# Testing *CPT* with Anomalous Magnetic Moments

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A theoretical framework is introduced that describes possible *CPT*-violating effects in the context of quantum electrodynamics. Experiments comparing the anomalous magnetic moments of the electron and the positron can place tight limits on *CPT* violation. The conventional figure of merit adopted in these experiments, involving the difference between the corresponding $g$ factors, is shown to provide a misleading measure of the precision of *CPT* limits. We introduce an alternative figure of merit, comparable to one commonly used in *CPT* tests with neutral mesons. To measure it, a straightforward extension of current experimental procedures is proposed. With current technology, a *CPT* bound better than about 1 part in $10^{20}$ is attainable. [S0031-9007(97)03884-2]

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The *CPT* theorem [1] is a powerful result holding for local relativistic quantum field theories of point particles in flat spacetime. It states that such theories must be invariant under the combined operations of charge conjugation *C*, parity reversal *P*, and time reversal *T*. Among the implications of the theorem are the equality of particle and antiparticle masses and lifetimes.

Invariance under *CPT* has been tested in a variety of experiments [2]. The tightest bound published to date arises from experiments with the neutral kaon system [3], where the *CPT* figure of merit

$$r_K \equiv |(m_K - m_{\overline{K}})/m_K| \qquad (1)$$

is known to be smaller than 2 parts in $10^{18}$. This remarkable precision is possible because neutral-kaon oscillations provide a natural interferometer with dimensionless sensitivity controlled by the mass difference between the physical $K_L$ and $K_S$ states: $|(m_L - m_S)/m_K| \simeq 10^{-14}$. The quoted precision for $r_K$ is thus attained via measurements with a precision of about 1 part in $10^4$.

Atomic experiments have also confirmed *CPT* symmetry. High-precision comparisons of the anomalous magnetic moments of the electron and positron currently provide the most stringent bounds on *CPT* violation in lepton systems [4]. Denote the electron and positron $g$ factors by $g_-$ and $g_+$, respectively. Then, a conventional figure of merit used in these experiments is [2]

$$r_g \equiv |(g_- - g_+)/g_{\mathrm{av}}|, \qquad (2)$$

which is known to be smaller than 2 parts in $10^{12}$. The experiments confine isolated single electrons or positrons in a Penning trap for the indefinite periods [4,5] and measure their cyclotron and anomaly frequencies to a precision of better than 1 part in $10^8$. These frequencies can be combined to determine $g - 2$, which is of order $10^{-3}$, and hence to yield the limit on $r_g$.

The figure of merit $r_g$ is poorer than $r_K$ by about 6 orders of magnitude, even though the experimental measurements involved in the $g - 2$ experiments are about 4 orders of magnitude sharper. This discrepancy originates in the difference between the quantities entering the dimensionless figures of merit. One is a mass (energy) difference while the other is a coupling difference. Indeed, all *CPT* tests to date have looked for differences between particles and antiparticle masses, lifetimes, or couplings. An important limiting factor in comparing bounds from various systems and in establishing new tests has been the absence of a theoretical framework for describing possible *CPT* violation.

The combination of the theoretical proof of *CPT* invariance in conventional field theory and high-precision tests in experiments has triggered investigations of possible *CPT* violation as a candidate signature for new physics beyond the standard model, such as string theory [6]. The current bounds in the kaon system are close to the scale of suppressed *CPT* violation possibly arising in strings [6,7], and new tests in other neutral-meson systems are feasible with analysis of existing data or in planned experiments [7,8]. There are also possible implications for baryogenesis [9].

Motivated by these ideas, a theoretical framework for the treatment of possible *CPT* and Lorentz violations at the level of the standard $SU(3) \times SU(2) \times U(1)$ model has recently been developed [10]. Within this framework, a general *CPT*- and Lorentz-violating extension to the standard model has been presented that appears to maintain desirable features of the quantum field theory, including gauge invariance, naive power-counting renormalizability, and microscopic causality. Possible *CPT* violations are controlled by parameters with values to be bounded by experiment.

The existence of this model suggests a variety of experimental approaches to testing *CPT* and makes possible a quantitative comparison of various figures of merit. In the present work, we consider a restriction of the model to quantum electrodynamics to investigate tests of *CPT* using the anomalous magnetic moments of the electron and positron. In what follows, we use this model to show that the conventional figure of merit $r_g$ adopted in $g - 2$ experiments is a misleading measure of *CPT* bounds in



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lepton systems. Instead, an alternative *CPT* figure of merit is introduced, and its value within our model is obtained. A straightforward experimental procedure to measure it is proposed, and an estimate is given of the likely resulting *CPT* bound.

In the present context, the dominant *CPT*-breaking terms from the model act to modify the Dirac equation. In natural units ($\hbar = c = 1$), the result is

$$(i\gamma^\mu \partial_\mu - eA_\mu\gamma^\mu - a_\mu\gamma^\mu - b_\mu\gamma_5\gamma^\mu - m)\psi = 0\,, \tag{3}$$

where $\psi$ is the electron-positron field, $A_\mu$ is the photon field, $e$ is the electron charge, and $m$ is its mass. The eight quantities $a_\mu$ and $b_\mu$ are (small) real constants that are invariant under *CPT* transformations and act as effective coupling constants. The standard *CPT*-transformation properties of $\psi$ can be used to show that the terms involving $a_\mu$ and $b_\mu$ break *CPT*. These features and Eq. (3) largely suffice to develop the results in the present work. Various issues concerning other symmetry transformations (including rotational and boost properties) and more general extensions of quantum electrodynamics are treated in Ref. [10] but are not directly relevant here.

In $g - 2$ experiments, the leading contributions to the energy spectrum originate in the particle interaction with the constant magnetic field of the Penning trap. The quadrupole electric field and other fields produce lesser effects. Since any possible *CPT* violation must be small, it suffices to work within a perturbative framework using relativistic quantum mechanics. The field $\psi$ can thus be regarded as a Dirac wave function for an electron, and $A_\mu$ can be treated as a background electromagnetic potential. We denote by $\hat{H}_0^-$ the conventional Dirac Hamiltonian operator for an electron in the potential $A_\mu$ for a constant magnetic field, including an anomaly term. The exact eigenenergies of $\hat{H}_0^-$ are the usual Landau levels, and the eigensolutions can be used as the basis for perturbative calculations. In the presence of the *CPT*-violating terms given in Eq. (3), the modified Dirac Hamiltonian for the electron wave function is $\hat{H}^- = \hat{H}_0^- + \hat{H}_{\rm int}^-$, where

$$\hat{H}_{\rm int}^- = a_\mu\gamma^0\gamma^\mu - b_\mu\gamma_5\gamma^0\gamma^\mu. \tag{4}$$

The wave function for a positron can be found using charge conjugation. Typically, experiments on positrons are performed in Penning traps with the same magnetic fields as used for electron experiments, with only the electric field changing polarity. We therefore solve for the positron wave function in the same field $A_\mu$ as for the electron. In the present case, this implies the usual Dirac Hamiltonian $\hat{H}_0^+$ for a positron is the same as $\hat{H}_0^-$ except that the coefficient of $A_\mu$ changes sign. Using the charge-conjugation transformation, the *CPT*-violating perturbation for the positron is found to be

$$\hat{H}_{\rm int}^+ = -a_\mu\gamma^0\gamma^\mu - b_\mu\gamma_5\gamma^0\gamma^\mu. \tag{5}$$

In investigating *CPT*-violating effects, it is unnecessary to include all possible perturbations that are relevant to $g - 2$ experiments. For example, the effects of the magnetron and axial motions and the usual higher-order relativistic corrections are all described within conventional Dirac theory and are the same for electrons and positrons. It therefore suffices to work with the electron and positron theories described by $H_0^\pm$. The point is that all perturbative corrections except those involving $a_\mu$ and $b_\mu$ vanish when the electron and positron energies are subtracted. Moreover, any interactions involving the coupling of $a_\mu$ and $b_\mu$ to other perturbative terms are of higher order and therefore can be neglected.

In what follows, we denote the relativistic electron and positron Landau-level wave functions by $\psi_{n,s}^-$ and $\psi_{n,s}^+$, respectively. The corresponding lowest-order eigenenergies are denoted $E_{n,s}^-$ and $E_{n,s}^+$, where $n = 0, 1, 2, \ldots$ labels the level number and $s = \pm 1$ labels the spin. In the electron case the spin-up and spin-down states form two ladders of levels, for which the spin-down states with given $n = n_0 > 0$ are almost degenerate with the spin-up states with $n = n_0 - 1$. The degeneracy is broken due to the anomalous magnetic moment. A similar situation holds for the positron case, except that the spin labels are reversed. The lowest-order cyclotron and anomaly frequencies $\omega_c^-$ and $\omega_a^-$ for the electron and the corresponding frequencies $\omega_c^+$ and $\omega_a^+$ for the positron can be expressed in terms of the lowest eigenenergies as

$$\omega_c^\mp = E_{1,\mp1}^\mp - E_{0,\mp1}^\mp, \quad \omega_a^\mp = E_{0,\pm1}^\mp - E_{1,\mp1}^\mp\,. \tag{6}$$

We orient our coordinate system so that the magnetic field $\vec{B} = B\hat{z}$ lies along the positive $z$ axis, and we choose the gauge $A^\mu = (0, -yB, 0, 0)$. The lowest-order *CPT*-violating corrections to the electron energies from $\hat{H}_{\rm int}^-$ then are

$$\delta E_{n,\pm1}^- = a_0 + a_3\frac{p_z}{E_{n,\pm1}^-} \mp b_3\left[1 - \frac{|eB|(2n + 1 \pm 1)}{E_{n,\pm1}^-(E_{n,\pm1}^- + m)}\right] \mp b_0\frac{p_z}{E_{n,\pm1}^-}\,, \tag{7}$$

where $p_z \equiv p^3$ is the third component of the momentum. For the positron, we find a similar expression but with the replacements $a_\mu \to -a_\mu$, $E_{n,\pm1}^- \to E_{n,\pm1}^+$, and $\pm1 \to \mp1$ in the numerator of the third term.

At first sight, it might appear from these equations that both $a_\mu$ and $b_\mu$ have physically observable consequences. However, the corrections due to $a_\mu$ correspond to a redefinition of the zero of the energy and momentum, $E \to E - a^0$ and $\vec{p} \to \vec{p} - \vec{a}$, in the dispersion relation for $E_{n,s}^-(\vec{p})$. The corresponding shifts for positrons would have opposite signs for $a_\mu$. Although the electron and positron four-momentum shifts are of opposite signs, they cannot be detected in $g - 2$ experiments because the double tower of states in each case is shifted so that all level spacings are constant. The cyclotron and anomaly frequencies remain unchanged for both cases, and hence $a_\mu$ has no observable effect [11]. Without loss of generality, we can therefore set $a_\mu$ to zero in what follows.

For Penning-trap configurations typically used in $g - 2$ experiments, the axial momentum replaces $p_z$. Since the energy of the axial motion is several orders of magnitude smaller than $E^-_{n,s}$, the terms in Eq. (7) involving the product of $b_0$ with $p_z/E^{\pm}_{n,s}$ can safely be neglected provided the ratio $b_0/b_3$ is not too large [13]. For the typical magnetic fields of $B \simeq 5$ T, $|eB|/m^2 \simeq 10^{-9}$, so the correction terms involving the product of $b_3$ with $|eB|$ can also be ignored. The dominant *CPT*-violating contributions therefore depend only on $b_3$. It follows that there are no corrections to the cyclotron frequencies, while the electron and positron anomaly frequencies shift by $-2b_3$ and $2b_3$, respectively. This gives

$$\Delta\omega_c \equiv \omega_c^- - \omega_c^+ = 0\,, \quad \Delta\omega_a \equiv \omega_a^- - \omega_a^+ = -4b_3\,. \tag{8}$$

The leading-order signal for *CPT* breaking in Penning-trap $g - 2$ experiments with fixed magnetic field is therefore a difference between the electron and positron anomaly frequencies. Note that the signature (8) for *CPT* violation is sensitive only to the spatial components of $\vec{b}$ in the direction of $\vec{B}$. However, since the relative directions of the two vectors can be probed experimentally, for example by changing the orientation of $\vec{B}$ or by performing measurements at different times, bounds on the different spatial components of $\vec{b}$ are in principle accessible.

At this point, we can address the issue of the appropriateness of the figure of merit $r_g$ given in Eq. (2) as a suitable measure of *CPT* violation. Recall that the $g$ factor of an elementary particle is essentially the strength of the gyromagnetic ratio, which is the ratio of the magnitudes of the magnetic moment and the angular momentum. Conventional quantum electrodynamics for an electron in a Penning trap predicts $(g - 2) = 2\omega_a/\omega_c$, and *CPT* invariance predicts $g_- = g_+$. The latter relation holds to within the measurement accuracy of two parts in $10^{12}$. It therefore appears tempting to use the figure of merit $r_g$ of Eq. (2) as a measure of *CPT* violation. However, within our framework, *CPT* is broken without affecting the electron or positron gyromagnetic ratios. This means that the theoretical value of $r_g$ is zero even though *CPT* is broken.

One might be tempted to fix this problem by adopting as fundamental the conventional experimentally based definition $(g_{\rm expt} - 2) \equiv 2\omega_a/\omega_c$, where $\omega_a$ and $\omega_c$ are experimental frequencies. This definition of $g$ would make $r_g$ nonzero if *CPT* is violated, but it would be different from the theoretical definition based on the gyromagnetic ratio. Moreover, $r_g$ would then depend on the field $B$ and might not be well defined. For example, our result (8) means that $r_g$ would become $r_g = |\Delta\omega_a/\omega_a^{\rm av}| \approx |4b_3/\omega_a^-|$, which diverges in the weak-field limit $B \to 0$. This provides an explicit counterexample to the thesis that $r_g$ is a suitable *CPT* figure of merit.

A more appropriate figure of merit can be introduced theoretically in a general context as the ratio of a *CPT*-violating electron-positron energy-level difference and the basic energy scale:

$$r_e \equiv |(\mathcal{E}^-_{n,s} - \mathcal{E}^+_{n,-s})/\mathcal{E}^-_{n,s}|\,, \tag{9}$$

taken as usual in the weak-field, zero-momentum limit. Here, $\mathcal{E}^-_{n,s}$ and $\mathcal{E}^+_{n,s}$ denote energy eigenvalues for the full Penning-trap Hamiltonians. Within our particular framework $\mathcal{E}^-_{n,s} \to m$ in this limit, and the difference of energies in the numerator becomes half the difference between the two measured anomaly frequencies, $\Delta\omega_a/2 \approx -2b_3$, independent of $n$ and $s$. Thus, in our model the definition (9) reduces to $r_e = |\Delta\omega_a/2m| = |2b_3/m|$. This shows that, unlike the conventional quantity $r_g$, the figure of merit $r_e$ is a well-defined measure of *CPT* violation. Moreover, since it is a ratio of energies, it is comparable to the measure $r_K$ in Eq. (1) conventionally used for *CPT* tests with the neutral-kaon system.

Within the framework of scenarios involving spontaneous *CPT* and Lorentz breaking from a higher-dimensional fundamental model such as a string theory [6,7,14], the natural suppression scale for *CPT* violation is the ratio of a light scale $m_l$ to a large (Planck or compactification) scale $M$. It is therefore plausible that $r_e \approx m_l/M$. Some intuition as to the range of possible values for $r_e$ can be found by choosing various values for $m_l$. If $m_l \approx m$ and taking $M \approx M_{\rm Planck}$, we find $r_e \simeq 5 \times 10^{-23}$. If instead $m_l \simeq 250$ GeV, which is of the order of the electroweak scale, then $r_e \simeq 2 \times 10^{-17}$.

We have seen that any existing *CPT* violation generated by $\vec{b}$ would induce a potentially measurable shift between the energy levels of electrons and positrons in a Penning trap. Indeed, the ratio $r_e$ could be bounded in experiments using current techniques. We have investigated several possible experimental procedures that could be adopted. The most effective one would involve taking advantage of the predicted vanishing of the difference $\Delta\omega_c$ in the electron and positron cyclotron frequencies. Since $\omega_a^{\mp}$ both depend on the magnitude of the magnetic field, it would be important to maintain the calibration of $B$ in the measurements of $\Delta\omega_a$. This could be accomplished by using the equality of the cyclotron frequencies to verify that the magnetic field remains the same for both electrons and positrons. The ratio $r_e$ could then be obtained from measurements of $\Delta\omega_a$ at equal values of the magnetic field. These measurements could be repeated using different values of the magnetic field to verify that $\Delta\omega_a$ is independent of the magnitude $B$ for a fixed orientation of the field axis. Since the Penning trap configuration selects the component of $\vec{b}$ in the direction of $\vec{B}$, an additional check would involve looking for diurnal variations in the difference $\Delta\omega_a$.

We can estimate the bound on $r_e$ that could be attained. Suppose the angular anomaly frequencies can be measured to an accuracy of approximately 10 Hz. This would seem feasible, for example, using the line-fitting procedure described in Ref. [4]. At the same time, the equality of the cyclotron frequencies would have to be maintained to an accuracy of one part in $10^8$ to account for possible drifts in the magnetic field. Using Eq. (8), $b_3 = -\Delta\omega_a/4$. Assuming no differences in the angular frequency are observed to this level of precision, then the bound $|b_3| \lesssim 2 \times 10^{-15}$ eV can be obtained. This corresponds to a *CPT* figure of merit of $r_e \lesssim 10^{-20}$ in the electron-positron sector.

This estimate suggests a somewhat tighter bound for $r_e$ would be attainable than that for the corresponding figure of merit $r_K$ arising from experiments with the neutral-kaon system. However, performing the latter tests would continue to be essential because neutral-meson *CPT* violation is controlled by distinct *CPT*-violating parameters appearing in the quark sector. In any event, a bound of the estimated magnitude for $r_e$ in the electron-positron sector would be in line with the greater precision that is experimentally accessible in a Penning trap using measurements of atomic transition frequencies.

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[11] Note that the coefficients $a_\mu$ *cannot* be removed by a gauge transformation on the electron and photon fields because the full Lagrangian, including the term involving $a_\mu$, is gauge invariant. Instead, in any theory with only one fermion field, $a_\mu$ can be eliminated by a field redefinition coupled with a background gauge change. This is equivalent to the redefinition of the energy and momentum as described in the text. These results have been proved in a general context in [10], where it was shown that interactions between two fermion fields can produce effects depending on the difference between couplings in two distinct terms of this type. This is irrelevant for separately trapped electrons or positrons. Incidentally, the effects of the $a_\mu$ term for the electron-positron field are unobservable even in Penning-trap experiments determining the electron-positron mass ratio [12], since the separate cyclotron-frequency measurements for the electron and positron remain unaffected by the $a_\mu$ coupling.

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[13] This always holds if $b_\mu$ is spacelike or lightlike, and it also holds for a large range of timelike $b_\mu$. In fact, a stronger result can be obtained. The axial motion can be incorporated into the analysis via a Foldy-Wouthuysen diagonalization of the full relativistic Hamiltonian allowing for both magnetic and electric fields. The resulting Hamiltonian appears more complicated than the expression in the text; for example, involving an operator momentum $\vec{p}$ rather than the constant linear momentum $p_z$ of Eq. (7). It can be shown by explicit calculation or by using $P$ symmetry that terms linear in $b_0$ produce no contributions to the energy levels $E^{\pm}_{n,s}$. This means experiments are sensitive only to $b_0^2$ at best, and the corresponding limits on $b_0$ would therefore be of more limited interest.

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