Lorentz and *CPT* Tests with Spin-Polarized Solids

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Experiments using macroscopic samples of spin-polarized matter offer exceptional sensitivity to Lorentz and *CPT* violation in the electron sector. Data from existing experiments with a spin-polarized torsion pendulum provide sensitivity in this sector rivaling that of all other existing experiments and could reveal spontaneous violation of Lorentz symmetry at the Planck scale.

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The standard model of particle physics is invariant under Lorentz and *CPT* transformations [1,2]. However, in an underlying theory combining the standard model with gravity, Lorentz and *CPT* symmetry might be spontaneously broken [3]. Small low-energy signals of Lorentz and *CPT* breaking might then be detectable in high-precision tests. The dimensionless suppression factor for such effects would be the ratio of a low-energy scale to the Planck scale, perhaps combined with dimensionless coupling constants.

Many high-precision tests of Lorentz and *CPT* symmetry in matter are spectroscopic in the sense that they involve measuring or monitoring frequencies associated with particles or atoms. Examples include comparative studies of anomaly and cyclotron frequencies of trapped particles and antiparticles [4,5] and clock-comparison experiments [6,7]. These are often regarded as the sharpest tests of Lorentz and *CPT* symmetry in matter. In the electron sector, for instance, it is possible with some theoretical assumptions to bound frequency differences due to Lorentz and *CPT* violation at the level of about 10^{-27} GeV.

In this Letter, we examine an alternative class of experiments involving studies of the behavior of macroscopic solid matter. The idea is to search for Lorentz- and *CPT*-violating spin couplings using materials with a net spin polarization, produced by the combined effects of many electrons. We show that a particular type of experiment is presently capable of testing Lorentz and *CPT* symmetry in the electron sector with a precision rivaling that of spectroscopic experiments.

A variety of experiments using spin-polarized matter exist. They include, for example, studies of torques on a spin-polarized torsion pendulum [8–11] and measurements of the induced magnetization in a paramagnetic salt using a dc SQUID [12]. Except for the experiment in Ref. [8], which was designed to test spatial isotropy, the primary motivation of these experiments has been to search for anomalous spin couplings associated with spin-gravitational effects and axion couplings [13], for which recently attained sensitivities exceed those of spectroscopic searches [14].

To investigate the sensitivity to Lorentz and *CPT* violation of experiments with spin-polarized matter, we use a general standard-model extension [15] describing effects arising in any fundamental theory in which spontaneous Lorentz and *CPT* breaking occurs. The theory provides a consistent microscopic description of these effects in the context of an otherwise conventional renormalizable quantum gauge field theory. In addition to the trapped-particle and clock-comparison tests mentioned above [4–7], the theory has been applied to spectroscopic comparisons of hydrogen and antihydrogen [16], experiments with muons [17], tests with neutral-meson oscillations [18,19], searches for cosmic birefringence [15,20,21], measurements of the baryon asymmetry [22], and observations of cosmic rays [23].

The macroscopic samples of spin-polarized matter used in the experiments of interest here, such as a spin-polarized torsion pendulum or a paramagnetic salt crystal, have a large net electron spin and negligible net nuclear spin. According to the above general theoretical framework, a sample of this type experiences an effective potential arising from the coupling of the electron angular momenta to spacetime-independent background tensors generating the Lorentz and *CPT* violation. The first step in determining this potential is to extract an appropriate quantum-electrodynamics limit of the standard-model extension describing the Lorentz- and *CPT*-violating effects on electrons. In units with $\hbar = c = 1$, the relevant perturbative Lorentz-violating Lagrangian terms are

$$\mathcal{L} = -a^e_\mu \bar{\psi}\gamma^\mu\psi - b^e_\mu \bar{\psi}\gamma_5\gamma^\mu\psi - \tfrac{1}{2}H^e_{\mu\nu}\bar{\psi}\sigma^{\mu\nu}\psi + \tfrac{1}{2}ic^e_{\mu\nu}\bar{\psi}\gamma^\mu \overleftrightarrow{D}^\nu\psi + \tfrac{1}{2}id^e_{\mu\nu}\bar{\psi}\gamma_5\gamma^\mu \overleftrightarrow{D}^\nu\psi\,, \quad (1)$$

where ψ denotes the electron field and $iD_\mu \equiv i\partial_\mu - qA_\mu$ with charge $q = -|e|$. The five parameters a^e_μ, b^e_μ, $H^e_{\mu\nu}$, $c^e_{\mu\nu}$, and $d^e_{\mu\nu}$ govern the (small) magnitudes of the Lorentz violation, with the *CPT*-odd terms being associated with the first two.

The electrons in the spin-polarized materials are nonrelativistic. The appropriate perturbative Hamiltonian δh_n for the nth electron can be derived from the Lagrangian (1) using established procedures involving field redefinitions and a Foldy-Wouthuysen transformation [24]. The multiparticle perturbative Hamiltonian δh describing leading-order Lorentz- and *CPT*-violating effects in the macroscopic spin-polarized material can be obtained by summing over n. Various physical properties can then be deduced from δh. For example, energy-level shifts induced by Lorentz and *CPT* violation can be found by taking expectation values in an appropriate multiparticle quantum state.

Although the form of δh is lengthy in detail, the dominant components relevant here can be shown to have the form

$$\delta h \supset -\tilde{b}_j^e \sum_n \sigma_n^j \,. \tag{2}$$

This equation describes the coupling of the electron spins σ_n^j to a combination $\tilde{b}_j^e$ of *CPT*-even and *CPT*-odd parameters for Lorentz violation given by

$$\tilde{b}_j^e \equiv b_j^e - m d_{j0}^e - \tfrac{1}{2}\varepsilon_{jkl} H_{kl}^e \,. \tag{3}$$

In these expressions, Lorentz indices are separated into timelike and spacelike Cartesian components ($\mu = 0$ and $j = 1, 2, 3$), and repeated indices are understood to be summed. Other pieces of δh generate at most suppressed contributions to the effective potential for spin-polarized matter. For example, although components involving the orbital angular momenta may appear, their expectation values and hence their contributions are suppressed by factors of order $\alpha^2 \simeq 5 \times 10^{-5}$ relative to those in Eq. (2) and so can be disregarded.

The form of δh has some immediate implications for experiments with macrosopic spin-polarized materials. For example, one type of experiment searches for anomalous spin-spin couplings by seeking effects when the relative orientation of two nearby spin-polarized masses is changed. However, δh contains no terms coupling electron spin to an external spin, and so no signal for Lorentz or *CPT* violation can be expected in experiments of this type. Other types of experiments search for spin-monopole couplings, with which Eq. (2) is certainly compatible. Nonetheless, even these experiments are insensitive to the effects in Eq. (2) unless the spin-polarized material is directly monitored. For example, the experiment of Ref. [9] studies the behavior of an unpolarized torsion pendulum in the presence of an external spin-polarized mass and therefore cannot detect couplings of the form (2). In contrast, experiments studying the behavior of a spin-polarized torsion pendulum [8,10,11] or measuring changes in magnetization in a paramagnetic salt [12] can be exquisitely sensitive to the couplings (2).

Consider first experiments with a spin-polarized torsion pendulum. Choosing the direction $\hat{z}$ in the laboratory frame as vertically upwards along the pendulum rotation axis, the explicit expression for the perturbative contribution to the potential energy of the pendulum is

$$U(\phi) = 2S\sqrt{(\tilde{b}_1^e)^2 + (\tilde{b}_2^e)^2}\cos\phi \,. \tag{4}$$

Here, S is the net electron spin of the polarized pendulum, and ϕ is the angle between the spin vector S^j and the projection of the vector $\tilde{b}^{ej}$ on the x-y plane. The factor $\sqrt{(\tilde{b}_1^e)^2 + (\tilde{b}_2^e)^2}$ is the magnitude of this projection.

Experimental determination of the behavior of a spin-polarized pendulum typically requires data collection over many hours. During this time, the sidereal rotation of the Earth changes the orientation of the laboratory-frame coordinates relative to the background tensors a_μ^e, b_μ^e, $H_{\mu\nu}^e$, $c_{\mu\nu}^e$, and $d_{\mu\nu}^e$. In the laboratory frame, the parameters $\tilde{b}_j^e$ in Eq. (3) therefore appear to be time dependent. To determine the corresponding time dependence of the potential U, it is useful to work with quantities defined with respect to a nonrotating frame. A suitable choice of basis $\{\hat{X}, \hat{Y}, \hat{Z}\}$ for a nonrotating frame can be introduced in terms of celestial equatorial coordinates [7]. With this choice, the $\hat{Z}$ direction lies along the Earth's rotational north pole, subtending an angle χ with the pendulum rotational axis $\hat{z}$. The time dependence of the laboratory-frame components $\tilde{b}_j^e$ can then be displayed explicitly in terms of nonrotating-frame components as

$$\begin{aligned}
\tilde{b}_1^e &= \tilde{b}_X^e \cos\chi \cos\Omega t + \tilde{b}_Y^e \cos\chi \sin\Omega t - \tilde{b}_Z^e \sin\chi \,, \\
\tilde{b}_2^e &= -\tilde{b}_X^e \sin\Omega t + \tilde{b}_Y^e \cos\Omega t \,, \\
\tilde{b}_3^e &= \tilde{b}_X^e \sin\chi \cos\Omega t + \tilde{b}_Y^e \sin\chi \sin\Omega t + \tilde{b}_Z^e \cos\chi \,.
\end{aligned} \tag{5}$$

Here, the angular frequency Ω is the Earth's sidereal (not solar) rotational frequency, $\Omega \simeq 2\pi/(23\ \mathrm{h}\ 56\ \mathrm{m})$.

At present, the spin-polarized torsion pendulum most sensitive to Lorentz- and *CPT*-violating effects is the one used with the Eöt-Wash II instrument at the University of Washington [10,11]. It has four stacked layers of toroidal magnets with alternating sections made of Alnico and SmCo, producing a large net electron spin (of approximately 8×10^{22} aligned spins) but a negligible magnetic moment. The apparatus is shielded from external magnetic fields, so any signal would represent a nonmagnetic interaction coupling to the electron spins. To search for a spin-monopole coupling, the torsion pendulum is mounted on a turntable that rotates about the suspension axis with angular frequency ω, and a time-varying signal harmonically related to ω is sought. Assuming an initial alignment of $\vec{S}$ along the $\hat{x}$ axis defined in the laboratory frame, the orientation of the spin vector $\vec{S}$ changes with the rotation as

$$\vec{S} = S(\cos\omega t\,\hat{x} + \sin\omega t\,\hat{y}) \,. \tag{6}$$

This provides a second source of time dependence for the potential U in Eq. (4).

The potential U induces a torque τ on the pendulum about the $\hat{z}$ axis. The overall time dependence of the torque can be calculated from the potential U as the cross product of the projection of the vector $\tilde{b}^{ej}$ onto the x-y plane with the spin vector $\vec{S}$. The resulting expression in terms of parameters in the nonrotating frame involves a sum of three harmonic terms with angular frequencies ω and $\omega \pm \Omega$:

$$\tau = 2\tilde{b}_Z^e S \sin\chi \sin\omega t + 2\tilde{b}_\perp^e S\{\sin^2 \tfrac{1}{2}\chi \sin[(\omega - \Omega)t + \beta] - \cos^2 \tfrac{1}{2}\chi \sin[(\omega + \Omega)t - \beta]\}, \quad (7)$$

where $\tilde{b}_\perp^e \equiv \sqrt{(\tilde{b}_X^e)^2 + (\tilde{b}_Y^e)^2}$ and $\beta \equiv \tan^{-1}(\tilde{b}_Y^e/\tilde{b}_X^e)$.

The torque generates a pendulum twist angle θ given by $\theta = \tau/\kappa$, where κ is the pendulum spring constant. As a function of time, θ can be obtained as the solution of the differential equation

$$I\ddot{\theta} + 2I\gamma\dot{\theta} + \kappa\theta = \tau, \quad (8)$$

where I is the moment of inertia and γ is the damping constant of the torsion pendulum. Provided the rotational frequency ω is much smaller than the natural frequency $\omega_0 = \sqrt{\kappa/I}$, oscillations with angular frequency ω_0 can be treated as irrelevant and the signal becomes the steady-state solution for $\theta(t)$. For the applied torque (7), the steady-state solution is

$$\theta(t) = \frac{2S}{\kappa}\{\tilde{b}_Z^e \mathcal{A}_\omega \sin\chi \sin(\omega t - \delta_\omega) + \tilde{b}_\perp^e[\mathcal{A}_{\omega-\Omega} \sin^2 \tfrac{1}{2}\chi \sin[(\omega - \Omega)t - \delta_{\omega-\Omega} + \beta] - \mathcal{A}_{\omega+\Omega} \cos^2 \tfrac{1}{2}\chi \sin[(\omega + \Omega)t - \delta_{\omega+\Omega} - \beta]]\}, \quad (9)$$

where $\mathcal{A}_z \equiv \omega_0^2[(\omega_0^2 - z^2)^2 + 4\gamma^2 z^2]^{-1/2}$ is the attenuation factor and $\delta_z \equiv \tan^{-1}[2\gamma z/(\omega_0^2 - z^2)]$ is the phase shift due to the harmonic response of the pendulum at frequency z.

The exact shape of $\theta(t)$ is uncertain because the relative sizes of the components $\tilde{b}_X^e$, $\tilde{b}_Y^e$, $\tilde{b}_Z^e$ are unknown. However, possible limiting cases can provide some insight. Consider the Eöt-Wash experiment, for which $\chi \simeq 42.3°$. If $\tilde{b}_Z^e \approx \tilde{b}_\perp^e$, then $\theta(t)$ approximately vanishes every sidereal period $T = 2\pi/\Omega$, and $\theta(t)$ oscillates at frequency ω under an envelope with sidereal periodicity. If instead $\tilde{b}_Z^e \ll \tilde{b}_\perp^e$, then the first term in $\theta(t)$ is largely negligible, and $\theta(t)$ exhibits beats with approximate period $\frac{1}{2}T$. Finally, if large $\tilde{b}_Z^e \gg \tilde{b}_\perp^e$, then the first term in $\theta(t)$ dominates and the sidereal variations disappear, so $\theta(t)$ merely oscillates with approximate frequency ω.

Given data taken with a rotating spin-polarized torsion pendulum of the Eöt-Wash type, a test of Lorentz and *CPT* violation could proceed by extraction of the harmonic components with frequencies ω and $\omega \pm \Omega$. The amplitudes of these Fourier components would determine values of all three parameters $\tilde{b}_X^e$, $\tilde{b}_Y^e$, and $\tilde{b}_Z^e$. A compelling nonzero signal would provide evidence of Lorentz violation. In the data analysis, any summation or averaging process used to increase the statistics would need to allow for the sidereal variation to maintain the phases in the different terms in Eq. (9). The data already taken with the Eöt-Wash II instrument are sensitive to the amplitude of twist-angle variations with frequency ω at a level better than 10 nrads [11]. If this accuracy can be achieved for all three Fourier components, then impressive bounds of about 10^{-28} GeV could be attained on the components $\tilde{b}_X^e$, $\tilde{b}_Y^e$, and $\tilde{b}_Z^e$.

In a search for spin-monopole couplings, a preliminary analysis of data taken with the Eöt-Wash II apparatus has been performed [11]. This analysis involves averaging results obtained at different sidereal times and extracting the amplitude of the harmonic components with frequencies equal to multiples of ω (but not $\omega \pm \Omega$). The averaging process maintains the phase associated with the frequency ω but not those associated with $\omega \pm \Omega$. In the context of Eq. (9), terms other than the first would therefore tend to average to zero in the large-statistics limit and so only the sensitivity to $\tilde{b}_Z^e$ remains.

The analysis yields the preliminary measurement of a time-varying signal for $\theta(t)$ with angular frequency ω and amplitude $8.9 \pm 2.1 \pm 4.6$ nrad. This time-varying signal provides a measurement of $|\tilde{b}_Z^e| \simeq (1.4 \pm 0.8) \times 10^{-28}$ GeV, where the two errors have been combined in quadrature. Note that this value is almost an order of magnitude below the best bound on Lorentz violation in the electron sector obtained to date in clock-comparison experiments [6,7]. Also, the ratio $r_{\rm spin}^e \equiv |\tilde{b}_Z^e|/m \simeq 3 \times 10^{-25}$ of this value to the electron mass compares favorably to the dimensionless suppression factor $m/M_{\rm Planck} \simeq 5 \times 10^{-20}$ that might be expected to govern spontaneous Lorentz and *CPT* breaking arising from the Planck scale [25]. Confirmation that this preliminary result is a signal for Lorentz and *CPT* violation could emerge from a data reanalysis extracting the amplitude of the harmonic components with frequencies $\omega \pm \Omega$ if nonzero amplitudes are detected in the ratio predicted by Eq. (9). This would also yield a measurement of $\tilde{b}_\perp^e$.

We conclude with some remarks about a different type of experiment using macroscopic spin-polarized matter, in which the induced magnetization in a magnetic substance is studied. An experiment of this type has recently been performed at the National Tsing Hua University in Taiwan [12]. Small changes in the induced magnetization in a sample of paramagnetic TbF_3 salt are measured using a dc SQUID. The salt is shielded in a field-free environment, and a copper mass is rotated about it with frequency f. The experiment searches for a time variation in the induced magnetization with frequency f.

The apparatus functions as a magnetometer with exceptional sensitivity to an effective potential per volume

$$u_{\rm eff} = \vec{M} \cdot \vec{B}_{\rm eff} \quad (10)$$

for anomalous couplings between the magnetization $\vec{M}$ and an effective field $\vec{B}_{\rm eff}$. With the correspondence $(B_{\rm eff})_j = \tilde{b}^e_j/\mu_B$ in the laboratory frame, where μ_B is the Bohr magneton, the form of this potential matches that obtained for Lorentz and CPT violation via Eq. (2).

Analysis of data taken with this apparatus provides [12] an upper bound on $B_{\rm eff}$ of approximately 10^{-12} G. This precision is achieved by accumulating large statistics, which is made possible by rotating the copper mass around the salt crystal at the relatively high frequency of $f \simeq 0.96$ Hz. However, in the context of the standard-model extension, no variation in the magnetization is caused by rotating a copper mass around the salt crystal.

A test of Lorentz and CPT symmetry with this apparatus could nonetheless be performed by searching for sidereal time variations in the magnetization of the salt crystal. An alternative possibility might be to rotate the entire apparatus on a turntable as in the Eöt-Wash II instrument. If the above sensitivity of 10^{-12} G could be achieved for an effective field $(B_{\rm eff})_j$ due to Lorentz and CPT violation, it would make attainable bounds on a combination of $\tilde{b}^e_X$, $\tilde{b}^e_Y$, and $\tilde{b}^e_Z$ at the level of 10^{-29} GeV. Another option for improving the sensitivity of experiments of this type might be their inclusion in satellite-based tests of Lorentz symmetry, perhaps in conjunction with a program for testing the equivalence principle [26].

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