

Prospects for Large Relativity Violations in Matter-Gravity Couplings

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Deviations from relativity are tightly constrained by numerous experiments. A class of unmeasured and potentially large violations is presented that can be tested in the laboratory only via weak-gravity couplings. Specialized highly sensitive experiments could achieve measurements of the corresponding effects. A single constraint of 1×10^{-11} GeV is extracted on one combination of the 12 possible effects in ordinary matter. Estimates are provided for attainable sensitivities in existing and future experiments.

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Einstein's theories of special and general relativity form the underpinning of our best existing description of nature at the fundamental level. The key idea behind relativity is the notion of Lorentz symmetry: the invariance of the laws of physics under rotations and boosts of the system. Experimental testing of these ideas has achieved impressive sensitivities to hypothetical tiny deviations from Lorentz symmetry in special relativity [1], with several tests using matter and light now well below parts in 10^{30} . For general relativity, the situation is more challenging because gravity is a weak force on small scales [2]. Recently, sensitive new constraints on violations of Lorentz symmetry in general relativity have been obtained [3].

Given the remarkable experimental sensitivities attained and the breadth of the studies, a question of immediate interest is whether any types of comparatively large relativity violations could have evaded detection to date. Here, we show the answer is affirmative. We demonstrate the existence of a type of Lorentz violation that is natural, challenging to observe in tests of special relativity, and directly detectable in laboratory experiments only when suppressed by weak gravitational effects. A general framework is given for studying this hidden type of violation, a constraint is obtained on one combination of the 12 possible effects in ordinary matter, and prospects for future measurements in specialized experiments are examined.

An arbitrary Lorentz violation represents a violation of rotation or boost symmetry and hence can be characterized via a nonzero vector or tensor quantity in the vacuum [4]. The specific violation of interest here involves an observer 4-vector a_μ that couples to a fermion field ψ as a term $\mathcal{L}_a = -a_\mu \bar{\psi} \gamma^\mu \psi$ in the Lagrange density. This coupling is comparatively simple and theoretically natural. As it is quadratic in ψ , it modifies the fermion dispersion relation. For example, for a constant a_μ in Minkowski spacetime, a free fermion of mass m , energy E , and momentum \vec{p} acquires the dispersion relation [5]

$$(E - a_0)^2 = m^2 c^4 + (\vec{p} - \vec{a})^2 c^2. \quad (1)$$

A fermion at rest can be shown to have $\vec{p} = \vec{a}$ and would therefore satisfy a modification of Einstein's famous equa-

tion relating matter and energy: $E = mc^2 + a_0$. More generally, the coupling a_μ can depend on the species w of fermion and is denoted a_μ^w .

Although at first glance a nonzero a_μ appears to be a substantial modification of known physics, in fact it is challenging to observe experimentally. In Minkowski spacetime, where gravity is irrelevant, a constant a_μ could be arbitrarily large because the coupling \mathcal{L}_a is unobservable [5] in experiments with a single fermion flavor. Under these circumstances, a_μ can be absorbed by a phase shift of the fermion field, which is a canonical transformation reflecting the inherent ambiguity of measuring absolute values of energy and momentum. Only the difference Δa_μ between two fermion flavors is potentially observable, and even if nonzero this requires special experiments involving flavor-changing fermions such as neutral-meson oscillations [6] or neutrino oscillations [7].

In weak gravitational fields such as those in our solar system, the effects of gravity can be understood as a perturbation $h_{\mu\nu}$ in a background Minkowski spacetime. A constant a_μ could still be absorbed by a phase shift and would remain strictly undetectable. However, a_μ cannot be constant generically because it must be compatible with the geometrical structure of gravity [8]. In essence, the interaction of a_μ with the gravitational field ensures that a_μ varies with spacetime position, and this implies only a single component of a_μ can be absorbed.

It is convenient to separate a_μ into a constant piece \bar{a}_μ and a fluctuation piece \tilde{a}_μ arising from the gravitational interaction: $a_\mu = \bar{a}_\mu + \tilde{a}_\mu$. In the vacuum, the fluctuation \tilde{a}_μ is tightly constrained by the requirements of geometric compatibility and coordinate independence of the physics. These give rise to \tilde{a}_μ of the form

$$\tilde{a}_\mu = \frac{1}{2} \alpha h_{\mu\nu} \bar{a}^\nu - \frac{1}{4} \alpha \bar{a}_\mu h^\nu{}_\nu \quad (2)$$

in harmonic coordinates, where the constant α is determined by the strength of the coupling of a_μ to gravity. The gravitational field $h_{\mu\nu}$ itself also acquires a correction $\tilde{h}_{\mu\nu}$, given at leading order by $\tilde{h}_{00} = 2\alpha \bar{a}_0 h_{00}/m$, which avoids

self-accelerations and ensures that Newton's third law holds between gravitating bodies.

The key point is that, although a nonzero constant \bar{a}_μ remains directly unobservable, its existence can be indirectly established through the effects of the \tilde{a}_μ coupling. In particular, since fluctuations in $h_{\mu\nu}$ are tiny in the solar system, the coefficient \bar{a}_μ can be enormous compared to other effects, while having evaded detection in all experimental tests of relativity to date. The validity of perturbation theory requires \bar{a}_μ to be less than the fermion mass m , but this still leaves room for effects some 10^{30} times greater than the best existing constraints [1] on other types of relativity violation. Indeed, the theoretically allowed values of \bar{a}_μ are large enough to obviate the Lorentz hierarchy problem [4], since they could lie within a few orders of the fermion mass. Radiative corrections involving two powers of \bar{a}_μ and a gravitational coupling could in principle produce effects in nongravitational experiments searching for other coefficients for Lorentz violation such as an observer two-tensor $\bar{c}_{\mu\nu}$, but even for large \bar{a}_μ the resulting signals would be far below current sensitivities. Evidently, the detection of \bar{a}_μ requires specialized gravitational experiments of high sensitivity.

What kind of field theory can produce an a_μ coupling? The geometric structure of gravity constrains the violation of Lorentz symmetry to be spontaneous rather than explicit [8], so the theory involves a Lorentz-tensor field that acquires a nonzero vacuum value. Since a_μ has a single index, the simplest choice is a vector field, denoted B_μ , although other tensor fields can be considered [9]. Vector theories with spontaneous breaking of Lorentz symmetry, generically called bumblebee theories, exist in many forms. Here, it suffices to suppose that the bumblebee field B_μ has a curvature coupling $\mathcal{L}_B \supset \xi B^\mu B^\nu R_{\mu\nu}$ and a coupling to the fermion field [10] $\mathcal{L}_\psi \supset -\zeta B_\mu \bar{\psi} \gamma^\mu \psi$, where ξ and ζ are coupling constants. In this class of models, the bumblebee vacuum value $\langle B_\mu \rangle = b_\mu$ produces a relativity violation of the a_μ type, with the identification $\alpha = -4\xi$, $\bar{a}_\mu = \zeta b_\mu$.

Spontaneous symmetry breaking is accompanied by massless modes called Nambu-Goldstone (NG) modes, which in the present context can be identified with vacuum fluctuations \mathcal{E}_μ of the bumblebee field [11] or equivalently with the fluctuation $\tilde{a}_\mu = \zeta \mathcal{E}_\mu$. In typical models, the NG modes play the role of a long-range force. They have previously been interpreted as the photon [11], the graviton [12], and a spin-dependent interaction [13]. Here, the NG modes play a different role: mediating a spin-independent force between fermions, with coupling constant ζ also controlling Lorentz violation. New spin-independent forces are constrained by experiments [14], which in this context limit the strength of ζ but not the size of the Lorentz violation \bar{a}_μ .

Numerous scenarios for \bar{a}_μ can be considered, depending on properties of the coupling ζ and the vacuum value

b_μ , and there is a correspondingly wide variety of potentially observable signals. The coupling ζ and hence the coefficient \bar{a}_μ may be flavor independent or may depend on properties of the fermion. For example, it could be proportional to the fermion mass m , in analogy with the usual Yukawa couplings. Alternatively, it may depend on other quantum numbers such as baryon number B , lepton number L , or combinations of these such as the difference $B - L$ that is conserved in many grand unified theories. It could be proportional to the fermion charge Q , as occurs in bumblebee electrodynamics [11]. This further hides the Lorentz violation because effects cancel in charge-neutral matter, so observable signals in this case require specialized experiments designed to study the effects of gravity on charged matter, such as electron interferometry [15]. Another scenario has effects from \bar{a}_μ canceling against those from different unmeasured coefficients for Lorentz violation such as an observer two-tensor $\bar{c}_{\mu\nu}$, so that signals in ordinary matter would be absent. Since \bar{a}_μ violates CPT symmetry while $\bar{c}_{\mu\nu}$ is invariant, this cancellation implies an observable enhancement in future gravitational experiments with antihydrogen [16] or antiparticles [17].

The vacuum value b_μ and hence the coefficient \bar{a}_μ could be timelike, lightlike, or spacelike, with different observable signals in each case. Substantial differences between the magnitudes of components of \bar{a}_μ can be generated naturally. For example, if \bar{a}_μ is timelike, then there exists an observer frame O in which it is purely timelike. Provided the domain size is cosmological, it may be appropriate to identify O with the rest frame U of the cosmic microwave background radiation. In effect, this aligns the Lorentz violation with the cosmological expansion, thereby preserving isotropy [8]. However, experiments are performed and reported locally in the solar system, for which it is appropriate and conventional to adopt a Sun-centered frame S [18]. Since S differs from U by a boost, in S the spatial components \bar{a}_J are nonzero but suppressed by a factor of about 1000 relative to the temporal component \bar{a}_T . As another example, if \bar{a}_μ is spacelike instead, then there exists an observer frame O' in which it is purely spacelike. If O' happens to coincide with U , then in S the temporal component \bar{a}_T is nonzero but suppressed by a factor of about 1000 relative to the spatial components \bar{a}_J .

To detect effects from \bar{a}_μ , the relevant experiments must be sensitive to gravity. In a laboratory frame L , it suffices to achieve sensitivity to modifications of the dominant local gravitational acceleration g . The effects predicted by \mathcal{L}_a can be extracted in the weak-gravity approximation and at leading order in \bar{a}_μ and $h_{\mu\nu}$. For a test body T moving in the gravitational field of Earth as the source S , the presence of nonzero \bar{a}_μ^w induces an additional contribution \tilde{F}_z to the usual vertical component F_z of the laboratory gravitational force in Newton's second law:

$$\tilde{F}_z = -2g(\alpha \bar{a}_T^T + \alpha \bar{a}_I^S m^T/m^S). \quad (3)$$

Here, m^T and m^S are the masses of T and S, while \bar{a}_i^T and \bar{a}_i^S are the time components of effective coefficients for Lorentz violation for T and S in the frame L . For a macroscopic test body T containing N_w^T particles of species w and negligible binding energy, the effective coefficient for Lorentz violation is $\bar{a}_\mu^T = \sum_w N_w^T \bar{a}_\mu^w$. Similarly, the effective coefficient for S with N_w^S particles of species w is $\bar{a}_\mu^S = \sum_w N_w^S \bar{a}_\mu^w$. Values of N_w^T can be computed exactly for atoms and well approximated for laboratory test bodies, while for N_w^S recent studies of the bulk Earth composition [19] yield the estimates $N_e^S = N_p^S \approx N_n^S = 1.8 \times 10^{51}$. Note that \bar{a}_i^T , \bar{a}_i^S are time dependent because the components \bar{a}_T^w , \bar{a}_J^w of \bar{a}_μ^w are constant in the frame S , and hence the rotation and the revolution of the Earth induce sidereal and annual time dependences in the component \bar{a}_i^w in the frame L .

The observable effects from nonzero \tilde{F}_z are of two basic kinds. One arises from the flavor dependence of \bar{a}_i^T and hence of \tilde{F}_z . This would produce a signal in experiments testing the weak equivalence principle (WEP), which compare the gravitational accelerations of two test bodies. The other effect arises from the time dependence of the laboratory-frame components \bar{a}_i^T and \bar{a}_i^S and hence of \tilde{F}_z . It would produce a signal in gravimeter or other experiments searching for time variations in the Newton gravitational coupling G_N . The transformation between the frames S and L expresses \bar{a}_i^w in terms of \bar{a}_T^w and the product of \bar{a}_J^w and the relevant boost, which is about 10^{-4} for the Earth's revolution and about 10^{-6} for its rotation. It follows that WEP tests can achieve sensitivity to all components \bar{a}_T^w with instantaneous signals and also to all components \bar{a}_J^w with signals involving sidereal or annual variations, with the latter suppressed by the boost factor. In contrast, the single-flavor gravimeter tests are insensitive to \bar{a}_T^w , which in this context causes an effect equivalent to an unobservable constant rescaling of G_N , but they have boost-suppressed sensitivity to the spatial components \bar{a}_J^w via annual and sidereal variations.

Comparatively few experiments sensitive to \bar{a}_μ^w exist, and so large values of \bar{a}_μ^w could have remained undetected to date even for generic models. If attention is restricted to the constituents of ordinary matter, up to 12 measurements are needed to constrain the 12 components \bar{a}_μ^w ($w = e, p,$

$n; \mu = T, X, Y, Z$). One constraint on the time components \bar{a}_T^w can be deduced from published data from WEP tests using a torsion pendulum with beryllium and titanium test masses [14]. For this experiment, calculating with Eq. (3) yields the constraint

$$|\alpha \bar{a}_T^e + \alpha \bar{a}_T^p - 0.8 \alpha \bar{a}_T^n| < 1 \times 10^{-11} \text{ GeV} \quad (4)$$

in natural units ($c = \hbar = 1$) at the 90% confidence level, where a generic scenario without cancellations is adopted. Somewhat weaker constraints on similar combinations of coefficients are implied at order 10^{-8} GeV by older data from WEP tests with falling corner cubes [20] and at order 10^{-5} GeV by data from WEP tests with atom interferometers [21]. However, these constraints can be evaded or suppressed in specific models. For example, if the coefficients \bar{a}_μ^w are proportional to the charge Q , no constraints exist because the effects cancel in neutral matter. If instead the coefficients are proportional to baryon number B or to the mass m_w , then the strongest constraints come from considerations of the binding energy in the test-body atoms, and these are weaker than the generic case by about an order of magnitude [22].

In contrast to the time components \bar{a}_T^w , the space components \bar{a}_J^w are presently unconstrained. Certain existing experiments and data could in principle yield sensitivity to some combinations of \bar{a}_J^w for generic scenarios. Analysis of sidereal and annual variations in the acceleration of falling corner cubes could reach 10^{-2} and 10^{-4} GeV, respectively. Sidereal measurements with matter interferometers at established sensitivities [23] could achieve 10^{-5} GeV on various components $\alpha \bar{a}_J^w$ using different atomic species. Experiments with torsion pendula could attain 10^{-7} GeV via sidereal variations and 10^{-6} GeV via annual effects, the latter being weaker due to centrifugal forces. Sidereal and annual studies with existing types of superconducting gravimeters could reach 10^{-4} GeV and 10^{-6} GeV on various combinations of $\alpha \bar{a}_J^w$, assuming sensitivities already attained in classic tests [24]. Analysis of annual modulations in available lunar laser ranging data [25] could achieve 10^{-6} GeV on some combinations of $\alpha \bar{a}_J^w$.

The prospects for improved measurements of $\alpha \bar{a}_\mu^w$ in future experiments are excellent, with gains of several orders of magnitude on the above estimates being plau-

TABLE I. Actual (this work), currently feasible (brackets), and future attainable (braces) estimated experimental sensitivities.

Experiment	$\alpha \bar{a}_T^w$, actual	$\alpha \bar{a}_J^w$, actual	$\alpha \bar{a}_J^w$, feasible	$\alpha \bar{a}_T^w$, future
Torsion pendulum [14]	10^{-11} GeV	...	$[10^{-7} \text{ GeV}]$...
Falling corner cube [20]	10^{-8} GeV	...	$[10^{-4} \text{ GeV}]$...
Atom interferometry [21,23,26]	10^{-5} GeV	...	$[10^{-5} \text{ GeV}]$	$\{10^{-15} \text{ GeV}\}$
Superconducting gravimeter [24]	$[10^{-6} \text{ GeV}]$...
Lunar laser ranging [25]	$[10^{-6} \text{ GeV}]$...
Drop tower [27]	$\{10^{-10} \text{ GeV}\}$
Balloon drop [28]	$\{10^{-13} \text{ GeV}\}$
Bouncing masses [29]	$\{10^{-14} \text{ GeV}\}$
Space-based WEP [30–33]	$\{10^{-13} - 10^{-16} \text{ GeV}\}$

sible. For $\alpha\bar{a}_T^w$, anticipated advances in atom interferometry [26] could make 10^{-15} GeV attainable. Estimated sensitivities for free-fall experiments imply sensitivities of 10^{-10} GeV using a drop tower [27], 10^{-13} GeV via balloon drop tests [28], and of 10^{-14} GeV using bouncing masses in the laboratory [29]. Various space-based WEP tests are also currently under development [30], with estimated sensitivities to $\alpha\bar{a}_T^w$ of 10^{-13} GeV for microSCOPE [31], of 10^{-15} GeV for Galileo Galilei [32], and of 10^{-16} GeV for STEP [33]. All these experiments also offer potential improvements in measurements of $\alpha\bar{a}_J^w$. The existing limits and estimated attainable sensitivities on $\alpha\bar{a}_\mu^w$ are summarized in Table I. Since the space components are presently unconstrained and only one combination of the time components is measured, there is considerable room for experimental investigation.

The relativity violations involving \bar{a}_μ discussed in this work are potentially large, possibly some 30 orders of magnitude greater than the best existing sensitivities, while being countershaded from most experimental observations. However, they may not be unique. Other coefficients for relativity violations exist that are unobservable in Minkowski spacetime but are observable through gravity couplings [8]. This offers interesting prospects for the existence of a realistic model with comparatively large relativity violations, generating signals that would be detectable in gravitational experiments with current or near-future technology.

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