Lorentz violation and short-baseline neutrino experiments

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A general discussion is given of signals for broken Lorentz symmetry in short-baseline neutrino experiments. Among the effects that Lorentz violation can introduce are a dependence on energy differing from that of the usual massive-neutrino solution and a dependence on the direction of neutrino propagation. Using the published result of the Liquid Scintillator Neutrino Detector experiment, analysis of the effects of broken Lorentz symmetry yields an estimated nonzero value $(3 \pm 1) \times 10^{-19}$ GeV for a combination of coefficients for Lorentz violation. This lies in the range expected for effects originating from the Planck scale in an underlying unified theory.

Evidence for neutrino oscillations from the Liquid Scintillator Neutrino Detector (LSND) experiment represents an open puzzle in neutrino physics [1,2]. The results of this experiment are consistent with muon antineutrinos $\bar{\nu}_\mu$ oscillating into electron antineutrinos $\bar{\nu}_e$ with a small probability of $P_{\bar{\nu}_\mu \to \bar{\nu}_e} \simeq 0.26 \pm 0.08\%$ as the neutrinos propagate over a distance $L \simeq 30$ m. Typically, the oscillations are explained by ascribing small masses to neutrinos. The puzzle exists because it appears difficult to match the LSND result to the combined results of all other neutrino-oscillation experiments within the simplest scenario of three massive neutrinos [3,4]. More exotic oscillation scenarios may help solve this puzzle. In the present work, we consider the possibility that the LSND oscillations are induced by violation of Lorentz symmetry. We show that the size of the violation required to account for the anomaly is consistent with the size of effects that would emerge from an underlying unified theory at the Planck scale.

The principle of Lorentz symmetry is deeply ingrained in our best theories of nature at the fundamental level, which are Einstein's general relativity and the standard model of particle physics. The former accounts for gravitation at the classical level, while the latter describes all other physical phenomena involving elementary particles and forces down to the quantum level. However, a unification of these two theories is expected to occur at the Planck scale, $m_P \simeq 10^{19}$ GeV, about 17 orders of magnitude above the electroweak scale. Among the candidate experimental signals of new physics arising from this underlying unified theory are minuscule violations of Lorentz symmetry [5,6].

Despite an expected suppression by 10^{-17} or more, Lorentz violation may be detectable with existing technology in experiments on a variety of systems, including neutrinos [7–16]. Under favorable circumstances, attainable sensitivities in the neutrino sector are competitive with those in other Lorentz tests, including ones with mesons [17], baryons [18], electrons [19–21], photons [22–25], muons [26], and the Higgs [27]. Indeed, the

current evidence for neutrino oscillations is compatible with small Lorentz violation, perhaps even without the introduction of neutrino mass [13]. The novel features introduced by Lorentz violation may also help to reconcile the LSND result and other neutrino experiments within the three-generation scenario. The confirmation of oscillations from Lorentz violation in LSND and other neutrino experiments would offer the first glimpse of nature at its most basic level.

General Lorentz violation is described by a theoretical framework known as the Standard-Model Extension (SME) [7,28]. It connects to the Planck scale through operators of nonrenormalizable dimension [29]. In the minimal form of the SME, neutrinos are massless and the oscillations are determined by a set of 102 real constant coefficients controlling the Lorentz violation [13]. These coefficients can be collected into matrices $(a_L)^\mu$ and $(c_L)^{\mu\nu}$ of mass dimension 1 and 0, respectively, corresponding to *CPT*-odd and *CPT*-even operators. Setting all the coefficients to zero yields the usual minimal standard model of particle physics. Assuming quantum-gravity origins for $(a_L)^\mu$ and $(c_L)^{\mu\nu}$ suggests their magnitude is suppressed by a factor of 10^{-17} or more.

The coefficients for Lorentz violation predict a plethora of unconventional signals in oscillation experiments. For example, the usual mass-induced oscillations vary as L/E with the neutrino energy E and the experimental baseline L. In contrast, the simplest Lorentz-violating oscillations vary as L or EL, and more complicated functional dependence on E can occur. Another unconventional feature of oscillations due to Lorentz violation is dependence on the direction of neutrino propagation. This causes several novel effects, which can include varying oscillation signals as the Earth rotates. For experiments like LSND, where both source and detector are fixed on the Earth's surface, the entire apparatus makes a full rotation each sidereal day $\simeq 23$ h 56 min. The direction of propagation of the detected neutrinos is therefore also changing, and in the presence of Lorentz violation the

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consequent change in oscillation probabilities provides a definite signal that cannot arise from mass-induced oscillations [13]. Note that neutrinos in distinct Earth-based experiments may propagate in different directions, so they can naturally behave differently in Lorentz-violating scenarios.

A more exotic possibility, absent in the minimal form of the SME but allowed in the general SME framework, is oscillations between neutrinos and antineutrinos [13]. Oscillations with $\nu \leftrightarrow \bar{\nu}$ mixing offer alternative Lorentz-violating modes that could explain the excess of $\bar{\nu}_e$ observed by LSND, since the numbers of ν_e, ν_μ, and $\bar{\nu}_\mu$ involved are comparable. In what follows, we restrict attention to the minimal SME scenario.

The large number of coefficients involved, even in the minimal SME, makes a general analysis challenging. However, in experiments like LSND, the short baseline offers the possibility of a valuable simplifying approximation. When the baseline is small compared to the oscillation lengths given by the Hamiltonian, the transition amplitudes can be expanded about the identity as a perturbation on the oscillation-free case. It turns out that the general leading-order result for the corresponding transition probabilities differs from the oscillation-free case by terms proportional to the squared modulus of Hamiltonian elements, as we show next.

In the minimal SME, the oscillatory behavior of the three generations of left-handed neutrinos is governed by the leading-order effective Hamiltonian [13]

$$h_{\text{eff}} = \frac{1}{E}[(a_L)^\mu p_\mu - (c_L)^{\mu\nu} p_\mu p_\nu]. \tag{1}$$

In this equation, $(a_L)^\mu$ and $(c_L)^{\mu\nu}$ are coefficients for Lorentz violation that are Hermitian 3×3 complex matrices of mass dimension 1 and 0, respectively. The energy E is assumed to be large compared to the elements of $(a_L)^\mu$ and $E(c_L)^{\mu\nu}$. The four-momentum $p_\mu \simeq E(1; -\hat{p})$ introduces both energy dependence through E and direction dependence through $\hat{p}$. Since the antisymmetric and trace pieces of $(c_L)^{\mu\nu}$ do not contribute to Eq. (1), we also assume in what follows the properties $(c_L)^{\mu\nu} = (c_L)^{\nu\mu}$ and $\eta_{\mu\nu}(c_L)^{\mu\nu} = 0$. The effective Hamiltonian for antineutrinos is obtained by complex conjugating Eq. (1) and reversing the sign of the $(a_L)^\mu$ term.

Under suitable experimental conditions, it is an excellent approximation to expand the oscillation amplitudes in powers of h_{eff}: $S(L) \simeq 1 - ih_{\text{eff}}L - \frac{1}{2}h_{\text{eff}}^2 L^2 + \cdots$. The validity of this expansion requires that the baseline L be short compared to the oscillation lengths given by h_{eff}. However, since h_{eff} varies with the neutrino energy E, the designation of a given experiment as short baseline in this context depends on the ranges of both L and E. At leading order in this short-baseline approximation, the oscillation probabilities are

$$P_{\nu_b \to \nu_a} \simeq \begin{cases} 1 - \sum\limits_{c,c \neq a} P_{\nu_a \to \nu_c}, & a = b, \\ |(h_{\text{eff}})_{ab}|^2 L^2, & a \neq b, \end{cases} \tag{2}$$

where the indices a, b range over the neutrino flavors e, μ, τ. The probabilities $P_{\bar{\nu}_b \to \bar{\nu}_a}$ for antineutrinos are obtained by changing the sign of $(a_L)^\mu$. Note that Eq. (2) can readily be modified for the nonminimal SME, including $\nu \leftrightarrow \bar{\nu}$ mixing [13].

In reporting results from experimental tests of Lorentz invariance, it is necessary to specify the frame of reference. In principle, any inertial frame can be adopted, but convention and convenience dictate the use of a Sun-centered celestial-equatorial frame. For experiments with both source and detector fixed on the Earth's surface, the sidereal rotation causes the direction of neutrino propagation $\hat{p}$ to change with respect to the Sun-centered frame. This causes the components of $\hat{p}$ to vary at the sidereal frequency $\omega_\oplus = 2\pi/(23\text{ h }56\text{ min})$, unless $\hat{p}$ happens to point along the Earth's rotation axis. This time dependence can be displayed explicitly in the effective Hamiltonian h_{eff}, which can be written in the form

$$(h_{\text{eff}})_{ab} = (C)_{ab} + (\mathcal{A}_s)_{ab} \sin\omega_\oplus T_\oplus + (\mathcal{A}_c)_{ab} \cos\omega_\oplus T_\oplus$$
$$+ (\mathcal{B}_s)_{ab} \sin2\omega_\oplus T_\oplus + (\mathcal{B}_c)_{ab} \cos2\omega_\oplus T_\oplus, \tag{3}$$

where $T_\oplus$ is the time measured from a standard origin [24]. This expression is independent of the short-baseline approximation, so Eq. (3) and what follows also apply more generally.

The energy dependence in Eq. (3) is given by further decomposition:

$$\begin{aligned}
(C)_{ab} &= (C^{(0)})_{ab} + E(C^{(1)})_{ab}, \\
(\mathcal{A}_s)_{ab} &= (\mathcal{A}_s^{(0)})_{ab} + E(\mathcal{A}_s^{(1)})_{ab}, \\
(\mathcal{A}_c)_{ab} &= (\mathcal{A}_c^{(0)})_{ab} + E(\mathcal{A}_c^{(1)})_{ab}, \\
(\mathcal{B}_s)_{ab} &= E(\mathcal{B}_s^{(1)})_{ab}, \qquad (\mathcal{B}_c)_{ab} = E(\mathcal{B}_c^{(1)})_{ab}.
\end{aligned} \tag{4}$$

The combinations $(\mathcal{A}_s^{(0)})_{ab}$, $(\mathcal{A}_c^{(0)})_{ab}$, $(C^{(0)})_{ab}$ contain the coefficients $(a_L)^\mu$, while $(\mathcal{A}_s^{(1)})_{ab}$, $(\mathcal{A}_c^{(1)})_{ab}$, $(\mathcal{B}_s^{(1)})_{ab}$, $(\mathcal{B}_c^{(1)})_{ab}$, $(C^{(1)})_{ab}$ involve the coefficients $(c_L)^{\mu\nu}$. The analogous decomposition for the antineutrino effective Hamiltonian generates combinations that can be obtained from their neutrino equivalents by complex conjugation and a sign reversal for $(\mathcal{A}_s^{(0)})_{ab}$, $(\mathcal{A}_c^{(0)})_{ab}$, $(C^{(0)})_{ab}$.

The explicit relationships between these quantities and the SME coefficients $(a_L)^\mu$ and $(c_L)^{\mu\nu}$ for Lorentz violation are

$$(C^{(0)})_{ab} = (a_L)_{ab}^T - \hat{N}^Z (a_L)_{ab}^Z, \tag{5}$$

$$(C^{(1)})_{ab} = -\frac{1}{2}(3 - \hat{N}^Z\hat{N}^Z)(c_L)^{TT}_{ab} + 2\hat{N}^Z(c_L)^{TZ}_{ab}$$
$$+ \frac{1}{2}(1 - 3\hat{N}^Z\hat{N}^Z)(c_L)^{ZZ}_{ab}, \qquad (6)$$

$$(\mathcal{A}^{(0)}_s)_{ab} = \hat{N}^Y(a_L)^X_{ab} - \hat{N}^X(a_L)^Y_{ab}, \qquad (7)$$

$$(\mathcal{A}^{(1)}_s)_{ab} = -2\hat{N}^Y(c_L)^{TX}_{ab} + 2\hat{N}^X(c_L)^{TY}_{ab} + 2\hat{N}^Y\hat{N}^Z(c_L)^{XZ}_{ab}$$
$$- 2\hat{N}^X\hat{N}^Z(c_L)^{YZ}_{ab}, \qquad (8)$$

$$(\mathcal{A}^{(0)}_c)_{ab} = -\hat{N}^X(a_L)^X_{ab} - \hat{N}^Y(a_L)^Y_{ab}, \qquad (9)$$

$$(\mathcal{A}^{(1)}_c)_{ab} = 2\hat{N}^X(c_L)^{TX}_{ab} + 2\hat{N}^Y(c_L)^{TY}_{ab} - 2\hat{N}^X\hat{N}^Z(c_L)^{XZ}_{ab}$$
$$- 2\hat{N}^Y\hat{N}^Z(c_L)^{YZ}_{ab}, \qquad (10)$$

$$(\mathcal{B}^{(1)}_s)_{ab} = \hat{N}^X\hat{N}^Y[(c_L)^{XX}_{ab} - (c_L)^{YY}_{ab}]$$
$$- (\hat{N}^X\hat{N}^X - \hat{N}^Y\hat{N}^Y)(c_L)^{XY}_{ab}, \qquad (11)$$

$$(\mathcal{B}^{(1)}_c)_{ab} = -\frac{1}{2}(\hat{N}^X\hat{N}^X - \hat{N}^Y\hat{N}^Y)[(c_L)^{XX}_{ab} - (c_L)^{YY}_{ab}]$$
$$- 2\hat{N}^X\hat{N}^Y(c_L)^{XY}_{ab}. \qquad (12)$$

In these expressions, $\hat{N}^X, \hat{N}^Y, \hat{N}^Z$ are directional factors containing information about the neutrino-beam direction with respect to the Earth. At the detector location, let θ be the angle between the beam and the vertical upward direction, let ϕ be the angle between the beam and south measured towards the east, and let χ be the colatitude of the detector. Then, the directional factors are given explicitly as

$$\begin{pmatrix} \hat{N}^X \\ \hat{N}^Y \\ \hat{N}^Z \end{pmatrix} = \begin{pmatrix} \cos\chi\sin\theta\cos\phi + \sin\chi\cos\theta \\ \sin\theta\sin\phi \\ -\sin\chi\sin\theta\cos\phi + \cos\chi\cos\theta \end{pmatrix}. \qquad (13)$$

Note that the decomposition (3) is general, so Eqs. (4)–(12) apply in contexts other than the short-baseline approximation. For example, a useful test model for sidereal variations in atmospheric neutrinos is the simple vector theory in which only the real coefficients $(a_L)^X_{\mu\tau}, (a_L)^Y_{\mu\tau}, (c_L)^{TX}_{\mu\tau}, (c_L)^{TY}_{\mu\tau}$ are nonzero, and for which the oscillation probability has time dependence $P_{\nu_\mu \leftrightarrow \nu_\tau} = \sin^2[(\mathcal{A}_s)_{\mu\tau}L\sin\omega_\oplus T_\oplus + (\mathcal{A}_c)_{\mu\tau}L\cos\omega_\oplus T_\oplus]$, with $(\mathcal{A}_s)_{\mu\tau}$ and $(\mathcal{A}_c)_{\mu\tau}$ given via Eqs. (7)–(10).

Any given short-baseline experiment is sensitive to three complex combinations of $(a_L)^\mu$ coefficients, $(\mathcal{A}^{(0)}_s)_{ab}, (\mathcal{A}^{(0)}_c)_{ab}, (C^{(0)})_{ab}$, and five complex combinations of $(c_L)^{\mu\nu}$ coefficients, $(\mathcal{A}^{(1)}_s)_{ab}, (\mathcal{A}^{(1)}_c)_{ab}, (\mathcal{B}^{(1)}_s)_{ab}, (\mathcal{B}^{(1)}_c)_{ab}, (C^{(1)})_{ab}$. However, the directional dependence implies that a combination of experiments testing a specific oscillation mode $\nu_a \to \nu_b$ can provide access to all

components of $(a_L)^\mu_{ab}$ and $(c_L)^{\mu\nu}_{ab}$, provided the directions of the associated neutrino beams differ.

For the special case of the transition mode relevant to LSND, the probability takes the form

$$P_{\bar{\nu}_\mu \to \bar{\nu}_e} \simeq L^2|(C)_{\bar{e}\,\bar{\mu}} + (\mathcal{A}_s)_{\bar{e}\,\bar{\mu}}\sin\omega_\oplus T_\oplus$$
$$+ (\mathcal{A}_c)_{\bar{e}\,\bar{\mu}}\cos\omega_\oplus T_\oplus + (\mathcal{B}_s)_{\bar{e}\,\bar{\mu}}\sin2\omega_\oplus T_\oplus$$
$$+ (\mathcal{B}_c)_{\bar{e}\,\bar{\mu}}\cos2\omega_\oplus T_\oplus|^2, \qquad (14)$$

where $\omega_\oplus \simeq 2\pi/(23\text{ h }56\text{ min})$ is the Earth's sidereal frequency and $T_\oplus$ is a standardized time [24]. The time variation is a direct consequence of the directional dependence. In the short-baseline approximation, we find harmonics up to $2\omega_\oplus$, but more generally all higher harmonics can occur.

In Eq. (14), the complex factors $(\mathcal{A}_s)_{\bar{e}\,\bar{\mu}}, (\mathcal{A}_c)_{\bar{e}\,\bar{\mu}}, (\mathcal{B}_s)_{\bar{e}\,\bar{\mu}}, (\mathcal{B}_c)_{\bar{e}\,\bar{\mu}}$, and $(C)_{\bar{e}\,\bar{\mu}}$ are experiment-dependent linear combinations of the SME coefficients $(a_L)^\mu$ and $(c_L)^{\mu\nu}$ for Lorentz violation. These combinations depend on the energy of the neutrinos. Their decomposition into energy-independent quantities takes a form analogous to that of Eq. (4):

$$(C)_{\bar{e}\,\bar{\mu}} = (C^{(0)})_{\bar{e}\,\bar{\mu}} + E(C^{(1)})_{\bar{e}\,\bar{\mu}},$$
$$(\mathcal{A}_s)_{\bar{e}\,\bar{\mu}} = (\mathcal{A}^{(0)}_s)_{\bar{e}\,\bar{\mu}} + E(\mathcal{A}^{(1)}_s)_{\bar{e}\,\bar{\mu}},$$
$$(\mathcal{A}_c)_{\bar{e}\,\bar{\mu}} = (\mathcal{A}^{(0)}_c)_{\bar{e}\,\bar{\mu}} + E(\mathcal{A}^{(1)}_c)_{\bar{e}\,\bar{\mu}}, \qquad (15)$$
$$(\mathcal{B}_s)_{\bar{e}\,\bar{\mu}} = E(\mathcal{B}^{(1)}_s)_{\bar{e}\,\bar{\mu}}, \qquad (\mathcal{B}_c)_{\bar{e}\,\bar{\mu}} = E(\mathcal{B}^{(1)}_c)_{\bar{e}\,\bar{\mu}}.$$

There are therefore a total of eight complex experiment-dependent coefficients: $(\mathcal{A}^{(0)}_s)_{\bar{e}\,\bar{\mu}}, (\mathcal{A}^{(0)}_c)_{\bar{e}\,\bar{\mu}}, (C^{(0)})_{\bar{e}\,\bar{\mu}}, (\mathcal{A}^{(1)}_s)_{\bar{e}\,\bar{\mu}}, (\mathcal{A}^{(1)}_c)_{\bar{e}\,\bar{\mu}}, (\mathcal{B}^{(1)}_s)_{\bar{e}\,\bar{\mu}}, (\mathcal{B}^{(1)}_c)_{\bar{e}\,\bar{\mu}}, (C^{(1)})_{\bar{e}\,\bar{\mu}}$. A comprehensive analysis of the LSND data for the above energy and sidereal dependence would in principle yield measurements of 16 of the possible 102 real degrees of freedom in the neutrino sector of the minimal SME. We remark in passing that the inclusion of a mass-squared matrix $(\tilde{m}^2)_{ab}$ for neutrinos in the present formalism is straightforward. For example, in Eq. (15) it suffices to extend the definition of $(C)_{\bar{e}\,\bar{\mu}}$ to $(C)_{\bar{e}\,\bar{\mu}} = (2E)^{-1} \times (\tilde{m}^2)^*_{\bar{e}\,\bar{\mu}} + (C^{(0)})_{\bar{e}\,\bar{\mu}} + E(C^{(1)})_{\bar{e}\,\bar{\mu}}$. It turns out that the general two-generation model with a mass-squared matrix and both $(a_L)^\mu$ and $(c_L)^{\mu\nu}$ coefficients has 41 degrees of freedom, while its rotation-invariant restriction has eight [13].

Under the above assumption of Lorentz violation, the published LSND results permit an estimate for one combination of these degrees of freedom. In the experiment, copious numbers of $\bar{\nu}_\mu$ were produced. An excess of $\bar{\nu}_e$ over background was observed, which was interpreted as $\bar{\nu}_\mu$ oscillating into $\bar{\nu}_e$. The corresponding oscillation probability is $P_{\bar{\nu}_\mu \to \bar{\nu}_e} \simeq 0.26 \pm 0.08\%$. Since this published result involves all events irrespective of sidereal

time, it represents an average over the run time of the experiment. To a good approximation, it can be taken as representing the expectation over a sidereal day, $\langle P_{\bar{\nu}_\mu \to \bar{\nu}_e} \rangle \simeq 0.26 \pm 0.08\%$. Using Eq. (14) and this result, we can estimate the corresponding nonzero value for a combination of SME coefficients for Lorentz violation:

$$|(C)_{\bar{e}\bar{\mu}}|^2 + \frac{1}{2}|(\mathcal{A}_s)_{\bar{e}\bar{\mu}}|^2 + \frac{1}{2}|(\mathcal{A}_c)_{\bar{e}\bar{\mu}}|^2 + \frac{1}{2}|(\mathcal{B}_s)_{\bar{e}\bar{\mu}}|^2$$

$$+ \frac{1}{2}|(\mathcal{B}_c)_{\bar{e}\bar{\mu}}|^2 \simeq L^{-2}\langle P_{\bar{\nu}_\mu \to \bar{\nu}_e} \rangle$$

$$\simeq [(3 \pm 1) \times 10^{-19} \text{ GeV}]^2. \qquad (16)$$

Since the LSND neutrino energy lies in the range 10 MeV $\lesssim$ 50 MeV, this result corresponds to values of the SME coefficients for Lorentz violation of order 10^{-19} GeV for $(a_L)^\mu$ and 10^{-17} for $(c_L)^{\mu\nu}$. Remarkably, these values are in the range predicted for quantum-gravity effects.

Establishing which specific combinations of the 16 possible degrees of freedom could be predominantly responsible for the LSND signal is more challenging and requires information about sidereal and energy dependences. Figure 1 illustrates some of the various possibilities. The probability $P_{\bar{\nu}_\mu \to \bar{\nu}_e}$ is displayed as a function of sidereal time for three situations with distinct combinations of nonzero coefficients $(C)_{\bar{e}\bar{\mu}}$ and $(\mathcal{A}_s)_{\bar{e}\bar{\mu}}$. The probabilities differ in detail, but all yield the result (16).

Other short-baseline experiments [30–35] could perform similar analyses to obtain further information about the space of coefficients for Lorentz violation. If any of
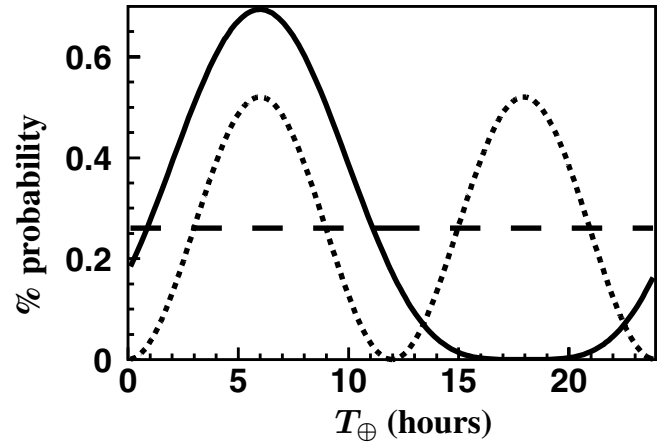

FIG. 1. Variations of the percent probability $P_{\bar{\nu}_\mu \to \bar{\nu}_e}$ over one sidereal day for three sample configurations with averaged probability $\langle P_{\bar{\nu}_\mu \to \bar{\nu}_e} \rangle = 0.26\%$: $(C)_{\bar{e}\bar{\mu}} \neq 0$ (dashed line), $(\mathcal{A}_s)_{\bar{e}\bar{\mu}} \neq 0$ (dotted line), and $(C)_{\bar{e}\bar{\mu}} = (\mathcal{A}_s)_{\bar{e}\bar{\mu}} \neq 0$ (solid line).

these analyses or a reexamination of the LSND data reveals a key signal for Lorentz violation such as the sidereal variations or compass asymmetries, it would constitute convincing evidence for Lorentz violation and could offer the first experimental glimpse of Planck-scale physics.

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