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Perturbative Lorentz and *CPT* violation for neutrino and antineutrino oscillations

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The effects of perturbative Lorentz and *CPT* violation on neutrino oscillations are studied. Features include neutrino-antineutrino oscillations, direction dependence, and unconventional energy behavior. Leading-order corrections arising from renormalizable operators are derived in the general three-flavor effective field theory. The results are applied to neutrino-beam experiments with long baselines, which offer excellent sensitivity to the accompanying effects. Key signatures of Lorentz and *CPT* violation using neutrino beams include sidereal variations in the oscillation probabilities arising from the breakdown of rotational symmetry, and *CPT* asymmetries comparing neutrino and antineutrino modes. Attainable sensitivities to coefficients for Lorentz violation are estimated for several existing and future experiments.

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I. INTRODUCTION

Experimental investigations of neutrino properties have provided crucial insights into particle physics since the existence of neutrinos was first proposed in 1930 by Pauli [1] to explain the spectrum of beta decay. In recent years, the confirmed observation of neutrino oscillations has established the existence of physics beyond the minimal standard model (SM) [2]. The interferometric nature of the oscillations makes them highly sensitive to new physics, including potential low-energy signals that may originate in a fundamental theory unifying quantum physics and gravity at the Planck scale $m_P \simeq 10^{19}$ GeV.

In this work, we investigate the experimental implications for neutrino oscillations of Lorentz and *CPT* violation, which is a promising category of Planck-scale signals [3]. The SM is known to provide a successful description of observed phenomena at energies well below m_P. As a consequence, the manifestation of Planck-scale effects involving Lorentz and *CPT* violation is expected to be well described at accessible energy scales by an effective field theory containing the SM [4,5].

The comprehensive effective field theory describing general Lorentz violation at attainable energies is the standard-model extension (SME) [6,7]. It incorporates both the SM and general relativity, serving as a realistic theory for analyses of experimental data. In the SME Lagrange density, each Lorentz-violating term is an observer scalar density constructed as the product of a Lorentz-violating operator with a controlling coefficient. Under mild assumptions, *CPT* violation in effective field theory is accompanied by Lorentz violation [8], so the SME also describes general breaking of *CPT* symmetry. These ideas have triggered a wide variety of tests over the past decade [9]. Several experimental searches have been performed using neutrino oscillations, yielding high sensitivities to SME coefficients for Lorentz and *CPT* violation [10–12].

Since both Lorentz-violating operators and mass terms can induce neutrino mixing, one way to classify neutrino models with Lorentz and *CPT* violation is in terms of their neutrino-mass content. Three categories exist: massless Lorentz-violating models, in which no neutrinos have mass; hybrid Lorentz-violating models, with mass terms for a subset of neutrinos; and massive Lorentz-violating models, where all neutrinos have conventional masses.

In massless Lorentz-violating models, all observed neutrino oscillations are attributed to nonzero coefficients for Lorentz violation rather than to masses. Certain coefficients can combine via a Lorentz-seesaw mechanism to produce pseudomasses that mimic the behavior of mass terms for a range of neutrino energies [13]. The prototypical example is the bicycle model [14] which uses two nonzero coefficients for Lorentz violation to reproduce the expected behavior of atmospheric neutrinos. This model agrees well with atmospheric data from the Super-Kamiokande experiment [11]. However, a combined analysis of neutrino data excludes both the bicycle model in its simplest form and a five-parameter generalization [15]. Massless models may also predict sidereal signals arising from the violation of rotation invariance [16]. The corresponding coefficients for Lorentz violation have been constrained in experimental analyses by the Liquid Scintillator Neutrino Detector (LSND) [10] and the Main Injector Neutrino Oscillation Search (MINOS) [12]. At present, it is an interesting open challenge to construct a massless Lorentz-violating model that is globally compatible with existing neutrino data.

For hybrid Lorentz-violating models, the tandem model [17] is the sole exemplar. It is a three-parameter model containing one neutrino mass, one coefficient for *CPT*-even Lorentz violation, and one coefficient for *CPT*-odd Lorentz violation. The model appears globally compatible with existing experimental data, including the LSND anomaly [18]. The tandem model predicted a low-energy excess in the Mini Booster Neutrino Experiment

(MiniBooNE) prior to its discovery, although the observed excess is quantitatively greater [19]. This success suggests that further theoretical investigations of hybrid Lorentz-violating models would be of definite interest.

Massive Lorentz-violating models are the primary focus of the present work. Most existing data from neutrino oscillations are consistent with oscillation phases proportional to the baseline L and inversely proportional to the energy E. This is conventionally interpreted as a consequence of mixing induced by a nondegenerate mass matrix. In massive Lorentz-violating models, the mixing due to mass is assumed to dominate over that due to Lorentz violation. Our goal here is to present a general study of perturbative Lorentz and CPT violation on mass-induced mixing, valid over a wide range of L and E.

The analysis presented here incorporates all coefficients for Lorentz violation associated with quadratic operators of renormalizable dimension in the neutrino sector [13]. Using notation reviewed in Sec. II, these coefficients are $(a_L)^\alpha_{ab}$, $(c_L)^{\alpha\beta}_{ab}$, $\tilde{g}^{\alpha\beta}_{a\bar{b}}$, and $\tilde{H}^\alpha_{a\bar{b}}$. Both $(a_L)^\alpha_{ab}$ and $\tilde{g}^{\alpha\beta}_{a\bar{b}}$ also control CPT violation. Taken alone, the coefficients $(a_L)^\alpha_{ab}$ and $\tilde{H}^\alpha_{a\bar{b}}$ generate oscillation phases proportional to L but independent of E, while $(c_L)^{\alpha\beta}_{ab}$ and $\tilde{g}^{\alpha\beta}_{a\bar{b}}$ produce phases proportional to the product LE. This indicates that experiments with long baselines or high energies are of special interest for studies of massive Lorentz-violating models. However, the techniques outlined in this work apply for any baseline for which the perturbative approximation is valid, including ones where oscillations due to mass are negligible. Indeed, the expressions for oscillation probabilities presented here reduce to those obtained for massless Lorentz-violating models [16] in the limit of vanishing mass mixing.

Massive Lorentz-violating models can exhibit effects lying in any of the six classes of physical effects due to Lorentz and CPT violation [13]. All coefficients affect the spectral dependence in at least some part of the energy range. Many of the associated operators violate rotation invariance, which can produce direction-dependent oscillations. For some experiments, including ones with neutrino beams, the daily rotation of the Earth induces variations in time of the probabilities at multiples of the sidereal frequency. Both CPT violation and neutrino-antineutrino mixings can occur.

In this work, we show that for massive Lorentz-violating models the coefficients $(a_L)^\alpha_{ab}$ and $(c_L)^{\alpha\beta}_{ab}$ primarily affect neutrino-neutrino and antineutrino-antineutrino mixings, with $(a_L)^\alpha_{ab}$ controlling first-order differences between the two mixings due to perturbative CPT violation. Since the original introduction of these SME coefficients [6], a substantial theoretical literature has developed concerning their implication for neutrino behavior in the context of massive Lorentz-violating models. Many works restrict attention to the special isotropic limits with only $(a_L)^T_{ab}$ or $(c_L)^{TT}_{ab}$ nonzero and real [20–33], and in some cases also to two flavors. A few consider also anisotropic effects [13,34–38]. Here, we treat the general case, allowing all components of $(a_L)^\alpha_{ab}$ and $(c_L)^{\alpha\beta}_{ab}$ to be nonzero.

In contrast, the dominant effects from $\tilde{g}^{\alpha\beta}_{a\bar{b}}$ and $\tilde{H}^\alpha_{a\bar{b}}$ in massive Lorentz-violating models arise only at second order. They involve neutrino-antineutrino mixing and also nonconservation of lepton number. A single flavor can therefore suffice to produce oscillations. Indeed, a simple analytical form is known for the mixing probability for the general one-flavor case including mass [13]. A few theoretical works have considered special massive Lorentz-violating models of this type [39,40]. At present, there are no published experimental constraints on any of the coefficients $\tilde{g}^{\alpha\beta}_{a\bar{b}}$ and $\tilde{H}^\alpha_{a\bar{b}}$. In this work, we investigate the general case and identify some potential signals for experimental searches.

Overall, most massive Lorentz-violating models remain viable. Only a few percent of the available coefficient space has been explored experimentally [10–12]. The methods described in the present work demonstrate that access to essentially the whole coefficient space is available via a combination of existing and future experiments.

This paper is organized as follows. The basic theory and notation is presented in Sec. II. Section II A reviews the properties of the Hamiltonian governing Lorentz and CPT violation in neutrino oscillations. The perturbation series for the transition amplitude is derived in Sec. II B, while the resulting oscillation probabilities are obtained in Sec. II C. Section III considers first-order effects involving the coefficients $(a_L)^\alpha$ and $(c_L)^{\alpha\beta}$. The directional and sidereal dependences of the probabilities are discussed in Sec. III A. Examples are provided for the case of three generations and its two-generation limit in Sec. III B. Asymmetries characterizing violations of the discrete symmetries CP and CPT are discussed in Sec. III C. Section IV investigates the second-order effects involving the coefficients $\tilde{g}^{\alpha\beta}$ and $\tilde{H}^\alpha$. Oscillations with lepton-number violation are studied in Sec. IV A, while others are considered in Sec. IV B. Section V concludes with a summary.

II. BASIC THEORY

This section begins with a brief review of the description of Lorentz and CPT violation in neutrino oscillations, assuming three generations of left-handed neutrinos and their antineutrinos. We then use time-dependent perturbation theory to derive expressions for the transition amplitudes and oscillation probabilities valid for small Lorentz and CPT violation.

A. Hamiltonian

Violations of Lorentz and CPT invariance in oscillations of left-handed neutrinos and their antineutrinos can be characterized by a 6×6 effective Hamiltonian $(h_{\rm eff})_{AB}$ taking the form [13]

$$(h_{\rm eff})_{AB} = (h_0)_{AB} + \delta h_{AB}. \tag{1}$$

Here, h_0 describes conventional Lorentz-invariant neutrino oscillations, while δh includes the Lorentz-violating contributions. The uppercase indices take six values, $A, B, \ldots = e, \mu, \tau, \bar{e}, \bar{\mu}, \bar{\tau}$, spanning the three flavors of neutrinos and antineutrinos.

Under typical assumptions, the conventional term h_0 induces no mixing between neutrinos and antineutrinos. It is therefore block diagonal, and we write it as

$$h_0 = \begin{pmatrix} (h_0)_{ab} & 0 \\ 0 & (h_0)_{\bar{a}\bar{b}} \end{pmatrix} = \frac{1}{2E}\begin{pmatrix} \Delta m^2_{ab} & 0 \\ 0 & \Delta m^2_{\bar{a}\bar{b}} \end{pmatrix}, \tag{2}$$

where E is the neutrino energy, lowercase indices $a, b, \ldots = e, \mu, \tau$ indicate neutrinos, and lowercase barred indices $\bar{a}, \bar{b}, \ldots = \bar{e}, \bar{\mu}, \bar{\tau}$ indicate antineutrinos. The two 3×3 mass matrices are related by

$$\Delta m^2_{\bar{a}\bar{b}} = \Delta m^{2*}_{ab}, \tag{3}$$

as required by the *CPT* theorem [8]. Note that contributions to the Hamiltonian proportional to the unit matrix generate no oscillation effects, but they may nonetheless be relevant to stability and causality of the underlying theory [41].

The Lorentz-invariant Hamiltonian h_0 can be diagonalized using a 6×6 unitary matrix U,

$$(h_0)_{A'B'} = \sum_{AB} U_{A'A} U^*_{B'B} (h_0)_{AB}, \tag{4}$$

where primed indices indicate the diagonal mass basis. The absence of mixing between neutrinos and antineutrinos implies the mixing matrix is block diagonal,

$$U = \begin{pmatrix} U_{a'b} & 0 \\ 0 & U_{\bar{a}'\bar{b}} \end{pmatrix}, \tag{5}$$

with vanishing 3×3 off-diagonal blocks,

$$U_{a'\bar{b}} = U_{\bar{a}'b} = 0. \tag{6}$$

Since the mass matrices for neutrinos and antineutrinos are related by complex conjugation, we also have

$$U_{\bar{a}'\bar{b}} = U^*_{a'b}. \tag{7}$$

The diagonal 3×3 blocks of h_0 can therefore be written as

$$(h_0)_{ab} = (h_0)^*_{\bar{a}\bar{b}} = \sum_{a'=1,2,3} U^*_{a'a} U_{a'b} E_{a'}, \tag{8}$$

where $E_{a'}$ are the usual three neutrino eigenenergies. In what follows, we assume that these three eigenenergies are nondegenerate. Note that this implies there are three twofold degeneracies in the full 6×6 Hamiltonian $(h_0)_{AB}$.

The Lorentz-violating term δh in Eq. (1) can be written in the form

$$\delta h = \begin{pmatrix} \delta h_{ab} & \delta h_{a\bar{b}} \\ \delta h_{\bar{a}b} & \delta h_{\bar{a}\bar{b}} \end{pmatrix}. \tag{9}$$

For Lorentz-violating operators of renormalizable dimension, the upper-left diagonal block takes the form

$$\delta h_{ab} = \frac{1}{E}[(a_L)^\alpha p_\alpha - (c_L)^{\alpha\beta} p_\alpha p_\beta]_{ab} \tag{10}$$

and leads to mixing between neutrinos, where the neutrino energy-momentum 4-vector is denoted $p_\alpha = (E, -\vec{p}) \approx E(1, -\hat{p})$ and $(a_L)^\alpha_{ab}$, $(c_L)^{\alpha\beta}_{ab}$ are complex coefficients for Lorentz violation [13]. Hermiticity implies

$$\delta h_{ab} = \delta h^*_{ba}, \tag{11}$$

which imposes the conditions

$$\begin{aligned} (a_L)^\alpha_{ab} &= (a_L)^{\alpha *}_{ba}, \\ (c_L)^{\alpha\beta}_{ab} &= (c_L)^{\alpha\beta *}_{ba}. \end{aligned} \tag{12}$$

Similarly, the lower-right diagonal block of δh produces mixing between antineutrinos,

$$\begin{aligned} \delta h_{\bar{a}\bar{b}} &= \frac{1}{E}[(a_R)^\alpha p_\alpha - (c_R)^{\alpha\beta} p_\alpha p_\beta]_{\bar{a}\bar{b}} \\ &= \frac{1}{E}[-(a_L)^\alpha p_\alpha - (c_L)^{\alpha\beta} p_\alpha p_\beta]^*_{ab}. \end{aligned} \tag{13}$$

The off-diagonal 3×3 blocks of δh, $\delta h_{a\bar{b}}$ and $\delta h_{\bar{b}a}$, lead to neutrino-antineutrino mixing, an unconventional effect. These blocks obey the Hermiticity condition

$$\delta h_{a\bar{b}} = \delta h^*_{\bar{b}a} \tag{14}$$

and can be written as [13]

$$\begin{aligned} \delta h_{a\bar{b}} &= -i\sqrt{2}(\epsilon_+)_\alpha [\tilde{g}^{\alpha\beta} p_\beta - \tilde{H}^\alpha]_{a\bar{b}}, \\ \delta h_{\bar{a}b} &= i\sqrt{2}(\epsilon_+)^*_\alpha [\tilde{g}^{\alpha\beta} p_\beta - \tilde{H}^\alpha]_{\bar{a}b} \\ &= i\sqrt{2}(\epsilon_+)^*_\alpha [\tilde{g}^{\alpha\beta} p_\beta + \tilde{H}^\alpha]^*_{a\bar{b}}. \end{aligned} \tag{15}$$

In these equations, the complex coefficients for Lorentz violation $\tilde{g}^{\alpha\beta}_{a\bar{b}}$ and $\tilde{H}^\alpha_{a\bar{b}}$ obey the relations

$$\begin{aligned} \tilde{g}^{\alpha\beta}_{a\bar{b}} &= \tilde{g}^{\alpha\beta}_{b\bar{a}} = \tilde{g}^{\alpha\beta *}_{\bar{b}a}, \\ \tilde{H}^\alpha_{a\bar{b}} &= -\tilde{H}^\alpha_{b\bar{a}} = \tilde{H}^{\alpha *}_{\bar{b}a}. \end{aligned} \tag{16}$$

The complex 4-vector $(\epsilon_+)_\alpha = (0, -\vec{\epsilon}_+)$ represents the helicity state. Introducing the local beam direction $\hat{e}_r$ and other unit vectors associated with local spherical coordinates as

$$\begin{aligned} \hat{e}_r &= (\sin\theta\cos\phi, \sin\theta\sin\phi, \cos\theta), \\ \hat{e}_\theta &= (\cos\theta\cos\phi, \cos\theta\sin\phi, -\sin\theta), \\ \hat{e}_\phi &= (-\sin\phi, \cos\phi, 0), \end{aligned} \tag{17}$$

the 3-vector $\vec{\epsilon}_+$ can be expressed as

$$\vec{\epsilon}_+ = \frac{1}{\sqrt{2}}(\hat{e}_\theta + i\hat{e}_\phi). \tag{18}$$

The coefficients $(a_L)^\alpha_{ab}$ and $\tilde{H}^\alpha_{a\bar{b}}$ have dimensions of mass, and each taken alone leads to oscillation effects that are energy independent. In contrast, the coefficients $(c_L)^{\alpha\beta}_{ab}$ and $\tilde{g}^{\alpha\beta}_{a\bar{b}}$ are dimensionless, so their effects naturally scale with energy. Note, however, that combinations of coefficients can produce involved energy dependences, including mimicking conventional mass terms via the Lorentz-violating seesaw mechanism [13,14,17].

The coefficients $(a_L)^\alpha_{ab}$ and $\tilde{g}^{\alpha\beta}_{a\bar{b}}$ control CPT-odd effects, while $(c_L)^{\alpha\beta}_{ab}$ and $\tilde{H}^\alpha_{a\bar{b}}$ govern CPT-even ones. Consequently, CPT symmetry holds when $(a_L)^\alpha_{ab}$ and $\tilde{g}^{\alpha\beta}_{a\bar{b}}$ vanish, and we find the oscillation probabilities obey the relationship

$$P_{\nu_a\to\nu_b} = P_{\bar{\nu}_b\to\bar{\nu}_a} \quad (CPT \text{ invariance}). \tag{19}$$

The CP symmetry may nonetheless be violated, so the relation $P_{\nu_a\to\nu_b} = P_{\bar{\nu}_a\to\bar{\nu}_b}$ may fail. Further discussion of CP and CPT tests is provided in Sec. III C below.

All the coefficients discussed here are taken to be spacetime constants, so that translational symmetry and energy-momentum conservation hold. If the Lorentz violation is spontaneous [3], which may be ubiquitous in effective field theories [42], then the coefficients can be understood as expectation values of operators in the fundamental theory. Under these circumstances, requiring constancy of the coefficients is equivalent to disregarding soliton solutions and massive or Nambu-Goldstone (NG) modes [43]. When gravity is included, the NG modes can play the role of the graviton [44], the photon in Einstein-Maxwell theory [45], or various new spin-dependent [46] or spin-independent [47] forces. The presence of gravity can also produce additional Lorentz-violating effects on neutrino oscillations [7,48].

When neutrinos propagate in matter, the resulting forward scattering on electrons, protons, and neutrons can affect neutrino oscillations [49]. In the rest frame of the matter, this adds to the effective Hamiltonian $(h_{\text{eff}})_{AB}$ terms equivalent to CPT-odd coefficients given by [13]

$$\begin{aligned}(a_{L,\text{eff}})^0_{ee} &= G_F(2n_e - n_n)/\sqrt{2},\\ (a_{L,\text{eff}})^0_{\mu\mu} &= (a_{L,\text{eff}})^0_{\tau\tau} = -G_F n_n/\sqrt{2},\end{aligned} \tag{20}$$

where n_e and n_n are the number densities of electrons and neutrons in the matter and G_F is the Fermi coupling constant. For example, in neutrino-oscillation experiments with long baselines, the propagation is over comparatively long distances in the Earth's crust. In this case, the densities n_e and n_n can be taken equal and constant to a good approximation, with $\sqrt{2}G_F n_e \simeq 2.1\times10^{-22}$ GeV $\simeq$ $(940\text{ km})^{-1}$. In the perturbative analysis of Lorentz violation that follows, any matter effects can be taken as part of the unperturbed Hamiltonian $(h_0)_{AB}$. In situations where mass oscillations dominate, the matter effects could alternatively be treated as perturbative and included as part of the Lorentz-violating term δh_{AB}.

B. Perturbation series

In this subsection, we use standard techniques of time-dependent perturbation theory to derive a perturbative series for the transition amplitudes. We treat the Hamiltonian component δh describing Lorentz and CPT violation as small compared to $1/L$.

The time-evolution operator $S(t)$ is written in the form

$$\begin{aligned}S(t) &\equiv e^{-ih_{\text{eff}}t}\\ &= (e^{-ih_{\text{eff}}t}e^{ih_0t})S^{(0)}(t)\\ &= S^{(0)}(t) + S^{(1)}(t) + S^{(2)}(t) + \cdots,\end{aligned} \tag{21}$$

where $S^{(n)}$ is the nth-order perturbation in δh. The conventional term is given by

$$S^{(0)} = e^{-ih_0t}. \tag{22}$$

The higher-order terms are obtained using the integral relation

$$\begin{aligned}e^{-ih_{\text{eff}}t}e^{ih_0t} = 1 &+ \int_0^t dt_1(-i)\Delta h(t_1)\\ &+ \int_0^t dt_2\int_0^{t_2} dt_1(-i)\Delta h(t_1)(-i)\Delta h(t_2)\\ &+ \cdots,\end{aligned} \tag{23}$$

where

$$\Delta h(t) = e^{-ih_0t}\delta h e^{ih_0t}. \tag{24}$$

The integrals (23) can conveniently be performed in the mass-diagonal basis. To second order in Lorentz-violating coefficients, the results take the form

$$\begin{aligned}S^{(0)}_{A'B'} &= \delta_{A'B'}\tau^{(0)}_{A'}(t),\\ S^{(1)}_{A'B'} &= -it\delta h_{A'B'}\tau^{(1)}_{A'B'}(t),\\ S^{(2)}_{A'B'} &= -\frac{1}{2}t^2\sum_{C'}\delta h_{A'C'}\delta h_{C'B'}\tau^{(2)}_{A'B'C'}(t).\end{aligned} \tag{25}$$

All sums over flavor indices are written explicitly throughout this work. The time dependence is contained in the factors $\tau^{(0)}_{A'}$, $\tau^{(1)}_{A'B'}$, and $\tau^{(2)}_{A'B'C'}$. The zeroth-order factor is the usual expression

$$\tau^{(0)}_{A'}(t) = \exp(-iE_{A'}t). \tag{26}$$

The first-order term is given by

$$\begin{aligned}\tau^{(1)}_{A'B'}(t) &= \frac{1}{t}\exp(-iE_{B'}t)\int_0^t dt_1\exp(-i\Delta_{A'B'}t_1)\\ &= \begin{cases}\exp(-iE_{B'}t), & E_{A'} = E_{B'},\\ \frac{\exp(-iE_{A'}t)-\exp(-iE_{B'}t)}{-i\Delta_{A'B'}t}, & \text{otherwise,}\end{cases}\end{aligned} \tag{27}$$

where

$$\Delta_{A'B'} = E_{A'} - E_{B'} \tag{28}$$

are the standard eigenenergy differences. The second-order factor is given by the integral

$$\begin{aligned}\tau^{(2)}_{A'B'C'}(t) &= \frac{2}{t^2}\exp(-iE_{B'}t)\\ &\quad\times\int_0^t dt_2\int_0^{t_2} dt_1 \exp(-i\Delta_{A'C'}t_1)\\ &\quad\times\exp(-i\Delta_{C'B'}t_2)\\ &= \begin{cases}\exp(-iE_{B'}t), & E_{A'}=E_{B'}=E_{C'},\\ 2\frac{\tau^{(1)}_{A'B'}-\tau^{(1)}_{C'B'}}{-i\Delta_{A'C'}t} = 2\frac{\tau^{(1)}_{A'C'}-\tau^{(1)}_{A'B'}}{-i\Delta_{C'B'}t}, & \text{otherwise.}\end{cases}\end{aligned} \tag{29}$$

We give two expressions in the last case so that expressions for the limiting cases $E_{A'} = E_{C'}$ and $E_{B'} = E_{C'}$ can readily be extracted. Note that both $\tau^{(1)}_{A'B'}$ and $\tau^{(2)}_{A'B'C'}$ are dimensionless functions of $E_{A'}t$ that are totally symmetric in mass-basis indices.

Transforming to the flavor basis, the Lorentz-invariant transition amplitude is found to be

$$S^{(0)}_{AB} = \sum_{A'} U^*_{A'A}U_{A'B}e^{-iE_{A'}t}. \tag{30}$$

This leads to the usual oscillation probabilities for the Lorentz-invariant case of massive neutrinos. At first order, we choose to express the transition amplitude in the convenient form

$$\begin{aligned}S^{(1)}_{AB}(t) &\equiv -it\mathcal{H}^{(1)}_{AB}(t)\\ &= -it\sum_{CD}(\mathcal{M}^{(1)}_{AB})_{CD}\delta h_{CD},\end{aligned} \tag{31}$$

where the factors

$$\begin{aligned}(\mathcal{M}^{(1)}_{AB})_{CD}(t) &= \frac{1}{t}\int_0^{t_1} dt_1 S^{(0)}_{AC}(t_1)S^{(0)}_{DB}(t-t_1)\\ &= \sum_{A'B'}\tau^{(1)}_{A'B'}(t)U^*_{A'A}U_{A'C}U^*_{B'D}U_{B'B}\end{aligned} \tag{32}$$

depend on the energy and baseline of the experiment and also on the conventional masses and mixing angles. For given mass spectrum and mixing angles, these factors determine the sensitivity of an experiment. They are independent of the direction of the neutrino propagation and of the coefficients for Lorentz violation. As a result, they remain unchanged as the Earth rotates. The quantity $\mathcal{H}^{(1)}_{AB}$ defined in Eq. (31) is a linear combination of these factors and the Lorentz-violating perturbation δh. It plays a key role in the expressions for the oscillation probabilities derived in the next subsection. Note that $\mathcal{H}^{(1)}_{AB}$ reduces to δh_{AB} in the limit of negligible mass mixing.

The second-order result for the transition amplitude can be written in a similar form. We define

$$\begin{aligned}S^{(2)}_{AB}(t) &\equiv -\frac{1}{2}t^2\mathcal{H}^{(2)}_{AB}\\ &= -\frac{1}{2}t^2\sum_{CDEF}(\mathcal{M}^{(2)}_{AB})_{CDEF}\delta h_{CD}\delta h_{EF},\end{aligned} \tag{33}$$

where the experiment-dependent factors

$$\begin{aligned}&(\mathcal{M}^{(2)}_{AB})_{CDEF}(t)\\ &= \frac{2}{t^2}\int_0^t dt_2\int_0^{t_2} dt_1 S^{(0)}_{AC}(t_1)S^{(0)}_{DE}(t_2-t_1)S^{(0)}_{FB}(t-t_2)\\ &= \sum_{A'B'C'}\tau^{(2)}_{A'B'C'}(t)U^*_{A'A}U_{A'C}U^*_{C'D}U_{C'E}U^*_{B'F}U_{B'B}\end{aligned} \tag{34}$$

again determine the combinations of coefficients relevant for oscillation effects. In this case, however, the quantity $\mathcal{H}^{(2)}_{AB}$ defined in Eq. (33) is a quadratic combination of coefficients. This leads to sidereal variations in neutrino oscillations at higher multiples of the Earth's rotation frequency. Note that $\mathcal{H}^{(2)}_{AB}$ reduces to $(\delta h^2)_{AB}$ in the limit of negligible mass mixing.

C. Probabilities

Using the above results for the transition amplitudes, we can derive the oscillation probabilities. At zeroth order, the transition amplitudes are Lorentz invariant and take the usual block-diagonal form, $S^{(0)}_{a\bar{b}} = S^{(0)}_{\bar{a}b} = 0$. So the zeroth-order probabilities for neutrino-antineutrino oscillations vanish,

$$P^{(0)}_{\bar{\nu}_b\to\nu_a} = P^{(0)}_{\nu_b\to\bar{\nu}_a} = 0. \tag{35}$$

Since *CPT* is conserved whenever Lorentz symmetry holds [8], we have $S^{(0)}_{ab} = S^{(0)}_{\bar{b}\,\bar{a}}$. This implies

$$P^{(0)}_{\nu_b\to\nu_a} = P^{(0)}_{\bar{\nu}_a\to\bar{\nu}_b} = |S^{(0)}_{ab}|^2, \tag{36}$$

which leads to the usual results for Lorentz-invariant oscillation probabilities in terms of mass-squared differences and mixing angles.

The full mixing probability is given by

$$P_{\nu_B\to\nu_A} = |S^{(0)}_{AB} + S^{(1)}_{AB} + S^{(2)}_{AB} + \cdots|^2. \tag{37}$$

At second order in δh, this gives

$$\begin{aligned}P^{(0)}_{\nu_B\to\nu_A} &= |S^{(0)}_{AB}|^2,\\ P^{(1)}_{\nu_B\to\nu_A} &= 2\,\mathrm{Re}((S^{(0)}_{AB})^*S^{(1)}_{AB})\\ &= 2t\,\mathrm{Im}((S^{(0)}_{AB})^*\mathcal{H}^{(1)}_{AB}),\\ P^{(2)}_{\nu_B\to\nu_A} &= 2\,\mathrm{Re}((S^{(0)}_{AB})^*S^{(2)}_{AB}) + |S^{(1)}_{AB}|^2,\\ &= -t^2\,\mathrm{Re}((S^{(0)}_{AB})^*\mathcal{H}^{(2)}_{AB}) + t^2|\mathcal{H}^{(1)}_{AB}|^2.\end{aligned} \tag{38}$$

These equations involve the six-dimensional space spanned by A. They can be decomposed into oscillation probabilities expressed in terms of the neutrino and antineutrino subspaces spanned by a and $\bar{a}$.

At first order, a short calculation shows that the probabilities can be written

$$\begin{aligned} P^{(1)}_{\nu_b\to\nu_a} &= 2t\,\mathrm{Im}((S^{(0)}_{ab})^*\mathcal{H}^{(1)}_{ab}), \\ P^{(1)}_{\bar\nu_b\to\bar\nu_a} &= 2t\,\mathrm{Im}((S^{(0)}_{\bar a\bar b})^*\mathcal{H}^{(1)}_{\bar a\bar b}), \\ P^{(1)}_{\nu_b\to\bar\nu_a} &= P^{(1)}_{\bar\nu_b\to\nu_a} = 0. \end{aligned} \tag{39}$$

No neutrino-antineutrino mixing occurs because $S^{(0)}_{a\bar b} = S^{(0)}_{\bar a b} = 0$. Only the combinations $\mathcal{H}^{(1)}_{ab}$ and $\mathcal{H}^{(1)}_{\bar a\bar b}$ obtained from the definition (31) contribute to these probabilities. Explicitly, we find

$$\begin{aligned} \mathcal{H}^{(1)}_{ab} &= \sum_{cd}(\mathcal{M}^{(1)}_{ab})_{cd}\delta h_{cd}, \\ \mathcal{H}^{(1)}_{\bar a\bar b} &= \sum_{\bar c\bar d}(\mathcal{M}^{(1)}_{\bar a\bar b})_{\bar c\bar d}\delta h_{\bar c\bar d}, \\ \mathcal{H}^{(1)}_{\bar a b} &= \sum_{\bar c d}(\mathcal{M}^{(1)}_{\bar a b})_{\bar c d}\delta h_{\bar c d}, \\ \mathcal{H}^{(1)}_{a\bar b} &= \sum_{c\bar d}(\mathcal{M}^{(1)}_{a\bar b})_{c\bar d}\delta h_{c\bar d}. \end{aligned} \tag{40}$$

Although $\mathcal{H}^{(1)}_{\bar a b}$ and $\mathcal{H}^{(1)}_{a\bar b}$ are absent from the first-order probabilities, we include their expressions here because they enter the second-order probabilities below. Since the first-order results involve the diagonal blocks of δh, only the coefficients $(a_L)^\alpha_{ab}$ and $(c_L)^{\alpha\beta}_{ab}$ play a role.

Decomposing the results (38) reveals that the second-order probabilities are

$$\begin{aligned} P^{(2)}_{\nu_b\to\nu_a} &= -t^2\,\mathrm{Re}((S^{(0)}_{ab})^*\mathcal{H}^{(2)}_{ab}) + t^2|\mathcal{H}^{(1)}_{ab}|^2, \\ P^{(2)}_{\bar\nu_b\to\bar\nu_a} &= -t^2\,\mathrm{Re}((S^{(0)}_{\bar a\bar b})^*\mathcal{H}^{(2)}_{\bar a\bar b}) + t^2|\mathcal{H}^{(1)}_{\bar a\bar b}|^2, \\ P^{(2)}_{\nu_b\to\bar\nu_a} &= t^2|\mathcal{H}^{(1)}_{\bar a b}|^2, \\ P^{(2)}_{\bar\nu_b\to\nu_a} &= t^2|\mathcal{H}^{(1)}_{a\bar b}|^2, \end{aligned} \tag{41}$$

where $S^{(0)}_{a\bar b} = S^{(0)}_{\bar a b} = 0$ is used to simplify the last two. The probabilities $P^{(2)}_{\nu_b\to\nu_a}$ and $P^{(2)}_{\bar\nu_b\to\bar\nu_a}$ include both leading-order contributions from $\mathcal{H}^{(2)}_{ab}$ and $\mathcal{H}^{(2)}_{\bar a\bar b}$ as well as higher-order contributions from the combinations $\mathcal{H}^{(1)}_{ab}$ and $\mathcal{H}^{(1)}_{\bar a\bar b}$. Also, nonzero mixing between neutrinos and antineutrinos appears, giving sensitivity to the linear combinations $\mathcal{H}^{(1)}_{\bar a b}$ and $\mathcal{H}^{(1)}_{a\bar b}$. This shows that the dominant effects of the coefficients $\tilde g^{\alpha\beta}_{a\bar b}$ and $\tilde H^{\alpha}_{a\bar b}$ appear only at second order. Moreover, $(a_L)^\alpha_{ab}$ or $(c_L)^{\alpha\beta}_{ab}$ play no role in neutrino-antineutrino mixing at this order.

Explicit expressions for $\mathcal{H}^{(2)}_{ab}$ and $\mathcal{H}^{(2)}_{\bar a\bar b}$ can be obtained by decomposing the quadratic combinations $\mathcal{H}^{(2)}_{AB}$ defined in Eq. (33). The structure of the factors $(\mathcal{M}^{(2)}_{AB})_{CDEF}$ and the form of the mixing matrix U reduce the number of terms that contribute. In particular, we find that $(\mathcal{M}^{(2)}_{AB})_{CDEF}$ vanishes unless the index pairs $\{AC\}$, $\{BF\}$, and $\{DE\}$ lie in the same subspace. This leads to

$$\begin{aligned} \mathcal{H}^{(2)}_{ab} &= \sum_{cdef}(\mathcal{M}^{(2)}_{ab})_{cdef}\delta h_{cd}\delta h_{ef} \\ &\quad + \sum_{c\bar d\bar e f}(\mathcal{M}^{(2)}_{ab})_{c\bar d\bar e f}\delta h_{c\bar d}\delta h_{\bar e f}, \\ \mathcal{H}^{(2)}_{\bar a\bar b} &= \sum_{\bar c\bar d\bar e\bar f}(\mathcal{M}^{(2)}_{\bar a\bar b})_{\bar c\bar d\bar e\bar f}\delta h_{\bar c\bar d}\delta h_{\bar e\bar f} \\ &\quad + \sum_{\bar c de\bar f}(\mathcal{M}^{(2)}_{\bar a\bar b})_{\bar c de\bar f}\delta h_{\bar c d}\delta h_{e\bar f}, \\ \mathcal{H}^{(2)}_{\bar a b} &= \sum_{\bar c def}(\mathcal{M}^{(2)}_{\bar a b})_{\bar c def}\delta h_{\bar c d}\delta h_{ef} \\ &\quad + \sum_{\bar c\bar d\bar e f}(\mathcal{M}^{(2)}_{\bar a b})_{\bar c\bar d\bar e f}\delta h_{\bar c\bar d}\delta h_{\bar e f}, \\ \mathcal{H}^{(2)}_{a\bar b} &= \sum_{cde\bar f}(\mathcal{M}^{(2)}_{a\bar b})_{cde\bar f}\delta h_{cd}\delta h_{e\bar f} \\ &\quad + \sum_{c\bar d\bar e\bar f}(\mathcal{M}^{(2)}_{a\bar b})_{c\bar d\bar e\bar f}\delta h_{c\bar d}\delta h_{\bar e\bar f}. \end{aligned} \tag{42}$$

Note that $\mathcal{H}^{(2)}_{\bar a b}$ and $\mathcal{H}^{(2)}_{a\bar b}$ are absent from the second-order probabilities but are included here for completeness. This implies that no cross terms between δh_{ab}, $\delta h_{\bar a\bar b}$, and $\delta h_{a\bar b} = \delta h^*_{\bar b a}$ appear at second order. In particular, all appearances of the coefficients $\tilde g^{\alpha\beta}_{a\bar b}$ and $\tilde H^{\alpha}_{a\bar b}$ arise as squares or as quadratic products with each other, without accompanying factors of $(a_L)^\alpha_{ab}$ or $(c_L)^{\alpha\beta}_{ab}$.

III. COEFFICIENTS $(a_L)^\alpha_{ab}$ AND $(c_L)^{\alpha\beta}_{ab}$

In this section, we consider effects originating from the coefficients $(a_L)^\alpha_{ab}$ and $(c_L)^{\alpha\beta}_{ab}$. These contribute only to neutrino-neutrino mixing and to antineutrino-antineutrino mixing. We focus here on the dominant signals, which arise from the first-order oscillation probabilities $P^{(1)}_{\nu_b\to\nu_a}$ and $P^{(1)}_{\bar\nu_b\to\bar\nu_a}$ given in Eq. (39).

The theoretical analysis presented in the previous section applies to any scenario involving neutrino propagation. However, the key experimental signals are different for beam experiments, solar-neutrino studies, and cosmological observations. For definiteness in this work, we provide results in the context of beam experiments.

Section III A establishes the key expressions describing sidereal variations in the oscillation probabilities. Illustrative examples involving all three generations are provided in Sec. III B, while the two-flavor case is considered in Sec. III B 3. The construction of CP and CPT

asymmetries to characterize the effects is considered in Sec. III C.

A. Sidereal variations

The combinations of coefficients for Lorentz and *CPT* violation entering the nonzero first-order probabilities (39) are controlled by the experimental factors $(\mathcal{M}^{(1)}_{ab})_{cd}$ and $(\mathcal{M}^{(1)}_{\bar{a}\bar{b}})_{\bar{c}\bar{d}}$ entering Eq. (40). These factors can be calculated from Eq. (32). The time t can be set equal to the baseline distance L because any difference between the two is small and leads only to suppressed higher-order corrections. For given values of E and L, the nine complex constants $(\mathcal{M}^{(1)}_{ab})_{cd}$ determine the coefficient combinations relevant for $\nu \leftrightarrow \nu$ mixing, while the nine complex constants $(\mathcal{M}^{(1)}_{\bar{a}\bar{b}})_{\bar{c}\bar{d}}$ determine those for $\bar{\nu} \leftrightarrow \bar{\nu}$ mixing. If CP is conserved in the usual mass matrix, as occurs in the two-generation limit, then the mixing matrices are real and obey $U_{ab} = U_{\bar{a}\bar{b}}$. We then find $(\mathcal{M}^{(1)}_{ab})_{cd} = (\mathcal{M}^{(1)}_{\bar{a}\bar{b}})_{\bar{c}\bar{d}}$, so a single set of nine constants determines the experimentally relevant combinations for both neutrinos and antineutrinos.

The specific combinations of $(a_L)^\alpha_{ab}$ and $(c_L)^{\alpha\beta}_{ab}$ appearing in the transition probabilities can be found by considering the forms of the relevant blocks of the Hamiltonian, Eqs. (10) and (13). In this context, it is convenient to define the linear combinations

$$\begin{aligned}
(\tilde{a}_L)^\alpha_{ab} &= \sum_{cd} (\mathcal{M}^{(1)}_{ab})_{cd} (a_L)^\alpha_{cd}, \\
(\tilde{c}_L)^{\alpha\beta}_{ab} &= \sum_{cd} (\mathcal{M}^{(1)}_{ab})_{cd} (c_L)^{\alpha\beta}_{cd}, \\
(\tilde{a}_R)^\alpha_{\bar{a}\bar{b}} &= \sum_{\bar{c}\bar{d}} (\mathcal{M}^{(1)}_{\bar{a}\bar{b}})_{\bar{c}\bar{d}} (a_R)^\alpha_{\bar{c}\bar{d}}, \\
(\tilde{c}_R)^{\alpha\beta}_{\bar{a}\bar{b}} &= \sum_{\bar{c}\bar{d}} (\mathcal{M}^{(1)}_{\bar{a}\bar{b}})_{\bar{c}\bar{d}} (c_R)^{\alpha\beta}_{\bar{c}\bar{d}}.
\end{aligned} \tag{43}$$

These are experiment-dependent combinations of the fundamental coefficients $(a_L)^\alpha_{ab}$ and $(c_L)^{\alpha\beta}_{ab}$. Using $(\tilde{a}_L)^\alpha_{ab}$, $(\tilde{a}_R)^\alpha_{ab}$, $(\tilde{c}_L)^{\alpha\beta}_{ab}$, and $(\tilde{c}_R)^{\alpha\beta}_{ab}$, the combinations $\mathcal{H}^{(1)}_{ab}$ and $\mathcal{H}^{(1)}_{\bar{a}\bar{b}}$ controlling the first-order probabilities can be written in a form that mimics the Hamiltonian perturbations δh_{ab} and $\delta h_{\bar{a}\bar{b}}$,

$$\begin{aligned}
\mathcal{H}^{(1)}_{ab} &= \frac{1}{E}[(\tilde{a}_L)^\alpha p_\alpha - (\tilde{c}_L)^{\alpha\beta} p_\alpha p_\beta]_{ab}, \\
\mathcal{H}^{(1)}_{\bar{a}\bar{b}} &= \frac{1}{E}[(\tilde{a}_R)^\alpha p_\alpha - (\tilde{c}_R)^{\alpha\beta} p_\alpha p_\beta]_{\bar{a}\bar{b}}.
\end{aligned} \tag{44}$$

This form reveals the explicit 4-momentum dependence of the transition probabilities.

The momentum dependence implies that the mixing behavior can depend on the direction of neutrino propagation. For Earth-based experiments, the source and detector rotate at the sidereal frequency $\omega_\oplus \simeq 2\pi/(23\text{ h }56\text{ min})$, which can induce sidereal variations in the oscillation probabilities. Since the first-order probabilities are linear in $\mathcal{H}^{(1)}_{ab}$ or $\mathcal{H}^{(1)}_{\bar{a}\bar{b}}$, each of which has both linear and quadratic terms in the 3-momentum, sidereal variations controlled by the coefficients $(\tilde{a}_L)^\alpha_{ab}$ and $(\tilde{c}_L)^{\alpha\beta}_{ab}$ can occur at the frequencies $\omega_\oplus$ and $2\omega_\oplus$.

To display explicitly these variations, a choice of inertial frame must be specified. By convention and convenience, the standard inertial frame is taken as a Sun-centered celestial-equatorial frame with coordinates (T, X, Y, Z) [9,50]. The Z axis of this frame is directed north and parallel to the rotational axis of the Earth. The X axis points from the Sun toward the vernal equinox, while the Y axis completes a right-handed system. The origin of the time coordinate is chosen as the vernal equinox 2000. The Earth's rotation causes the neutrino 3-momentum to vary in local sidereal time $T_\oplus$ at the frequency $\omega_\oplus$ in the Sun-centered frame, unless it happens to lie along the Z axis.

For neutrino-neutrino mixing, we can display explicitly the sidereal variation by expanding $\mathcal{H}^{(1)}_{ab}$ as

$$\begin{aligned}
\mathcal{H}^{(1)}_{ab} = (\mathcal{C}^{(1)})_{ab} &+ (\mathcal{A}^{(1)}_s)_{ab} \sin\omega_\oplus T_\oplus + (\mathcal{A}^{(1)}_c)_{ab} \cos\omega_\oplus T_\oplus \\
&+ (\mathcal{B}^{(1)}_s)_{ab} \sin 2\omega_\oplus T_\oplus + (\mathcal{B}^{(1)}_c)_{ab} \cos 2\omega_\oplus T_\oplus.
\end{aligned} \tag{45}$$

Suppose the neutrinos of interest are emitted in a definite direction relative to the Earth, perhaps as a neutrino beam from an accelerator. Let the vector $(\hat{N}^X, \hat{N}^Y, \hat{N}^Z)$ represent the propagation direction in the Sun-centered frame at local sidereal time $T_\oplus = 0$. We can write this vector in terms of local spherical coordinates at the detector. Denote by χ the colatitude of the detector. Introduce at the detector the angle θ between the beam direction and vertical, and also the angle ϕ between the beam and east of south. The components of the vector can then be written as

$$\begin{aligned}
\hat{N}^X &= \cos\chi \sin\theta \cos\phi + \sin\chi \cos\theta, \\
\hat{N}^Y &= \sin\theta \sin\phi, \\
\hat{N}^Z &= -\sin\chi \sin\theta \cos\phi + \cos\chi \cos\theta.
\end{aligned} \tag{46}$$

Using these expressions, the amplitudes in the expansion (45) are specified in terms of coefficients for Lorentz violation in the Sun-centered frame as

$$
\begin{aligned}
(\mathcal{C}^{(1)})_{ab} &= (\tilde{a}_L)^T_{ab} - \hat{N}^Z(\tilde{a}_L)^Z_{ab} - \frac{1}{2}(3 - \hat{N}^Z\hat{N}^Z)E(\tilde{c}_L)^{TT}_{ab} + 2\hat{N}^Z E(\tilde{c}_L)^{TZ}_{ab} + \frac{1}{2}(1 - 3\hat{N}^Z\hat{N}^Z)E(\tilde{c}_L)^{ZZ}_{ab},\\
(\mathcal{A}_s^{(1)})_{ab} &= \hat{N}^Y(\tilde{a}_L)^X_{ab} - \hat{N}^X(\tilde{a}_L)^Y_{ab} - 2\hat{N}^Y E(\tilde{c}_L)^{TX}_{ab} + 2\hat{N}^X E(\tilde{c}_L)^{TY}_{ab} + 2\hat{N}^Y\hat{N}^Z E(\tilde{c}_L)^{XZ}_{ab} - 2\hat{N}^X\hat{N}^Z E(\tilde{c}_L)^{YZ}_{ab},\\
(\mathcal{A}_c^{(1)})_{ab} &= -\hat{N}^X(\tilde{a}_L)^X_{ab} - \hat{N}^Y(\tilde{a}_L)^Y_{ab} + 2\hat{N}^X E(\tilde{c}_L)^{TX}_{ab} + 2\hat{N}^Y E(\tilde{c}_L)^{TY}_{ab} - 2\hat{N}^X\hat{N}^Z E(\tilde{c}_L)^{XZ}_{ab} - 2\hat{N}^Y\hat{N}^Z E(\tilde{c}_L)^{YZ}_{ab}, \qquad (47)\\
(\mathcal{B}_s^{(1)})_{ab} &= \hat{N}^X\hat{N}^Y E((\tilde{c}_L)^{XX}_{ab} - (\tilde{c}_L)^{YY}_{ab}) - (\hat{N}^X\hat{N}^X - \hat{N}^Y\hat{N}^Y)E(\tilde{c}_L)^{XY}_{ab},\\
(\mathcal{B}_c^{(1)})_{ab} &= -2\hat{N}^X\hat{N}^Y E(\tilde{c}_L)^{XY}_{ab} - \frac{1}{2}(\hat{N}^X\hat{N}^X - \hat{N}^Y\hat{N}^Y)E((\tilde{c}_L)^{XX}_{ab} - (\tilde{c}_L)^{YY}_{ab}).
\end{aligned}
$$

The form of the above expansion matches that used in Ref. [16] in the context of short-baseline neutrino experiments.

The sidereal variation in $\mathcal{H}^{(1)}_{ab}$ leads to a corresponding variation in the probabilities. We parametrize these variations as

$$
\begin{aligned}
\frac{P^{(1)}_{\nu_b\to\nu_a}}{2L} &= \mathrm{Im}((S^{(0)}_{ab})^*\mathcal{H}^{(1)}_{ab})\\
&= (P^{(1)}_{\mathcal{C}})_{ab} + (P^{(1)}_{\mathcal{A}_s})_{ab}\sin\omega_\oplus T_\oplus + (P^{(1)}_{\mathcal{A}_c})_{ab}\cos\omega_\oplus T_\oplus + (P^{(1)}_{\mathcal{B}_s})_{ab}\sin2\omega_\oplus T_\oplus + (P^{(1)}_{\mathcal{B}_c})_{ab}\cos2\omega_\oplus T_\oplus, \qquad (48)
\end{aligned}
$$

where

$$
\begin{aligned}
(P^{(1)}_{\mathcal{C}})_{ab} &= \mathrm{Im}((S^{(0)}_{ab})^*(\mathcal{C}^{(1)})_{ab}),\\
(P^{(1)}_{\mathcal{A}_s})_{ab} &= \mathrm{Im}((S^{(0)}_{ab})^*(\mathcal{A}_s^{(1)})_{ab}),\\
(P^{(1)}_{\mathcal{A}_c})_{ab} &= \mathrm{Im}((S^{(0)}_{ab})^*(\mathcal{A}_c^{(1)})_{ab}), \qquad (49)\\
(P^{(1)}_{\mathcal{B}_s})_{ab} &= \mathrm{Im}((S^{(0)}_{ab})^*(\mathcal{B}_s^{(1)})_{ab}),\\
(P^{(1)}_{\mathcal{B}_c})_{ab} &= \mathrm{Im}((S^{(0)}_{ab})^*(\mathcal{B}_c^{(1)})_{ab})
\end{aligned}
$$

are combination of coefficients for Lorentz violations. Note that the sidereal amplitudes $(P^{(1)}_{\mathcal{C}})_{ab}$, $(P^{(1)}_{\mathcal{A}_s})_{ab}$, $(P^{(1)}_{\mathcal{A}_c})_{ab}$, $(P^{(1)}_{\mathcal{B}_s})_{ab}$, and $(P^{(1)}_{\mathcal{B}_c})_{ab}$ are tiny, with size determined by $(a_L)^{\alpha}_{ab}$ and $(c_L)^{\alpha\beta}_{ab}$. The expression (48) reveals that the experimental sensitivity to perturbative Lorentz and CPT violation increases with the baseline L.

For antineutrino-antineutrino oscillations, analogous results hold. We can expand $\mathcal{H}^{(1)}_{\bar{a}\bar{b}}$ in the form (45), replacing the indices $\{ab\}$ with $\{\bar{a}\bar{b}\}$. The amplitudes again take the form (47), but with the substitutions $(\tilde{a}_L)\to(\tilde{a}_R)$, $(\tilde{c}_L)\to(\tilde{c}_R)$, $\{ab\}\to\{\bar{a}\bar{b}\}$. Similarly, the sidereal variation in the probabilities can be written as Eq. (48) by replacement of the indices.

B. Illustrations

In this subsection, the first-order perturbative formalism derived above is applied to several illustrative situations. Following some comments about the methodology, we consider an example involving mixing of all three flavors of neutrinos and then discuss the limiting case of two flavors.

1. Methodology

Since the effects from Lorentz and CPT violation are perturbative, an explicit analysis must specify the conventional mass spectrum and mixing angles. In the standard three-neutrino massive model [2], the usual 3×3 effective Hamiltonian $(h_0)_{ab}$ for neutrino-neutrino vacuum mixing appears as the upper-left block of Eq. (2). It can be written as

$$(h_0)_{ab} = \frac{1}{2E}\Delta m^2_{ab} = \frac{1}{2E}\sum_{a'b'}U^*_{a'a}U_{b'b}\Delta m^2_{a'b'}, \qquad (50)$$

where $\Delta m^2_{a'b'}$ is the diagonal mass matrix. Only two mass-squared differences contribute to oscillations. Without loss of generality, we can therefore express the diagonal mass matrix as

$$m^2_{a'b'} = \begin{pmatrix} 0 & 0 & 0\\ 0 & \Delta m^2_\odot & 0\\ 0 & 0 & \Delta m^2_{\mathrm{atm}}\end{pmatrix}. \qquad (51)$$

For a normal mass hierarchy, the quantity $\Delta m^2_\odot$ is the smaller of the two mass-squared differences. It is of particular relevance in situations involving low-energy oscillations such as solar-neutrino measurements. The larger mass-squared difference $\Delta m^2_{\mathrm{atm}}$ plays a central role where mixing of high-energy neutrinos occurs, such as atmospheric-neutrino experiments. The mixing matrix $U_{a'b}$ can be written as

$$
\begin{aligned}
U_{a'b} = &\begin{pmatrix} c_{12} & -s_{12} & 0\\ s_{12} & c_{12} & 0\\ 0 & 0 & 1\end{pmatrix}\begin{pmatrix} c_{13} & 0 & -s_{13}e^{-i\delta}\\ 0 & 1 & 0\\ s_{13}e^{i\delta} & 0 & c_{13}\end{pmatrix}\\
&\times\begin{pmatrix} 1 & 0 & 0\\ 0 & c_{23} & -s_{23}\\ 0 & s_{23} & c_{23}\end{pmatrix}, \qquad (52)
\end{aligned}
$$

where $c_{ij}=\cos\theta_{ij}$, $s_{ij}=\sin\theta_{ij}$, and δ is the CP-violating phase. For antineutrino-antineutrino mixing, the conventional effective Hamiltonian for vacuum mixing is obtained by complex conjugation of the above results, which is equivalent to changing the sign of δ.

In the presence of matter, the 3×3 effective Hamiltonian $(h_0)_{ab}$ for neutrino-neutrino mixing acquires an additional term from Eq. (20) and becomes

$$(h_0)_{ab} = \frac{1}{2E}\Delta m^2_{ab} + (a_{L,\mathrm{eff}})^T_{ee}\delta_{ae}\delta_{be}, \qquad (53)$$

where the index e refers to the electron-neutrino flavor. The additional term changes the eigenvalues of $(h_0)_{ab}$ and hence the explicit values of the components of the overall mixing matrix. Note that the expression (53) is strictly valid only in the rest frame of the matter. For the specific scenarios considered below, this frame comoves with the Earth as it rotates on its axis and revolves about the Sun. These motions are nonrelativistic, however, so the comoving frame can be identified with the Sun-centered frame of Sec. III A to an accuracy of parts in 10^4. We adopt this identification in what follows.

In performing an analysis for Lorentz and *CPT* violation, the appropriate methodology depends on the location of the experiment in L-E space and on the presently unknown value of θ_{13}. Consider, for example, three hypothetical beam experiments with the same long baseline $L \sim 200$ km but with differing energies $E_1 \sim 10$ MeV, $E_2 \sim 1$ GeV, $E_3 \sim 100$ GeV. The first lies in a region where mass oscillations involve all three flavors, so the treatment of Lorentz and *CPT* violation requires the three-flavor formalism of the previous section. The same is true of the second experiment for large θ_{13}. However, if θ_{13} is small or zero, then significant mass mixings in this second experiment involve only two generations. Lorentz-violating effects within these two generations can then be studied using a two-flavor limit of the previous section, while effects involving the third flavor are well described using the procedures for negligible mass mixing presented in Ref. [16]. For the third experiment, no significant mass mixings occur and so the methods of Ref. [16] are applicable for Lorentz and *CPT* violation in all flavors of oscillations.

High sensitivity to operators for Lorentz and *CPT* violation of mass dimension three, such as those controlled by the coefficients $(a_L)^\alpha_{ab}$, can be achieved in experiments with long baselines L. Similarly, high sensitivity to operators of mass dimension four, such as those governed by $(c_L)^{\alpha\beta}_{ab}$, can be obtained via long baselines L, high energies E, or both. At present, most existing or planned long-baseline experiments have baselines $L \sim 200$–1500 km and energies $E \sim 1$–10 GeV and hence lie in a region of L-E space analogous to that of the second hypothetical experiment above.

For illustrative purposes, we consider here a variety of beam experiments involving long baselines L and seeking ν_e appearance in ν_μ beams or studying ν_μ disappearance. Existing experiments in this category include KEK to Super-Kamiokande (K2K) with baseline $L \simeq 250$ km [51], the MINOS far detector with baseline $L \simeq 750$ km [52], and the Oscillation Project with Emulsion-Tracking Apparatus (OPERA) [53]. The latter has essentially identical baseline $L \simeq 750$ km to the Imaging Cosmic and Rare Underground Signals experiment (ICARUS) [54]. The Fermilab E929 experiment (NOνA) [55] with baseline $L \simeq 800$ km is currently under construction, while the Tokai to Kamioka (T2K) experiment [56] with baseline $L \simeq 300$ km has recently begun data taking. Other experiments with even longer baselines are under consideration, including one at the Deep Underground Science and Engineering Lab (DUSEL) [57] using a neutrino beam from Fermilab and baseline $L \sim 1300$ km, and the Tokai to Kamioka and Korea (T2KK) experiment [58] using the same neutrino beam as T2K but with baseline $L \simeq 1000$ km. All of these experiments have excellent sensitivity to perturbative Lorentz and *CPT* violation.

For definiteness in what follows, we consider two explicit scenarios. In the first, discussed in Sec. III B 2, we take a comparatively large value of θ_{13} and consider the effects of Lorentz and *CPT* violation for studies of ν_e appearance. For this situation, mass mixing involving all three flavors occurs and so the full formalism of the previous section is appropriate for the analysis. In the second scenario, considered in Sec. III B 3, we suppose θ_{13} is negligible and investigate the effects of Lorentz and *CPT* violation on ν_μ disappearance. For this case, only $\nu_\mu \leftrightarrow \nu_\tau$ involves significant mass mixing and so a two-flavor limit of the previous section can be applied.

We can also identify two interesting regions of L-E space that could benefit from the development of new experiments. The first is the region of long baselines $L \gtrsim 200$ km with low energies $E \lesssim 100$ MeV. This is of particular interest if $\theta_{13} \simeq 0°$, since it provides an opportunity for clean studies of three-flavor mixings that are otherwise challenging to perform. A comparatively intense source is needed due to the long baseline and the cross-section falloff with energy. One possibility is a setup similar to the Kamioka Liquid Scintillator Antineutrino Detector (KamLAND) [59] but with a single source, directional sensitivity, or both. For longer baselines, a beta beam [60] may be an interesting option. A low-energy beta beam has been studied in the context of short-baseline experiments [61]. Another possibility might be an intense pulsed neutrino beam such as that proposed for a neutrino facility at the Spallation Neutron Source (ν-SNS) [62] and lying in the 10–50 MeV energy range.

The second interesting region lies at high energies $E \gtrsim 100$ GeV, even for comparatively short baselines $L \lesssim 10$ km. Neutrino beams at these energies have been used in the Neutrinos at the Tevatron (NuTeV) [63] and Chicago-Columbia-Fermilab-Rochester (CCFR) [64] experiments. For studies of the coefficients $(c_L)^{\alpha\beta}_{ab}$ for Lorentz and *CPT* violation, a high energy compensates for a shorter baseline and so oscillation experiments in this region could have a competitive reach. Since mass mixing is negligible, the methods of Ref. [16] are applicable for analyzing Lorentz and *CPT* violation in this case.

2. Example: ν_e appearance

Consider first searches for Lorentz and *CPT* violation via ν_e appearance in a ν_μ beam, within the assumption of a comparatively large θ_{13}. This requires a full three-flavor analysis. We incorporate effects from matter-induced mixing via Eq. (53), and we adopt explicit parameter values for vacuum mixing compatible with the observed three-flavor oscillations in solar, atmospheric, reactor, and accelerator experiments [2],

$$\begin{gathered}\Delta m^2_\odot \simeq 8.0\times 10^{-5}\ \mathrm{eV}^2,\\ \Delta m^2_{\mathrm{atm}} \simeq 2.5\times 10^{-3}\ \mathrm{eV}^2,\\ \theta_{12}\simeq 34^\circ,\qquad \theta_{23}\simeq 45^\circ,\qquad \theta_{13}\simeq 12^\circ,\qquad \delta\simeq 0^\circ.\end{gathered} \tag{54}$$

Together with the matter effects, the values (54) determine the linear combinations of coefficients $(a_L)^\alpha_{ab}$ and $(c_L)^{\alpha\beta}_{ab}$ relevant for any given experiment. Note that both magnitudes and signs are significant. For example, the above values hold for a normal mass hierarchy. Our general expressions for oscillation probabilities are valid for any magnitudes and signs, but the numerical results for the illustrative examples below assume the specific choices (54). For a comprehensive exploration of the space of coefficients for Lorentz and *CPT* violation, distinct analyses of the same data must be performed for each acceptable choice of parameter values and must be reported as such. Note also that the approximation $\delta\simeq 0^\circ$ made in Eq. (54) implies that there is little or no *CP* violation in standard oscillations, although perturbative *CP* violation from Lorentz and *CPT* violation can still appear.

The anisotropies introduced by nonzero coefficients for Lorentz violation can lead to sidereal variations, which are the primary signals of interest here. The sidereal decomposition of the probability takes the form (48) with $\{ab\} = \{e\mu\}$. The four amplitudes $(P^{(1)}_{\mathcal{A}_s})_{e\mu}$, $(P^{(1)}_{\mathcal{A}_c})_{e\mu}$, $(P^{(1)}_{\mathcal{B}_s})_{e\mu}$, and $(P^{(1)}_{\mathcal{B}_c})_{e\mu}$ are linear combinations of coefficients for Lorentz violation given by Eqs. (47) and (49), and they can be measured by studying the variations of the neutrino mixing with sidereal time. The effects from the combination $(P^{(1)}_{\mathcal{C}})_{e\mu}$ are more challenging to detect experimentally because they have no accompanying time variation.

The amplitudes of the sidereal-variation probabilities depend on the quantities $(\tilde{a}_L)^\alpha_{e\mu}$ and $(\tilde{c}_L)^{\alpha\beta}_{e\mu}$, which are linear combinations of fundamental coefficients for Lorentz and *CPT* violation,

$$\begin{aligned}(\tilde{a}_L)^\alpha_{e\mu} &= \sum_{cd}(\mathcal{M}^{(1)}_{e\mu})_{cd}(a_L)^\alpha_{cd},\\ (\tilde{c}_L)^{\alpha\beta}_{e\mu} &= \sum_{cd}(\mathcal{M}^{(1)}_{e\mu})_{cd}(c_L)^{\alpha\beta}_{cd}.\end{aligned} \tag{55}$$

The results of an experimental analysis can therefore be expressed in terms of the fundamental coefficients $(a_L)^\alpha_{cd}$ and $(c_L)^{\alpha\beta}_{cd}$ by calculating the relevant complex factors $(\mathcal{M}^{(1)}_{e\mu})_{cd}$ for the given experiment.

Table I presents approximate numerical values of the real and imaginary parts of these factors for the eight long-baseline experiments K2K, MINOS, OPERA, ICARUS, NOνA, T2K, DUSEL, and T2KK. The entries are obtained assuming the parameter values (54) and incorporating effects of matter-induced mixing. The factors vary with the experiment, reflecting their dependence on the baseline L and the neutrino energy E. Since the experiments are clustered in a single region of L-E space, the corresponding numerical values for each factor are roughly comparable and so the four sets of $(\tilde{a}_L)^\alpha_{e\mu}$, $(\tilde{c}_L)^{\alpha\beta}_{e\mu}$ obtained from Eq. (55) are roughly comparable combinations of the fundamental coefficients $(a_L)^\alpha_{ab}$, $(c_L)^{\alpha\beta}_{ab}$. The table reveals that each experiment can measure particular combinations of $(\tilde{a}_L)^\alpha_{e\mu}$ and $(\tilde{c}_L)^{\alpha\beta}_{e\mu}$, which can then be used to constrain the coefficient space of $(a_L)^\alpha_{ab}$ and $(c_L)^{\alpha\beta}_{ab}$. The large number of coefficients for Lorentz and *CPT* violation and the limited number of observables for a given experiment imply that multiple experiments are needed to explore the entire coefficient space. In performing the analysis, it is of practical value to obtain an estimated maximal sensitivity to each individual component of the fundamental coefficients $(a_L)^\alpha_{ab}$ and $(c_L)^{\alpha\beta}_{ab}$ in turn, by allowing only that component to be nonzero and using the data to constrain it.

We can use Eqs. (47) and (49) to calculate estimated first-order sensitivities to $(\tilde{a}_L)^\alpha_{e\mu}$ and $(\tilde{c}_L)^{\alpha\beta}_{e\mu}$ for each of these experiments. Table II lists the four amplitudes $(P^{(1)}_{\mathcal{A}_s})_{e\mu}$, $(P^{(1)}_{\mathcal{A}_c})_{e\mu}$, $(P^{(1)}_{\mathcal{B}_s})_{e\mu}$, and $(P^{(1)}_{\mathcal{B}_c})_{e\mu}$ as explicit linear combinations of the real and imaginary parts of $(\tilde{a}_L)^\alpha_{e\mu}$ and $(\tilde{c}_L)^{\alpha\beta}_{e\mu}$. The linear combinations typically differ for each experiment and for each amplitude because they depend on the beam energy E and direction $\hat{p}$. Note that the two experiments OPERA and ICARUS have approximately the same baseline, orientation, and energy, so they can be listed together for our purposes. As an example, the table reveals that in GeV units the amplitude $(P^{(1)}_{\mathcal{A}_s})_{e\mu}$ for the K2K experiment is approximately

$$\begin{aligned}(P^{(1)}_{\mathcal{A}_s})_{e\mu} \approx{}& -0.1\,\mathrm{Re}(\tilde{a}_L)^X_{e\mu} + 0.4\,\mathrm{Re}(\tilde{c}_L)^{TX}_{e\mu}\\ &+ 0.1\,\mathrm{Im}(\tilde{a}_L)^X_{e\mu} - 0.3\,\mathrm{Im}(\tilde{c}_L)^{TX}_{e\mu}.\end{aligned} \tag{56}$$

Using Table I, all these combinations can be written in

TABLE I. Approximate numerical values of experiment-dependent dimensionless factors $(\mathcal{M}^{(1)}_{e\mu})_{cd}$ for the experiments K2K, MINOS, OPERA, ICARUS, NOνA, T2K, DUSEL, and T2KK. Numerical values are computed via Eq. (32) adopting the parameters (54), using estimated beam baselines L and neutrino energies E for each experiment. Within this approximation, the antineutrino factors $(\mathcal{M}^{(1)}_{\bar{e}\bar{\mu}})_{\bar{c}\bar{d}}$ are identical.

Experiment	K2K	MINOS	OPERA, ICARUS	NOνA	T2K	DUSEL	T2KK
$\mathrm{Re}(\mathcal{M}^{(1)}_{e\mu})_{ee}$	−0.05	−0.10	−0.01	−0.17	−0.16	−0.08	−0.11
$\mathrm{Re}(\mathcal{M}^{(1)}_{e\mu})_{e\mu}$	0.84	0.63	0.88	0.38	0.48	0.38	0.41
$\mathrm{Re}(\mathcal{M}^{(1)}_{e\mu})_{e\tau}$	−0.13	−0.24	−0.02	−0.46	−0.46	−0.35	−0.39
$\mathrm{Re}(\mathcal{M}^{(1)}_{e\mu})_{\mu e}$	0.00	0.00	0.00	0.00	0.00	0.01	0.01
$\mathrm{Re}(\mathcal{M}^{(1)}_{e\mu})_{\mu\mu}$	−0.05	−0.09	−0.01	−0.15	−0.15	−0.09	−0.11
$\mathrm{Re}(\mathcal{M}^{(1)}_{e\mu})_{\mu\tau}$	−0.01	−0.02	0.00	−0.01	−0.01	0.04	0.03
$\mathrm{Re}(\mathcal{M}^{(1)}_{e\mu})_{\tau e}$	0.00	0.00	0.00	0.00	0.00	0.00	0.00
$\mathrm{Re}(\mathcal{M}^{(1)}_{e\mu})_{\tau\mu}$	−0.05	−0.08	−0.01	−0.14	−0.14	−0.06	−0.08
$\mathrm{Re}(\mathcal{M}^{(1)}_{e\mu})_{\tau\tau}$	−0.01	−0.01	0.00	0.00	0.00	0.03	0.02
$\mathrm{Im}(\mathcal{M}^{(1)}_{e\mu})_{ee}$	−0.08	−0.06	−0.02	−0.03	−0.08	0.10	0.07
$\mathrm{Im}(\mathcal{M}^{(1)}_{e\mu})_{e\mu}$	−0.39	−0.62	−0.44	−0.61	−0.43	−0.34	−0.32
$\mathrm{Im}(\mathcal{M}^{(1)}_{e\mu})_{e\tau}$	−0.24	−0.24	−0.06	−0.21	−0.27	0.17	0.12
$\mathrm{Im}(\mathcal{M}^{(1)}_{e\mu})_{\mu e}$	0.00	0.01	0.00	0.02	0.03	−0.04	−0.03
$\mathrm{Im}(\mathcal{M}^{(1)}_{e\mu})_{\mu\mu}$	−0.07	−0.06	−0.02	−0.02	−0.02	−0.05	−0.05
$\mathrm{Im}(\mathcal{M}^{(1)}_{e\mu})_{\mu\tau}$	0.01	0.02	0.00	0.07	0.08	−0.08	−0.06
$\mathrm{Im}(\mathcal{M}^{(1)}_{e\mu})_{\tau e}$	0.00	0.01	0.00	0.02	0.02	−0.05	−0.04
$\mathrm{Im}(\mathcal{M}^{(1)}_{e\mu})_{\tau\mu}$	−0.06	−0.05	−0.02	0.00	0.00	−0.02	−0.02
$\mathrm{Im}(\mathcal{M}^{(1)}_{e\mu})_{\tau\tau}$	0.01	0.02	0.00	0.06	0.07	−0.12	−0.10

terms of the fundamental coefficients $(a_L)^\alpha_{ab}$ and $(c_L)^{\alpha\beta}_{ab}$ for Lorentz and *CPT* violation.

To obtain a crude estimate of the sensitivities for each experiment, we suppose that a 10% sidereal variation in the oscillation probability can be detected. This leads to a sensitivity of order $10\%/2L$. The last row of Table II lists these values for each experiment. In conjunction with the other entries in Table II and with the factors listed in Table I, these values can be used to obtain the estimated first-order reach for any desired coefficient for Lorentz and *CPT* violation.

Table II shows that long-baseline experiments have the potential to achieve extreme sensitivities to Lorentz and *CPT* violation in the $\nu_\mu \to \nu_e$ appearance mode if there is appreciable mass mixing arising from a comparatively large value of θ_{13}. Since some of the predicted effects are small, second-order effects may also play a role and may need to be incorporated in a comprehensive analysis of real data. If instead θ_{13} is tiny or zero, studies of Lorentz and *CPT* violation in the $\nu_\mu \to \nu_e$ appearance mode can be performed using the methodology of Ref. [16], as discussed in the previous subsection.

In the event that the listed experiments run in antineutrino mode, the attainable reach can be estimated similarly. In the vacuum, the factors $(\mathcal{M}^{(1)}_{\bar{a}\bar{b}})_{\bar{c}\bar{d}}$ are unaffected because the parameter values (54) imply *CP* invariance, so the corresponding amplitudes of the sidereal-variation probabilities can be found by replacing $(\tilde{a}_L)^\alpha_{e\mu}$ and $(\tilde{c}_L)^{\alpha\beta}_{e\mu}$ with $(\tilde{a}_R)^\alpha_{\bar{e}\bar{\mu}}$ and $(\tilde{c}_R)^{\alpha\beta}_{\bar{e}\bar{\mu}}$. However, the contribution from mass-induced mixing in Eq. (53) enters with opposite sign, so the estimated amplitudes in Table II acquire corresponding changes.

3. Example: ν_μ disappearance

Beams of ν_μ also provide opportunities to search for ν_μ disappearance. The probability of ν_μ oscillating into other neutrinos is given by

$$P_{\nu_\mu\to\nu_X} = 1 - P_{\nu_\mu\to\nu_\mu}. \tag{57}$$

The correction introduced by $(a_L)^\alpha_{ab}$ and $(c_L)^{\alpha\beta}_{ab}$ coefficients is therefore given by

$$P^{(1)}_{\nu_\mu\to\nu_X} = -P^{(1)}_{\nu_\mu\to\nu_\mu} = -2L\,\mathrm{Im}((S^{(0)}_{\mu\mu})^*\mathcal{H}^{(1)}_{\mu\mu}), \tag{58}$$

where $\mathcal{H}^{(1)}_{\mu\mu}$ is defined in Eq. (44). Again, sidereal variations can arise from the anisotropies introduced by Lorentz violation, and $\mathcal{H}^{(1)}_{\mu\mu}$ can be expanded in sidereal time according to Eq. (45).

TABLE II. Estimated amplitudes of sidereal-variation probabilities $(P^{(1)}_{\mathcal{A}_s})_{e\mu}$, $(P^{(1)}_{\mathcal{A}_c})_{e\mu}$, $(P^{(1)}_{\mathcal{B}_s})_{e\mu}$, and $(P^{(1)}_{\mathcal{B}_c})_{e\mu}$ for appearance experiments with $\nu_\mu \to \nu_e$. The numerical value is listed for the estimated contribution to each amplitude from the real and imaginary parts of the combinations $(\tilde{a}_L)^\alpha_{e\mu}$ and $(\tilde{c}_L)^{\alpha\beta}_{e\mu}$ of fundamental coefficients for Lorentz and *CPT* violation. Values are given to one decimal place, in dimensionless units for $(\tilde{a}_L)^\alpha_{e\mu}$ and in units of GeV for $(\tilde{c}_L)^{\alpha\beta}_{e\mu}$. A value of 0.0 indicates rounding to zero at this precision, while a dash implies the value is identically zero. The last row lists the approximate sensitivity in GeV of each experiment.

Experiment	K2K				MINOS				OPERA, ICARUS			
Amplitude	$P^{(1)}_{\mathcal{A}_s}$	$P^{(1)}_{\mathcal{A}_c}$	$P^{(1)}_{\mathcal{B}_s}$	$P^{(1)}_{\mathcal{B}_c}$	$P^{(1)}_{\mathcal{A}_s}$	$P^{(1)}_{\mathcal{A}_c}$	$P^{(1)}_{\mathcal{B}_s}$	$P^{(1)}_{\mathcal{B}_c}$	$P^{(1)}_{\mathcal{A}_s}$	$P^{(1)}_{\mathcal{A}_c}$	$P^{(1)}_{\mathcal{B}_s}$	$P^{(1)}_{\mathcal{B}_c}$
$\mathrm{Re}(\tilde{a}_L)^X_{e\mu}$	−0.1	0.0	—	—	0.0	0.1	—	—	0.0	0.0	—	—
$\mathrm{Re}(\tilde{a}_L)^Y_{e\mu}$	0.0	0.1	—	—	0.1	0.0	—	—	0.0	0.0	—	—
$\mathrm{Re}(\tilde{c}_L)^{TX}_{e\mu}$	0.4	0.0	—	—	0.2	−0.3	—	—	−1.0	0.6	—	—
$\mathrm{Re}(\tilde{c}_L)^{TY}_{e\mu}$	0.0	−0.4	—	—	−0.3	−0.2	—	—	0.6	1.0	—	—
$\mathrm{Re}(\tilde{c}_L)^{XX}_{e\mu}$	—	—	0.0	0.1	—	—	0.1	0.0	—	—	0.2	0.1
$\mathrm{Re}(\tilde{c}_L)^{XY}_{e\mu}$	—	—	0.0	−0.1	—	—	−0.1	0.0	—	—	−0.2	−0.1
$\mathrm{Re}(\tilde{c}_L)^{XZ}_{e\mu}$	0.0	0.0	—	—	−0.1	0.2	—	—	−0.4	0.2	—	—
$\mathrm{Re}(\tilde{c}_L)^{YY}_{e\mu}$	—	—	0.2	0.0	—	—	0.0	−0.1	—	—	0.3	−0.4
$\mathrm{Re}(\tilde{c}_L)^{YZ}_{e\mu}$	0.0	0.0	—	—	0.2	0.1	—	—	0.2	0.4	—	—
$\mathrm{Im}(\tilde{a}_L)^X_{e\mu}$	0.1	0.0	—	—	0.1	−0.1	—	—	0.0	0.0	—	—
$\mathrm{Im}(\tilde{a}_L)^Y_{e\mu}$	0.0	−0.1	—	—	−0.1	−0.1	—	—	0.0	0.0	—	—
$\mathrm{Im}(\tilde{c}_L)^{TX}_{e\mu}$	−0.3	0.0	—	—	−0.4	0.5	—	—	0.5	−0.3	—	—
$\mathrm{Im}(\tilde{c}_L)^{TY}_{e\mu}$	0.0	0.0	—	—	0.2	−0.3	—	—	0.2	−0.1	—	—
$\mathrm{Im}(\tilde{c}_L)^{XX}_{e\mu}$	—	—	0.0	−0.1	—	—	−0.1	0.0	—	—	−0.1	−0.1
$\mathrm{Im}(\tilde{c}_L)^{XY}_{e\mu}$	—	—	0.0	0.1	—	—	0.1	0.0	—	—	0.1	0.1
$\mathrm{Im}(\tilde{c}_L)^{XZ}_{e\mu}$	0.0	0.0	—	—	0.2	−0.3	—	—	0.2	−0.1	—	—
$\mathrm{Im}(\tilde{c}_L)^{YY}_{e\mu}$	—	—	−0.1	0.0	—	—	0.1	0.2	—	—	−0.1	0.3
$\mathrm{Im}(\tilde{c}_L)^{YZ}_{e\mu}$	0.0	0.0	—	—	−0.3	−0.2	—	—	−0.1	−0.2	—	—
Sensitivity	$<8 \times 10^{-23}$				$<3 \times 10^{-23}$				$<3 \times 10^{-23}$			

Experiment	NOνA				T2K				DUSEL				T2KK			
Amplitude	$P^{(1)}_{\mathcal{A}_s}$	$P^{(1)}_{\mathcal{A}_c}$	$P^{(1)}_{\mathcal{B}_s}$	$P^{(1)}_{\mathcal{B}_c}$	$P^{(1)}_{\mathcal{A}_s}$	$P^{(1)}_{\mathcal{A}_c}$	$P^{(1)}_{\mathcal{B}_s}$	$P^{(1)}_{\mathcal{B}_c}$	$P^{(1)}_{\mathcal{A}_s}$	$P^{(1)}_{\mathcal{A}_c}$	$P^{(1)}_{\mathcal{B}_s}$	$P^{(1)}_{\mathcal{B}_c}$	$P^{(1)}_{\mathcal{A}_s}$	$P^{(1)}_{\mathcal{A}_c}$	$P^{(1)}_{\mathcal{B}_s}$	$P^{(1)}_{\mathcal{B}_c}$
$\mathrm{Re}(\tilde{a}_L)^X_{e\mu}$	0.0	0.0	—	—	0.0	0.0	—	—	−0.1	0.0	—	—	−0.1	0.0	—	—
$\mathrm{Re}(\tilde{a}_L)^Y_{e\mu}$	0.0	0.0	—	—	0.0	0.0	—	—	0.0	0.1	—	—	0.0	0.1	—	—
$\mathrm{Re}(\tilde{c}_L)^{TX}_{e\mu}$	0.0	0.0	—	—	0.0	0.0	—	—	0.1	0.0	—	—	0.1	0.0	—	—
$\mathrm{Re}(\tilde{c}_L)^{TY}_{e\mu}$	0.0	0.0	—	—	0.0	0.0	—	—	0.0	−0.1	—	—	0.0	−0.1	—	—
$\mathrm{Re}(\tilde{c}_L)^{XX}_{e\mu}$	—	—	0.0	0.0	—	—	0.0	0.0	—	—	0.0	0.0	—	—	0.0	0.0
$\mathrm{Re}(\tilde{c}_L)^{XY}_{e\mu}$	—	—	0.0	0.0	—	—	0.0	0.0	—	—	0.0	0.0	—	—	0.0	0.0
$\mathrm{Re}(\tilde{c}_L)^{XZ}_{e\mu}$	0.0	0.0	—	—	0.0	0.0	—	—	0.0	0.0	—	—	0.0	0.0	—	—
$\mathrm{Re}(\tilde{c}_L)^{YY}_{e\mu}$	—	—	0.0	0.0	—	—	0.0	0.0	—	—	0.1	0.0	—	—	0.1	0.0
$\mathrm{Re}(\tilde{c}_L)^{YZ}_{e\mu}$	0.0	0.0	—	—	0.0	0.0	—	—	0.0	0.0	—	—	0.0	0.0	—	—
$\mathrm{Im}(\tilde{a}_L)^X_{e\mu}$	0.2	−0.2	—	—	0.3	0.0	—	—	0.1	0.0	—	—	0.2	0.0	—	—
$\mathrm{Im}(\tilde{a}_L)^Y_{e\mu}$	−0.2	−0.2	—	—	0.0	−0.3	—	—	0.0	−0.1	—	—	0.0	−0.2	—	—
$\mathrm{Im}(\tilde{c}_L)^{TX}_{e\mu}$	−0.6	0.8	—	—	−0.4	0.0	—	—	−0.3	0.0	—	—	−0.3	0.0	—	—
$\mathrm{Im}(\tilde{c}_L)^{TY}_{e\mu}$	0.4	−0.5	—	—	0.0	0.0	—	—	0.0	0.0	—	—	0.0	0.0	—	—
$\mathrm{Im}(\tilde{c}_L)^{XX}_{e\mu}$	—	—	−0.2	0.1	—	—	0.0	−0.1	—	—	0.0	−0.1	—	—	0.0	−0.1
$\mathrm{Im}(\tilde{c}_L)^{XY}_{e\mu}$	—	—	0.2	−0.1	—	—	0.0	0.1	—	—	0.0	0.1	—	—	0.0	0.1
$\mathrm{Im}(\tilde{c}_L)^{XZ}_{e\mu}$	0.4	−0.5	—	—	0.0	0.0	—	—	0.0	0.0	—	—	0.0	0.0	—	—
$\mathrm{Im}(\tilde{c}_L)^{YY}_{e\mu}$	—	—	0.1	0.4	—	—	−0.2	0.0	—	—	−0.1	0.0	—	—	−0.2	0.0
$\mathrm{Im}(\tilde{c}_L)^{YZ}_{e\mu}$	−0.5	−0.4	—	—	0.0	0.0	—	—	0.0	0.0	—	—	0.0	0.0	—	—
Sensitivity	$<2 \times 10^{-23}$				$<7 \times 10^{-23}$				$<2 \times 10^{-23}$				$<2 \times 10^{-23}$			

For ν_μ disappearance experiments, the sidereal decomposition of the probability is given by Eq. (48) with $\{ab\} = \{\mu\mu\}$. The four amplitudes $(P^{(1)}_{\mathcal{A}_s})_{\mu\mu}$, $(P^{(1)}_{\mathcal{A}_c})_{\mu\mu}$, $(P^{(1)}_{\mathcal{B}_s})_{\mu\mu}$, $(P^{(1)}_{\mathcal{B}_c})_{\mu\mu}$ of the sidereal-variation probabilities are provided in Eqs. (47) and (49). Their exact expressions in terms of the combinations $(\tilde{a}_L)^\alpha_{\mu\mu}$ and $(\tilde{c}_L)^{\alpha\beta}_{\mu\mu}$ of fundamental coefficients for Lorentz and *CPT* violation take the form

$$(\tilde{a}_L)^\alpha_{\mu\mu} = \sum_{cd}(\mathcal{M}^{(1)}_{\mu\mu})_{cd}(a_L)^\alpha_{cd},$$
$$(\tilde{c}_L)^{\alpha\beta}_{\mu\mu} = \sum_{cd}(\mathcal{M}^{(1)}_{\mu\mu})_{cd}(c_L)^{\alpha\beta}_{cd}. \tag{59}$$

If needed, the experiment-dependent complex factors $(\mathcal{M}^{(1)}_{\mu\mu})_{cd}$ controlling these linear combinations can be obtained using Eq. (32).

In scenarios with oscillations occurring primarily between ν_μ and ν_τ, the probability for ν_μ disappearance can be well approximated by restricting attention to two-flavor vacuum mixing. This limit involves only one mass-squared difference and one mixing angle, and it offers another useful illustration of the general analysis given in Sec. III A. In the event of tiny or zero θ_{13}, it is also the relevant limit for the eight experiments considered above. We adopt this case as our second illustrative example.

In the Lorentz-invariant two-flavor limit, an overall diagonal term can be removed from the Hamiltonian $(h_0)_{ab}$ because it is irrelevant for oscillations. This gives a 2×2 mass matrix of the form

$$(h_0)_{ab} \simeq \frac{1}{2E} U^\dagger \begin{pmatrix} 0 & 0 \\ 0 & \Delta m^2_{32} \end{pmatrix} U. \tag{60}$$

The flavor indices are now restricted to two generations, $a, b, \ldots = \mu, \tau$. The mixing matrix U depends on the mixing angle θ_{23} according to

$$U_{a'a} = \begin{pmatrix} c_{23} & -s_{23} \\ s_{23} & c_{23} \end{pmatrix}. \tag{61}$$

Note that *CP* violation due to mass mixing is strictly unobservable in this limit. The single mass difference is then given by

$$\Delta m^2_{32} = \Delta m^2_{\text{atm}} - \Delta m^2_{\odot} \simeq \Delta m^2_{\text{atm}}. \tag{62}$$

For the explicit estimations in this subsection, we choose for Δm^2_{atm} and θ_{23} the values

$$\Delta m^2_{\text{atm}} \simeq 2.5 \times 10^{-3}\ \text{eV}^2,$$
$$\theta_{23} \simeq 45°, \tag{63}$$

which are consistent with the three-flavor parameter values (54).

In the presence of Lorentz and *CPT* violation, the two-generation approximation simplifies the expression (39) for the first-order oscillation probabilities. With the two flavors being ν_μ and ν_τ, we have

$$S^{(0)}_{e\mu} = S^{(0)}_{e\tau} = S^{(0)}_{\mu e} = S^{(0)}_{\tau e} = 0. \tag{64}$$

This implies no mixing with electron neutrinos occurs in the first-order perturbation. Also, inspection of the form of $(\mathcal{M}^{(1)}_{ab})_{cd}(t)$ given in Eq. (32) reveals that only those coefficients for Lorentz violation lying in the two-flavor $\{\mu\tau\}$ subspace can lead to first-order effects.

Explicitly, we find the first-order oscillation probabilities are

$$\begin{aligned} P^{(1)}_{\nu_\mu\leftrightarrow\nu_\tau} &= -P^{(1)}_{\nu_\mu\to\nu_\mu} = -P^{(1)}_{\nu_\tau\to\nu_\tau} \\ &= 2L\,\text{Im}((S^{(0)}_{\tau\mu})^* \mathcal{H}^{(1)}_{\tau\mu}) \\ &\approx \text{Re}(\delta h_{\mu\tau})L \sin(\Delta m^2_{32}L/2E). \end{aligned} \tag{65}$$

We assume maximal mixing in the last expression, in accordance with the parameter values (63). Note that only the real part of $(\delta h)_{\mu\tau}$ contributes to first-order mixing in this limit. Also, the corresponding antineutrino mixing probabilities are found by the index replacements $\{\mu\tau\} \to \{\bar{\mu}\,\bar{\tau}\}$, which is equivalent to changing the sign of the coefficient $(a_L)^\alpha_{\mu\tau}$ in $(\delta h)_{\mu\tau}$.

The oscillation probability (65) can be decomposed into sidereal amplitudes according to Eq. (48). The amplitudes take the form (49) with the definitions (47). The two-flavor approximation makes it straightforward to express the latter directly in terms of the real parts of the fundamental coefficients $(a_L)^\alpha_{\mu\tau}$ and $(c_L)^{\alpha\beta}_{\mu\tau}$ rather than the intermediate combinations $(\tilde{a}_L)^\alpha_{\mu\tau}$ and $(\tilde{c}_L)^{\alpha\beta}_{\mu\tau}$. We can use these results to estimate experimental sensitivities to the real parts of $(a_L)^\alpha_{\mu\tau}$ and $(c_L)^{\alpha\beta}_{\mu\tau}$ for any specified experiment.

Table III presents the results of estimates for the eight long-baseline beam experiments considered above. In each experiment, numerical values are listed for the weighting of the real parts $(a_L)^\alpha_{\mu\tau}$ and $(c_L)^{\alpha\beta}_{\mu\tau}$ in the four amplitudes $(P^{(1)}_{\mathcal{A}_s})_{\mu\tau}$, $(P^{(1)}_{\mathcal{A}_c})_{\mu\tau}$, $(P^{(1)}_{\mathcal{B}_s})_{\mu\tau}$, and $(P^{(1)}_{\mathcal{B}_c})_{\mu\tau}$. The last row provides a rough approximation to the attainable sensitivity, based on assuming that a 10% sidereal variation in the oscillation probability can be detected. The results indicate that all these experiments can achieve impressive sensitivities to perturbative Lorentz and *CPT* violation.

C. *CP* and *CPT* asymmetries

In this subsection, we discuss the effects of Lorentz violation on tests of the discrete symmetries *CP* and *CPT* using neutrino oscillations. Experimentally, nature is known to break *CP* invariance in the weak interactions, although no *CP* violation in neutrino oscillations has yet been detected. In contrast, compelling evidence for violation of *CPT* symmetry in any system is lacking to date. On the theoretical front, *CPT* invariance has a profound connection to Lorentz invariance in quantum field theory, where the *CPT* theorem shows that under mild assumptions *CPT* violation is accompanied by Lorentz violation

TABLE III. Estimated amplitudes of sidereal-variation probabilities $(P^{(1)}_{\mathcal{A}_s})_{\mu\tau}$, $(P^{(1)}_{\mathcal{A}_c})_{\mu\tau}$, $(P^{(1)}_{\mathcal{B}_s})_{\mu\tau}$, and $(P^{(1)}_{\mathcal{B}_c})_{\mu\tau}$ within the two-generation approximation. The numerical value is listed for the estimated contribution to each amplitude from the real parts of the fundamental coefficients $(a_L)^\alpha_{\mu\tau}$ and $(c_L)^{\alpha\beta}_{\mu\tau}$ for Lorentz and CPT violation. Values are given to one decimal place, in dimensionless units for $(a_L)^\alpha_{\mu\tau}$ and in units of GeV for $(c_L)^{\alpha\beta}_{\mu\tau}$. A value of 0.0 indicates rounding to zero at this precision, while a dash implies the value is identically zero. The last row lists the approximate sensitivity in GeV of each experiment. Analogous results for possible experiments with antineutrinos can be obtained by replacing $(a_L)^\alpha_{\mu\tau}$ and $(c_L)^{\alpha\beta}_{\mu\tau}$ with $(a_R)^\alpha_{\bar\mu\bar\tau}$ and $(c_R)^{\alpha\beta}_{\bar\mu\bar\tau}$.

Experiment	K2K				MINOS				OPERA, ICARUS			
Amplitude	$P^{(1)}_{\mathcal{A}_s}$	$P^{(1)}_{\mathcal{A}_c}$	$P^{(1)}_{\mathcal{B}_s}$	$P^{(1)}_{\mathcal{B}_c}$	$P^{(1)}_{\mathcal{A}_s}$	$P^{(1)}_{\mathcal{A}_c}$	$P^{(1)}_{\mathcal{B}_s}$	$P^{(1)}_{\mathcal{B}_c}$	$P^{(1)}_{\mathcal{A}_s}$	$P^{(1)}_{\mathcal{A}_c}$	$P^{(1)}_{\mathcal{B}_s}$	$P^{(1)}_{\mathcal{B}_c}$
$\mathrm{Re}(a_L)^X_{\mu\tau}$	−0.5	0.0	—	—	−0.2	0.2	—	—	0.1	−0.1	—	—
$\mathrm{Re}(a_L)^Y_{\mu\tau}$	0.0	0.5	—	—	0.2	0.2	—	—	−0.1	−0.1	—	—
$\mathrm{Re}(c_L)^{TX}_{\mu\tau}$	1.2	−0.1	—	—	0.7	−1.0	—	—	−3.5	2.0	—	—
$\mathrm{Re}(c_L)^{TY}_{\mu\tau}$	−0.1	−1.2	—	—	−1.0	−0.7	—	—	2.0	3.5	—	—
$\mathrm{Re}(c_L)^{XX}_{\mu\tau}$	—	—	0.0	0.3	—	—	0.2	−0.1	—	—	0.8	0.5
$\mathrm{Re}(c_L)^{XY}_{\mu\tau}$	—	—	0.0	−0.3	—	—	−0.2	0.1	—	—	−0.8	−0.5
$\mathrm{Re}(c_L)^{XZ}_{\mu\tau}$	−0.1	0.0	—	—	−0.4	0.6	—	—	−1.4	0.8	—	—
$\mathrm{Re}(c_L)^{YY}_{\mu\tau}$	—	—	0.6	−0.1	—	—	−0.1	−0.4	—	—	0.9	−1.6
$\mathrm{Re}(c_L)^{YZ}_{\mu\tau}$	0.0	0.1	—	—	0.6	0.4	—	—	0.8	1.4	—	—
Sensitivity	$<8\times10^{-23}$				$<3\times10^{-23}$				$<3\times10^{-23}$			

Experiment	NOνA				T2K				DUSEL				T2KK			
Amplitude	$P^{(1)}_{\mathcal{A}_s}$	$P^{(1)}_{\mathcal{A}_c}$	$P^{(1)}_{\mathcal{B}_s}$	$P^{(1)}_{\mathcal{B}_c}$	$P^{(1)}_{\mathcal{A}_s}$	$P^{(1)}_{\mathcal{A}_c}$	$P^{(1)}_{\mathcal{B}_s}$	$P^{(1)}_{\mathcal{B}_c}$	$P^{(1)}_{\mathcal{A}_s}$	$P^{(1)}_{\mathcal{A}_c}$	$P^{(1)}_{\mathcal{B}_s}$	$P^{(1)}_{\mathcal{B}_c}$	$P^{(1)}_{\mathcal{A}_s}$	$P^{(1)}_{\mathcal{A}_c}$	$P^{(1)}_{\mathcal{B}_s}$	$P^{(1)}_{\mathcal{B}_c}$
$\mathrm{Re}(a_L)^X_{\mu\tau}$	−0.1	0.2	—	—	−0.2	0.0	—	—	−0.5	0.0	—	—	−0.5	0.0	—	—
$\mathrm{Re}(a_L)^Y_{\mu\tau}$	0.2	0.1	—	—	0.0	0.2	—	—	0.0	0.5	—	—	0.0	0.5	—	—
$\mathrm{Re}(c_L)^{TX}_{\mu\tau}$	0.6	−0.8	—	—	0.2	0.0	—	—	1.0	0.0	—	—	0.8	0.1	—	—
$\mathrm{Re}(c_L)^{TY}_{\mu\tau}$	−0.8	−0.6	—	—	0.0	−0.2	—	—	0.0	−1.0	—	—	0.1	−0.8	—	—
$\mathrm{Re}(c_L)^{XX}_{\mu\tau}$	—	—	0.2	−0.1	—	—	0.0	0.1	—	—	0.0	0.2	—	—	0.0	0.2
$\mathrm{Re}(c_L)^{XY}_{\mu\tau}$	—	—	−0.2	0.1	—	—	0.0	−0.1	—	—	0.0	−0.2	—	—	0.0	−0.2
$\mathrm{Re}(c_L)^{XZ}_{\mu\tau}$	−0.4	0.5	—	—	0.0	0.0	—	—	−0.2	0.0	—	—	0.1	0.0	—	—
$\mathrm{Re}(c_L)^{YY}_{\mu\tau}$	—	—	−0.1	−0.3	—	—	0.1	0.0	—	—	0.5	0.0	—	—	0.4	0.1
$\mathrm{Re}(c_L)^{YZ}_{\mu\tau}$	0.5	0.4	—	—	0.0	0.0	—	—	0.0	0.2	—	—	0.0	−0.1	—	—
Sensitivity	$<2\times10^{-23}$				$<7\times10^{-23}$				$<2\times10^{-23}$				$<2\times10^{-23}$			

[8]. No such relationship exists for CP, and indeed CP may be violated even when Lorentz invariance holds.

Consider first CP violation. The CP transformation interchanges oscillation probabilities according to

$$P_{\nu_a\to\nu_b} \overset{CP}{\leftrightarrow} P_{\bar\nu_a\to\bar\nu_b}, \tag{66}$$

so CP violation can be revealed as differences in neutrino and antineutrino probabilities. A generic measure of CP violation for mixing involving flavors $\{a, b\}$ is the asymmetry

$$\mathcal{A}^{CP}_{ab} = \frac{P_{\nu_a\to\nu_b} - P_{\bar\nu_a\to\bar\nu_b}}{P_{\nu_a\to\nu_b} + P_{\bar\nu_a\to\bar\nu_b}}, \tag{67}$$

which is zero when CP is a symmetry of the oscillations.

Similar results hold for CPT. Under the CPT transformation, the probabilities exchange according to

$$P_{\nu_a\to\nu_b} \overset{CPT}{\leftrightarrow} P_{\bar\nu_b\to\bar\nu_a}. \tag{68}$$

For mixing involving flavors $\{a, b\}$, we can therefore define the asymmetry

$$\mathcal{A}^{CPT}_{ab} = \frac{P_{\nu_a\to\nu_b} - P_{\bar\nu_b\to\bar\nu_a}}{P_{\nu_a\to\nu_b} + P_{\bar\nu_b\to\bar\nu_a}}. \tag{69}$$

This asymmetry vanishes if CPT invariance holds. Note, however, that the converse is false: models can be constructed in which CPT is violated even when the asymmetry $\mathcal{A}^{CPT}_{ab}$ vanishes [13]. In such cases, detailed studies of energy and direction dependences may be required to reveal CPT violation.

For the special case of two-flavor models, the probabilities are blind to possible T violation, so $P_{\nu_a\to\nu_b} = P_{\nu_b\to\nu_a}$ and $P_{\bar\nu_a\to\bar\nu_b} = P_{\bar\nu_b\to\bar\nu_a}$. Consequently, the CP and CPT asymmetries are identical in the two-flavor limit,

$$\mathcal{A}^{CP}_{ab} = \mathcal{A}^{CPT}_{ab}, \qquad a, b = \mu, \tau. \tag{70}$$

Note, however, that *CPT* violation may present itself in other ways, such as unconventional energy and direction dependence.

The above asymmetries can be used to test both *CP* and *CPT* in neutrino-oscillation experiments. In the Lorentz-invariant case, violation of *CP* symmetry occurs when both the mixing angle θ_{13} and the *CP* phase δ are nonzero. Measuring θ_{13} and searching for *CP* violation are major goals of many forthcoming oscillation experiments. Some experiments can change polarity, choosing to focus either positively or negatively charged mesons into the decay pipe, and hence can run in both neutrino and antineutrino modes. This feature may permit high-statistics direct searches for *CP* violation. The nature of the beam or other properties may also lead to accumulation of neutrino and antineutrino data. In all these cases, both Lorentz-conserving and Lorentz-violating situations can be accessed, thereby enabling also searches for *CPT* violation.

The interpretation of asymmetries constructed from experimental data requires a theoretical framework. One phenomenological approach to *CPT* violation assumes different masses and mixing angles for neutrinos and antineutrinos. In the two-flavor case, for example, this approach takes a set of parameters $(\Delta m^2, \theta)$ for neutrinos and a second set $(\Delta\bar{m}^2, \bar{\theta})$ for antineutrinos. It is tempting to adopt the resulting explicit expression for the asymmetry $\mathcal{A}^{CPT}_{ab} \equiv \mathcal{A}^{CP}_{ab}$ for purposes of data analysis and interpretation, treating the parameters Δm^2, $\Delta\bar{m}^2$, θ, and $\bar{\theta}$ as Lorentz-scalar constants. However, according to the *CPT* theorem this procedure is inconsistent with quantum field theory because under mild assumptions *CPT* violation in field theory must come with Lorentz violation [8], so the parameters Δm^2, $\Delta\bar{m}^2$, θ, and $\bar{\theta}$ cannot be Lorentz scalars. Instead, they must depend on the 4-momentum of the neutrino, including both the energy E and the propagation direction $\hat{p}$ relative to the Sun-centered frame. A typical experiment involves neutrinos spanning a spectrum of values for E and $\hat{p}$. The 4-momentum dependence of the asymmetry therefore entails significant consequences for data analysis and its interpretation in searches for *CPT* violation.

As an illustration, we derive here the explicit first-order form of the two-flavor *CPT* asymmetries $\mathcal{A}^{CPT}_{\mu\tau} = \mathcal{A}^{CP}_{\mu\tau}$ and $\mathcal{A}^{CPT}_{\mu\mu} = \mathcal{A}^{CP}_{\mu\mu}$ in the field-theoretic context. For definiteness we assume maximal mixing, which is consistent with the parameter values (63). At first order, calculation reveals these asymmetries depend on the coefficients $(a_L)^\alpha_{\mu\tau}$ for Lorentz and *CPT* violation but are independent of $(c_L)^{\alpha\beta}_{\mu\tau}$.

To present the asymmetries, it is convenient to introduce the *CPT*-odd part $(\delta h)^{CPT}_{\mu\tau}$ of the perturbative Hamiltonian $(\delta h)_{\mu\tau}$ with coefficients expressed in the Sun-centered frame,

$$\begin{aligned}(\delta h)^{CPT}_{\mu\tau} &\equiv (\delta h)_{\mu\tau}|_{c_L\to 0}\\ &= (a_L)^T_{\mu\tau} - \hat{N}^Z(a_L)^Z_{\mu\tau}\\ &\quad + (\hat{N}^Y(a_L)^X_{\mu\tau} - \hat{N}^X(a_L)^Y_{\mu\tau})\sin\omega_\oplus T_\oplus\\ &\quad - (\hat{N}^X(a_L)^X_{\mu\tau} + \hat{N}^Y(a_L)^Y_{\mu\tau})\cos\omega_\oplus T_\oplus.\end{aligned} \tag{71}$$

In terms of this quantity, we find that the *CPT* asymmetry $\mathcal{A}^{CPT}_{\mu\tau}$ is

$$\mathcal{A}^{CPT}_{\mu\tau} = \mathcal{A}^{CP}_{\mu\tau} \approx 2L\cot\left(\frac{\Delta m^2_{32}L}{4E}\right)\mathrm{Re}(\delta h)^{CPT}_{\mu\tau}. \tag{72}$$

This result is valid provided the experiment operates away from the region of parameter space leading to small oscillations, $\sin(\Delta m^2_{32}L/4E) \not\approx 0$. For the second *CPT* asymmetry $\mathcal{A}^{CPT}_{\mu\mu}$, we obtain

$$\mathcal{A}^{CPT}_{\mu\mu} = \mathcal{A}^{CP}_{\mu\mu} \approx -2L\tan\left(\frac{\Delta m^2_{32}L}{4E}\right)\mathrm{Re}(\delta h)^{CPT}_{\mu\tau}, \tag{73}$$

where now we assume the experiment operates away from the region of parameter space leading to large oscillations, $\sin(\Delta m^2_{32}L/4E) \not\approx 1$. Inspection of these results reveals that the two asymmetries $\mathcal{A}^{CPT}_{\mu\tau}$ in Eq. (72) and $\mathcal{A}^{CPT}_{\mu\mu}$ in Eq. (73) contain the same essential information about *CPT* violation but are valid in different regions of parameter space. In practice, at least one of the two asymmetries can be applied for a given experiment.

The results (72) and (73) display several interesting features. The asymmetries grow with baseline L, so experiments with comparable statistical power but longer baselines have improved sensitivity. According to Eq. (71), the asymmetries also vary with sidereal time $T_\oplus$ and depend on the direction of the neutrino beam. Both these effects are features of *CPT* violation and its accompanying Lorentz breaking. We remark in passing that the structure of the above equations bears a close similarity to that of the analogous measures for *CPT* violation in studies of neutral mesons. For example, the dependence of $(\delta h)^{CPT}_{\mu\tau}$ on the coefficients $(a_L)^\alpha_{\mu\tau}$ in Eq. (71) parallels that of the measure of *CPT* violation given in Eq. (14) of Ref. [65].

In a given experiment, measuring the amplitudes of the sidereal variations in the asymmetries (72) and (73) may produce interesting sensitivities to the coefficient combinations $(\hat{N}^Y(a_L)^X_{\mu\tau} - \hat{N}^X(a_L)^Y_{\mu\tau})$ and $(\hat{N}^X(a_L)^X_{\mu\tau} + \hat{N}^Y(a_L)^Y_{\mu\tau})$. These combinations are independent of the coefficients $(c_L)^{\alpha\beta}_{\mu\tau}$ for *CPT*-even Lorentz violation. Inspection of Eq. (71) reveals that each asymmetry also depends on the coefficients $(a_L)^T_{\mu\tau}$ and $(a_L)^Z_{\mu\tau}$, which are inaccessible via direct sidereal decomposition of the oscillation probabilities or asymmetries. One way to extract sensitivity to these coefficients is to average the data over time, in analogy to the extraction of the corresponding coefficients for *CPT* violation in experiments with neutral mesons [66]. The time-averaged asymmetry $\overline{\mathcal{A}^{CPT}_{\mu\tau}}$ is

$$\overline{\mathcal{A}^{CPT}_{\mu\tau}} \approx 2L\cot\left(\frac{\Delta m^2_{32}L}{4E}\right)\mathrm{Re}[(a_L)^T_{\mu\tau} - \hat{N}^Z(a_L)^Z_{\mu\tau}], \tag{74}$$

while the time-averaged asymmetry $\overline{\mathcal{A}^{CPT}_{\mu\mu}}$ is

$$\overline{\mathcal{A}^{CPT}_{\mu\mu}} \approx -2L\tan\left(\frac{\Delta m^2_{32}L}{4E}\right)\mathrm{Re}[(a_L)^T_{\mu\tau} - \hat{N}^Z(a_L)^Z_{\mu\tau}]. \tag{75}$$

Note that these results remain dependent on the beam direction despite the time averaging. Each asymmetry therefore typically has distinct physical meanings for different experiments. For example, the directional factor $\hat{N}^Z$ is $\hat{N}^Z \simeq 0.1$ for K2K, $\hat{N}^Z \simeq 0.6$ for MINOS, $\hat{N}^Z \simeq -0.4$ for OPERA and ICARUS, $\hat{N}^Z \simeq 0.6$ for NOνA, $\hat{N}^Z \simeq -0.01$ for T2K, $\hat{N}^Z \simeq 0.2$ for DUSEL, and $\hat{N}^Z \simeq -0.1$ for T2KK.

While experiments capable of CP tests necessarily test for CPT signals in the two-flavor approximation, the CPT signature $P_{\nu_a\to\nu_b} \neq P_{\bar{\nu}_b\to\bar{\nu}_a}$ may be more challenging to detect in three-neutrino scenarios. Data from the accelerator experiments discussed above or from next-generation studies using a beta beam [60] or a dedicated neutrino factory [67] could be well suited for seeking three-flavor CPT violation through direct comparisons of neutrinos and antineutrinos using the asymmetry $\mathcal{A}^{CPT}_{ab}$ of Eq. (69). Suitable comparisons of neutrinos and antineutrinos, perhaps including time averaging as above, could also lead to measurements of coefficient combinations without accompanying sidereal variations.

IV. COEFFICIENTS $\tilde{g}^{\alpha\beta}_{a\bar{b}}$ AND $\tilde{H}^{\alpha}_{a\bar{b}}$

In this section, we discuss the dominant effects on neutrino oscillations arising from the coefficients $\tilde{g}^{\alpha\beta}_{a\bar{b}}$ and $\tilde{H}^{\alpha}_{a\bar{b}}$. As shown in Eq. (39), these coefficients leave the oscillation probabilities unaffected at first order. The dominant effects appear at second order, where the probabilities are given by Eq. (41).

The features introduced by $\tilde{g}^{\alpha\beta}_{a\bar{b}}$ and $\tilde{H}^{\alpha}_{a\bar{b}}$ include unconventional energy and directional dependences. However, some key differences arise compared to the case of the coefficients $(a_L)^\alpha_{ab}$ and $(c_L)^{\alpha\beta}_{ab}$. For example, the dominant sidereal variations include higher harmonics with frequencies up to $4\omega_\oplus$. Another example is mixing between neutrinos and antineutrinos [13], which violates lepton-number conservation. This feature arises because $\tilde{g}^{\alpha\beta}_{a\bar{b}}$ and $\tilde{H}^{\alpha}_{a\bar{b}}$ lie in the off-diagonal blocks of the perturbative Hamiltonian (9).

In Sec. IV A, we focus on the second-order contributions to the neutrino-antineutrino oscillation probability $P^{(2)}_{\bar{\nu}_b\to\nu_a}$, which involves lepton-number violation. The second-order effects on the neutrino-neutrino and antineutrino-antineutrino mixing probabilities $P^{(2)}_{\nu_b\to\nu_a}$ and $P^{(2)}_{\bar{\nu}_b\to\bar{\nu}_a}$ are considered in Sec. IV B.

A. Oscillations violating lepton number

In this subsection, we derive the sidereal behavior of the second-order oscillation probability $P^{(2)}_{\bar{\nu}_b\to\nu_a}$ for neutrino-antineutrino mixing. Equations (40) and (41) specify this probability in terms of the perturbative Hamiltonian $\delta h_{a\bar{b}}$, which itself depends on the coefficients $\tilde{g}^{\alpha\beta}_{a\bar{b}}$ and $\tilde{H}^{\alpha}_{a\bar{b}}$ according to Eq. (15). Note that neutrino-antineutrino oscillations are independent of the coefficients $(a_L)^\alpha_{ab}$ and $(c_L)^{\alpha\beta}_{ab}$ at this order.

To determine the sidereal decomposition of the off-diagonal block $\delta h_{a\bar{b}}$ of the perturbative Hamiltonian, we note that $\tilde{H}^{\alpha}_{a\bar{b}}$ is an observer vector and hence induces effects at frequency $\omega_\oplus$, while $\tilde{g}^{\alpha\beta}_{a\bar{b}}$ is a 2-tensor and hence induces effects at $\omega_\oplus$ and $2\omega_\oplus$. The sidereal decomposition of $\delta h_{a\bar{b}}$ therefore takes the form

$$\begin{aligned}\delta h_{a\bar{b}} &\equiv -i\sqrt{2}(\epsilon_+)_\alpha[\tilde{g}^{\alpha\beta}p_\beta - \tilde{H}^\alpha]_{a\bar{b}}\\ &= (\mathcal{C})_{a\bar{b}} + (\mathcal{A}_s)_{a\bar{b}}\sin\omega_\oplus T_\oplus + (\mathcal{A}_c)_{a\bar{b}}\cos\omega_\oplus T_\oplus\\ &\quad + (\mathcal{B}_s)_{a\bar{b}}\sin 2\omega_\oplus T_\oplus + (\mathcal{B}_c)_{a\bar{b}}\cos 2\omega_\oplus T_\oplus.\end{aligned} \tag{76}$$

In this expression, the amplitudes $(\mathcal{C})_{a\bar{b}}$, $(\mathcal{A}_s)_{a\bar{b}}$, $(\mathcal{A}_c)_{a\bar{b}}$, $(\mathcal{B}_s)_{a\bar{b}}$, and $(\mathcal{B}_c)_{a\bar{b}}$ are direction-dependent linear combinations of the coefficients $\tilde{g}^{\alpha\beta}_{a\bar{b}}$ and $\tilde{H}^{\alpha}_{a\bar{b}}$ for Lorentz violation.

The direction dependence is governed by two vectors, the momentum $\vec{p}$ and the polarization $\vec{\epsilon}_+$. The momentum is determined by the beam direction, which varies sidereally and is specified at time $T_\oplus = 0$ in the Sun-centered frame by the vector $(\hat{N}^X, \hat{N}^Y, \hat{N}^Z)$ given in local spherical coordinates by Eq. (46). We denote the analogous vector for $\vec{\epsilon}_+$ by $(\hat{\mathcal{E}}^X_+, \hat{\mathcal{E}}^Y_+, \hat{\mathcal{E}}^Z_+)$. In the same local spherical coordinates, this vector has components

$$\begin{aligned}\hat{\mathcal{E}}^X_+ &= \frac{1}{\sqrt{2}}(\cos\chi(\cos\theta\cos\phi - i\sin\phi) - \sin\chi\sin\theta),\\ \hat{\mathcal{E}}^Y_+ &= \frac{1}{\sqrt{2}}(\cos\theta\sin\phi + i\cos\phi),\\ \hat{\mathcal{E}}^Z_+ &= -\frac{1}{\sqrt{2}}(\sin\chi(\cos\theta\cos\phi - i\sin\phi) + \cos\chi\sin\theta),\end{aligned} \tag{77}$$

where we have used the expression (18) for $(\epsilon_+)^j$.

Some calculation reveals that the sidereal amplitudes in Eq. (76) are given as

$$(\mathcal{C})_{a\bar{b}} = -i\sqrt{2}\bigg[\hat{\mathcal{E}}_+^Z \tilde{H}_{a\bar{b}}^Z - \hat{\mathcal{E}}_+^Z E\tilde{g}_{a\bar{b}}^{ZT} + \hat{\mathcal{E}}_+^Z \hat{N}^Z E\bigg(\tilde{g}_{a\bar{b}}^{ZZ} - \frac{1}{2}\tilde{g}_{a\bar{b}}^{XX} - \frac{1}{2}\tilde{g}_{a\bar{b}}^{YY}\bigg) + \frac{i}{2}\hat{\mathcal{E}}_+^Z E(\tilde{g}_{a\bar{b}}^{XY} - \tilde{g}_{a\bar{b}}^{YX})\bigg],$$

$$(\mathcal{A}_s)_{a\bar{b}} = -i\sqrt{2}[-\hat{\mathcal{E}}_+^Y \tilde{H}_{a\bar{b}}^X + \hat{\mathcal{E}}_+^X \tilde{H}_{a\bar{b}}^Y + \hat{\mathcal{E}}_+^Y E\tilde{g}_{a\bar{b}}^{XT} - \hat{\mathcal{E}}_+^X E\tilde{g}_{a\bar{b}}^{YT} - \hat{\mathcal{E}}_+^Y \hat{N}^Z E\tilde{g}_{a\bar{b}}^{XZ} + \hat{\mathcal{E}}_+^X \hat{N}^Z E\tilde{g}_{a\bar{b}}^{YZ} - \hat{\mathcal{E}}_+^Z \hat{N}^Y E\tilde{g}_{a\bar{b}}^{ZX} + \hat{\mathcal{E}}_+^Z \hat{N}^X E\tilde{g}_{a\bar{b}}^{ZY}],$$

$$(\mathcal{A}_c)_{a\bar{b}} = -i\sqrt{2}[\hat{\mathcal{E}}_+^X \tilde{H}_{a\bar{b}}^X + \hat{\mathcal{E}}_+^Y \tilde{H}_{a\bar{b}}^Y - \hat{\mathcal{E}}_+^X E\tilde{g}_{a\bar{b}}^{XT} - \hat{\mathcal{E}}_+^Y E\tilde{g}_{a\bar{b}}^{YT} + \hat{\mathcal{E}}_+^X \hat{N}^Z E\tilde{g}_{a\bar{b}}^{XZ} + \hat{\mathcal{E}}_+^Y \hat{N}^Z E\tilde{g}_{a\bar{b}}^{YZ} + \hat{\mathcal{E}}_+^Z \hat{N}^X E\tilde{g}_{a\bar{b}}^{ZX} + \hat{\mathcal{E}}_+^Z \hat{N}^Y E\tilde{g}_{a\bar{b}}^{ZY}],$$

$$(\mathcal{B}_s)_{a\bar{b}} = -i\sqrt{2}\bigg[\frac{1}{2}(\hat{\mathcal{E}}_+^X \hat{N}^X - \hat{\mathcal{E}}_+^Y \hat{N}^Y)E(\tilde{g}_{a\bar{b}}^{XY} + \tilde{g}_{a\bar{b}}^{YX}) - \frac{1}{2}(\hat{\mathcal{E}}_+^X \hat{N}^Y + \hat{\mathcal{E}}_+^Y \hat{N}^X)E(\tilde{g}_{a\bar{b}}^{XX} - \tilde{g}_{a\bar{b}}^{YY})\bigg],$$

$$(\mathcal{B}_c)_{a\bar{b}} = -i\sqrt{2}\bigg[\frac{1}{2}(\hat{\mathcal{E}}_+^X \hat{N}^Y + \hat{\mathcal{E}}_+^Y \hat{N}^X)E(\tilde{g}_{a\bar{b}}^{XY} + \tilde{g}_{a\bar{b}}^{YX}) + \frac{1}{2}(\hat{\mathcal{E}}_+^X \hat{N}^X - \hat{\mathcal{E}}_+^Y \hat{N}^Y)E(\tilde{g}_{a\bar{b}}^{XX} - \tilde{g}_{a\bar{b}}^{YY})\bigg]. \quad (78)$$

This completes the decomposition of $\delta h_{a\bar{b}}$ in terms of the sidereal time $T_\oplus$, the coefficients $\tilde{g}_{a\bar{b}}^{\alpha\beta}$, $\tilde{H}_{a\bar{b}}^{\alpha}$, and the components $\hat{N}^J$, $\hat{\mathcal{E}}_+^J$. The analogous decomposition for $\delta h_{\bar{b}a}$ is obtained by taking the Hermitian conjugate, following Eq. (15).

At dominant order, the neutrino-antineutrino mixing is controlled by the linear combinations $\mathcal{H}_{a\bar{b}}^{(1)}$ and $\mathcal{H}_{\bar{a}b}^{(1)}$ given in Eq. (40). The sidereal dependence of $\delta h_{a\bar{b}}$ transfers to these combinations, leading to the expansion

$$\begin{aligned}\mathcal{H}_{a\bar{b}}^{(1)} = (\mathcal{C}^{(1)})_{a\bar{b}} + (\mathcal{A}_s^{(1)})_{a\bar{b}} \sin\omega_\oplus T_\oplus \\ + (\mathcal{A}_c^{(1)})_{a\bar{b}} \cos\omega_\oplus T_\oplus + (\mathcal{B}_s^{(1)})_{a\bar{b}} \sin 2\omega_\oplus T_\oplus \\ + (\mathcal{B}_c^{(1)})_{a\bar{b}} \cos 2\omega_\oplus T_\oplus,\end{aligned} \quad (79)$$

with a similar expression for $\mathcal{H}_{\bar{a}b}^{(1)}$. The coefficients $\tilde{g}_{a\bar{b}}^{\alpha\beta}$ and $\tilde{H}_{a\bar{b}}^{\alpha}$ appear in this expansion in linear combinations weighted by the complex experiment-dependent factors $(\mathcal{M}_{a\bar{b}}^{(1)})_{c\bar{d}}$ and $(\mathcal{M}_{\bar{a}b}^{(1)})_{\bar{c}d}$ given in Eq. (32). It is convenient to introduce the definitions

$$\begin{aligned}\tilde{\tilde{g}}_{a\bar{b}}^{\alpha\beta} &= \sum_{c\bar{d}} (\mathcal{M}_{a\bar{b}}^{(1)})_{c\bar{d}} \tilde{g}_{c\bar{d}}^{\alpha\beta}, \\ \tilde{\tilde{H}}_{a\bar{b}}^{\alpha} &= \sum_{c\bar{d}} (\mathcal{M}_{a\bar{b}}^{(1)})_{c\bar{d}} \tilde{H}_{c\bar{d}}^{\alpha}, \\ \tilde{\tilde{g}}_{\bar{a}b}^{\alpha\beta} &= \sum_{\bar{c}d} (\mathcal{M}_{\bar{a}b}^{(1)})_{\bar{c}d} \tilde{g}_{\bar{c}d}^{\alpha\beta} = \sum_{\bar{c}d} (\mathcal{M}_{\bar{a}b}^{(1)})_{\bar{c}d} \tilde{g}_{d\bar{c}}^{\alpha\beta *}, \\ \tilde{\tilde{H}}_{\bar{a}b}^{\alpha} &= \sum_{\bar{c}d} (\mathcal{M}_{\bar{a}b}^{(1)})_{\bar{c}d} \tilde{H}_{\bar{c}d}^{\alpha} = \sum_{\bar{c}d} (\mathcal{M}_{\bar{a}b}^{(1)})_{\bar{c}d} \tilde{H}_{d\bar{c}}^{\alpha *}.\end{aligned} \quad (80)$$

In terms of these, the sidereal amplitudes $(\mathcal{C}^{(1)})_{a\bar{b}}$, $(\mathcal{A}_s^{(1)})_{a\bar{b}}$, $(\mathcal{A}_c^{(1)})_{a\bar{b}}$, $(\mathcal{B}_s^{(1)})_{a\bar{b}}$, $(\mathcal{B}_c^{(1)})_{a\bar{b}}$ take the same form as the corresponding amplitudes in Eq. (78) but with $\tilde{g}_{a\bar{b}}^{\alpha\beta}$ and $\tilde{H}_{a\bar{b}}^{\alpha}$ replaced with $\tilde{\tilde{g}}_{a\bar{b}}^{\alpha\beta}$ and $\tilde{\tilde{H}}_{a\bar{b}}^{\alpha}$. For the coefficient combinations $\mathcal{H}_{\bar{b}a}^{(1)}$, we can define analogous sidereal amplitudes $(\mathcal{C}^{(1)})_{\bar{b}a}$, $(\mathcal{A}_s^{(1)})_{\bar{b}a}$, $(\mathcal{A}_c^{(1)})_{\bar{b}a}$, $(\mathcal{B}_s^{(1)})_{\bar{b}a}$, $(\mathcal{B}_c^{(1)})_{\bar{b}a}$. The forms of these can also be obtained from Eq. (78), by first taking the Hermitian conjugates of the expressions on the right-hand side and then replacing $\tilde{g}_{\bar{b}a}^{\alpha\beta}$, $\tilde{H}_{\bar{b}a}^{\alpha\beta}$ with $\tilde{\tilde{g}}_{\bar{b}a}^{\alpha\beta}$, $\tilde{\tilde{H}}_{\bar{b}a}^{\alpha}$.

According to Eq. (41), the combinations $\mathcal{H}_{a\bar{b}}^{(1)}$ contribute quadratically to the second-order neutrino-antineutrino probabilities $P_{\bar{\nu}_b \to \nu_a}^{(2)}$. This implies that sidereal variations at frequencies up to $4\omega_\oplus$ are observable. Consequently, we expand $P_{\bar{\nu}_b \to \nu_a}^{(2)}$ as

$$\begin{aligned}\frac{P_{\bar{\nu}_b \to \nu_a}^{(2)}}{L^2} &\equiv |\mathcal{H}_{a\bar{b}}^{(1)}|^2 \\ &= (P_{\mathcal{C}}^{(2)})_{a\bar{b}} + (P_{\mathcal{A}_s}^{(2)})_{a\bar{b}} \sin\omega_\oplus T_\oplus \\ &\quad + (P_{\mathcal{A}_c}^{(2)})_{a\bar{b}} \cos\omega_\oplus T_\oplus + (P_{\mathcal{B}_s}^{(2)})_{a\bar{b}} \sin 2\omega_\oplus T_\oplus \\ &\quad + (P_{\mathcal{B}_c}^{(2)})_{a\bar{b}} \cos 2\omega_\oplus T_\oplus + (P_{\mathcal{D}_s}^{(2)})_{a\bar{b}} \sin 3\omega_\oplus T_\oplus \\ &\quad + (P_{\mathcal{D}_c}^{(2)})_{a\bar{b}} \cos 3\omega_\oplus T_\oplus + (P_{\mathcal{F}_s}^{(2)})_{a\bar{b}} \sin 4\omega_\oplus T_\oplus \\ &\quad + (P_{\mathcal{F}_c}^{(2)})_{a\bar{b}} \cos 4\omega_\oplus T_\oplus.\end{aligned} \quad (81)$$

Each of the nine amplitudes in this equation is a quadratic combination of the coefficients $\tilde{g}_{a\bar{b}}^{\alpha\beta}$ and $\tilde{H}_{a\bar{b}}^{\alpha}$ for Lorentz violation. These combinations depend on the mass matrix and also vary with the experimental scenario through the neutrino energy and the direction of propagation.

The explicit forms of the amplitudes in Eq. (81) are somewhat lengthy and are omitted here. However, we can obtain compact expressions in terms of the amplitudes for the sidereal decomposition of $\mathcal{H}_{a\bar{b}}^{(1)}$, which are defined in Eq. (79). For the harmonics up to $2\omega_\oplus$, some calculation yields the results

$$\begin{aligned}(P_{\mathcal{C}}^{(2)})_{a\bar{b}} &= |(\mathcal{C}^{(1)})_{a\bar{b}}|^2 + \frac{1}{2}|(\mathcal{A}_s^{(1)})_{a\bar{b}}|^2 + \frac{1}{2}|(\mathcal{A}_c^{(1)})_{a\bar{b}}|^2 \\ &\quad + \frac{1}{2}|(\mathcal{B}_s^{(1)})_{a\bar{b}}|^2 + \frac{1}{2}|(\mathcal{B}_c^{(1)})_{a\bar{b}}|^2, \\ (P_{\mathcal{A}_s}^{(2)})_{a\bar{b}} &= \mathrm{Re}[2(\mathcal{C}^{(1)})_{a\bar{b}}^*(\mathcal{A}_s^{(1)})_{a\bar{b}} + (\mathcal{A}_c^{(1)})_{a\bar{b}}^*(\mathcal{B}_s^{(1)})_{a\bar{b}} \\ &\quad - (\mathcal{A}_s^{(1)})_{a\bar{b}}^*(\mathcal{B}_c^{(1)})_{a\bar{b}}], \\ (P_{\mathcal{A}_c}^{(2)})_{a\bar{b}} &= \mathrm{Re}[2(\mathcal{C}^{(1)})_{a\bar{b}}^*(\mathcal{A}_c^{(1)})_{a\bar{b}} + (\mathcal{A}_s^{(1)})_{a\bar{b}}^*(\mathcal{B}_s^{(1)})_{a\bar{b}} \\ &\quad + (\mathcal{A}_c^{(1)})_{a\bar{b}}^*(\mathcal{B}_c^{(1)})_{a\bar{b}}], \\ (P_{\mathcal{B}_s}^{(2)})_{a\bar{b}} &= \mathrm{Re}[2(\mathcal{C}^{(1)})_{a\bar{b}}^*(\mathcal{B}_s^{(1)})_{a\bar{b}} + (\mathcal{A}_s^{(1)})_{a\bar{b}}^*(\mathcal{A}_c^{(1)})_{a\bar{b}}], \\ (P_{\mathcal{B}_c}^{(2)})_{a\bar{b}} &= 2\,\mathrm{Re}[(\mathcal{C}^{(1)})_{a\bar{b}}^*(\mathcal{B}_c^{(1)})_{a\bar{b}}] - |(\mathcal{A}_s^{(1)})_{a\bar{b}}|^2 \\ &\quad + |(\mathcal{A}_c^{(1)})_{a\bar{b}}|^2,\end{aligned} \quad (82)$$

while for the harmonics at $3\omega_\oplus$ and $4\omega_\oplus$ we obtain

$$\begin{aligned}
(P^{(2)}_{\mathcal{D}_s})_{a\bar{b}} &= \mathrm{Re}[(\mathcal{A}^{(1)}_s)^*_{a\bar{b}}(\mathcal{B}^{(1)}_c)_{a\bar{b}} + (\mathcal{A}^{(1)}_c)^*_{a\bar{b}}(\mathcal{B}^{(1)}_s)_{a\bar{b}}],\\
(P^{(2)}_{\mathcal{D}_c})_{a\bar{b}} &= \mathrm{Re}[(\mathcal{A}^{(1)}_c)^*_{a\bar{b}}(\mathcal{B}^{(1)}_c)_{a\bar{b}} - (\mathcal{A}^{(1)}_s)^*_{a\bar{b}}(\mathcal{B}^{(1)}_s)_{a\bar{b}}],\\
(P^{(2)}_{\mathcal{F}_s})_{a\bar{b}} &= \mathrm{Re}[(\mathcal{B}^{(1)}_s)^*_{a\bar{b}}(\mathcal{B}^{(1)}_c)_{a\bar{b}}],\\
(P^{(2)}_{\mathcal{F}_c})_{a\bar{b}} &= |(\mathcal{B}^{(1)}_c)_{a\bar{b}}|^2 - |(\mathcal{B}^{(1)}_s)_{a\bar{b}}|^2.
\end{aligned}\tag{83}$$

The structure of these equations reflects the frequency dependence in the sidereal decomposition (79) of $\mathcal{H}^{(1)}_{a\bar{b}}$. For example, the amplitudes $(P^{(2)}_{\mathcal{F}_s})_{a\bar{b}}$, $(P^{(2)}_{\mathcal{F}_c})_{a\bar{b}}$ for the fourth harmonic $4\omega_\oplus$ of the probability $P^{(2)}_{\bar{\nu}_b\to\nu_a}$ involve quadratic products of the amplitudes $(\mathcal{B}^{(1)}_s)_{a\bar{b}}$, $(\mathcal{B}^{(1)}_c)_{a\bar{b}}$ for the second harmonic $2\omega_\oplus$ of $\mathcal{H}^{(1)}_{a\bar{b}}$, as expected.

Comparable expressions for the CP-conjugate transition probability $P^{(2)}_{\nu_b\to\bar{\nu}_a}$ can readily be obtained following the same procedure. The results take the same form as Eqs. (81) and (83), but with the index replacement $\{a\bar{b}\}\to\{\bar{a}b\}$.

In the event that neutrino-antineutrino oscillations are observed in nature, the sidereal decomposition of the probability $P^{(2)}_{\bar{\nu}_b\to\nu_a}$ and its CP conjugate offers a powerful approach to identifying the relevant coefficients $\tilde{g}^{\alpha\beta}_{a\bar{b}}$ and $\tilde{H}^{\alpha}_{a\bar{b}}$. Each experimental analysis separating the available sidereal harmonics would generate eight independent measurements, with multiple experiments able to constrain much of the available coefficient space.

B. Oscillations conserving lepton number

The analysis in the previous subsection demonstrates that the detection of $\nu\leftrightarrow\bar{\nu}$ oscillations is a unique signal for nonzero coefficients $\tilde{g}^{\alpha\beta}_{a\bar{b}}$ and $\tilde{H}^{\alpha}_{a\bar{b}}$. However, these coefficients also contribute at second order to the more conventional $\nu\leftrightarrow\nu$ and $\bar{\nu}\leftrightarrow\bar{\nu}$ mixings. For completeness, we present the associated equations in this subsection. Effects quadratic in the coefficients $(a_L)^\alpha_{ab}$ and $(c_L)^{\alpha\beta}_{ab}$ also appear at this order. Inspection of Eq. (41) reveals that these contribute independently to the oscillation probabilities, so we set them to zero here for simplicity.

The probabilities for $\nu\leftrightarrow\nu$ and $\bar{\nu}\leftrightarrow\bar{\nu}$ mixing are affected at second order by $\delta h_{a\bar{b}}$ through its quadratic appearance in the quantities $\mathcal{H}^{(2)}_{ab}$ and $\mathcal{H}^{(2)}_{\bar{a}\bar{b}}$ defined in Eq. (42). This produces sidereal variations at harmonics up to frequency $4\omega_\oplus$. We can therefore decompose $\mathcal{H}^{(2)}_{ab}$ as the sidereal expansion

$$\begin{aligned}
\mathcal{H}^{(2)}_{ab} &\equiv \sum_{c\bar{d}\bar{e}f}(\mathcal{M}^{(2)}_{ab})_{c\bar{d}\bar{e}f}\delta h_{c\bar{d}}\delta h_{\bar{e}f}\\
&= (\mathcal{C}^{(2)})_{ab} + (\mathcal{A}^{(2)}_s)_{ab}\sin\omega_\oplus T_\oplus + (\mathcal{A}^{(2)}_c)_{ab}\cos\omega_\oplus T_\oplus\\
&\quad + (\mathcal{B}^{(2)}_s)_{ab}\sin 2\omega_\oplus T_\oplus + (\mathcal{B}^{(2)}_c)_{ab}\cos 2\omega_\oplus T_\oplus\\
&\quad + (\mathcal{D}^{(2)}_s)_{ab}\sin 3\omega_\oplus T_\oplus + (\mathcal{D}^{(2)}_c)_{ab}\cos 3\omega_\oplus T_\oplus\\
&\quad + (\mathcal{F}^{(2)}_s)_{ab}\sin 4\omega_\oplus T_\oplus + (\mathcal{F}^{(2)}_c)_{ab}\cos 4\omega_\oplus T_\oplus.
\end{aligned}\tag{84}$$

The nine amplitudes in the above expression can be written as combinations of the experiment-dependent factors $(\mathcal{M}^{(2)}_{ab})_{c\bar{d}\bar{e}f}$ and the five sidereal coefficients for the perturbative Hamiltonian $\delta h_{a\bar{b}}$ listed in Eq. (78). For the amplitudes of the harmonics with frequencies $2\omega_\oplus$ or less in the expansion (84), we find the results

$$\begin{aligned}
(\mathcal{C}^{(2)})_{ab} &= \sum_{c\bar{d}\bar{e}f}(\mathcal{M}^{(2)}_{ab})_{c\bar{d}\bar{e}f}\Big[(\mathcal{C})_{c\bar{d}}(\mathcal{C})_{\bar{e}f} + \tfrac{1}{2}(\mathcal{A}_s)_{c\bar{d}}(\mathcal{A}_s)_{\bar{e}f} + \tfrac{1}{2}(\mathcal{A}_c)_{c\bar{d}}(\mathcal{A}_c)_{\bar{e}f} + \tfrac{1}{2}(\mathcal{B}_s)_{c\bar{d}}(\mathcal{B}_s)_{\bar{e}f} + \tfrac{1}{2}(\mathcal{B}_c)_{c\bar{d}}(\mathcal{B}_c)_{\bar{e}f}\Big],\\
(\mathcal{A}^{(2)}_s)_{ab} &= \sum_{c\bar{d}\bar{e}f}(\mathcal{M}^{(2)}_{ab})_{c\bar{d}\bar{e}f}\Big[(\mathcal{C})_{c\bar{d}}(\mathcal{A}_s)_{\bar{e}f} + (\mathcal{A}_s)_{c\bar{d}}(\mathcal{C})_{\bar{e}f} - \tfrac{1}{2}(\mathcal{A}_s)_{c\bar{d}}(\mathcal{B}_c)_{\bar{e}f} + \tfrac{1}{2}(\mathcal{A}_c)_{c\bar{d}}(\mathcal{B}_s)_{\bar{e}f} + \tfrac{1}{2}(\mathcal{B}_s)_{c\bar{d}}(\mathcal{A}_c)_{\bar{e}f}\\
&\qquad - \tfrac{1}{2}(\mathcal{B}_c)_{c\bar{d}}(\mathcal{A}_s)_{\bar{e}f}\Big],\\
(\mathcal{A}^{(2)}_c)_{ab} &= \sum_{c\bar{d}\bar{e}f}(\mathcal{M}^{(2)}_{ab})_{c\bar{d}\bar{e}f}\Big[(\mathcal{C})_{c\bar{d}}(\mathcal{A}_c)_{\bar{e}f} + (\mathcal{A}_c)_{c\bar{d}}(\mathcal{C})_{\bar{e}f} + \tfrac{1}{2}(\mathcal{A}_s)_{c\bar{d}}(\mathcal{B}_s)_{\bar{e}f} + \tfrac{1}{2}(\mathcal{A}_c)_{c\bar{d}}(\mathcal{B}_c)_{\bar{e}f} + \tfrac{1}{2}(\mathcal{B}_s)_{c\bar{d}}(\mathcal{A}_s)_{\bar{e}f}\\
&\qquad + \tfrac{1}{2}(\mathcal{B}_c)_{c\bar{d}}(\mathcal{A}_c)_{\bar{e}f}\Big],\\
(\mathcal{B}^{(2)}_s)_{ab} &= \sum_{c\bar{d}\bar{e}f}(\mathcal{M}^{(2)}_{ab})_{c\bar{d}\bar{e}f}\Big[(\mathcal{C})_{c\bar{d}}(\mathcal{B}_s)_{\bar{e}f} + (\mathcal{B}_s)_{c\bar{d}}(\mathcal{C})_{\bar{e}f} + \tfrac{1}{2}(\mathcal{A}_s)_{c\bar{d}}(\mathcal{A}_c)_{\bar{e}f} + \tfrac{1}{2}(\mathcal{A}_c)_{c\bar{d}}(\mathcal{A}_s)_{\bar{e}f}\Big],\\
(\mathcal{B}^{(2)}_c)_{ab} &= \sum_{c\bar{d}\bar{e}f}(\mathcal{M}^{(2)}_{ab})_{c\bar{d}\bar{e}f}\Big[(\mathcal{C})_{c\bar{d}}(\mathcal{B}_c)_{\bar{e}f} + (\mathcal{B}_c)_{c\bar{d}}(\mathcal{C})_{\bar{e}f} - \tfrac{1}{2}(\mathcal{A}_s)_{c\bar{d}}(\mathcal{A}_s)_{\bar{e}f} + \tfrac{1}{2}(\mathcal{A}_c)_{c\bar{d}}(\mathcal{A}_c)_{\bar{e}f}\Big].
\end{aligned}\tag{85}$$

For the remaining harmonics in the expansion (84) with frequencies $3\omega_\oplus$ and $4\omega_\oplus$, the results for the amplitudes are

$$
\begin{aligned}
(\mathcal{D}_s^{(2)})_{ab} &= \sum_{c\bar{d}\bar{e}f} (\mathcal{M}_{ab}^{(2)})_{c\bar{d}\bar{e}f} \times \frac{1}{2}[(\mathcal{A}_s)_{c\bar{d}}(\mathcal{B}_c)_{\bar{e}f} + (\mathcal{A}_c)_{c\bar{d}}(\mathcal{B}_s)_{\bar{e}f} + (\mathcal{B}_s)_{c\bar{d}}(\mathcal{A}_c)_{\bar{e}f} + (\mathcal{B}_c)_{c\bar{d}}(\mathcal{A}_s)_{\bar{e}f}],\\
(\mathcal{D}_c^{(2)})_{ab} &= \sum_{c\bar{d}\bar{e}f} (\mathcal{M}_{ab}^{(2)})_{c\bar{d}\bar{e}f} \times \frac{1}{2}[-(\mathcal{A}_s)_{c\bar{d}}(\mathcal{B}_s)_{\bar{e}f} + (\mathcal{A}_c)_{c\bar{d}}(\mathcal{B}_c)_{\bar{e}f} - (\mathcal{B}_s)_{c\bar{d}}(\mathcal{A}_s)_{\bar{e}f} + (\mathcal{B}_c)_{c\bar{d}}(\mathcal{A}_c)_{\bar{e}f}],\\
(\mathcal{F}_s^{(2)})_{ab} &= \sum_{c\bar{d}\bar{e}f} (\mathcal{M}_{ab}^{(2)})_{c\bar{d}\bar{e}f} \times \frac{1}{2}[(\mathcal{B}_s)_{c\bar{d}}(\mathcal{B}_c)_{\bar{e}f} + (\mathcal{B}_c)_{c\bar{d}}(\mathcal{B}_s)_{\bar{e}f}],\\
(\mathcal{F}_c^{(2)})_{ab} &= \sum_{c\bar{d}\bar{e}f} (\mathcal{M}_{ab}^{(2)})_{c\bar{d}\bar{e}f} \times \frac{1}{2}[-(\mathcal{B}_s)_{c\bar{d}}(\mathcal{B}_s)_{\bar{e}f} + (\mathcal{B}_c)_{c\bar{d}}(\mathcal{B}_c)_{\bar{e}f}].
\end{aligned}
\tag{86}
$$

Analogous expressions for the sidereal decomposition of the quantities $\mathcal{H}_{\bar{a}\bar{b}}^{(2)}$ and the resulting amplitudes can be obtained by substituting barred for unbarred indices and vice versa.

The second-order probability for neutrino-neutrino oscillations inherit the same sidereal-frequency structure. Introducing the expansion

$$
\begin{aligned}
\frac{P^{(2)}_{\nu_b\to\nu_a}}{L^2} &\equiv -\mathrm{Re}((S_{ab}^{(0)})^* \mathcal{H}_{ab}^{(2)})\\
&= (P_{\mathcal{C}}^{(2)})_{ab} + (P_{\mathcal{A}_s}^{(2)})_{ab}\sin\omega_\oplus T_\oplus + (P_{\mathcal{A}_c}^{(2)})_{ab}\cos\omega_\oplus T_\oplus + (P_{\mathcal{B}_s}^{(2)})_{ab}\sin 2\omega_\oplus T_\oplus + (P_{\mathcal{B}_c}^{(2)})_{ab}\cos 2\omega_\oplus T_\oplus\\
&\quad + (P_{\mathcal{D}_s}^{(2)})_{ab}\sin 3\omega_\oplus T_\oplus + (P_{\mathcal{D}_c}^{(2)})_{ab}\cos 3\omega_\oplus T_\oplus + (P_{\mathcal{F}_s}^{(2)})_{ab}\sin 4\omega_\oplus T_\oplus + (P_{\mathcal{F}_c}^{(2)})_{ab}\cos 4\omega_\oplus T_\oplus,
\end{aligned}
\tag{87}
$$

we find the nine corresponding amplitudes for the probability are given by the equations

$$
\begin{aligned}
(P_{\mathcal{C}}^{(2)})_{ab} &= -\mathrm{Re}((S_{ab}^{(0)})^*(\mathcal{C}^{(2)})_{ab}),\\
(P_{\mathcal{A}_s}^{(2)})_{ab} &= -\mathrm{Re}((S_{ab}^{(0)})^*(\mathcal{A}_s^{(2)})_{ab}),\\
(P_{\mathcal{A}_c}^{(2)})_{ab} &= -\mathrm{Re}((S_{ab}^{(0)})^*(\mathcal{A}_c^{(2)})_{ab}),\\
(P_{\mathcal{B}_s}^{(2)})_{ab} &= -\mathrm{Re}((S_{ab}^{(0)})^*(\mathcal{B}_s^{(2)})_{ab}),\\
(P_{\mathcal{B}_c}^{(2)})_{ab} &= -\mathrm{Re}((S_{ab}^{(0)})^*(\mathcal{B}_c^{(2)})_{ab}),\\
(P_{\mathcal{D}_s}^{(2)})_{ab} &= -\mathrm{Re}((S_{ab}^{(0)})^*(\mathcal{D}_s^{(2)})_{ab}),\\
(P_{\mathcal{D}_c}^{(2)})_{ab} &= -\mathrm{Re}((S_{ab}^{(0)})^*(\mathcal{D}_c^{(2)})_{ab}),\\
(P_{\mathcal{F}_s}^{(2)})_{ab} &= -\mathrm{Re}((S_{ab}^{(0)})^*(\mathcal{F}_s^{(2)})_{ab}),\\
(P_{\mathcal{F}_c}^{(2)})_{ab} &= -\mathrm{Re}((S_{ab}^{(0)})^*(\mathcal{F}_c^{(2)})_{ab}).
\end{aligned}
\tag{88}
$$

The probability for antineutrino-antineutrino oscillations can be found from the above equations by replacing all indices $\{ab\}$ with $\{\bar{a}\bar{b}\}$.

The calculations in this subsection demonstrate that searches for sidereal variations in $\nu \leftrightarrow \nu$ and $\bar{\nu} \leftrightarrow \bar{\nu}$ oscillations at the higher frequencies $3\omega_\oplus$ and $4\omega_\oplus$ can offer access to the coefficients $\tilde{g}^{\alpha\beta}_{a\bar{b}}$ and $\tilde{H}^{\alpha}_{a\bar{b}}$ for Lorentz violation without the need to study $\nu \leftrightarrow \bar{\nu}$ oscillations. Moreover, in addition to studies based on the above direct sidereal decompositions, investigation of the *CP* and *CPT* asymmetries (67) and (69) introduced in Sec. III C provides another avenue for data analysis. As before, the time-averaged versions of these asymmetries offer sensitivities to coefficients that are challenging to detect in searches for sidereal variations. In all these studies, the second-order effects enter in conjunction with a factor of L^2, so the large baselines associated with the experiments considered in Sec. III imply that their intrinsic sensitivities to the coefficients $\tilde{g}^{\alpha\beta}_{a\bar{b}}$ and $\tilde{H}^{\alpha}_{a\bar{b}}$ are only mildly suppressed relative to the sensitivities to $(a_L)^\alpha_{ab}$ and $(c_L)^{\alpha\beta}_{ab}$.

V. SUMMARY

In this paper, we study the effects of perturbative Lorentz and *CPT* violation on neutrino oscillations dominated by mass mixing. The primary focus is on corrections arising from renormalizable operators for Lorentz violation within effective field theory. In the neutrino sector, these operators are controlled by SME coefficients for Lorentz violation denoted $(a_L)^\alpha_{ab}$, $(c_L)^{\alpha\beta}_{ab}$, $\tilde{g}^{\alpha\beta}_{a\bar{b}}$, and $\tilde{H}^{\alpha}_{a\bar{b}}$. They can affect conventional oscillations in $\nu \leftrightarrow \nu$ and $\bar{\nu} \leftrightarrow \bar{\nu}$ mixing. They also can induce $\nu \leftrightarrow \bar{\nu}$ mixing, which violates lepton number.

Using time-dependent perturbation theory, a series expansion for the oscillation probabilities is derived in Sec. II. To second order in coefficients for Lorentz and *CPT* violation, the probabilities for a nondegenerate mass spectrum are presented in Eqs. (35), (36), (39), and (41). At first order, only the coefficients $(a_L)^\alpha_{ab}$ and $(c_L)^{\alpha\beta}_{ab}$ contribute, and lepton number is preserved. Oscillations involving $\nu \leftrightarrow \bar{\nu}$ mixing appear at second order, governed by the coefficients $\tilde{g}^{\alpha\beta}_{a\bar{b}}$ and $\tilde{H}^{\alpha}_{a\bar{b}}$.

A key feature introduced by Lorentz and *CPT* violation is variations in the oscillation probabilities with sidereal time. The sidereal dependence arising from the coefficients $(a_L)^\alpha_{ab}$ and $(c_L)^{\alpha\beta}_{ab}$ is discussed in Sec. III A. It is described by the expansion (48), which involves first and second harmonics in the sidereal frequency. At this order, the amplitudes for each harmonic are linear combinations of $(a_L)^\alpha_{ab}$ and $(c_L)^{\alpha\beta}_{ab}$. Data analyses using binning in sidereal time can therefore measure these coefficients.

Section III B addresses the methodology for data analyses and provides illustrative estimates of numerical quantities relevant for sidereal investigations in several long-baseline experiments. The results are summarized in Tables I through III. For the three-generation case, we demonstrate the procedure to identify the relevant linear combinations of $(a_L)^\alpha_{ab}$ and $(c_L)^{\alpha\beta}_{ab}$, using the K2K, MINOS, OPERA, ICARUS, NOνA, T2K, DUSEL, and T2KK experiments as examples. The two-flavor limit is also considered. In this case, the sidereal expansion is considerably simplified and the oscillation probability takes the comparatively elegant form (65).

In addition to direct sidereal studies, Lorentz and *CPT* violation can be sought through analysis of *CP* and *CPT* asymmetries in experimental data. This topic is addressed in Sec. III C. Suitable *CP* and *CPT* asymmetries are defined in Eqs. (67) and (69). In the two-flavor limit, these coincide and take the comparatively simple form (72) or (73). Experiments running in both neutrino and antineutrino modes can probe *CP* and *CPT* via this route. Analyses along the lines proposed here could provide access to different combinations of coefficients for Lorentz and *CPT* violation, including ones that are challenging to detect via studies of sidereal variations.

In Sec. IV, we consider effects arising from nonzero coefficients $\tilde{g}^{\alpha\beta}_{a\bar{b}}$ and $\tilde{H}^{\alpha}_{a\bar{b}}$. Among the features is mixing between neutrinos and antineutrinos, implying violations of lepton number. These coefficients have no first-order perturbative effects. Their dominant contributions arise at second order, where the probabilities involve quadratic combinations of $\tilde{g}^{\alpha\beta}_{a\bar{b}}$ and $\tilde{H}^{\alpha}_{a\bar{b}}$. This induces sidereal variations with harmonics up to 4 times the sidereal frequency in all three kinds of mixings, $\nu \leftrightarrow \nu$, $\bar{\nu} \leftrightarrow \bar{\nu}$, and $\nu \leftrightarrow \bar{\nu}$. The probabilities for neutrino-antineutrino mixing are given in Eq. (81), while those for neutrino-neutrino mixing take the similar form (87).

Overall, we find that the dominant effects of renormalizable operators for Lorentz and *CPT* violation in the neutrino sector generate variations in oscillation probabilities up to 4 times the sidereal frequency. Subdominant perturbative effects may also offer useful information. These higher-order perturbations cause sidereal effects at higher harmonics, with signals suppressed compared to the ones discussed here. We remark that other harmonics can also arise from Lorentz-violating operators of nonrenormalizable dimensions [13]. A comprehensive SME-based study in analogy to that performed for electrodynamics [68] could establish the corresponding signals of Lorentz and *CPT* violation in neutrinos.

The results in this work demonstrate that excellent sensitivity to Lorentz and *CPT* violation is attainable by studying neutrino oscillations with high energies and long baselines. Our primary focus has been beam experiments, where existing constraints [10–12] span only a few percent of the available coefficient space. The procedures outlined in this work provide access to essentially all the coefficient space, and moreover at sensitivities that can exceed the current ones by about 2 orders of magnitude.

An interesting direction for further work using a longer baseline is a systematic investigation of perturbative effects of Lorentz and *CPT* violation on solar neutrinos, for which day-night and annual signals play a role analogous to sidereal effects in beam experiments. Future searches for Lorentz and *CPT* violation using extreme baselines could also include studies of oscillations and dispersion for supernova neutrinos, for which a sufficient population over a substantial solid angle would offer interesting sensitivity to a significant portion of the coefficient space. A more speculative possibility using a cosmological baseline would be the search for anisotropies in eventual observations of the cosmic neutrino background. The maximal baseline makes this an ideal arena for studying low-dimension operators for Lorentz and *CPT* violation, in analogy to the tight limits achieved on low-dimension operators in the photon sector using observations of the cosmic microwave background [69]. In the meanwhile, the long baselines involved in the many current and near-future beam experiments on the Earth imply impressive potential sensitivities to the effects of Lorentz and *CPT* violation, rivaling the best tests in other sectors of the SME.

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