I n d i a n a U n i v e r s i t y

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Regression Models for Ordinal and Nominal Dependent Variables Using SAS, Stata, LIMDEP, and SPSS*

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This document summarizes logit and probit regression models for ordinal and nominal dependent variables and illustrates how to estimate individual models using SAS 9.2, Stata 11, LIMDEP 9, and SPSS 17.

1. Introduction

A categorical variable here refers to a variable that is binary, ordinal, or nominal. Event count data are discrete (categorical) but often treated as continuous variables. When a dependent variable is categorical, the ordinary least squares (OLS) method can no longer produce the best linear unbiased estimator (BLUE); that is, OLS is biased and inefficient. Consequently, researchers have developed various regression models for categorical dependent variables. The nonlinearity of categorical dependent variable models makes it difficult to fit the models and interpret their results.

1.1 Regression Models for Categorical Dependent Variables

In categorical dependent variable models, the left-hand side (LHS) variable or dependent variable is neither interval nor ratio, but rather categorical. The level of measurement and data generation process (DGP) of a dependent variable determine a proper model for data analysis. Binary responses (0 or 1) are modeled with binary logit and probit regressions, ordinal responses (1st, 2nd, 3rd, …) are formulated into (generalized) ordinal logit/probit regressions, and nominal responses are analyzed by the multinomial logit (probit), conditional logit, or nested logit model depending on specific circumstances. Independent variables on the right-hand side (RHS) are interval, ratio, and/or binary (dummy).

Table 1.1 Ordinary Least Squares and Categorical Dependent Variable Models

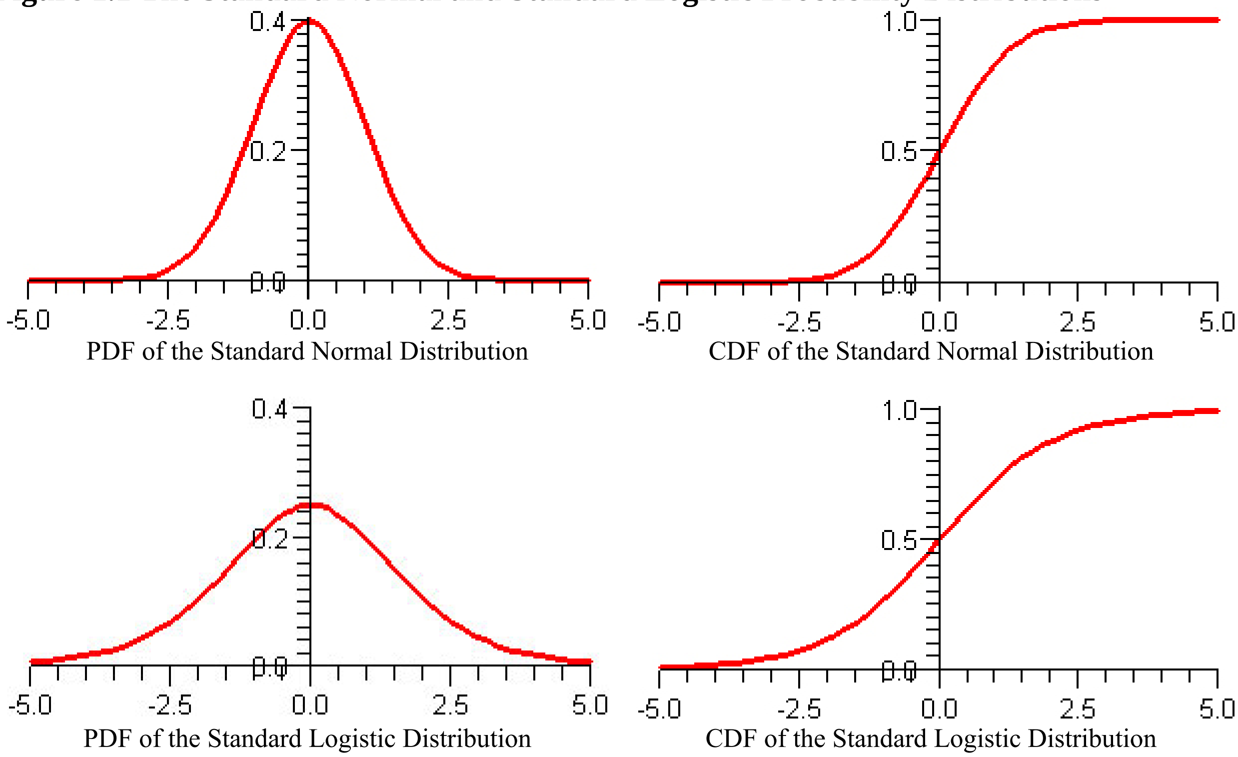
	Model	Dependent (LHS)	Estimation	Independent (RHS)
OLS	Ordinary least squares	Interval or ratio	Moment based method	A linear function of interval/ratio or binary variables $\beta_0 + \beta_1 X_1 + \beta_2 X_2 \ldots$
Categorical DV Models	Binary response	Binary (0 or 1)	Maximum likelihood method	
	Ordinal response	Ordinal (1st, 2nd, 3rd…)		
	Nominal response	Nominal (A, B, C …)		
	Event count data	Count (0, 1, 2, 3…)		

Categorical dependent variable models adopt the maximum likelihood (ML) estimation method, whereas OLS uses the moment based method. The ML method requires an assumption about probability distribution functions, such as the logistic function and the complementary log-log function. Logit models use the standard logistic probability distribution, while probit models assume the standard normal distribution. This document focuses on logit and probit models only, excluding regression models for event count data (e.g., negative binomial regression model and zero-inflated or zero-truncated regression models). Table 1.1 summarizes categorical dependent variable models in comparison with OLS.

1.2 Logit Models versus Probit Models

How do logit models differ from probit models? The core difference lies in the distribution of errors (disturbances). In the logit model, errors are assumed to follow the standard logistic distribution with mean 0 and variance $\frac{\pi^2}{3}$, $\lambda(\varepsilon)=\frac{e^{\varepsilon}}{(1+e^{\varepsilon})^2}$. The errors of the probit model are assumed to follow the standard normal distribution, $\phi(\varepsilon)=\frac{1}{\sqrt{2\pi}}e^{-\frac{\varepsilon^2}{2}}$ with variance 1.

Figure 1.1 The Standard Normal and Standard Logistic Probability Distributions



The probability density function (PDF) of the standard normal probability distribution has a higher peak and thinner tails than the standard logistic probability distribution (Figure 1.1). The standard logistic distribution looks as if someone has weighed down the peak of the standard normal distribution and strained its tails. As a result, the cumulative density function (CDF) of the standard normal distribution is steeper in the middle than the CDF of the standard logistic distribution and quickly approaches zero on the left and one on the right.

The two models, of course, produce different parameter estimates. In binary response models, the estimates of a logit model are roughly $\pi/\sqrt{3}$ times larger than those of the probit model. These estimators, however, end up with almost the same standardized impacts of independent variables (Long 1997).

The choice between logit and probit models is more closely related to estimation and familiarity rather than theoretical and interpretive aspects. In general, logit models reach convergence fairly well. Although some (multinomial) probit models may take a long time to reach convergence, a probit model works well for bivariate models. As computing power improves and new algorithms are developed, importance of this issue is diminishing. For discussion on choosing logit and probit models, see Cameron and Trivedi (2009: 471-474).

1.3 Estimation in SAS, Stata, LIMDEP, R, and SPSS

SAS provides several procedures for categorical dependent variable models, such as PROC LOGISTIC, PROBIT, GENMOD, QLIM, MDC, PHREG, and CATMOD. Since these procedures support various models, a categorical dependent variable model can be estimated by multiple procedures. For example, you may run a binary logit model using PROC LOGISTIC, QLIM, GENMOD, and PROBIT. PROC LOGISTIC and PROC PROBIT of SAS/STAT have been commonly used, but PROC QLIM and PROC MDC of SAS/ETS have advantages over other procedures. PROC LOGISTIC reports factor changes in the odds and tests key hypotheses of a model.

Table 1.2 Procedures and Commands for Categorical Dependent Variable Models

	Model	SAS 9.2	Stata 11	LIMDEP 9	SPSS17
OLS (Ordinary least squares)		REG	`.regress`	Regress$	Regression
Binary	Binary logit	QLIM, LOGISTIC, GENMOD, PROBIT	`.logit`, `.logistic`	Logit$	Logistic regression
	Binary probit	QLIM, LOGISTIC, GENMOD, PROBIT	`.probit`	Probit$	Probit
Bivariate	Bivariate probit	QLIM	`.biprobit`	Bivariateprobit$	-
Ordinal	Ordinal logit	QLIM, LOGISTIC, GENMOD, PROBIT	`.ologit`	Ordered$, Logit$	Plum
	Generalized logit	-	`.gologit2`*	-	-
	Ordinal probit	QLIM, LOGISTIC, GENMOD, PROBIT	`.oprobit`	Ordered$	Plum
Nominal	Multinomial logit	LOGISTIC, CATMOD	`.mlogit`	Mlogit$, Logit$	Nomreg
	Conditional logit	LOGISTIC, MDC, PHREG	`.clogit`	Clogit$, Logit$	Coxreg
	Nested logit	MDC	`.nlogit`	Nlogit$**	-
	Multinomial probit	-	`.mprobit`	-	-

* A user-written command written by Williams (2005)
** The `Nlogit$` command is supported by NLOGIT, a stand-alone package, which is sold separately.

The QLIM (Qualitative and LImited dependent variable Model) procedure analyzes various categorical and limited dependent variable regression models such as censored, truncated, and sample-selection models. PROC QLIM also handles Box-Cox regression and the bivariate probit model. The MDC (Multinomial Discrete Choice) procedure can estimate conditional logit and nested logit models.

Another advantage of using SAS is the Output Delivery System (ODS), which makes it easy to manage SAS output. ODS enables users to redirect the output to HTML (Hypertext Markup Language) and RTF (Rich Text Format) formats. Once SAS output is generated in a HTML document, users can easily handle tables and graphics especially when copying and pasting them into a wordprocessor document.

Unlike SAS, Stata has individualized commands for corresponding categorical dependent variable models. For example, the `.logit` and `.probit` commands respectively fit the binary logit and probit models, while `.mlogit` and `.nlogit` estimate the mulitinomial logit and nested logit models. Stata enables users to perform post-hoc analyses such as marginal effects and discrete changes in an easy manner.

The LIMDEP `Logit$` and `Probit$` commands support a variety of categorical dependent variable models that are addressed in Greene's *Econometric Analysis* (2003). The output format of LIMDEP 9 is slightly different from that of previous version, but key statistics remain unchanged. The nested logit model and multinomial probit model in LIMDEP are estimated by NLOGIT, a separate package. In R, `glm()` fits binary logit and probit models in the object-oriented programming concept. SPSS also supports some categorical dependent variable models and its output is often messy and hard to read. Stata and R are case-sensitive, but SAS, LIMDEP, and SPSS are not. Table 1.2 summarizes the procedures and commands used for categorical dependent variable models.

1. 4 Long and Freese's SPost

Stata users may benefit from user-written commands such as J. Scott Long and Jeremy Freese's SPost. This collection of user-written commands conducts many follow-up analyses of various categorical dependent variable models including event count data models (See section 2.2).

In order to install SPost, execute the following commands consecutively. Visit J. Scott Long's Web site at http://www.indiana.edu/~jslsoc/ to get further information.

```
. net from http://www.indiana.edu/~jslsoc/stata/
. net install spost9_ado, replace
. net get spost9_do, replace
```

If a Stata command, function, or user-written command does not work in version 11, run the `.version` command to switch the interpreter to old one and execute that command again. For example, `normal()` was `norm()` in old versions. Also you may update Stata or reinstall user-written models to get their latest version installed.

```
. version 9
```

You may use Vincent Kang Fu's `gologit` (1998) and Richard Williams' `gologit2` (2005) for the generalized ordinal logit model. `.mfx2` is a related command written by Williams to compute marginal effects (discrete changes) in (generalized) ordinal logit and multinomial logit models. Visit http://www.nd.edu/~rwilliam/gologit2/tsfaq.html for more information.

```
. net install gologit, from(http://www.stata.com/users/jhardin) replace
. ssc install gologit2, replace
. ssc install mfx2, replace
```

2. Ordinal Logit and Probit Regression Models

Suppose we have an ordinal dependent variable such as religious intensity (0=no religion, 1=somewhat strong, 2=not very strong, and 3=strong). Ordinal logit and probit models have the parallel regression assumption or proportional odds assumption, which in practice is often violated.

2.1 Ordinal Logit Model in Stata (.ologit)

Stata has `.ologit` and `.oprobit` commands to estimate ordinal logit and probit regression models, respectively. Their output looks like the result of `.logit` except for cut points and the intercept. Stata estimates τ_m, `/cut1`, `/cut2`, and `/cut3`, assuming $\beta_0 = 0$ (Long and Freese 2003: 148-149). Accordingly, the output below does not report the intercept. By contrast, PROC QLIM, PROC PROBIT, and LIMDEP have different parameterization and assume $\tau_1 = 0$; therefore, `(0-/cut1)` is reported as their intercept.

```
. use http://www.indiana.edu/~statmath/stat/all/cdvm/gss_cdvm.dta, clear

. ologit belief educate income age male www

Iteration 0:   log likelihood = -1499.6929
Iteration 1:   log likelihood = -1480.3168
Iteration 2:   log likelihood = -1480.2738
Iteration 3:   log likelihood = -1480.2738

Ordered logistic regression                       Number of obs   =       1174
                                                  LR chi2(5)      =      38.84
                                                  Prob > chi2     =     0.0000
Log likelihood = -1480.2738                       Pseudo R2       =     0.0129

------------------------------------------------------------------------------
      belief |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
     educate |  -.0020145   .0220039    -0.09   0.927    -.0451414    .0411124
      income |  -.0059213   .0089976    -0.66   0.510    -.0235563    .0117137
         age |   .0186456   .0042123     4.43   0.000     .0103897    .0269015
        male |  -.4661952   .1085422    -4.30   0.000    -.6789339   -.2534564
         www |   .1264832   .1357087     0.93   0.351    -.1395009    .3924673
-------------+----------------------------------------------------------------
       /cut1 |  -1.183894   .3674989                     -1.904178    -.463609
       /cut2 |  -.4989643   .3648623                     -1.214081    .2161526
       /cut3 |   1.186547    .366256                      .4686988    1.904396
------------------------------------------------------------------------------
```

The model fairly fits the date although only age and gender are statistically significant. SPost `.fitstat` returns a list of goodness-of-fit measures. `D(1166)` indicates that this model estimates eight parameters (five regressors and three cut points): 1,166=1,174-8.

```
. fitstat

Measures of Fit for ologit of belief

Log-Lik Intercept Only:      -1499.693   Log-Lik Full Model:          -1480.274
D(1166):                      2960.548   LR(5):                          38.838
                                         Prob > LR:                       0.000
McFadden's R2:                   0.013   McFadden's Adj R2:               0.008
ML (Cox-Snell) R2:               0.033   Cragg-Uhler(Nagelkerke) R2:      0.035
McKelvey & Zavoina's R2:         0.033
Variance of y*:                  3.403   Variance of error:               3.290
```

```
Count R2:                          0.407   Adj Count R2:                   0.031
AIC:                               2.535   AIC*n:                       2976.548
BIC:                           -5280.941   BIC':                          -3.497
BIC used by Stata:              3017.093   AIC used by Stata:           2976.548
```

Ordinal logit and probit models are not as easy to interpret the output as binary response models. Factor changes in the odds are better for interpretation than marginal effects and discrete changes in the ordinal logit model. *The factor change in the odds of a lower versus a higher outcome* is exp(b) in binary response models (0 versus 1), but exp(-b) in the ordinal logit model. For the sake of convenience in interpretation, however, *the factor change in the odds of a higher outcome compared to a lower outcome*, exp(b), can be considered an alternative (Long and Freese 2003: 165-168). Also see Long (1997: 138-140). Although numerically different, both factor changes are equivalent.

The following `.listcoef` produces factor changes in the odds of a higher compared to a lower outcome. For instance, the factor change in the odds of age is 1.0188=exp(b)=exp(.0187)= =1/exp(-.0187)=1/.9815, holding all other covariates constant. For a unit increase in age, *the odds of having stronger religious belief* change (increase in this case) by the factor of 1.0188, holding all other variables constant. For a standard deviation increase in age, the odds of having stronger religious belief compared to weaker belief increase by the factor of 1.2840= exp(.01865*13.4071)=1/exp(-.01865*13.4071)=1/.7788. The odds of having stronger religious belief are .6274=exp(-.4662)=1/exp(.4662)=1/1.5939 times smaller for men than for women.

```
. listcoef, help

ologit (N=1174): Factor Change in Odds

  Odds of: >m vs <=m

----------------------------------------------------------------------
      belief |      b         z     P>|z|    e^b    e^bStdX      SDofX
-------------+--------------------------------------------------------
     educate |  -0.00201   -0.092   0.927   0.9980   0.9948     2.5697
      income |  -0.00592   -0.658   0.510   0.9941   0.9640     6.1943
         age |   0.01865    4.427   0.000   1.0188   1.2840    13.4071
        male |  -0.46620   -4.295   0.000   0.6274   0.7929     0.4978
         www |   0.12648    0.932   0.351   1.1348   1.0533     0.4108
----------------------------------------------------------------------
       b = raw coefficient
       z = z-score for test of b=0
   P>|z| = p-value for z-test
     e^b = exp(b) = factor change in odds for unit increase in X
 e^bStdX = exp(b*SD of X) = change in odds for SD increase in X
   SDofX = standard deviation of X
```

The `reverse` option in `.listcoef` computes factor changes in the odds of a lower outcome compared to a higher outcome. The factor changes in the odds of having weaker religious belief with respect to age is .9815=exp(-b)=exp(-.0187)=1/1.0188. For a unit increase in age, the *odds of having weaker belief* decrease by a factor of .9815. The odds of having weaker religious belief are 1.5939 = exp(-(-.4662)) = 1/.6274 times larger for men than for women, holding all other variables constant.

```
. listcoef, reverse

ologit (N=1174): Factor Change in Odds

  Odds of: <=m vs >m
```

```
-------------------------------------------------------------------
      belief |      b         z     P>|z|    e^b    e^bStdX      SDofX
-------------+-----------------------------------------------------
     educate |  -0.00201   -0.092   0.927   1.0020   1.0052     2.5697
      income |  -0.00592   -0.658   0.510   1.0059   1.0374     6.1943
         age |   0.01865    4.427   0.000   0.9815   0.7788    13.4071
        male |  -0.46620   -4.295   0.000   1.5939   1.2612     0.4978
         www |   0.12648    0.932   0.351   0.8812   0.9494     0.4108
-------------------------------------------------------------------
```

Alternatively, you may also compute the percentage changes in the odds using the `percent` option. The odds of having stronger religious belief are 37.3 percent smaller for men than for woman, holding all other variables constant.

```
. listcoef, percent help

ologit (N=1174): Percentage Change in Odds

  Odds of: >m vs <=m

-------------------------------------------------------------------
      belief |      b         z     P>|z|      %      %StdX      SDofX
-------------+-----------------------------------------------------
     educate |  -0.00201   -0.092   0.927     -0.2     -0.5     2.5697
      income |  -0.00592   -0.658   0.510     -0.6     -3.6     6.1943
         age |   0.01865    4.427   0.000      1.9     28.4    13.4071
        male |  -0.46620   -4.295   0.000    -37.3    -20.7     0.4978
         www |   0.12648    0.932   0.351     13.5      5.3     0.4108
-------------------------------------------------------------------
       b = raw coefficient
       z = z-score for test of b=0
   P>|z| = p-value for z-test
       % = percent change in odds for unit increase in X
   %StdX = percent change in odds for SD increase in X
   SDofX = standard deviation of X
```

Marginal effects (discrete changes) are used to interpret the output substantively. Use either `.mfx` or `.prchange` with, if you want, particular reference points other than the default means of covariates specified. `.mfx` reports standard errors of marginal effects and discrete changes, but `.prchange` does not.

`.prchange` reports the predicted probability of having no religion (`belief=0`) and list marginal effects (discrete changes for binary variables). For female WWW users at the average age of 41 who graduated a college (16 years of education) and have the average family income of 25 thousand dollars (see reference points under the last column *x* below), the predicted probability of having no religion is 12.98 percent.

```
. mfx, at(mean educate=16 male=0 www=1)

Marginal effects after ologit
      y  = Pr(belief==0) (predict)
         =  .12983744
------------------------------------------------------------------------------
variable |      dy/dx    Std. Err.     z    P>|z|  [    95% C.I.   ]      X
---------+--------------------------------------------------------------------
 educate |   .0002276      .00249    0.09   0.927  -.004655  .005111        16
  income |    .000669      .00102    0.66   0.510  -.001322  .002659   24.6486
     age |  -.0021066      .00049   -4.27   0.000  -.003075 -.001139   41.3075
   male*|   .0622968      .01503    4.15   0.000   .032845  .091748         0
    www*|   -.014971       .0166   -0.90   0.367  -.047509  .017567         1
------------------------------------------------------------------------------
(*) dy/dx is for discrete change of dummy variable from 0 to 1
```

Marginal effects and discrete changes are more intuitive than factor changes in the odds. For 10 unit increase in age from its mean 41, the probability of having no religion is expected to decrease by 2.1 percent (.21*10), holding all other variables constant at the reference points. Men are 6.23 percent more likely than women to have no religion at the same reference points.

`.prchange` reports predicted probabilities of four religious intensity and produces marginal effects (`-+1/2` or `MargEfct`) and discrete changes (`0->1`) of covariates in probabilities of all four outcomes. This command computes marginal effects for a standard deviation change (`-+sd/2`) as well.

```
. prchange age male, x(educate=16 male=0 www=1) rest(mean)

ologit: Changes in Probabilities for belief

age
           Avg|Chg|    No_relig    Somewhat    Not_very      Strong
Min->Max   .1509029  -.12637663  -.07334894  -.10208026   .30180579
   -+1/2  .00220756  -.00210658  -.00117922  -.00112933   .00441512
  -+sd/2  .02956489  -.02826677  -.01577979  -.01508322   .05912977
MargEfct  .00220758  -.00210658  -.00117923  -.00112935   .00441516

male
       Avg|Chg|    No_relig    Somewhat    Not_very      Strong
0->1  .05150692   .06229679   .02986491   .01085213  -.10301384

         No_relig   Somewhat   Not_very     Strong
Pr(y|x)  .12983744  .09854499   .3865383  .38507926

       educate   income      age     male      www
    x=      16  24.6486  41.3075        0        1
sd_x=  2.56971  6.19427  13.4071  .497765  .410755
```

Find the same marginal effect of age -.0021 at the `MargEfct` or `-+1/2` row under the label `No_relig`. Interestingly, only marginal effects on having strong intensity are positive. For a standard deviation increases in age (13.4071) from the mean 41, the probability of having strong religious belief is expected to increase by 5.91 percent, holding all other variables constant at their reference points. By contrast, signs of discrete changes of gender are opposite. The probability that men WWW users have strong belief is 10.30 percent lower than that of women counterparts, holding all other variables at their reference points.

Williams' `.mfx2` is very useful especially for ordinal and multinomial response models. This command produces marginal effects (discrete changes) and their standard errors for all outcomes, whereas `.mfx` reports marginal effects for the first outcome (0 in this case) only. But they share the same output format. Therefore, `.prchange` in fact summarizes the output of `.mfx2`. Compare the following output with what `.prchange` produced above.

```
. mfx2, at(mean educate=16 male=0 www=1)

Frequencies for belief...

      Religious |
      Intensity |      Freq.     Percent        Cum.
----------------+-----------------------------------
    No religion |        192       16.35       16.35
Somewhat strong |        134       11.41       27.77
Not very strong |        456       38.84       66.61
         Strong |        392       33.39      100.00
----------------+-----------------------------------
```

```
          Total |      1,174      100.00

Computing marginal effects after ologit for belief == 0...

Marginal effects after ologit
      y  = Pr(belief==0) (predict, o(0))
         =  .12983744
------------------------------------------------------------------------------
variable |      dy/dx    Std. Err.     z    P>|z|  [    95% C.I.   ]      X
---------+--------------------------------------------------------------------
 educate |   .0002276      .00249    0.09   0.927  -.004655  .005111        16
  income |    .000669      .00102    0.66   0.510  -.001322  .002659   24.6486
     age |  -.0021066      .00049   -4.27   0.000  -.003075 -.001139   41.3075
    male*|   .0622968      .01503    4.15   0.000   .032845  .091748         0
     www*|   -.014971       .0166   -0.90   0.367  -.047509  .017567         1
------------------------------------------------------------------------------
(*) dy/dx is for discrete change of dummy variable from 0 to 1

Computing marginal effects after ologit for belief == 1...

Marginal effects after ologit
      y  = Pr(belief==1) (predict, o(1))
         =  .09854499
------------------------------------------------------------------------------
variable |      dy/dx    Std. Err.     z    P>|z|  [    95% C.I.   ]      X
---------+--------------------------------------------------------------------
 educate |   .0001274      .00139    0.09   0.927  -.002602  .002857        16
  income |   .0003745      .00057    0.66   0.511  -.000742  .001491   24.6486
     age |  -.0011792      .00028   -4.17   0.000  -.001733 -.000625   41.3075
    male*|   .0298649       .0073    4.09   0.000   .015564  .044166         0
     www*|  -.0080795      .00874   -0.92   0.355  -.025211  .009052         1
------------------------------------------------------------------------------
(*) dy/dx is for discrete change of dummy variable from 0 to 1

Computing marginal effects after ologit for belief == 2...

Marginal effects after ologit
      y  = Pr(belief==2) (predict, o(2))
         =   .3865383
------------------------------------------------------------------------------
variable |      dy/dx    Std. Err.     z    P>|z|  [    95% C.I.   ]      X
---------+--------------------------------------------------------------------
 educate |    .000122      .00132    0.09   0.927  -.002472  .002716        16
  income |   .0003586      .00055    0.65   0.517  -.000727  .001444   24.6486
     age |  -.0011294      .00036   -3.15   0.002  -.001833 -.000426   41.3075
    male*|   .0108521       .0057    1.90   0.057  -.000329  .022033         0
     www*|   -.006432      .00619   -1.04   0.299  -.018568  .005704         1
------------------------------------------------------------------------------
(*) dy/dx is for discrete change of dummy variable from 0 to 1

Computing marginal effects after ologit for belief == 3...

Marginal effects after ologit
      y  = Pr(belief==3) (predict, o(3))
         =  .38507927
------------------------------------------------------------------------------
variable |      dy/dx    Std. Err.     z    P>|z|  [    95% C.I.   ]      X
---------+--------------------------------------------------------------------
 educate |   -.000477      .00521   -0.09   0.927  -.010682  .009728        16
  income |  -.0014021      .00213   -0.66   0.511   -.00558  .002776   24.6486
     age |   .0044152        .001    4.41   0.000   .002455  .006375   41.3075
    male*|  -.1030138      .02374   -4.34   0.000  -.149547 -.056481         0
     www*|   .0294825      .03126    0.94   0.346  -.031777  .090743         1
------------------------------------------------------------------------------
(*) dy/dx is for discrete change of dummy variable from 0 to 1
```

Now, move on to the interpretation using predicted probabilities. Like `.prchange` and `.mfx2`, the `.prvalue` command returns the predicted probabilities for other categories. The predicted probability of no religion is 12.98, 9.85 for somewhat strong, 38.65 for not very strong, and 38.51 for strong religious belief.

```
. prvalue, x(educate=16 male=0 www=1) rest(mean)

ologit: Predictions for belief

Confidence intervals by delta method

                                95% Conf. Interval
     Pr(y=No_relig|x): 0.1298  [ 0.1063,    0.1533]
     Pr(y=Somewhat|x): 0.0985  [ 0.0805,    0.1166]
     Pr(y=Not_very|x): 0.3865  [ 0.3577,    0.4154]
     Pr(y=Strong|x):  0.3851   [ 0.3437,    0.4265]

      educate     income        age       male        www
x=         16  24.648637  41.307496          0          1
```

The `.prtab` command constructs the tables of predicted probabilities for combinations of different values of independent variables. The following tables suggest that gender appears to make difference in religious intensity.

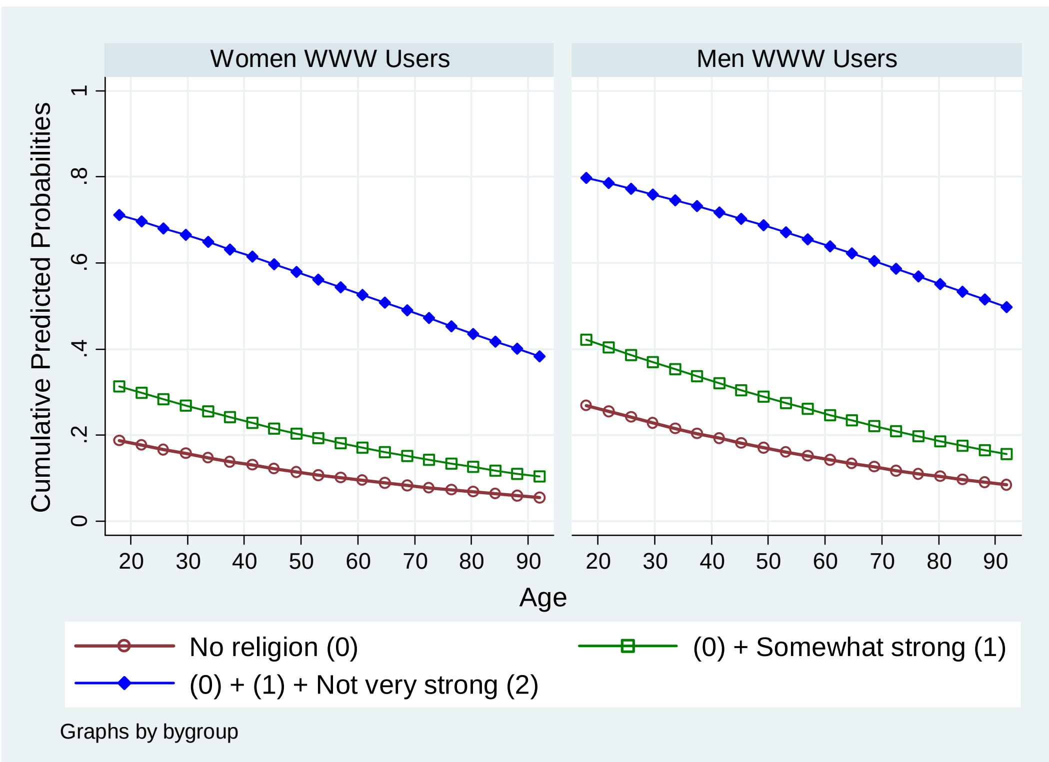
```
. prtab male www, x(educate=16 male=0 www=1) rest(mean)

ologit: Predicted probabilities for belief

Predicted probability of outcome 0 (No_religion)

--------------------------------
          |       WWW Use
   Gender | Non-users      Users
----------+---------------------
   Female |    0.1448     0.1298
     Male |    0.2125     0.1921
--------------------------------

Predicted probability of outcome 1 (Somewhat_strong)

--------------------------------
          |       WWW Use
   Gender | Non-users      Users
----------+---------------------
   Female |    0.1066     0.0985
     Male |    0.1362     0.1284
--------------------------------

Predicted probability of outcome 2 (Not_very_strong)

--------------------------------
          |       WWW Use
   Gender | Non-users      Users
----------+---------------------
   Female |    0.3930     0.3865
     Male |    0.3941     0.3974
--------------------------------

Predicted probability of outcome 3 (Strong)

--------------------------------
          |       WWW Use
   Gender | Non-users      Users
----------+---------------------
   Female |    0.3556     0.3851
     Male |    0.2572     0.2821
--------------------------------

      educate     income        age       male        www
```

```
x=          16   24.648637   41.307496          0          1
```

SPost `.prgen` is very useful when visualizing predicted probabilities. The following commands produce a series of predicted probabilities as age changes from 18 to 92. `ncases(20)` computes predicted probabilities at the 20 different points of age, holding other independent variables at the reference points. See the attached Stata script for data manipulation for Figure 2.1. As we found in the above tables, women are more likely to have strong belief and less likely to have no religions than men.

```
. prgen age, from(18) to(92) ncases(20) x(educate=16 male=1 www=1) rest(mean) gen(Logit_age1)

ologit: Predicted values as age varies from 18 to 92.

      educate      income         age        male         www
x=         16   24.648637   41.307496           1           1

. prgen age, from(18) to(92) ncases(20) x(educate=16 male=0 www=1) rest(mean) gen(Logit_age0)

ologit: Predicted values as age varies from 18 to 92.

      educate      income         age        male         www
x=         16   24.648637   41.307496           0           1
```

Figure 2.1 Predicted Probabilities of Religious Intensity (Ordinal Logit Model)



2.2 Ordinal Probit Model in Stata (.oprobit)

Let us fit the ordinal probit model using the same specification. Logit and probit models produce similar parameter estimates and goodness-of-fit measures. For example, their likelihood ratios are 38.84 versus 40.13 and pseudo R^2 are .0129 versus .0134, respectively.

```
. oprobit belief educate income age male www

Iteration 0:   log likelihood = -1499.6929
Iteration 1:   log likelihood =   -1479.63
Iteration 2:   log likelihood = -1479.6279
Iteration 3:   log likelihood = -1479.6279

Ordered probit regression                         Number of obs   =       1174
                                                  LR chi2(5)      =      40.13
                                                  Prob > chi2     =     0.0000
Log likelihood = -1479.6279                       Pseudo R2       =     0.0134

------------------------------------------------------------------------------
      belief |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
     educate |  -.0015194   .0130701    -0.12   0.907    -.0271362    .0240974
      income |  -.0027382   .0053709    -0.51   0.610    -.0132649    .0077886
         age |   .0109693   .0024755     4.43   0.000     .0061175    .0158211
        male |   -.290305   .0646295    -4.49   0.000    -.4169764   -.1636335
         www |   .0642404   .0809186     0.79   0.427    -.0943572    .2228379
-------------+----------------------------------------------------------------
       /cut1 |  -.7138045   .2182722                       -1.14161   -.2859989
       /cut2 |  -.3178217   .2172398                      -.7436038    .1079604
       /cut3 |   .7199238    .217734                        .293173    1.146675
------------------------------------------------------------------------------

. fitstat

Measures of Fit for oprobit of belief

Log-Lik Intercept Only:      -1499.693   Log-Lik Full Model:          -1479.628
D(1166):                      2959.256   LR(5):                          40.130
                                         Prob > LR:                       0.000
McFadden's R2:                   0.013   McFadden's Adj R2:               0.008
ML (Cox-Snell) R2:               0.034   Cragg-Uhler(Nagelkerke) R2:      0.036
McKelvey & Zavoina's R2:         0.040
Variance of y*:                  1.041   Variance of error:               1.000
Count R2:                        0.414   Adj Count R2:                    0.042
AIC:                             2.534   AIC*n:                        2975.256
BIC:                         -5282.233   BIC':                           -4.789
BIC used by Stata:            3015.801   AIC used by Stata:            2975.256
```

In a probit model, `.listcoef` produces standardized coefficients instead of factor changes (or percent changes) of the odds.

```
. listcoef, help

oprobit (N=1174): Unstandardized and Standardized Estimates

 Observed SD: 1.044809
   Latent SD: 1.020498

-------------------------------------------------------------------------------
      belief |      b         z     P>|z|    bStdX    bStdY   bStdXY      SDofX
-------------+-----------------------------------------------------------------
     educate |  -0.00152   -0.116   0.907  -0.0039  -0.0015  -0.0038     2.5697
      income |  -0.00274   -0.510   0.610  -0.0170  -0.0027  -0.0166     6.1943
         age |   0.01097    4.431   0.000   0.1471   0.0107   0.1441    13.4071
        male |  -0.29030   -4.492   0.000  -0.1445  -0.2845  -0.1416     0.4978
         www |   0.06424    0.794   0.427   0.0264   0.0630   0.0259     0.4108
-------------------------------------------------------------------------------
       b = raw coefficient
       z = z-score for test of b=0
   P>|z| = p-value for z-test
   bStdX = x-standardized coefficient
   bStdY = y-standardized coefficient
  bStdXY = fully standardized coefficient
   SDofX = standard deviation of X
```

Let us compute predicted probabilities and marginal effects (discrete changes) at the same reference points. The following `.mfx` command reports that 12.73 percent of female WWW users have no religion (12.98 percent in the logit model above).

```
. mfx, at(mean educate=16 male=0 www=1)

Marginal effects after oprobit
      y  = Pr(belief==0) (predict)
         =  .12727708
------------------------------------------------------------------------------
variable |      dy/dx    Std. Err.     z    P>|z|  [    95% C.I.   ]      X
---------+--------------------------------------------------------------------
 educate |   .0003167      .00273    0.12   0.908  -.005037   .00567        16
  income |   .0005708      .00112    0.51   0.610  -.001622  .002764   24.6486
     age |  -.0022867      .00053   -4.28   0.000  -.003335 -.001238   41.3075
   male*|     .070649      .01616    4.37   0.000   .038981  .102317         0
    www*|  -.0138841      .01793   -0.77   0.439   -.04903  .021262         1
------------------------------------------------------------------------------
(*) dy/dx is for discrete change of dummy variable from 0 to 1
```

`.prvalue` reports other predicted probabilities as well: 10.14 percent for somewhat strong, 38.71 for not very strong, and 38.42 for the strong religious belief (9.85, 38.65, and 38.51 in the logit model above).

```
. prvalue, x(educate=16 male=0 www=1) rest(mean)

oprobit: Predictions for belief

Confidence intervals by delta method

                                95% Conf. Interval
     Pr(y=No_relig|x): 0.1273  [ 0.1028,    0.1518]
     Pr(y=Somewhat|x): 0.1014  [ 0.0823,    0.1204]
     Pr(y=Not_very|x): 0.3871  [ 0.3561,    0.4182]
     Pr(y=Strong|x):  0.3842   [ 0.3438,    0.4247]

      educate     income        age       male        www
x=         16  24.648637  41.307496          0          1
```

The following output of `.prchange` reports that marginal effect and discrete change on having strong belief are .42 percent for age and -10.49 percent for gender, which are respectively very similar to .44 and -10.30 percent in the logit model above.

```
. prchange age male, x(educate=16 male=0 www=1) rest(mean)

oprobit: Changes in Probabilities for belief

age
            Avg|Chg|    No_relig    Somewhat    Not_very      Strong
Min->Max   .14324967  -.13683906  -.06736732  -.08229297   .28649932
   -+1/2   .00209527  -.00228667  -.00103298  -.00087088   .00419053
  -+sd/2   .02806862  -.03066584   -.0138232  -.01164818   .05613723
MargEfct   .00209528  -.00228667  -.00103299  -.00087091   .00419057

male
       Avg|Chg|    No_relig   Somewhat   Not_very     Strong
0->1  .05242721  .07064901  .02597284  .00823256  -.1048544

          No_relig   Somewhat   Not_very     Strong
Pr(y|x)  .12727708  .10135041  .38713527  .38423723

       educate   income       age     male      www
   x=       16  24.6486   41.3075        0        1
```

```
sd_x=  2.56971  6.19427  13.4071  .497765  .410755
```

Williams' `.mfx2` produces predicted probabilities, marginal effects (discrete changes), and standard errors for all four categories in a single command. Compare the output of `.prchange` and `.mfx2`.

```
. mfx2, at(mean educate=16 male=0 www=1)

Frequencies for belief...

      Religious |
      Intensity |      Freq.     Percent        Cum.
----------------+-----------------------------------
    No religion |        192       16.35       16.35
Somewhat strong |        134       11.41       27.77
Not very strong |        456       38.84       66.61
         Strong |        392       33.39      100.00
----------------+-----------------------------------
          Total |      1,174      100.00

Computing marginal effects after oprobit for belief == 0...

Marginal effects after oprobit
      y  = Pr(belief==0) (predict, o(0))
         =  .12727708
------------------------------------------------------------------------------
variable |      dy/dx    Std. Err.     z    P>|z|  [    95% C.I.   ]      X
---------+--------------------------------------------------------------------
 educate |   .0003167      .00273    0.12   0.908  -.005037   .00567        16
  income |   .0005708      .00112    0.51   0.610  -.001622  .002764   24.6486
     age |  -.0022867      .00053   -4.28   0.000  -.003335 -.001238   41.3075
    male*|    .070649      .01616    4.37   0.000   .038981  .102317         0
     www*|  -.0138841      .01793   -0.77   0.439   -.04903  .021262         1
------------------------------------------------------------------------------
(*) dy/dx is for discrete change of dummy variable from 0 to 1

Computing marginal effects after oprobit for belief == 1...

Marginal effects after oprobit
      y  = Pr(belief==1) (predict, o(1))
         =  .10135041
------------------------------------------------------------------------------
variable |      dy/dx    Std. Err.     z    P>|z|  [    95% C.I.   ]      X
---------+--------------------------------------------------------------------
 educate |   .0001431      .00123    0.12   0.907  -.002269  .002555        16
  income |   .0002579      .00051    0.51   0.611  -.000734   .00125   24.6486
     age |   -.001033      .00025   -4.16   0.000   -.00152 -.000546   41.3075
    male*|   .0259728      .00613    4.24   0.000   .013958  .037987         0
     www*|  -.0060148      .00755   -0.80   0.426  -.020822  .008792         1
------------------------------------------------------------------------------
(*) dy/dx is for discrete change of dummy variable from 0 to 1

Computing marginal effects after oprobit for belief == 2...

Marginal effects after oprobit
      y  = Pr(belief==2) (predict, o(2))
         =  .38713527
------------------------------------------------------------------------------
variable |      dy/dx    Std. Err.     z    P>|z|  [    95% C.I.   ]      X
---------+--------------------------------------------------------------------
 educate |   .0001206      .00103    0.12   0.907  -.001895  .002136        16
  income |   .0002174      .00043    0.50   0.614  -.000627  .001062   24.6486
     age |  -.0008709      .00028   -3.15   0.002  -.001412 -.000329   41.3075
    male*|   .0082326      .00456    1.81   0.071  -.000701  .017166         0
     www*|  -.0043953      .00504   -0.87   0.383   -.01428  .005489         1
------------------------------------------------------------------------------
(*) dy/dx is for discrete change of dummy variable from 0 to 1
```

```
Computing marginal effects after oprobit for belief == 3...

Marginal effects after oprobit
      y  = Pr(belief==3) (predict, o(3))
         =  .38423723
------------------------------------------------------------------------------
variable |      dy/dx    Std. Err.     z    P>|z|  [    95% C.I.   ]      X
---------+--------------------------------------------------------------------
 educate |  -.0005804      .00499   -0.12   0.907   -.01036    .0092        16
  income |   -.001046      .00205   -0.51   0.610  -.005069  .002977   24.6486
     age |   .0041906      .00095    4.43   0.000   .002335  .006046   41.3075
    male*|  -.1048544      .02315   -4.53   0.000  -.150222 -.059487         0
     www*|   .0242943      .03037    0.80   0.424  -.035234  .083822         1
------------------------------------------------------------------------------
(*) dy/dx is for discrete change of dummy variable from 0 to 1
```

You may present predicted probabilities computed at different values of key variables. The following predicted probabilities suggest that women are less likely to have no religion (12.73 versus 19.79 percent for WWW users) and more likely to have strong belief (38.42 versus 27.94 percent for WWW users) than men, and that there is no substantial difference in religious intensity between WWW users and non-users. Find the same predicted probabilities (12.73, 10.14, 38.71, and 38.42) in the following four tables generated by `.prtab`.

```
. prtab male www, x(educate=16 male=0 www=1) rest(mean)

oprobit: Predicted probabilities for belief

Predicted probability of outcome 0 (No_religion)

--------------------------------
          |       WWW Use
   Gender | Non-users      Users
----------+---------------------
   Female |    0.1412     0.1273
     Male |    0.2163     0.1979
--------------------------------

Predicted probability of outcome 1 (Somewhat_strong)

--------------------------------
          |       WWW Use
   Gender | Non-users      Users
----------+---------------------
   Female |    0.1074     0.1014
     Male |    0.1324     0.1273
--------------------------------

Predicted probability of outcome 2 (Not_very_strong)

--------------------------------
          |       WWW Use
   Gender | Non-users      Users
----------+---------------------
   Female |    0.3915     0.3871
     Male |    0.3931     0.3954
--------------------------------

Predicted probability of outcome 3 (Strong)

--------------------------------
          |       WWW Use
   Gender | Non-users      Users
----------+---------------------
   Female |    0.3599     0.3842
     Male |    0.2582     0.2794
--------------------------------
```

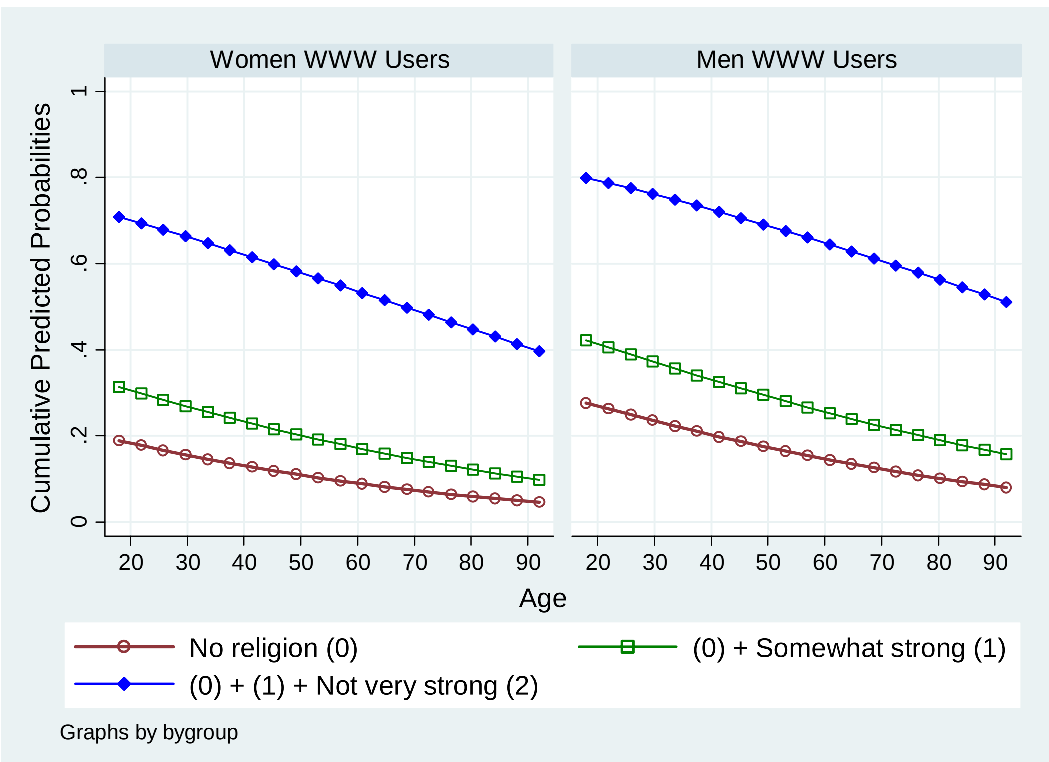
```
      educate      income         age        male         www
x=         16   24.648637   41.307496           0           1
```

Visualizing cumulative predicted probabilities is another effective way to present the result (Figure 2.2). Three curves segment each plane into four parts from no religion (bottom), somewhat strong, not very strong, to strong belief (top). Strong belief holds a larger portion in the women's plane than in the men's. Men are more likely to have no religion than women when controlling age and other covariates. As people get older, they are more likely to have strong belief and less likely to have no religion. Age does not appear to affect somewhat strong and not very strong categories significantly. Figure 2.1 and 2.2 are almost identical.

The following `.prgen` produces a series of predicted probabilities as age changes from 18 to 92. `ncases(20)` computes predicted probabilities at the 20 different points of age, holding other independent variables at the reference points.

```
. prgen age, from(18) to(92) ncases(20) x(educate=16 male=1 www=1) rest(mean) gen(Page1)

oprobit: Predicted values as age varies from 18 to 92.

      educate      income         age        male         www
x=         16   24.648637   41.307496           1           1

. prgen age, from(18) to(92) ncases(20) x(educate=16 male=0 www=1) rest(mean) gen(Page0)

oprobit: Predicted values as age varies from 18 to 92.

      educate      income         age        male         www
x=         16   24.648637   41.307496           0           1
```

Figure 2.2 Predicted Probabilities of Religious Intensity (Ordinal Probit Model)



2.3 Parallel Regression Assumption and Generalized Ordinal Logit Models

The `.brant` command of SPost conducts the Brant test after the `.ologit` command. This command tests the parallel regression assumption (or proportional odds assumption) of the ordinal logit regression model. The test suggests that age and gender may have different slopes across categories. The large chi-squared of 21.94 rejects the null hypothesis of the parallel regression assumption at the .05 level.

```
. quietly ologit belief educate income age male www

. brant, detail

Estimated coefficients from j-1 binary regressions

                 y>0         y>1         y>2
educate  -.01683738  -.01987509   .01376747
 income   .00437285  -.00678136  -.00665741
    age   .01549009   .01092697   .02364093
   male   -.6489834  -.34446179  -.51696936
    www  -.03895167    .2059211   .10840812
  _cons   1.4968083   .96044435  -1.5775835

Brant Test of Parallel Regression Assumption

    Variable |      chi2   p>chi2    df
-------------+--------------------------
         All |     21.94    0.015    10
-------------+--------------------------
     educate |      1.46    0.482     2
      income |      1.67    0.434     2
         age |      6.59    0.037     2
        male |      7.99    0.018     2
         www |      2.66    0.264     2
----------------------------------------

A significant test statistic provides evidence that the parallel
regression assumption has been violated.
```

The parallel regression assumption is often violated. If this is the case, you may use the multinomial logit model or estimate the generalized ordinal logit model using either the `.gologit` command written by Fu (1998) or the `.gologit2` command by Williams (2005). Notice that Fu's command does not impose the restriction of $(\tau_j - x\beta_j) \geq (\tau_{j-1} - x\beta_{j-1})$ (Long's class note 2003). Let us begin with Fu's `.gologit`.

```
. gologit belief educate income age male www

Iteration 0:  Log Likelihood = -1499.6929
Iteration 1:  Log Likelihood = -1476.9406
Iteration 2:  Log Likelihood = -1469.3715
Iteration 3:  Log Likelihood = -1469.3215
Iteration 4:  Log Likelihood = -1469.3214
Iteration 5:  Log Likelihood = -1469.3214

Generalized Ordered Logit Estimates                Number of obs    =    1174
                                                   Model chi2(15)   =   60.74
                                                   Prob > chi2      =  0.0000
Log Likelihood =  -1469.3214457                    Pseudo R2        =  0.0203

------------------------------------------------------------------------------
      belief |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
```

```
mleq1        |
     educate |  -.0215025   .0313129    -0.69   0.492    -.0828747    .0398696
      income |  -.0011176   .0123599    -0.09   0.928    -.0253425    .0231073
         age |   .0165176   .0062489     2.64   0.008       .00427    .0287652
        male |  -.6447443   .1602963    -4.02   0.000    -.9589192   -.3305693
         www |  -.0326311   .2046955    -0.16   0.873     -.433827    .3685648
       _cons |   1.651383   .5292113     3.12   0.002     .6141477    2.688618
-------------+----------------------------------------------------------------
mleq2        |
     educate |  -.0226758   .0270061    -0.84   0.401    -.0756068    .0302553
      income |   -.006108   .0109703    -0.56   0.578    -.0276093    .0153933
         age |   .0108099    .005133     2.11   0.035     .0007494    .0208705
        male |  -.3500519    .131329    -2.67   0.008     -.607452   -.0926518
         www |   .2117636   .1658317     1.28   0.202    -.1132606    .5367877
       _cons |   .9875713   .4478875     2.20   0.027     .1097279    1.865415
-------------+----------------------------------------------------------------
mleq3        |
     educate |   .0160895   .0256324     0.63   0.530    -.0341492    .0663282
      income |  -.0072066   .0106401    -0.68   0.498    -.0280609    .0136477
         age |   .0238357   .0048312     4.93   0.000     .0143667    .0333046
        male |  -.5126078   .1285168    -3.99   0.000    -.7644962   -.2607194
         www |   .1149432   .1613481     0.71   0.476    -.2012932    .4311796
       _cons |  -1.612449   .4276243    -3.77   0.000    -2.450577   -.7743211
------------------------------------------------------------------------------
```

Williams' `.gologit2` fits another version of the generalized ordinal logit regression model. `autofit` tests if the proportional odds assumption is satisfied. This test reports that education, family income, and WWW use have parallel lines (slopes) but age and gender may not. The Wald test does not reject the null hypothesis of the parallel regression assumption at the .05 level. This result conflicts with the Brant test that rejects the null hypothesis.

```
. gologit2 belief educate income age male www, autofit

------------------------------------------------------------------------------
Testing parallel lines assumption using the .05 level of significance...

Step  1:  Constraints for parallel lines imposed for income (P Value = 0.7820)
Step  2:  Constraints for parallel lines imposed for educate (P Value = 0.3893)
Step  3:  Constraints for parallel lines imposed for www (P Value = 0.2635)
Step  4:  Constraints for parallel lines are not imposed for
          age (P Value = 0.01066)
          male (P Value = 0.01923)

Wald test of parallel lines assumption for the final model:

 ( 1)  [No_religion]income - [Somewhat_strong]income = 0
 ( 2)  [No_religion]educate - [Somewhat_strong]educate = 0
 ( 3)  [No_religion]www - [Somewhat_strong]www = 0
 ( 4)  [No_religion]income - [Not_very_strong]income = 0
 ( 5)  [No_religion]educate - [Not_very_strong]educate = 0
 ( 6)  [No_religion]www - [Not_very_strong]www = 0

           chi2(  6) =    5.07
         Prob > chi2 =    0.5350

An insignificant test statistic indicates that the final model
does not violate the proportional odds/ parallel lines assumption

If you re-estimate this exact same model with gologit2, instead
of autofit you can save time by using the parameter

pl(income educate www)

------------------------------------------------------------------------------

Generalized Ordered Logit Estimates               Number of obs   =       1174
                                                  Wald chi2(9)    =      54.08
                                                  Prob > chi2     =     0.0000
```

```
Log likelihood = -1471.9949                       Pseudo R2       =     0.0185

 ( 1)  [No_religion]income - [Somewhat_strong]income = 0
 ( 2)  [No_religion]educate - [Somewhat_strong]educate = 0
 ( 3)  [No_religion]www - [Somewhat_strong]www = 0
 ( 4)  [Somewhat_strong]income - [Not_very_strong]income = 0
 ( 5)  [Somewhat_strong]educate - [Not_very_strong]educate = 0
 ( 6)  [Somewhat_strong]www - [Not_very_strong]www = 0
------------------------------------------------------------------------------
      belief |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
No_religion  |
     educate |  -.0015707   .0219905    -0.07   0.943    -.0446712    .0415298
      income |  -.0057882   .0090514    -0.64   0.523    -.0235287    .0119522
         age |   .0170225    .006218     2.74   0.006     .0048354    .0292097
        male |  -.6494197   .1601173    -4.06   0.000    -.9632438   -.3355956
         www |   .1248686   .1357661     0.92   0.358    -.1412281    .3909653
       _cons |   1.341287   .4162863     3.22   0.001     .5253808    2.157193
-------------+----------------------------------------------------------------
Somewhat_s~g |
     educate |  -.0015707   .0219905    -0.07   0.943    -.0446712    .0415298
      income |  -.0057882   .0090514    -0.64   0.523    -.0235287    .0119522
         age |   .0102271   .0050627     2.02   0.043     .0003045    .0201498
        male |   -.346836   .1312805    -2.64   0.008    -.6041411   -.0895309
         www |   .1248686   .1357661     0.92   0.358    -.1412281    .3909653
       _cons |   .7696745   .3873154     1.99   0.047     .0105502    1.528799
-------------+----------------------------------------------------------------
Not_very_s~g |
     educate |  -.0015707   .0219905    -0.07   0.943    -.0446712    .0415298
      income |  -.0057882   .0090514    -0.64   0.523    -.0235287    .0119522
         age |   .0238896   .0047821     5.00   0.000     .0145169    .0332624
        male |  -.5092498    .128288    -3.97   0.000    -.7606898   -.2578099
         www |   .1248686   .1357661     0.92   0.358    -.1412281    .3909653
       _cons |  -1.406625   .3819714    -3.68   0.000    -2.155275   -.6579751
------------------------------------------------------------------------------
```

`.gologit` and `.gologit2` produce different parameter estimates and their standard errors. Like ordinal logit and probit models, the generalized ordinal logit model suggests that age and gender are only good predictors for religious intensity. This model does not fit the data well.

Since `.mfx` does not work in this user-written command, you need to run Williams' `.mfx2` to compute marginal effects for the generalized ordinal logit model.

```
. mfx2, at(mean educate=16 male=0 www=1)

Frequencies for belief...

      Religious |
      Intensity |      Freq.     Percent        Cum.
----------------+-----------------------------------
    No religion |        192       16.35       16.35
Somewhat strong |        134       11.41       27.77
Not very strong |        456       38.84       66.61
         Strong |        392       33.39      100.00
----------------+-----------------------------------
          Total |      1,174      100.00

Computing marginal effects after gologit2 for belief == 0...

Marginal effects after gologit2
      y  = Pr(belief==0) (predict, o(0))
         =  .11904434
------------------------------------------------------------------------------
variable |      dy/dx    Std. Err.     z    P>|z|  [    95% C.I.   ]      X
---------+--------------------------------------------------------------------
 educate |   .0001647      .00231    0.07   0.943  -.004363  .004693        16
  income |    .000607      .00095    0.64   0.522  -.001252  .002466   24.6486
```

```
     age |  -.0017852      .00066    -2.70   0.007  -.003079 -.000491   41.3075
    male*|   .0864843       .0216     4.00   0.000   .044153  .128815         0
     www*|  -.0137306      .01546    -0.89   0.374  -.044031   .01657         1
------------------------------------------------------------------------------
(*) dy/dx is for discrete change of dummy variable from 0 to 1

Computing marginal effects after gologit2 for belief == 1...

Marginal effects after gologit2
      y  = Pr(belief==1) (predict, o(1))
         =  .12159128
------------------------------------------------------------------------------
variable |      dy/dx    Std. Err.     z    P>|z|  [    95% C.I.   ]      X
---------+--------------------------------------------------------------------
 educate |   .0001223      .00171    0.07   0.943  -.003235   .00348        16
  income |   .0004507      .00071    0.64   0.523  -.000933  .001834   24.6486
     age |  -.0000836      .00066   -0.13   0.899  -.001373  .001206   41.3075
    male*|  -.0175994      .01826   -0.96   0.335  -.053384  .018185         0
     www*|  -.0098187      .01078   -0.91   0.362  -.030947   .01131         1
------------------------------------------------------------------------------
(*) dy/dx is for discrete change of dummy variable from 0 to 1

Computing marginal effects after gologit2 for belief == 2...

Marginal effects after gologit2
      y  = Pr(belief==2) (predict, o(2))
         =  .37302809
------------------------------------------------------------------------------
variable |      dy/dx    Std. Err.     z    P>|z|  [    95% C.I.   ]      X
---------+--------------------------------------------------------------------
 educate |   .0000854      .00119    0.07   0.943  -.002243  .002414        16
  income |   .0003146       .0005    0.63   0.530  -.000668  .001297   24.6486
     age |   -.003795      .00107   -3.54   0.000  -.005897 -.001693   41.3075
    male*|   .0429671      .02891    1.49   0.137  -.013693  .099627         0
     www*|  -.0056029      .00543   -1.03   0.302  -.016246   .00504         1
------------------------------------------------------------------------------
(*) dy/dx is for discrete change of dummy variable from 0 to 1

Computing marginal effects after gologit2 for belief == 3...

Marginal effects after gologit2
      y  = Pr(belief==3) (predict, o(3))
         =  .38633629
------------------------------------------------------------------------------
variable |      dy/dx    Std. Err.     z    P>|z|  [    95% C.I.   ]      X
---------+--------------------------------------------------------------------
 educate |  -.0003724      .00521   -0.07   0.943  -.010585  .009841        16
  income |  -.0013723      .00215   -0.64   0.523   -.00558  .002836   24.6486
     age |   .0056638      .00113    4.99   0.000    .00344  .007887   41.3075
    male*|   -.111852      .02772   -4.04   0.000  -.166181 -.057523         0
     www*|   .0291523      .03133    0.93   0.352  -.032252  .090556         1
------------------------------------------------------------------------------
(*) dy/dx is for discrete change of dummy variable from 0 to 1
```

2.4 Ordinal Logit Model in SAS

QLIM, LOGISTIC, and PROBIT procedures estimate ordinal logit and probit models. As shown in Tables 2.1 and 3.2, PROC QLIM is most recommended. The DIST=LOGISTIC below fits the ordinal logit egression model using the standard logistic probability distribution. Stata and PROC QLIM report same goodness-of-fit measures, parameter estimates, and standard errors.

```
PROC QLIM DATA=masil.gss_cdvm;
     MODEL belief = educate income age male www /DISCRETE (DIST=LOGISTIC);
RUN;
```

The QLIM Procedure

Discrete Response Profile of belief

Index	Value	Frequency	Percent
1	0	192	16.35
2	1	134	11.41
3	2	456	38.84
4	3	392	33.39

Model Fit Summary

Number of Endogenous Variables	1
Endogenous Variable	belief
Number of Observations	1174
Log Likelihood	-1480
Maximum Absolute Gradient	5.69774E-6
Number of Iterations	15
Optimization Method	Quasi-Newton
AIC	2977
Schwarz Criterion	3017

Goodness-of-Fit Measures

Measure	Value	Formula
Likelihood Ratio (R)	38.838	2 * (LogL - LogL0)
Upper Bound of R (U)	2999.4	- 2 * LogL0
Aldrich-Nelson	0.032	R / (R+N)
Cragg-Uhler 1	0.0325	1 - exp(-R/N)
Cragg-Uhler 2	0.0353	(1-exp(-R/N)) / (1-exp(-U/N))
Estrella	0.0327	1 - (1-R/U)^(U/N)
Adjusted Estrella	0.0193	1 - ((LogL-K)/LogL0)^(-2/N*LogL0)
McFadden's LRI	0.0129	R / U
Veall-Zimmermann	0.0446	(R * (U+N)) / (U * (R+N))
McKelvey-Zavoina	0.1019	

N = # of observations, K = # of regressors

Algorithm converged.

Parameter Estimates

Parameter	DF	Estimate	Standard Error	t Value	Approx Pr > |t|
Intercept	1	1.183894	0.367498	3.22	0.0013
educate	1	-0.002015	0.022004	-0.09	0.9271
income	1	-0.005921	0.008998	-0.66	0.5105
age	1	0.018646	0.004212	4.43	<.0001
male	1	-0.466195	0.108542	-4.30	<.0001
www	1	0.126483	0.135709	0.93	0.3513

```
          _Limit2        1         0.684929        0.056692       12.08       <.0001
          _Limit3        1         2.370441        0.085565       27.70       <.0001
```

However, Stata and PROC QLIM present cut points in a different way. Unlike Stata, PROC QLIM estimates the intercept, τ_2, and τ_3, assuming $\tau_1 = 0$. The estimated intercept (1.1839) of PROC QLIM is the same as `-/cut1` in Stata: - (-1.1839). The `_Limit2` above is the deviation of τ_1 from τ_2, .6849 = $\hat{\tau}_2 - \hat{\tau}_1$=-.4990-(-1.1839); $\hat{\tau}_2$ -.4990 is the value of `/cut2` in Stata (see Section 5.1). Similarly, `_Limit2` is 2.3704= $\hat{\tau}_3 - \hat{\tau}_1$=1.1865-(-1.1839), where 1.1865 is the value of `/cut3` in Stata. See Long and Freese (2003: 148-149) for discussion on this issue.

PROC LOGISTIC and PROC PROBIT estimate ordinal logit and probit models when a ordinal dependent variable is specified. The DESCENDING option is used to switch the signs of coefficients. PROC LOGISTIC conducts the Brant test on the parallel regression assumption, although the chi-squared 22.64 is slightly larger than 21.94 of `.brant` in Section 5.3 (22.64 versus 21.94). The hypothesis of the proportional odds assumption is rejected (p<.0122).

```
PROC LOGISTIC DATA = masil.gss_cdvm DESC;
     MODEL belief = educate income age male www /LINK=LOGIT;
RUN;
```

```
                                 The LOGISTIC Procedure

                                   Model Information

                    Data Set                      MASIL.GSS_CDVM
                    Response Variable             belief              belief
                    Number of Response Levels     4
                    Model                         cumulative logit
                    Optimization Technique        Fisher's scoring


                         Number of Observations Read        1174
                         Number of Observations Used        1174


                                    Response Profile

                            Ordered                      Total
                              Value        belief     Frequency

                                  1             3           392
                                  2             2           456
                                  3             1           134
                                  4             0           192

               Probabilities modeled are cumulated over the lower Ordered Values.


                                Model Convergence Status

                        Convergence criterion (GCONV=1E-8) satisfied.
```

Score Test for the Proportional Odds Assumption

Chi-Square	DF	Pr > ChiSq
22.6404	10	0.0122

Model Fit Statistics

Criterion	Intercept Only	Intercept and Covariates
AIC	3005.386	2976.548
SC	3020.590	3017.093
-2 Log L	2999.386	2960.548

Testing Global Null Hypothesis: BETA=0

Test	Chi-Square	DF	Pr > ChiSq
Likelihood Ratio	38.8383	5	<.0001
Score	38.2773	5	<.0001
Wald	38.2220	5	<.0001

Analysis of Maximum Likelihood Estimates

Parameter	DF	Estimate	Standard Error	Wald Chi-Square	Pr > ChiSq
Intercept 3	1	-1.1865	0.3648	10.5771	0.0011
Intercept 2	1	0.4990	0.3633	1.8863	0.1696
Intercept 1	1	1.1839	0.3655	10.4906	0.0012
educate	1	-0.00201	0.0218	0.0085	0.9265
income	1	-0.00592	0.00903	0.4303	0.5119
age	1	0.0186	0.00417	19.9857	<.0001
male	1	-0.4662	0.1088	18.3660	<.0001
www	1	0.1265	0.1355	0.8704	0.3508

Odds Ratio Estimates

Effect	Point Estimate	95% Wald Confidence Limits	
educate	0.998	0.956	1.042
income	0.994	0.977	1.012
age	1.019	1.011	1.027
male	0.627	0.507	0.776
www	1.135	0.870	1.480

Association of Predicted Probabilities and Observed Responses

Percent Concordant	57.9	Somers' D	0.168

```
Percent Discordant      41.1    Gamma         0.169
Percent Tied             0.9    Tau-a         0.117
Pairs                 480928    c             0.584
```

Stata `.ologit` and PROC LOGISTIC produce the same parameter estimates and similar (slightly different) standard errors. Intercept 1 (1.1839) through 3 (-1.1865) are equivalent to `/cut1` (-1.1839) through `/cut3` (1.1865) but their signs are switched. If you omit DESC, you will get the same cut points but parameter estimates of regressors will have opposite signs instead.

PROC GENMOD also fits the ordinal logit model with /DIST=MULTINOMIAl and /LINK=CLOGIT (or CUMLOGIT). Two options respectively indicate the multinomial probability distribution and cumulative logit function. This procedure with DESC produces the same parameter estimates and goodness-of-fit statistics. All cut points have opposite signs and cut point 1 and 3 are switched. Indeed, it is confusing. The output for parameter estimates is selectively displayed below.

```
PROC GENMOD DATA = masil.gss_cdvm DESC;
     CLASS belief;
     MODEL belief = educate income age male www /DIST=MULTINOMIAL LINK=CLOGIT;
RUN;
```

The GENMOD Procedure

Analysis Of Maximum Likelihood Parameter Estimates

Parameter	DF	Estimate	Standard Error	Wald 95% Confidence Limits		Wald Chi-Square	Pr > ChiSq
Intercept1	1	-1.1865	0.3663	-1.9044	-0.4687	10.50	0.0012
Intercept2	1	0.4990	0.3649	-0.2162	1.2141	1.87	0.1715
Intercept3	1	1.1839	0.3675	0.4636	1.9042	10.38	0.0013
educate	1	-0.0020	0.0220	-0.0451	0.0411	0.01	0.9271
income	1	-0.0059	0.0090	-0.0236	0.0117	0.43	0.5105
age	1	0.0186	0.0042	0.0104	0.0269	19.59	<.0001
male	1	-0.4662	0.1085	-0.6789	-0.2535	18.45	<.0001
www	1	0.1265	0.1357	-0.1395	0.3925	0.87	0.3513
Scale	0	1.0000	0.0000	1.0000	1.0000		

PROC PROBIT produces the same parameter estimates and standard errors with opposite signs. This command returns the same cut points as those of PROC QLIM except for the sign of the intercept. PROC QLIM and PROC PROBIT report 1.1839 and -1.1839, respectively.

```
PROC PROBIT DATA = masil.gss_cdvm;
     CLASS belief;
     MODEL belief = educate income age male www /DIST=LOGISTIC;
RUN;
```

The Probit Procedure

Analysis of Maximum Likelihood Parameter Estimates

Standard   95% Confidence   Chi-

```
Parameter   DF Estimate    Error       Limits        Square Pr > ChiSq

Intercept    1  -1.1839   0.3675  -1.9042  -0.4636   10.38      0.0013
Intercept2   1   0.6849   0.0567   0.5738   0.7960  145.97      <.0001
Intercept3   1   2.3704   0.0856   2.2027   2.5381  767.49      <.0001
educate      1   0.0020   0.0220  -0.0411   0.0451    0.01      0.9271
income       1   0.0059   0.0090  -0.0117   0.0236    0.43      0.5105
age          1  -0.0186   0.0042  -0.0269  -0.0104   19.59      <.0001
male         1   0.4662   0.1085   0.2535   0.6789   18.45      <.0001
www          1  -0.1265   0.1357  -0.3925   0.1395    0.87      0.3513
```

2.5 Ordinal Probit Model in SAS

PROC QLIM by default estimates a probit model. The DIST=NORMAL in the following procedure can be omitted.

```
PROC QLIM DATA=masil.gss_cdvm;
     MODEL belief = educate income age male www /DISCRETE (DIST=NORMAL);
RUN;
```

```
                              The QLIM Procedure

                     Discrete Response Profile of belief

              Index          Value           Frequency     Percent

                1              0                   192       16.35
                2              1                   134       11.41
                3              2                   456       38.84
                4              3                   392       33.39


                              Model Fit Summary

                 Number of Endogenous Variables                 1
                 Endogenous Variable                       belief
                 Number of Observations                      1174
                 Log Likelihood                             -1480
                 Maximum Absolute Gradient              0.0004222
                 Number of Iterations                          15
                 Optimization Method                 Quasi-Newton
                 AIC                                         2975
                 Schwarz Criterion                           3016


                           Goodness-of-Fit Measures

      Measure                      Value    Formula

      Likelihood Ratio (R)         40.13    2 * (LogL - LogL0)
      Upper Bound of R (U)        2999.4    - 2 * LogL0
      Aldrich-Nelson              0.0331    R / (R+N)
      Cragg-Uhler 1               0.0336    1 - exp(-R/N)
      Cragg-Uhler 2               0.0364    (1-exp(-R/N)) / (1-exp(-U/N))
      Estrella                    0.0338    1 - (1-R/U)^(U/N)
      Adjusted Estrella           0.0204    1 - ((LogL-K)/LogL0)^(-2/N*LogL0)
```

```
          McFadden's LRI              0.0134    R / U
          Veall-Zimmermann             0.046    (R * (U+N)) / (U * (R+N))
          McKelvey-Zavoina            0.0397

          N = # of observations, K = # of regressors

Algorithm converged.
```

Parameter Estimates

Parameter	DF	Estimate	Standard Error	t Value	Approx Pr > |t|
Intercept	1	0.713805	0.218273	3.27	0.0011
educate	1	-0.001519	0.013070	-0.12	0.9075
income	1	-0.002738	0.005371	-0.51	0.6102
age	1	0.010969	0.002475	4.43	<.0001
male	1	-0.290305	0.064630	-4.49	<.0001
www	1	0.064241	0.080919	0.79	0.4273
_Limit2	1	0.395983	0.032090	12.34	<.0001
_Limit3	1	1.433728	0.048873	29.34	<.0001

PROC QLIM and `.oprobit` produce almost the same parameter estimates and standard errors but present τ_m in a different manner. The intercept .7138 is the value of `/cut1` in Stata with an opposite sign. `_Limit2` is the deviation of τ_1 from τ_2: .3960 = $\tau_2 - \tau_1$=-.3178-(-.7138). Similarly, `_Limit3` is 1.4337 = $\tau_3 - \tau_1$=.7199-(-.7138).

PROC LOGISTIC also estimates the ordinal probit model with /LINK=PROBIT. The test for the parallel regression assumption reports a large chi-squared of 21.3229 and reject the null hypothesis (p<.0190). PROC LOGISTIC returns the same parameter estimates but slightly different standard errors, compared to PROC QLIM and Stata.

```
PROC LOGISTIC DATA = masil.gss_cdvm DESC;
     MODEL belief = educate income age male www /LINK=PROBIT;
RUN;
```

The LOGISTIC Procedure

Model Information

Data Set	MASIL.GSS_CDVM	
Response Variable	belief	belief
Number of Response Levels	4	
Model	cumulative probit	
Optimization Technique	Fisher's scoring	

Number of Observations Read	1174
Number of Observations Used	1174

Response Profile

Ordered Value	belief	Total Frequency
1	3	392
2	2	456
3	1	134
4	0	192

Probabilities modeled are cumulated over the lower Ordered Values.

Model Convergence Status

Convergence criterion (GCONV=1E-8) satisfied.

Score Test for the Equal Slopes Assumption

Chi-Square	DF	Pr > ChiSq
21.3229	10	0.0190

Model Fit Statistics

Criterion	Intercept Only	Intercept and Covariates
AIC	3005.386	2975.256
SC	3020.590	3015.801
-2 Log L	2999.386	2959.256

Testing Global Null Hypothesis: BETA=0

Test	Chi-Square	DF	Pr > ChiSq
Likelihood Ratio	40.1299	5	<.0001
Score	39.6928	5	<.0001
Wald	39.6600	5	<.0001

Analysis of Maximum Likelihood Estimates

Parameter	DF	Estimate	Standard Error	Wald Chi-Square	Pr > ChiSq
Intercept 3	1	-0.7199	0.2175	10.9547	0.0009
Intercept 2	1	0.3178	0.2170	2.1449	0.1430
Intercept 1	1	0.7138	0.2177	10.7509	0.0010
educate	1	-0.00152	0.0130	0.0136	0.9072
income	1	-0.00274	0.00538	0.2587	0.6110
age	1	0.0110	0.00248	19.5717	<.0001
male	1	-0.2903	0.0647	20.1340	<.0001
www	1	0.0642	0.0809	0.6307	0.4271

```
                Association of Predicted Probabilities and Observed Responses

                     Percent Concordant       57.9    Somers' D    0.167
                     Percent Discordant       41.2    Gamma        0.169
                     Percent Tied              0.9    Tau-a        0.117
                     Pairs                  480928    c            0.584
```

PROC GENMOD with /LINK=CUMPROBIT (CPROBIT) fits the ordinal probit regression model and reports the same parameter estimates and standard errors. Compared to Stata, this procedure returns the same cut points with different signs and order. The intercept 1 of -.7199 is equivalent to `/cut3` of .7199 in Stata.

```
PROC GENMOD DATA = masil.gss_cdvm DESC;
     CLASS belief;
     MODEL belief = educate income age male www /DIST=MULTINOMIAL LINK=CPROBIT;
RUN;
```

```
                                        The GENMOD Procedure

                         Analysis Of Maximum Likelihood Parameter Estimates

                                         Standard     Wald 95% Confidence          Wald
  Parameter       DF      Estimate          Error           Limits             Chi-Square    Pr > ChiSq

  Intercept1       1       -0.7199         0.2177     -1.1467       -0.2932         10.93        0.0009
  Intercept2       1        0.3178         0.2172     -0.1080        0.7436          2.14        0.1435
  Intercept3       1        0.7138         0.2183      0.2860        1.1416         10.69        0.0011
  educate          1       -0.0015         0.0131     -0.0271        0.0241          0.01        0.9075
  income           1       -0.0027         0.0054     -0.0133        0.0078          0.26        0.6102
  age              1        0.0110         0.0025      0.0061        0.0158         19.64        <.0001
  male             1       -0.2903         0.0646     -0.4170       -0.1636         20.18        <.0001
  www              1        0.0642         0.0809     -0.0944        0.2228          0.63        0.4273
  Scale            0        1.0000         0.0000      1.0000        1.0000
```

PROC PROBIT also fit the ordinal probit model and produces the same parameter estimates with their signs switched. Other parts of the output are skipped.

```
PROC PROBIT DATA = masil.gss_cdvm;
     CLASS belief;
     MODEL belief = educate income age male www /DIST=NORMAL;
RUN;
```

```
                         Analysis of Maximum Likelihood Parameter Estimates

                                    Standard    95% Confidence      Chi-
             Parameter   DF Estimate    Error       Limits        Square Pr > ChiSq

             Intercept    1  -0.7138   0.2183  -1.1416  -0.2860    10.69     0.0011
             Intercept2   1   0.3960   0.0321   0.3331   0.4589   152.27     <.0001
             Intercept3   1   1.4337   0.0489   1.3379   1.5295   860.62     <.0001
             educate      1   0.0015   0.0131  -0.0241   0.0271     0.01     0.9075
             income       1   0.0027   0.0054  -0.0078   0.0133     0.26     0.6102
             age          1  -0.0110   0.0025  -0.0158  -0.0061    19.64     <.0001
             male         1   0.2903   0.0646   0.1636   0.4170    20.18     <.0001
```

```
              www         1  -0.0642   0.0809  -0.2228   0.0944    0.63     0.4273
```

2.6 Ordinal Logit and Probit Models in LIMDEP (Ordered$)

In LIMDEP, the `Ordered$` command estimates ordinal logit and probit models. The `Logit` subcommand fits the ordinal logit model. In `Ordered$`, the values of the dependent variable need to begin with zero; otherwise, this command does not work.

```
ORDERED;Lhs=BELIEF;
    Rhs=ONE,EDUCATE,INCOME,AGE,MALE,WWW;
    Logit$

Normal exit from iterations. Exit status=0.
+---------------------------------------------+
| Ordered Probability Model                   |
| Maximum Likelihood Estimates                |
| Model estimated: Sep 09, 2009 at 04:30:47PM.|
| Dependent variable               BELIEF     |
| Weighting variable                 None     |
| Number of observations             1174     |
| Iterations completed                 12     |
| Log likelihood function       -1480.274     |
| Number of parameters                  8     |
| Info. Criterion: AIC =          2.53539     |
|   Finite Sample: AIC =          2.53550     |
| Info. Criterion: BIC =          2.56993     |
| Info. Criterion:HQIC =          2.54841     |
| Restricted log likelihood     -1499.693     |
| McFadden Pseudo R-squared      .0129488     |
| Chi squared                    38.83830     |
| Degrees of freedom                    5     |
| Prob[ChiSqd > value] =         .0000000     |
| Underlying probabilities based on Logistic  |
+---------------------------------------------+
+---------------------------------------------+
| Ordered Probability Model                   |
| Cell frequencies for outcomes               |
|  Y Count Freq  Y Count Freq  Y Count Freq   |
|  0   192 .163  1   134 .114  2   456 .388   |
|  3   392 .333                               |
+---------------------------------------------+

+--------+--------------+----------------+--------+--------+----------+
|Variable| Coefficient  | Standard Error |b/St.Er.|P[|Z|>z]| Mean of X|
+--------+--------------+----------------+--------+--------+----------+
---------+Index function for probability
 Constant|    1.18389368       .36502662     3.243   .0012
 EDUCATE |    -.00201453       .02200365     -.092   .9271   14.2427598
 INCOME  |    -.00592127       .00899848     -.658   .5105   24.6486371
 AGE     |     .01864562       .00422023     4.418   .0000   41.3074957
 MALE    |    -.46619519       .10864254    -4.291   .0000    .45059625
 WWW     |     .12648324       .13572220      .932   .3514    .78534923
---------+Threshold parameters for index
 Mu(1)   |     .68492936       .04678905    14.639   .0000
 Mu(2)   |    2.37044102       .07061820    33.567   .0000

+-----------------------------------------------------------------------+
|   Cross tabulation of predictions. Row is actual, column is predicted.    |
|   Model = Logistic  .  Prediction is number of the most probable cell.    |
+-------+-------+-----+-----+-----+-----+-----+-----+-----+-----+-----+-----+
| Actual|Row Sum|  0  |  1  |  2  |  3  |  4  |  5  |  6  |  7  |  8  |  9  |
+-------+-------+-----+-----+-----+-----+-----+-----+-----+-----+-----+-----+
|      0|    192|    0|    0|  157|   35|
|      1|    134|    0|    0|  102|   32|
|      2|    456|    0|    0|  352|  104|
|      3|    392|    0|    0|  266|  126|
+-------+-------+-----+-----+-----+-----+-----+-----+-----+-----+-----+-----+
```

```
|Col Sum|   1174|    0|    0|  877|  297|    0|    0|    0|    0|    0|    0|
+-------+-------+-----+-----+-----+-----+-----+-----+-----+-----+-----+-----+
```

LIMDEP and PROC QLIM produce the same parameter estimates but a bit different standard errors for cut points. `Mu(1)` and `Mu(2)` are equivalent to PROC QLIM's `_Limit2` and `_Limit3`, respectively. Their goodness-of-fit measures are slightly different. LIMDEP's AIC 2,969= 2.5354*1,174 and BIC 3,009=2.5699*1,174 are slightly different from those of SAS and Stata (2,976.548 and 3,017.0929, respectively).

The ordinal probit model is estimated by the `Ordered$` command without Logit. This command by default fits the ordinal probit model. You may find the same parameter estimates and slightly different standard errors for cut points.

```
ORDERED;Lhs=BELIEF;
    Rhs=ONE,EDUCATE,INCOME,AGE,MALE,WWW$

Normal exit from iterations. Exit status=0.
+---------------------------------------------+
| Ordered Probability Model                   |
| Maximum Likelihood Estimates                |
| Model estimated: Sep 09, 2009 at 04:37:36PM.|
| Dependent variable              BELIEF      |
| Weighting variable                None      |
| Number of observations            1174      |
| Iterations completed                11      |
| Log likelihood function       -1479.628     |
| Number of parameters                 8      |
| Info. Criterion: AIC =          2.53429     |
|   Finite Sample: AIC =          2.53439     |
| Info. Criterion: BIC =          2.56883     |
| Info. Criterion:HQIC =          2.54731     |
| Restricted log likelihood     -1499.693     |
| McFadden Pseudo R-squared      .0133794     |
| Chi squared                    40.12995     |
| Degrees of freedom                   5      |
| Prob[ChiSqd > value] =         .0000000     |
| Underlying probabilities based on Normal    |
+---------------------------------------------+
+---------------------------------------------+
| Ordered Probability Model                   |
| Cell frequencies for outcomes               |
|  Y Count Freq  Y Count Freq  Y Count Freq   |
|  0   192 .163  1   134 .114  2   456 .388   |
|  3   392 .333                               |
+---------------------------------------------+

+--------+--------------+----------------+--------+--------+----------+
|Variable| Coefficient  | Standard Error |b/St.Er.|P[|Z|>z]| Mean of X|
+--------+--------------+----------------+--------+--------+----------+
---------+Index function for probability
 Constant|     .71380448       .21714136     3.287   .0010
 EDUCATE |    -.00151940       .01306980     -.116   .9075   14.2427598
 INCOME  |    -.00273816       .00537112     -.510   .6102   24.6486371
 AGE     |     .01096931       .00247696     4.429   .0000   41.3074957
 MALE    |    -.29030497       .06462851    -4.492   .0000    .45059625
 WWW     |     .06424039       .08092358      .794   .4273    .78534923
---------+Threshold parameters for index
 Mu(1)   |     .39598281       .02776738    14.261   .0000
 Mu(2)   |    1.43372824       .04228906    33.903   .0000

+-----------------------------------------------------------------------+
|   Cross tabulation of predictions. Row is actual, column is predicted.  |
|   Model = Probit    .  Prediction is number of the most probable cell.  |
+-------+-------+-----+-----+-----+-----+-----+-----+-----+-----+-----+-----+
| Actual|Row Sum|  0  |  1  |  2  |  3  |  4  |  5  |  6  |  7  |  8  |  9  |
```

```
+-------+-------+-----+-----+-----+-----+-----+-----+-----+-----+-----+-----+
|      0|    192|    0|    0|  158|   34|
|      1|    134|    0|    0|  101|   33|
|      2|    456|    0|    0|  359|   97|
|      3|    392|    0|    0|  265|  127|
+-------+-------+-----+-----+-----+-----+-----+-----+-----+-----+-----+-----+
|Col Sum|   1174|    0|    0|  883|  291|    0|    0|    0|    0|    0|    0|
+-------+-------+-----+-----+-----+-----+-----+-----+-----+-----+-----+-----+
```

2.7 Ordinal Logit and Probit Models in SPSS

The `Plum` command estimates ordinal logit and probit models in SPSS. The `/LINK=LOGIT` and `/LINK=PROBIT` command fit ordinal logit and probit models, respectively. SPSS and Stata produce the same parameter estimates and cut points. Stata, SAS, LIMDEP, and SPSS report the same parameter estimates with some differences in standard errors.

```
PLUM belief WITH educate income age male www
   /CRITERIA = CIN(95) DELTA(0) LCONVERGE(0) MXITER(100) MXSTEP(5)
               PCONVERGE(1.0E-6) SINGULAR(1.0E-8)
   /LINK = LOGIT
   /PRINT = FIT PARAMETER SUMMARY.
```

Parameter Estimates

		Estimate	Std. Error	Wald	df	Sig.	95% Confidence Interval Lower Bound	95% Confidence Interval Upper Bound
Threshold	[belief = 0]	-1.184	.366	10.490	1	.001	-1.900	-.467
	[belief = 1]	-.499	.363	1.886	1	.170	-1.211	.213
	[belief = 2]	1.187	.365	10.578	1	.001	.472	1.902
Location	educate	-.002	.022	.009	1	.927	-.045	.041
	income	-.006	.009	.430	1	.512	-.024	.012
	age	.019	.004	19.988	1	.000	.010	.027
	male	-.466	.109	18.365	1	.000	-.679	-.253
	www	.126	.136	.871	1	.351	-.139	.392

Link function: Logit.

```
PLUM belief WITH educate income age male www
   /CRITERIA = CIN(95) DELTA(0) LCONVERGE(0) MXITER(100) MXSTEP(5)
               PCONVERGE(1.0E-6) SINGULAR(1.0E-8)
   /LINK = PROBIT
   /PRINT = FIT PARAMETER SUMMARY .
```

Parameter Estimates

		Estimate	Std. Error	Wald	df	Sig.	95% Confidence Interval Lower Bound	95% Confidence Interval Upper Bound
Threshold	[belief = 0]	-.714	.218	10.751	1	.001	-1.140	-.287
	[belief = 1]	-.318	.217	2.145	1	.143	-.743	.107
	[belief = 2]	.720	.218	10.955	1	.001	.294	1.146

Location	educate	-.002	.013	.014	1	.907	-.027	.024
	income	-.003	.005	.259	1	.611	-.013	.008
	age	.011	.002	19.572	1	.000	.006	.016
	male	-.290	.065	20.134	1	.000	-.417	-.164
	www	.064	.081	.631	1	.427	-.094	.223

Link function: Probit.

Table 2.1 summarizes the results of the ordinal logit model that Stata, SAS, and LIMDEP produced. You will get the similar results in the ordinal probit model.

Table 2.1 Parameter Estimates and Goodness-of-fit of the Ordinal Logit Model

	SAS				Stata	LIMDEP
	LOGISTIC	PROBIT	GENMOD	QLIM	(.ologit)	(Ordered$)
Education	-.0020	.0020	-.0020	-.0020	-.0020	-.0020
	(.0218)	(.0220)	(.0220)	(.0220)	(.0220)	(.0220)
Family income	-.0059	.0059	-.0059	-.0059	-.0059	-.0059
	(.0090)	(.0090)	(.0090)	(.0090)	(.0090)	(.0090)
Age	.0186	-.0186	.0186	.0186	.0186	.0186
	(.0042)	(.0042)	(.0042)	(.0042)	(.0042)	(.0042)
Gender (male)	-.4662	.4662	-.4662	-.4662	-.4662	-.4662
	(.1088)	(.1085)	(.1085)	(.1085)	(.1085)	(.1086)
WWW use	.1265	-.1265	.1265	.1265	.1265	.1265
	(.1355)	(.1357)	(.1357)	(.1357)	(.1357)	(.1357)
Cut point 1	1.1839	-1.1839	-1.1865	1.1839	-1.1839	1.1839
	(.3655)	(.3675)	(.3663)	(.3675)	(.3675)	(.3650)
Cut point 2	.4990	.6849	.4990	.6849	-.4990	.6849
	(.3633)	(.0567)	(.3649)	(.0567)	(.3649)	(.0468)
Cut point 3	-1.1865	2.3704	1.1839	2.3704	1.1865	2.3704
	(.3648)	(.0856)	(.3675)	(.0856)	(.3663)	(.0762)
Log likelihood	-1480.2738	-1480.2738	-1480.2738	-1480.	-1480.2738	-1480.274
Likelihood test	38.8383			38.838	38.84	38.8383
Pseudo R^2				.0129	.0129	.0129
AIC	2976.548		2976.5475	2977.	*2976.548*	2968.9417
Schwarz	3017.093			3017.		
BIC			3017.0929		*3017.093*	3009.388

* PROC LOGISTIC reports (-2*Log-likelihood).

PROC LOGISTIC, PROC QLIM, and Stata are recommended for ordinal response models. Despite slightly different standard errors and opposite signs of threshold points, PROC LOGISTIC returns the comparable statistics to Stata and PROC QLIM. The beauty of PROC LOGISTIC is the feature that tests the parallel regression assumption (proportional odds assumption in a logit model) in both logit and probit models. In Stata, you can conduct the Brant test using SPost `.brant` for the logit model (not available in the probit model) and estimate a generalized ordinal logit model using Williams' `.gologit2`. You may also benefit from other SPost commands such as `.listcoef`, `.prchange`, and `.prgen` and Williams' `.mfx2` in Stata.

3. Multinomial Logit Regression Model

Let us examine the model of religious intensity in the multinomial logit model without changing specification. Remember that the Brant test rejects the null hypothesis of the proportional odds assumption and thus the ordinal logit model in chapter 2 is not theoretically valid. Stata has the `.mprobit` command to fit the multinomial probit model but this model is less often used than the logit counterpart mainly due to its practical difficulty in estimation.

In a multinomial logit model, independent variables contain characteristics of individuals, while they are attributes of the choices in a conditional logit model, which will be discussed in chapter 4.

3.1 Multinomial Logit and Probit in Stata (.mlogit and .mprobit)

In Stata, the `.mlogit` command fits the multinomial logit model. This command by default uses most frequent category (not very strong in this case) as the base outcome when estimating the model. SAS PROC LOGISTIC and LIMDEP use the smallest value as the base outcome, while PROC CATMOD fits the model on the basis of the largest value in the dependent variable. SPSS can change a base outcome. In order to compare Stata with other software packages, let us fit the same model using two different base outcomes. The `base()` option indicates a value of the dependent variable other than the default of the most frequent outcome. The following `base(3)` fits the model using the last category (strong in this case).

```
. mlogit belief educate income age male www, base(3)

Iteration 0:   log likelihood = -1499.6929
Iteration 1:   log likelihood = -1469.6341
Iteration 2:   log likelihood = -1469.4492
Iteration 3:   log likelihood = -1469.4492

Multinomial logistic regression                   Number of obs   =       1174
                                                  LR chi2(15)     =      60.49
                                                  Prob > chi2     =     0.0000
Log likelihood = -1469.4492                       Pseudo R2       =     0.0202

------------------------------------------------------------------------------
      belief |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
No_religion  |
     educate |   .0041038   .0364791     0.11   0.910     -.067394    .0756016
      income |   .0005614   .0149146     0.04   0.970    -.0286708    .0297935
         age |  -.0288972   .0070994    -4.07   0.000    -.0428118   -.0149827
        male |   .8967689   .1827037     4.91   0.000     .5386761    1.254862
         www |  -.0347578   .2318055    -0.15   0.881    -.4890883    .4195727
       _cons |   .0141817   .6060507     0.02   0.981    -1.173656    1.202019
-------------+----------------------------------------------------------------
Somewhat_s~g |
     educate |   .0060908    .041313     0.15   0.883    -.0748812    .0870628
      income |   .0231701   .0184093     1.26   0.208    -.0129116    .0592517
         age |  -.0161198   .0077715    -2.07   0.038    -.0313517   -.0008878
        male |   .1738551   .2064474     0.84   0.400    -.2307744    .5784847
         www |  -.4482836   .2417881    -1.85   0.064    -.9221795    .0256124
       _cons |  -.7764871   .7036746    -1.10   0.270    -2.155664    .6026898
-------------+----------------------------------------------------------------
Not_very_s~g |
     educate |  -.0269446   .0284494    -0.95   0.344    -.0827043    .0288151
      income |   .0048478     .01171     0.41   0.679    -.0181035    .0277991
         age |  -.0237972   .0053893    -4.42   0.000    -.0343599   -.0132344
```

```
        male |   .4602734   .1429313     3.22   0.001     .1801332    .7404135
         www |  -.0252644   .1785439    -0.14   0.887    -.3752041    .3246753
       _cons |   1.237746   .4728153     2.62   0.009     .3110455    2.164447
-------------+----------------------------------------------------------------
Strong       |  (base outcome)
------------------------------------------------------------------------------
```

Now, fit the model using the smallest value of the outcome variable. Two outcomes produce the same goodness-of-fit measures but their parameter estimates are different each other. They estimate exactly the same model but present it in different ways.

```
. mlogit belief educate income age male www, base(0)

Iteration 0:   log likelihood = -1499.6929
Iteration 1:   log likelihood = -1469.6341
Iteration 2:   log likelihood = -1469.4492
Iteration 3:   log likelihood = -1469.4492

Multinomial logistic regression                   Number of obs   =       1174
                                                  LR chi2(15)     =      60.49
                                                  Prob > chi2     =     0.0000
Log likelihood = -1469.4492                       Pseudo R2       =     0.0202

------------------------------------------------------------------------------
      belief |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
No_religion  |  (base outcome)
-------------+----------------------------------------------------------------
Somewhat_s~g |
     educate |    .001987   .0465735     0.04   0.966    -.0892955    .0932695
      income |   .0226087   .0202724     1.12   0.265    -.0171245    .0623419
         age |   .0127774   .0090102     1.42   0.156    -.0048822     .030437
        male |  -.7229137   .2307764    -3.13   0.002    -1.175227   -.2706002
         www |  -.4135258   .2781579    -1.49   0.137    -.9587052    .1316536
       _cons |  -.7906688   .7863491    -1.01   0.315    -2.331885    .7505472
-------------+----------------------------------------------------------------
Not_very_s~g |
     educate |  -.0310484   .0352445    -0.88   0.378    -.1001264    .0380297
      income |   .0042864   .0142516     0.30   0.764    -.0236462    .0322191
         age |      .0051   .0069699     0.73   0.464    -.0085608    .0187608
        male |  -.4364955     .17508    -2.49   0.013    -.7796459   -.0933451
         www |   .0094934   .2233075     0.04   0.966    -.4281812     .447168
       _cons |   1.223565   .5817148     2.10   0.035     .0834247    2.363705
-------------+----------------------------------------------------------------
Strong       |
     educate |  -.0041038   .0364791    -0.11   0.910    -.0756016     .067394
      income |  -.0005614   .0149146    -0.04   0.970    -.0297935    .0286708
         age |   .0288972   .0070994     4.07   0.000     .0149827    .0428118
        male |  -.8967689   .1827037    -4.91   0.000    -1.254862   -.5386761
         www |   .0347578   .2318055     0.15   0.881    -.4195727    .4890883
       _cons |  -.0141817   .6060507    -0.02   0.981    -1.202019    1.173656
------------------------------------------------------------------------------
```

This multinomial logit model returns a large likelihood ratio statistic (χ^2=60.49) but most individual parameters are not statistically discernable from zero. This model does not fit the data well.

```
. fitstat

Measures of Fit for mlogit of belief

Log-Lik Intercept Only:      -1499.693   Log-Lik Full Model:           -1469.449
D(1150):                      2938.898   LR(15):                          60.487
                                         Prob > LR:                        0.000
McFadden's R2:                   0.020   McFadden's Adj R2:                0.004
ML (Cox-Snell) R2:               0.050   Cragg-Uhler(Nagelkerke) R2:       0.054
```

```
Count R2:                        0.428   Adj Count R2:                  0.064
AIC:                             2.544   AIC*n:                      2986.898
BIC:                         -5189.499   BIC':                         45.535
BIC used by Stata:            3066.126   AIC used by Stata:          2974.898
```

Before interpreting the output, you need to check if the independence of irrelevant alternatives (IIA) assumption is satisfied. The SPost `.mlogtest` command conducts a variety of statistical tests for the multinomial logit model. This command conducts the Hausman and Small-Hsiao tests for a multinomial logit model.

```
. mlogtest, hausman smhsiao base

**** Hausman tests of IIA assumption (N=40)

 Ho: Odds(Outcome-J vs Outcome-K) are independent of other alternatives.

 Omitted |      chi2   df   P>chi2   evidence
---------+------------------------------------
Somewhat |    -0.044   11     ---     ---
Not_very |  4671.304   11    0.000   against Ho
  Strong |  9621.685   11    0.000   against Ho
No_relig |     1.075   11    1.000   for Ho
----------------------------------------------
 Note: If chi2<0, the estimated model does not
 meet asymptotic assumptions of the test.

**** Small-Hsiao tests of IIA assumption (N=40)

 Ho: Odds(Outcome-J vs Outcome-K) are independent of other alternatives.

 Omitted |  lnL(full)  lnL(omit)    chi2   df   P>chi2   evidence
---------+---------------------------------------------------------
Somewhat |    -12.762     -8.407   8.711   12    0.727   for Ho
Not_very |     -4.528     -2.794   3.467   12    0.991   for Ho
  Strong |     -9.813     -6.151   7.325   12    0.835   for Ho
No_relig |     -7.977     -1.556  12.840   12    0.381   for Ho
-------------------------------------------------------------------
```

In Hausman test, two tests reject the null hypothesis that IIA holds. Despite a negative chi-squared, IIA does not appear to be hold in this model. However, none of tests in Small-Hsiao rejects the null hypothesis; the IIA assumption is not violated. Both tests report inconsistent and mixed results. See Long and Freese (2003:188-191) for the discussion on the Hausman and Small-Hsiao tests.

Let us fit the multinomial probit model using the `.mprobit` command and compare with the multinomial logit model. Most parameter estimates and standard errors are smaller than those of the multinomial logit model. This multinomial probit model took longer time to converge than the logit model.

```
. mprobit belief educate income age male www, base(0)

Iteration 0:   log likelihood =  -1470.818
Iteration 1:   log likelihood = -1469.3687
Iteration 2:   log likelihood = -1469.3674
Iteration 3:   log likelihood = -1469.3674

Multinomial probit regression                     Number of obs   =       1174
                                                  Wald chi2(15)   =      59.20
Log likelihood = -1469.3674                       Prob > chi2     =     0.0000

------------------------------------------------------------------------------
      belief |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
```

```
-------------+----------------------------------------------------------------
No_religion  |  (base outcome)
-------------+----------------------------------------------------------------
Somewhat_s~g |
     educate |  -.0015242    .029834    -0.05   0.959    -.0599978    .0569494
      income |   .0130568   .0126041     1.04   0.300    -.0116469    .0377605
         age |   .0085334   .0057111     1.49   0.135    -.0026602    .0197269
        male |  -.4719833   .1480945    -3.19   0.001    -.7622431   -.1817235
         www |   -.265123   .1815288    -1.46   0.144     -.620913    .0906669
       _cons |  -.4473774   .5004132    -0.89   0.371    -1.428169    .5334145
-------------+----------------------------------------------------------------
Not_very_s~g |
     educate |  -.0254642    .025485    -1.00   0.318    -.0754138    .0244854
      income |   .0029475   .0103607     0.28   0.776    -.0173591    .0232541
         age |   .0020936   .0048947     0.43   0.669    -.0074998     .011687
        male |  -.2794787   .1251762    -2.23   0.026    -.5248194   -.0341379
         www |   .0111551   .1592424     0.07   0.944    -.3009543    .3232645
       _cons |   .9806248   .4175688     2.35   0.019     .1622049    1.799045
-------------+----------------------------------------------------------------
Strong       |
     educate |  -.0027547   .0258363    -0.11   0.915    -.0533929    .0478835
      income |  -.0008257   .0107118    -0.08   0.939    -.0218203    .0201689
         age |   .0210121   .0049422     4.25   0.000     .0113257    .0306986
        male |  -.6416372   .1287642    -4.98   0.000    -.8940103    -.389264
         www |   .0321726   .1626568     0.20   0.843    -.2866288    .3509741
       _cons |  -.0220019   .4275629    -0.05   0.959    -.8600097    .8160059
------------------------------------------------------------------------------
```

Both multinomial logit and probit models produce similar goodness-of-fit measures. Their likelihood ratios are 60.487 and 59.201 and AIC*Ns are 2986.898 and 2974.735, respectively.

```
. fitstat

Measures of Fit for mprobit of belief

Log-Lik Full Model:          -1469.367   D(1156):                    2938.735
Wald X2(15):                    59.201   Prob > X2:                     0.000
Count R2:                        0.428   Adj Count R2:                  0.064
AIC:                             2.534   AIC*n:                      2974.735
BIC:                         -5232.072   BIC':                         46.822
BIC used by Stata:            3065.962   AIC used by Stata:          2974.735
```

3.2 Interpretation of the Multinomial Logit Model in Stata

Since multinomial logit and probit models produce many parameter estimates and other statistics, their interpretation is not as easy as that of binary logit and probit models. Let us interpret the result using factor changes in the odds, predicted probabilities, and marginal effects (discrete changes). For theoretical discussion on this issue, see Long (1997: 164-178).

. `listcoef` compares all possible pairs of responses (outcomes) to compute factor changes in odds with respect to variables listed.

```
. listcoef age male, factor help

mlogit (N=1174): Factor Change in the Odds of belief

Variable: age (sd=13.407127)

Odds comparing    |
Alternative 1     |
to Alternative 2  |      b         z     P>|z|     e^b   e^bStdX
------------------+---------------------------------------------
Somewhat-Not_very |   0.00768    0.990   0.322   1.0077   1.1084
Somewhat-Strong   |  -0.01612   -2.074   0.038   0.9840   0.8056
```

```
Somewhat-No_relig |   0.01278    1.418   0.156   1.0129   1.1869
Not_very-Somewhat |  -0.00768   -0.990   0.322   0.9924   0.9022
Not_very-Strong   |  -0.02380   -4.416   0.000   0.9765   0.7268
Not_very-No_relig |   0.00510    0.732   0.464   1.0051   1.0708
Strong  -Somewhat |   0.01612    2.074   0.038   1.0163   1.2413
Strong  -Not_very |   0.02380    4.416   0.000   1.0241   1.3758
Strong  -No_relig |   0.02890    4.070   0.000   1.0293   1.4732
No_relig-Somewhat |  -0.01278   -1.418   0.156   0.9873   0.8426
No_relig-Not_very |  -0.00510   -0.732   0.464   0.9949   0.9339
No_relig-Strong   |  -0.02890   -4.070   0.000   0.9715   0.6788
----------------------------------------------------------------

Variable: male (sd=.49776532)

Odds comparing    |
Alternative 1     |
to Alternative 2  |      b         z     P>|z|     e^b   e^bStdX
------------------+---------------------------------------------
Somewhat-Not_very |  -0.28642   -1.426   0.154   0.7509   0.8671
Somewhat-Strong   |   0.17386    0.842   0.400   1.1899   1.0904
Somewhat-No_relig |  -0.72291   -3.133   0.002   0.4853   0.6978
Not_very-Somewhat |   0.28642    1.426   0.154   1.3316   1.1532
Not_very-Strong   |   0.46027    3.220   0.001   1.5845   1.2575
Not_very-No_relig |  -0.43650   -2.493   0.013   0.6463   0.8047
Strong  -Somewhat |  -0.17386   -0.842   0.400   0.8404   0.9171
Strong  -Not_very |  -0.46027   -3.220   0.001   0.6311   0.7952
Strong  -No_relig |  -0.89677   -4.908   0.000   0.4079   0.6399
No_relig-Somewhat |   0.72291    3.133   0.002   2.0604   1.4331
No_relig-Not_very |   0.43650    2.493   0.013   1.5473   1.2427
No_relig-Strong   |   0.89677    4.908   0.000   2.4517   1.5626
----------------------------------------------------------------
       b = raw coefficient
       z = z-score for test of b=0
   P>|z| = p-value for z-test
     e^b = exp(b) = factor change in odds for unit increase in X
 e^bStdX = exp(b*SD of X) = change in odds for SD increase in X
```

Sample interpretations are as follows. For a unit increase in age, *the odds of having strong belief (3) versus no religion (0)* is expected to increase by a factor of 1.0293=exp(.0289) or *the odds of having no religion relative to strong belief* will decrease by a factor of .9715=exp(-.0289) =1/1.0293, holding all other variables constant. For a standard deviation increase in age, the odds of having somewhat strong belief (1) relative to not very strong belief (2) will increase by a factor of 1.1084=exp(.0077*13.4071) or the odds of having not very strong belief versus somewhat strong belief is expected to decrease by a factor of .9022= exp(.0077*13.4071)= 1/1.1084. The odds of having strong belief relative to no religion are .4079=exp(-.8968) times smaller for men than for women, holding all other covariates constant; the odds of having no religion relative strong belief are 2.4517 (=1/.4079) times larger for men than for women.

Alternative way is to report percent changes of the odds. For a unit increase in age, the odds of having strong belief relative to no religion is expected to increase by 2.9 percent or the odds of having no religion versus strong belief will decrease by 2.8 percent. The odds of having strong belief versus no religion are 59.2 percent smaller for men than for women; the odds of having no religion relative to strong belief are 145.2 percent larger for men than for women. Women are more likely to have religion and, if any, have strong belief than men.

```
. listcoef age male, percent help

mlogit (N=1174): Percentage Change in the Odds of belief

Variable: age (sd=13.407127)
```

```
Odds comparing    |
Alternative 1     |
to Alternative 2  |      b         z     P>|z|       %     %StdX
------------------+---------------------------------------------
Somewhat-Not_very |   0.00768    0.990   0.322      0.8     10.8
Somewhat-Strong   |  -0.01612   -2.074   0.038     -1.6    -19.4
Somewhat-No_relig |   0.01278    1.418   0.156      1.3     18.7
Not_very-Somewhat |  -0.00768   -0.990   0.322     -0.8     -9.8
Not_very-Strong   |  -0.02380   -4.416   0.000     -2.4    -27.3
Not_very-No_relig |   0.00510    0.732   0.464      0.5      7.1
Strong  -Somewhat |   0.01612    2.074   0.038      1.6     24.1
Strong  -Not_very |   0.02380    4.416   0.000      2.4     37.6
Strong  -No_relig |   0.02890    4.070   0.000      2.9     47.3
No_relig-Somewhat |  -0.01278   -1.418   0.156     -1.3    -15.7
No_relig-Not_very |  -0.00510   -0.732   0.464     -0.5     -6.6
No_relig-Strong   |  -0.02890   -4.070   0.000     -2.8    -32.1
----------------------------------------------------------------

Variable: male (sd=.49776532)

Odds comparing    |
Alternative 1     |
to Alternative 2  |      b         z     P>|z|       %     %StdX
------------------+---------------------------------------------
Somewhat-Not_very |  -0.28642   -1.426   0.154    -24.9    -13.3
Somewhat-Strong   |   0.17386    0.842   0.400     19.0      9.0
Somewhat-No_relig |  -0.72291   -3.133   0.002    -51.5    -30.2
Not_very-Somewhat |   0.28642    1.426   0.154     33.2     15.3
Not_very-Strong   |   0.46027    3.220   0.001     58.5     25.7
Not_very-No_relig |  -0.43650   -2.493   0.013    -35.4    -19.5
Strong  -Somewhat |  -0.17386   -0.842   0.400    -16.0     -8.3
Strong  -Not_very |  -0.46027   -3.220   0.001    -36.9    -20.5
Strong  -No_relig |  -0.89677   -4.908   0.000    -59.2    -36.0
No_relig-Somewhat |   0.72291    3.133   0.002    106.0     43.3
No_relig-Not_very |   0.43650    2.493   0.013     54.7     24.3
No_relig-Strong   |   0.89677    4.908   0.000    145.2     56.3
----------------------------------------------------------------
       b = raw coefficient
       z = z-score for test of b=0
   P>|z| = p-value for z-test
       % = percent change in odds for unit increase in X
   %StdX = percent change in odds for SD increase in X
```

Predicted probabilities are more intuitive than changes in the odds. You may report predicted probabilities in a table or a plot. `.prvalue` computes the predicted probabilities of all outcome categories given a set of reference points. For example, the predicted probability that female WWW users with 16 years of education have strong religious belief (`belief=3`) is 39.41 percent, holding family income and age at their means (25 thousands and age 41). The predicted probability of having no religion is 12.67 percent, 11.61 for somewhat strong, and 36.30 for not very strong.

```
. prvalue, x(educate=16 male=0 www=1) rest(mean)

mlogit: Predictions for belief

Confidence intervals by delta method

                                95% Conf. Interval
  Pr(y=Somewhat|x):   0.1161   [ 0.0871,    0.1451]
  Pr(y=Not_very|x):   0.3630   [ 0.3193,    0.4068]
  Pr(y=Strong|x):     0.3941   [ 0.3490,    0.4392]
  Pr(y=No_relig|x):   0.1267   [ 0.0971,    0.1564]

      educate     income        age       male        www
x=         16  24.648637  41.307496          0          1
```

The following `.prtab` command returns a series of tables of predicted probabilities for the combination of WWW use and gender. Find the four predicted probabilities above in the following tables. There appear to be significant gender difference in intensity of religious belief but WWW use does not make any significant difference.

```
. prtab male www, x(educate=16 male=0 www=1) rest(mean)

mlogit: Predicted probabilities for belief

Predicted probability of outcome 1 (Somewhat_strong)

--------------------------------
          |        WWW Use
   Gender | Non-users      Users
----------+---------------------
   Female |    0.1684     0.1161
     Male |    0.1421     0.0974
--------------------------------

Predicted probability of outcome 2 (Not_very_strong)

--------------------------------
          |        WWW Use
   Gender | Non-users      Users
----------+---------------------
   Female |    0.3449     0.3630
     Male |    0.3876     0.4056
--------------------------------

Predicted probability of outcome 3 (Strong)

--------------------------------
          |        WWW Use
   Gender | Non-users      Users
----------+---------------------
   Female |    0.3651     0.3941
     Male |    0.2589     0.2779
--------------------------------

Predicted probability of outcome 0 (No_religion)

--------------------------------
          |        WWW Use
   Gender | Non-users      Users
----------+---------------------
   Female |    0.1215     0.1267
     Male |    0.2113     0.2191
--------------------------------

      educate     income        age       male        www
x=         16  24.648637  41.307496          0          1
```

Now, let us see how predicted probabilities change as a continuous covariate increases. The `.prgen` command makes it easy to generate such predicted probabilities. The following commands generate a series of predicted probabilities that male and female WWW users, who graduated a college, fall in each category of religious intensity at the average family income.

```
. quietly mlogit belief educate income age male www, base(3)

. prgen age, from(18) to(92) ncases(20) x(educate=16 male=1 www=1) rest(mean) gen(age1)

mlogit: Predicted values as age varies from 18 to 92.

      educate     income        age       male        www
x=         16  24.648637  41.307496          1          1
```

```
. prgen age, from(18) to(92) ncases(20) x(educate=16 male=0 www=1) rest(mean) gen($age0)

mlogit: Predicted values as age varies from 18 to 92.

      educate     income        age       male        www
x=         16  24.648637  41.307496          0          1
```

Figure 3.1 is based on the predicted probabilities generated by `.prgen` above. Notice that we are using the same reference points when computing predicted probabilities in binary, ordinal, and multinomial response models. See the Stata script for the detail about data manipulation.

Figure 3.1 Predicted Probabilities of Religious Intensity (Multinomial Logit Model)

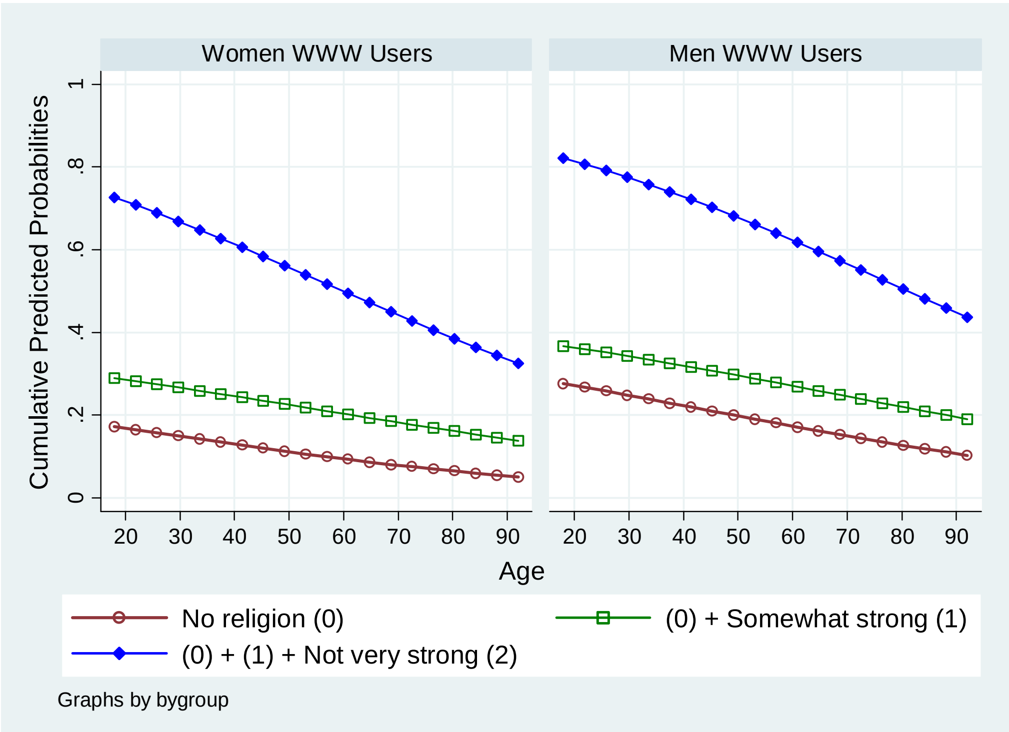


Figure 3.1 is very similar to Figure 2.1 for the ordinal logit model and 2.2 for the ordinal probit model. Pay attention to the proportions of areas segmented by three curves in each plane. As people get older, they are more likely to have strong religious belief and less likely to have no religion and not very strong belief. However, age does not influence the category of somewhat strong belief; the first two curves from the bottom run parallel and the area between the curves (virtually lines) remains unchanged regardless of age in both planes. Obviously, gender makes big difference; women WWW users are more likely to have strong belief than their men counterparts, holding all other covariates at their reference points. More than half of women WWW users have strong religious belief if they are older than 60, while more than half of men who are older than 80 have strong belief.

Finally, you may interpret the output of a multinomial logit model using marginal changes and discrete changes. `.mfx` reports that the predicted probability that female WWW users with 16 years of education do not have any religion is 12.67 percent at the reference points (cross-check in the output of `.prvalue` and `.prtab` above).

```
. mfx, at(mean educate=16 male=0 www=1)

Marginal effects after mlogit
      y  = Pr(belief==No_religion) (predict)
         =  .12671241
------------------------------------------------------------------------------
variable |      dy/dx    Std. Err.     z    P>|z|  [    95% C.I.   ]      X
---------+--------------------------------------------------------------------
 educate |    .001604      .00366    0.44   0.661  -.005574  .008782        16
  income |  -.0005018      .00147   -0.34   0.732  -.003377  .002374   24.6486
     age |  -.0018658      .00072   -2.58   0.010  -.003284 -.000447   41.3075
   male*|   .0923384       .0229    4.03   0.000   .047445  .137231         0
    www*|   .0051744      .02219    0.23   0.816  -.038324  .048673         1
------------------------------------------------------------------------------
(*) dy/dx is for discrete change of dummy variable from 0 to 1
```

For example, for a unit increase in age, the predicted probability of having no religion is expected to decrease by .19 percent, holding all other variables at their reference points (education=16 years, income=25 thousands). Men are 9.23 percent more likely to have no religion than women at the same reference points. These results are consistent with your conclusion in the ordinal logit model (see Section 5.1 and 5.2). Next `.prchange` reports marginal changes for all outcomes (no religion through strong belief).

```
. prchange age male, x(educate=16 male=0 www=1) rest(mean)

mlogit: Changes in Probabilities for belief

age
            Avg|Chg|    Somewhat    Not_very      Strong    No_relig
Min->Max   .18615842   -.0245965  -.23263715   .37231684  -.11508319
   -+1/2   .00279291  -.00022607  -.00349399   .00558585  -.00186574
  -+sd/2   .03737569  -.00302599  -.04674432   .07475138  -.02498107
MargEfct   .00279294  -.00022607  -.00349406   .00558589  -.00186576

male
        Avg|Chg|    Somewhat    Not_very      Strong    No_relig
0->1    .0674563  -.01869338   .04257423  -.11621922   .09233837

          Somewhat   Not_very     Strong   No_relig
Pr(y|x)  .11611661  .36304542  .39412558   .1267124

       educate   income      age     male      www
   x=       16  24.6486  41.3075        0        1
sd_x=  2.56971  6.19427  13.4071  .497765  .410755
```

For a unit increase in age, the probability of having strong belief is expected to increase by .56 percent, holding all other variables constant at their reference points. Male are 11.62 percent less likely than women to have strong religious belief. `.mfx2` produces more detail information including standard errors for all outcomes. Find the corresponding marginal effects and discrete changes discussed so far in the following tables.

```
. mfx2, at(mean educate=16 male=0 www=1)

Frequencies for belief...

      Religious |
      Intensity |      Freq.     Percent        Cum.
----------------+-----------------------------------
    No religion |        192       16.35       16.35
Somewhat strong |        134       11.41       27.77
Not very strong |        456       38.84       66.61
         Strong |        392       33.39      100.00
```

```
----------------+-----------------------------------
          Total |      1,174      100.00

Computing marginal effects after mlogit for belief == 0...

Marginal effects after mlogit
      y  = Pr(belief==0) (predict, o(0))
         =   .1267124
------------------------------------------------------------------------------
variable |      dy/dx    Std. Err.     z    P>|z|  [    95% C.I.   ]      X
---------+--------------------------------------------------------------------
 educate |    .001604      .00366    0.44   0.661  -.005573  .008781        16
  income |  -.0005018      .00147   -0.34   0.732  -.003377  .002374   24.6486
     age |  -.0018658      .00072   -2.58   0.010  -.003284 -.000447   41.3075
    male*|   .0923384       .0229    4.03   0.000   .047445  .137231         0
     www*|   .0051744      .02219    0.23   0.816  -.038324  .048673         1
------------------------------------------------------------------------------
(*) dy/dx is for discrete change of dummy variable from 0 to 1

Computing marginal effects after mlogit for belief == 1...

Marginal effects after mlogit
      y  = Pr(belief==1) (predict, o(1))
         =  .11611661
------------------------------------------------------------------------------
variable |      dy/dx    Std. Err.     z    P>|z|  [    95% C.I.   ]      X
---------+--------------------------------------------------------------------
 educate |   .0017006      .00398    0.43   0.669  -.006091  .009493        16
  income |   .0021654      .00175    1.24   0.216  -.001267  .005598   24.6486
     age |  -.0002261      .00074   -0.31   0.760  -.001677  .001225   41.3075
    male*|  -.0186934       .0178   -1.05   0.294   -.05358  .016193         0
     www*|  -.0522979      .02949   -1.77   0.076    -.1101  .005504         1
------------------------------------------------------------------------------
(*) dy/dx is for discrete change of dummy variable from 0 to 1

Computing marginal effects after mlogit for belief == 2...

Marginal effects after mlogit
      y  = Pr(belief==2) (predict, o(2))
         =  .36304541
------------------------------------------------------------------------------
variable |      dy/dx    Std. Err.     z    P>|z|  [    95% C.I.   ]      X
---------+--------------------------------------------------------------------
 educate |  -.0066763      .00567   -1.18   0.239  -.017784  .004432        16
  income |   .0001184      .00236    0.05   0.960  -.004512  .004749   24.6486
     age |  -.0034941      .00111   -3.16   0.002  -.005662 -.001326   41.3075
    male*|   .0425742      .02889    1.47   0.141  -.014053  .099201         0
     www*|   .0181153      .03496    0.52   0.604  -.050399   .08663         1
------------------------------------------------------------------------------
(*) dy/dx is for discrete change of dummy variable from 0 to 1

Computing marginal effects after mlogit for belief == 3...

Marginal effects after mlogit
      y  = Pr(belief==3) (predict, o(3))
         =  .39412557
------------------------------------------------------------------------------
variable |      dy/dx    Std. Err.     z    P>|z|  [    95% C.I.   ]      X
---------+--------------------------------------------------------------------
 educate |   .0033717      .00615    0.55   0.583  -.008681  .015424        16
  income |  -.0017821      .00255   -0.70   0.485  -.006785  .003221   24.6486
     age |   .0055859      .00115    4.85   0.000   .003326  .007846   41.3075
    male*|  -.1162192      .02817   -4.13   0.000  -.171437 -.061002         0
     www*|   .0290082      .03749    0.77   0.439  -.044477  .102493         1
------------------------------------------------------------------------------
(*) dy/dx is for discrete change of dummy variable from 0 to 1
```

Now, let us compare marginal effects and discrete changes between the multinomial logit and probit models. Fit the probit model again.

```
. quietly mprobit belief educate income age male www, base(0)
```

In this multinomial probit model, the predicted probabilities at the reference points are 12.76 percent for having no religion, 11.56 for somewhat strong, 36.19 for not very strong, and 39.48 for strong belief. These probabilities are very similar to 12.67, 11.61, 36.30, and 39.41 percent, respectively.

```
. prchange age male, x(educate=16 male=0 www=1) rest(mean)

mprobit: Changes in Probabilities for belief

age
            Avg|Chg|    No_relig    Not_very      Strong    Somewhat
Min->Max   .18373563  -.11792623   -.2287672   .36747125  -.02077785
   -+1/2   .00275265  -.00190124  -.00342152   .00550529  -.00018255
  -+sd/2    .0368494  -.02545384  -.04580182   .07369879  -.00244313

male
        Avg|Chg|    No_relig    Not_very      Strong    Somewhat
0->1   .06728141   .09236868   .04219413  -.11648223   -.0180806

          No_relig   Not_very     Strong   Somewhat
Pr(y|x)  .12760836  .36192566  .39484766  .11561833

       educate   income      age     male      www
   x=       16  24.6486  41.3075        0        1
sd_x=  2.56971  6.19427  13.4071  .497765  .410755
```

Marginal changes and discrete changes are also very similar in both logit and probit models. The marginal changes of age with respect to having strong belief, for instance, are .56 percent in the logit model and .55 in the probit model. The discrete changes of gender with respect to having no religion are 9.23 and 9.24 percent, respectively. The probability of having strong belief is 11.65 percent (11.62 in the logit model) larger for women than for men, holding all other variables constant at their reference points. Find the corresponding marginal effects and discrete change in the following output of `.mfx2`.

```
. mfx2, at(mean educate=16 male=0 www=1)

Frequencies for belief...

      Religious |
      Intensity |      Freq.     Percent        Cum.
----------------+-----------------------------------
    No religion |        192       16.35       16.35
Somewhat strong |        134       11.41       27.77
Not very strong |        456       38.84       66.61
         Strong |        392       33.39      100.00
----------------+-----------------------------------
          Total |      1,174      100.00

Computing marginal effects after mprobit for belief == 0...

Marginal effects after mprobit
      y  = Pr(belief==No religion) (predict, o(0))
         =  .12760835
------------------------------------------------------------------------------
variable |      dy/dx    Std. Err.     z    P>|z|  [    95% C.I.   ]      X
---------+--------------------------------------------------------------------
 educate |   .0019664      .00385    0.51   0.609  -.005578  .009511        16
  income |  -.0005142      .00156   -0.33   0.741  -.003562  .002534   24.6486
     age |  -.0019013      .00074   -2.56   0.010  -.003356 -.000447   41.3075
   male*|   .0923687      .02284    4.04   0.000   .047603  .137135         0
```

```
     www*|   .0056651      .02319    0.24   0.807  -.039789  .051119         1
------------------------------------------------------------------------------
(*) dy/dx is for discrete change of dummy variable from 0 to 1

Computing marginal effects after mprobit for belief == 1...

Marginal effects after mprobit
      y  = Pr(belief==Somewhat strong) (predict, o(1))
         =  .11561833
------------------------------------------------------------------------------
variable |      dy/dx    Std. Err.     z    P>|z|  [    95% C.I.   ]      X
---------+--------------------------------------------------------------------
 educate |   .0015321      .00387    0.40   0.692  -.006059  .009123        16
  income |   .0019272      .00164    1.18   0.240  -.001287  .005141   24.6486
     age |  -.0001825      .00073   -0.25   0.802  -.001607  .001242   41.3075
    male*|  -.0180806      .01794   -1.01   0.313  -.053235  .017074         0
     www*|  -.0507386      .02831   -1.79   0.073  -.106226  .004749         1
------------------------------------------------------------------------------
(*) dy/dx is for discrete change of dummy variable from 0 to 1

Computing marginal effects after mprobit for belief == 2...

Marginal effects after mprobit
      y  = Pr(belief==Not very strong) (predict, o(2))
         =  .36192565
------------------------------------------------------------------------------
variable |      dy/dx    Std. Err.     z    P>|z|  [    95% C.I.   ]      X
---------+--------------------------------------------------------------------
 educate |  -.0070131      .00567   -1.24   0.217  -.018136   .00411        16
  income |   .0001992      .00236    0.08   0.933  -.004424  .004822   24.6486
     age |  -.0034215      .00109   -3.13   0.002  -.005567 -.001276   41.3075
    male*|   .0421941      .02882    1.46   0.143  -.014291  .098679         0
     www*|   .0171076      .03521    0.49   0.627  -.051898  .086113         1
------------------------------------------------------------------------------
(*) dy/dx is for discrete change of dummy variable from 0 to 1

Computing marginal effects after mprobit for belief == 3...

Marginal effects after mprobit
      y  = Pr(belief==Strong) (predict, o(3))
         =  .39484766
------------------------------------------------------------------------------
variable |      dy/dx    Std. Err.     z    P>|z|  [    95% C.I.   ]      X
---------+--------------------------------------------------------------------
 educate |   .0035145      .00599    0.59   0.557  -.008218  .015247        16
  income |  -.0016122       .0025   -0.64   0.520  -.006519  .003294   24.6486
     age |   .0055053      .00113    4.87   0.000   .003291  .007719   41.3075
    male*|  -.1164822      .02805   -4.15   0.000  -.171456 -.061508         0
     www*|   .0279659      .03683    0.76   0.448  -.044215  .100147         1
------------------------------------------------------------------------------
(*) dy/dx is for discrete change of dummy variable from 0 to 1
```

Therefore, we can conclude that logit and probit models, despite different parameter estimates and standard errors, report similar goodness-of fit measures and effects of covariates on each category of the dependent variable.

3.3 Multinomial Logit Model in SAS: PROC LOGISTIC and PROC CATMOD

SAS LOGISTRIC and CATMOD procedures fit the multinomial logit model.[1] /LINK=GLOGIT below specifies the generalized logit function as a link function. Keep in mind that you will get the opposite signs of coefficients if you do not specify DESCENDING.

[1] http://support.sas.com/kb/22/598.html

```
PROC LOGISTIC DATA = masil.gss_cdvm DESC;
     MODEL belief = educate income age male www /LINK=GLOGIT;
     UNITS educate=SD income=SD age=SD;
RUN;
```

PROC LOGISTIC and `.mlogit` with `base(0)` produce same goodness-of-fit measures, parameter estimates, and standard errors, but they return a bit different AIC (2974.898 versus 2986.898=2.544*1,174).

```
                              The LOGISTIC Procedure

                                Model Information

                  Data Set                      MASIL.GSS_CDVM
                  Response Variable             belief                belief
                  Number of Response Levels     4
                  Model                         generalized logit
                  Optimization Technique        Newton-Raphson


                       Number of Observations Read        1174
                       Number of Observations Used        1174


                                 Response Profile

                        Ordered                      Total
                          Value        belief     Frequency

                              1             3           392
                              2             2           456
                              3             1           134
                              4             0           192

                   Logits modeled use belief=0 as the reference category.


                             Model Convergence Status

                     Convergence criterion (GCONV=1E-8) satisfied.


                               Model Fit Statistics

                                                   Intercept
                                      Intercept          and
                       Criterion           Only   Covariates

                       AIC             3005.386     2974.898
                       SC              3020.590     3066.126
                       -2 Log L        2999.386     2938.898


                       Testing Global Null Hypothesis: BETA=0

                   Test                 Chi-Square       DF      Pr > ChiSq
```

Likelihood Ratio	60.4874	15	<.0001
Score	59.9903	15	<.0001
Wald	57.8319	15	<.0001

The LOGISTIC Procedure

Type 3 Analysis of Effects

Effect	DF	Wald Chi-Square	Pr > ChiSq
educate	3	1.4316	0.6982
income	3	1.6983	0.6373
age	3	25.7958	<.0001
male	3	26.5658	<.0001
www	3	3.9190	0.2703

Analysis of Maximum Likelihood Estimates

Parameter	belief	DF	Estimate	Standard Error	Wald Chi-Square	Pr > ChiSq
Intercept	3	1	-0.0142	0.6061	0.0005	0.9813
Intercept	2	1	1.2236	0.5817	4.4242	0.0354
Intercept	1	1	-0.7907	0.7863	1.0110	0.3147
educate	3	1	-0.00410	0.0365	0.0127	0.9104
educate	2	1	-0.0310	0.0352	0.7761	0.3783
educate	1	1	0.00199	0.0466	0.0018	0.9660
income	3	1	-0.00056	0.0149	0.0014	0.9700
income	2	1	0.00429	0.0143	0.0905	0.7636
income	1	1	0.0226	0.0203	1.2438	0.2647
age	3	1	0.0289	0.00710	16.5680	<.0001
age	2	1	0.00510	0.00697	0.5354	0.4643
age	1	1	0.0128	0.00901	2.0110	0.1562
male	3	1	-0.8968	0.1827	24.0916	<.0001
male	2	1	-0.4365	0.1751	6.2157	0.0127
male	1	1	-0.7229	0.2308	9.8127	0.0017
www	3	1	0.0348	0.2318	0.0225	0.8808
www	2	1	0.00949	0.2233	0.0018	0.9661
www	1	1	-0.4135	0.2782	2.2102	0.1371

Odds Ratio Estimates

Effect	belief	Point Estimate	95% Wald Confidence Limits	
educate	3	0.996	0.927	1.070
educate	2	0.969	0.905	1.039
educate	1	1.002	0.915	1.098
income	3	0.999	0.971	1.029
income	2	1.004	0.977	1.033
income	1	1.023	0.983	1.064
age	3	1.029	1.015	1.044
age	2	1.005	0.991	1.019

age	1	1.013	0.995	1.031
male	3	0.408	0.285	0.584
male	2	0.646	0.459	0.911
male	1	0.485	0.309	0.763
www	3	1.035	0.657	1.631
www	2	1.010	0.652	1.564
www	1	0.661	0.383	1.141

Odds Ratios

Effect	belief	Unit	Estimate
educate	3	2.5697	0.990
educate	2	2.5697	0.923
educate	1	2.5697	1.005
income	3	6.1943	0.997
income	2	6.1943	1.027
income	1	6.1943	1.150
age	3	13.4071	1.473
age	2	13.4071	1.071
age	1	13.4071	1.187

PROC LOGISTIC produces factor changes in odds of each category versus the base outcome (no religion, `belief=0`). For a unit increase in age, the odds of having somewhat strong belief (1) relative to no religion (0) are expected to increase by a factor of 1.013 = exp(.0128). The odds of having not very strong (2) versus no religion are .646=exp(-.4365) times smaller for men than for women. The optional UNIT statement reports odds ratios (see the last part of the above output) for a standard deviation increase in covariates listed. For a standard deviation increase in age, the odds of having strong belief relative to no religion are expected to increase by a factor of 1.473=exp(.0289*13.4071). Double-check with odds ratios that Stata produced in Section 3.2.

PROC LOGISTIC with DESC by default uses the last ordered value (0 in this case) as a base outcome, whereas PROC CATMOD fits the model on the basis of the largest value. But PROC LOGISTIC can specify a base outcome other than the default last outcome using /REFERENCE. In the following PROC LOGISTIC, /DESC sorts the dependent variable in the descending order (3, 2, 1, 0) and /REFERENCE=FIRST uses 3 (the first ordered value) as a reference. You may specify a particular value of the outcome like /REFERENCE='3' as well.

```
PROC LOGISTIC DATA = masil.gss_cdvm DESC REFERENCE=FIRST;
     MODEL belief = educate income age male www /LINK=GLOGIT;
     UNITS age=SD;
RUN;

PROC LOGISTIC DATA = masil.gss_cdvm DESC REFERENCE='3';
     MODEL belief = educate income age male www /LINK=GLOGIT;
     UNITS age=SD;
RUN;
```

The LOGISTIC Procedure

Model Information

```
Data Set                      MASIL.GSS_CDVM
Response Variable             belief                belief
Number of Response Levels     4
Model                         generalized logit
Optimization Technique        Newton-Raphson


          Number of Observations Read        1174
          Number of Observations Used        1174


                        Response Profile

             Ordered                      Total
               Value       belief     Frequency

                   1            3           392
                   2            2           456
                   3            1           134
                   4            0           192

  Logits modeled use belief=3 as the reference category.


                   Model Convergence Status

      Convergence criterion (GCONV=1E-8) satisfied.


                     Model Fit Statistics

                                         Intercept
                           Intercept           and
          Criterion             Only    Covariates

          AIC               3005.386      2974.898
          SC                3020.590      3066.126
          -2 Log L          2999.386      2938.898


          Testing Global Null Hypothesis: BETA=0

  Test                 Chi-Square       DF     Pr > ChiSq

  Likelihood Ratio        60.4874       15         <.0001
  Score                   59.9903       15         <.0001
  Wald                    57.8319       15         <.0001


                  Type 3 Analysis of Effects

                                  Wald
        Effect       DF    Chi-Square    Pr > ChiSq

        educate       3        1.4316        0.6982
        income        3        1.6983        0.6373
```

age	3	25.7958	<.0001
male	3	26.5658	<.0001
www	3	3.9190	0.2703

Analysis of Maximum Likelihood Estimates

Parameter	belief	DF	Estimate	Standard Error	Wald Chi-Square	Pr > ChiSq
Intercept	2	1	1.2377	0.4728	6.8530	0.0088
Intercept	1	1	-0.7765	0.7037	1.2177	0.2698
Intercept	0	1	0.0142	0.6061	0.0005	0.9813
educate	2	1	-0.0269	0.0284	0.8970	0.3436
educate	1	1	0.00609	0.0413	0.0217	0.8828
educate	0	1	0.00410	0.0365	0.0127	0.9104
income	2	1	0.00485	0.0117	0.1714	0.6789
income	1	1	0.0232	0.0184	1.5841	0.2082
income	0	1	0.000561	0.0149	0.0014	0.9700
age	2	1	-0.0238	0.00539	19.4982	<.0001
age	1	1	-0.0161	0.00777	4.3023	0.0381
age	0	1	-0.0289	0.00710	16.5680	<.0001
male	2	1	0.4603	0.1429	10.3700	0.0013
male	1	1	0.1739	0.2064	0.7092	0.3997
male	0	1	0.8968	0.1827	24.0916	<.0001
www	2	1	-0.0253	0.1785	0.0200	0.8875
www	1	1	-0.4483	0.2418	3.4374	0.0637
www	0	1	-0.0348	0.2318	0.0225	0.8808

Odds Ratio Estimates

Effect	belief	Point Estimate	95% Wald Confidence Limits	
educate	2	0.973	0.921	1.029
educate	1	1.006	0.928	1.091
educate	0	1.004	0.935	1.079
income	2	1.005	0.982	1.028
income	1	1.023	0.987	1.061
income	0	1.001	0.972	1.030
age	2	0.976	0.966	0.987
age	1	0.984	0.969	0.999
age	0	0.972	0.958	0.985
male	2	1.585	1.197	2.097
male	1	1.190	0.794	1.783
male	0	2.452	1.714	3.507
www	2	0.975	0.687	1.384
www	1	0.639	0.398	1.026
www	0	0.966	0.613	1.521

Odds Ratios

Effect	belief	Unit	Estimate
age	2	13.4071	0.727

```
                    age          1              13.4071         0.806
                    age          0              13.4071         0.679
```

The above PROC LOGISTIC and PROC CATMOD fit the multinomial logit model using the largest value as a base outcome; by contrast, PROC LOGISTIC uses the smallest value. The RESPONSE statement specifies the function of response probabilities. PROC CATMOD and `.mlogit` with `base(3)` produce the same result including parameter estimates and standard errors. Compare the following output with corresponding output in Section 6.1.

```
PROC CATMOD DATA = masil.gss_cdvm;
   DIRECT educate income age male www;
   RESPONSE LOGITS;
   MODEL belief = educate income age male www /NOPROFILE;
RUN;
```

```
                                    The CATMOD Procedure

                                        Data Summary

                      Response           belief       Response Levels     4
                      Weight Variable    None         Populations       862
                      Data Set           GSS_CDVM     Total Frequency  1174
                      Frequency Missing  0            Observations     1174


                                Maximum Likelihood Analysis

                         Maximum likelihood computations converged.

                          Maximum Likelihood Analysis of Variance

                       Source              DF    Chi-Square    Pr > ChiSq
                       --------------------------------------------------
                       Intercept            3         12.61        0.0056
                       educate              3          1.43        0.6982
                       income               3          1.70        0.6372
                       age                  3         25.80        <.0001
                       male                 3         26.57        <.0001
                       www                  3          3.92        0.2703

                       Likelihood Ratio   3E3       2292.18        1.0000

                          Analysis of Maximum Likelihood Estimates

                        Function                Standard         Chi-
               Parameter  Number      Estimate      Error       Square     Pr > ChiSq
               -----------------------------------------------------------------------
               Intercept     1          0.0142     0.6061         0.00         0.9813
                             2         -0.7765     0.7036         1.22         0.2698
                             3          1.2377     0.4728         6.85         0.0088
               educate       1         0.00410     0.0365         0.01         0.9104
                             2         0.00609     0.0413         0.02         0.8828
                             3         -0.0269     0.0284         0.90         0.3436
               income        1        0.000561     0.0149         0.00         0.9700
                             2          0.0232     0.0184         1.58         0.2081
                             3         0.00485     0.0117         0.17         0.6789
```

```
age         1      -0.0289    0.00710      16.57        <.0001
            2      -0.0161    0.00777       4.30        0.0381
            3      -0.0238    0.00539      19.50        <.0001
male        1       0.8968     0.1827      24.09        <.0001
            2       0.1739     0.2064       0.71        0.3997
            3       0.4603     0.1429      10.37        0.0013
www         1      -0.0348     0.2318       0.02        0.8808
            2      -0.4483     0.2418       3.44        0.0637
            3      -0.0253     0.1785       0.02        0.8875
```

Both PROC LOGISTIC and PROC CATMOD fit the multinomial logit model, but PROC LOGISTIC is recommended for its simpler syntax and ability to report goodness-of-fit measures and factor changes in the odds.

3.4 Multinomial Logit Model in LIMDEP (Mlogit$)

In LIMDEP, you may use either the `Mlogit$` or simply the `Logit$` commands to fit the multinomial logit model. Like SAS PROC LOGISTIC, LIMDEP by default uses the smallest value as the base outcome. Both procedure and command produce the same result. Compare the following output with what PROC LOGISTIC and and `.mlogit` with `base(0)` produced in Section 6.1 and 6.3. AIC 2,974(=2.5340*1,174) and BIC 3,066 (=2.6117*1,174) are similar to those of PROC LOGISTIC.

```
LOGIT;Lhs=BELIEF;
    Rhs=ONE,EDUCATE,INCOME,AGE,MALE,WWW$

MLOGIT;Lhs=BELIEF;
    Rhs=ONE,EDUCATE,INCOME,AGE,MALE,WWW;
    Marginal Effect$
```

```
Normal exit from iterations. Exit status=0.

+---------------------------------------------+
| Multinomial Logit Model                     |
| Maximum Likelihood Estimates                |
| Model estimated: Sep 13, 2009 at 09:19:08PM.|
| Dependent variable              BELIEF      |
| Weighting variable                None      |
| Number of observations            1174      |
| Iterations completed                 4      |
| Log likelihood function      -1469.449      |
| Number of parameters                18      |
| Info. Criterion: AIC =         2.53399      |
|   Finite Sample: AIC =         2.53449      |
| Info. Criterion: BIC =         2.61169      |
| Info. Criterion:HQIC =         2.56329      |
| Restricted log likelihood    -1499.693      |
| McFadden Pseudo R-squared      .0201666     |
| Chi squared                   60.48737      |
| Degrees of freedom                  15      |
| Prob[ChiSqd > value] =        .0000000      |
+---------------------------------------------+
+--------+--------------+----------------+--------+--------+----------+
|Variable| Coefficient  | Standard Error |b/St.Er.|P[|Z|>z]| Mean of X|
+--------+--------------+----------------+--------+--------+----------+
---------+Characteristics in numerator of Prob[Y = 1]
 Constant|    -.79066875       .78634914    -1.005   .3147
 EDUCATE |     .00198700       .04657354      .043   .9660   14.2427598
 INCOME  |     .02260869       .02027240     1.115   .2647   24.6486371
 AGE     |     .01277744       .00901017     1.418   .1562   41.3074957
 MALE    |    -.72291372       .23077644    -3.133   .0017    .45059625
```

```
 WWW     |    -.41352579       .27815788    -1.487   .1371    .78534923
---------+Characteristics in numerator of Prob[Y = 2]
 Constant|    1.22356470       .58171478     2.103   .0354
 EDUCATE |    -.03104837       .03524454     -.881   .3783   14.2427598
 INCOME  |     .00428642       .01425161      .301   .7636   24.6486371
 AGE     |     .00510002       .00696993      .732   .4643   41.3074957
 MALE    |    -.43649550       .17507997    -2.493   .0127    .45059625
 WWW     |     .00949342       .22330746      .043   .9661    .78534923
---------+Characteristics in numerator of Prob[Y = 3]
 Constant|    -.01418167       .60605065     -.023   .9813
 EDUCATE |    -.00410379       .03647913     -.112   .9104   14.2427598
 INCOME  |    -.00056137       .01491463     -.038   .9700   24.6486371
 AGE     |     .02889721       .00709939     4.070   .0000   41.3074957
 MALE    |    -.89676886       .18270372    -4.908   .0000    .45059625
 WWW     |     .03475780       .23180552      .150   .8808    .78534923

+---------------------------------------------------------------------+
| Information Statistics for Discrete Choice Model.                   |
|                            M=Model MC=Constants Only   M0=No Model |
| Criterion F (log L)    -1469.44924       -1499.69292   -1627.50958 |
| LR Statistic vs. MC       60.48737            .00000        .00000 |
| Degrees of Freedom        15.00000            .00000        .00000 |
| Prob. Value for LR          .00000            .00000        .00000 |
| Entropy for probs.      1469.44924        1499.69292    1627.50958 |
| Normalized Entropy          .90288            .92146       1.00000 |
| Entropy Ratio Stat.      316.12067         255.63332        .00000 |
| Bayes Info Criterion       2.59363           2.64515       2.86290 |
| BIC(no model) - BIC         .26927            .21775        .00000 |
| Pseudo R-squared            .02017            .00000        .00000 |
| Pct. Correct Pred.        42.75980            .00000      25.00000 |
| Means:       y=0    y=1    y=2    y=3    y=4    y=5     y=6   y>=7 |
| Outcome     .1635  .1141  .3884  .3339  .0000  .0000  .0000  .0000 |
| Pred.Pr     .1635  .1141  .3884  .3339  .0000  .0000  .0000  .0000 |
| Notes: Entropy computed as Sum(i)Sum(j)Pfit(i,j)*logPfit(i,j).      |
|        Normalized entropy is computed against M0.                   |
|        Entropy ratio statistic is computed against M0.              |
|        BIC = 2*criterion - log(N)*degrees of freedom.               |
|        If the model has only constants or if it has no constants,  |
|        the statistics reported here are not useable.                |
+---------------------------------------------------------------------+
+--------------------------------------------+
| Partial derivatives of probabilities with |
| respect to the vector of characteristics. |
| They are computed at the means of the Xs. |
| Observations used for means are All Obs.  |
| A full set is given for the entire set of |
| outcomes, BELIEF   = 0 to BELIEF   =  3.  |
| Probabilities at the mean vector are      |
|  0= .159 1= .115 2= .395 3= .331          |
+--------------------------------------------+
+--------+-------------+----------------+--------+--------+----------+
|Variable| Coefficient  | Standard Error |b/St.Er.|P[|Z|>z]|Elasticity|
+--------+-------------+----------------+--------+--------+----------+
---------+Marginal effects on Prob[Y = 0]
 Constant|    -.06182477       .07165730     -.863   .3883
 EDUCATE |     .00213458       .00435008      .491   .6236    .19083307
 INCOME  |    -.00065373       .00176920     -.370   .7118   -.10114321
 AGE     |    -.00207732       .00085129    -2.440   .0147   -.53861532
 MALE    |     .08795034       .02117039     4.154   .0000    .24875522
 WWW     |     .00513317       .02761131      .186   .8525    .02530439
---------+Marginal effects on Prob[Y = 1]
 Constant|    -.13530400       .06469038    -2.092   .0365
 EDUCATE |     .00176607       .00385793      .458   .6471    .21913338
 INCOME  |     .00212417       .00172893     1.229   .2192    .45613009
 AGE     |   -.300424D-04      .00073100     -.041   .9672   -.01081111
 MALE    |    -.01961205       .01903919    -1.030   .3030   -.07698699
 WWW     |    -.04376894       .02202321    -1.987   .0469   -.29945777
---------+Marginal effects on Prob[Y = 2]
 Constant|     .33016424       .09821107     3.362   .0008
 EDUCATE |    -.00697469       .00588769    -1.185   .2362   -.25138145
 INCOME  |    .723265D-04      .00242151      .030   .9762    .00451133
```

```
 AGE     |    -.00313733       .00112798    -2.781   .0054   -.32794614
 MALE    |     .04566702       .02909600     1.570   .1165    .05207198
 WWW     |     .01648419       .03660286      .450   .6525    .03276004
---------+Marginal effects on Prob[Y = 3]
 Constant|    -.13303547       .09431272    -1.411   .1584
 EDUCATE |     .00307405       .00566946      .542   .5877    .13238370
 INCOME  |    -.00154277       .00235855     -.654   .5130   -.11498031
 AGE     |     .00524469       .00106287     4.934   .0000    .65505589
 MALE    |    -.11400532       .02846801    -4.005   .0001   -.15532546
 WWW     |     .02215157       .03554447      .623   .5331    .05260140

Marginal Effects Averaged Over Individuals
--------+---------+---------+---------+---------+
Variable|    Y=00 |    Y=01 |    Y=02 |    Y=03 |
--------+---------+---------+---------+---------+
ONE     |  -.0651 |  -.1327 |   .3240 |  -.1263 |
EDUCATE |   .0022 |   .0017 |  -.0069 |   .0029 |
INCOME  |  -.0007 |   .0021 |   .0001 |  -.0015 |
AGE     |  -.0020 |  -.0001 |  -.0030 |   .0051 |
MALE    |   .0869 |  -.0190 |   .0418 |  -.1098 |
WWW     |   .0051 |  -.0433 |   .0160 |   .0222 |
--------+---------+---------+---------+---------+

Averages of Individual Elasticities of Probabilities
--------+---------+---------+---------+---------+
Variable|    Y=00 |    Y=01 |    Y=02 |    Y=03 |
--------+---------+---------+---------+---------+
ONE     |  -.3803 | -1.1709 |   .8433 |  -.3945 |
EDUCATE |   .1867 |   .2150 |  -.2556 |   .1282 |
INCOME  |  -.1012 |   .4561 |   .0045 |  -.1150 |
AGE     |  -.5641 |  -.0363 |  -.3535 |   .6295 |
MALE    |   .2253 |  -.1004 |   .0286 |  -.1788 |
WWW     |   .0221 |  -.3027 |   .0295 |   .0494 |
--------+---------+---------+---------+---------+

Frequencies of actual & predicted outcomes
Predicted outcome has maximum probability.

            Predicted
------  --------------------  +  -----
Actual      0    1    2    3  |  Total
------  --------------------  +  -----
  0         0    0  147   45  |    192
  1         0    0   90   44  |    134
  2         0    0  330  126  |    456
  3         0    0  220  172  |    392
------  --------------------  +  -----
Total       0    0  787  387  |   1174
```

`Marginal Effect` subcommand computes marginal effects and discrete changes by default at the means of independent variables. Compare them with marginal changes (discrete changes) produced by the following `.prchange`. The marginal effect of age on having strong belief is, for example, .52 percent and men are 11.40 percent (11.30 percent in Stata) less likely to have strong religious belief than women, holding all variables at their means. LIMDEP and Stata produce same marginal effects but slightly different discrete changes.

```
. quietly mlogit belief educate income age male www, base(0)

. prchange, rest(mean)

mlogit: Changes in Probabilities for belief

educate
            Avg|Chg|    Somewhat    Not_very      Strong    No_relig
Min->Max   .06350871   .03140458  -.12701744   .05719805   .03841479
   -+1/2    .0034873   .00176604  -.00697461   .00307399   .00213455
  -+sd/2   .00896056    .0045379  -.01792112   .00789851   .00548472
```

```
MargEfct   .00348735   .00176607  -.00697469   .00307405   .00213458

income
            Avg|Chg|    Somewhat    Not_very      Strong    No_relig
Min->Max   .02769286   .04849818   .00688753  -.03914931  -.01623641
   -+1/2   .00109825   .00212418   .00007233  -.00154275  -.00065373
  -+sd/2   .00680311     .013161   .00044522  -.00955665  -.00404958
MargEfct   .00109825   .00212417   .00007233  -.00154277  -.00065373

age
            Avg|Chg|    Somewhat    Not_very      Strong    No_relig
Min->Max   .18139681  -.01300404  -.21853864   .36279362  -.13125092
   -+1/2   .00262233  -.00003005  -.00313732   .00524464  -.00207731
  -+sd/2   .03510485  -.00041036  -.04198521   .07020971  -.02781411
MargEfct   .00262234  -.00003004  -.00313733   .00524469  -.00207732

male
        Avg|Chg|    Somewhat    Not_very      Strong    No_relig
0->1   .06641669  -.01983403    .0433048  -.11299935   .08952859

www
        Avg|Chg|    Somewhat    Not_very      Strong    No_relig
0->1   .02407742  -.04815486   .01855594   .02359158    .0060073

          Somewhat   Not_very     Strong   No_relig
Pr(y|x)  .11478714  .39517197  .33072728  .15931362

       educate   income      age     male      www
   x=  14.2428  24.6486  41.3075  .450596  .785349
sd_x=  2.56971  6.19427  13.4071  .497765  .410755
```

3.5 Multinomial Logit Model in SPSS

SPSS has the `Nomreg` command to estimate the multinomial logit model. Like SAS PROC CATMOD, SPSS by default uses the largest value as the base outcome. Like Stata and PROC LOGISTIC, you may change the baseline by specifying FIRST or any particular value of the response variable at the `Base=` option.

```
NOMREG belief (BASE=LAST ORDER=ASCENDING) WITH educate income age male www
   /CRITERIA CIN(95) DELTA(0) MXITER(100) MXSTEP(5) CHKSEP(20) LCONVERGE(0)
   /MODEL
   /STEPWISE=PIN(.05) POUT(0.1) MINEFFECT(0) RULE(SINGLE) ENTRYMETHOD(LR) REMOVALMETHOD(LR)
   /INTERCEPT=INCLUDE
   /PRINT=PARAMETER SUMMARY LRT CPS STEP MFI.
```

Parameter Estimates

belief[a]		B	Std. Error	Wald	df	Sig.	Exp(B)	95% Confidence Interval for Exp(B)	
								Lower Bound	Upper Bound
0	Intercept	.014	.606	.001	1	.981			
	educate	.004	.036	.013	1	.910	1.004	.935	1.079
	income	.001	.015	.001	1	.970	1.001	.972	1.030
	age	-.029	.007	16.568	1	.000	.972	.958	.985
	male	.897	.183	24.092	1	.000	2.452	1.714	3.507
	www	-.035	.232	.022	1	.881	.966	.613	1.521
1	Intercept	-.776	.704	1.218	1	.270			

	educate	.006	.041	.022	1	.883	1.006	.928	1.091
	income	.023	.018	1.584	1	.208	1.023	.987	1.061
	age	-.016	.008	4.302	1	.038	.984	.969	.999
	male	.174	.206	.709	1	.400	1.190	.794	1.783
	www	-.448	.242	3.437	1	.064	.639	.398	1.026
2	Intercept	1.238	.473	6.853	1	.009			
	educate	-.027	.028	.897	1	.344	.973	.921	1.029
	income	.005	.012	.171	1	.679	1.005	.982	1.028
	age	-.024	.005	19.498	1	.000	.976	.966	.987
	male	.460	.143	10.370	1	.001	1.585	1.197	2.097
	www	-.025	.179	.020	1	.887	.975	.687	1.384

a. The reference category is: 3.

The above table is selected form the SPSS output. PROC LOGISTIC with /REFERENCE='3', PROC CATMOD, and `.mlogit` with `base(3)`, and SPSS with `BASE=LAST` produce the same goodness-of-fit measures, parameter estimates, and standard errors except for rounding errors. Since the base outcome is strong belief, you need to interpret the odds ratios with caution. Or fit the model with `BASE=FIRST` again and then interpret the output.

For a unit increase in age, the odds of having not very strong belief (2) relative to strong belief (3) are expected to decrease by a factor of .976=exp(-.024). The odds of having somewhat strong belief (1) versus strong belief are 1.190=exp(.174) times larger for men than for women. Compare the above the odds ratios with what is produced by `.listcoef` in Section 6.1.

Table 3.1 Parameter Estimates and Goodness-of-fit of the Multinomial Response Models

	SAS			Stata w/ `base(0)`		LIMDEP
	LOGISTIC	LOGISTIC	CATMOD	`.mlogit`	`.mprobit`	`Mlogit$`
Education	Base outcome	.0041	.0041	Base outcome	Base outcome	Base outcome
		(.0365)	(.0365)			
Family income		.0006	.0006			
		(.0149)	(.0149)			
Age		-.0289	-.0289			
		(.0071)	(.0071)			
Gender (male)		.8968	.8968			
		(.1827)	(.1827)			
WWW use		-.0348	-.0348			
		(.2318)	(.2318)			
Education	.0020	.0061	.0061	.0020	-.0015	.0020
	(.0466)	(.0413)	(.0413)	(.0466)	(.0298)	(.0466)
Family income	.0226	.0232	.0232	.0226	.0131	.0226
	(.0203)	(.0184)	(.0184)	(.0203)	(.0126)	(.0203)
Age	.0128	-.0161	-.0161	.0128	.0085	.0128
	(.0090)	(.0078)	(.0078)	(.0090)	(.0057)	(.0090)
Gender (male)	-.7229	.1739	.1739	-.7229	-.4720	-.7229
	(.2308)	(.2064)	(.2064)	(.2308)	(.1481)	(.2308)
WWW use	-.4135	-.4483	-.4483	-.4135	-.2651	-.4135
	(.2782)	(.2418)	(.2418)	(.2782)	(.1815)	(.2782)
Education	-.0310	-.0269	-.0269	-.0310	-.0255	-.0310
	(.0352)	(.0284)	(.0284)	(.0352)	(.0255)	(.0352)
Family income	.0043	.0049	.0049	.0043	.0029	.0043
	(.0143)	(.0117)	(.0117)	(.0143)	(.0104)	(.0143)
Age	.0051	-.0238	-.0238	.0051	.0021	.0051
	(.0070)	(.0054)	(.0054)	(.0070)	(.0049)	(.0070)
Gender (male)	-.4365	.4603	.4603	-.4365	-.2795	-.4365
	(.1751)	(.1429)	(.1429)	(.1751)	(.1252)	(.1751)
WWW use	.0095	-.0253	-.0253	.0095	.0112	.0095
	(.2233)	(.1785)	(.1785)	(.2233)	(.1592)	(.2233)

Education	-.0041 (.0365)	Base outcome	Base outcome	-.0041 (.0365)	-.0028 (.0258)	-.0041 (.0365)
Family income	-.0006 (.0149)			-.0006 (.0149)	-.0008 (.0107)	-.0006 (.0149)
Age	.0289 (.0071)			.0289 (.0071)	.0210 (.0049)	.0289 (.0071)
Gender (male)	-.8968 (.1827)			-.8968 (.1827)	-.6416 (.1288)	-.8968 (.1827)
WWW use	.0348 (.2318)			.0348 (.2318)	.0322 (.1627)	.0348 (.2318)
Log likelihood	-1469.449	-1469.449		-1469.4492	-1469.3674	-1469.4492
Likelihood test	60.4874	60.4874		60.49	59.201	60.4874
Pseudo R^2				.0202		.0202
AIC	2974.898	2974.898		*2986.898*	*2974.735*	2974.9043
Schwarz	3066.126	3066.126				
BIC				*3066.126*	*3065.962*	3066.1241

* PROC LOGISTIC and SPSS report (-2*Log-likelihood).

Table 3.1 summarizes the results that Stata, SAS, and LIMDEP produced. From the top, parameter estimates except for the intercept of category 0 (no religion) through 3 (strong belief) are listed. Notice that the largest value of the dependent variable is used as a base outcome in PROC LOGISTIC with /REFERENCE='3' and PROC CATMOD.

All software packages report the same parameter estimates and standard errors. Also they produce very similar goodness-of-fit measures except for the log likelihood of -1,288.500 in SPSS. SAS and SPSS conduct the Wald test (chi-squared), while Stata and LIMDEP report *z* score; however, they return the same p-values. PROC LOGISTIC and Stata `.mlogit` are recommended for the multinomial logit model.

4. Conditional Logit Regression Model

Suppose you are choosing a travel mode among air flight, train, bus, and car. We will replicate the conditional logit model discussed in Greene (2003), which examines how generalized cost measure (`cost`), terminal waiting time (`time`), and interaction of air flight and household income (`air_inc`) affect the choice of travel mode.

$$\Pr ob(y_i = c \mid z_i) = \frac{\exp(z_{ic}\gamma)}{\sum_{j=1} \exp(z_{ij}\gamma)}$$

Where z_{ij} is the *j*th alternative of subject *i*, z_{ic} is the choice of alternative *c* of subject *i*.

In a conditional logit model, independent variables are not characteristics of subjects (individuals), but attributes of the alternatives. In other words, the conditional logit model, unlike the multinomial logit model, estimates how alternative-specific, not individual-specific, variables affect the likelihood of observing a given outcome (Long 2003). Since units of analysis (more specifically, units of observations in this case) are different from each other, the conditional logit model differs in data arrangement from the multinomial logit model (Figure 4.1).

Figure 4.1 Data Arrangement for the Conditional Logit Model

subject	mode	choice	air	train	bus	car	cost	time	income	air_inc
1	1	0	1	0	0	0	70	69	35	35
1	2	0	0	1	0	0	71	34	35	0
1	3	0	0	0	1	0	70	35	35	0
1	4	1	0	0	0	1	30	0	35	0
2	1	0	1	0	0	0	68	64	30	30
2	2	0	0	1	0	0	84	44	30	0
2	3	0	0	0	1	0	85	53	30	0
2	4	1	0	0	0	1	50	0	30	0
3	1	0	1	0	0	0	129	69	40	40
3	2	0	0	1	0	0	195	34	40	0
3	3	0	0	0	1	0	149	35	40	0
3	4	1	0	0	0	1	101	0	40	0
4	1	0	1	0	0	0	59	64	70	70
4	2	0	0	1	0	0	79	44	70	0
4	3	0	0	0	1	0	81	53	70	0
…	…	…	…	…	…	…	…	…	…	…

The data set has four observations per subject, each of which contains attributes of using air flight, train, bus, and car. The dependent variable `choice` is coded 1 only if a subject chooses that travel mode. The four dummy variables, `air`, `train`, `bus`, and `car`, are flagging the corresponding modes of transportation.

4.1 Conditional Logit Model in Stata (.clogit)

In Stata, the `.clogit` command to estimate the condition logit model. The `group()` option specifies the variable (e.g., identification number) that identifies unique individuals.

```
. use http://www.indiana.edu/~statmath/stat/all/cdvm/travel.dta, clear

. clogit choice air train bus cost time air_inc, group(subject)

Iteration 0:   log likelihood =  -205.8187
Iteration 1:   log likelihood = -199.23679
Iteration 2:   log likelihood = -199.12851
Iteration 3:   log likelihood = -199.12837
Iteration 4:   log likelihood = -199.12837

Conditional (fixed-effects) logistic regression   Number of obs   =        840
                                                  LR chi2(6)      =     183.99
                                                  Prob > chi2     =     0.0000
Log likelihood = -199.12837                       Pseudo R2       =     0.3160

------------------------------------------------------------------------------
      choice |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
         air |   5.207443   .7790551     6.68   0.000     3.680523    6.734363
       train |   3.869043   .4431269     8.73   0.000      3.00053    4.737555
         bus |   3.163194   .4502659     7.03   0.000     2.280689    4.045699
        cost |  -.0155015    .004408    -3.52   0.000     -.024141   -.006862
        time |  -.0961248   .0104398    -9.21   0.000    -.1165865   -.0756631
     air_inc |    .013287   .0102624     1.29   0.195    -.0068269     .033401
------------------------------------------------------------------------------
```

A large likelihood ratio of 184 and McFadden's R^2 (pseudo R^2) .316 suggest that this conditional logit model fits the data well.

```
. fitstat

Measures of Fit for clogit of choice

Log-Lik Intercept Only:       -291.122   Log-Lik Full Model:           -199.128
D(204):                        398.257   LR(6):                         183.987
                                         Prob > LR:                       0.000
McFadden's R2:                   0.316   McFadden's Adj R2:               0.295
ML (Cox-Snell) R2:               0.584   Cragg-Uhler(Nagelkerke) R2:      0.623
Count R2:                        0.690
AIC:                             1.954   AIC*n:                         410.257
BIC:                          -692.553   BIC':                         -151.904
BIC used by Stata:             438.657   AIC used by Stata:             410.257
```

Run the `.listcoef` command to get factor changes in the odds. For a one unit increase in the waiting time for a given travel mode, we can expect a decrease in the odds of using that travel by a factor of .9084=exp(-.0961), holding other variables constant.

```
. listcoef, help

clogit (N=840): Factor Change in Odds

  Odds of: 1 vs 0

--------------------------------------------------
      choice |      b         z     P>|z|    e^b
-------------+------------------------------------
         air |   5.20744    6.684   0.000 182.6265
       train |   3.86904    8.731   0.000  47.8965
         bus |   3.16319    7.025   0.000  23.6460
        cost |  -0.01550   -3.517   0.000   0.9846
        time |  -0.09612   -9.207   0.000   0.9084
     air_inc |   0.01329    1.295   0.195   1.0134
--------------------------------------------------
       b = raw coefficient
       z = z-score for test of b=0
   P>|z| = p-value for z-test
```

```
     e^b = exp(b) = factor change in odds for unit increase in X
   SDofX = standard deviation of X
```

Let us conduct the Hausman specification test by running a full model and encompassed model without one choice (airline in this case). However, the test in this case is not reliable since the variance matrix is not positive definite

```
. quietly clogit choice air train bus cost time air_inc, group(subject)

. estimates store full

. quietly clogit choice train bus cost time air_inc, group(subject)

. hausman full .

                 ---- Coefficients ----
             |      (b)          (B)            (b-B)     sqrt(diag(V_b-V_B))
             |      full          .           Difference          S.E.
-------------+----------------------------------------------------------------
       train |    3.869043     2.065398        1.803645        .3252505
         bus |    3.163194     1.331226        1.831968        .3137705
        cost |   -.0155015    -.0150573       -.0004442          .00118
        time |   -.0961248    -.0498026       -.0463222        .0080997
     air_inc |     .013287     .0621491       -.0488621        .0056885
------------------------------------------------------------------------------
                          b = consistent under Ho and Ha; obtained from clogit
           B = inconsistent under Ha, efficient under Ho; obtained from clogit

    Test:  Ho:  difference in coefficients not systematic

                  chi2(5) = (b-B)'[(V_b-V_B)^(-1)](b-B)
                          =       27.87
                Prob>chi2 =      0.0000
                (V_b-V_B is not positive definite)
```

The .mfx and other SPost commands such as .prchange and .prgen do not work for this model.

4.2 Conditional Logit Model in SAS: PROC LOGISTIC and PROC MDC

In SAS, PROC LOGISTIC and PROC MDC fit the conditional logit model. In PROC LOGISTIC, you need to add the STRATA statement and specify individuals (subjects). Stata and PROC LOGISTIC produce same likelihood ratio (183.9869), AIC (410.257), and BIC (438.657). Their parameter estimates and standard errors are also identical.

```
PROC LOGISTIC DATA=masil.travel DESCENDING;
   MODEL choice = air train bus cost time air_inc;
   STRATA subject;
RUN;
```

```
                                    The LOGISTIC Procedure

                                     Conditional Analysis

                                      Model Information

                       Data Set                        MASIL.TRAVEL
                       Response Variable               choice
                       Number of Response Levels       2
                       Number of Strata                210
```

```
Model                        binary logit
Optimization Technique       Newton-Raphson ridge


      Number of Observations Read         840
      Number of Observations Used         840


                  Response Profile

         Ordered                      Total
           Value       choice     Frequency

               1            1           210
               2            0           630

         Probability modeled is choice=1.


                  Strata Summary

                 choice
   Response    ------     Number of
    Pattern    1    0       Strata    Frequency

         1     1    3          210          840

         Newton-Raphson Ridge Optimization

             Without Parameter Scaling

  Convergence criterion (GCONV=1E-8) satisfied.


               Model Fit Statistics

                     Without          With
      Criterion     Covariates     Covariates

      AIC              582.244        410.257
      SC               582.244        438.657
      -2 Log L         582.244        398.257


      Testing Global Null Hypothesis: BETA=0

Test                 Chi-Square       DF     Pr > ChiSq

Likelihood Ratio       183.9869        6         <.0001
Score                  173.4374        6         <.0001
Wald                   103.7695        6         <.0001


        Analysis of Maximum Likelihood Estimates

                                Standard          Wald
Parameter    DF    Estimate        Error    Chi-Square    Pr > ChiSq
```

```
air          1     5.2074     0.7791     44.6800     <.0001
train        1     3.8690     0.4431     76.2344     <.0001
bus          1     3.1632     0.4503     49.3530     <.0001
cost         1    -0.0155    0.00441     12.3671     0.0004
time         1    -0.0961     0.0104     84.7779     <.0001
air_inc      1     0.0133     0.0103      1.6763     0.1954


                    Odds Ratio Estimates

                        Point          95% Wald
            Effect     Estimate      Confidence Limits

            air         182.627      39.667     840.808
            train        47.897      20.096     114.155
            bus          23.646       9.783      57.151
            cost          0.985       0.976       0.993
            time          0.908       0.890       0.927
            air_inc       1.013       0.993       1.034
```

PROC MDC fits the conditional logit model using TYPE=CLOGIT (or TYPE=CL). The ID statement specifies an identification variable and NCHOICE=4 indicates that there are four choices for transportation.

```
PROC MDC DATA=masil.travel;
   MODEL choice = air train bus cost time air_inc /TYPE=CLOGIT NCHOICE=4;
   ID subject;
RUN;
```

PROC MDC returns the Schwarz Information Criterion of 430.3394 slightly different from BIC 438.657 that PROC LOGISTIC reported above. Other goodness-of-fit measures and parameter estimates remain unchanged.

```
                         The MDC Procedure

                    Conditional Logit Estimates

Algorithm converged.


                         Model Fit Summary

          Dependent Variable                          choice
          Number of Observations                         210
          Number of Cases                                840
          Log Likelihood                          -199.12837
          Log Likelihood Null (LogL(0))           -291.12182
          Maximum Absolute Gradient               2.73164E-8
          Number of Iterations                             5
          Optimization Method                 Newton-Raphson
          AIC                                      410.25674
          Schwarz Criterion                        430.33938


                    Discrete Response Profile
```

Index	CHOICE	Frequency	Percent
0	1	58	27.62
1	2	63	30.00
2	3	30	14.29
3	4	59	28.10

Goodness-of-Fit Measures

Measure	Value	Formula
Likelihood Ratio (R)	183.99	2 * (LogL - LogL0)
Upper Bound of R (U)	582.24	- 2 * LogL0
Aldrich-Nelson	0.467	R / (R+N)
Cragg-Uhler 1	0.5836	1 - exp(-R/N)
Cragg-Uhler 2	0.6225	(1-exp(-R/N)) / (1-exp(-U/N))
Estrella	0.6511	1 - (1-R/U)^(U/N)
Adjusted Estrella	0.6212	1 - ((LogL-K)/LogL0)^(-2/N*LogL0)
McFadden's LRI	0.316	R / U
Veall-Zimmermann	0.6354	(R * (U+N)) / (U * (R+N))

N = # of observations, K = # of regressors

Conditional Logit Estimates

Parameter Estimates

Parameter	DF	Estimate	Standard Error	t Value	Approx Pr > |t|
air	1	5.2074	0.7791	6.68	<.0001
train	1	3.8690	0.4431	8.73	<.0001
bus	1	3.1632	0.4503	7.03	<.0001
cost	1	-0.0155	0.004408	-3.52	0.0004
time	1	-0.0961	0.0104	-9.21	<.0001
air_inc	1	0.0133	0.0103	1.29	0.1954

PROC LOGISTIC and PROC MDC do not conduct the Hausman's specification test. If you are interested in the test, take a look at the following document and run a macro script http://support.sas.com/documentation/cdl/en/etsug/60372/HTML/default/etsug_mdc_sect038.htm.

PROC PHREG can estimate the Cox proportional hazards model for survival data and the conditional logit model as well. You need to create a failure time variable, `failure=1-choice` in order to make the data set consistent with the survival analysis data. An identification variable is specified in the STRATA statement. NOSUMMARY suppresses the display of event and censored observation frequencies.

```
PROC PHREG DATA=masil.travel NOSUMMARY;
   STRATA subject;
   MODEL failure*choice(0) = air train bus cost time air_inc;
RUN;
```

```
                         The PHREG Procedure

                          Model Information

              Data Set                  MASIL.TRAVEL
              Dependent Variable        failure
              Censoring Variable        choice
              Censoring Value(s)        0
              Ties Handling             BRESLOW


             Number of Observations Read         840
             Number of Observations Used         840


                         Convergence Status

         Convergence criterion (GCONV=1E-8) satisfied.


                        Model Fit Statistics

                                Without           With
             Criterion       Covariates     Covariates

             -2 LOG L           582.244        398.257
             AIC                582.244        410.257
             SBC                582.244        430.339


             Testing Global Null Hypothesis: BETA=0

     Test                 Chi-Square       DF     Pr > ChiSq

     Likelihood Ratio       183.9869        6         <.0001
     Score                  173.4374        6         <.0001
     Wald                   103.7695        6         <.0001


            Analysis of Maximum Likelihood Estimates

                  Parameter      Standard                                     Hazard
Parameter    DF    Estimate         Error    Chi-Square    Pr > ChiSq          Ratio

air           1     5.20743       0.77905       44.6799        <.0001        182.625
train         1     3.86904       0.44313       76.2343        <.0001         47.896
bus           1     3.16319       0.45027       49.3530        <.0001         23.646
cost          1    -0.01550       0.00441       12.3671        0.0004          0.985
time          1    -0.09612       0.01044       84.7778        <.0001          0.908
air_inc       1     0.01329       0.01026        1.6763        0.1954          1.013
```

Both PROC MDC and PROC PHREG produce same goodness-of-fit measures, parameter estimates, and standard errors. While PROC MDC reports t statistics, PROC PHREG computes chi-squared (e.g., 12.3671=-3.52^2). But they produce same p-values. PROC PHREG presents the hazard ratio at the last column of the output, which is equivalent to the factor changes in the odds in Section 4.1.

4.3 Conditional Logit Model in LIMDEP (Clogit$)

In LIMDEP, the `Clogit$` or `Logit$` commands fit the conditional logit model. The `Clogit$` command has the `Choices` subcommand to list available choices (i.e., airline, train, bus, and car). Stata, SAS, and LIMDEP reports same parameter estimates and standard errors.

```
CLOGIT;
    Lhs=choice;
    Rhs=air,train,bus,cost,time,air_inc;
    Choices=air,train,bus,car$
```

```
+---------------------------------------------+
| Discrete choice and multinomial logit models|
+---------------------------------------------+
Normal exit from iterations. Exit status=0.
+---------------------------------------------+
| Discrete choice (multinomial logit) model   |
| Maximum Likelihood Estimates                |
| Model estimated: Sep 07, 2009 at 00:34:10PM.|
| Dependent variable              Choice      |
| Weighting variable                None      |
| Number of observations             210      |
| Iterations completed                 6      |
| Log likelihood function      -199.1284      |
| Number of parameters                 6      |
| Info. Criterion: AIC =         1.95360      |
|   Finite Sample: AIC =         1.95557      |
| Info. Criterion: BIC =         2.04924      |
| Info. Criterion:HQIC =         1.99226      |
| R2=1-LogL/LogL*  Log-L fncn  R-sqrd  RsqAdj |
| Constants only    -283.7588  .29825  .29150 |
| Response data are given as ind. choice.     |
| Number of obs.=   210, skipped   0 bad obs. |
+---------------------------------------------+

+---------------------------------------------+
| Notes No coefficients=> P(i,j)=1/J(i).      |
|       Constants only => P(i,j) uses ASCs    |
|         only. N(j)/N if fixed choice set.   |
|         N(j) = total sample frequency for j |
|         N    = total sample frequency.      |
|       These 2 models are simple MNL models. |
|       R-sqrd = 1 - LogL(model)/logL(other)  |
|       RsqAdj=1-[nJ/(nJ-nparm)]*(1-R-sqrd)   |
|         nJ   = sum over i, choice set sizes |
+---------------------------------------------+
+--------+--------------+----------------+--------+--------+
|Variable| Coefficient  | Standard Error |b/St.Er.|P[|Z|>z]|
+--------+--------------+----------------+--------+--------+
 AIR     |    5.20744330       .77905514     6.684   .0000
 TRAIN   |    3.86904270       .44312685     8.731   .0000
 BUS     |    3.16319421       .45026593     7.025   .0000
 COST    |    -.01550153       .00440799    -3.517   .0004
 TIME    |    -.09612480       .01043985    -9.207   .0000
 AIR_INC |     .01328703       .01026241     1.295   .1954
```

The `Clogit$` command has the `Ias` subcommand to conduct the Hausman's specification test for the IIA assumption (e.g., `Ias=air, bus$`). Unfortunately, the subcommand does not work in this model because the Hessian is not positive definite.

The `Logit$` command takes the panel data analysis approach. The `Pds` subcommand specifies the number of time periods. The two commands produce same log likelihood, parameter estimates, and standard errors but report different AIC and BIC.

```
LOGIT;
    Lhs=choice;
    Rhs=air,train,bus,cost,time,air_inc;
    Pds=4$
```

```
+--------------------------------------------------+
| Panel Data Binomial Logit Model                  |
| Number of individuals         =     210          |
| Number of periods             =       4          |
| Conditioning event is the sum of CHOICE          |
| Distribution of sums over the  4 periods:        |
| Sum        0     1     2     3     4     5     6 |
| Number     0   210     0     0     0     5    10 |
| Pct.     .00100.00   .00   .00   .00   .00   .00 |
+--------------------------------------------------+
Normal exit from iterations. Exit status=0.

+---------------------------------------------+
| Logit Model for Panel Data                  |
| Maximum Likelihood Estimates                |
| Model estimated: Sep 07, 2009 at 00:35:10PM.|
| Dependent variable               CHOICE     |
| Weighting variable                 None     |
| Number of observations              840     |
| Iterations completed                  6     |
| Log likelihood function       -199.1284     |
| Number of parameters                  6     |
| Info. Criterion: AIC =           .48840     |
|   Finite Sample: AIC =           .48852     |
| Info. Criterion: BIC =           .52221     |
| Info. Criterion:HQIC =           .50136     |
| Hosmer-Lemeshow chi-squared = 251.24482     |
| P-value=  .00000 with deg.fr. =       8     |
| Fixed Effects Logit Model for Panel Data    |
+---------------------------------------------+
+--------+--------------+----------------+--------+--------+
|Variable| Coefficient  | Standard Error |b/St.Er.|P[|Z|>z]|
+--------+--------------+----------------+--------+--------+
 AIR     |    5.20744330       .77905514     6.684   .0000
 TRAIN   |    3.86904270       .44312685     8.731   .0000
 BUS     |    3.16319421       .45026593     7.025   .0000
 COST    |    -.01550153       .00440799    -3.517   .0004
 TIME    |    -.09612480       .01043985    -9.207   .0000
 AIR_INC |     .01328703       .01026241     1.295   .1954
```

4.4 Conditional Logit Model in SPSS

Like PROC PHREG, the SPSS `Coxreg` command, which was designed for survival analysis data, provides a backdoor way of estimating the conditional logit model. Like PROC PHREG and SPSS `Probit`, SPSS `Coxreg` for the conditional logit model asks you to create a variable indicating failure as opposed to success. The following `Compute` command generates a variable `failure` by subtracting `choice` from 1 so that success and failure are respectively recoded as 0 and 1.

```
COMPUTE failure = 1 - choice.
```

```
COXREG failure WITH air train bus cost time air_inc
   /STATUS=choice(1)
   /STRATA=subject.
```

SPSS also produces the same parameter estimates and standard errors. Like PROC PHREG, SPSS `Coxreg` reports Wald statistics.

Variables in the Equation

	B	SE	Wald	Df	Sig.	Exp(B)
air	5.207	.779	44.680	1	.000	182.627
train	3.869	.443	76.234	1	.000	47.897
bus	3.163	.450	49.353	1	.000	23.646
cost	-.016	.004	12.367	1	.000	.985
time	-.096	.010	84.778	1	.000	.908
air_inc	.013	.010	1.676	1	.195	1.013

5. Nested Logit Regression Model

Consider a nested structure of choices. The first choice is made and the second choice then follows conditional on the first choice. When the IIA assumption is violated, one of the alternatives is the nested logit model. This chapter replicates the nested logit model discussed in Greene (2003). The model is formulated,

$$P(choice, branch) = P(choice \mid branch) * P(branch)$$
$$P(choice \mid branch) = P_{child}(\alpha_1 air + \alpha_2 train + \alpha_3 bus + \beta_1 \cos t + \beta_2 time)$$
$$P(branch) = P_{parent}(\gamma_{income} air\_inc + \tau_{fly} IV_{fly} + \tau_{ground} IV_{ground})$$

A LIMDEP example is skipped here since the nested logit model is fitted by NLOGIT, a stand-alone package to be purchased separately.

5.1 Nested Logit Model in Stata (.nlogit)

In Stata, the `.nlogit` command fits the nested logit model using the full information maximum-likelihood (FIML) method. You need to create a variable based on the specification of the tree using the `.nlogitgen` command. From the top, the parent-level has fly and ground branches; the fly branch at the child-level has air flight (1); the ground branch has train (2), bus (3), and car (4). `fly` and `ground` below are not variable names but arbitrary names you prefer.

```
. nlogitgen tree = mode(fly: 1, ground: 2 | 3 | 4)

new variable tree is generated with 2 groups
label list lb_tree
lb_tree:
           1 fly
           2 ground
```

The `.nlogittree` command displays the tree-structure defined by the `.nlogitgen` command.

```
. nlogittree mode tree, choice(choice)

tree structure specified for the nested logit model

 tree    N      mode  N   k
-----------------------------
 fly    210 --- 1    210  58
 ground 630 --- 2    210  63
             |- 3    210  30
             +- 4    210  59
-----------------------------
              total  840 210

k = number of times alternative is chosen
N = number of observations at each level
```

In Stata 10, `.nlogit` by default uses parameterization consistent with random utility maximization and introduces new syntax different from one in previous edition (Stata 2007: 434). This command is followed by a binary dependent variable, a list of independent variables, specifications of each level, and options. `case()` is required to specify an identification variable and `nonnormalized` is needed to request unscaled parameterization. Remind that the variable `tree` was defined by `.nlogitgen` above.

```
. nlogit choice air train bus cost time || tree: air_inc ||  ///
         mode:, case(subject) nonnormalized nolog noconstant notree
```

The `notree` option does not show the tree-structure and `nolog` suppresses an iteration log of the log likelihood. Remember that /// joins the next command line to the current line.

```
note: ground:air~c dropped because of collinearity

Nonnormalized nested logit regression          Number of obs      =        840
Case variable: subject                         Number of cases    =        210

Alternative variable: mode                     Alts per case: min =          4
                                                              avg =        4.0
                                                              max =          4

                                                  Wald chi2(6)    =      80.11
Log likelihood = -193.65615                       Prob > chi2     =     0.0000

------------------------------------------------------------------------------
      choice |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
mode         |
         air |   6.041827   1.198628     5.04   0.000      3.69256    8.391095
       train |   5.063954   .6619239     7.65   0.000     3.766607    6.361301
         bus |   4.095842   .6150907     6.66   0.000     2.890287    5.301398
        cost |  -.0315757   .0081541    -3.87   0.000    -.0475575   -.0155938
        time |  -.1126084   .0141277    -7.97   0.000    -.1402981   -.0849187
------------------------------------------------------------------------------
tree equations
------------------------------------------------------------------------------
fly          |
     air_inc |   .0153323   .0093813     1.63   0.102    -.0030548    .0337193
-------------+----------------------------------------------------------------
ground       |
     air_inc |     (base)
------------------------------------------------------------------------------
inclusive-value parameters
------------------------------------------------------------------------------
tree         |
    /fly_tau |   .5861148   .1406178                      .3105089    .8617207
 /ground_tau |    .389015   .1236901                      .1465869    .6314432
------------------------------------------------------------------------------
LR test for IIA (tau = 1):           chi2(2) =    10.94   Prob > chi2 = 0.0042
------------------------------------------------------------------------------
```

Hausman's specification test for this model reject the null hypothesis of IIA at the .01 level (p<.0042). `.mfx` and SPost commands do not work for this model. The following postestimation command computes AIC and BIC.

```
. estat ic

-----------------------------------------------------------------------------
       Model |    Obs    ll(null)   ll(model)     df          AIC         BIC
-------------+---------------------------------------------------------------
           . |    840           .   -193.6561      8     403.3123    441.1795
-----------------------------------------------------------------------------
               Note:  N=Obs used in calculating BIC; see [R] BIC note
```

If you prefer old style, list a binary dependent or choice variable, utility functions of the parent and child-levels, and options. The `group()`option is equivalent to `case()` in version 10 and higher. Do not forget to run the `.version` command to use a previsions version of command interpreter.

```
. version 9

. nlogit choice (mode=air train bus cost time) (tree=air_inc), ///
        group(subject) notree nolog

Nested logit regression
Levels                 =          2                 Number of obs      =        840
Dependent variable =      choice                    LR chi2(8)         =   194.9313
Log likelihood      = -193.65615                    Prob > chi2        =     0.0000

------------------------------------------------------------------------------
      choice |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
mode         |
         air |   6.042255   1.198907     5.04   0.000     3.692441     8.39207
       train |   5.064679   .6620317     7.65   0.000     3.767121    6.362237
         bus |   4.096302   .6151582     6.66   0.000     2.890614     5.30199
        cost |  -.0315888   .0081566    -3.87   0.000    -.0475754   -.0156022
        time |  -.1126183   .0141293    -7.97   0.000    -.1403111   -.0849254
-------------+----------------------------------------------------------------
tree         |
     air_inc |   .0153337   .0093814     1.63   0.102    -.0030534    .0337209
-------------+----------------------------------------------------------------
(incl. value |
 parameters) |
tree         |
        /fly |   .5859993   .1406199     4.17   0.000     .3103894    .8616092
     /ground |   .3889488   .1236623     3.15   0.002     .1465753    .6313224
------------------------------------------------------------------------------
LR test of homoskedasticity (iv = 1): chi2(2)=   10.94    Prob > chi2 = 0.0042
------------------------------------------------------------------------------
```

5.2 Nested Logit Model in SAS: PROC MDC

In SAS, PROC MDC fits the conditional logit model as well as the nested logit model. For the nested logit model, you have to use the UTILITY statement to specify utility functions of the parent (level 2) and child level (level 1), and the NEST statement to construct the decision-tree structure. "`2 3 4 @ 2`" reads that there are three nodes at the child level under the branch 2 at the parent-level.

```
PROC MDC DATA=masil.travel;
   MODEL choice = air train bus cost time air_inc /TYPE=NLOGIT CHOICE=(mode);
   ID subject;
   UTILITY U(1,) = air train bus cost time,
           U(2, 1 2) = air_inc;
   NEST LEVEL(1) = (1 @ 1, 2 3 4 @ 2),
        LEVEL(2) = (1 2 @ 1);
RUN;
```

```
                                        The MDC Procedure

                                     Nested Logit Estimates

Algorithm converged.


                                        Model Fit Summary
```

Dependent Variable	choice
Number of Observations	210
Number of Cases	840
Log Likelihood	-193.65615
Log Likelihood Null (LogL(0))	-291.12182
Maximum Absolute Gradient	0.0000147
Number of Iterations	15
Optimization Method	Newton-Raphson
AIC	403.31230
Schwarz Criterion	430.08916

Discrete Response Profile

Index	mode	Frequency	Percent
0	1	58	27.62
1	2	63	30.00
2	3	30	14.29
3	4	59	28.10

Goodness-of-Fit Measures

Measure	Value	Formula
Likelihood Ratio (R)	194.93	2 * (LogL - LogL0)
Upper Bound of R (U)	582.24	- 2 * LogL0
Aldrich-Nelson	0.4814	R / (R+N)
Cragg-Uhler 1	0.6048	1 - exp(-R/N)
Cragg-Uhler 2	0.6451	(1-exp(-R/N)) / (1-exp(-U/N))
Estrella	0.6771	1 - (1-R/U)^(U/N)
Adjusted Estrella	0.6485	1 - ((LogL-K)/LogL0)^(-2/N*LogL0)
McFadden's LRI	0.3348	R / U
Veall-Zimmermann	0.655	(R * (U+N)) / (U * (R+N))

N = # of observations, K = # of regressors

Parameter Estimates

Parameter	DF	Estimate	Standard Error	t Value	Approx Pr > |t|
air_L1	1	6.0423	1.1989	5.04	<.0001
train_L1	1	5.0646	0.6620	7.65	<.0001
bus_L1	1	4.0963	0.6152	6.66	<.0001
cost_L1	1	-0.0316	0.008156	-3.87	0.0001
time_L1	1	-0.1126	0.0141	-7.97	<.0001
air_inc_L2G1	1	0.0153	0.009381	1.63	0.1022
INC_L2G1C1	1	0.5860	0.1406	4.17	<.0001
INC_L2G1C2	1	0.3890	0.1237	3.15	0.0017

The `/fly_tau` (or `/fly`) and `/ground_tau` (or `/ground`) in the Stata output are equivalent to the INC_L2G1C1 and INC_L2G1C2 in the PROC MDC output. SAS and Stata produce goodness-of-fit measures, parameter estimates, and standard errors. Stata produces BIC of 441.1795 and PROC MDC computes Schwarz criterion 430.0892. Both return the same AIC 403.3123.

6. Conclusion

The regression models discussed so far are of categorical dependent variables (binary, ordinal, and nominal responses). An appropriate regression model is determined largely by the measurement level of a categorical dependent variable of interest. The level of measurement should be, however, considered in conjunction with your theory and research questions (Long 1997). You must also examine the data generation process (DGP) of a dependent variable to understand its "behavior." Experienced researchers pay special attention to censoring, truncation, sample selection, and other particular patterns of the DGP although these limited dependent variable issues are not addressed here.

Generally speaking, if your dependent variable is a binary variable, you may use the binary logit or probit regression model. For ordinal responses, try to fit either ordered logit or probit regression model. If you have a nominal response variable, investigate the DGP carefully and then choose one of the multinomial logit, conditional logit, and nested logit models. In order to use the conditional logit and nested logit, you need to reshape the data set in advance.

You should check key assumptions of a model when fitting the model. Examples are the parallel regression assumption in ordered logit and probit models and the independence of irrelevant alternatives (IIA) assumption in the multinomial logit model. You may respectively conduct the Brant test and Hausman test for these assumptions. If an assumption of an ordered or nominal response model is violated, find alternative models or think carefully if a dependent variable can be explored in a binary response model by dichotomizing the variable.

Since logit and probit models are nonlinear, their parameter estimates are difficult to interpret intuitively. The situation becomes even worse in generalized ordered logit and multinomial logit models, where many parameter estimates and related statistics are produced. Consequently, researchers need to spend more time and effort interpreting the results substantively. Simply reporting parameter estimates and goodness-of-fit statistics is not sufficient. J. Scott Long (1997) and Long and Freese (2003) provide good examples of meaningful interpretations using predicted probabilities, factor changes in odds, and marginal effects (discrete changes) of predicted probabilities. It is highly recommended to visualize marginal effects and discrete changes using a plot of predicted probabilities.

In general, logit and probit models require larger N than do linear regression models. Like the Bayesian estimation method, the maximum likelihood estimation method depends on data. You need to check if you have sufficient valid observations especially when your data contain many missing values. Scott Long's rule of thumb says 500 observations and at least additional 10 per independent variable are required in ML estimation. If you have small N, DO NOT include a large number of independent variables. This is the so called "small N and large parameter" problem; you may not be able to reach convergence in estimation (you are just torturing SAS or Stata to get nothing) and/or may not get reliable results with desirable asymptotic ML properties. What if 10 parameters are estimated on the basis of 50 observations? By contrast, an extremely large N, say millions to estimate only two parameters, is not always a virtue since it absurdly boosts the statistical power of a test without adding new information. Even a tiny effect, which should have been negligible in a normal situation, may be mistakenly reported as statistically significant.

Regarding statistical software packages, I would recommend the SAS LOGISTIC, QLIM, and MDC procedures of SAS/ETS (see Table 2.1 and 3.1). SAS also has PROC GENMOD and PROC PROBIT, but PROC LOGISTIC and PROC QLIM appear to be best for binary and ordinal response models, and PROC MDC is good for nominal dependent variable models. ODS is another advantage of using SAS. I also strongly recommend Stata since it provides handy ways to fit various models and also can be assisted by SPost, which has various useful commands such as `.fitstat`, `.prchange`, `.listcoef`, `.prtab`, and `.prgen`. I encourage SAS Institute to develop additional statements similar to, in particular, `.prchange` and `.prgen`.

LIMDEP supports various regression models for categorical dependent variables addressed in Greene (2003) but does not seem as user-friendly and stable as SAS and Stata. However, LIMDEP computes direct and indirect effects in the recursive bivariate probit model and helps researchers interpret the result in more detail. You may benefits from R's object-oriented programming concept and analyze data flexibly in your own way. SPSS is least recommended mainly due to its limited support for categorical dependent variable models and messy syntax and output.

If you are interested in logit and probit models for binary outcome variables, see Park, Hun Myoung. 2009. *Regression Models for Binary Dependent Variables Using Stata, SAS, R, LIMDEP, and SPSS*. Working Paper. The University Information Technology Services (UITS) Center for Statistical and Mathematical Computing, Indiana University."
http://www.indiana.edu/~statmath/stat/all/cdvm/index.html

Appendix: Data Sets

The first data set is a subset of the 2002 General Social Survey compiled by the National Opinion Research Center at the University of Chicago, http://www.norc.org.

http://www.indiana.edu/~statmath/stat/all/cdvm/gss_cdvm.csv
http://www.indiana.edu/~statmath/stat/all/cdvm/gss_cdvm.sas7bdat
http://www.indiana.edu/~statmath/stat/all/cdvm/gss_cdvm.dta

- `trust`: 1 if a respondent trust most people
- `belief`: Religious intensity: no religion (0) through strong (3)
- `educate`: respondent's education (years)
- `income`: family income ($1,000.00)
- `age`: respondent's age
- `male`: 1 for male and 0 for female
- `www`: 1 if a respondent have used WWW

```
. sum trust belief educate income age male www, sep(20)

    Variable |       Obs        Mean    Std. Dev.       Min        Max
-------------+--------------------------------------------------------
       trust |        40        .375    .4902903          0          1
      belief |        40        1.55    1.131144          0          3
     educate |        40      14.775    2.235925         11         20
      income |        40      24.325    7.566415          2       27.5
         age |        40      41.825    10.76053         20         65
        male |        40         .55    .5038315          0          1
         www |        40          .7    .4640955          0          1

. tab trust male, miss

    Social |        Gender
     Trust |    Female       Male |     Total
-----------+----------------------+----------
         0 |        11         14 |        25
         1 |         7          8 |        15
-----------+----------------------+----------
     Total |        18         22 |        40

. tab trust www, miss

    Social |        WWW Use
     Trust | Non-users      Users |     Total
-----------+----------------------+----------
         0 |        10         15 |        25
         1 |         2         13 |        15
-----------+----------------------+----------
     Total |        12         28 |        40

. tab male www, miss

           |        WWW Use
    Gender | Non-users      Users |     Total
-----------+----------------------+----------
    Female |         7         11 |        18
      Male |         5         17 |        22
-----------+----------------------+----------
     Total |        12         28 |        40

. tab belief male, miss
```

```
       Religious |        Gender
       Intensity |    Female       Male |     Total
-----------------+----------------------+----------
     No religion |         5          6 |        11
 Somewhat strong |         1          4 |         5
 Not very strong |         4         11 |        15
          Strong |         8          1 |         9
-----------------+----------------------+----------
           Total |        18         22 |        40
```

```
. tab belief www, miss

       Religious |        WWW Use
       Intensity | Non-users      Users |     Total
-----------------+----------------------+----------
     No religion |         3          8 |        11
 Somewhat strong |         2          3 |         5
 Not very strong |         5         10 |        15
          Strong |         2          7 |         9
-----------------+----------------------+----------
           Total |        12         28 |        40
```

The second data set is of travel mode choice (Greene 2003). You may get the data from http://pages.stern.nyu.edu/~wgreene/Text/tables/tablelist5.htm

http://www.indiana.edu/~statmath/stat/all/cdvm/travel.csv
http://www.indiana.edu/~statmath/stat/all/cdvm/travel.sas7bdat
http://www.indiana.edu/~statmath/stat/all/cdvm/travel.dta

- `subject`: identification number
- `mode`: 1=Air, 2=Train, 3=Bus, 4=Car
- `choice`: 1 if the travel mode is chosen
- `time`: terminal waiting time, 0 for car
- `cost`: generalized cost measure
- `income`: household income
- `air_inc`: interaction of air flight and household income, air*income
- `air`: 1 for the air flight mode, 0 for others
- `train`: 1 for the train mode, 0 for others
- `bus`: 1 for the bus mode, 0 for others
- `car`: 1 for the car mode, 0 for others
- `failure`: failure time variable, `1-choice`

```
. tab choice mode

           |                      mode
    choice |         1          2          3          4 |     Total
-----------+--------------------------------------------+----------
         0 |       152        147        180        151 |       630
         1 |        58         63         30         59 |       210
-----------+--------------------------------------------+----------
     Total |       210        210        210        210 |       840

. sum time income air_inc

    Variable |       Obs        Mean    Std. Dev.       Min        Max
-------------+--------------------------------------------------------
        time |       840    34.58929    24.94861          0         99
      income |       840    34.54762    19.67604          2         72
     air_inc |       840    8.636905    17.91206          0         72
```

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Revision History

- 2003. 04 First draft
- 2004. 07 Second draft
- 2005. 09 Third draft (Added bivariate logit/probit and nested logit models)
- 2008. 10 Fourth draft (Added SAS ODS and SPSS output)
- 2009. 09 Fifth draft (Estimated models using different data and rewrote chapter 2, 3)