

CPT and Lorentz tests in Penning traps

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A theoretical analysis is performed of Penning-trap experiments testing *CPT* and Lorentz symmetry through measurements of anomalous magnetic moments and charge-to-mass ratios. Possible *CPT* and Lorentz violations arising from spontaneous symmetry breaking at a fundamental level are treated in the context of a general extension of the $SU(3)\times SU(2)\times U(1)$ standard model and its restriction to quantum electrodynamics. We describe signals that might appear in principle, introduce suitable figures of merit, and estimate *CPT* and Lorentz bounds attainable in present and future Penning-trap experiments. Experiments measuring anomaly frequencies are found to provide the sharpest tests of *CPT* symmetry. Bounds are attainable of approximately 10^{-20} in the electron-positron case and of 10^{-23} for a suggested experiment with protons and antiprotons. Searches for diurnal frequency variations in these experiments could also limit certain types of Lorentz violation to the level of 10^{-18} in the electron-positron system and others at the level of 10^{-21} in the proton-antiproton system. In contrast, measurements comparing cyclotron frequencies are sensitive within the present theoretical framework to different kinds of Lorentz violation that preserve *CPT*. Constraints could be obtained on one figure of merit in the electron-positron system at the level of 10^{-16} , on another in the proton-antiproton system at 10^{-24} , and on a third at 10^{-25} using comparisons of H^- ions with antiprotons.

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I. INTRODUCTION

Invariance under the combined discrete symmetry *CPT* is a fundamental symmetry of the $SU(3)\times SU(2)\times U(1)$ standard model and of quantum electrodynamics. The *CPT* theorem [1] predicts that various quantities such as masses, lifetimes, charge-to-mass ratios, and gyromagnetic ratios are equal for particles and antiparticles. Typically, experimental tests of *CPT* are comparative measurements of one or more of these quantities for a particular particle and antiparticle [2].

Several high-precision tests of this type have been performed in experiments confining single particles or antiparticles in a Penning trap for indefinite times. A comparison of the electron and positron gyromagnetic ratios can be obtained from measurements of their cyclotron and anomaly frequencies [3,4], producing the bound

$$r_g \equiv |(g_- - g_+)/g_{av}| \leq 2 \times 10^{-12}, \quad (1)$$

where g_- and g_+ denote the electron and positron g factors, respectively. Similarly, measurements of the proton and antiproton cyclotron frequencies allow a comparison of their charge-to-mass ratios [5]. The result can be presented as the bound

$$r_{q/m}^p \equiv |[(q_p/m_p) - (q_{\bar{p}}/m_{\bar{p}})]/(q/m)_{av}| \leq 1.5 \times 10^{-9}. \quad (2)$$

Analogous experiments performed with electrons and positrons [6] yield the bound

$$r_{q/m}^e \equiv |[(q_{e^-}/m_{e^-}) - (q_{e^+}/m_{e^+})]/(q/m)_{av}| \leq 1.3 \times 10^{-7}. \quad (3)$$

It has recently been shown that the conventional figure of merit r_g of Eq. (1) can provide a misleading measure of *CPT* violation in $g-2$ experiments [7]. In the context of a general theoretical framework that describes possible *CPT*- and Lorentz-violating effects in an extension of the $SU(3)\times SU(2)\times U(1)$ standard model and in quantum electrodynamics [8], the predicted value of r_g is zero whether or not *CPT* is violated. However, an alternative figure of merit that is sensitive to *CPT* violation does exist, and it could be bounded to 1 part in 10^{20} with existing technology [7].

In the present work, we generalize this analysis to a larger class of experiments on charged fermions confined within a Penning trap, including comparative measurements of anomaly and cyclotron frequencies in the electron-positron, proton-antiproton, and H^- -antiproton systems. Since the dominant interactions are electromagnetic, we consider the pure-fermion sector of a *CPT*- and Lorentz-violating extension of quantum electrodynamics [8] emerging as a limit of the general standard-model extension. This broadens the scope relative to that of Ref. [7], since it also includes terms breaking Lorentz symmetry but preserving *CPT*.

Our primary goal is to determine the sensitivity of the Penning-trap experiments to possible *CPT*- and Lorentz-violating effects in the extension of quantum electrodynamics. We investigate the suitability of the conventional figures of merit as measures of *CPT* violation. Where necessary, more appropriate figures of merit and corresponding experiments are suggested. Estimates are also made of the magnitude of bounds accessible to experiments with existing technology.

Section II introduces various topics necessary for the analysis, including descriptions of the relevant *CPT*- and

Lorentz-violating terms, issues concerning their perturbative treatment in Penning-trap experiments, and the possible signals they might engender. Section III considers experiments with electrons and positrons and contains three subsections: one describing theoretical issues, one discussing experiments on anomalous magnetic moments, and one treating experiments on charge-to-mass ratios. Section IV is concerned with protons and antiprotons and has a similar structure, but includes a fourth subsection treating experiments with hydrogen ions. We summarize in Sec. V.

II. BASICS

A. Theoretical framework

The framework for the extension of the $SU(3)\times SU(2)\times U(1)$ standard model and quantum electrodynamics originates from the idea of spontaneous CPT and Lorentz breaking in a more fundamental model such as string theory [9,10]. It lies within the context of conventional quantum field theory and appears to preserve various desirable features of the standard model such as gauge invariance, power-counting renormalizability, and microcausality. Possible violations of CPT and Lorentz symmetry are parametrized by quantities that can be bounded by experiments, including interferometric tests with neutral mesons [9,11,12] as well as the $g-2$ comparisons mentioned above. There are also implications for baryogenesis [13].

Within this framework, the modified Dirac equation obeyed by a four-component spinor field ψ describing a particle with charge q and mass m is given by

$$\left(i\gamma^\mu D_\mu - m - a_\mu \gamma^\mu - b_\mu \gamma_5 \gamma^\mu - \frac{1}{2} H_{\mu\nu} \sigma^{\mu\nu} + i c_{\mu\nu} \gamma^\mu D^\nu + i d_{\mu\nu} \gamma_5 \gamma^\mu D^\nu \right) \psi = 0. \quad (4)$$

Here, $iD_\mu \equiv i\partial_\mu - qA_\mu$, with A^μ being the electromagnetic potential. The quantities a_μ , b_μ , $H_{\mu\nu}$, $c_{\mu\nu}$, $d_{\mu\nu}$ are real and act as effective coupling constants, with $H_{\mu\nu}$ antisymmetric and $c_{\mu\nu}$, $d_{\mu\nu}$ traceless. Some properties of these quantities are discussed in Ref. [8]. For our present purposes, it suffices to note that the transformation properties of ψ imply that the terms involving a_μ , b_μ break CPT while those involving $H_{\mu\nu}$, $c_{\mu\nu}$, $d_{\mu\nu}$ preserve it, and that Lorentz invariance is broken by all five terms.

Since no CPT or Lorentz breaking has been observed to date, the quantities a_μ , b_μ , $H_{\mu\nu}$, $c_{\mu\nu}$, $d_{\mu\nu}$ must all be small. Within the framework of spontaneous CPT and Lorentz breaking arising from a more fundamental model, a natural suppression scale for these quantities is the ratio of a light scale m_l to a scale of order of the Planck mass M . For example, this could range from $m_l/M \approx 5 \times 10^{-23}$ for $m_l \approx m_e$ to $m_l/M \approx 3 \times 10^{-17}$ for $m_l \approx 250$ GeV, the latter being roughly the electroweak scale. Since in natural units with $\hbar = c = 1$ the quantities a_μ , b_μ , $H_{\mu\nu}$ have dimensions of mass while $c_{\mu\nu}$, $d_{\mu\nu}$ are dimensionless, it is plausible that a_μ , b_μ , $H_{\mu\nu}$ might be of order $m_l m/M$, while $c_{\mu\nu}$, $d_{\mu\nu}$ might be of order m_l/M .

B. Application to the Penning trap

The effects of the small quantities a_μ , b_μ , $H_{\mu\nu}$, $c_{\mu\nu}$, $d_{\mu\nu}$ can be determined within a perturbative framework in relativistic quantum mechanics, with A_μ chosen as an appropriate background potential. The first step is therefore to extract a suitable quantum-mechanical Hamiltonian from Eq. (4).

The appearance of time-derivative couplings in Eq. (4) means that the standard procedure fails to produce a Hermitian quantum-mechanical Hamiltonian operator generating time translations on the wave function. This technical difficulty can be overcome in several ways. The simplest method is to perform a field redefinition at the Lagrangian level, chosen to eliminate the additional time derivatives. In this case, we find that the appropriate redefinition is

$$\psi \equiv \left(1 - \frac{1}{2} c_{\mu 0} \gamma^0 \gamma^\mu - \frac{1}{2} d_{\mu 0} \gamma^0 \gamma_5 \gamma^\mu \right) \chi. \quad (5)$$

Rewriting the Lagrangian in terms of the new field χ cannot affect the physics. However, the quantum-mechanical Dirac wave function corresponding to χ does have conventional time evolution. The physics associated with the original time-derivative couplings is reflected instead in additional interactions in the rewritten Dirac Hamiltonian, appearing as a consequence of the redefinition (5).

We denote the Dirac wave function corresponding to the field χ by χ^q , where $q \equiv e^-$ for a trapped electron and $q \equiv p$ for a trapped proton. The corresponding quantum-mechanical Dirac Hamiltonian is denoted \hat{H}^q . The rewritten Dirac equation then takes the form

$$i\partial_0 \chi^q = \hat{H}^q \chi^q. \quad (6)$$

This equation remains invariant under gauge transformations involving χ^q and A_μ .

Loop effects arising at the level of the quantum field theory imply that the true quantum-mechanical Dirac Hamiltonian is the sum of \hat{H}^q and other terms that could be constructed in an effective-action approach. In the present work, we are interested in leading-order effects in the CPT - and Lorentz-violating quantities a_μ , b_μ , $H_{\mu\nu}$, $c_{\mu\nu}$, $d_{\mu\nu}$. We therefore work in the context of an effective quantum-mechanical Hamiltonian \hat{H}_{eff}^q that by definition incorporates all-orders quantum corrections in the fine-structure constant induced from the quantum field theory but that keeps only first-order terms in CPT - and Lorentz-breaking quantities. For perturbative calculations, we then write

$$\hat{H}_{\text{eff}}^q = \hat{H}_0^q + \hat{H}_{\text{pert}}^q, \quad (7)$$

where \hat{H}_0^q is a conventional Dirac Hamiltonian representing a charged particle in a Penning trap in the absence of CPT - and Lorentz-violating perturbations but including quantum corrections such as an anomaly term. The perturbative Hamiltonian \hat{H}_{pert}^q and its analogue $\hat{H}_{\text{pert}}^{\bar{q}}$ for the antiparticle are both linear in the CPT - and Lorentz-breaking quantities a_μ , b_μ , $H_{\mu\nu}$, $c_{\mu\nu}$, $d_{\mu\nu}$.

In a Penning trap, a strong magnetic field along the axis of the trap provides the primary radial confinement while axial

trapping is imposed with a quadrupole electric field. The presence of the electric field induces a shift in the physical cyclotron frequency relative to its value ω_c in the pure magnetic field, but an invariance relation [4] permits the value of ω_c to be deduced directly from measurements of the physical cyclotron, axial, and magnetron frequencies in the trap. The measurements are complicated in practice by various experimental issues [14]. These include the disentanglement of induced couplings between the axial and cyclotron motions, the elimination of cyclotron-frequency shifts due to resonances with cavity modes inside the trap, and the treatment of temporal drifts in the trapping fields. Various techniques have been developed for controlling the latter, with accuracies of parts per 10^9 attained in frequency measurements [3,15].

For the experiments of interest here, the dominant contributions to the energy spectrum arise from the interaction of the particle or antiparticle with the constant magnetic field of the trap. Except for certain situations discussed in Sec. III A below, the quadrupole electric and other fields generate smaller effects. In a perturbative calculation, the dominant corrections due to *CPT*- and Lorentz-violating effects can therefore be obtained by taking A_μ as the potential for a constant magnetic field only. Since the signals of interest are energy-level shifts rather than transition probabilities, this means it suffices to use relativistic Landau-level wave functions as the unperturbed basis set and to calculate within first-order perturbation theory in \hat{H}_{pert}^q or $\hat{H}_{\text{pert}}^{\bar{q}}$. However, the unperturbed energy levels must be taken as the relativistic Landau levels shifted by an anomaly term and other quantum corrections.

As usual, the spin-up and spin-down states form two ladders of levels. The anomalous magnetic moment of the trapped particle breaks the degeneracy of the excited states. The energy-level ladder pairs for particles and antiparticles are similar, except that spin labels are reversed. Let the level number be labeled by $n=0,1,2,3,\dots$ and the spin by $s=\pm 1$. We denote the relativistic Landau-level wave functions for the particle and antiparticle by $\chi_{n,s}^q$ and $\chi_{n,s}^{\bar{q}}$, respectively. The corresponding energy levels, including the anomaly shift and all conventional perturbative effects, are denoted $E_{n,s}^q$ and $E_{n,s}^{\bar{q}}$. Corrections to these energy levels due to *CPT* and Lorentz breaking are denoted by $\delta E_{n,s}^q$ and $\delta E_{n,s}^{\bar{q}}$ and are well approximated by

$$\delta E_{n,s}^q = \int \chi_{n,s}^{q\dagger} \hat{H}_{\text{pert}}^q \chi_{n,s}^q d^3r, \quad \delta E_{n,s}^{\bar{q}} = \int \chi_{n,s}^{\bar{q}\dagger} \hat{H}_{\text{pert}}^{\bar{q}} \chi_{n,s}^{\bar{q}} d^3r. \quad (8)$$

In what follows, the exact physical energies incorporating all perturbative corrections are denoted $\mathcal{E}_{n,s}^q$ and $\mathcal{E}_{n,s}^{\bar{q}}$. For calculational definiteness in the subsequent sections, we orient the instantaneous coordinate system so that the magnetic field $\vec{B} = B\hat{z}$ lies along the positive z axis, and we choose the gauge $A^\mu = (0, -yB, 0, 0)$.

To lowest order in the fine-structure constant, we find that the perturbative Hamiltonian \hat{H}_{pert}^q for a particle is

$$\begin{aligned} \hat{H}_{\text{pert}}^q = & a_\mu \gamma^0 \gamma^\mu - b_\mu \gamma_5 \gamma^0 \gamma^\mu - c_{00} m \gamma^0 - i(c_{0j} + c_{j0}) D^j \\ & + i(c_{00} D_j - c_{jk} D^k) \gamma^0 \gamma^j - d_{j0} m \gamma_5 \gamma^j \\ & + i(d_{0j} + d_{j0}) D^j \gamma_5 + i(d_{00} D_j - d_{jk} D^k) \gamma^0 \gamma_5 \gamma^j \\ & + \frac{1}{2} H_{\mu\nu} \gamma^0 \sigma^{\mu\nu}. \end{aligned} \quad (9)$$

For the antiparticle, the Dirac wave function $\chi^{\bar{q}}$ and Hamiltonian $\hat{H}^{\bar{q}}$ can be found via charge conjugation. Experimental procedures for replacing particles with antiparticles in Penning traps typically reverse the electric field but leave unchanged the magnetic field described by A_μ . We therefore choose the same potential A_μ in the Dirac Hamiltonians for the particle and antiparticle. The resulting perturbative Hamiltonian $\hat{H}_{\text{pert}}^{\bar{q}}$ for an antiparticle is

$$\begin{aligned} \hat{H}_{\text{pert}}^{\bar{q}} = & -a_\mu \gamma^0 \gamma^\mu - b_\mu \gamma_5 \gamma^0 \gamma^\mu - c_{00} m \gamma^0 - i(c_{0j} + c_{j0}) D^j \\ & + i(c_{00} D_j - c_{jk} D^k) \gamma^0 \gamma^j + d_{j0} m \gamma_5 \gamma^j \\ & - i(d_{0j} + d_{j0}) D^j \gamma_5 - i(d_{00} D_j - d_{jk} D^k) \gamma^0 \gamma_5 \gamma^j \\ & - \frac{1}{2} H_{\mu\nu} \gamma^0 \sigma^{\mu\nu}. \end{aligned} \quad (10)$$

Here, the covariant derivative is given as $iD_\mu = i\partial_\mu - (-q)A_\mu$, as is appropriate for an antiparticle of charge $-q$.

In the above discussion, the electromagnetic potential A_μ is treated as the usual classical background field solving the conventional Maxwell equations. In principle, effects beyond those considered here might arise from possible *CPT*- and Lorentz-breaking modifications of the Maxwell equations [8]. A plausible argument indicates that any changes directly involving the potential A_μ would be irrelevant in the situations considered here and that the source for the extended classical theory would still be the classical current density, in which case a uniform magnetic field can be produced by conventional experimental techniques and the results we obtain below are unaffected. In any event, a detailed treatment of these issues lies outside the scope of the present work.

C. Experimental signatures

In high-precision comparative tests using nonrelativistic particles or antiparticles confined in a Penning trap, the relevant experimental observables are frequencies. The effects requiring theoretical investigation are therefore possible energy-level shifts, which can be obtained in perturbation theory using Eq. (8). This subsection contains some general comments on features to be expected and corresponding experimental signatures.

In the present context, the perturbative corrections to a given energy level could in principle depend on several variables, including the quantum numbers of the state, the strength of the applied field, and its orientation. Indeed, all of these appear in the calculational results presented below.

A given energy level lies in one of four stacks of levels, according to whether the state describes a particle or antiparticle and whether it has spin up or spin down. Comparative

tests sensitive to *CPT*- and Lorentz-breaking effects could involve either states from different stacks or states from a given stack. For instance, one possible effect involving different stacks is a relative energy shift between particle states of one spin and antiparticle states of the opposite spin. The *CPT* theorem predicts that this difference should vanish, assuming the trap magnetic field is the same for the particle and antiparticle cases. A possible effect involving states within a given stack is an energy shift that varies with spatial orientation. This would conventionally be excluded by the rotational component of Lorentz symmetry.

The various types of *CPT*- and Lorentz-violating effects might in principle produce several kinds of observable signal in Penning-trap experiments. For example, comparative measurements of anomaly frequencies could reveal the presence of energy-level shifts that differ between particles and antiparticles. Another possibility associated with level shifts depending on spatial orientation is the occurrence of cyclic time variations in either the cyclotron or anomaly frequencies. The point is that for a given experiment the magnetic field of the Penning trap establishes a spatial orientation and hence defines an instantaneous coordinate system. This coordinate system rotates as the Earth does, and so certain non-vanishing components of the quantities a_μ , b_μ , $H_{\mu\nu}$, $c_{\mu\nu}$, $d_{\mu\nu}$ could have values that appear to vary diurnally with a definite period determined by the associated multipolarity. Note that observing an effect would require the absence of corresponding diurnal variations of the magnetic field, which might conceivably arise from diurnal variations of the source in the effective classical Maxwell equations. We disregard this possibility in what follows. Note also that the magnitude of any signal would be affected by various geometrical factors, including the latitude at which the experiment is performed and a projection of the observable onto the equatorial plane of the Earth. For the order-of-magnitude estimates of bounds obtained in the sections that follow, we treat these factors as being of order one.

Since experiments measure frequencies rather than energy levels, observable signals can only arise from differential energy-level shifts, i.e., shifts producing changes in spacings between pairs of levels. Furthermore, experiments involving comparisons of frequencies between two systems are sensitive only to double-differential level shifts, i.e., level shifts that produce *different* frequency shifts for each system. The requirement of differential or double-differential level shifts for the generation of observable signals means that any given Penning-trap experiment is expected to be sensitive to only a subset of the possible *CPT*- and Lorentz-breaking effects described by Eq. (4). This is confirmed by explicit calculation, as is shown in the following sections. In particular, since the conventional figures of merit r_g , $r_{q/m}^p$, $r_{q/m}^e$ discussed in the Introduction are defined directly as comparative measures of fundamental quantities, it is unclear *a priori* whether they are sensitive to any *CPT*- and Lorentz-breaking effects and hence whether they are appropriate measures of invariance. This question is also addressed in the following sections.

As an important example illustrating the issue of *CPT* sensitivity, consider experiments involving comparative measurements of cyclotron frequencies of a particle and antiparticle. In the absence of a definite theoretical framework,

it might be expected *a priori* that these could reveal *CPT*-violating energy-level shifts. As described above, a *CPT*-breaking signal would require double-differential level shifts. However, there is a further constraint: in the idealized comparative experiment the particle and antiparticle anomaly and cyclotron frequencies are related not only by *CPT* but also by *CT*, which means that their comparison is sensitive only to *CPT*-violating effects that also break *CT*.

In the context of the present theoretical framework, the only terms in Eq. (4) breaking both *CPT* and *CT* are those involving the quantities a_0 and \vec{b} . It has previously been shown [8,7] that corrections involving a_μ can be reinterpreted via a redefinition of the zeros of energy and momentum, $E \rightarrow E - a_0$ and $\vec{p} \rightarrow \vec{p} - \vec{a}$, in the dispersion relation for $E_{n,s}^q(\vec{p})$. Since all energy-level spacings and hence the anomaly and cyclotron frequencies remain unaffected, these four-momentum shifts have no measurable effects even though the particle and antiparticle shifts are of opposite sign. All observable quantities in Penning-trap experiments are therefore independent of a_μ . To show this explicitly, a_μ is kept in the calculations that follow.

These results imply that leading-order comparisons of particle and antiparticle anomaly and cyclotron frequencies can at most depend on \vec{b} . However, the leading-order effect of a nonzero \vec{b} is to shift by a constant the energy of all states with one spin relative to those with the other [8,7]. This means that at leading order a nonzero \vec{b} is expected to modify anomaly-frequency comparisons but leaves unaffected cyclotron-frequency comparisons. In particular, it follows that comparisons of particle and antiparticle cyclotron frequencies are insensitive to all leading-order *CPT*-violating effects within the present theoretical framework.

Using a related argument, comparative Penning-trap experiments searching for Lorentz-violating but *CPT*-preserving effects can be shown to be sensitive only to effects that also preserve *CT* and that couple differentially to the spin. In the present framework, the corresponding parameters are H_{jk} , d_{0j} , and d_{j0} . Furthermore, a field redefinition can be found that at first order in the Lorentz-breaking parameters allows H_{jk} to be absorbed into the antisymmetric component of d_{j0} [8]. Physical effects in the present case must therefore involve only a particular linear combination of H_{jk} and d_{j0} . All the above results for comparative experiments are confirmed by the calculations that follow.

Another interesting issue is the relative sensitivity to possible *CPT* and Lorentz violation of Penning-trap versus various other experiments. Addressing this would require a detailed study of the latter in the context of the present theoretical framework and lies well outside the scope of the present work. We note, however, that the analyses in Refs. [7,8,11] and the following sections show that certain comparative Penning-trap measurements produce *CPT* bounds similar in precision to those from experiments on neutral-meson oscillations, widely regarded as the best available *CPT* limits [2]. The analysis in the present work also suggests that the Penning-trap sensitivity to possible Lorentz violation is likely to compare favorably with many tests of special relativity. A few such tests, including experiments of

the Hughes-Drever type [16], are believed under suitable circumstances to provide exceptionally sensitive measures of certain kinds of Lorentz violation, although care is required with interpretation of the results within specific models [17]. With some theoretical assumptions, these experiments might place correspondingly stringent bounds on the parameters of interest here. This issue is being investigated in a separate work.

III. ELECTRONS AND POSITRONS

In this section, we consider some tests of *CPT* and Lorentz violation involving comparative experiments with single electrons or positrons confined in a Penning trap. The treatment is separated into three subsections, one describing calculations of energy-level and frequency shifts, one for experiments on anomalous magnetic moments, and one for experiments on charge-to-mass ratios.

A. Theory

The Dirac Hamiltonian \hat{H}^{e^-} describing the electron is identified with \hat{H}^q of Eq. (6), while for positrons $\hat{H}^{e^+} \equiv \hat{H}^{\bar{q}}$. The energy levels without *CPT*- and Lorentz-violating perturbations are denoted $E_{n,s}^{e^-}$ and $E_{n,s}^{e^+}$. The corresponding electron cyclotron and anomaly frequencies are defined as

$$\omega_c = E_{1,-1}^{e^-} - E_{0,-1}^{e^-}, \quad \omega_a = E_{0,+1}^{e^-} - E_{1,-1}^{e^-}. \quad (11)$$

By the *CPT* theorem, they have the same values as those of the positron.

To distinguish the quantities parametrizing *CPT* and Lorentz breaking for electrons and positrons from those for other particles introduced below, we add superscripts: a_μ^e , b_μ^e , $H_{\mu\nu}^e$, $c_{\mu\nu}^e$, $d_{\mu\nu}^e$. The dominant energy-level corrections that are first order in these quantities can be calculated using Eq. (8). For electrons, we find

$$\begin{aligned} \delta E_{n,\pm 1}^{e^-} = & a_0^e + a_3^e \frac{p_z}{E_{n,\pm 1}^{e^-}} \mp b_3^e \left(1 - \frac{(2n+1\pm 1)|eB|}{E_{n,\pm 1}^{e^-}(E_{n,\pm 1}^{e^-} + m_e)} \right) \mp b_0^e \frac{p_z}{E_{n,\pm 1}^{e^-}} - c_{00}^e E_{n,\pm 1}^{e^-} - (c_{03}^e + c_{30}^e) p_z - (c_{11}^e + c_{22}^e) \frac{(2n+1\pm 1)|eB|}{2E_{n,\pm 1}^{e^-}} \\ & - c_{33}^e \frac{p_z^2}{E_{n,\pm 1}^{e^-}} \pm d_{00}^e p_z \pm d_{30}^e m_e \left(1 - \frac{p_z^2}{E_{n,\pm 1}^{e^-}(E_{n,\pm 1}^{e^-} + m_e)} \right) \pm (d_{03}^e + d_{30}^e) \frac{p_z^2}{E_{n,\pm 1}^{e^-}} \pm (d_{11}^e + d_{22}^e) p_z \frac{(2n+1\pm 1)|eB|}{2E_{n,\pm 1}^{e^-}(E_{n,\pm 1}^{e^-} + m_e)} \\ & \pm d_{33}^e p_z \left(1 - \frac{(2n+1\pm 1)|eB|}{E_{n,\pm 1}^{e^-}(E_{n,\pm 1}^{e^-} + m_e)} \right) \pm H_{12}^e \left(1 - \frac{p_z^2}{E_{n,\pm 1}^{e^-}(E_{n,\pm 1}^{e^-} + m_e)} \right). \end{aligned} \quad (12)$$

Here, $p_z \equiv p^3$ is the third component of the momentum. The corresponding result for positrons, $\delta E_{n,\pm 1}^{e^+}$, has the same structure as for the electron but with the substitutions $a_\mu^e \rightarrow -a_\mu^e$, $d_{\mu\nu}^e \rightarrow -d_{\mu\nu}^e$, $H_{\mu\nu}^e \rightarrow -H_{\mu\nu}^e$, $E_{n,\pm 1}^{e^-} \rightarrow E_{n,\pm 1}^{e^+}$, and $(2n+1\pm 1) \rightarrow (2n+1\mp 1)$.

In Eq. (12), corrections proportional to the magnetic field B are suppressed because the typical fields of $B \approx 5$ T generate only a small ratio $|eB|/m_e^2 \approx 10^{-9}$. Also, axial confinement in the Penning-trap context is implemented by an electric field, which means the Landau momentum p_z appearing in Eq. (12) physically corresponds to an effective momentum for the axial motion. The axial frequency is several orders of magnitude smaller than the cyclotron frequency, and so in the analysis it is tempting to neglect terms involving powers of the ratio $p_z/E_{n,\pm 1}^{e^-}$. If the electric field is explicitly incorporated, the linear terms in p_z are replaced with expectation values involving the axial momentum. These would vanish for stable trapping and hence can indeed be safely ignored. However, in experimental situations the cooling process can equipartition the axial and cyclotron energies, producing large axial quantum numbers, so that expectation values of terms quadratic in the axial momentum can be comparable in magnitude to the cyclotron frequency and therefore cannot be disregarded *a priori*. Despite this, as is explicitly evident in the calculation that follows, terms of this type give no leading-order contribution to experimental observables.

Using Eq. (12), we find that the leading-order energy corrections are given by

$$\begin{aligned} \delta E_{n,\pm 1}^{e^-} \approx & a_0^e \mp b_3^e - c_{00}^e m_e \pm d_{30}^e m_e \pm H_{12}^e \\ & - \frac{1}{2}(c_{00}^e + c_{11}^e + c_{22}^e)(2n+1\pm 1)\omega_c \\ & - \left(\frac{1}{2}c_{00}^e + c_{33}^e \mp d_{03}^e \mp d_{30}^e \right) \frac{p_z^2}{m_e} \end{aligned} \quad (13)$$

for the electron, and by

$$\begin{aligned} \delta E_{n,\pm 1}^{e^+} \approx & -a_0^e \mp b_3^e - c_{00}^e m_e \mp d_{30}^e m_e \mp H_{12}^e \\ & - \frac{1}{2}(c_{00}^e + c_{11}^e + c_{22}^e)(2n+1\mp 1)\omega_c \\ & - \left(\frac{1}{2}c_{00}^e + c_{33}^e \pm d_{03}^e \pm d_{30}^e \right) \frac{p_z^2}{m_e} \end{aligned} \quad (14)$$

for the positron. Keeping only resulting leading-order shifts in the cyclotron and anomaly frequencies arising from *CPT* and Lorentz breaking, we find

$$\omega_c^{e^-} \approx \omega_c^{e^+} \approx (1 - c_{00}^e - c_{11}^e - c_{22}^e)\omega_c, \quad (15)$$

$$\omega_a^{e^\mp} \approx \omega_a \mp 2b_3^e + 2d_{30}^e m_e + 2H_{12}^e. \quad (16)$$

In these expressions, ω_c and ω_a denote the unperturbed frequencies given in Eq. (11), while $\omega_c^{e\mp}$ and $\omega_a^{e\mp}$ represent the frequencies including the corrections.

As mentioned in Sec. II C, any cyclotron-frequency shifts must of necessity involve double-differential effects, which means they depend on the quantum number n and hence on the cyclotron frequency itself. The corrections in Eq. (15) are therefore the leading ones in the *CPT*- and Lorentz-breaking quantities, in the magnetic field, and in the fine-structure constant. Similarly, Eq. (16) includes all dominant terms. For example, the contributions to the anomaly frequencies from Eqs. (13) and (14) that vary as p_z^2/m_e are suppressed relative to the ones displayed and hence have been omitted.

The above derivation allows for possible relativistic effects and quantum corrections but treats the Penning-trap electric field only indirectly. However, the same result would be obtained from a more complete calculation. One approach would be to treat the electric field and the associated axial and magnetron motions via a Foldy-Wouthuysen diagonalization of the full relativistic Hamiltonian. Restricting for simplicity our attention to effects depending on b_μ^e , for example, we find that the contribution to the fourth-order Foldy-Wouthuysen Hamiltonian is

$$H_{b^e}^{''''} = -\frac{b_0^e}{m_e} \vec{\pi} \cdot (\gamma^0 \vec{\Sigma}) - \frac{b_0^e}{2m_e^3} (\vec{\pi}^2 + |e| \vec{B} \cdot \vec{\Sigma}) (\vec{\pi} \cdot \gamma^0 \vec{\Sigma}) + \vec{b}^e \cdot \vec{\Sigma} + \frac{|e|}{2m_e^3} \vec{E} \cdot (\vec{b}^e \times \vec{\pi}) \gamma^0 - \frac{|e|}{2m_e^2} \vec{b}^e \cdot \left(\vec{B} - \frac{1}{2} i \vec{B} \times \vec{\Sigma} \right) - \frac{1}{2m_e^2} [(\vec{b}^e \cdot \vec{\Sigma}) \vec{\pi}^2 - (\vec{\pi} \cdot \vec{\Sigma}) (\vec{b}^e \cdot \vec{\pi})]. \quad (17)$$

Here, $\vec{\pi} = \vec{p} - q\vec{A}$ and $\vec{\Sigma} = I \otimes \vec{\sigma}$, where I is the 2×2 unit matrix.

The Hamiltonian $H_{b^e}^{''''}$ involves an operator momentum \vec{p} instead of the constant linear momentum p_z . Expectation values of the unperturbed wave functions determine the energy shifts. Inspection shows that neglecting the electric-field contributions is justified and confirms the suppression of the magnetic-field and other relativistic corrections compared with the term $\vec{b}^e \cdot \vec{\Sigma}$, which generates the contribution $\mp 2b_3^e$ in Eq. (16).

The form of $H_{b^e}^{''''}$ means that terms linear in b_0^e generate no contributions to the energy correction $\delta E_{n,\pm 1}^{e\mp}$, and so experiments can be sensitive at best to $(b_0^e)^2$. In fact, this result holds to all orders in the Foldy-Wouthuysen diagonalization, as follows. The full Hamiltonian $\hat{H}_{\text{eff}}^{e\mp}$ is invariant under conventional parity transformations together with a change in sign of b_0^e . The coefficient of the linear term in b_0^e in the diagonalized Hamiltonian must therefore be odd under parity. Since parity is a symmetry of the *CPT*- and Lorentz-invariant Hamiltonian $\hat{H}_0^{e\mp}$, the corresponding wave functions must be eigenstates of parity, and hence the expectation values of terms linear in b_0^e must vanish. Note in particular

that there are no corrections to the anomalous magnetic moment at first order in b_μ^e , since the only term dependent on the combination $\vec{B} \cdot \vec{\Sigma}$ is proportional to b_0^e and produces no contribution to $\delta E_{n,\pm 1}^{e\mp}$.

The expressions obtained from a complete Foldy-Wouthuysen treatment would depend on cyclotron, axial, and magnetron quantum numbers. The present work focuses on potentially observable shifts in the cyclotron and anomaly frequencies, as derived in Eqs. (15) and (16). However, we note that possible future precision experiments on axial or magnetron frequencies might in principle also produce new tests of *CPT* and Lorentz symmetry.

B. Anomalous magnetic moments

High-precision comparisons of the anomalous magnetic moments of electrons and positrons [3] currently provide the most stringent bounds on *CPT* violation in lepton systems. These Penning-trap experiments measure cyclotron and anomaly frequencies to a precision of better than 1 part in 10^8 . Combining the measurements gives the $g-2$ factors, which are of order 10^{-3} , and produces the bound on the conventional figure of merit r_g given in Eq. (1).

The effects on $g-2$ measurements of possible *CPT* and Lorentz violations can be obtained from the results in the previous subsection. Using Eqs. (15) and (16), we find the electron-positron differences for the cyclotron and anomaly frequencies to be

$$\Delta \omega_c^e \equiv \omega_c^{e-} - \omega_c^{e+} \approx 0, \quad \Delta \omega_a^e \equiv \omega_a^{e-} - \omega_a^{e+} \approx -4b_3^e. \quad (18)$$

The dominant signal for *CPT* breaking in Penning-trap $g-2$ experiments is therefore a difference between the electron and positron anomaly frequencies. No leading-order contributions appear from terms that preserve *CPT* but break Lorentz invariance.

Since the g factors of the electron and positron are unaffected by the *CPT* violation to this order, the theoretical value of r_g in Eq. (1) is zero whether or not *CPT* is broken. Instead, a model-independent figure of merit providing a well-defined measure of *CPT* violation in the weak-field, zero-momentum limit can be introduced as [7]

$$r_{\omega_a}^e \equiv \frac{|\mathcal{E}_{n,s}^{e-} - \mathcal{E}_{n,-s}^{e+}|}{\mathcal{E}_{n,s}^{e-}}. \quad (19)$$

Within the present framework for *CPT* violation, it can be shown that

$$r_{\omega_a}^e \approx |\Delta \omega_a^e| / 2m_e \approx |2b_3^e| / m_e. \quad (20)$$

Note that since the frequency difference $\Delta \omega_a^e$ depends only on the projection of \vec{b}^e along \hat{B} while the direction of \hat{B} can be changed, bounds on different spatial components of \vec{b}^e are possible in principle. With the cyclotron frequency as a magnetometer, experiments using existing techniques could place an estimated bound on this figure of merit [7]:

$$r_{\omega_a}^e \lesssim 10^{-20}. \quad (21)$$

As mentioned in Sec. II C, there exists another class of possible experimental signals, involving a diurnal variation of anomaly-frequency measurements. In particular, the energy corrections $\delta E_{n,\pm 1}^{e^-}$ and $\delta E_{n,\pm 1}^{e^+}$ could change as the Earth rotates, producing variations in $\omega_c^{e^\mp}$ and $\omega_a^{e^\mp}$ in Eqs. (15) and (16). However, $g-2$ experiments typically determine the ratio $2\omega_a^{e^\mp}/\omega_c^{e^\mp}$ rather than obtaining absolute measurements of $\omega_a^{e^\mp}$. This avoids problems with drifting magnetic fields. Using the cyclotron frequency for controlling and monitoring such drifts in a search for diurnal variations is problematic in principle since it too could contain signal time variations, as might other possible monitoring devices.

Nonetheless, even under circumstances where sizable field drifts cannot be excluded, a relatively stringent bound on Lorentz violation can be obtained. Consider the average $(\omega_a^{e^-} + \omega_a^{e^+})/2$ of the electron and positron anomaly frequencies. Using Eq. (16) with equal magnetic fields, we find

$$\frac{1}{2}(\omega_a^{e^-} + \omega_a^{e^+}) \approx \omega_a + 2d_{30}^e m_e + 2H_{12}^e. \quad (22)$$

Suppose field-drift effects, including systematic effects such as diurnal temperature changes, cannot be excluded, and assume no significant Lorentz violation is detected. Then, as electrons and positrons are alternately loaded in the Penning trap during the course of the experiment, we conservatively estimate that the time variation of the measured value of the anomaly-frequency average would be confined at least to within a 1 kHz band centered on the mean value. This corresponds to a maximal field drift limited to 5 parts in 10^6 for the typical superconducting solenoids used.

As before, a suitable model-independent figure of merit can be introduced theoretically in terms of differences between exact energy levels. Define

$$\Delta_{\omega_a}^e \equiv \frac{|\mathcal{E}_{0,+1}^{e^-} - \mathcal{E}_{1,-1}^{e^-}|}{2\mathcal{E}_{0,-1}^{e^-}} + \frac{|\mathcal{E}_{0,-1}^{e^+} - \mathcal{E}_{1,+1}^{e^+}|}{2\mathcal{E}_{0,+1}^{e^+}}. \quad (23)$$

If diurnal variations arise due to Lorentz-violating effects, then $\Delta_{\omega_a}^e$ would display a periodic time dependence. The appropriate figure of merit would be the (dimensionless) amplitude of this oscillation, which we denote $r_{\omega_a, \text{diurnal}}^e$. In the context of the present framework, we find using Eqs. (22) and (23) that this figure of merit depends on a combination of Lorentz-violating quantities,

$$r_{\omega_a, \text{diurnal}}^e \approx |d_{30}^e m_e + H_{12}^e|/m_e, \quad (24)$$

expressed in the comoving laboratory frame on the Earth. The restriction to a 1 kHz band mentioned above then yields an estimated bound of

$$r_{\omega_a, \text{diurnal}}^e \lesssim 10^{-18}. \quad (25)$$

With magnetic fields stable to 1 part in 10^9 , a 1000-fold improvement in this bound would be plausible.

C. Charge-to-mass ratios

Experiments measuring cyclotron frequencies also provide high-precision comparisons of isolated electrons and positrons confined in a Penning trap. These measurements are conventionally interpreted as determining charge-to-mass ratios. The associated conventional figure of merit, given in Eq. (3), is related to experimentally measured quantities by $r_{q/m}^e = |\Delta\omega_c^e/\omega_c^e|$, where $\Delta\omega_c^e$ is the electron-positron cyclotron-frequency difference.

The present theoretical framework for treating *CPT* and Lorentz violation can be used to examine possible effects on the electron and positron cyclotron frequencies. These acquire corrections given in Eq. (15). An immediate result is that to leading order the frequencies $\omega_c^{e^\mp}$ are independent of *CPT*-violating quantities. Since the electron and positron cyclotron frequencies can remain unchanged even in the presence of *CPT* violation, it would be misleading to regard comparisons of these frequencies as appropriate measures of *CPT* breaking. In particular, this applies to the figure of merit $r_{q/m}^e$ in Eq. (3), which is controlled by the frequency difference $\Delta\omega_c^e$.

The leading-order cyclotron-frequency shifts in Eq. (15) do display a dependence on the Lorentz-breaking but *CPT*-preserving quantity $c_{\mu\nu}^e$. However, the instantaneous equality of the electron and positron cyclotron frequencies means that it would also be misleading to regard their difference as an appropriate signal for Lorentz violation.

Another possibility is to search for diurnal variations in either $\omega_c^{e^-}$ or $\omega_c^{e^+}$, which might arise from the dependence of these frequencies on the combination of spatial components $|c_{11}^e + c_{22}^e|$ of $c_{\mu\nu}^e$, appearing in Eq. (15). Note that the component c_{00}^e cannot be bounded by such measurements, since it remains unchanged as the orientation of the magnetic field changes. Together with the trace condition $c_{\mu\mu}^e = 0$, this implies that a bound on the combination $|c_{11}^e + c_{22}^e|$ can also constrain $|c_{33}^e|$.

For possible diurnal variations of the electron cyclotron frequency, an appropriate model-independent theoretical figure of merit can be introduced as follows. Define for the electron

$$\Delta_{\omega_c}^{e^-} \equiv \frac{|\mathcal{E}_{1,-1}^{e^-} - \mathcal{E}_{0,-1}^{e^-}|}{\mathcal{E}_{0,-1}^{e^-}}. \quad (26)$$

An analogous definition could be introduced for the positron case. Diurnal variations due to Lorentz violations would appear as periodic fluctuations in $\Delta_{\omega_c}^{e^-}$. We take their amplitude as a suitable figure of merit, $r_{\omega_c, \text{diurnal}}^e$. In the context of the present framework, we find

$$r_{\omega_c, \text{diurnal}}^e \approx |c_{11}^e + c_{22}^e| \omega_c / m_e, \quad (27)$$

again in the comoving Earth frame. This figure of merit depends on the magnetic field through ω_c , which is appropriate because the associated types of level shift are explicitly dependent on ω_c , as can be seen from Eq. (13). As the applied field is increased, the level shifts grow.

The results of Ref. [6] can be used to estimate an upper bound on $r_{\omega_c, \text{diurnal}}^e$. During the 10-h period in which data were taken, the cyclotron frequencies varied by approximately 5 parts in 10^7 . Attributing the whole of this to a hypothetical diurnal variation in ω_c^e arising from the contribution $|c_{11}^e + c_{22}^e| \omega_c$ produces an estimated upper bound

$$r_{\omega_c, \text{diurnal}}^e \lesssim 10^{-16}. \quad (28)$$

More recent techniques for stabilizing the magnetic field might sharpen this bound by two orders of magnitude. The bound could also be improved by monitoring the cyclotron frequencies over a longer time scale, together with a search for signals with a diurnally related period.

IV. PROTONS AND ANTIPROTONS

In this section, we investigate some tests of *CPT* and Lorentz symmetry using comparative Penning-trap experiments with protons and antiprotons. The discussion is divided into four subsections. The first treats some issues for the underlying theory, while the second and third consider experiments on anomalous magnetic moments and charge-to-mass ratios, respectively. The fourth subsection examines comparative experiments with hydrogen ions and antiprotons.

A. Theory

At the level of the $SU(3) \times SU(2) \times U(1)$ standard model, protons and antiprotons are composite particles formed as bound states of quarks and antiquarks, respectively. Possible *CPT*- and Lorentz-violating effects in the extension of the model appear as perturbations involving the basic fields [8]. For example, a distinct set of parameters a_μ , b_μ , $H_{\mu\nu}$, $c_{\mu\nu}$, $d_{\mu\nu}$ is assigned to each quark flavor, and suitable combinations of these determine the *CPT*- and Lorentz-violating features of the proton.

For our present investigation involving electromagnetic interactions of protons and antiprotons in a Penning trap, it suffices to work instead within the usual effective theory in which the protons and antiprotons are regarded as basic objects described by a four-component Dirac quantum field with dynamics governed by a minimally coupled Lagrangian. We therefore introduce effective parameters a_μ^p , b_μ^p , $H_{\mu\nu}^p$, $c_{\mu\nu}^p$, $d_{\mu\nu}^p$ controlling possible *CPT*- and Lorentz-breaking effects for the proton, and we take the Lagrangian to be the standard one for proton-antiproton quantum electrodynamics but extended to include possible small *CPT*- and Lorentz-violating terms. The corresponding Dirac equation has the form of Eq. (4). The analysis of this model is analogous to the treatment presented in Sec. II.

We identify the Dirac Hamiltonian \hat{H}^p for the proton with \hat{H}^q given in Eq. (6), with perturbative terms as in Eq. (9) except for superscripts p on all *CPT*- and Lorentz-violating parameters and the replacement $m \rightarrow m_p$ for the proton mass. Similarly, for the antiproton we identify $\hat{H}^{\bar{p}} \equiv \hat{H}^{\bar{q}}$. The wave functions for perturbative calculations are well approximated as relativistic Landau eigenfunctions for protons and antiprotons. We denote the associated energies, including anomaly

terms and other quantum effects but excluding *CPT*- and Lorentz-breaking shifts, by $E_{n,s}^p$ and $E_{n,s}^{\bar{p}}$. The corresponding proton cyclotron and anomaly frequencies are defined as

$$\omega_c = E_{1,+1}^p - E_{0,+1}^p, \quad \omega_a = E_{0,-1}^p - E_{1,+1}^p. \quad (29)$$

The *CPT* theorem implies that they have the same values as those of the antiproton.

Proceeding as in Sec. III A, we can calculate perturbative energy corrections that are first order in *CPT*- and Lorentz-breaking parameters. Contributions proportional to the magnetic field are now suppressed by a factor of order 10^{-16} . Terms involving the axial or magnetron motions are treated as before. Keeping only leading-order perturbations, we find that the corrections to the proton energies are

$$\begin{aligned} \delta E_{n,\pm 1}^p &\approx a_0^p \mp b_3^p - c_{00}^p m_p \pm d_{30}^p m_p \pm H_{12}^p \\ &\quad - \frac{1}{2} (c_{00}^p + c_{11}^p + c_{22}^p) (2n + 1 \mp 1) \omega_c \\ &\quad - \left(\frac{1}{2} c_{00}^p + c_{33}^p \mp d_{03}^p \mp d_{30}^p \right) \frac{p_z^2}{m_p}. \end{aligned} \quad (30)$$

The energy shifts $\delta E_{n,\pm 1}^{\bar{p}}$ for the antiproton can be obtained by the substitutions $a_\mu^p \rightarrow -a_\mu^{\bar{p}}$, $d_{\mu\nu}^p \rightarrow -d_{\mu\nu}^{\bar{p}}$, $H_{\mu\nu}^p \rightarrow -H_{\mu\nu}^{\bar{p}}$, $E_{n,\pm 1}^p \rightarrow E_{n,\pm 1}^{\bar{p}}$, and $(2n + 1 \mp 1) \rightarrow (2n + 1 \pm 1)$. These results produce corrected cyclotron and anomaly frequencies. At leading order in the *CPT*- and Lorentz-breaking quantities, in the electromagnetic fields, and in the fine-structure constant, the modified frequencies are given by

$$\omega_c^p \approx \omega_c^{\bar{p}} \approx (1 - c_{00}^p - c_{11}^p - c_{22}^p) \omega_c, \quad (31)$$

$$\omega_a^p \approx \omega_a + 2b_3^p - 2d_{30}^p m_p - 2H_{12}^p,$$

$$\omega_a^{\bar{p}} \approx \omega_a - 2b_3^p - 2d_{30}^p m_p - 2H_{12}^p. \quad (32)$$

Here, ω_c and ω_a are the unperturbed frequencies of Eq. (29). Note that much of the discussion associated with the theoretical derivation in Sec. III A applies here. Note also that the ratio of proton and electron cyclotron frequencies is about 10^{-3} , whereas the proton and electron anomaly frequencies are roughly comparable in magnitude because the corresponding $g - 2$ values differ by a factor of about 10^3 .

B. Anomalous magnetic moments

Currently, the best measurements of the antiproton magnetic moment are accurate to only about 3 parts in 10^3 and are extracted from experiments with exotic atoms [18]. In principle, precision measurements of the anomalous magnetic moments of protons and antiprotons could be obtained in Penning traps, in analogy with the electron-positron experiments discussed in Sec. III B, provided sufficient cooling to temperatures below 4 K can be achieved.

A comparison of the experimental ratios $2\omega_a^p/\omega_c^p$ and $2\omega_a^{\bar{p}}/\omega_c^{\bar{p}}$ would then provide a stringent test of *CPT* and Lorentz violation. No such experiments have been performed to date, although the possibility has received some attention in the literature [19,20].

Using the present theoretical framework, we can investigate the sensitivity of possible future $g-2$ experiments to CPT and Lorentz violations. To leading order, we find that the proton-antiproton differences for the cyclotron and anomaly frequencies are

$$\Delta \omega_c^p \equiv \omega_c^p - \omega_c^{\bar{p}} = 0, \quad \Delta \omega_a^p \equiv \omega_a^p - \omega_a^{\bar{p}} = 4b_3^p. \quad (33)$$

Just as in the electron-positron case, the leading-order signal for CPT breaking is thus an anomaly-frequency difference. The corresponding figure of merit providing a well-defined measure of CPT violation is

$$r_{\omega_a}^p \equiv \frac{|\mathcal{E}_{n,s}^p - \mathcal{E}_{n,-s}^{\bar{p}}|}{\mathcal{E}_{n,s}^p}, \quad (34)$$

where the weak-field, zero-momentum limit is understood. Within the present theoretical framework, we find

$$r_{\omega_a}^p \approx |\Delta \omega_a^p|/2m_p \approx |2b_3^p|/m_p. \quad (35)$$

Assuming an experiment could be made sensitive enough to measure ω_a^p and $\omega_a^{\bar{p}}$ with a precision similar to that of electron $g-2$ experiments, we can estimate the bound on $r_{\omega_a}^p$ that would be attainable. For example, supposing in analogy with the electron-positron experiments that a frequency accuracy of about 2 Hz can be attained in the measurements of $\omega_a^p, \omega_a^{\bar{p}}$ and equality of $\omega_c^p, \omega_c^{\bar{p}}$ is observed to 1 part in 10^8 , a bound of $|b_3^p| \lesssim 10^{-15}$ eV becomes possible. This corresponds to an estimated bound on the figure of merit of

$$r_{\omega_a}^p \lesssim 10^{-23}. \quad (36)$$

It is evident that this experiment has the potential to provide a particularly stringent CPT bound in a baryon system.

Just as for the electron-positron case in Sec. III B, experiments of this type could also bound diurnal variations in the average anomaly frequency. An appropriate theoretical figure of merit in this case can be introduced in terms of the quantity

$$\Delta_{\omega_a}^p \equiv \frac{|\mathcal{E}_{0,-1}^p - \mathcal{E}_{1,+1}^p|}{2\mathcal{E}_{0,+1}^p} + \frac{|\mathcal{E}_{0,+1}^{\bar{p}} - \mathcal{E}_{1,-1}^{\bar{p}}|}{2\mathcal{E}_{0,-1}^{\bar{p}}}. \quad (37)$$

The figure of merit is the amplitude $r_{\omega_a}^p, \text{diurnal}$ of diurnal variations observed in $\Delta_{\omega_a}^p$. In the present framework, these depend on Lorentz-violating but CPT -preserving terms, and we find

$$r_{\omega_a}^p, \text{diurnal} \approx |d_{30}^p m_p + H_{12}^p|/m_p, \quad (38)$$

in the comoving Earth frame. Assuming observations confine diurnal variations of the anomaly-frequency average to within a 1 kHz band as before, we obtain an estimated bound on the figure of merit of

$$r_{\omega_a}^p, \text{diurnal} \lesssim 10^{-21}. \quad (39)$$

C. Charge-to-mass ratios

Experiments confining single protons and antiprotons in an open-access Penning trap provide high-precision comparisons of their cyclotron frequencies [5], yielding the limit $|\Delta \omega_c^p|/\omega_c^p \lesssim 10^{-9}$. The corresponding conventional figure of merit $r_{q/m}^p$ and its current bound are given in Eq. (2).

Within the present theoretical framework, Eq. (30) demonstrates that the CPT - and Lorentz-violating terms introduce nonzero energy-level shifts, even in the weak-field zero-momentum limit. The perturbations of the cyclotron frequencies are given in Eq. (33). To leading order, the proton and antiproton cyclotron frequencies are independent of CPT -violating quantities, just as for the electron-positron case discussed in Sec. III C. As the cyclotron frequencies are unaffected even if CPT is broken, a comparison of these frequencies would represent a misleading measure of CPT violation. For example, the figure of merit $r_{q/m}^p$ in Eq. (2), which is proportional to the frequency difference $\Delta \omega_c^p$, may vanish even though the model contains explicit CPT violation.

The Lorentz-breaking but CPT -preserving parameters induce identical shifts in the proton and antiproton cyclotron frequencies. In analogy with the electron-positron case, this indicates that the frequency difference $\Delta \omega_c^p$ would be an inappropriate measure of Lorentz violation.

Another possibility is the occurrence of diurnal variations in the cyclotron frequencies, which could be induced by the Earth's rotation during the course of an experiment. Such variations would arise in the present context from the dependence of the cyclotron frequencies on the components $|c_{11}^p + c_{22}^p|$ of $c_{\mu\nu}^p$. As discussed for the electron-positron case in Sec. III C, the unobservability of the component c_{00}^p means that a bound on $|c_{11}^p + c_{22}^p|$ can also constrain $|c_{33}^p|$.

A suitable theoretical figure of merit can be introduced in analogy with the electron-positron case. Define for the proton

$$\Delta_{\omega_c}^p \equiv \frac{|\mathcal{E}_{1,-1}^p - \mathcal{E}_{0,-1}^p|}{\mathcal{E}_{0,-1}^p}. \quad (40)$$

The figure of merit is the amplitude $r_{\omega_c}^p, \text{diurnal}$ of periodic fluctuations in $\Delta_{\omega_c}^p$. In the comoving Earth frame, we find

$$r_{\omega_c}^p, \text{diurnal} \approx |c_{11}^p + c_{22}^p| \omega_c / m_p. \quad (41)$$

As for the corresponding electron-positron case, the appearance of ω_c implies that the value of this figure of merit depends on the magnetic field. This is appropriate, since the associated level shifts in Eq. (30) also explicitly depend on ω_c .

A crude estimated upper bound on $r_{\omega_c}^p, \text{diurnal}$ can be obtained from the data in Ref. [5], which represent alternate measurements of proton and antiproton cyclotron frequencies over a 12-h period. The slow drifts in these frequencies are confined to a band of approximate width 2 Hz. This suggests an upper bound on a possible diurnal variation in $r_{\omega_c}^p, \text{diurnal}$ arising from the contribution proportional to $|c_{11}^p + c_{22}^p|$, given by

$$r_{\omega_c}^p, \text{diurnal} \lesssim 10^{-24}. \quad (42)$$

Note that diurnal fluctuations in the antiproton cyclotron frequency could be treated similarly.

The bound (42) is better than the corresponding one for electrons and positrons given in Eq. (28). It might be sharpened through detailed analysis of the experimental data, perhaps including a fit for diurnal variations and compensation for known correlations with temperature fluctuations in the experimental hall.

D. Experiments with hydrogen ions

When protons and antiprotons are interchanged in the Penning-trap experiments of Ref. [5], the associated reversal of the electric field can lead to offset potentials affecting differently the proton and antiproton cyclotron frequencies. In an ingenious recent experiment [21], Gabrielse and co-workers have addressed this issue by comparing antiproton cyclotron frequencies with those of an H^- ion instead of a proton. The equality of the charges means that the same trap and fields can be used, and the experiment also allows relatively rapid interchanges between hydrogen ions and antiprotons. The expected theoretical value of the difference $\Delta\omega_c^{H^-} \equiv \omega_c^{H^-} - \omega_c^{\bar{p}}$ can be obtained in the context of conventional quantum theory using established precision measurements of the electron mass and the H^- binding energy. Comparison of this theoretical value with the experimental result for $\Delta\omega_c^{H^-}$ is expected to provide a symmetry test with a precision of about 1 part in 10^{10} .

Understanding the implications of this experiment within the present theoretical framework requires a description of the electromagnetic interactions of the hydrogen ion in a Penning trap in the presence of possible *CPT* and Lorentz violation. A hydrogen ion can be regarded as a charged composite fermion, and so its electromagnetic interactions can be discussed within an effective spinor electrodynamics producing a Dirac equation of the form (4) for a fermion of mass m_{H^-} . The corresponding effective *CPT*- and Lorentz-breaking parameters are denoted $a_\mu^{H^-}, b_\mu^{H^-}, H_{\mu\nu}^{H^-}, c_{\mu\nu}^{H^-}, d_{\mu\nu}^{H^-}$. The theoretical treatment then proceeds as in Sec. II.

For a hydrogen ion in a Penning trap, we obtain the leading-order energy shifts from *CPT* and Lorentz breaking following the method in Secs. III A and IV A. We find

$$\begin{aligned} \delta E_{n,\pm 1}^{H^-} &\approx a_0^{H^-} \mp b_3^{H^-} - c_{00}^{H^-} m_{H^-} \pm d_{30}^{H^-} m_{H^-} \pm H_{12}^{H^-} \\ &- \frac{1}{2} (c_{00}^{H^-} + c_{11}^{H^-} + c_{22}^{H^-}) (2n + 1 \pm 1) \omega_c^{H^-} \\ &- (c_{00}^{H^-} - c_{33}^{H^-} \mp d_{03}^{H^-} \mp d_{30}^{H^-}) \frac{p_z^2}{m_{H^-}}. \end{aligned} \quad (43)$$

The H^- cyclotron frequency is therefore shifted from its value $\omega_c^{H^-}$ in the absence of Lorentz violation to a perturbed value $\omega_{c,\text{pert}}^{H^-}$ given by

$$\omega_{c,\text{pert}}^{H^-} \approx (1 - c_{00}^{H^-} - c_{11}^{H^-} - c_{22}^{H^-}) \omega_c^{H^-}. \quad (44)$$

Much of the discussion in Secs. III A and IV A concerning the corresponding theoretical derivations also applies here.

The above result can be used to obtain limits on Lorentz-violating quantities for hydrogen ions and protons. Denote as before the difference between the cyclotron frequencies of the hydrogen ion and the antiproton by $\Delta\omega_c^{H^-}$. Then, the component $\Delta\omega_{c,\text{th}}^{H^-}$ of $\Delta\omega_c^{H^-}$ that is determined theoretically to arise purely from *CPT*- and Lorentz-violating effects can be obtained from Eqs. (31) and (44). We find

$$\Delta\omega_{c,\text{th}}^{H^-} \approx (c_{00}^p + c_{11}^p + c_{22}^p) \omega_c - (c_{00}^{H^-} + c_{11}^{H^-} + c_{22}^{H^-}) \omega_c^{H^-}. \quad (45)$$

As before, ω_c is the proton-antiproton cyclotron frequency in the absence of *CPT* or Lorentz perturbations.

The definition of a model-independent figure of merit proceeds in analogy with the treatments in preceding sections. We introduce the quantity

$$\Delta_{\omega_c}^{H^-} \equiv \frac{|\mathcal{E}_{1,-1}^{H^-} - \mathcal{E}_{0,-1}^{H^-}|}{2\mathcal{E}_{0,-1}^{H^-}} - \frac{|\mathcal{E}_{1,-1}^{\bar{p}} - \mathcal{E}_{0,-1}^{\bar{p}}|}{2\mathcal{E}_{0,-1}^{\bar{p}}}. \quad (46)$$

As defined, $\Delta_{\omega_c}^{H^-}$ is nonzero even if *CPT* and Lorentz symmetry is preserved. To obtain a measure that vanishes in the exact symmetry limit, we remove from the hydrogen-ion terms in $\Delta_{\omega_c}^{H^-}$ the conventional contributions arising from the differences between the H^- ion and a proton: the masses of the two electrons and the binding energy. The result is a suitable figure of merit for Lorentz violation, denoted by $r_{\omega_c}^{H^-}$. The calculations leading to Eq. (45) imply that within the present framework

$$r_{\omega_c}^{H^-} \approx |\Delta\omega_{c,\text{th}}^{H^-}|/m_p. \quad (47)$$

It is plausible that a precision of about 1 part in 10^{10} could be attained in measurements of the ratio $|\Delta\omega_c^{H^-}|/\omega_c^{H^-}$. Suppose the observed value agrees with conventional theory to within a certain accuracy. Then, this accuracy must be larger than the predicted shift ratio $|\Delta\omega_{c,\text{th}}^{H^-}|/\omega_c^{H^-}$. We thus obtain an estimated bound of

$$r_{\omega_c}^{H^-} \lesssim 10^{-25} \quad (48)$$

that might be attained in this class of experiment.

The above results involve a combination of the Lorentz-violating quantities for hydrogen ions and protons. However, all the effective *CPT*- and Lorentz-breaking parameters for a hydrogen ion are determined by appropriate combinations of the corresponding parameters for its constituents. Lowest-order perturbation theory can be used to find approximations to these relationships. The wave function of the hydrogen ion can be treated as a product of a proton wave function and a two-electron wave function, and the corresponding net *CPT*- and Lorentz-breaking energy shifts induced for the hydrogen ion can be estimated, neglecting nonperturbative issues involving binding effects.

In this approximation, we find

TABLE I. Estimated CPT - and Lorentz-violating bounds for electron-positron, proton-antiproton, and H^- -antiproton experiments. The first two columns specify the type of experiment. The third column lists figures of merit, while the fourth gives the corresponding bounds estimated from current or future experiments. The fifth column shows which of the quantities in Eq. (4) enter the constraint. Entries in the final column are the numbers for the equations in the text where the bound is presented.

Experiment	Figure of merit	Estimated bound	Parameters	Equation
e^-e^+	ω_a comparison	$r_{\omega_a}^e$	b_j^e	(21)
	diurnal ω_a variation	$r_{\omega_a, \text{diurnal}}^e$	d_{j0}^e, H_{jk}^e	(25)
	diurnal ω_c variation	$r_{\omega_c, \text{diurnal}}^e$	c_{jj}^e	(28)
$p\bar{p}$	ω_a comparison	$r_{\omega_a}^p$	b_j^p	(36)
	diurnal ω_a variation	$r_{\omega_a, \text{diurnal}}^p$	d_{j0}^p, H_{jk}^p	(39)
	diurnal ω_c variation	$r_{\omega_c, \text{diurnal}}^p$	c_{jj}^p	(42)
$H^-\bar{p}$	ω_c comparison	$r_{\omega_c}^{H^-}$	$c_{\mu\mu}^{H^-}, c_{\mu\omega}^p$	(48)

$$c_{\mu\mu}^{H^-} \approx c_{\mu\mu}^p + (c_{\mu\mu}^e - c_{\mu\mu}^p) \frac{2m_e}{m_p}, \quad (49)$$

where no sum is implied on repeated indices. Substitution of this result into Eq. (45) gives

$$\Delta \omega_{c, \text{th}}^{H^-} \approx (c_{00}^p + c_{11}^p + c_{22}^p)(\omega_c - \omega_c^{H^-}) - \frac{2m_e}{m_p} (c_{00}^e + c_{11}^e + c_{22}^e - c_{00}^p - c_{11}^p - c_{22}^p) \omega_c^{H^-}. \quad (50)$$

This result implies that the bound in Eq. (48) constrains a combination of Lorentz-violating but CPT -preserving quantities, including c_{00}^e and c_{00}^p . The latter would be inaccessible through the other experiments considered in the present work. Moreover, this experiment does not require searching for diurnal variations in the cyclotron frequency, which means potential systematics associated with diurnal field drifts are eliminated.

We remark in passing that in principle anomaly-frequency comparisons of H^- and antiprotons could also be envisaged. Leaving aside experimental issues, the theoretical motivation for such experiments seems somewhat lacking. One point is that perturbative calculation indicates $b_\mu^{H^-} \approx b_\mu^p$, and so bounds that might be obtained in this way would also be accessible in the experiments mentioned in Sec. III B.

V. SUMMARY

In this paper, we have used a general theoretical framework based on an extension of the standard model and quantum electrodynamics to establish and investigate possible signals of CPT and Lorentz breaking in certain Penning-trap experiments. We have focused on leading-order limits arising

from high-precision measurements of anomaly and cyclotron frequencies. Table I summarizes our results.

Our estimated bounds from experiments with the electron-positron system are given in Eqs. (21), (25), and (28). Bounds from the proton-antiproton system are in Eqs. (36), (39), and (42), while a bound from the H^- -antiproton system is given in Eq. (48).

Sharp tests of CPT symmetry emerge from $g-2$ experiments. We have introduced appropriate figures of merit with attainable bounds of approximately 10^{-20} using current methods in the electron-positron case and of 10^{-23} for a plausible experiment with protons and antiprotons. Other experimental signals originating from CPT -preserving Lorentz violations could occur, involving possible diurnal variations in frequency measurements. These could produce bounds at the level of 10^{-18} in the electron-positron system and 10^{-21} in the proton-antiproton system.

In contrast, comparative measurements of cyclotron frequencies for particles and antiparticles are insensitive to leading-order effects from CPT breaking within the present framework. However, diurnal variations of cyclotron frequencies and comparative measurements of cyclotron frequencies for hydrogen ions and antiprotons are affected by different CPT -preserving Lorentz-violating quantities. These experiments could generate bounds on various dimensionless figures of merit at the level of 10^{-16} in the electron-positron system, 10^{-24} in the proton-antiproton system, and 10^{-25} using the H^- -antiproton system.

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