# *CPT* and Lorentz Tests with Muons

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Precision experiments with muons are sensitive to Planck-scale *CPT* and Lorentz violation that is undetectable in other tests. Existing data on the muonium ground-state hyperfine structure and on the muon anomalous magnetic moment could be analyzed to provide dimensionless figures of merit for *CPT* and Lorentz violation at the levels of $4 \times 10^{-21}$ and $10^{-23}$.

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The minimal standard model of particle physics is *CPT* and Lorentz invariant. However, spontaneous breaking of these symmetries may occur in a more fundamental theory incorporating gravity [1,2]. Minuscule low-energy signals of *CPT* and Lorentz breaking could then emerge in experiments sensitive to effects suppressed by the ratio of a low-energy scale to the Planck scale. At presently attainable energies, the resulting effects would be described by a general standard-model extension [3] that allows for *CPT* and Lorentz violation but otherwise maintains conventional properties of quantum field theory, including gauge invariance, renormalizability, and energy conservation.

In the present work, we study the sensitivity of different muon experiments to *CPT* and Lorentz violation. Planck-scale sensitivity to possible effects is known to be attainable in certain experiments without muons. These include, for example, tests with neutral-meson oscillations [4,5], searches for cosmic birefringence [3,6,7], clock-comparison experiments [8,9], comparisons of particles and antiparticles in Penning traps [10,11], spectroscopic comparisons of hydrogen and antihydrogen [12], measurements of the baryon asymmetry [13], and observations of high-energy cosmic rays [14]. However, in the context of the standard-model extension, dominant effects in the muon sector would be disjoint from those in any of the above experiments because the latter involve only photons, hadrons, and electrons. Moreover, if the size of *CPT* and Lorentz violation scales with mass, high-precision experiments with muons would represent a particularly promising approach to detecting lepton-sector effects from the Planck scale.

The standard *CPT* test involving muons compares the $g$ factors for $\mu^-$ and $\mu^+$, with a bound [15,16] given by the figure of merit

$$r_g^\mu \equiv |g_{\mu^+} - g_{\mu^-}|/g_{\rm av} \lesssim 10^{-8}. \qquad (1)$$

We show here that data from experiments normally not associated with *CPT* or Lorentz tests, including muonium microwave spectroscopy [17] and $g-2$ experiments on $\mu^+$ alone [18], can indeed provide Planck-scale sensitivity to *CPT* and Lorentz violation.

For the experiments considered here, it suffices to consider a quantum-electrodynamics limit of the standard-model extension incorporating only muons, electrons, and photons. Other terms in the full standard-model extension would be irrelevant or lead only to subdominant effects. In natural units with $\hbar = c = 1$, the Lorentz-violating Lagrangian terms of interest are

$$\begin{aligned}\mathcal{L} = &-a_{\kappa AB}\bar{l}_A\gamma^\kappa l_B - b_{\kappa AB}\bar{l}_A\gamma_5\gamma^\kappa l_B \\ &- \tfrac{1}{2}H_{\kappa\lambda AB}\bar{l}_A\sigma^{\kappa\lambda}l_B + \tfrac{1}{2}ic_{\kappa\lambda AB}\bar{l}_A\gamma^\kappa \overleftrightarrow{D}^\lambda l_B \\ &+ \tfrac{1}{2}id_{\kappa\lambda AB}\bar{l}_A\gamma_5\gamma^\kappa \overleftrightarrow{D}^\lambda l_B\,. \end{aligned} \qquad (2)$$

Here, the lepton fields are denoted by $l_A$ with $A = 1,2$ corresponding to $e^-$, $\mu^-$, respectively, and $iD_\lambda \equiv i\partial_\lambda - qA_\lambda$ with charge $q = -|e|$. To avoid confusion with four-vector indices, the symbol $\mu$ is reserved in this Letter solely as a label for the muon.

The terms associated with the parameters $a_{\kappa AB}$, $b_{\kappa AB}$ are *CPT* odd, while the others are *CPT* even. All the parameters in Eq. (2) are assumed small, and they all are Hermitian $2 \times 2$ matrices in flavor space. For example,

$$b_\kappa = \begin{pmatrix} b_\kappa^e & b_\kappa^{e\mu} \\ b_\kappa^{\mu e} & b_\kappa^\mu \end{pmatrix}, \qquad (3)$$

where $b_\kappa^e$, $b_\kappa^\mu$ are associated with terms preserving lepton number while the others are associated with terms violating it. Since the usual standard model conserves lepton number, leading-order rates for processes that violate lepton number in the standard-model extension must be quadratic in the flavor-nondiagonal parameters $b_\kappa^{e\mu}$, etc. In contrast, processes violating Lorentz symmetry but preserving lepton number can depend linearly on flavor-diagonal parameters $b_\kappa^e$, $b_\kappa^\mu$, etc. This means that experimental bounds from processes preserving lepton number are typically many orders of magnitude sharper than bounds involving lepton-number violation.

Consider first spectroscopic studies of muonium $M$, which is a $\mu^+$-$e^-$ bound state. In experiments at RAL and LANL, precisions of about 20 ppb have been attained both for the $1S$-$2S$ transition [19] and for the ground-state Zeeman hyperfine transitions [17]. However, we restrict attention here to the latter because the hyperfine transition frequencies are much smaller than the $1S$-$2S$ ones, which implies better absolute energy resolution and corresponding sensitivity to *CPT* and Lorentz violation [20].

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The four hyperfine ground states of $M$ can be labeled 1, 2, 3, 4 in order of decreasing energy. The Zeeman hyperfine transitions $\nu_{12}$, $\nu_{34}$ have been measured in a 1.7 T magnetic field [21] with a precision of about 40 Hz (~20 ppb), and the hyperfine interval has been extracted.

Since electromagnetic transitions in $M$ conserve lepton number, dominant effects in the standard-model extension arise from flavor-diagonal terms in Eq. (2). For the case of an antimuon $\mu^+$, the modified Dirac equation is

$$(i\gamma^\lambda D_\lambda - m_\mu + a^\mu_\lambda \gamma^\lambda - b^\mu_\lambda \gamma_5 \gamma^\lambda + \tfrac{1}{2} H^\mu_{\kappa\lambda}\sigma^{\kappa\lambda} + ic^\mu_{\kappa\lambda}\gamma^\kappa D^\lambda + id^\mu_{\kappa\lambda}\gamma_5\gamma^\kappa D^\lambda)\psi = 0\,, \tag{4}$$

where $\psi$ is a four-component $\mu^+$ field of mass $m_\mu$. A similar equation exists for the $e^-$, containing parameters $a^e_\lambda$, $b^e_\lambda$, $H^e_{\kappa\lambda}$, $c^e_{\kappa\lambda}$, $d^e_{\kappa\lambda}$. The associated Hamiltonians are found using established procedures [11]. The Coulomb potential in $M$ is $A^\lambda = (|e|/4\pi r, \vec{0})$.

The leading-order Lorentz-violating energy shifts in $M$ can be obtained from these Hamiltonians using perturbation theory and relativistic two-fermion techniques [22]. For the four Zeeman hyperfine levels in a 1.7 T magnetic field, we thereby can determine the corresponding shifts $\delta\nu_{12}$, $\delta\nu_{34}$ in the frequencies $\nu_{12}$, $\nu_{34}$. We find

$$\delta\nu_{12} \approx -\delta\nu_{34} \approx -\tilde{b}^\mu_3/\pi\,, \tag{5}$$

where $\tilde{b}^\mu_3 \equiv b^\mu_3 + d^\mu_{30}m_\mu + H^\mu_{12}$. Although in a weak or zero field [23] the results would depend on a combination of both muon and electron parameters for Lorentz violation, only the muon parameters appear in Eq. (5) because in a 1.7 T field the relevant transitions essentially involve pure muon-spin flips. Note that subleading-order Lorentz-violating effects are further suppressed by powers of $\alpha$ or $\mu_\mu B/m_\mu \simeq 5 \times 10^{-15}$ and can therefore be neglected.

Since the laboratory frame rotates with the Earth, and since the frequency shifts (5) depend on spatial components of the parameters for *CPT* and Lorentz violation, the frequencies $\nu_{12}$, $\nu_{34}$ oscillate about a mean value with frequency equal to the Earth's sidereal frequency $\Omega \simeq 2\pi/(23\text{ h }56\text{ m})$. Note that no signal of this type emerges at any perturbative order in the usual standard model without Lorentz violation. Also, the anticorrelation of the variations of $\delta\nu_{12}$ and $\delta\nu_{34}$ could help exclude environmental systematic effects in analyzing real data.

The result (5) could directly be used to place a bound on *CPT* and Lorentz violation in the laboratory frame. However, for purposes of comparison among experiments it is much more useful to work with quantities defined with respect to a nonrotating frame. A suitable choice of basis $\{\hat{X}, \hat{Y}, \hat{Z}\}$ for a nonrotating frame is standard celestial equatorial axes, with the $\hat{Z}$ direction oriented along the Earth's rotational north pole [9]. Then, the laboratory-frame quantity $\tilde{b}^\mu_3$ can be written as

$$\tilde{b}^\mu_3 = \tilde{b}^\mu_Z \cos\chi + (\tilde{b}^\mu_X \cos\Omega t + \tilde{b}^\mu_Y \sin\Omega t)\sin\chi\,, \tag{6}$$

where the nonrotating-frame quantity $\tilde{b}^\mu_J$ with $J = X, Y, Z$ is defined by $\tilde{b}^\mu_J \equiv b^\mu_J + m_\mu d^\mu_{J0} + \frac{1}{2}\epsilon_{JKL}H^\mu_{KL}$, and where $\chi$ is the angle between $\hat{Z}$ and the quantization axis defined by the laboratory magnetic field.

Suppose, for definiteness, that a reanalysis of the data in Ref. [17] using time stamps on the frequency measurements places a bound of 100 Hz on the amplitude of sidereal variations $\delta\nu_{12}$. In terms of nonrotating-frame components, this corresponds to the constraint

$$|\sin\chi|\sqrt{(\tilde{b}^\mu_X)^2 + (\tilde{b}^\mu_Y)^2} \lesssim 2 \times 10^{-22}\ \text{GeV}\,. \tag{7}$$

An appropriate dimensionless figure of merit for this result is the ratio $(r^\mu_{\rm hf})_{\rm sidereal}$ of the amplitude of energy variations to the relativistic energy of $M$. The bound (7) gives

$$(r^\mu_{\rm hf})_{\rm sidereal} \approx 2\pi|\delta\nu_{12}|/m_\mu \approx 2\pi|\delta\nu_{34}|/m_\mu \lesssim 4 \times 10^{-21}, \tag{8}$$

which is comparable to the dimensionless ratio of the $\mu^+$ mass to the Planck scale $M_P$, $m_\mu/M_P \simeq 10^{-21}$.

We consider next measurements of the muon anomalous magnetic moment [15,16,18]. The most recent experiment [18] measures the angular anomaly frequency $\omega_a$, which is the difference between the spin-precession frequency $\omega_s$ and the cyclotron frequency $\omega_c$. This BNL experiment uses relativistic polarized $\mu^+$ moving in a constant 1.45 T magnetic field. The $\mu^+$ have momentum $p \simeq 3.09$ GeV and "magic" $\gamma \simeq 29.3$, which eliminates the dependence of $\omega_a$ on the electric field. Positrons from the decay $\mu^+ \to e^+ + \nu_e + \bar{\nu}_\mu$ are detected and their decay spectrum is fitted to a specified time function. The anomaly frequency $\omega_a$, which in conventional theory is proportional to $(g-2)/2$, is measured to about 10 ppm. An accuracy below 1 ppm is expected in the near future.

In the standard-model extension, the relativistic Hamiltonian for a $\mu^+$ with an anomalous magnetic moment in a magnetic field $\vec{B}$ is

$$\hat{H} = \gamma^0\vec{\gamma}\cdot\vec{\pi} + m_\mu(1 - c^\mu_{00})\gamma^0 + \tfrac{1}{2}(g-2)\mu_\mu\gamma^0\vec{\Sigma}\cdot\vec{B} - a^\mu_0 - (c^\mu_{0j} + c^\mu_{j0})\pi^j - [b^\mu_0 + (d^\mu_{0j} + d^\mu_{j0})\pi^j]\gamma_5 - [a^\mu_j + (c^\mu_{jk} + c^\mu_{00}\delta_{jk})\pi^k]\gamma^0\gamma^j - iH^\mu_{0j}\gamma^j - [b^\mu_j + (d^\mu_{jk} + d^\mu_{00}\delta_{jk})\pi^k]\gamma_5\gamma^0\gamma^j + [\tfrac{1}{2}\varepsilon_{jkl}H^\mu_{kl} + md^\mu_{j0}]\gamma_5\gamma^j, \tag{9}$$

where $\Sigma^j = \gamma_5\gamma^0\gamma^j$, $\mu_\mu$ is the muon magneton, and $\vec{\pi} = \vec{p} - q\vec{A}$, with $q = +|e|$ for $\mu^+$. This Hamiltonian contains no terms that provide leading-order corrections to the $g$ factors for $\mu^+$ or $\mu^-$. Instead, the dominant sensitivity to *CPT* violation results from the sensitivity to small frequency shifts associated with the spin precession. The conventional figure of merit $r^\mu_g$ therefore is zero at leading order despite the presence of explicit *CPT* violation, which means alternative figures of merit are needed [11].

A Foldy-Wouthuysen transformation [24] can be used to convert the Hamiltonian $\hat{H}$ to another Hamiltonian $\hat{H}'$ in which the $2 \times 2$ off-diagonal blocks contain only first-order terms in the magnetic field $\vec{B}$ [25] and in the parameters for *CPT* and Lorentz violation. We find $\hat{H}' = \exp(\gamma^0\gamma_5\phi)\hat{H}\exp(-\gamma^0\gamma_5\phi)$, with $\tan 2\phi = |\vec{\Sigma}\cdot\vec{\pi}|/m_\mu$ and $|\vec{\Sigma}\cdot\vec{\pi}|^2 = \vec{\pi}^2 - q\vec{\Sigma}\cdot\vec{B}$. The off-diagonal blocks in $\hat{H}'$ are irrelevant at leading order since here they produce effects that are at least quadratic in small parameters.

The upper-left $2 \times 2$ block of $\hat{H}'$ is the relevant relativistic Hamiltonian for the $\mu^+$ in the laboratory frame [26]. It has the form

$$\hat{H}' = \mathcal{E}_0 + \mathcal{E}_1 + \tfrac{1}{2}\vec{\sigma}\cdot(\vec{\omega}_{s,0} + f_1\vec{\beta} + \vec{f}_2), \tag{10}$$

where $\mathcal{E}_0 = \gamma m$ and $\gamma = (1 - \beta^2)^{-1/2}$ with three-velocity $\vec{\beta}$. The term $\mathcal{E}_1$ contains irrelevant spin-independent corrections. The quantity $\vec{\omega}_{s,0} = (g - 2 + 2/\gamma)\mu_\mu\vec{B}$ is the usual spin-precession frequency. The term $f_1\vec{\beta}$ is proportional to $\vec{\beta}$, and its contributions average to zero since the detectors in the $(g - 2)$ experiments are spread around the ring and their data are summed. The term $\vec{f}_2$ depends on the parameters for *CPT* and Lorentz violation and partially on $\vec{\beta}$, but again only the $\vec{\beta}$-independent terms are relevant here.

The spin-precession frequency $\omega_s$ is calculated as $\frac{d\vec{\sigma}}{dt} = i[\hat{H}', \vec{\sigma}] = \vec{\omega}_s \times \vec{\sigma}$. Since the detectors are in the $\hat{x}$-$\hat{y}$ plane in the laboratory frame, only the vertical component $\omega_s$ is measured. Substituting for $\hat{H}'$ and keeping only the velocity-independent terms along the $\hat{z}$ direction gives for $\mu^+$ the result $\omega_s \approx \omega_{s,0} + 2\check{b}_3^\mu$, where $\check{b}_3^\mu \equiv b_3^\mu/\gamma + m_\mu d_{30}^\mu + H_{12}^\mu$. Note that $\check{b}_3^\mu$ reduces to $\tilde{b}_3^\mu$ in the nonrelativistic limit [27].

The cyclotron frequency $\omega_c$ is obtained from $[\hat{H}', \vec{r}] = \vec{\pi}/\mathcal{E}_0$, which contains a term $\vec{\omega}_c \times \vec{r}$. However, no leading-order corrections to the usual cyclotron frequency appear: $\omega_c \approx \omega_{c,0} = 2\mu_\mu B/\gamma$. Subleading-order terms do in fact contribute but are of lower order than those in $\omega_s$ and therefore can be ignored.

Combining the above results and converting to the nonrotating frame as in Eq. (6), we find the correction to the $\mu^+$ anomaly frequency $\omega_a = \omega_s - \omega_c$ due to *CPT* and Lorentz violation is

$$\delta\omega_a^{\mu^+} \approx 2\check{b}_Z^\mu\cos\chi + 2(\check{b}_X^\mu\cos\Omega t + \check{b}_Y^\mu\sin\Omega t)\sin\chi, \tag{11}$$

where $\chi$ is now the colatitude of the experiment. The corresponding expression $\delta\omega_a^{\mu^-}$ for $\mu^-$ is obtained by the substitution $b_J^\mu \rightarrow -b_J^\mu$ in the expressions for $\check{b}_X, \check{b}_Y, \check{b}_Z$.

These results suggest two interesting types of experimental signal. The first involves the difference $\Delta\omega_a^\mu \equiv \delta\omega_a^{\mu^+} - \delta\omega_a^{\mu^-}$, which is $\Delta\omega_a^\mu \approx 4b_3^\mu/\gamma$ in the laboratory frame [28]. It is impractical to measure $g - 2$ for both $\mu^+$ and $\mu^-$ simultaneously, so instead one can directly consider the time-averaged difference $\overline{\Delta\omega_a^\mu}$. In the nonrotating frame,

$$\overline{\Delta\omega_a^\mu} \approx \frac{4}{\gamma}\, b_Z^\mu \cos\chi\,. \tag{12}$$

An appropriate figure of merit $r_{\Delta\omega_a}^\mu$ here is the relative energy difference between $\mu^+$ and $\mu^-$ caused by their different spin precessions:

$$r_{\Delta\omega_a}^\mu \approx \overline{\Delta\omega_a^\mu}/m_\mu\,. \tag{13}$$

The CERN $g - 2$ experiments compared average $\mu^+$ and $\mu^-$ anomaly frequencies, finding [15] $\overline{\Delta\omega_a^\mu}/2\pi \simeq 5 \pm 3$ Hz. This gives a value of $r_{\Delta\omega_a}^\mu$ on the order of $2 \times 10^{-22}$, corresponding to $b_Z^\mu \simeq (2 \pm 1) \times 10^{-22}$ GeV. A subsequent measurement at BNL [18] provides a $\mu^+$ result within 1 standard deviation of the CERN $\mu^-$ result. If the BNL experiment eventually limits the frequency difference to 1 ppm, it would provide a sensitivity at the level of $r_{\Delta\omega_a}^\mu \lesssim 10^{-23}$, corresponding to $b_Z^\mu \lesssim 10^{-23}$ GeV.

The second interesting type of experimental signal involves sidereal variations in the anomaly frequency. It can be studied using $\mu^+$ alone, in which case time stamps on frequency measurements would permit a bound on sidereal variations of $\omega_a^{\mu^+}$. An appropriate figure of merit $(r_{\omega_a}^\mu)_{\text{sidereal}}$ is the relative size of the amplitude of energy variations compared to the total energy. Assuming a precision of 1 ppm, we estimate an attainable bound of

$$(r_{\omega_a}^\mu)_{\text{sidereal}} \approx |\delta\omega_a^{\mu^+}|/m_\mu \lesssim 10^{-23}. \tag{14}$$

The associated bound on parameters in the nonrotating frame is

$$|\sin\chi|\sqrt{(\check{b}_X^\mu)^2 + (\check{b}_Y^\mu)^2} \lesssim 5 \times 10^{-25}\ \text{GeV}\,, \tag{15}$$

which again represents sensitivity to the Planck scale. Note that this test involves different sensitivity to *CPT* violation than the previous one: the two figures of merit $r_{\Delta\omega_a}^\mu$, $(r_{\omega_a}^\mu)_{\text{sidereal}}$ depend on independent components of parameters for *CPT* and Lorentz violation.

In addition to effects in flavor-diagonal processes, off-diagonal terms of the type in Eq. (3) arising in the standard-model extension allow Lorentz-violating contributions to flavor-changing processes. For example, precision searches have been performed for the radiative muon decay $\mu \rightarrow e\gamma$, which has a branching ratio below $5 \times 10^{-11}$ [29]. This decay has previously been analyzed using a *CPT*- and rotation-invariant model with Lorentz and lepton-number violation that involves terms equivalent (up to field renormalizations) to those of the form $c_{00}^{e\mu}$ and $d_{00}^{e\mu}$ in Eq. (2) [14]. The results of this analysis indicate that combinations of the dimensionless parameters $c_{00}^{e\mu}$ and $d_{00}^{e\mu}$ are bounded at the level of about $10^{-12}$ by rest-frame muon decays or by muon lifetime measurements in the CERN $g - 2$ experiments, and at the level of about $10^{-19}$ by constraints from horizontal air showers of cosmic-ray muons. As expected from the discussion following Eq. (3), these bounds are several orders of magnitude weaker than those from lepton-number preserving processes. An extension of this analysis to include all types of term in Eq. (2) would provide the best existing bounds on the flavor-nondiagonal parameters in the electron-muon sector of the standard-model extension. Useful constraints on these parameters could also be extracted from other future experiments. These include the proposed tests for muon-electron conversion [30], which have an estimated sensitivity to the process $\mu^- + N \rightarrow e^- + N$ of $2 \times 10^{-17}$, and the various precision tests that might be envisaged at a future muon collider.

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