slightly. So, even when the overtones are aligned, the fundamentals are not going to conform to theoretical values, and will tend towards meantone values. Again, the concept is not to match a specific interval size, but a sound concept.

Developing the system: notes of a higher position

When venturing out of first position, some things change while many remain the same. Any good rule needs an exception, and for the inviolacy of the purity of fifths and octaves of open strings, it is yet again the B_2 played on the D-string. As the G is freed to resonate at its fifth partial, there is a theoretical discrepancy of about 16 cents between the notated B of the G-string and that of the E-string. Because the B partial of the E-string is stronger and recurs more frequently, it will still tend to govern the intonation of B_2; however, because the discrepancy is so small in terms of bandwidth, it is still possible to activate both series without violating perceptual cleanliness. Similarly, though without a second string to interfere, $F\#_2$ and $F\#_3$ will both be tied to the D-string, matching the fifth partial. While this produces a lowered (purer) major third, it yields a brightness, due to sympathetic vibration, which would make it acceptable as a harmonic or melodic third.

F_2 and F_3 are interesting cases because they show the utility of tuning in perfect fourths, with the strings' common harmonics aligning. F_1 and F_2, when played in half and first positions, are supposed to align their A partials with those of the open A-string. When played in higher positions, those partials are not available in the case of F_2, or very weak in comparison in the case of F_3. However, the A partials of the open D-string are comparably frequent, strong, available, and the same frequency as those of the A-string's. Thus, a great consistency between the F's of various octaves is maintained. Similarly, when Bb_2 is played on the D-string, the D partials of the G-string can be used to tune the Bb_2.

C_3 is a bit of a conundrum, because it should theoretically ring very well with the E partials of the E- and A-strings. However, in practice, I have not found this to be the case, but find it still rings very well with the G-string when played on the D-string. The reason I propose this is that C_3 is of a high enough register that the nearest coinciding partials is number 12 for the A-string and number 16 for the E-string, by which time the partials of each are so weak as to not make any significant contribution to the tonal color.[382] However, C_3 interacts with the G-string at its relatively strong fourth partial. To solve the problem of where to put the C on the G-string, I would follow the precedent of matching octaves and unisons, keeping the same pitch used on C_2 and C_3 on the D-string. The only caveat is when C_3 is played as a double stop with A_2, when, to get as smooth of a matching as possible, the C will probably need to be lowered.

The last bothersome tone is C#$_3$. If we were to follow Eb on the A-string straight across by perfect fourths, we would get a very high C#, which might be usable in some circumstances but would not resonate well with the rest of the instrument. However, since the A-string is available and the fifth partial of the A-string coincides with the second partial of C#$_3$, there is a strong argument for lowering the C# so that is resonates with the open A-string. Melodically, the rote interval may be lower than some may wish to use, but as with the F#, the added overtone serves to clarify and brighten the C#, which makes the pitch perception sharper than the frequency count would indicate.

Evaluation and application: the numbers of the system

If the theoretical positions of all of the closed tones are calculated by equalizing the appropriate harmonics of the closed tone to the harmonics of ideal strings, the fundamental frequencies of the respective tones can be generated, and I've added the

[382] Benade1976, 522ff even hypothesizes that partials above 8 are damped by bow interaction anyways, and that any partials heard above 8 are the result of heterodyne interaction of the strings and body resonances.

distances from each other in cents.[383] While an exemplar table of ratios is included below, a table of 3.5 octaves-worth is found in Appendix A.

[383] The cents measurements are derived from the standard frequency ratio to cents formula of 1200 x $\log_2(freq_a/freq_b)$.

Note Name	Note Freq.(Hz)	Distance in cents					
		E1	**F1**	**F#1**	**G1**	**G#1 low**	**G#1 high**
E1	**41.25**	0					
F1	**44**	111.7313	0				
F#1	**46.40625**	203.91	92.17872	0			
G1	**48.88889**	294.135	182.4037	90.22498	0		
G#1 low	**51.5625**	386.3137	274.5824	182.4037	92.17868	0	
G#1 high	**52.14815**	405.8663	294.135	201.9563	111.7312	19.55256	0

Table 1: Exemplar table of frequency ratios projected by the system, expressed in cents

The point of this table is to show the ideal interval sizes generated by the system, so that they can be easy analyzed. Of course, the positions of tones and the interval sizes between them will be different on a real instrument, but this abstraction allows for further discussion. What one may notice is an almost even alternation of major and minor semitones, which are about 22 cents apart. While this may seem like a large discrepancy (and in absolute terms it is), several mitigating factors exist. The first is that both fall well within the categorical boundaries of a semitone, if one takes the center of a semitone schema as being one-hundred cents. The major semitone is about 12 cents sharp of 100 cents, the minor is about 8 cents flat, and both sit on the edge between discrepancies perceived but usually ignored and a discrepancy which is perceived as a variant in a musical context, as both the general model and practical study has indicated.[384] Secondly, as we have seen, much string pedagogy has cited the utility, if not necessity of multiple sizes of semitones.[385] The last regards the caveat about ideal strings being used to generate the frequency chart. The main difference between ideal and real strings is that while the overtone series of an ideal string is perfectly harmonic, real strings have

[384] e.g. Shackford 1961 and Rakowski 1990
[385] c.f. Flesch 1939, Mozart 1951, and Borup 2008, citing Spohr

nearly harmonic overtones. This leads to two implications: the pitch of the real string might not exactly match the fundamental frequency, as pitch is gleaned from the virtual pitch generated by the specific location of the overtones, and the discrepancy of the overtones will cause a discrepancy between the interval size as measured by the pitch perception and by the absolute fundamental frequency ratio. When worked out mathematically, this tends to normalize the intervals towards equally tempered intervals, though the extent to which that happens is dependent on the degree to which the string harmonics are mistuned, which varies across strings and instruments. An extension of this final reasoning is that real strings bowed by real persons rarely have a steady state narrower than ±4 cents in any case,[386] so I find such a close discrepancy perceptually negligible for most purposes. That said, as so many others before me have done, I will retain the use of labeling tones by their fundamental frequency as shorthand for the tones' pitch.

The primary observation I would make about the values on the table is that most intervals fall within ±10 cents of the abstracted equal-tempered value, many fall within ±16 cents, and quite a few fall within ±4 cents. There are only a few rare and special-case intervals which fall outside of ±20 cents. Further, because of the ideal/real string differentiation, in practical terms the absolute distance of the intervals will trend even more towards equally tempered values. The main point of this analysis is that tuning intervals to the overtone series of the open strings, even in the mathematically worst cases, prevents schema failure: avoidance of that horrendously uncomfortable, tense feeling associated with poor intonation. Intonation which causes schema failure is

[386] Strange & Strange 2001

immediately brought into focused awareness, leading to a negatively valenced emotional response, and prolonged poor intonation, i.e., prolonged schema failure, can cause a feeling of distress.[387] Speaking purely from the mathematical standpoint, the system may not yield the most preferred interval sizes, but it should yield acceptably sized intervals within the categorical bounds of the tonal schema. If we return the considerations of timbral quality to the mix, because the interval sizes are within even expert-sized categorical bounds, the interval size may not even be considered, as things proper and normal are not brought into focused awareness. If looked through the Barbour-ian lens of temperament analysis, this system would fall somewhere in the "acceptable" to "good" range.[388]

The class of intervals which fall outside of ±22 cents is those intervals involving pitch classes with high/low options and is largely an artifact of the tabular design. It is simply unlikely that a low $C\#_2$ would be used in conjunction with a high $G\#_2$, for example. The key and context of the excerpt in question will usually determine the use of the high or low set of pitch-classes, as will be demonstrated below. Even if the coincidence of a low C# and high G# does occur and depending on the context, the large size of the interval may still go unnoticed, or at least unremarked, and its acceptance is largely related to how the context interacts with our echoic memory (trace), STM, and LTM. The most direct comparison between the two pitches would be a double stop, where the size of the interval would be apparent both categorically and psychoacoustically, causing failure of the schema of what it means to be a fifth and

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[387] See Chapter 2.
[388] Barbour 1951

severe beating, as both tones are able to be simultaneously and directly compared. Unless a special circumstance called for such a dissonant, nearly-perfect fifth, the interval would likely be rejected as out of tune.

When the tones are separated in time, the comparison progressively deteriorates. Tonal bass lines are particularly designed to suggest a harmony, i.e., activate a particular tonal schema, as the line unfolds and the precision required of a particular interval depends on its function within the tonal schema and its psychoacoustic limits. Part of a schema's function is to be predictive of what the next event (a pitch event in this case) is likely to occur, and the closer to the center of the tonal schema the events are, the more the prediction of the succeeding tone needs to be "right." In the context of C# minor, the C# and G# are very structural pitch classes forming the structural interval of the key. The G# will then need to be very close to the center of the schema of the $\hat{1}$ - $\hat{5}$ perfect fifth in C# minor, both schematically and (psycho)acoustically. If the C# and G# comprising the larger version of the interval are played melodically as separate tones, the instrument will most likely still be ringing from the C# as the G# is developing, which leads to acoustic overlap and subtle beating. As the ear begins to process the change in tones and as the two tones are more than a step apart, the trace of the C# will be retained in echoic memory to be directly compared to the incoming G# via recursive looping. In addition to the acoustic beating present in the change between the two tones, neural beating will also be present due to the incongruity of the two tones. Further down the chain and farther into background processing, as the G# is sounding, the C# would have already been categorized, placed into the STM queue, recalled or reinforced the C# minor schema from LTM, and primed for a following tone in C# minor. An image of the incoming G# bypasses the categorization process, is lead directly into focused awareness, and is directly compared to the G# (functioning as $\hat{5}$ in C# minor) stored in the LTM schema as a first order comparison of position. Because of the higher placement of the G# in our hypothetical example above, it would probably fail to meet the expectations of the schema and cause the failure itself to be brought into focused awareness, but the interval probably is not a large enough error to cause a failure of the schema. A mitigating factor is that although the G# is higher than the schema would predict and is high enough to be noticeable, it is still low enough to be recognizable as a G# versus C# and would be categorized as such for the purpose of schema evaluation and comparison to succeeding tones. Collecting all of the factors, an interval such as this, while not causing a total failure of the schema, would likely cause high memory loading as it would not directly conform to any of the memory system's expectations. It would also be brought to the attention of focused awareness and be judged as abnormal at the very least. Conversely, if the interval was played acoustically pure, the echoic memory loading would be reduced, due to the lack of physical and neural beating, and the G# would satisfy the schematic expectations of a G# in C# minor, reducing the memory loading on the STM and LTM, as stimulus highly in concordance with schematic expectation requires the least amount of processing power. This condition would not be brought into focused awareness as no memory system expectations would have been violated, and more than likely, an acoustically pure fifth between C# and G# in C# minor would also satisfy the emotional schema of "stability" implied be the occurrence of the structural fifth of C# minor.

If the tones are played with enough separation, because both the acoustic and trace signals decay exponentially, there will be very little signal remaining for a direct comparison of the tones, even after a relatively short time, the half-life of echoic memory believed to be on the order of .9 second.[389] The valuation of the interval would then be left strictly to the comparison of the categorized C# and the incoming G#. While it will probably be felt to be large, two mitigating factors will also come to play. Firstly, while the comparison in focused awareness of the pitch and the schema will return a sharp value, the G# is still low enough to be categorized as a G# in the parallel systems moving the auditory image into STM, and this gains prominence as there are not the negative contributions of the other memory systems. Secondly, the underlying timbral quality of each tone would become far more important to the interpretation of the intonation of the interval, as again there are not hierarchically more important failures of the memory systems in play.

If a tone, say E, were inserted between the C# and G# in the context of C# minor, then the first-order comparison switches from C#-G# to C#-E and E-G#, and the comparison of the C# and G# becomes a second-order comparison. As elements are added to the STM queue, the more difficult it becomes to track the change in position while simultaneously tracking amount of change.[390] Because the E and G# are adjacent, E will become the primary tone against which the G# is compared. It will also be compared at a lower level to C# as the third categorized tone stored in STM, but because

[389] Huron & Parncutt 1993
[390] See Huron 2006.

it is a higher order relation, it will have less of an impact on the valuation of the distance of the G# from the C#, leading to a slightly increased tolerance window for the exact position of G#, largely because the C# minor schema would be highly activated by the C#-E-G# succession of tones. The comparison would then be between the G#, the object C# stored in STM, and the schematic, idealized versions of C# and G# stored in the C# minor schema in LTM. Should there be more objects in STM intervening between the C# and G#, the object C# will degrade as time continues, shifting more of the comparison emphasis to the proximal tones and the schematic C#, increasing the tolerance window further. The other potential in this case is with that many intervening tones the object C# may totally drop out of the time/unit limits of STM, or the active schema may be changed to one other than C# minor, which would remove the severe restrictions on the size of the C#-G# interval. For example, if in A major, the size of an incidental fifth between C# and G# could be very different from the acoustically pure fifth and well tolerated because that fifth would not be functioning as a perfect fifth, structural or otherwise. The C# would likely be functioning as a tonic substitute and the G# as a dominant, both of which are very independent functionally and would have little cognitive relation or expectation of acoustic purity.

Notes on the sketches

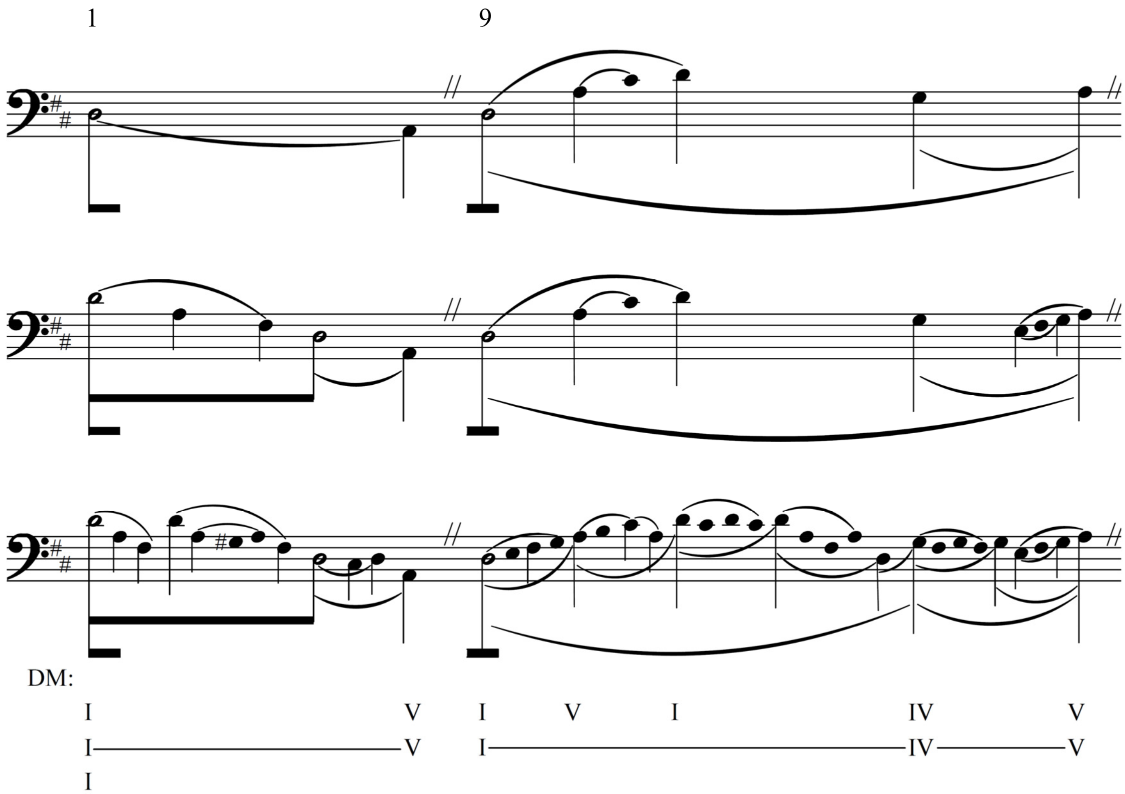
To help illustrate some of the points I will be discussing in applying the system to the performance literature, I've included a Schenkarian-style sketch of each excerpt, though please don't mistake it for an actual Schenkarian reading of the piece in question. The main distinction I wish to make is that a traditional Schenkarian analysis is at least somewhat retrospective, looking for the fundamental structure of a piece *after* it has been heard as a whole. So, some features which seem very important as the piece unfolds may not have as much significance when viewed retrospectively and in "reduction." However, as intonation is prospective, retrospective, and immediately accessible, some modification is needed. For comparison, I have included excerpts of the full scores for each double bass part examined in Appendix B (p. 236).

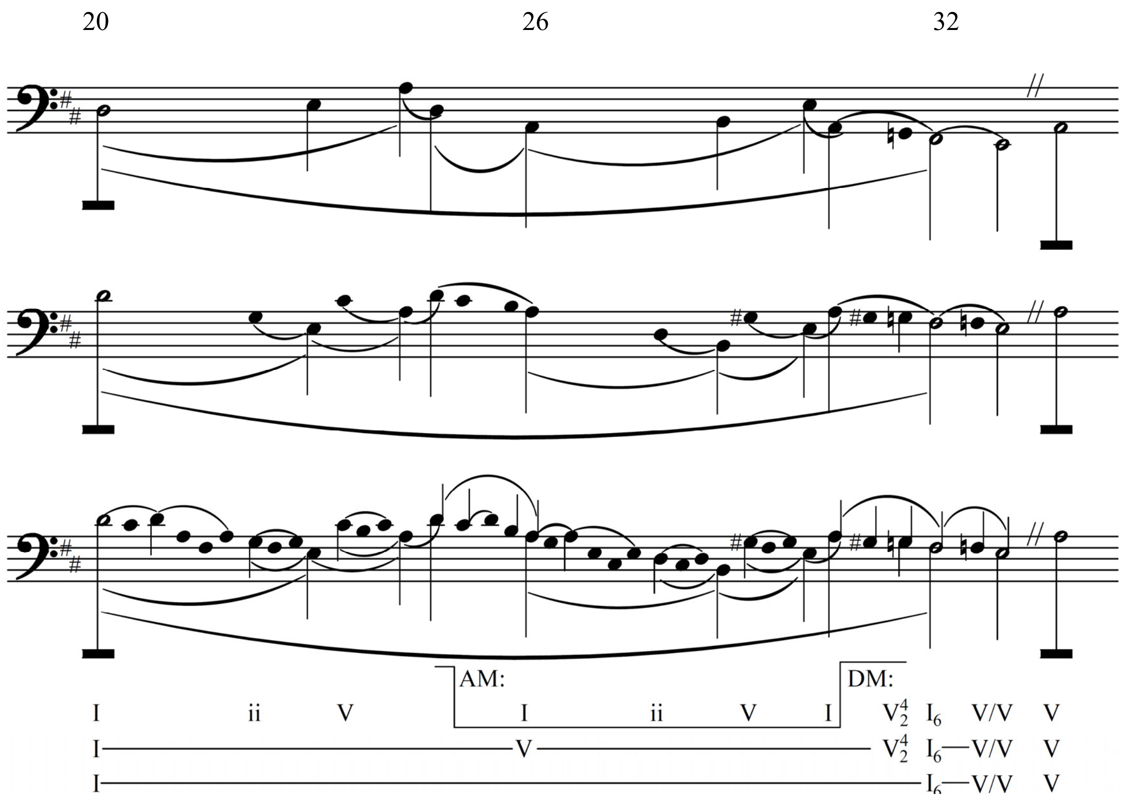
Reflecting the mainly immediate and prospective viewpoints I used as an approach thus far, the sketches outline major intonation features as the piece unfolds. Thus, some features which are not structurally as important are portrayed as having a significant impact, which merely reflects their significant impact on the intonation of the passage. For example, open note-heads don't necessarily imply high-level structural features, but imply tones which control the local, contextual intonation, though the two often intersect! Large-scale harmonic motion within a key or between key areas, which might be traditionally sketched with high-level symbols, might be sketched here with lower level symbols until the move is perceptually confirmed as the piece unfolds. A theorist looking retrospectively might see a modulation confirmed much earlier than an auditor might make schematic changes in real time, even if the piece is relatively well known. Also, sometimes there is a tension between the full context of an excerpt and how just the bassline of an excerpt can be heard. Because the context of this study is to examine bass intonation when played as a solo and ensemble instrument, the analysis bends towards hearing the implications of the bassline alone, but with at least some acknowledgement of the full context of the piece, as these are fairly well-known symphonic works.

In addition to the open note-heads mentioned above, a few other notes on the notation are in order. The general principle I tried to follow is that fundamental harmonic features are noted with stems below the note-heads and melodic features are noted with stems above the note-heads. So, strong harmonic (intonationally significant) features, such as a tonic prolongation, a circle of fifths motion, are noted with stems and slurs below the note-heads. Melodic groupings (intonationally less significant) are noted with stems and slurs above the note-heads. Flags on stems below the note-head retain their traditional meaning of indicating a structural pre-dominant because of the pre-dominant's role in schematic establishment. Flags on stems above the note-head also retain their indication of upper neighbor tones but also include lower neighbors, a change necessitated by treating a bassline simultaneously as a bassline and a melodic line in its own right. Beams below the notes indicate a strong association with a transition between one perceptual key area and another, transferring the intonation assignment from one area to the next.

Evaluation and application: Mozart, Symphony No. 35 in D Major (Haffner), K385, mvt. IV, mm. 1-38

Example 1: Sketch of Mozart, Symphony No. 35 in D Major (Haffner), K385, mvt. IV, mm. 1–38





The application in this excerpt is as much about the timbral brilliance possible using the system as it is about the intonation itself. However, it does offer a relatively simple set of challenges upon which to start. Contrabass instruments of the period and in the orchestra for whom Mozart wrote this piece would have most probably been tuned in what we now call Viennese tuning in D major (F’A’DF#A). The tuning fitting so perfectly the key and the variable fretting associated with it would have yielded a resonant, sparkling character, with all members of the major triad present as open strings with a doubling of the dominant. In a presentation adapted for modern double bass, this aesthetic can be carried forward, beginning with the matching of the octave and unison D’s, A’s, G’s, and E’s. As this excerpt stays fairly rooted in D major, early establishment of the tonic and dominant relative to the open strings will quickly establish the tonality and tone color of the excerpt, fully activate the tonal schema, and prevent any significant pitch excursions.

The main choice to be made in the excerpt is the use of the low or high F#. The excerpt can be read as the exposition of the primary theme with a large scale motion of I-V-I_6 before cadencing on V/V, the majority of the excerpt is devoted to the establishment of tonic in D major. The use of a lowered F# is supported by the arpeggiated context, the historical inference of the use of the lowered F# in extended meantone systems employed, and the use of the lowered $F\#_2$ implicit in Viennese tuning. The lowered $F\#_2$ on the double bass would mimic this by resonating its $C\#_4$ harmonic with the $C\#_4$ harmonic of the A-string, which can quickly convey timbre and pitch information and also provides adequate perceptual clearance between the arpeggiation of the $F\#_2$ and A_2 necessitated by the quick tempo of the excerpt. Similarly, the use of the lowered C# and G# is also called for, which provides not only for the added resonance and quick perception of these tones, but also the long term pure P4/P5 relationship between all of the active tones. The lowered C# is particularly advantageous in m. 9, where the A is active in the fore- and mid-ground and the line leading to the octave D is temporarily broken by an arpeggiation to A. The lowered C# will ring well against the active A and there is less pressure to demand that it be raised to point toward D, as it is functioning more as $\hat{3}$ of V, rather than a melodic leading tone to D.

These positions hold well even in the passage on V (mm. 26–31), which would retain D major as the underlying schematic paradigm, as there has not been sufficient effort placed in supplanting D major with, for example, a perfect authentic cadence in A major. The lowered position of G# is not an issue of its expected position in D major, as it is an augmented fourth from $\hat{1}$, one of the cognitively weakest positions for judging the size of an interval. However, as has been noted above, it retains its relative position versus the F# and C#, but also its harmonically important role as the third of V/V, which unlike the Viennese bass, exists as an open string on the modern bass. The rest of the pitches of this section would remain in the control of D major, as the only discrepancy would be with the G#.

The B's of this excerpt deserve a specific mention. In terms of the circle of fifths relations implied within the excerpt, the pitch classes E through A are grouped together, as are F# through G#. The pitch class connecting these two groups in the circle of fifths is of course B, and in the strictest sense must align with one group or the other. However, as $\hat{6}$ of D major, it is a weak interval, and given its context in the bulk of the excerpt as a passing or neighbor tone, it can play the role of "hiding the comma," serving as the transition between the two groups relatively unnoticed. The one exception to this is in mm. 27 and 29, where the B's function as ii/V, helping outline the ii-V-I progression tonicizing[391] V and relating most strongly to being a full perfect fourth from E. However, there are multiple elements between the B and E, which lessen the necessity of the strictness of an acoustically pure 498 cent interval. Either functioning as $\hat{6}$ in D major or

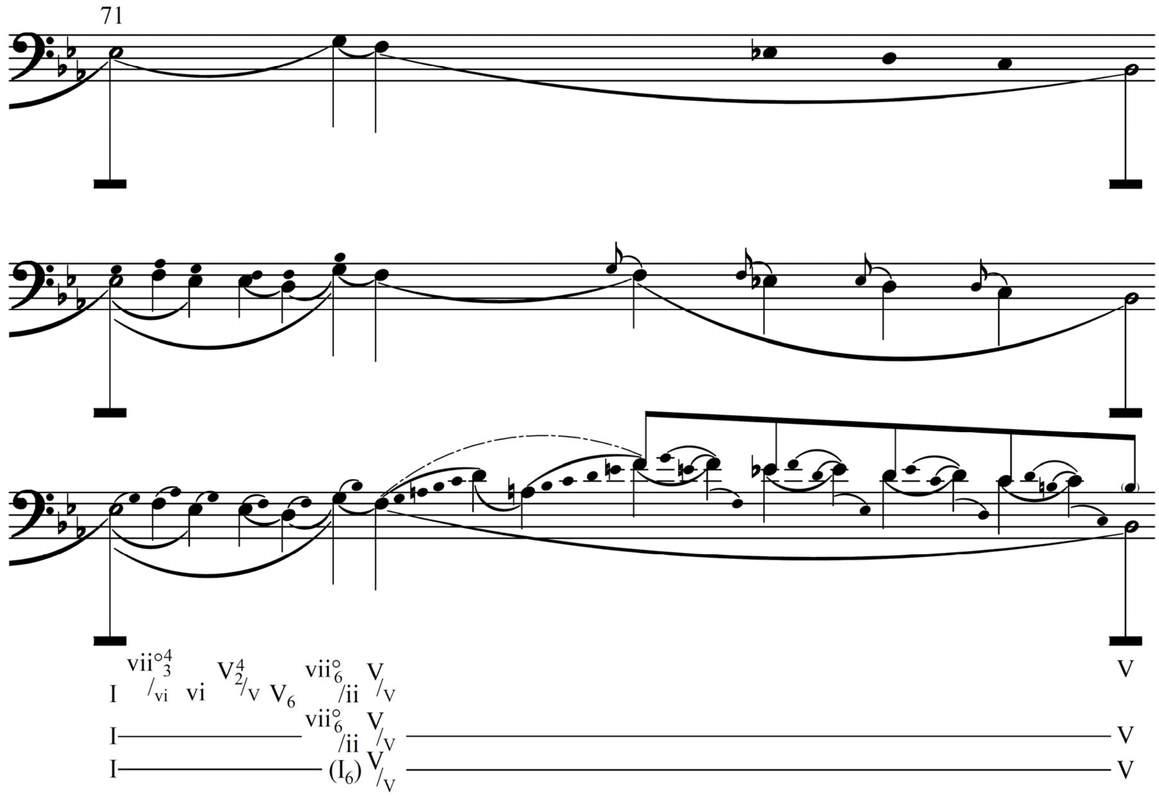
[391] Tonicization is the short-term establishment of a chord as a tonic within the context of a larger key area. It is distinct from a modulation to a new key area because of its shorter duration, from a few chords to no more than a phrase, and the retention of the sense of the centrality of the home key area.

$\hat{2}$ on A major, the B is in a schematically weak position and would have a wider range of acceptability.

In creating an intonation which emphasizes the timbral quality of the excerpt by minimizing the acoustic roughness associated with mistuned overtones, one also has eased intonation perception. Even at more reasonable tempi, this excerpt is quick for the auditory system to actually discriminate and evaluate the incoming signals, as most tones do not have enough cycles to be assigned a pitch under the traditional models of pitch assignment. By mimicking the timbral experience associated with good intonation, the ringing tone will convey a "good tone," both in quality of the signal itself and the lack of interference of overlapping tones, which is vital to the interpretation of intonation. The ringing overtones add needed information to assist the auditory system in identifying and clarifying the pitch of the incoming signals. The higher overtones are able to accomplish enough cycles to speed the assignment of the virtual pitch to the overall signal. The application of the system serves to stabilize and reinforce D major as the prevailing schema for the excerpt, with repetition of tones consistent with the schema. The structural pitches of A and G are held at consistent sizes of purely tuned fourths and fifths in relation to D throughout the excerpt, F# is likewise held in an acoustically pure position relative to D, and all three fall within the bounds of the general model and extended meantone system.

Evaluation and application: Mozart, Symphony No. 39, K543, mvt. I, mm. 40-97

Example 2: Sketch of Mozart, Symphony No. 39, K543, mvt. I, mm. 40–97



This second Mozart excerpt serves as another interesting example of the interaction of writing for Viennese tuning and modern tuning, as well as the use and stability of the system in flat keys. The dominate harmonic feature of this excerpt is the repetition of the establishment of tonic (Eb major) via arpeggiation and half-cadence motion and the large scale motion to V. Throughout much of the excerpt, the harmony remains standard and elegantly simple, which has its set of challenges and benefits for the system.

The practice of Viennese tuning being in D major is true for much 18th century literature. However, as has been noted, the Viennese contrabass was not terribly far removed in form or technique from the viol family, and this influence was felt in the practice of tuning the bass in different keys other than D major as a solo scordatura or to maximize the resonance in a particular key.[392] For this piece it would be conceivable that the bass would be tuned in Eb major rather than D major, providing a highly stable structure for the tonic. Following practice, the G would likely have been tuned to the acoustically pure (lowered) third against the Eb, and the Bb to an acoustically pure fifth. In the case of the system, this resonance profile would be mimicked by the Eb being set so as to resonate with the G-string, and the Bb with the D-string. This tuning of the Bb is incidentally an acoustically pure fifth above the Eb. This works to the great benefit of the melody's construction which is highly arpeggiated and suggests the use of the lowered third for increased acoustical and cognitive agreement. The clear delineation of the Eb major triad and chord progression also creates the problem of satisfying a highly activated schema. The targeting of the active resonance of Eb major returns both a timbrally satisfying and technically consistent tone. In addition to the overtone alignment of the Eb major triad, the same application to ii and V returns resonant D's and F's consistent with an extended meantone temperament and retaining structurally stable, acoustically pure fifths between C, F, Bb, and Eb.

A question arises between the occurrences of IV or $\hat{4}$, Ab, and $\#\hat{4}$, A♮. Generally, the system would predict the higher positions of Ab, generated from the position of Eb. This sits contrary to the lower position suggested by acoustic agreement with the E-string. Unfortunately, contextual cues are only helpful in two instances: m. 52, where the Ab is functioning as the root of the pre-dominant IV, arriving from a V/IV and moving towards the first half cadence, and m. 73, where the Ab occurs as the seventh

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392 See particularly Planyavsky 1998 and Chapman 2003.

above the root vii$^{\circ 4}_{3}$/vi but above an F bass. In these cases, the higher Ab is clearly indicated for acoustic agreement with its context, as a perfect fourth above Eb in the first case and as a large m3 above F, both indicating the higher of the position choices for Ab. The other cases are less clear: the Ab occurs as a linear coincidence, often alongside the chromatic A♮.

Laying aside the position of the remaining Ab's for a moment, it is helpful to note the positions of the A♮, Bb, and G. Any G's and A's are of course bound to their respective open strings: no matter the key, the acoustic agreement with each respective open string will be a dominating resonance or interference pattern if mistuned, and creates a slightly large M2 between them (204 cents). The Bb is also fairly fixed, as it is tied to the position of the Eb and the resonance of the D- and G-strings, resulting in a major semitone between the Bb and A♮. The position of the non-structural Ab's can then be seen as "floating" between the fixed points of the A♮ and the G, again "hiding the comma." From this perspective, it can be seen that either position of the Ab is valid, as both exist within the perceptual boundaries of the general theory of intonation, the Ab is not as strongly governed by acoustic considerations as the other contextual tones, and Ab is not governed by strong tonal schematic implications within the intonation horizon. The lower position has acoustic advantages, the higher more closely conforms to the underlying schematic expectations, extended meantone system, and the possibly advantageous major/minor semitone alternation suggested by Duffin,[393] but either of which fall within "generally acceptable parameters" for intonation in this study.

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[393] Duffin 2007, 145ff

To do a bit of interpreting, I would suggest that the system's assignment of the lowered Ab, resonating with the E-string, provides a satisfactory solution. A primary factor here is that this position stands the best chance for resonance and the projection of a good tone from a note, which on the bass is notorious for being wolfy, as it tends to resonate both the G- and A-strings simultaneously. Shading to the lower side allows margin from the bandwidth of the A-string and picks up some good resonance from the E-string. Perceptually, at the local (fore- and middle ground) levels, the questionable Ab's often function as the seventh of V in these cases, and the lowered Ab closely conforms to the natural seventh generated by the 7/4 ratio above the Bb. The second factor, breaking a bit from period practice and adding a bit of modern expressiveness, stems from the main occurrences of Ab being an upper neighbor to G. A melodic feature of this excerpt is the recurrence of a neighbor note resolving down by half-step, and in most contexts this can be heard as a "pressing down" feeling. To accomplish this expression, a modern performer would tighten the distance covered by the half-step, and given that the spot of the G would almost be rotely practiced to be in tune with the G-string, this would mean that the Ab would be lowered. If equal temperament is the abstract center, then the pitch would be acoustically tuned and lowered toward with the E-string. This lowering perceptually points the Ab down towards the G, emphasizing the G as a structural note, often as I_6 and melodically making it "belong" to the G. The lower position of the Ab sets the position of the Db in m. 66 for similar treatment when the Db points out the structural significance of the C in m. 67. In mm. 46–47, the low Ab accentuates the diminished fifth at the surface level[394], creating a great deal of tension

[394] The surface level represents the basic "notes on the page" reading of a passage and is opposed to deeper structural levels, such as the middle- and background levels.

before and direction towards the resolution on I_6.

A fair question would be if IV, I, and V (Ab major, Eb major, and Bb major chords in this case) are the structural, inviolate elements of the tonal schema regardless of stylistic concerns, how can the Ab be mutated as much as it has been in this case? The answer lies in the fact that most of the time in this excerpt, Ab is not functioning as IV, but most often as the seventh of V. As I noted, when it is function as IV, as in m. 52, the interval from Ab to Eb be in the region of 498 cents to be considered acceptably in tune, as there is little schematic or acoustic wiggle room in that context. If that high position of the Ab, and later transferred to the Db, were maintained throughout the excerpt, it would not violate the general schematic or period practice context. However, the other solution of tuning the Ab to the resonances of the E-string is an alternative which, while not totally consistent on the background level, does offer greater acoustic and expressive agreement with the immediate context. The reason this alternative is acceptable is that the lower Ab, when not in proximity to Eb or Bb and not otherwise functioning as IV, remains placed within the general "in-tune" bounds of the general intonation theory[395] and elements with which it could be more structurally compared are out of STM and the intonation horizon. In the foreground context, such as when the Ab appears as the seventh of V and is acoustically tuned to the E-string, the acoustic and expressive elements are immediately available and more salient to the tone's acceptably in tune judgment.

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[395] See Chapter 2.

Even with the complications of determining the position of the Ab, by aligning the overtones of each pitch to the overtones of the open strings, the system consistently returns pitches supporting the structural features of the tonal schema, particularly of $\hat{1}$, $\hat{5}$, and $\hat{3}$, resonating with the open strings, providing replication stability and increased timbral quality. When presented with the challenge of the position of the Ab, the system generated two options which were both perceptually consistent and would have likely returned "acceptably in tune" judgments. A deeper analysis of each option in specific contexts revealed different interpretive options and a confirmation that neither option violated neither the principles of the system nor the general model of acceptable intonation.[396]

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[396] See Chapter 2.

Evaluation and application: Beethoven, Symphony No. 5 in C Minor, Op. 67, mvt. III, mm. 1-97

Example 3: Sketch of Beethoven, Symphony No. 5 in C Minor, Op. 67, mvt. III, mm. 1–97


44
53
60
b♭m:
i V/V V i V i V/V V V/ii ii V/vi vi
i V/V V i V/V V
♭vii ♭vii/V ♭vii
cm:
V^4_3 V^6_5
V^4_3 V^6_5
V^4_3 V^6_5
72
80
fm:
i IV V i V
cm:
i
i iv i

One can hardly write a paper involving basses and orchestra excerpts without addressing the third movement of Beethoven's Fifth Symphony. While the Trio of the movement follows a relatively standard key scheme and would follow the principles already outlined fairly easily, one may kindly say the Scherzo display's Beethoven's virtuosity in easily reaching some rather remote key areas, which would seem to challenge the system. In terms of Lerhdal's pitch-space theory,[397] C minor-Eb minor-Bb minor-G dominant pedal-C minor/F minor/C minor represents quite a journey, indeed! One advantage of moving to such distant key areas is that there is little schematic overlap between areas. However, within each key area, the tonality is remarkably stable, even conventional, indicating a high schematic activation within each area. Thus, the influence of a preceding key area offers little if any influence on the subsequent area.

From the historical perspective, keyboard instruments were certainly pushing towards a concept of fixed equal temperament, but ensembles were still in a non-fixed meantone-esque mode of thought.[398] Beethoven's conception, even in, or perhaps especially because of, his deafness, would likely have been for each key area to be stable and consonant within itself, as the meantone approach suggests more acoustically consonant triads and the flexibility of non-fretted instruments would have afforded some adjustments towards the consonant or coloristic. The double bass of the time would have been very close to the modern form, though a question remains as to whether German basses had retained frets through this period.[399] While this may seem to be an academic question, the structure of the movement's bassline encourages a consistent placement of

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[397] See Lerdahl 1988.
[398] See Chapter 3: Performance Practice.
[399] c.f. Planyavsky 1998, Brun 2000, and Chapman 2003

each tone which would be easily accomplished with frets. This is particularly seen with the stability between key areas generated by consistent placement of common tones between each key area, the consistent placement of $\hat{3}$ in each area, and the perfect fifth/fourth arrangements, particularly when structurally significant.

The movement begins with great harmonic stability and unambiguously in C minor, with the unfolding of a i-V/V-V-i progression and an expansion of V by alternation of V and i, leading to a half-cadence caesura. The system reflects that stability by providing acoustically consonant returns of C, G, and D in acoustically pure fifths. The system also predicts a high Eb, which is stylistically consistent with a meantone approach and acoustically consonant with both the C and the G. This acoustic consonance is particularly important for the opening as the Eb is in close proximity to both notes and will engage both in acoustic and direct memory overlap. Because of the close relationship each significant tone has with the pitch class G, the intonation of the passage through m. 18 is controlled largely by the G-string.

In many ways, a move from C minor to an Eb tonal center is a fairly logical one, though moving to Eb minor is rather unusual! In making such a move, the C tonal center is almost completely supplanted by the Eb tonal center, but the relationship overall is maintained through the retention of the Eb's placement relative to the G-string. Because we've moved to Eb minor, though, there is a question as to where to put the Gb's in mm. 30 and 35. The two main options are low, agreeing acoustically with the A-string and expressively with the expectation of a low m3, or high, agreeing acoustically with the B (as Cb) and with the temperament system already established. A third option would be to treat each occurrence of Gb individually in this section of the excerpt. The placement of the Gb in m. 35 is a bit more clear because of its relation to the Cb, which should agree acoustically with the E-string. The Gb, being a fairly functional part of a circle of fifths motion, should be in close acoustic agreement with the neighboring Cb and Db. Because the Cb is in a fixed position, we can work backwards to set the Gb in the higher position and Db also in the higher position.

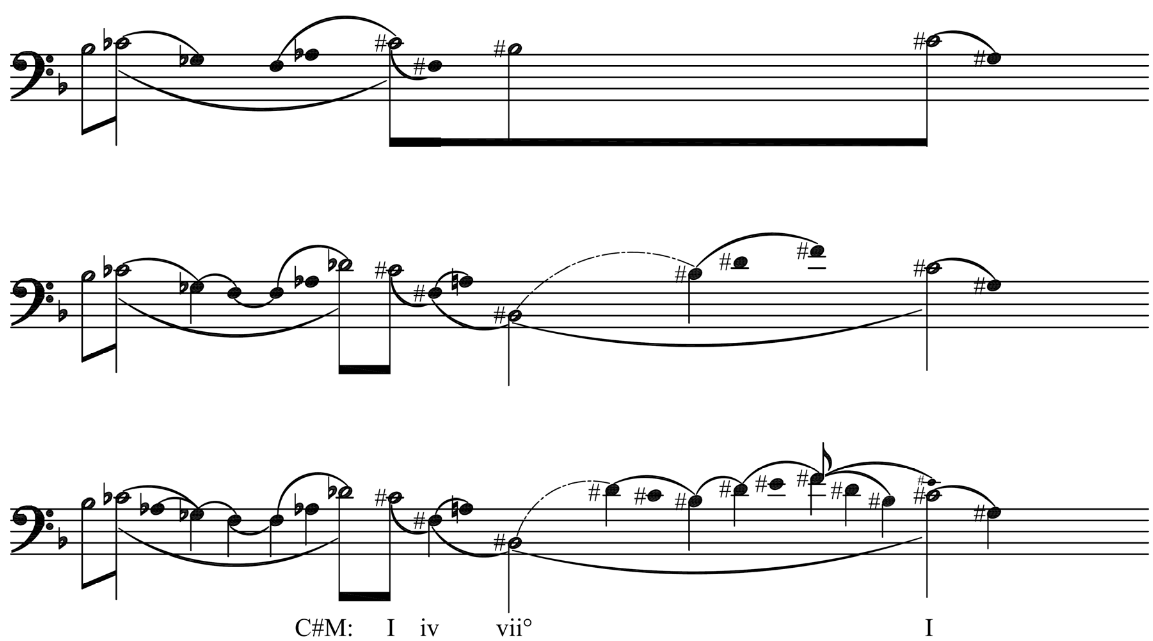
The placement of the Gb in m. 30 is a bit less clear. Looking to parallel the placement of the Gb in m. 35, the Gb in m. 30 would be of the higher position as well, but a direct parallel is not necessarily called for. The modern reading would have the Gb in the lower position to emphasize the minor tonality. However, from the C minor section, the position of $\hat{3}$ in the minor tonality of the excerpt was established as being of the higher position. This higher position of the Gb pairs well with the later Gb, and both are in prominent positions being the local maxima and minima of the passage and might benefit from a perfect octave relation. Even though they are separated by several pitches, they still occur within the same key area and are pitches of relatively high hierarchical importance locally. What is interesting to note from a broader perspective is that there is some flexibility of the placement of the Gb because it is of a very low hierarchical importance in C minor. Granting that intonation is viewed primarily at the local level but comparing the opposite case of a G♮ in place of the Gb, the proximity of the C minor key area would still hold some sway in establishing the position of the Eb and G, as they both hold high hierarchical positions in C minor and Eb major. Because there has not been significant time or chordal distance between leaving C minor and establishing Eb major, the schema would not have been fully supplanted. But, since the Gb is not part of the C minor diatonic collection and a full-scale schema change is in order, the tolerable variation of the first Gb especially could be significant.

By the time the Db is reached in mm. 85–88 we've been through C minor, Eb minor, Bb minor, a dominant pedal of C, back to C minor, leading to the midst of F minor. All the while, the intonation of each area, distant as it might seem, was stabilized by a common tone: Eb linked C and Eb minors, Bb linked Eb and Bb minors, G linked Bb minor and the dominant pedal, the G linked the pedal with C minor, and the C linked C and F minors. These tones are not only hierarchically important in their key areas, but are also strongly associated with their respective open string associations. However, some tones are not so easily defined, as was the case with the Gb's in the E minor key area and the Db's in the Bb minor section. In the case of these Db's, taking a similar approach as with the Gb's yields a similarly placed, higher variety of Db consistent with the hierarchical position of the Db and the placement of the Gb. In mm. 85–88, on the other hand, the Db's are of a much lower hierarchical position and embedded in a sequential move where they are less functionally significant. So, the question is where to put them!

Taking a cue from the parallel passage in Eb minor, it is helpful to start with the circle of fifths motion in mm. 86–88 and with keeping perfect fifths/fourths perfect. While in the group of Eb, Ab, and Db, the Ab is the most hierarchically important note, I think it a better idea to set this passage using the Eb, which has the strongest resonance of the three.

Evaluation and application: Beethoven, Symphony No. 9 in D Minor, Op. 125, mvt. IV, mm. 65-75

Sketch of Beethoven, Symphony No. 9 in D Minor, Op. 125, mvt. IV, mm. 65–75



This Recitative has been a notoriously difficult passage to play in tune, particularly because of the Ab in m. 67 and the cadential dominant in mm. 73-74. Much of this can be attributed to the ambiguous key identification. The reading I offer is not nearly the only possible solution, but it is one which works with the notes as presented and with the system. The initial Bb is a tempting tonic, as it is the first pitch present in the Recitative and is consistent with the tonal center of the previous woodwind interlude when listened to in context. However, in applying the intervallic rivalry model, this tonic is supplanted by the ascending minor second leading to Cb. This serves through the passing motion of the Ab to Gb, implying a tonic substitution (Ab major chord) leading to a pause on V of Cb major. However, fulfilling expectation generated by earlier similar figures in Recitatives 2 and 3, the line continues to pass down by half-step to F, which is not part of the Cb major tone collection, and the unexpected interval of a falling minor second at that particular juncture calls into question the assignment of Cb as tonic, and sets in motion the search for a new tonal center as the half step does not conform to the tonal schematic expectations in Cb. In this case and for the purposes of the system's illustration, I tend to read this F as a non-chord-tone anticipation of the iteration of the pitch in m. 69.

The passing of F, Ab, to Db in mm. 69–70 elucidates the motion to Db major. Especially after the ambiguous nature of the previous measures, the clarity of the minor third-perfect fourth sequence establishes the tonal center in Db/C#. The F# in m. 64 which follows the establishment of C# would seem to be a logical progression from tonic to subdominant in concert with the falling fifth motion; however, it is often heard as somewhat jarring. The F♮s in mm. 68–69 are perceptually very prominent, the first because it upsets the dominating schema and the second as it is the first tone heard after a rest and iterates the incongruous tone. They would be retained at a higher level and while within the limits of STM be readily compared to incoming tones. The F and F# fall within the same STM window and are available for direct comparison, when the expectation might have been for a return to an F♮ to begin a new sequence. Further, this is pointed to directly by the change in metric entrainment, which is also contrary to metric/rhythmic preference rules.[400] The entrainment of half note/quarter note per bar is

[400] See Lerdahl & Jackendoff 1996.

disrupted by a change to quarter/half, and the prototypical entrainment is not reestablished until m. 74, which attempts to establish some order for the cadence. The half-step ascension and change in metric entrainment contrary to expectation would be unsettling. The occurrence of A♮ in mm. 72-73 offers a similar comparison with the Ab in m. 71 as F and F# enjoy, but less attention is drawn to the comparison by their rhythmic considerations. Rather, the modal mixture is as much the disquieting factor here, further destabilizing the sense of which exact schema is to be activated.

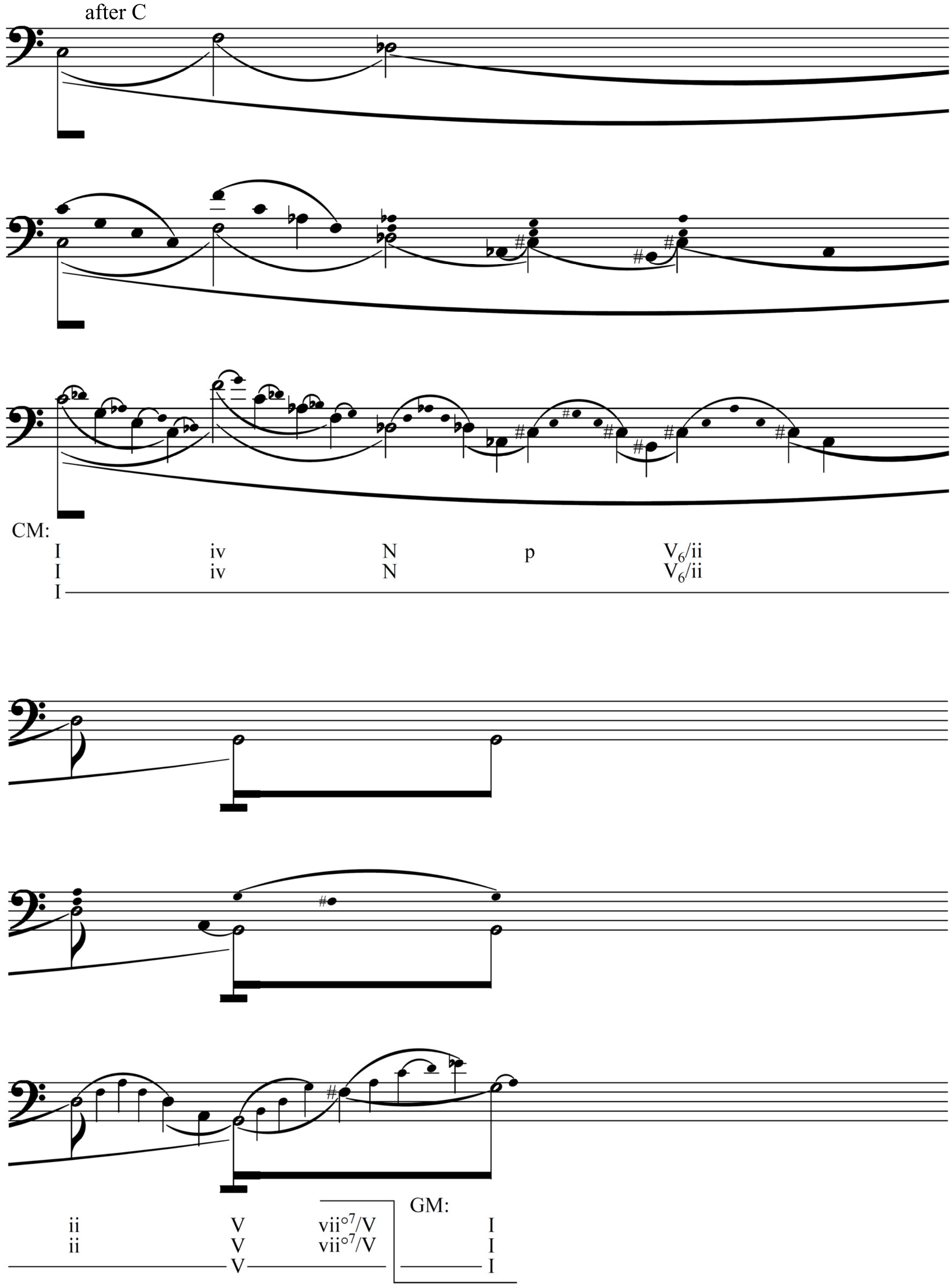
The perennial note of concern in this excerpt is the Ab in m. 67; stories abound on the audition circuit of jobs being won or lost on the perceived intonation of that one note, which tends to be sharp. Following the system, the Ab is tied to the resonance of G# on the E-string, but advantageously, this is a low position for the Ab, correcting the typical problem of it's being sharp. The Ab would also be in a position of melodically demonstrating its passing motion down towards the Gb; in a higher position, the Ab would be tied more to the Cb and blocking the step-wise motion from the Ab to F. This also illustrates the importance of audiation, being able to hear a tone as functioning in a certain manner, and the difficulty with this passage not having a clear tonal center. Without a clear idea of where the tonal center is, it can be legitimately difficult to hear where the Ab should go. But, reading the Ab as a passing tone to Gb (Ab functionally moving down to Gb) keeps the audiation of the Ab low enough and satisfies the system.

The lower positioning of the Ab fixes the position of the Db/C# in mm. 70–71, as the key centering in C# is solidified by an acoustically pure Ab to Db motion. As noted, the following F# in m. 71 is the subject to some cognitive consternation, but this can be somewhat alleviated by solid psychoacoustic grounding. The syncopated position draws attention to the pitch, and the lower position of the F# would form an acoustically pure fifth. The lack of acoustic and psychoacoustic beating would help convey control of the intonation and performance overall. Further, as the following A is definitely tied acoustically to the A-string, the interval between the F# and A would be of the wider, acoustically pure variety of minor third which would also lack a beating psychoacoustic trace, further providing stability through the harmonic twists of the modal mixture.

The B#° chord in mm. 73–72 looks intimidating, confounding many performers, but taking B# as C and D# as Eb for a start, the chord begins to fall into place. As such, the B# and A agree acoustically, even if not cognitively, and a stable reference point for the B# is created with the G-string. The Eb is also referenced to the G-string, placing the D#. The interesting aspect of this placement is that it creates a slightly large major second between D# and C# immediately following. In this case though, $\hat{1}$ is not functioning as tonic: it is at best a passing tone later suspended over the B#. The lower position indicated earlier offers a favorable expressive possibility, drawing the passing/suspension tone down towards the resolution tone of B#, which is a minor semitone from the C#. Lastly, the position of the F# in m. 74, so often flat, is also set by its resonance with the D-string, which while low by the system's standards, is higher than many bassists reach. This position is also tuned with position of the C#, providing long-term stability, as it occurs within the STM window including the C#'s of mm. 72–73 and the following C#'s in 74-75, and provides a narrow tritone with B#, especially as the interval closes into the C# major chord in mm. 74–75.

Evaluation and application: Schubert, Symphony No. 9 in C Major, D. 944 (the Great), mvt. III, rehearsal B – 2 m. after C

Example 5: Sketch of Schubert, Symphony No. 9 in C Major, D. 944 (the Great), mvt. III, rehearsal B–2 m. after C



Some teachers, smiling with glee about torturing their students, might term this a "fun" excerpt, as it is one which is not only technically difficult in placing the finger near the position of the intended pitch, but is also legitimately hard to hear internally because of its slippery chromaticism. For the time, the progression of I-iv-N-#i (passing)-V_6/ii-ii-V was highly chromatic, and even today it remains striking. That such a progression was comprehensible may in part be credited to the rise of equal temperament, imprecise though it was. There is still some weak support for the idea of orchestra key differentiation by color, but of more importance in this excerpt was the idea that the position of enharmonically equivalent sharp and flat notes had switched, such that F# was now higher than Gb, for instance, or that their position was now equal. The double basses of the period were also becoming more "standardized," greatly reflecting its present form.

This excerpt is difficult to play in tune not only because of odd chord progression, as mentioned, but also composed-in melodic features and specific method of chord change which destabilize the schema. The beginning of the excerpt would ostensibly set out the tone center with its clear outline of a C major triad; however that clear sense is disturbed by the chromatic half-tone upper neighbors creating ambiguity as to which set of tones are the stable tones, as the rare interval hypothesis might suggest a center of Db. So instead, a strongly activated C major schema is weakened. Similarly, the modal mixture of iv (an F minor chord instead of F major) weakly reinforces the activation of the C major schema. From the view of the cognitive process of identifying the key,[401] the C major schema is activated not because of a strong stimulus encouraging it, but rather

[401] in the combined view of Brown, Butler, & Jones and Krumhansl

because it's the best viable candidate. As the excerpt moves from the eighth-note passages to the half-note/quarter-note rhythms in 9 m. after rehearsal B, stability (though not necessarily schematic centrality!) is established with the Db minor arpeggio. That stability is short lived, as the chord shifts to V/ii via a passing chord. Linearly, with each chord in the progression the change occurs by a "false" circle of fifths motion (rising fourth of an upbeat leading to a downbeat) on the surface and half-step motion in the middle ground. This use of a stable gesture leading to a false promise in combination with linear sinuousness serves to destabilize the sense of centrality, and ultimately leads to more cognitive latitude for intonation.

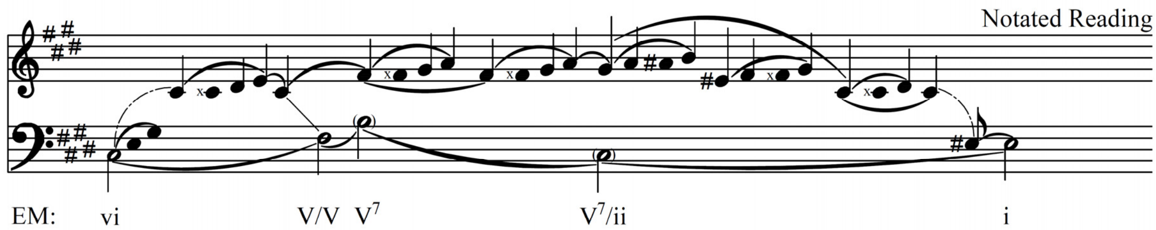
The goal then should be to stabilize the execution of the these passages so as to provide a constant reference point in pitch-space to which the system is well suited, since a stable reference in chordspace is not available. By aiming not for a particular position or spot on the fingerboard, or set of relations in pitch-space, but maximal resonance of a particular tone, the stability and repeatability of each tone is more assured. Because a particular tonal schema is not strongly activated for the excerpt as a whole and the surface level prevalence of arpeggios suggests its own schema, the acoustic agreement of the interplaying tones will also be more prevalent in the in/out of tune judgment. To accomplish this, the lower position of G#/Ab and C#/Db is indicated.

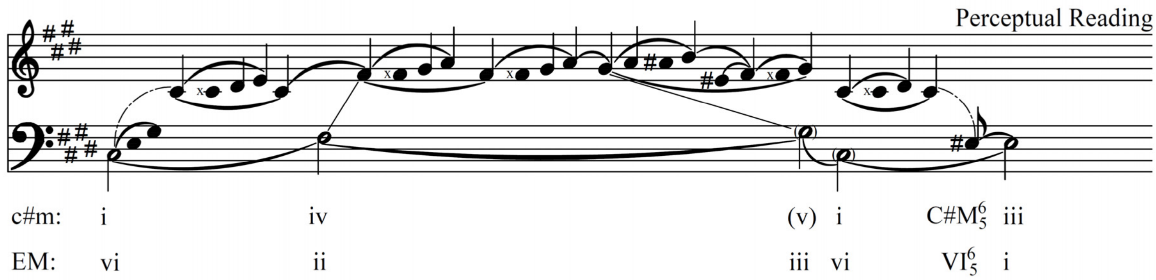
For instance, because the Neapolitan, Db major, is very distantly related in chordspace to the general key of C major and the surrounding chords (F minor and C# minor), its absolute relation to the home key plays less of a role in the judgment of its intonation, as two of the three tones contained within are very low on the tonal schema stability list. The judgment of the intonation would then rely much more heavily on the internal construction of the arpeggio itself, referencing the tonal schema immediately implied by the arpeggio and acoustic agreement of its tones, which will still be physically and psychoacoustically active after the player has moved on to the next tone. The resultant intonation of the passage would yield chords very close to acoustically pure, as the system would predict. But the system also provides stability in the larger context by providing constant reference points, e.g. the Db and C#'s will always be in the same position, as they activate the same harmonics on the instrument. As this is a prominent position melodically, harmonically, and rhythmically, the pitch will likely be maintained more strongly in STM and a discrepancy between successive, similar occurrences would most likely be noted. Any linear discrepancy between adjacent tones more in the background would be ameliorated by both the greater latitude implied by the harmonic fluidity in general and acoustic agreement of each tone's immediate surroundings. For example, the background interval between the G# and A in m. 19 and 22, which is an important linear relationship, matters less in intonation judgments as they agree acoustically with their immediate chord-tone neighbors.

Using the system to position the tones composing each chord helps portray the aesthetic quality implied in each chord. Starting with the opening, the C major chord is generally very bright and resonant, and tries to be so here. But, instead, the half-step neighbors knock some of the shine off the chord's resonance and serve to destabilize it from both the harmonic and intonation standpoints. With the arrival of the Neapolitan, the implication is of a dull, dark sound pointing downwards to the functional pre-dominant. Even using the system to its fullest advantage, a Db major chord will be a dark chord: each tone in the chord will resonate, but none very well. However, they will resonate well enough to not be considered out of tune purely based on their tone quality! When Db major changes to C# minor, the change in accidental sign indicates a slightly brighter feel. While not a particularly "happy-go-lucky" implication of D major, it turns the corner from the more decidedly brooding F minor and Neapolitan. A traditional response to this turn might be to sharpen the C# and G#, but I think the feeling could be served well enough by the increase in resonance of the E-string, which would bring a net increase in the brightness of the unit by the significant increase of resonance with the open E-string, as opposed to the less obvious resonances in Db major. The option to slightly raise the C# and G# for dramatic effect would of course still be available to an artist, but the point here is to illustrate that even unaltered, the system satisfactorily serves the aesthetic, as well as intonation requirements of the piece.

Evaluation and application: Strauss, *Don Juan*, Op. 20, rehearsal F – 21 m. after F

Example 6: Strauss, *Don Juan*, op. 20, rehearsal F–21 m. after F





If any one passage in the orchestral double bass literature represents difficulty in intonation, it would most likely be this passage. While there are some very difficult passages in the literature, the most striking features here are not only the hyperchromaticism at the surface level of the music, but the difference in the perceived single-line harmonic implications versus underlying contextual harmony (if one can really hear that in the first place), both of which contribute to the difficulty of solidifying and replicating an acceptable intonation for the passage. A further complication is that the most difficult parts of the passage begin to lie in the range of good pitch detection for most listeners; no hiding behind the low notes here! The system in this case provides stable reference points to create a relatively stable tonal structure when played as an unaccompanied line, which is also embellished with chromatic passing tones.

In unraveling the tangle of suspensions, anticipations, and non-chord tones presented in the orchestral context of the passage, the harmonic progression is not terribly complex. The passage begins fairly clearly on iv, moving to V/V in 6 after rehearsal F, to V^7 9 m. after rehearsal F. The next move in 13 m. after rehearsal F is doubly deceptive by flowing to an extended elaboration of vi in the guise of V^7/ii, but, instead follows the pattern of secondary dominants leading to an authentic cadence. The passage slides chromatically from V^7/ii to i, in one of the more anguish-ridden moments of the piece.

In comparing the contextual analysis to the single-line perceptual analysis, the divergence quickly becomes apparent. Looking at the single line alone, a quasi-normal tonal progression elaborating, if not outright tonicizing, vi can be observed. C# minor is established in the same manner, but in 6 m. after rehearsal F, we don't have any perceptual cues telling us that the C# presented in the foreground no longer represents the C# minor chord but now belongs to the F# major chord. Perceptually, the C# of 6 m. after rehearsal F still is strongly associated with the C# minor chord. The outlining of the F# octave in 7 and 9 mm. after rehearsal F satisfies an expected tonal possibility of standard circle of fifths motion, and the chromatic passing up to A in 9-12 mm. after rehearsal F serve to confirm F# minor as the active chord. This again stands in contrast to the contextual analysis, as the B making this section part of the V/i family is not present in the single line, and there is a perceptually satisfying simpler solution to the quandary of key finding in this passage.

In 13 m. after rehearsal F, that there is a chord change is fairly unmistakable, but its assignment as to what it should be is fairly weak. The entrainment to this point has been that the root of the chord occurs on the downbeat. The G# suggests itself as the root for this reason and that it satisfies the underlying tonal progression schema, but it is admittedly a weak assignment based only on there being few perceptual alternatives. This assignment is further complicated by the occurrence of the following B♮ and E#. Following the harmonies strictly, a B# would have been expected if the G# were acting as V/vi. However, the use of B♮ also follows the established pattern of only ascending a minor third across the chromatic intensification and might even be read as part of the chromatic foreshadowing of the collapse of the C# minor tonicization into the main tonic of E minor in m. 21 after rehearsal F. Following the downbeat entrainment strictly, an E# chord would be suggested 15 after rehearsal F. Belying this is the interval at the surface level of a tritone between the E# and the previous B♮, which schematically desires to be resolved, in this case upward, starting a sweeping chromatic rise into the G# of 16 after rehearsal F. Instead of being a chord-tone of V^7/ii, the E# can be seen as a further chromatic intensification, pointing upward toward and solidification of G# as the local chord center. The G# then falls as one would expect a dominant to do, by perfect fifth, to the tonic of C# for the excerpt.

These preceding four bars are interesting because they illustrate the levels of perceptual control of given schemas, the propensity for schemas to fill in unrequited data, and the ability to look back, allowing sense to be made of initially incongruous data. The G# chord locally controls 13-16 mm. after rehearsal F in part because of the entrainment mentioned above, but also because a better chord does not suggest itself to wrest control from G#, and a G# chord fits within the expectations of a progression in C#. While the broader expectation from a G# chord in a C# key would be for a B#, the B♮ presented still satisfies the expectations of a chord rooted in G#. So, locally the "G#-edness" of the passage is satisfied, and it is of little overall consequence in this case. A B# is most important linearly, as it would lead melodically to the tonic, but in this case, the B♮ does not lead to the tonic; it is, in fact being left hanging, unresolved through the passage and afterward. Because the B♮ still contributes to the local G# centricity, when the G# falls to the C#, it seems like a perfectly natural dominant-tonic function. Also because of the downbeat entrainment, there is a good bit of confusion created by the E# of 15 m. after rehearsal F. Not only does it create a surface tritone with the B before, but it is also outside of the schema of both G# and C#, and not strongly suggestive of a closely related schema. In fact, left to its own devises, the E# could lead to an abrupt failure of both the G# and C# schemas, starting the search for a new tonal center. The immediate passing of the E# to F# relives the immediate psychoacoustic tension by closing to a consonant interval, supplanting the trace of the E# with F# against the trace of the B, and allows for the pattern to unfold, revealing the G# at the end. Having returned to the G#, confirming the present chord and elaborating the broader harmony, the schema can look back at the preceding 5 pitches, which are still active in STM, and closely relate the B, which may even still be somewhat active in echoic memory through recurrent looping, to the G#, therefore reaffirming the G# tonality, and confirming the E#-F#-F-double-sharp figure as a chromatic figure leading to and pointing out the G#.

So, if C# controls the passage, what then controls the C#? The short answer is the E-string. By the time Strauss penned *Don Juan*, equal temperament had been firmly established in theory, if not totally in practice, and as such, the piece was likely written with that temperament in mind, especially given its intense chromaticism. However, if this passage were played in as exact agreement with the tuner as possible, I would argue that a great deal of resonance and clarity would be lost, especially on the C#'s and G#'s, leading to a less than positive judgment of such performer's intonation. Aligning the C#'s and G#'s in particular with the resonances of the E-string would ensure great resonance and stability of pitch throughout the excerpt. This would of course put the C# and G# as an acoustically pure fifth, in the lower position relative to the overall string length, and the C# in acoustic agreement with the A-string as well. This alignment results in a beneficial relationship to F#. When the F# first occurs in 6 after rehearsal F, it is a direct perfect fifth against the C#, which by most methods of fingering the passage would still be acoustically active and benefits from pure agreement. This again puts the F# in the lower position and also generates a pure octave with the following F#, which is placed in direct acoustic agreement with the D-string. This serves to outline the key of C# minor with the pillars of the key, I, IV, and V, being at acoustically pure distances, and the third also at an acoustically pure position, creating a stable tonal environment, brilliant resonance, and stable, easily recognizable reference points.

The remaining chord tones, A and B, are pretty well addressed with the A resonating with the D-string and the B with the E-string. In both cases, the acoustic congruence of the minor third is honored with the slightly larger spacing resulting from the lower F# and G#. The chromatic passing tones are a slightly harder matter and cause much of the consternation in this excerpt. Much of the issue is resolved simply by having the chord tones outlining the stable points of the passage. This way, pitch drift is far less likely to occur. The chromatic passing tones can then fit between the chordal "goal posts" at each end. Of the two chromatic pitches within each group of four ascending chromatic groups, one is a scale member elaborating the chord and one is a true chromatic passing tone. Ideally, one would be able to lock in the scale member, such as with F#, G#, or A, to an open string resonance, as it has more cognitive weight than the chromatic passing tone, and fit in the true chromatic passing tone in between. However, this leaves D# unaccounted for. The D# in that register is not strongly associated with any open string. The chromatic passing tone in that group, however, is C-double-sharp, or D♮, which is strongly associated with the D-string. In this case, instead of using the scale member as the resonant tone, the chromatic passing tone should be used as it has such a strong association with the open string, and the scale member between the C-double-sharp and E.

The remaining problem child is the E#, which is perhaps the most difficult pitch to play in tune in the passage. This is understandable purely from the standpoint that the tritone is an extremely difficult interval to hear, is among the weakest of intervals as categorized by Rakowski,[402] and most often involves a shift between the B and E#. The easy answer is to treat the E# as an F and allow it to resonate with the A- and D-strings. In addition to the physical reference point, the higher F♮ would provide a natural pointer,

[402] Rakowski 1990

resolving towards the F# at the surface and emphasizing the F#'s deeper structural significance. The slower tempo of this passage allows the E# to be heard as a pointer to F# with increase affect as we wait for the resolution and allows for fine adjustment to allow this. So, thought of as a complex of three tones, the background structural interval could be heard as a descending perfect fourth, supported from below by a rising half-step, and targeted by a resonance point below the F#, rather than aiming for a descending tritone followed by a chromatic ascension.

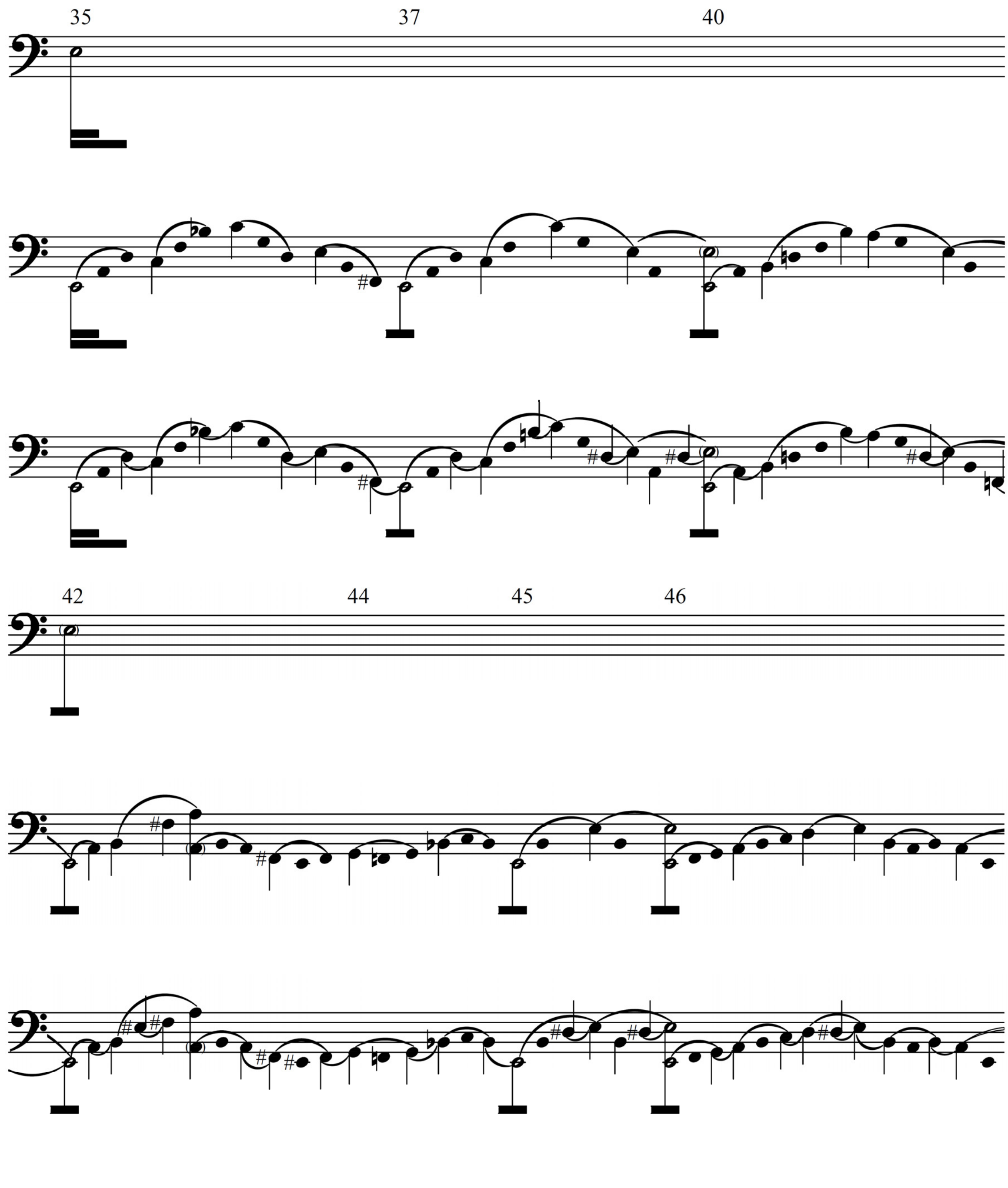
The cognitive underpinning for this lies in the higher hierarchical position of the chord tones, followed by the scale tones, then the chromatic tones, presuming no intervening timbral considerations. The chord tones must be in the more acoustically pure positions, as they are activated most schematically and are retained in more detail for reference. They are in turn the points of reference used to judge the overall intonation of the passage, provided the chromatics in between lie within the categorical bounds, and inter- and intrachord reference points are within the intonation horizon. The scalar tones can be "fudged" a bit more as their measurement relies almost entirely on note-to-note comparison of absolute pitch distance and categorized schematic comparison, without the benefit of substantive trace comparison. The chromatic passing tones would allow the greatest leeway, as they are activated only at a very low level, and would suffer the harshest judgment when not placed evenly between the scalar pitches. Further, they often occur on weak beats, which have the lowest level of attention and whose imprecise intonation is least likely to be noticed.

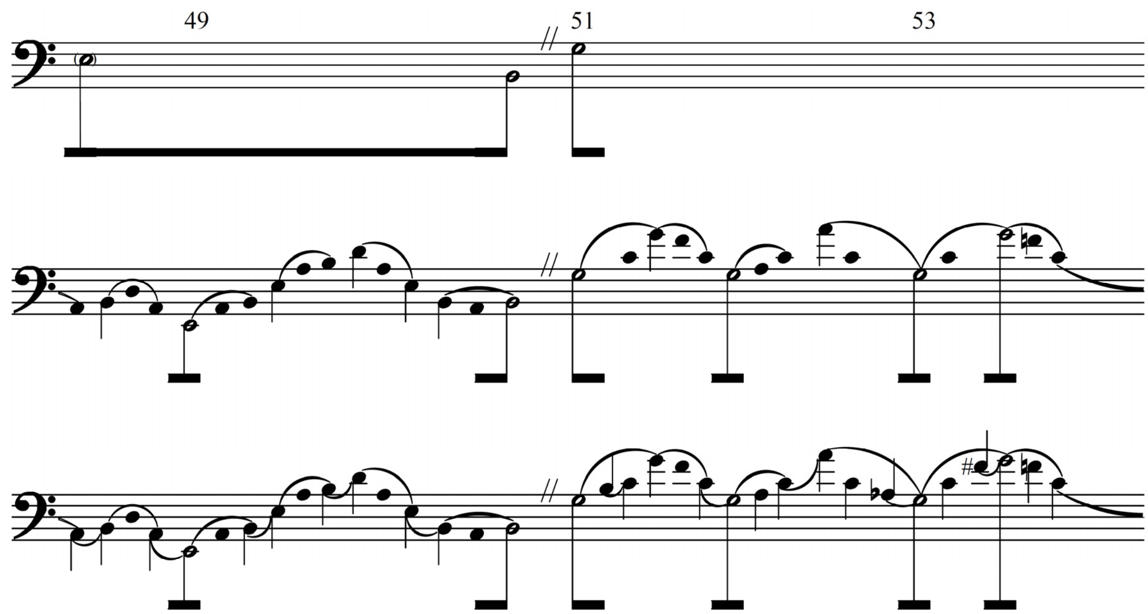
I realize that this puts me a bit at odds with Rakowski, who listed minor seconds as strong intervals.[403] A criticism relevant here is that Rakowski used total deviation in cents to generate his list. I argue that it is more appropriate to measure interval strength by its variance using the percent error rather than the total error, as he does. So a minor second might have a categorical center at 100 cents, ±5 cents, whereas a perfect fifth might have a categorical center at 700 cents, ±14 cents. Under Rakowski's analysis, the minor second would be seen to be a much stronger interval than the perfect fifth. However, in terms of percent error, the perfect fifth has an error of only ±2%, while the minor second has an error of ±5%. This turns the tables, with the perfect fifth being more stable than the minor second, which seems to align more with common practice.

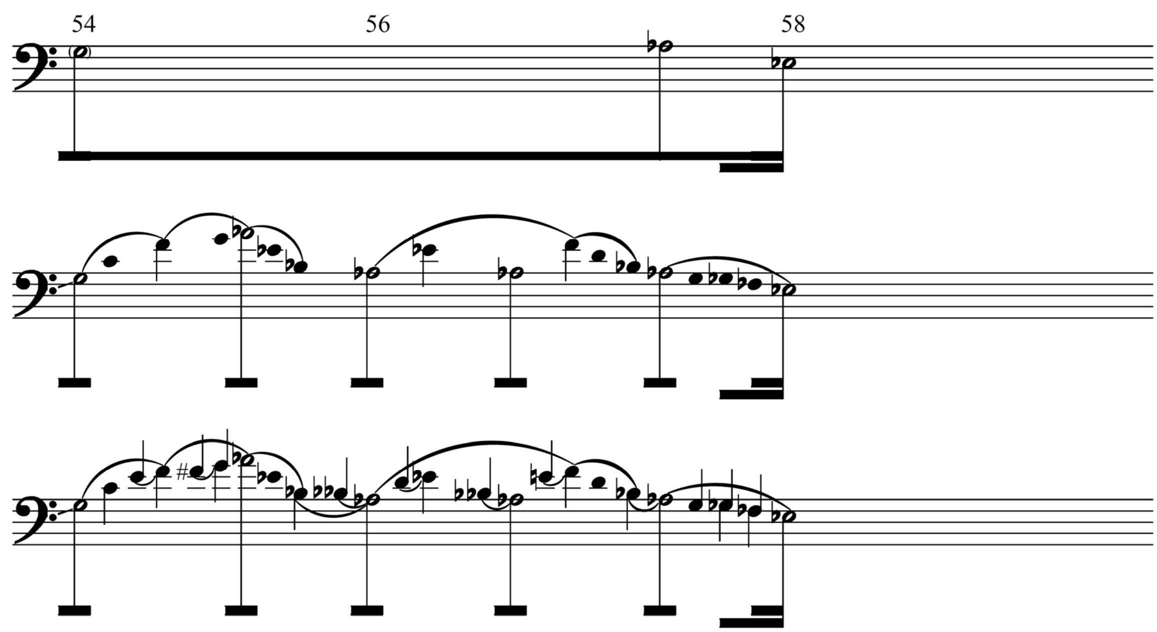
[403] Rakowski 1990

Evaluation and application: Bartok, *Concerto for Orchestra*, Sz. 116, mvt. I, mm. 35-58

Example 7: Bartok, *Concerto for Orchestra*, Sz. 116, mvt. I, mm. 35–58







One could safely say that the intonation is far down the list of concerns for this passage. Indeed, there are few very complex intonation choices to be made here. However, it does examine the system to determine how it performs with a modern, non-tonal piece. By the time *Concerto for Orchestra* was written, equal temperament had almost totally taken over the musical landscape, verging on universal acceptability. Compositional thought had moved from the purely functional tertian harmonies, embracing atonal techniques or at least novel ways of establishing key centers.

The construction of this passage is based on three note groups of perfect fourths linked together by a whole step, most clearly exposed in the opening mm. 35–36, though this principle is mutated through the course of the excerpt. Half-steps perform the particular function of the first note in the group pointing towards the resolving second note of the group, thereby serving to melodically emphasize more structural pitches. An effect is that the "pointer" elides two three-note groupings, as the pointer is not a structural pitch, but transfers the terminus of the one three-note group to the beginning of the next. For example, this occurs in mm. 38–39, where the B is a melodic pointer to the C. The half-step resolution makes the C more structural than the B, closing the group in m. 38 as it begins the group in m. 39. In comparing this to m. 35, this is also an example of Bartok slowly manipulating the melodic and structural material towards that which is used in m. 58.

The central pitch-class for mm. 35–50 is E, defined by its placement, emphasis, and references to tonal implications. The E is, of course, the lowest pitch sounding in the excerpt, as well as the pitch to which the line continually returns just about every other measure, forming a two-measure long hypermetrical grouping. In fact, every measure save two, mm. 43 and 44, contains an E of some register. The E's of the mid-staff are often emphasized by a preceding D# pointing to the E, as mentioned above, or in the case of mm. 41–42, the F is pointing down to the low E by half-step (the previous returns to E had been by whole-step and major seventh leap). A subdominant-tonic-dominant relationship is also gently whispered through much of the excerpt, but is most clearly seen in mm. 45–46, which contains a more direct V-I implication. Measures 51-58 have more transitory centers, moving from G, to Ab, and finally to Eb, but each center is established using the same tools, particularly the half-step emphasis.

With the pitch-class E clearly dominating much of this passage and one of the structural relationships being the perfect fourth, the usual concern of where the system would place the B and F# is equally clear: tuned with the E-string! This highlights that one of the key benefits to the system are purely tuned fourths and fifths across the fingerboard. This excerpt exercises any manner of fourths, played from the lowest to the highest strings, directly against one another and composed out over several notes. But, the cognitive, tonal, and acoustic stability of the acoustically pure fourth is retained through almost the full excerpt.[404] An interesting consequence of the system's predicted pitch placements is that a majority of the half-steps are of the smaller variety, which has directional and expressive implications. D# (as Eb) is tuned to the open G-string making the half-step between D# and E smaller; if B is tuned to the E-string, it is consequently close to the C, which is tuned to the G-string; Ab as G# is also tuned to the E-string, and therefore low against the open G-string; and finally, Fb to Eb is an inverted form of E to D#. What is interesting is that no affective alteration was needed: these narrow minor seconds are a natural resultant of the system and incidentally would be fairly close to

[404] Two possible exceptions will be discussed below.

their equally tempered counterparts. Similarly, most surface-level major thirds are also fairly close to an equally tempered value and major seconds tend to be of the nearly equally tempered size. While at this point one may expect me to rail against the acoustic impurity of the larger intervals, I take a different view: the system is mimicking the function of equal temperament, which is completely appropriate given that this piece was conceived in an equally tempered world and the piece is structured more around the perfect fourth than the third. Here, the system retains the acoustic consonance of the fourth at the expense of a larger third, but that is exactly what equal temperament does and the major thirds are almost always melodically active in this piece, pushing out towards a perfect fourth, as in m. 45-46.

While I would argue that the system works well with this particular piece of literature, I by no means say it responds perfectly! In m. 42, we have a slight bit of a conundrum of how to interpret the F# as it relates to the notes around it. The A's and B's are particularly fixed to the A- and E-strings, respectively, as the E# is to the A-string, which leaves the question as to which governs the placement of the F#: the B structurally or the A acoustically, and, where does that leave the relationship between the E# and F#? As a precedent, the B had been set to the higher position tuning with the E-string, which the F# would generally follow. Measure 40 began a series of measures in which groups based on A or B alternated. So structurally, the B and the F# are much more closely associated than the F# and the A, which belong to structurally different groupings, and thus, the acoustic preference would go to relating the B and F# because of their structural connection, as well as the tighter tolerance for perfect fifths over minor thirds. Perhaps not so incidentally, the higher position on the F# makes the minor third between it and the following A much closer to the equally tempered value.

The positioning of the F# in the higher position does leave a bit of a quandary about the size of the minor second between E# and F#, which would be a larger minor second, as predicted by the system, as would the minor second between E and F in m. 54. In both cases, a determination needs to be made as whether those minor seconds are directional. The E# is hard to hear as structural, as every other chromatically altered fourth has been directional, and the one must ask if Bartok would have really used a C major$^{6}_{4}$ chord at that particular moment in time, especially if there has been recent precedence for a G-C-F structure. To practically apply Fyk's intonation horizon model,[405] the difference between a large or small minor second would not be that great. In both cases, the intonation of the structural elements surrounding the passing tone is much more important to the judgment of the intonation of the unit. As both the large and small minor seconds fall within the categorical bounds and the structural elements are acoustically tuned, the intonation of the unit will at least reach the "acceptable" level. However, to illustrate how the system is only a basic starting point of how to listen to intonation and execute acceptable intonation, an astute listener/player might notice that the absolute size of those two minor seconds does not match the previous examples of functionally similar intervals. Even though the two pitches in question may conform to the system, an artist-level player may decide to break the system here to achieve a more consistent, unified presentation by decreasing the size of the minor second to match those of the previous directional minor seconds.

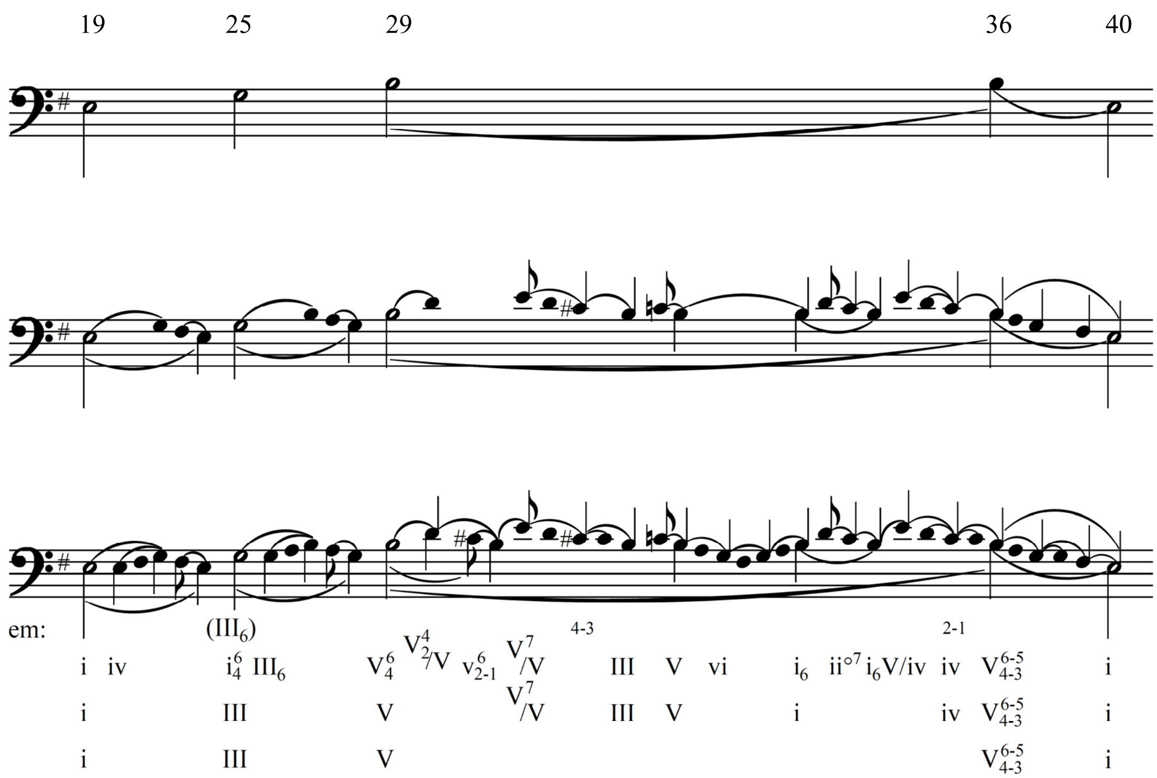
[405] Fyk 1995

The Eb's in mm. 55–56 more fully break the positioning predicted by the system, but this instance further illustrates the flexibility and pedagogical power of the principles underlying the system. As we've seen multiple times, Ab's tend to be positioned to resonate with the E-string; however, when juxtaposed with an Eb typically placed by the G-string, the resultant interval is a fifth plus a syntonic comma. In this case, the Ab is the more structural of the two pitches and will tend to be given tuning preference. This places the player in the position of choosing to accept the predicted position of the Eb or seeking a new one. Accepting the predicted position would tend to be a less favorable choice: the Eb is tuned to the open G-string, which is currently closed by the Eb and very little alternative resonance is available to ameliorate the acoustic and psychoacoustic disturbance caused by the incongruous Ab/Eb combination. However, breaking the placement of the system to put the Eb in a position acoustically tuned to the Ab would in effect be consistent with the spirit of the system. Foundationally, the system is designed to allow maximal resonance through the agreement of the overtone structure of two pitches, and tone placement is designed around allowing a stopped tone to agree with an open tone. By tying the Eb to the Ab, we have essentially substituted the physical resonance agreement of an open string for the psychoacoustic and residual acoustic agreement of two closed pitches. Having practiced tuning tones to be in acoustical agreement with an open string, the premise should be implanted of how two consonant intervals should agree, even if not directly resonating. Thus, the ear should be trained to "be the guide" when it detects, even at a subconscious level, that two structural tones are not in agreement with each other.

This excerpt is perhaps the most powerful one of the group examined, not because of how well it fits the system, but because of how it doesn't fit and how the underlying principles can be used to solve those discrepancies. That said, the system does hold up remarkably well for the excerpt as a whole: out of approximately 130 pitches, there were only about 5 discrepancies which required solution, and maybe 5 more which could benefit modification. Even taking 10 discrepancies, a 92% "batting average" is still much better than my hoped-for 85%! While the system isn't perfect, it still is remarkably workable for a non-tonal piece.

Evaluation and application: Koussevitzky, Concerto Op. 3, mvt I, mm. 19-40

Example 9: Koussevitzky, Concerto Op. 3, mvt I, mm. 19–40



This is a piece theorists and audition committees fear to hear: theorists because, as one theory professor allegedly put it, "it is the worst piece of tonal music written," and audition committees because it is so often wretchedly out of tune. While I have tried and failed to defend against the former, the latter is a bit more manageable. Many know Koussevitzky as one of the master conductors of the early 20th century, but most bassists know him as the foremost double bass virtuosi of his time. His concerto was written in a highly Romantic style for a modern double bass. It seems as if the placements predicted by the system serve the dual purpose of affording the maximal resonance of the instrument for the passage and simultaneously maximal affective conveyance. While we are not able to ask Koussevitzky about these points, as a master of the double bass and purveyor of the Romantic style, I find it hard to believe this a coincidence. As a note, I will reference the bass part at its notated pitch and the accompaniment by Roman numeral to avoid too much confusion about the double bass solo tuning versus standard orchestral tuning.

The larger arch of the primary theme is rather obviously elaborating E minor. At the foreground, the key is centered with melodic material outlining $\hat{1}$, $\hat{3}$, and $\hat{5}$, and at the background level, the melody arpeggiates up to $\hat{5}$ and returns to $\hat{1}$ via scalar motion. It should come as no surprise at this point that the E-string will control a large portion of the intonation of this passage. The primary assistance that the system can give to a performer here is practicing for resonance. The particular set of tones in this register have fundamentals which are in the range of strongest partials of the E-string and as the fundamental will be acting directly on the overtone, will cause a great deal of excitement of the open string. This is also true for the A’s, C#’s and D’s of the passage which will interact with the partials of their respective strings. The only tone which one would provide some difficulty would be the C♮ of m. 33, but this is a neighbor tone, in which one could read an expressive quality of pushing the line down and which would be accentuated with a duller sound. Its surrounding tones would likely be heard as very resonant, if not brilliant, due to the sympathetic vibration of every other tone. Having the E as the center would be supported by the continual return to the same pitch and resonance, the B being an acoustically pure fifth away, the A, a pure fourth, outlining the pillars of the key and preventing pitch drift, provided the performer returned to the same resonance each time.

A consequence of the assertion that the E-string controls the intonation of the passage is that placement predicted by the system would support at least one expressive reading of the passage. The opening has been read as a call of sadness or of mourning, reflected generically in the minor tonality. A traditional expressive gesture to further convey that feeling would be to compress the interval between the E and the G to a smaller size than its acoustically pure counterpart, tuned by the alignment of the B overtones of each fundamental. Here, the system predicts the small size, as the E is placed higher when tuned to the E-string than it would be if tuned to the G-string. Since the reference point is the E, it sounds as if the G is being kept low, conveying the feeling of sadness.

With the arpeggiation to G, the mood in the accompaniment lightens somewhat, though it can't quite seem to decide if it wishes to move to a more subtle version of i or fully to III. However, the system helps settle a bit of the discussion in the solo part at least with the larger major third between the G and B. This larger third is often heard as being brighter, if not outright happy, conveying a brief levity implied by a move to G major. However, this gayety is overtaken by not just a move to V, but a move to v^6_4 in m. 29. Not only was the schematic expectation to return to i or at least V thwarted, but the use of $\flat\hat{7}$ of E minor in such a prominent position has been read to be particularly anguishing. Again, the system predicts a lower placement of the D relative to the B, creating a small minor third to accentuate the affect conveyed.

Chapter 6: Conclusion and Ideals for Further Research

"I'll tell you what my advisor told me: it's a *document*, not a dissertation" –Tom Weiligman, Orchestra Manager, Indiana University-Bloomington

For what is supposed to be a document, this study has expanded beyond reasonable expectations of that term, as most who advised me *not* to undertake this subject knew it would! However, unlike many other instrumental disciplines, the double bass has very little writing about how to connect the dots of ear training, finger placement, and tone production, and how these three basic components interact to form this broad concept we know as intonation. Combining this with my personal struggle to develop acceptable intonation in my own playing led to a compelling interest in developing this study. I selected a cross-disciplinary approach involving acoustics, psychoacoustics and cognition, performance practice, and musical pedagogy in an attempt to be informed about as many aspects of intonation as possible in order to derive a systematic approach to acceptable double bass intonation based on those findings.

Acoustics taught us about the basic interaction of waves in fluid materials, and particularly about tuning and instrument functioning. Psychoacoustics I defined particularly as the early processing from the bottom-up of incoming acoustic signals into a raw neural signal, generally without the aid of memory-based, learned, categorized processing, generally defined as cognition. This neat division is somewhat oversimplified, but then again entire books could be written on the psychoacoustic and cognitive sides of intonation and their precise interaction. This partly explains on multiple levels why such flexibility of intonation exists and why we have emotional reactions to intonation. The study of performance practice led to insight of issues of temperaments employed at various periods of music, the function of bass lines and their harmonic implications, how performers of various periods and instruments viewed the concept of intonation, and how the lineage of the double bass could influence its performance today. In close conjunction, pedagogy was able to tell us how different instrument groups thought about intonation, how improvement was achieved, and particularly of the influence of the various models of intonation on the violin family.

The systematic approach that emerged was based heavily on the acoustic agreement of the overtones of stopped tones with the overtone series of the open strings (resonance system), the low-memory loading caused by lack of neural beating, the harmonic and relatively non-melodic function of tonal bass lines, the categorical latitude afforded the intonation of intervals whose tones were considered of good quality, the general guidelines of a model of acceptable intonation, and what would return a stable reference pitch for a given passage. The system was also heavily informed by the performance practice standards of a given piece. It was designed to establish a pitch which would generally be considered acceptably in tune within a given context.

However, let me reiterate: it's a *system*, representing a complex set of variables in simple terms in an attempt to allow an individual to comprehend the entire system at once. Details are rendered out and simplified. Predictions based on them may not come to fruition in the real world exactly as envisioned: just look to your local meteorologist's weather models. I don't claim that the intonation system presented here will return a perfect solution for every situation. But, it allows for a starting point and returns a theoretically acceptable result in almost every case when applied to actual literature. In an actual performance, where detail can be included and comprehended as the piece unfolds, the predicted tone may prove inadequate, but it will at least be close to the needed tone and generate a resonant timbre. Fyk makes a distinction between static and dynamic melodic intonation, between ridged, predetermined tones and a system with relatively fixed points of reference and flexible interior positions, and makes a strong argument for the dynamic model being the contemporary artistic standard.[406] While my system is admittedly on the static side, the point of its development was to fill the pedagogical gap I perceived between the broad within-category placements, which might or might not acceptable to a discriminating auditor, and the artistic-level, dynamic intonation.

Pedagogical implications: a possible course

One of the next logical steps is pedagogical implementation. Instead of developing a year-by-year syllabus for teaching the system, I prefer to think of stages of development, as so much of the reproduction task of "correct" pitch is interdependent with the pitch discrimination and recognition tasks which are the domain of ear training,

[406] Fyk 1995

more or less formal as it may be! As such, I hesitate in this study to proscribe too much of what the pace of intonational development "should" be, as it in part depends on the pace of development of the student's tonal categories. However, proper and consistent implementation of the system should also aid in establishing those categorical boundaries and centers by the constant repetition of acceptable pitches, both by listening/identification and reproduction.

Stage 1 would involve focusing on stimulating the unisons and octaves of the open strings. Because of the wider bandwidth of the fundamental and second harmonic, this is easily accomplished with even the most basic of students. One teacher even refers to it as her "magic trick" to her pre- and early teenaged students.[407] This is also a lower level cognitive task, as the student is only asked to discriminate if the pitch played is the same or different from the reference pitch, without really being asked to describe the absolute distance from the reference pitch.

Stage 2 would involve the tuning of the fourths/fifths of the open strings and octaves of the open strings, which also have relatively wide bandwidths, but tighter than the open fundamental. This is also a slightly higher level of ear training as the student must be able to discriminate the size of the interval from the reference pitch to the target pitch and have a conception of what a perfect fifth should sound like acoustically and categorically. Fortunately, this task is still basically a bottom-up processing question, but it is more complex than a simple pitch discrimination task.

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[407] Personal communication D. Taylor, December 2013.

Stage 3 would begin the tuning of the thirds/sixths. At this point, several difficulties arise as the tolerances for thirds acoustically decrease and cognitively increase. In order for a student to successfully execute this stage, he or she must have attained enough skill to execute a finger placement in a very narrow window of overtone alignment and have very sensitive discrimination and identification abilities to pick out the disturbance that is a misalignment of partials 5 and 6. In terms of absolute size, while fourths and fifths have a relatively narrow range of acceptability thanks in part to a preponderance of acoustic cues, thirds have a much larger range, and some experience is needed to determine how large a third is because of the relative lack of acoustic cues.

Stage 4 would begin the process of breaking down the "put finger here" mentality of a fixed-pitch methodology somewhat implied above by having the students tune pitches to the tonal center of a passage, which may or may not align with an open string overtone and not lie neatly on a position predicted by the system. Adding yet another layer of ear training, the student must be able to visually or aurally identify at least the local key center, which implies the development of a strong tonal schema, and compare the target pitch to the reference pitch in a more abstract way than comparing it to an open string. A perfect example of this is the modification required of the Eb in the Bartok example above: had the internal reference not changed from the G-string to the Ab, the fifth between the Ab and the Eb would have been large by a comma, which would be unacceptable from the listener's perspective. To alleviate that, the system needed to be broken to accommodate the modification of the Eb to a standard closer to an acoustically pure fifth from Ab.

Stage 5 would include tuning of dissonant intervals. I use "tuning" here loosely; as the adage goes, the tones of dissonant intervals don't just disagree, they disagree in a very particular way! Again, it takes some experience to understand how the tuning of an augmented fourth and a diminished fifth differ, both in terms of identification and discrimination. This also begins a bit of a discussion of functional (expressive) intonation, as not all minor seconds are created equally, for example. This in part takes some additional understanding of written theory or very good ear training with an emphasis on normative tonal functioning.

Stage 6 would introduce expressive variants explicitly. While some could be introduced earlier, I prefer to save them for the end because at this point a student should have a fairly set grasp of what is a normal execution and what deviations can be made for an expressive purpose. The problem I see with exposing students to expressive variants early on is that they lack the sharply defined categorical underpinning to really know what is expressive from what is just out of tune. Recall that to an expert listener a 16-20 cent deviation could be either an extremely expressive gesture or simply out of tune depending on the context of the tonal function at that particular moment. Prior to this point, a student would not necessarily have the experience to discriminate the difference, which is why I place the expressive variants after the student goes through much grounding in various degrees of intonation certainty. Even so, the teacher, with expertise far beyond the student to that point, would be needed to guide the student through what is not only acceptable, but artistic.

Another general principle which can be observed from the application of the system and which would have bearing on the interim steps of the system's pedagogical implementation is that Flesch's previously mentioned observation of the lower, acoustically-tuned position of two tones is normal, also applies to the double bass. F#'s, C#'s, B's and Ab's tended to be positioned in the lower position and Eb's and Bb's tended to be higher than expected when applied to real literature as a consequence of how the keys and interval patterns used fit into the resonances of the double bass. That said, I would still contend that students often do not play F#'s high enough nor Bb's low enough, for example. While some tones the system generates do not exactly conform to an equally tempered scheme, they at least fall within the categorically acceptable range, whereas an out of tune pitch falls outside the categorical boundaries. From a teaching perspective, the point would be to train students to place tones in a nominal position which would offer flexibility to vary the pitch as needed for a given musical situation instead of a constantly high tone or half position fingering which abuts the nut. Are there occasions in which those placements might be useful? Certainly, but to use them all of the time would deprive them of their greater potential.

What I have proposed above is only the barest of outlines of a pedagogy of the system; a comprehensive discussion would be my next document! But, I do think the proposal sets out how one might approach teaching the system and intonation in a more systematic way than has been outlined in the current double bass pedagogical material. As outlined, the approach is still, and may always be, heavily dependent upon the vigilance of the teacher in enforcing not only correct placement of the resonant tones in question, but of supporting the intonation of the tones not yet covered. The pace of a student's physical technical development could and probably would outstrip the growth in ear training. For example, F♮s will probably appear in a student's technical and orchestral literature before getting to stage 3, where thirds and sixths are placed. It would be up to the teacher to either enforce the idea that the F♮ is a perfect fourth away from C or to offer "make it higher/lower" corrections to the student, which, as noted, remains one of the most long-term effective methods of bettering a student's intonation. Part of the teacher's role is to offer, iterate, and reiterate exemplars of both absolute pitch distances, point out how various tones agree or disagree acoustically, and how to use that to one's advantage in performance.

Lastly, if the principles underlying the system are followed, the student will more quickly learn to play as in tune as possible with an ensemble. The primary consideration in ensemble intonation is matching the overtones of the bass with the extant tones of the ensemble. Training in the resonance system will enable the student to learn to listen to the overtones which are in the same register as the other instruments and the need to agree with those tones.

Pedagogical implications: tools of the trade

I have never claimed that my idea for a system of double bass intonation was particularly new to double bass performance practice specifically or string pedagogy generally, but I reasonably believe I am the first to assemble the elements into one system. Many of the tools already in use to develop good intonation, such as scales in major thirds, actually return harmonically pure, small major thirds, so the concept of their use and acceptability is not new. However, to really think about intonation in terms of it being acceptable when the overtones are aligned to encourage maximal resonance is not something I have seen in printed double bass pedagogical literature.

The most important tools are the teachers. They have the experience to know what a given interval is supposed to sound like, how the bass is supposed to ring, and how the overtones are supposed to agree. The student, by definition, doesn't know that initially and must be given exemplars to establish the categorical bounds of pitch distance, tone quality, and acceptable roughness. The student must also be taught how to work internally with the measurements of vibrating air we know as divisions of the octave, how to identify a pitch, how to discriminate one pitch from another, and how to reproduce a desired pitch. This process, broken down into its cognitive components, is the particular subset of ear training most commonly known as audiation. A student must have a general idea of where he or she is going in order to get there (i.e., arrive at the desired pitch within the categorical bounds), and the system could then be used to fine-tune the final finger placement. As another common adage goes, "if you can sing it, you can play it," may be true provided one can discriminate between the internally audiated desired pitch and the pitch of the tone physically produced.

One tool which has yet to receive much serious work is the tactile feedback through the bow and the left hand. As noted, the paper by Askenfelt & Jansson showed that vibrations through the bow are of a correct frequency and of great enough extent to be perceived by the hand.[408] This lends credence to much of the anecdotal evidence of performers who say that the instrument doesn't "feel right" when it is not played in tune.[409] Mistuned partials of a tone and a sympathetically vibrating string can produce a beating pattern which can be felt under the bow as an odd variable resistance pattern, an extreme example being bowing of a wolf-tone. Conversely, if the partials are aligned, the interference pattern dies out, resulting in a smooth feel under the bow.

Drones of one form or another have been a staple of intonation pedagogy, whether it be an organ drone, one generated electronically, or doublestopping either an open string or another stopped tone. This is of course using the principle of retaining a stable reference point and having acoustic feedback provided. What is so interesting about the resultant tones and intonation patterns which develop is that they reflect harmonic interval sizes, not expressive ones. The system very much resembles droning an open string without it having to be directly stimulated or adjacent at all. One might say that it is a "ghost drone" of only the part of the string's harmonic series needed to maintain reference.

The last refuge of those totally unaware is the ubiquitous electronic tuner. To soften my harshness a bit, for the times when one simply cannot get close to an intended pitch, or an interval is extremely hard to hear, or the registral extremes are causing interference, or if one just needs a quick tuning reference in a noisy room, it can be a

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[408] Askenfelt & Jansson 1992
[409] e.g. Vance 1984 and Tirado 2002

useful tool. But as I noted previously, every tool can be abused, and part of the impetus for this study was my perception of the student bass community's overreliance on the electronic tuner, turning an aid into a crutch.[410] If one finds a situation where one is totally at a loss about pitch orientation, a tuner can be a great aid to getting one's bearings. However, what I've seen occur more often than not is the musician's ears becoming totally swayed by the swing of the electronic needle and not developing an independent sense of what it means for something to be "in tune."

Ideas for further research

If you've read this far, it might be not too much of a surprise to learn that I have a few things to say about the state of intonation research. A constant tension in any field of research is between purely controlled academic research, and how things work in the real world. Intonation is no exception. There has been much truly excellent work done in our understanding of how tones relate and are judged in isolation, but so often in the past, these experiments were (at least seemingly) out of touch with the performance side of the music business. While this work absolutely needed to be done, the flavor of the results often seems to lack the sweet taste of the practical. I think we are to the point, both in terms of our knowledge base and technological analysis abilities, where we can follow the lead of Shackford, Fyk, Brown, and Leukel & Stoffer[411] in looking at intonation in performance both in terms of what we do as performers and what we accept as expert listeners, as well as a research line into practical intonation as opposed to abstract intonation.

[410] Before anyone accuses me of being too much of a Luddite, remember today's bassists are participating in an analogue art form with an instrument which basically hasn't changed in the last 2 centuries.
[411] Schackford 1961 and 1962, Fyk 1995, Brown 1996, and Leukel & Stoffer 2004

The first step in that direction would be to replicate the studies of Seashore, Greene, Shackford, Brown, Fyk,[412] or any study which has been in the direction of practical intonation in string playing in particular, with the benefit of modern audio signal processing. A very specific change in methodology would be to follow Sundberg[413] in use modern computing with Fast Fourier Transformation software to evaluate the tones by the region of the spectra from which pitch is derived, not pool all intervals by similarity of size but by similar function, and attempt to determine the influence of the open strings on intonation. Partly as a result of the technological aides of the time and partly under the assumption that the fundamental frequency could always stand in place of the perceived pitch, most previous studies used the fundamental frequency of a tone to compare to other tones. From Plomp and Levelt, we know that this is not always the case if the upper partials are not perfectly harmonic against the fundamental.[414] The problem is we just don't know if reading the upper partials for the pitch would change the results of the study. It might also be interesting to look at the envelope of relation, testing Fyk's intonation horizon theory, through this perspective.

Not pooling all similarly sized intervals, say major thirds as an example, together when conducting statistical analysis could have a major impact on the variance of the data sets. As we've discussed, not all intervals of the same interval class have the same function, and different sizes of the same interval class have different functional implications. In the past all of the data regarding the size of one interval class was pooled

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[412] Seashore 1937a and b, Greene 1936, Schackford 1961 and 1962, Brown 1996, Fyk 1982 and 1995
[413] especially Sundberg 1999
[414] Plomp and Levelt 1965

together and analyzed as a class grouping. It might be that the major third of tonic at the end of a piece is in practice verifiably lower than the major third of the dominant or of even tonic at the beginning of the piece, as Shackford implies.[415] Again, there is very little data available here, aside from a few bits saved in research and the anecdotal.

The specific role of sympathetic vibration in intonation performance has been speculated and hypothesized about by Fyk in particular,[416] but even how sympathetic vibrations change the sound of an instrument has not been rigorously measured in a performance context. Anecdotally, we all know it does and there is some correlation between spectral change and descriptor change, but its precise influence has not been measured. It would be interesting to measure the size of various interval classes in comparison to how they might be tuned to various strings.

More generally, there is good work to be done as to what the categorical bounds are for intervals to be acceptably in tune. As I noted, much of the work on the boundary issue has involved finding the limits of identification or categorical center/preference. From this perspective, Hall has done some good preliminary work, but as his experiments weren't designed to test specifically what the acceptability limits are, they can't be extended to answer this question.[417] A specific question to be answered is if the assumption about a person not being able to distinguish the intonation of a tone ±10 cents of the categorical center is indeed true universally for a musically trained population. Again, there is plenty of anecdotal evidence to support this claim, but the specific

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[415] Shackford 1961 and 1962
[416] Fyk 1995
[417] Hall 2002

research has yet to be done. Tied in with this line could be an analysis of the acceptability of expressive variants and an analysis of acceptability versus key area.

A couple very specific research questions concern how the lowest tones are heard, which generally is not a spectrum of the hearing range usually tested, i.e., below 100 Hz. One is the role of outer hairs in resolving, controlling, or confusing roughness caused by low tones and their role in low frequency pitch extraction. Because this frequency range is not one commonly used by auditors, even specifically most musicians, how we physiologically hear contra register tones has not been studied in detail and may shed new light on intonation performance practice for bassists. Quickly following that would be to question if bass players physiologically hear the contra pitches differently than most other auditors, using a lower part of the partial spectrum than current pitch extraction models would predict or using some other method. The converse, if treble instrument players hear the contra range differently, would also be of significant interest. Rosner has already observed that listeners "become inefficient" when listening to tones below 200 Hz.[418]

A line of thought which has come up in the literature is the difference in preference/acceptability of intonation of a known versus and unknown example, where the unknown example usually has a higher intonation acceptability tolerance. The question which occurs to me is if the known example generates a low-level audiation response, with specific pitch expectation associated with that, and if that is what causes the tighter intonation tolerances for a known piece. Assuming that is generally true, the

[418] Rosner 1994

extension would be to ask if that simply causes a categorical failure associated with a highly active prototype, or if it causes some form of neural beating at a much lower level. More generally, what is the role of audiation in our expectation and tolerance of intonation?

Of course, I would be particularly challenged to initiate and conduct a controlled study as to where the system is in fact viable in the abstract, hopefully with one run of just probe tones and another which would include a simulated sympathetic vibration. In practice, the system seems to work, both in my own playing and in my students', but to show its viability in the abstract would be a great boon.

The concluding remarks

I felt it was important to show the development of the system from root principles to its fleshed out form. Clearly there is still much to learn about how intonation actually works; however, the musical community has a good start on the project and its practitioners have been doing it for roughly five-hundred years in the tonal tradition. Based on the research literature and performance practice, I think I have developed a viable system of intonation for the double bass. I make no claims that it is the perfect or a static solution to all intonation quandaries, especially as I think Fyk is right that we have moved from an age of static intonation into a world of dynamic intonation.[419] It is, however, a starting place from which a dynamic intonation of the double bass can evolve.

[419] Fyk 1995

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Appendix A: Table of Ratios Predicted by the System Expressed in Cents

Note Name	Note Frequency (Hz)	Distance in cents				
		E1	F1	F#1	G1	G#1 low
E1	41.25	0				
F1	44	111.7313	0			
F#1	46.40625	203.91	92.17872	0		
G1	48.88889	294.135	182.4037	90.22498	0	
G#1 low	51.5625	386.3137	274.5824	182.4037	92.17868	0
G#1 high	52.14815	405.8663	294.135	201.9563	111.7312	19.55256
A1	55	498.045	386.3137	294.135	203.91	111.7313
Bb1	58.66667	609.7763	498.045	405.8663	315.6412	223.4626
B1	61.875	701.955	590.2237	498.045	407.82	315.6413
C2	65.18518	792.18	680.4487	588.27	498.0449	405.8663
C#2 low	68.75	884.3587	772.6274	680.4487	590.2237	498.045
C#2 high	69.60938	905.865	794.1337	701.955	611.73	519.5513
D2	73.33333	996.09	884.3587	792.18	701.955	609.7763
Eb2	78.22222	1107.821	996.09	903.9113	813.6862	721.5076
E2	82.5	1200	1088.269	996.09	905.865	813.6863
F2	88	1311.731	1200	1107.821	1017.596	925.4176
F#2 low	91.66667	1382.404	1270.672	1178.494	1088.269	996.09
F#2 high	92.8125	1403.91	1292.179	1200	1109.775	1017.596
G2	97.77778	1494.135	1382.404	1290.225	1200	1107.821
G#2	104.2963	1605.866	1494.135	1401.956	1311.731	1219.553
A2	110	1698.045	1586.314	1494.135	1403.91	1311.731
Bb2	117.3333	1809.776	1698.045	1605.866	1515.641	1423.463
B2 low	122.2222	1880.449	1768.717	1676.539	1586.314	1494.135
B2 high	123.75	1901.955	1790.224	1698.045	1607.82	1515.641
C3	130.3704	1992.18	1880.449	1788.27	1698.045	1605.866
C#3	137.5	2084.359	1972.627	1880.449	1790.224	1698.045
D3	146.6667	2196.09	2084.359	1992.18	1901.955	1809.776
Eb3	156.4444	2307.821	2196.09	2103.911	2013.686	1921.508
E3	165	2400	2288.269	2196.09	2105.865	2013.686
F3	176	2511.731	2400	2307.821	2217.596	2125.418
F#3	183.3333	2582.404	2470.672	2378.494	2288.269	2196.09
G3	195.5556	2694.135	2582.404	2490.225	2400	2307.821
G#3	206.25	2786.314	2674.582	2582.404	2492.179	2400
A3	220	2898.045	2786.314	2694.135	2603.91	2511.731
Bb3	234.6667	3009.776	2898.045	2805.866	2715.641	2623.463
B3	247.5	3101.955	2990.224	2898.045	2807.82	2715.641

	G#1 high	A1	Bb1	B1	C2	C#2 low
E1						
F1						
F#1						
G1						
G#1 low						
G#1 high	0					
A1	92.17865	0				
Bb1	203.9099	111.7313	0			
B1	296.0887	203.91	92.17862	0		
C2	386.3136	294.135	182.4036	90.22498	0	
C#2 low	478.4924	386.3137	274.5823	182.4037	92.17885	0
C#2 high	499.9987	407.82	296.0886	203.91	113.6851	21.50629
D2	590.2236	498.045	386.3136	294.135	203.9101	111.7313
Eb2	701.9549	609.7763	498.0449	405.8663	315.6414	223.4626
E2	794.1337	701.955	590.2236	498.045	407.8201	315.6413
F2	905.8649	813.6863	701.9549	609.7763	519.5514	427.3726
F#2 low	976.5374	884.3587	772.6273	680.4487	590.2238	498.045
F#2 high	998.0437	905.865	794.1336	701.955	611.7301	519.5513
G2	1088.269	996.09	884.3586	792.18	701.9551	609.7763
G#2	1200	1107.821	996.0899	903.9113	813.6864	721.5076
A2	1292.179	1200	1088.269	996.09	905.8651	813.6863
Bb2	1403.91	1311.731	1200	1107.821	1017.596	925.4176
B2 low	1474.582	1382.404	1270.672	1178.494	1088.269	996.09
B2 high	1496.089	1403.91	1292.179	1200	1109.775	1017.596
C3	1586.314	1494.135	1382.404	1290.225	1200	1107.821
C#3	1678.492	1586.314	1474.582	1382.404	1292.179	1200
D3	1790.224	1698.045	1586.314	1494.135	1403.91	1311.731
Eb3	1901.955	1809.776	1698.045	1605.866	1515.641	1423.463
E3	1994.134	1901.955	1790.224	1698.045	1607.82	1515.641
F3	2105.865	2013.686	1901.955	1809.776	1719.551	1627.373
F#3	2176.537	2084.359	1972.627	1880.449	1790.224	1698.045
G3	2288.269	2196.09	2084.359	1992.18	1901.955	1809.776
G#3	2380.447	2288.269	2176.537	2084.359	1994.134	1901.955
A3	2492.179	2400	2288.269	2196.09	2105.865	2013.686
Bb3	2603.91	2511.731	2400	2307.821	2217.596	2125.418
B3	2696.089	2603.91	2492.179	2400	2309.775	2217.596

	C#2 high	D2	Eb2	E2	F2	F#2 low
E1						
F1						
F#1						
G1						
G#1 low						
G#1 high						
A1						
Bb1						
B1						
C2						
C#2 low						
C#2 high	0					
D2	90.22486	0				
Eb2	201.9561	111.7314	0			
E2	294.1349	203.9101	92.17877	0		
F2	405.8662	315.6414	203.9101	111.7313	0	
F#2 low	476.5386	386.3138	274.5825	182.4037	70.67242	0
F#2 high	498.0449	407.8201	296.0888	203.91	92.17872	21.50623
G2	588.2699	498.0451	386.3137	294.135	182.4037	111.7312
G#2	700.0011	609.7763	498.045	405.8663	294.135	223.4625
A2	792.1799	701.9551	590.2238	498.045	386.3137	315.6412
Bb2	903.9111	813.6864	701.955	609.7763	498.045	427.3725
B2 low	974.5836	884.3588	772.6275	680.4487	568.7174	498.0449
B2 high	996.0899	905.8651	794.1338	701.955	590.2237	519.5512
C3	1086.315	996.0901	884.3587	792.18	680.4487	609.7762
C#3	1178.494	1088.269	976.5375	884.3587	772.6274	701.9549
D3	1290.225	1200	1088.269	996.0899	884.3586	813.6861
Eb3	1401.956	1311.731	1200	1107.821	996.09	925.4175
E3	1494.135	1403.91	1292.179	1200	1088.269	1017.596
F3	1605.866	1515.641	1403.91	1311.731	1200	1129.328
F#3	1676.539	1586.314	1474.582	1382.404	1270.672	1200
G3	1788.27	1698.045	1586.314	1494.135	1382.404	1311.731
G#3	1880.449	1790.224	1678.492	1586.314	1474.582	1403.91
A3	1992.18	1901.955	1790.224	1698.045	1586.314	1515.641
Bb3	2103.911	2013.686	1901.955	1809.776	1698.045	1627.373
B3	2196.09	2105.865	1994.134	1901.955	1790.224	1719.551

	F#2 high	G2	G#2	A2	Bb2	B2 low
E1						
F1						
F#1						
G1						
G#1 low						
G#1 high						
A1						
Bb1						
B1						
C2						
C#2 low						
C#2 high						
D2						
Eb2						
E2						
F2						
F#2 low						
F#2 high	0					
G2	90.22498	0				
G#2	201.9563	111.7312	0			
A2	294.135	203.91	92.17865	0		
Bb2	405.8663	315.6412	203.9099	111.7313	0	
B2 low	476.5387	386.3137	274.5824	182.4037	70.67246	0
B2 high	498.045	407.82	296.0887	203.91	92.17877	21.5066
C3	588.27	498.0449	386.3136	294.135	182.4037	111.7316
C#3	680.4487	590.2237	478.4924	386.3137	274.5825	203.9103
D3	792.1799	701.9549	590.2236	498.0449	386.3137	315.6415
Eb3	903.9113	813.6862	701.9549	609.7763	498.045	427.3729
E3	996.09	905.865	794.1337	701.955	590.2238	519.5516
F3	1107.821	1017.596	905.8649	813.6863	701.9551	631.2829
F#3	1178.494	1088.269	976.5374	884.3587	772.6275	701.9553
G3	1290.225	1200	1088.269	996.09	884.3587	813.6866
G#3	1382.404	1292.179	1180.447	1088.269	976.5375	905.8653
A3	1494.135	1403.91	1292.179	1200	1088.269	1017.597
Bb3	1605.866	1515.641	1403.91	1311.731	1200	1129.328
B3	1698.045	1607.82	1496.089	1403.91	1292.179	1221.507

	B2 high	C3	C#3	D3	Eb3	E3
E1						
F1						
F#1						
G1						
G#1 low						
G#1 high						
A1						
Bb1						
B1						
C2						
C#2 low						
C#2 high						
D2						
Eb2						
E2						
F2						
F#2 low						
F#2 high						
G2						
G#2						
A2						
Bb2						
B2 low						
B2 high	0					
C3	90.22498	0				
C#3	182.4037	92.17832	0			
D3	294.1349	203.9095	111.7312	0		
Eb3	405.8663	315.6409	223.4626	111.7309	0	
E3	498.045	407.8196	315.6413	203.9096	92.17921	0
F3	609.7763	519.5509	427.3726	315.6409	203.9105	111.7313
F#3	680.4487	590.2233	498.045	386.3133	274.5829	182.4037
G3	792.18	701.9546	609.7763	498.0446	386.3142	294.135
G#3	884.3587	794.1333	701.955	590.2233	478.4929	386.3137
A3	996.09	905.8646	813.6863	701.9546	590.2242	498.045
Bb3	1107.821	1017.596	925.4176	813.6859	701.9555	609.7763
B3	1200	1109.775	1017.596	905.8646	794.1342	701.955

	F3	F#3	G3	G#3	A3	Bb3
E1						
F1						
F#1						
G1						
G#1 low						
G#1 high						
A1						
Bb1						
B1						
C2						
C#2 low						
C#2 high						
D2						
Eb2						
E2						
F2						
F#2 low						
F#2 high						
G2						
G#2						
A2						
Bb2						
B2 low						
B2 high						
C3						
C#3						
D3						
Eb3						
E3						
F3	0					
F#3	70.67242	0				
G3	182.4037	111.7316	0			
G#3	274.5824	203.9103	92.17832	0		
A3	386.3137	315.6416	203.9096	111.7313	0	
Bb3	498.045	427.3729	315.6409	223.4626	111.7313	0
B3	590.2237	519.5516	407.8196	315.6413	203.91	92.17869

Table of Equal Temperament Intervals in Cents for Comparison

Note Name	Note Freq (Hz)	Distance in cents E	F1	F#1	G1	G#	A	Bb	B	C	C#	D	D#	E
E1	41.2	0												
F1	43.7	100	0											
F#1	46.2	200	100	0										
G1	49	300	200	100	0									
G#1	51.9	400	300	200	100	0								
A1	55	500	400	300	200	100	0							
Bb1	58.3	600	500	400	300	200	100	0						
B1	61.7	700	600	500	400	300	200	100	0					
C2	65.4	800	700	600	500	400	300	200	100	0				
C#2	69.3	900	800	700	600	500	400	300	200	100	0			
D2	73.4	1000	900	800	700	600	500	400	300	200	100	0		
D#2	77.8	1100	1000	900	800	700	600	500	400	300	200	100	0	
E2	82.4	1200	1100	1000	900	800	700	600	500	400	300	200	100	0

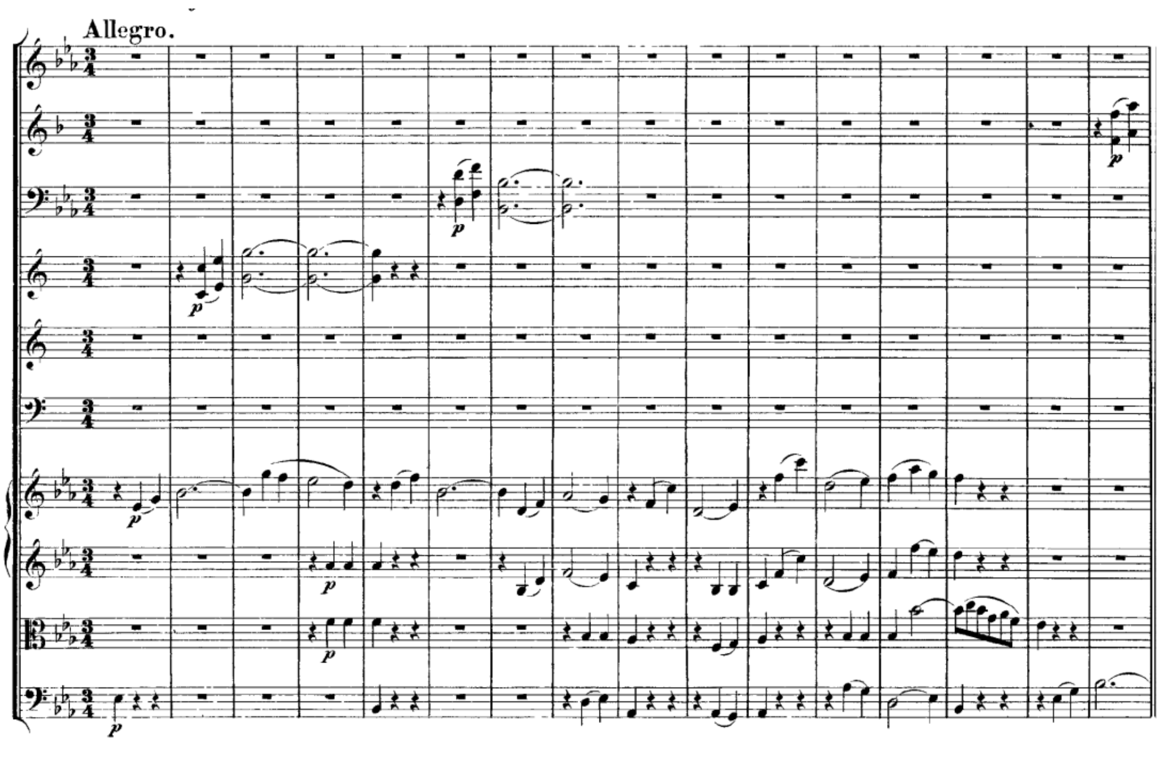
Appendix B: Scores of Musical Examples

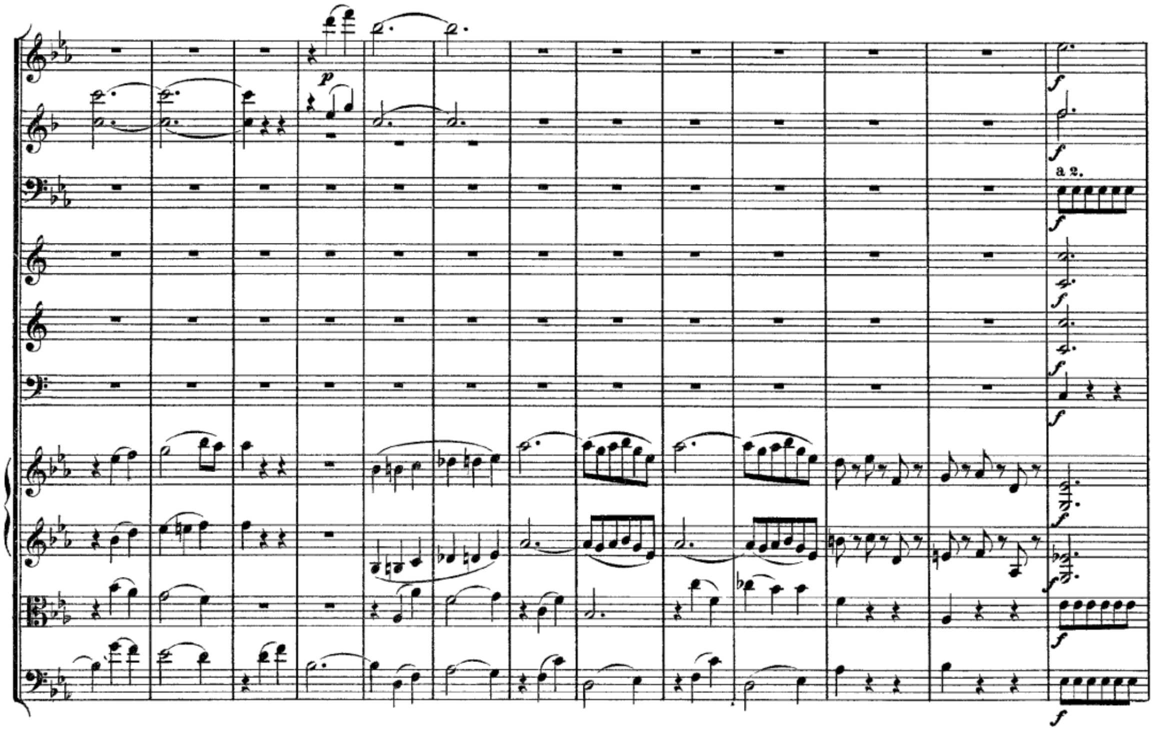
Mozart, Symphony No. 35 in D Major (Haffner), K385, mvt. IV, mm. 1–38 (Leipzig: Breitkopf & Härtel, 1880)


a 2.
a 2.


sf
p

Mozart, Symphony No. 39, K543, mvt. I, mm. 26–97 (Leipzig: Breitkopf & Härtel, 1880)





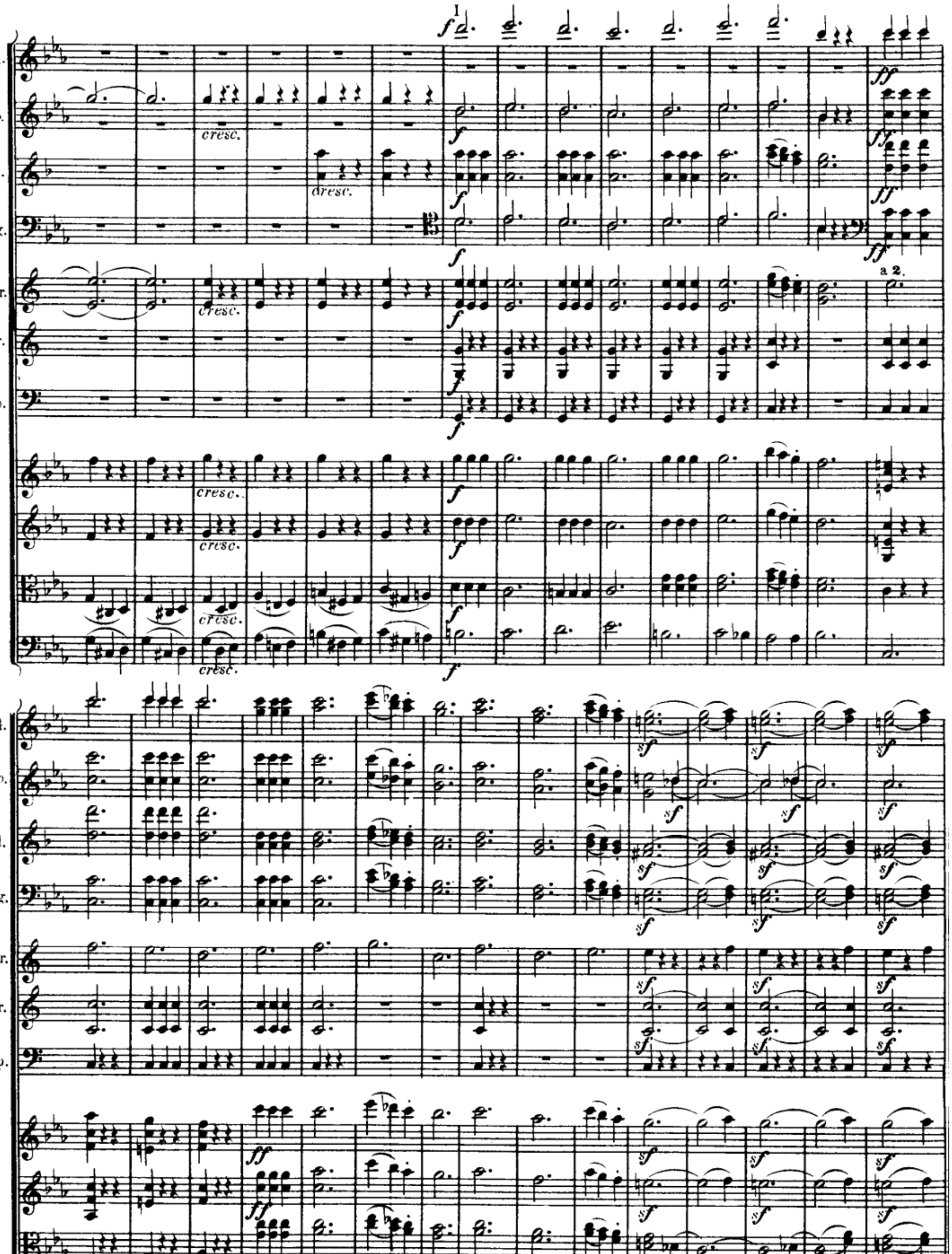

ten.
ten.
ten.
ten.
ten.
ten.
ten.
a 2.
tr
tr
tr
tr


a 2.


sfp
sfp
sfp
sfp
sfp
sfp
sfp
sfp
sfp
sfp
p
p
Cello
Bass

Beethoven, Symphony No. 5 in C Minor, Op. 67, mvt. III, mm. 1–97 (Leipzig: Breitkopf & Härtel, 1862)


Fl.
Ob.
Cl.
Fag.
Cor.
sf
dim.
pp
a Tempo.
poco rit.
I
II
p


Fl.
Ob.
Cl.
Fag.
Cor.
Tr.
Tp.
cresc.
a 2.


Fl.
Ob.
Cl.
Fag.
Cor.
Tr.
Tp.
sf
dim.
pp
p

Beethoven, Symphony No. 9 in D Minor, Op. 125 cello and double bass part, mvt. IV, mm. 65–75 (Leipzig: Breitkopf & Härtel, 1863)

Schubert, Symphony No. 9 in C Major, D. 944 (the Great), mvt. III, rehearsal B – 2 m. after C (Leipzig: Breitkopf & Härtel, c. 1850)


p
cresc.
cresc.
cresc.
p
cresc.
cresc.
cresc.
cresc.


C

Strauss, *Don Juan*, Op. 20, rehearsal F – 21 m. after F (New York: Dover 1979)


in E.
in E.
in E.
in E.
cresc.
espr.
p Solo
weich

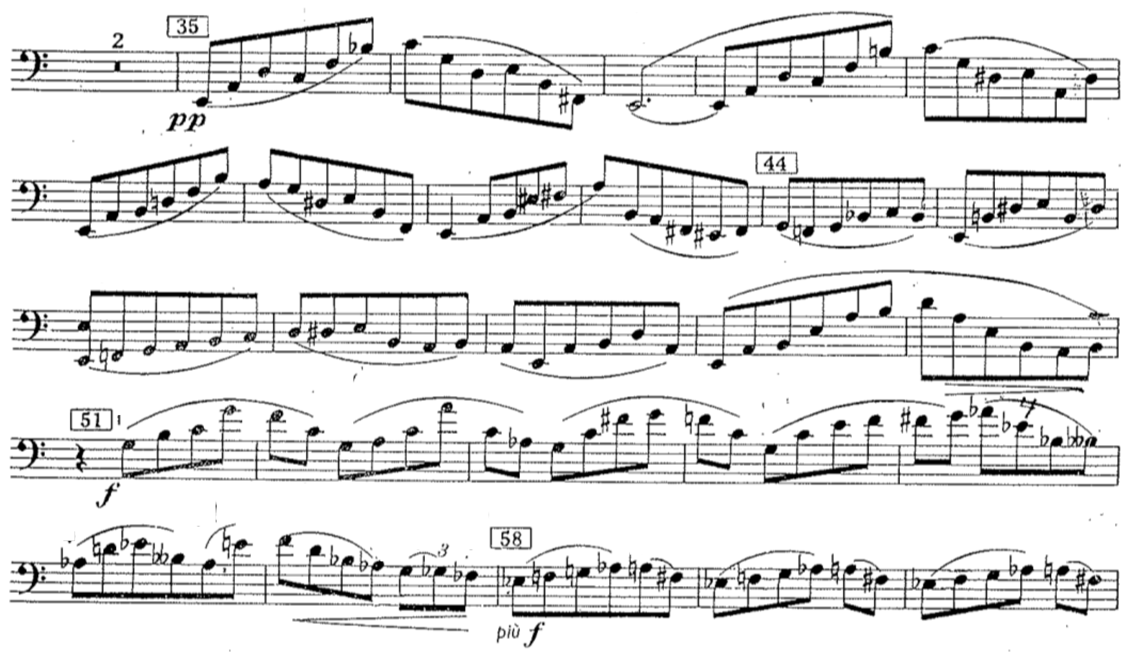

espr.
in E.
in E.
in E.
in E.
dim.
dim.
cresc.
Fis nach E umstimmen
espr.
cresc.


espr.
in E.
in E.
in E.
in E.
cresc.
mf
espr.
cresc.
molto appassionato e
sempre cresc.
sempre cresc.
cresc.
cresc.
cresc.


sempre un poco stringendo
espr.
cresc.
in E.
in E.
cresc.
in E. cresc.
molto espr.
in E.
sempre un poco stringendo
molto espr.
molto espr.
cresc.

𝅗𝅥 = 60.
un poco più lento.
in E.
in E.
in E.
in E.
cresc.
cresc.
un poco più lento.

Bartok, *Concerto for Orchestra,* Sz. 116 double bass part, mvt. I, mm. 35–58 (London: Boosey & Hawkes 1946)



Koussevitzky, Concerto Op. 3 piano score, mvt I, mm. 19–40 (Leipzig: Robert Forberg, c. 1910)


mf

Appendix C: Glossary

Audiate: To internally generate and hear a pitch not physically present in a tonally functional context. Based on Gordon's aural-oral feedback theories (see Gordon 1989).

Beat: The interference pattern (both physical and psychological) caused by two or more nearly-coinciding partials of a tone, usually perceived as a rise and fall of the volume or throbbing of the combined sound.

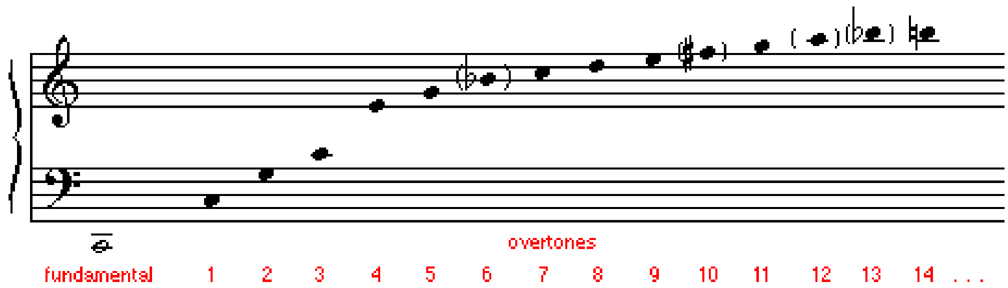
Chunk: A group of individual pieces of coded information grouped and processed as a single unit. Individual chunks can also be grouped into a single chunk, a super-chunk, as it were.

Cognition: The high-level processing of stimuli of raw neural signals through memory-based systems.

dB: decibels: The logarithmic unit of measurement of sound pressure levels, also used as a surrogate for perceived loudness.

Echoic Memory: A memory system which retains an unprocessed, high resolution image of raw, uncategorized perceptual data for a brief period, on the order of 1-2 seconds. This image is retained for immediate direct comparison to new incoming perceptual data and is available to the focused awareness.

Harmonic: This is analogous to a partial and used rather interchangeably. Also used to describe partials with frequencies which are whole-number multiples of the fundamental frequency of a complex tone. Partials are numbered from the fundamental as partial 1.



Retrieved from www.compositionsecrets.com/chord-voiceing showing the numbering of a normal harmonic overtone series.

Harmonic Series: This is the collection of the partials of a complex sound above the fundamental pitch, also known as the overtone series.

Hz: Hertz: The measurement of frequency in cycles per second.

Inharmonicity: The description applied to a tone with inharmonic partials, which are not whole-number multiples of the fundamental frequency.

Intervallic Rivalry Theory: Developed primarily by Butler, Brown, and Jones, this theory proposed that the sensation of a tonal/key center is engendered by the succession of tones forming intervals which are specific to a particular key.

JND: Just Noticeable Difference: The point at which two stimuli are perceived as two separate percepts. Below the JND, the two stimuli are perceived as one percept.

Key Profile Theory: Developed primarily by Krumhansl and Shepard, this theory proposed initially that the sensation of a tonal/key center is engendered by the experience of a collection of pitches activating a schema which was hierarchically organized according to importance and completion-satisfaction of a tone.

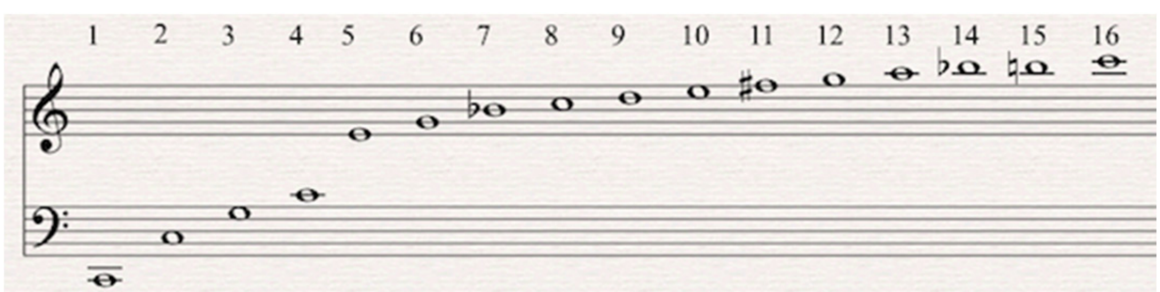
LTM: Long-Term Memory: a memory system which stores schemas and processes categorized data through those schemas.

Note: The written, unsounding reference to a pitch.

Overtone Series: This is the collection of the partials of a complex sound above the fundamental pitch, also known as the harmonic series. In difference to the numbering of partials, which are numbered from the fundamental as the first partial, overtones are numbered from the first overtone above the fundamental.

Retrieved from www.compositionsecrets.com/chord-voiceing showing the numbering of a normal harmonic overtone series.

Partial: The sine-wave component of a complex tone. Harmonic partials are those partials which are whole-number multiples of the fundamental frequency. Inharmonic partials are those which are not whole-number multiples of the fundamental. Partials are numbered from the fundamental as partial 1.



Retrieved from music.tutsplus.com/tutorials/quick-tip-the-overtone-series--audio-4672 showing the numbering of partials.

Phon: The standardized unit of measurement of perceived loudness.

Pitch: The categorized perception of a sound, which can be assigned a perceived fundamental frequency and a perceived register relative to other pitches.

Pitch Salience: The degree to which a tone can be perceived to have the qualities of a pitch.

Psychoacoustics: The field of study regarding the perception of sound. Psychoacoustics is limited to the early bottom-up processing of sound, largely limited to sound's transduction into neural impulse trains and not mediated by any higher brain functions.

Real: In acoustics, this term is used to separate actual physical systems from their theoretical counterparts. Theoretical systems such as a string fixed at two points can be explored with mathematical certainty and has great predictability. A real string system would not have such neat mathematics, as many other factors come into play such as the variable density of the string, the "fixed" ends of the string not really being fixed, etc., which are not accounted for in the theoretical model.

Recursive Looping: Neural pathways which lead from a higher-order processing region directly to locations at or near the sensory nerve endings. These transmit previously recorded information back to the lower-level processors for comparison to incoming information or to affect the processing of incoming information.

Rhythmic Entrainment: This is the imposition and synchronization of an external rhythm onto a subject's (auditor's) internal rhythmic sense. A regular pulse establishes the expectation of further regular pulses, develops a sense of meter, and governs the cycles of heightened attention/lowered attention within a metrical structure.

Roughness: In psychoacoustics, this is the physiological sensation of acoustic dissonance or two partials of a complex sound activating the same portion of the basilar membrane. This stands in contrast to the sense of this word often used in a musical context, which describes a poorly or imprecisely played piece.

Schema: A mental framework composed of categorized "slots" with default place-holders filled by incoming data. Schemas form the basic structure of long-term memory and aid in processing and contextualizing categorized information in short-term memory (STM). A schema can take several forms such as a temporal-order/procedural schema, an abstract concept, or a visual picture (e.g. "a summer's day").

Signal: The train of nerve impulses generated from an external stimulus.

STM: Short-Term Memory: A memory system which abstracts raw information into categorized data which can be processed at a much higher rate, but at a loss of resolution. The limits of STM are 7±2 units of information (bits or chunks) and 5-9 seconds of unrehearsed retention. The information contained within STM is indirectly accessible by the conscious mind

Temperament: A systematic or ad hoc modification of interval sizes away from their purely tuned values, usually in order to reduce or eliminate the diatonic comma for keyboard instruments. This stands in contrast to "tuning."

Tone: The physical sound produced in reference to a note and which an auditor can process for pitch information. In this context, the term tone has no relation to a sound's timbral quality. A tone can be simple (sine wave) or complex, having multiple sine wave components added to form one sound.

Trace: The high-resolution image of raw data retained, compared and processed by echoic memory.

Tuning: The adjustment of the distance between two tones so as to eliminate beating between the partials of each. This is in contrast to tempering, which expressly *de*tunes an interval.

Tuning in *X*: This refers to instruments of multiple strings, where the interval between each string is fixed, such as a perfect fourth or perfect fifth between each string's sounding tone.

Valence: Especially in emotion cognition research, an intrinsically positive (attraction) or negative (aversion) emotional reaction is said to be positively or negatively valenced. This allows emotional responses to be discussed without having to label that response with a particular emotion or emotional state.

WTS: Western Tonal System