Testing CPT invariance with the neutral-D system

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We investigate the issue of testing CPT invariance in the neutral-D system, using D events obtained either from fixed-target experiments or from a τ-charm factory. For both types of experiments we show that the expected suppression of mixing in the D system, normally viewed as a disadvantage for CP tests, allows unsuppressed measurement of certain parameters describing CPT violation. Asymmetries are presented that permit the extraction of parameters for direct CPT violation in the D system and for indirect CPT violation in the K system. We also show that experiments on the neutral-D system provide an alternative means for measuring conventional indirect T violation in the kaon system.

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I. INTRODUCTION

CPT invariance is believed to be a fundamental symmetry of local relativistic point-particle field theories [1–4]. Experimental studies of CPT symmetry probe the foundations of modern particle physics, and they can therefore provide tests of certain alternatives to conventional theories. For example, one measurable signature of an underlying string theory could be a violation of CPT appearing through a mechanism ultimately traceable to string nonlocality [5,6]. Another example is a possible CPT-violating effect that might arise from modifications of quantum mechanics due to quantum gravity [7–9], perhaps in the context of string theory [10].

The most stringent bound on CPT violation, which by one measure is a few parts in 10^{18} [11–13], arises from interferometric tests in the neutral-kaon system. Other neutral-meson systems may also provide interesting CPT bounds. For example, in the string-based scenario for CPT violation, effects could appear at levels accessible to experiment not only in the K system but also in the B and D systems [6]. Since the corresponding sizes of any CPT violation could be different, it is important to bound experimentally the CPT-violating parameters in all these systems.

To date, no experimental bounds have been placed on CPT violation in the B or D systems. The feasibility of placing experimental limits on the parameters for direct and indirect CPT violation in the B system has recently been demonstrated [14]. It is plausible that analysis of existing data could already set interesting bounds, and the B factories currently under construction should provide further improvements.

In contrast, despite significant advances in the understanding of charm physics (for a recent Proceedings, see Ref. [15]), there has been as yet neither an experimental study nor a systematic theoretical treatment of CPT violation in the D system. Indeed, measurements of *any* indirect CP violation in the D system are generally viewed as infeasible. This is because the decay time for the neutral D meson is much less than its characteristic mixing time, so any indirect CP violation is suppressed by a small mixing parameter x. Also, the short D lifetime means that only time-integrated rates can be observed. This raises a second issue: disentangling the various T and CPT effects so the quantities parametrizing CPT violation can be isolated.

In this paper, we address the issues of suppression and disentanglement with a model-independent treatment in the context of conventional quantum mechanics. Our framework can therefore handle the string-based CPT violation discussed in Refs. [5,6]. Additional CPT-violating effects involving some modified form of quantum mechanics might arise from the evolution of pure states into mixed ones in quantum gravity [7–9] or possibly in string theory [10]. One approach to modeling such effects is to modify the Schrödinger equation, which introduces additional parameters. In the context of the kaon system, a simple parametrization involving three additional quantities has been suggested [16,17]. A treatment of this topic in the present context lies outside the scope of the present paper. We restrict ourselves here to the observation that it would be of interest to generalize the present results and those of Ref. [14] to incorporate possible CPT violation in the B and D systems arising from modifications to quantum mechanics.

To address the issue of suppression, we show that the size of the mixing parameter x is irrelevant to the measurement of certain parameters describing direct CPT and indirect T violation. Moreover, the suppression of indirect D-system CP violation can in fact be viewed as an advantage because the D-decay modes to kaons are then accompanied by unsuppressed and measurable indirect K-system CPT and T violation. These results indicate that CP studies in the neutral-D system are of experimental interest despite the small size of the mixing parameter x.

To address the issue of disentangling T and CPT effects, we provide certain asymmetries that separate parameters describing CPT violation. In the analysis, we consider fixed-target experiments producing single tagged neutral D mesons (these would also include tagged D mesons from a B factory) and experiments at a τ-charm factory, which would produce large numbers of correlated $D^0\overline{D}^0$ pairs from the decay of the $\psi(3770)$ resonance. We obtain estimates for bounds on CPT violation that could be obtained from present and future experiments of both classes.

Current experimental limits on direct CP violation in Cabibbo-Kobayashi-Maskawa- (CKM-) suppressed decay modes of the D meson are attaining the 10% level [18].

About 10^5 fully reconstructed charm events already exist [19], and it appears feasible to obtain about 10^8 fully reconstructed D events by the turn of the century using fixed-target and factory experiments [20]. The analysis we present here suggests that some bounds on CPT violation might already be obtained from extant data and that results of experiments over the next few years would yield useful limits.

II. PRELIMINARIES

The eigenvectors of the effective Hamiltonian for the D^0-$\overline{D^0}$ system are[1]

$$|D_S\rangle=[(1+\epsilon_D+\delta_D)|D^0\rangle+(1-\epsilon_D-\delta_D)|\overline{D^0}\rangle]/\sqrt{2},$$
$$|D_L\rangle=[(1+\epsilon_D-\delta_D)|D^0\rangle-(1-\epsilon_D+\delta_D)|\overline{D^0}\rangle]/\sqrt{2}. \quad (1)$$

The CP-violating complex parameters ϵ_D and δ_D are measures of indirect T and indirect CPT violation, respectively. The analogous parameters ϵ_K and δ_K for the K system are defined by Eq. (1) but with the replacement $D\rightarrow K$.

We denote the decay rates of the physical particles D_S, D_L by γ_S, γ_L and their masses by m_S, m_L. Useful combinations of these basic parameters are $\Delta\gamma=\gamma_S-\gamma_L$, $\Delta m=m_L-m_S$, $\gamma=\gamma_S+\gamma_L$, $a^2=\Delta m^2+\Delta\gamma^2/4$, and $b^2=\Delta m^2+\gamma^2/4$.

In the D system, the mixing parameter $x=2\Delta m/\gamma$ is experimentally bounded[2] [21] to $|x|<0.08$. The theoretical value of x is uncertain. Calculations based on standard-model physics [22] suggest that $|x|$ is smaller than 10^{-2}, although the presence of long-distance dispersive effects makes accurate prediction difficult [23,24]. However, extensions to the standard model can generate larger values of x. We therefore keep terms to order x^2 in what follows. Also, in explicit estimates, we take $\Delta\gamma\simeq\Delta m$ for simplicity. Among the effects of scaling $\Delta\gamma$ relative to Δm is the scaling of $\mathrm{Re}\,\delta_D$ relative to $\mathrm{Im}\,\delta_D$. In any event, any such effects are straightforward to calculate from the general expressions we provide below.

Our analysis makes use of two different classes of D-meson decays. The first class, called *semileptonic-type f decays*, includes the usual semileptonic decays along with a special class of other modes $D^0\rightarrow f$ for which there is no lowest-order weak process allowing a significant contamination of either $\overline{D^0}\rightarrow f$ or $D^0\rightarrow\bar{f}$. The currently observed final states f of this type with significant branching ratios are the usual semileptonic ones and those that involve production of a $\bar{K}^*(892)^0$ along with other nonstrange mesons. For the remaining observed modes, a CKM-suppressed process contributes to the contaminating transitions. The second class, called *semileptonic-type* $\overline{K^0}X$ *decays*, includes all final states that contain a $\overline{K^0}$ but do *not* contain a K^0. This means there is no lowest-order weak process for $\overline{D^0}\rightarrow\overline{K^0}X$ or $D^0\rightarrow K^0\bar{X}$. Note that the above requirements for both types of decay are more stringent than imposing only that the conjugate mode is CKM suppressed. For example, the state $f\equiv K^-\pi^+$ is excluded.

[1]Throughout this paper we assume small CP violation, implying small T and CPT violation, and we neglect terms that are higher order in small quantities. Our phase conventions are discussed in more detail in Refs. [6,14].

[2]The analysis leading to this result assumes negligible CPT violation. The value of x could be larger if significant CPT violation is present. Our assumption of small CP violation makes it consistent to take x small also.

III. FIXED-TARGET EXPERIMENTS

This section presents our analysis of rates and asymmetries for fixed-target experiments. We first consider integrated decay rates involving semileptonic-type f decays. The associated transition amplitudes can be parametrized as [25,26]

$$\langle f|T|D^0\rangle=F_f(1-y_f),\quad \langle f|T|\overline{D^0}\rangle=x_fF_f(1-y_f),$$
$$\langle\bar{f}|T|\overline{D^0}\rangle=F_f^*(1+y_f^*),\quad \langle\bar{f}|T|D^0\rangle=\bar{x}_f^*F_f^*(1+y_f^*). \quad (2)$$

The independent quantities F_f, y_f, x_f, and $\bar{x}_f$ are all complex. The latter two vanish if $\Delta C=\Delta Q$, so in what follows we treat them as small. If T invariance holds, all four quantities are real. If CPT invariance holds, $x_f=\bar{x}_f$ and $y_f=0$. The parameter y_f therefore characterizes direct CPT violation in the decay to f and as such is of particular interest here.

Time-integrated rates for semileptonic-type f decays of D mesons can be expressed in terms of the above quantities. Denote by $|D(t)\rangle$ the time-evolved state arising from the state $|D^0\rangle$ at $t=0$ and by $|\overline{D}(t)\rangle$ the state arising from $|\overline{D^0}\rangle$ at $t=0$. Then, there are four time-integrated rates of interest, given by

$$R_f=\int_0^\infty dt|\langle f|T|D(t)\rangle|^2$$
$$=\left|\frac{F_f}{2}\right|^2\left[\gamma\left(\frac{1}{\gamma_S\gamma_L}+\frac{1}{b^2}\right)(1-2\,\mathrm{Re}y_f)-\frac{2\Delta\gamma}{\gamma_S\gamma_L}\mathrm{Re}(2\delta_D+x_f)-\frac{4\Delta m}{b^2}\mathrm{Im}(2\delta_D+x_f)\right], \quad (3)$$

$$\bar{R}_{\bar{f}}=\int_0^\infty dt|\langle\bar{f}|T|\overline{D}(t)\rangle|^2$$
$$=R_f(y_f\rightarrow -y_f,\delta_D\rightarrow -\delta_D,x_f\rightarrow\bar{x}_f^*, \quad (4)$$

$$R_{\bar{f}}=\int_0^\infty dt|\langle\bar{f}|T|D(t)\rangle|^2$$
$$=\left|\frac{F_f}{2}\right|^2\left[\gamma\left(\frac{1}{\gamma_S\gamma_L}-\frac{1}{b^2}\right)[1-2\,\mathrm{Re}(2\epsilon_D-y_f)]-\frac{2\Delta\gamma}{\gamma_S\gamma_L}\mathrm{Re}\bar{x}_f-\frac{4\Delta m}{b^2}\mathrm{Im}\bar{x}_f\right], \quad (5)$$

$$\bar{R}_f=\int_0^\infty dt|\langle f|T|\overline{D}(t)\rangle|^2$$
$$=R_{\bar{f}}(y_f\rightarrow -y_f,\epsilon_D\rightarrow -\epsilon_D,\bar{x}_f\rightarrow x_f^*). \quad (6)$$

The expressions for the rates $\bar{R}_{\bar{f}}$ and $\bar{R}_f$ are obtained by making the indicated replacements in R_f and $R_{\bar{f}}$, respectively.

From these rates, we can extract an asymmetry providing information about direct CPT violation. It is

$$A_f \equiv \frac{R_f-\bar{R}_{\bar{f}}}{R_f+\bar{R}_{\bar{f}}} = -2\ \mathrm{Re}y_f - \frac{1}{\gamma(b^2+\gamma_S\gamma_L)}\{\Delta\gamma b^2[4\ \mathrm{Re}\delta_D+\mathrm{Re}(x_f-\bar{x}_{\bar{f}})]+2\Delta m\gamma_S\gamma_L[4\ \mathrm{Im}\delta_D+\mathrm{Im}(x_f+\bar{x}_{\bar{f}})]\}$$

$$\simeq -2\ \mathrm{Re}y_f - x[\mathrm{Re}\delta_D+\tfrac{1}{4}\ \mathrm{Re}(x_f-\bar{x}_{\bar{f}})+2\ \mathrm{Im}\delta_D+\tfrac{1}{2}\ \mathrm{Im}(x_f+\bar{x}_{\bar{f}})]. \quad (7)$$

The first term in this expression is a measure of direct CPT violation in the D system. The remaining terms are suppressed by the mixing parameter x. If we further assume that violations of $\Delta C=\Delta Q$ are independent of CP violation, then $x_f=\bar{x}_{\bar{f}}^*$. This makes the asymmetry A_f independent of the particular final state.

Another asymmetry can be formed as

$$A_f' \equiv \frac{\bar{R}_f-R_{\bar{f}}}{\bar{R}_f+R_{\bar{f}}} \simeq 2\ \mathrm{Re}(2\epsilon_D-y_f). \quad (8)$$

In deriving the latter expression, we have taken violations of $\Delta C=\Delta Q$ to vanish for simplicity. This asymmetry is of lesser interest than that of Eq. (7) because the rates themselves are suppressed by at least one power of x. This means the statistics required for observation of this asymmetry are increased by a factor of at least $1/x^2$, so we disregard it in the following.

Next, we consider integrated decay rates involving semileptonic-type $\overline{K^0}X$ decays. Define transition amplitudes for these processes analogous to the definitions in Eq. (2):

$$\langle\overline{K^0}|T|D^0\rangle=F_K(1-y_K), \quad \langle\overline{K^0}|T|\overline{D^0}\rangle=x_KF_K(1-y_K),$$

$$\langle K^0|T|\overline{D^0}\rangle=F_K^*(1+y_K^*), \quad \langle K^0|T|D^0\rangle=\bar{x}_K^*F_K^*(1+y_K^*). \quad (9)$$

To simplify notation, in the above amplitudes the symbols $\langle K^0|$ and $\langle\overline{K^0}|$ are used to represent the full multiparticle final states containing the corresponding K meson. The complex parameters x_K, $\bar{x}_K$, F_K, and y_K depend on the specific final state involved. They have the same properties under T and CPT invariance as the analogous parameters for semileptonic-type f decays. Note that x_K and $\bar{x}_K$ are taken to be small because the associated amplitudes have no lowest-order weak contribution.

Observable final states involve K_S and K_L rather than K^0 and $\overline{K^0}$. We use the notation $\langle K_S|$ and $\langle K_L|$ to represent the linear combinations of the semileptonic-type $\overline{K^0}X$ final state and its charge conjugate given by the kaon equivalent of Eq. (1). With these amplitudes, there are again four integrated rates:

$$R_S=\int_0^\infty dt|\langle K_S|T|D(t)\rangle|^2=\frac{\mathrm{Re}^2F_K}{2}\left\{\frac{1}{\gamma_S}+\left(\frac{2}{\gamma_S}-\frac{\gamma}{b^2}\right)\mathrm{Re}\left(\delta_D-\epsilon_D+\frac{1}{2}x_K\right)+\left(\frac{2}{\gamma_S}+\frac{\gamma}{b^2}\right)\mathrm{Re}\ \frac{1}{2}\bar{x}_K-\frac{\gamma}{b^2}\ \mathrm{Re}(\epsilon_K+\delta_K+y_K)\right.$$

$$\left.+\frac{2\Delta m}{b^2}\left[\mathrm{Im}\left(\epsilon_K+\delta_K-\delta_D+\epsilon_D-\frac{1}{2}(\bar{x}_K+x_K)\right)+\frac{\mathrm{Im}F_K}{\mathrm{Re}F_K}\right]\right\}, \quad (10)$$

$$\bar{R}_S=\int_0^\infty dt|\langle K_S|T|\overline{D}(t)\rangle|^2=R_S(\epsilon_D\to-\epsilon_D^*,\delta_D\to-\delta_D^*,y_K\to-y_K,x_K\leftrightarrow\bar{x}_K,\epsilon_K\to-\epsilon_K^*,\delta_K\to-\delta_K^*), \quad (11)$$

$$R_L=\int_0^\infty dt|\langle K_L|T|D(t)\rangle|^2=\frac{\mathrm{Re}^2F_K}{2}\left\{\frac{1}{\gamma_L}-\left(\frac{2}{\gamma_L}-\frac{\gamma}{b^2}\right)\mathrm{Re}\left(\delta_D+\epsilon_D+\frac{1}{2}x_K\right)-\left(\frac{2}{\gamma_L}+\frac{\gamma}{b^2}\right)\mathrm{Re}\ \frac{1}{2}\bar{x}_K-\frac{\gamma}{b^2}\ \mathrm{Re}(\epsilon_K-\delta_K+y_K)\right.$$

$$\left.+\frac{2\Delta m}{b^2}\left[\mathrm{Im}\left(\epsilon_K-\delta_K+\delta_D+\epsilon_D+\frac{1}{2}(\bar{x}_K+x_K)\right)+\frac{\mathrm{Im}F_K}{\mathrm{Re}F_K}\right]\right\}, \quad (12)$$

$$\bar{R}_L=\int_0^\infty dt|\langle K_L|T|\overline{D}(t)\rangle|^2=R_L(\epsilon_D\to-\epsilon_D^*,\delta_D\to-\delta_D^*,y_K\to-y_K,x_K\leftrightarrow\bar{x}_K,\epsilon_K\to-\epsilon_K^*,\delta_K\to-\delta_K^*). \quad (13)$$

In the above a double arrow $\leftrightarrow$ is used to indicate an interchange of parameters rather than a substitution.

From these integrated rates, two useful asymmetries can be constructed:

$$A_S\equiv\frac{\bar{R}_S-R_S}{\bar{R}_S+R_S}=2\left(1-\frac{\gamma\gamma_S}{2b^2}\right)\mathrm{Re}(\epsilon_D-\delta_D)-\frac{\gamma\gamma_S}{2b^2}\ \mathrm{Re}(\bar{x}_K-x_K)+\frac{\gamma\gamma_S}{b^2}\ \mathrm{Re}(\epsilon_K+\delta_K+y_K)$$

$$\simeq 2(1+\tfrac{1}{2}x-x^2)\mathrm{Re}[\epsilon_K+\delta_K+y_K-\tfrac{1}{2}(\bar{x}_K-x_K)]-x(1-2x)\mathrm{Re}(\epsilon_D-\delta_D), \quad (14)$$

$$A_L \equiv \frac{\bar{R}_L - R_L}{\bar{R}_L + R_L} = 2\left(1 - \frac{\gamma\gamma_L}{2b^2}\right)\mathrm{Re}(\epsilon_D + \delta_D) + \frac{\gamma\gamma_L}{2b^2}\mathrm{Re}(\bar{x}_K - x_K) + \frac{\gamma\gamma_L}{b^2}\mathrm{Re}(\epsilon_K - \delta_K + y_K)$$

$$\simeq 2(1 - \tfrac{1}{2}x - x^2)\mathrm{Re}[\epsilon_K - \delta_K + y_K + \tfrac{1}{2}(\bar{x}_K - x_K)] + x(1+2x)\mathrm{Re}(\epsilon_D + \delta_D). \tag{15}$$

Their difference gives the combination

$$A_L - A_S = -4\ \mathrm{Re}\delta_K + 2\ \mathrm{Re}(\bar{x}_K - x_K) + 2x\ \mathrm{Re}(\epsilon_D - \epsilon_K - y_K) + 4x^2\ \mathrm{Re}[\delta_D + \delta_K - \tfrac{1}{2}(\bar{x}_K - x_K)], \tag{16}$$

while their sum is

$$A_L + A_S = 4\ \mathrm{Re}(\epsilon_K + y_K) + 2x\ \mathrm{Re}[\delta_D + \delta_K - \tfrac{1}{2}(\bar{x}_K - x_K)] + 4x^2\ \mathrm{Re}(\epsilon_D - \epsilon_K - y_K). \tag{17}$$

Assuming violations of $\Delta C = \Delta Q$ are independent of CPT violation, the terms containing $\bar{x}_K - x_K$ vanish. In any event, for negligible mixing x these expressions reduce to their first terms.

IV. EXPERIMENTS AT A τ-CHARM FACTORY

In this section, we turn to a discussion of rates and asymmetries for experiments that are feasible at a τ-charm factory. Several relevant integrated rates arise. One is the time-integrated rate $\Gamma(f_1,f_2)$ for the decay of the correlated $D^0\bar{D}^0$ pair into states f_1 and f_2. In discussing other rates, it is convenient to separate the rate $\Gamma(f_1,f_2)$ into the component $\Gamma^+(f_1,f_2)$ for which the decay into f_1 occurs first and the component $\Gamma^-(f_1,f_2)$ for which f_2 occurs first. Then, another useful quantity is the inclusive rate $\Gamma^+_{\mathrm{incl}}(f_1)$, obtained by summing $\Gamma^+(f_1,f_2)$ over final states f_2. We find

$$\Gamma(f_1,f_2) = \frac{1}{2\gamma_S\gamma_L}\left[|a_{1S}a_{2L}|^2 + |a_{1L}a_{2S}|^2 - \frac{\gamma_S\gamma_L}{b^2}(a^*_{1S}a^*_{2L}a_{1L}a_{2S} + \text{c.c.})\right], \tag{18}$$

$$\Gamma^+_{\mathrm{incl}}(f_1) = \sum_{f_2}\Gamma^+(f_1,f_2) = \frac{1}{2\gamma}\{|a_{1S}|^2 + |a_{1L}|^2 - 2[a^*_{1S}a_{1L}(\mathrm{Re}\epsilon_D + i\ \mathrm{Im}\delta_D) + \text{c.c.}]\}, \tag{19}$$

where the transition amplitudes are defined as

$$a_{\alpha S} = \langle f_\alpha|T|D_S\rangle, \quad a_{\alpha L} = \langle f_\alpha|T|D_L\rangle. \tag{20}$$

Further details about calculating these rates may be found in Refs. [6,14].

We begin by considering semileptonic-type f decays. The parameter $\mathrm{Re}y_f$ describing direct CPT violation can be extracted from the above rates using the asymmetry

$$A^+_f \equiv \frac{\Gamma^+_{\mathrm{incl}}(f) - \Gamma^+_{\mathrm{incl}}(\bar{f})}{\Gamma^+_{\mathrm{incl}}(f) + \Gamma^+_{\mathrm{incl}}(\bar{f})} = -2\ \mathrm{Re}y_f. \tag{21}$$

We see that this asymmetry provides a test of direct CPT violation that is independent of any T or indirect CPT violation.

Other asymmetries with suppressed component rates or suppressed magnitudes can be examined by restricting attention to semileptonic-type f decays in both channels. An asymmetry isolating parameters for indirect CPT violation in the D system can be constructed as

$$A_{f,\bar{f}} \equiv \frac{\Gamma^+(f,\bar{f}) - \Gamma^-(f,\bar{f})}{\Gamma^+(f,\bar{f}) + \Gamma^-(f,\bar{f})} = 4\frac{b^2\Delta\gamma\ \mathrm{Re}\delta_D + 2\Delta m\,\gamma_S\gamma_L\ \mathrm{Im}\delta_D}{\gamma(b^2 + \gamma_S\gamma_L)} \simeq x(\mathrm{Re}\delta_D + 2\ \mathrm{Im}\delta_D). \tag{22}$$

The derivation assumes violations of $\Delta C = \Delta Q$ are independent of CP violation, so that $x_f = \bar{x}^*_f$. The x dependence of the result means that the number of events required to measure a nonzero value for $A_{f,\bar{f}}$ is scaled by $1/x^2$. Another possible asymmetry is

$$A^{\mathrm{tot}}_{f,\bar{f}} \equiv \frac{\Gamma(f,f) - \Gamma(\bar{f},\bar{f})}{\Gamma(f,f) + \Gamma(\bar{f},\bar{f})} = 4\ \mathrm{Re}(\epsilon_D - y_f). \tag{23}$$

In this case, the rates themselves are suppressed by a factor of x^2, so the required statistics are scaled by $1/x^4$.

Next, we consider the case with one channel involving a semileptonic-type K_SX or K_LX decay while the other channel involves a semileptonic-type f decay. Transition amplitudes for the $\bar{K}X$ decays are defined in Eq. (9). With these, we obtain

$$\eta_{K_S} \equiv \frac{\langle K_S|T|D_L\rangle}{\langle K_S|T|D_S\rangle} = -\epsilon^*_K + \epsilon_D - \delta^*_K - \delta_D - \frac{\mathrm{Re}(F_K y_K)}{\mathrm{Re}F_K} - \frac{1}{2\ \mathrm{Re}F_K}(x_K F_K - \bar{x}^*_K F^*_K)$$
$$+ i\frac{\mathrm{Im}F_K}{\mathrm{Re}F_K}\left[1 - i\frac{\mathrm{Im}F_K}{\mathrm{Re}F_K}(-\epsilon^*_K + \epsilon_D - \delta^*_K + \delta_D) - \frac{1}{2\ \mathrm{Re}F_K}(x_K F_K + \bar{x}^*_K F^*_K) + i\frac{\mathrm{Im}(F_K y_K)}{\mathrm{Re}F_K}\right], \tag{24}$$

$$\eta_{K_L}\equiv\frac{\langle K_L|T|D_S\rangle}{\langle K_L|T|D_L\rangle}=\eta_{K_S}(\delta_D\to-\delta_D,\delta_K\to-\delta_K,x_K\to-x_K,\bar{x}_K\to-\bar{x}_K). \qquad (25)$$

Using these, we calculate two useful rate asymmetries:

$$A_{f,K_S}\equiv\frac{\Gamma(f,K_S)-\Gamma(\bar{f},K_S)}{\Gamma(f,K_S)+\Gamma(\bar{f},K_S)}$$
$$=2\ \mathrm{Re}(\epsilon_D-y_f-\delta_D)-\frac{2\gamma_S\gamma_L}{b^2}\mathrm{Re}(\eta_{K_S}), \qquad (26)$$

and

$$A_{f,K_L}\equiv A_{f,K_S}(K_S\to K_L)=A_{f,K_S}(\delta_D\to-\delta_D,\eta_{K_S}\to\eta_{K_L}). \qquad (27)$$

These two asymmetries have been derived under the assumption that violations of $\Delta C=\Delta Q$ are independent of CPT violation, so that $x_f=\bar{x}_f$ and $x_K=\bar{x}_K$. Since $\mathrm{Im}F_K$ controls the direct T violation in these processes, we have also treated it as a small quantity. This latter assumption is made more plausible by the current experimental bounds at about the 10% level on parameters describing direct T violation [18]. The above expressions can be approximated for the D system as

$$A_{f,K_S}\simeq-2\ \mathrm{Re}y_f+2\ \mathrm{Re}(\epsilon_K+\delta_K+y_K)+\tfrac{5}{2}x^2\ \mathrm{Re}(\epsilon_D-\delta_D), \qquad (28)$$

$$A_{f,K_L}\simeq-2\ \mathrm{Re}y_f+2\ \mathrm{Re}(\epsilon_K-\delta_K+y_K)+\tfrac{5}{2}x^2\ \mathrm{Re}(\epsilon_D+\delta_D). \qquad (29)$$

The difference between the two above equations is a function of CPT-violating parameters:[3]

$$A^-_{L,S}\equiv A_{f,K_L}-A_{f,K_S}\simeq-4\ \mathrm{Re}\delta_K+5x^2\ \mathrm{Re}\delta_D. \qquad (30)$$

The coefficient of $\mathrm{Re}\delta_D$ is of order no larger than 10^{-4}. The second term can therefore be neglected, and we are left with an asymmetry measuring the parameter $\mathrm{Re}\delta_K$ for indirect CPT violation in the K system. This result is independent of either of the final states, so the statistics can be made more favorable by summing over the class of relevant final states.

The sum of the two asymmetries gives the combination

$$A^+_{L,S}\equiv A_{f,K_L}+A_{f,K_S}\simeq4\ \mathrm{Re}(\epsilon_K+y_K-y_f)+5x^2\ \mathrm{Re}\epsilon_D. \qquad (31)$$

Here, the parameter measuring indirect T violation in the neutral-D system is suppressed by the factor x^2, thereby producing an asymmetry measuring the combination $\mathrm{Re}(\epsilon_K+y_K-y_f)$. Note that this quantity depends on both final states through the direct CPT-violation parameters y_K and y_f.

[3]It can be shown that Eq. (30) is in fact correct to terms simultaneously quadratic in $\mathrm{Im}F_K$ and linear in δ_D or δ_K even if the constraint of small direct T violation is relaxed.

V. ESTIMATES OF BOUNDS ATTAINABLE

The asymmetries given in the previous two sections demonstrate that CPT information can be extracted from the D system. Next, we investigate the bounds attainable using these asymmetries. For fixed-target experiments, we estimate the number of D^0 particles required to reduce the error in a given asymmetry to one standard deviation. For measurements at a τ-charm factory, we estimate the number of $\psi(3770)$ events required for a similar precision. The analysis follows the methods presented in Refs. [27,6,14].

Assuming a binomial event distribution and a general asymmetry $A=(N_+-N_-)/(N_++N_-)$, observation of a nonzero $\langle A\rangle$ at the $N\sigma$ level requires an expected number of events $\langle N_+\rangle=N^2(1+\langle A\rangle)(1-\langle A\rangle^2)/2\langle A\rangle^2$. Conversion of this to an expected number of D^0 events required in a fixed-target experiment involves multiplication by the inverse branching ratio for the D^0 decay into the relevant final state. Similarly, conversion to an expected number of $\psi(3770)$ events required in a τ-charm factory requires multiplication by two, to incorporate the branching ratio of $\psi(3770)$ into two neutral D mesons, and by the inverse branching ratio for the further decays into the relevant final states. Interference effects in the correlated decays can be neglected because T and CPT violation is assumed small.

We first consider the asymmetries arising from the fixed-target experiments discussed in Sec. III.

For the asymmetry A_f given by Eq. (7), assuming sufficient suppression by x, the number of D^0 events required to measure $\mathrm{Re}y_f$ to within one standard deviation σ is

$$N_D(\mathrm{Re}y_f)\simeq\frac{1}{8\sigma^2B(D^0\to f)}. \qquad (32)$$

The asymmetries A_S and A_L given in Eqs. (14) and (15) have the same general form as A_f, so the number of D^0 events needed to reduce the error in either to one standard deviation is given by an expression analogous to Eq. (32). However, the interesting information is contained in their sum and difference $A_L\pm A_S$ given in Eqs. (17) and (16). Combining errors in quadrature gives the number of D particles required to reduce the error in $A_L\pm A_S$ to one standard deviation as

$$N_D(A_L\pm A_S)\simeq\frac{1}{\sigma^2B(D^0\to\overline{K^0}+\mathrm{any})}. \qquad (33)$$

In particular, for small x we obtain

$$N_D(\mathrm{Re}\delta_K)\simeq N_D[\mathrm{Re}(\epsilon_K+y_K)]\simeq\frac{1}{16\sigma^2B(D^0\to\overline{K^0}+\mathrm{any})}. \qquad (34)$$

The branching ratios relevant to the above neutral-D decay are typically of the order of several percent. Disregarding (potentially important) experimental effects, the present availability of 10^5 reconstructed events suggests that bounds of the order of 10^{-2} to 10^{-3} could be placed on both direct CPT violation in the D system [using Eq. (32)] and indirect CPT violation in the K system [using Eq. (34)]. Direct bounds at this level would already be of interest, and the possibility of substantially increased numbers of reconstructed D events suggest significant improvement could be expected in the near future.

Next, we turn to bounds arising from experiments at a τ-charm factory.

The parameter $\mathrm{Re}y_f$ determining direct CPT violation can be measured using A_f^+ given in Eq. (21). The second final state is unrestricted, so it suffices to multiply by the inverse branching ratio for the process $D^0 \to f$. However, the asymmetry involves only those events for which the decay into f occurs first. Since $\gamma_S \approx \gamma_L$ in the D system, events with the decay into f occurring first are roughly equally frequent as those with the decay occurring second. An extra factor of two is therefore needed. Collecting these factors, we find that the number $N_{\psi(3770)}(\mathrm{Re}y_f)$ of $\psi(3770)$ events needed to reduce the error in $\mathrm{Re}y_f$ to within one standard deviation σ is

$$N_{\psi(3770)}(\mathrm{Re}y_f) \simeq \frac{1}{2\sigma^2 B(D^0 \to f)}. \tag{35}$$

The combination $A_{L,S}^-$ of asymmetries given by Eq. (30) measures $\mathrm{Re}\delta_K$, the parameter for indirect CPT violation in the K system. The independence of $A_{L,S}^-$ on the specific semileptonic-type f decay means that the corresponding branching ratios can be added. Using Ref. [21], we get $\Sigma_f B(D^0 \to f) \simeq 27\%$. The quantity $A_{L,S}^-$ is also independent of the specific final-state particles that appear with the K_S or K_L in the other channel in a general semileptonic-type $\overline{K^0}X$ final state. Summing over these gives $\Sigma_X B(D^0 \to \overline{K^0}X) \simeq 38\%$. Furthermore, in determining the final result it is reasonable to take as roughly equal the errors in the two asymmetries A_{f,K_L} and A_{f,K_S}. Collecting this information, we find that the number $N_{\psi(3770)}(\mathrm{Re}\delta_K)$ of $\psi(3770)$ events needed to reduce the error in $\mathrm{Re}\delta_K$ to within one standard deviation σ is

$$N_{\psi(3770)}(\mathrm{Re}\delta_K) \simeq \frac{1}{\sigma^2}. \tag{36}$$

This result is somewhat more favorable than the corresponding fixed-target result, Eq. (34).

The sum of the asymmetries $A_{L,S}^+$ measuring the combination $\mathrm{Re}(\epsilon_K + y_K - y_f)$, given in Eq. (31), depends on both final states. Otherwise the estimation is the same as for $A_{L,S}^-$ given above. Therefore, we get

$$N_{\psi(3770)}[\mathrm{Re}(\epsilon_K + y_K - y_f)] \simeq \frac{1}{8\sigma^2 B(D^0 \to \overline{K^0} + \mathrm{any}) B(D^0 \to f)}. \tag{37}$$

In this case, the result is less favorable than the corresponding fixed-target results, Eqs. (32) and (34), because two (relatively small) branching ratios enter instead of one.

Equations (34) and (37) involve ϵ_K, which measures indirect T violation in the K system, along with the parameters y_f and y_K for direct CPT violation in the D system. These equations can therefore be used either as measurements of direct CPT violation if ϵ_K is taken from other experiments in the K system, or as measurements providing a new check on ϵ_K if CPT violation is assumed small. In the latter case, the possibility of 10^8 reconstructed D events could in principle produce a measurement of ϵ_K to 10^{-4} or so.[4]

In the event that the magnitude of x is relatively close to the current experimental limit, as could happen in extensions of the standard model, it may be possible to use the above asymmetries to extract the parameters $\mathrm{Re}\delta_D$ and $\mathrm{Im}\delta_D$ describing D-system CPT violation and therefore to provide a test of the string-inspired relation [6]

$$\mathrm{Re}\delta_D = \pm \frac{2\Delta m}{\Delta \gamma} \mathrm{Im}\delta_D. \tag{38}$$

For illustrative purposes, we now neglect y_f, x_f, $\bar{x}_f$, and terms involving second and higher powers of x in the asymmetries, and we suppose ϵ_K is known to sufficient precision from other experiments. From Eq. (7) for the asymmetry A_f, summing over final states f we find

$$N_D(\mathrm{Re}\delta_D + 2\ \mathrm{Im}\delta_D) \simeq \frac{3}{2x^2\sigma^2} \simeq \frac{600}{\sigma^2}, \tag{39}$$

where in the final form we have taken a value $x \simeq 0.05$ close to the maximum possible. Similarly, summing over final states $\overline{K}X$ in Eq. (17) gives

$$N_D(\mathrm{Re}\delta_D) \simeq \frac{1}{2x^2\sigma^2} \simeq \frac{200}{\sigma^2}. \tag{40}$$

Finally, summing over final states in Eq. (22) gives

$$N_{\psi(3770)}(\mathrm{Re}\delta_D + 2\ \mathrm{Im}\delta_D) \simeq \frac{9}{x^2\sigma^2} \simeq \frac{3600}{\sigma^2}. \tag{41}$$

These equations show that both $\mathrm{Re}\delta_D$ and $\mathrm{Im}\delta_D$ can in principle be extracted if conditions are favorable. For this purpose, fixed-target experiments are somewhat better from the theoretical viewpoint. Under these circumstances, interesting bounds could already be placed on δ_D with existing data.

VI. SUMMARY

In this paper, the possibility of testing CPT invariance in the neutral-D system has been examined. We give asymmetries relevant to this issue that can be obtained from data taken at present and future fixed-target and factory experiments. They permit the determination of certain parameters governing CPT violation in the D and K systems. Unsuppressed measurements of direct D-system CPT violation are

[4]The possibility of measuring ϵ_K using reconstructed *charged* $D^{\pm}$ events has recently been suggested [28].

feasible. Moreover, suppression by the mixing parameter x of indirect D-system CPT violation makes unsuppressed measurements of indirect K-system CPT violation possible too. Under particularly favorable circumstances, indirect D-system CPT violation may also be measurable.

In the fixed-target case, assuming small x, Eq. (7) gives the parameter $\mathrm{Re}y_f$ controlling direct CPT violation in the D system. Similarly, Eq. (16) gives $\mathrm{Re}\delta_K$, involving indirect K-system CPT violation, and Eq. (17) gives the combination $\mathrm{Re}(\epsilon_K+y_K)$ of quantities measuring indirect K-system T violation and direct D-system CPT violation. All these asymmetries also contain terms higher than second order in the small mixing parameter x, involving indirect CP violation in the D system.

For experiments at a τ-charm factory, Eq. (21) gives $\mathrm{Re}y_f$, Eq. (30) gives $\mathrm{Re}\delta_K$, and Eq. (31) gives the combination $\mathrm{Re}(\epsilon_K+y_K-y_f)$. Again, parameters for indirect CPT violation in the D system are suppressed by some power of x. The only asymmetries requiring knowledge of the sign of the time difference between the two decays of the correlated pair are those in Eqs. (21) and (22).

For both types of experiment, excluding possible background or acceptance issues, estimates of the bounds attainable are given in Sec. IV. From a purely theoretical perspective, fixed-target experiments are preferable for measuring direct CPT violation because only one decay channel is involved. In contrast, experiments at a τ-charm factory are preferable for measuring indirect K-system CPT violation because the correlations between the D pairs make possible a sum over decay channels. In any event, interesting bounds on various types of CPT violation are attainable in the neutral-D system, using data already existing or likely to become available within a few years.

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