Fortran Program for the Upward and Downward Continuation and Derivatives of Potential Fields

By ALBERT J. RUDMAN *and* ROBERT F. BLAKELY

DEPARTMENT OF NATURAL RESOURCES
GEOLOGICAL SURVEY OCCASIONAL PAPER 10

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To the Geophysics Community

This report is one of a series of Geophysical Computer Programs that will be published in the Indiana Geological Survey Occasional Paper Series. Members of the geophysics section of the Indiana Geological Survey, with the advice and counsel of an Advisory Board, will select and edit submitted papers. At present, programs dealing with the calculation of gravity and magnetic fields over two- and three-dimensional bodies, depth calculations from seismic refraction data, digital filtering, and cross correlation and convolution processes are in preparation. Readers are invited to submit programs and manuscripts to the geophysics section. The primary purpose of this series will be to make readily availabe those programs that deal with established geophysical computations.

Although the editors of some journals solicit only new approaches, we will seek to publish programs that also deal with standard and classic problems. Our experience has shown that geophysicists, working alone or at relatively small laboratories, do not always have access to such programs. We also solicit programs implementing new geophysical procedures, but we anticipate that such material will be made available only rarely. Nevertheless, even large laboratories with extensive computer libraries may welcome a study of the other fellow's approach. In the same spirit, we hope that geophysicists will share both their new and standard programs.

The format for this series is intentionally kept simple to encourage others to submit manuscripts. It should contain: (1) a statement to establish the purpose of the program and some discussion of applications; (2) a brief summary of the theory that underlies the algorithm; (3) a discussion of the program, perhaps with the aid of a flow diagram; and (4) presentation of a test case.

Responsibility for distribution of the program cards or furnished tapes will be assumed by the Indiana Geological Survey.

—Albert J. Rudman and Robert F. Blakely, editors

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Fortran Program for the Upward and Downward Continuation and Derivatives of Potential Fields

By ALBERT J. RUDMAN *and* ROBERT F. BLAKELY[1]

Abstract

In 1960 Roland G. Henderson, of the U.S. Geological Survey, published a comprehensive system for computation of first and second derivatives of potential fields and the continuation of fields to levels above or below the plane of observation. In our study a Fortran IV program (HNDRSN2), based on Henderson's algorithm, uses map data digitized at an equally spaced grid interval. Output from program HNDRSN2 includes maps of the field continued upward or downward from one to five grid units and first and second derivative maps on the surface and on selected downward continued levels. Test cases demonstrate the reliability of the program in standard analyses of gravity and magnetic fields.

Introduction

Measurements of the earth's gravity and magnetic field in geophysical exploration do not directly yield a unique geologic picture of the surveyed region. Usually, the first step in the analysis of such potential fields involves routine and well-understood procedures to reduce observed values to a datum. Interpretation of these data may then follow any of the numerous analytic and graphic methods that are described in standard texts (Dobrin, 1960; Nettleton, 1940). The most important of these interpretive methods are those that use first and second derivatives, but all attempt to isolate and sharpen the anomaly and relate the resulting field to its geologic source. Another approach involves continuation of the observed field upward or downward to a new level. A field measured at any level above a geologic source, for example, may be transformed downward to a selected level in relation to the source. "Seeing" the anomaly at close range is advantageous in the interpretation of the data, especially in delineating the edges of the source.

In 1960 Roland G. Henderson, of the U.S. Geological Survey, published a comprehensive system for the calculation of first and second vertical derivatives of potential fields and the analytical continuation of fields to levels above and below the plane of observation. Although derivatives and continuation have long been recognized as a basic approach in the enhancement and analysis of gravity and magnetic fields, Henderson's method permits the geophysicist to implement this technique rapidly on the computer and has become one of the most popular approaches. The purpose of this paper is to present a Fortran IV program for computing the derivative and continued fields and to present a test case based on Henderson's algorithm.

One common application of the first derivative is to suppress the regional gradient and to increase the sharpness of an anomaly. The net effect of taking the first derivative is an isolation of the anomalous field. The second derivative also may be used to outline the shape of the source of an anomaly, especially in the magnetic case, provided the source is essentially a vertically sided, vertically polarized prism. In that case the cross section of the upper surface of the prism may be outlined by the zero contour of the second derivative (Vacquier and others, 1951). In the case of a gravitational field, the source is isolated but not outlined as definitively as in the magnetic case.

Upward continuation is used to smooth out irregularities in the observed field. As the field is continued upward, away from the source, sharp variations in the field are either eliminated or subdued. Thus prominent anomalies originating from shallow geologic sources are filtered out by the process of upward continuation. The opposite effects are observed in downward continuation. The effects of anomalies having shallow sources are emphasized, while regional variations originating from deeper sources are subdued. Recently, downward continuation also has been used to detect the top of the source of an anomaly (Rudman and others, 1971; LeMouel and others, 1974), although this application has not been tested widely.

The merit in Henderson's approach lies in its simplicity: a set of 11 values are computed once for each

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map point. From them all derivative or continued fields at that point can be approximated by a multiplication process, wherein coefficients may be selected to calculate first or second derivatives on any upward or downward continued surface. Multiplication by the coefficients is essentially a filtering process. Results of the calculations are not unique in that they approximate the analytic solution. Although recent papers have explored other methods for calculating derivatives and continuation (especially filtering in the frequency domain), Henderson's 1960 paper continues to be the classic approach and it remains one of the more popular procedures.

Theory

The following discussion of the continuation and derivative processes is intended only to introduce the basic equations used in the algorithm. For an extensive presentation the reader is referred to Henderson's 1960 paper, to the text by Grant and West (1965), or to numerous articles cross indexed in the "S.E.G. Cumulative Index of Geophysics" under the key words CONTINUATION and DERIVATIVE. Peters' (1949) article is a basic reference and is a good starting point for any comprehensive review.

Given the field $P(0)$ on a surface $z = 0$ (fig. 1), the field at a point $-z$ above the surface can be computed from the classic solution of the Dirichlet problem for a half space

$$P(-z) = \int_0^\infty \frac{(-z)\ \overline{P}(r)\ dr}{(r^2 + z^2)^{3/2}} \qquad \text{(Equation 1)}$$

where r is the radius of a circle about the central point. $\overline{P}(r)$ is the average value of the field on a circle of radius r and is given by

$$\overline{P}(r) = \frac{1}{2\pi} \int_0^{2\pi} P(r, \vartheta)\ d\vartheta \qquad \text{(Equation 2)}$$

In routine calculations, $\overline{P}(r)$ is the arithmetic average of values intercepted by the digitizing grid on a circle of specified radius r. For example, in figure 1 $\overline{P}(r_3)$ is the average of the eight values at the intersections of the grid system.

Henderson used a heuristic approach to obtain a working formula to compute the field at a specified height $z = ka$

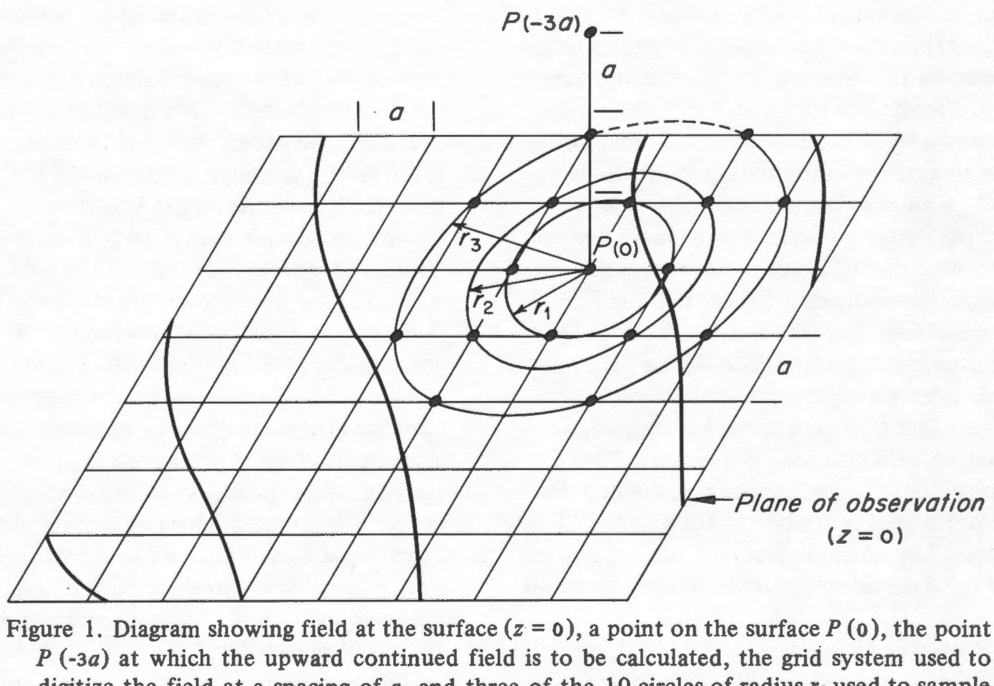

Figure 1. Diagram showing field at the surface ($z = 0$), a point on the surface $P(0)$, the point $P(-3a)$ at which the upward continued field is to be calculated, the grid system used to digitize the field at a spacing of a, and three of the 10 circles of radius r_i used to sample the field. Note that a value of $r_1 = a$ has four grid (field) points, that $r_2 = a\sqrt{2}$ also has four, and that $r_3 = a\sqrt{5}$ has eight. In some examples circles with large radii may extend into adjacent regions where the field is not defined. (See fig. 2 for additional discussion.) In our example circles with large radii would intersect grid points beyond the edge of the original map.

$$P(ka) \approx \sum_{i=0}^{10} \bar{P}(r_i)\ C(r_i, k) \qquad \text{(Equation 3)}$$

where a is the digitizing or grid interval and k is an integer. Tests of simple geometries show that k should not exceed 5 and that the field $P(0)$ is adequately sampled by the central point $r = 0$ and average $\bar{P}$ values at radii of $r = a, a\sqrt{2}, a\sqrt{5}, a\sqrt{8}, a\sqrt{13}, a\,5, a\sqrt{50}, a\sqrt{136}, a\sqrt{274}$, and $a\,25$. The coefficients $C(r_i, k)$ for upward continuation were given by Henderson from the assumption that a grid value of a = unity.

Equation 3 is also the fundamental working formula for calculation of downward continuation and first and second derivatives, with different coefficient sets specified for each quantity. Once the average field values are calculated for the 10 rings around a point, calculation of any quantity directly above or below that point depends only on multiplying these averages by the appropriate coefficients and summing them to obtain a final value. The method is efficient because the ring values need only be calculated once.

Reliability of the results depends on the grid interval selected, smoothness of the field, and the depth of continuation. For example, oscillations are generated if the sampling grid is too fine, if the original field is noisy, or if the field is continued downward to a depth too close to the source of the anomaly. Such oscillations are minimized if (1) a grid interval is selected equal to one-fourth the depth of the top of the anomalous source; (2) a filter is applied to smooth the original field; and (3) continuation does not extend below the source. Because downward continuation generally introduces some noise in the form of minor variations and oscillations in the field, differentiation of downward continued levels also presents special problems. The differentiation process emphasizes changes in the field and these are not always geologically meaningful on downward continued surfaces.

Underlying the basic computational formula (equation 3) is a generalized expression for the field fitted to a Lagrange interpolation formula

$$\bar{P}(z) = \sum_{k=0}^{n} \frac{(-1)^k\ z(z+a)\ (z+2a)\ldots(z+ka)}{a^n\ (z+ka)\ (n-k)!\ k!}\ P(-ka)$$

(Equation 4)

This equation may be modified to compute the field $\bar{P}(z)$ above and below the plane of observation. It can also be used to arrive at the derivative formula. Although Henderson's 1960 paper placed strong theoretical emphasis on equation 4, it does not play a role in the application of the algorithm in our report.

Algorithm for Program HNDRSN2

(See appendix 1)

1. Input Parameters (See appendix 2)

Read in identification card (HEADING); codes to select specified continuation levels and derivatives (ISELECT(L)); maximum x, y coordinate of the map input (IMAX, JMAX); base value to adjust original map data (BASE); and a scale factor (SCALE).

A generalized flow of program HNDRSN2 (appendix 1) is given in these sections. The first operation involves reading the parameters summarized above. The base value (selected by the user) is added to the original input data to adjust it for printing as a positive number on a map with format of F4.0. Program HNDRSN2 yields a compact map with each output map printed on one page. Because the F4.0 format accommodates only a three-digit positive number or a two-digit negative number, the scale factor (SCALE) is used to modify the output data to the map size. A second program with a larger output format is available in our files and does not need the scale factor. It is called HNDRSN1 and its output is printed on four separate line printer sheets with an F10.2 format. The user must assemble (clip or tape) these four sheets into one map. Details of the identification card, the code to select a specified operation on the data, and the exact map size for HNDRSN2 are given in appendices 2-5 for a test case of a gravitational field over a vertical prism. (HNDRSN1 will not be discussed.)

2. Input Map Data (See appendix 2)

Read in map data (P (I, J)); adjust to base value and appropriate scale factor; and print results in map form.

Map data (P (I, J)), one per card, is read in row by row, with the x coordinate (horizontal) governed by the I index and the y coordinate by the J index. (For orientation details see fig. 2.) Row length and number of rows are designated by IMAX, JMAX parameters. A maximum number of 25 X 25 map values can be read in for program HNDRSN2. (As the program is written, simply increasing the "dimensions" of the array will not increase the input capacity.) Map values are next modified by the base value and then multiplied by the scale factor and values stored in a two-dimensional array.

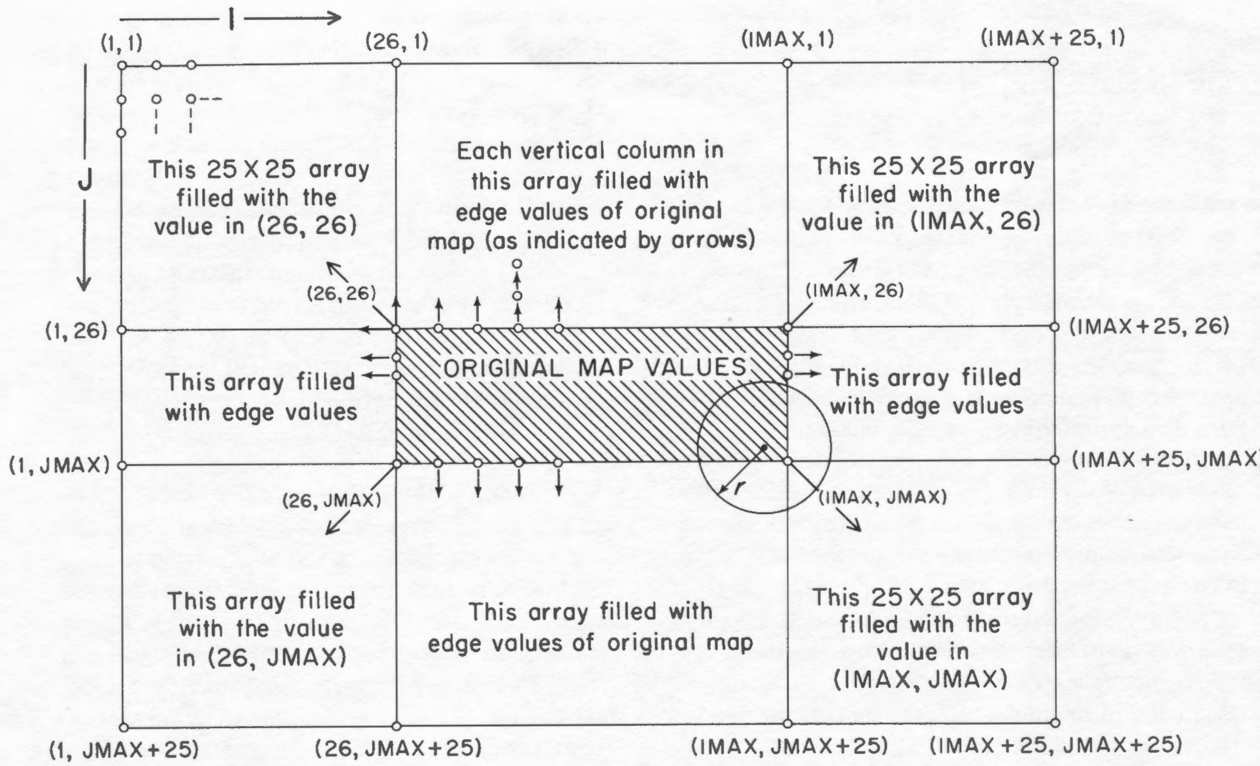

Figure 2. Sketch of original data and adjacent blocks filled with pseudo-map values. The average value around a circle of radius *r* may involve data from more than one adjacent block. Data in the adjacent blocks are obtained by extending edge values of the original data in the manner shown. Output maps are always the size of the input map.

3. Calculate Ring Averages (See fig. 2)

Generate pseudo-map data in regions adjacent to the original map. (See fig. 2.) Average values (R (I, J, K)) are calculated for 10 rings surrounding each map point.

Each map point has associated with it 11 values (R (I, J, K)): the original map value (K = 1) and average ring values calculated for 10 circles of varying radii surrounding the point (K = 2, 3. ., 11). These 11 values are stored in a three-dimensional array for later calculations to obtain the derivative and continued fields at that point. Circles surrounding a point near the edge of the map may extend beyond the original map. Figure 2 shows a circle that intercepts three adjacent "blocklike" areas. Rather than use "zero" values in these adjacent blocks, values along the edge of the original map are extended outward. Because the largest circle is 25 grid units in radius, adjacent areas are a maximum of 25 X 25. (Computations depend on the average value of each ring; this figure demonstrates why data near the edge of the original map is sometimes unreliable.)

4. Calculate and Print Output (See appendices 3-5)

The 11 values (R (I, J, K)) associated with each point (I, J) are multiplied by 11 coefficients (C (K, L)) and then summed to yield 11 numbers (equation 3). The sum of these yield one value appropriate to the continuation and (or) derivative designated by the L code. Output maps are printed to present the final results.

Because the 11 R (I, J, K) values for each point are stored, computation for each continuation level and derivative requires only one step: multiplication by the appropriate coefficient and summing the 11 numbers. Output maps include (1) the original map data (modified by the base and scale factors) and (2) all the maps specified by the L code. Each map is identified by the identification parameter and a statement describing the derivative and (or) the continuation level.

5. Possible Modifications of the Algorithm

Every programmer knows that there are always better ways to write a program. So, too, with HNDRSN2. Some of the possible modifications are desirable enough that a few comments are in order. In our original approach, both map and computational data were stored in either two- or three-dimensional arrays. Programming for data displayed in this manner is simple to visualize and manipulate, but both space and computer time could be saved if data were handled in smaller arrays. For example, once the 11 ring values around a point are calculated, it would be possible to compute all the desired values at that point and then discard the ring values. Instead, we store these ring values in a three-dimensional array R (I, J, K) and make all computations separately.

Similarly, the pseudo-map values in adjacent map areas are created by extending the edge values. Because potential fields are smooth functions, a polynomial trend surface fit to the real map data might be used to predict better adjacent values. Writing such a program and the fitting process are time consuming and the advantages of the system have not been investigated. Moreover, as now programmed, these larger 25 X 25 arrays are all stored for use during the "ring" computations. A program could be written to assign the appropriate values (without stored arrays) by using the (I, J) value as an identifier of the current ring calculation.

Coefficients for each derivative and continuation operation are presently incorporated into the Fortran program. For each calculation, all the coefficients need to be repeatedly read in. A rewriting of the program should include these coefficients as part of the initialization process under the Fortran command:

```
DATA CC (1, 1), CC (2, 2), . . . ./0.11193, 0.32193, . . . ./
```

The experienced programmer will find numerous other major and minor changes to make; however, we leave the program in its present state for expediency. It is a working program!

Concluding Statement

Appendices 1-5 discuss the operating procedures and present a test case. In our discussion of the output maps (appendices 3, 4, and 5), little mention has been made of the reliability of the results. The input data was a field computed over a cylinder. All the parameters of this model are known, and selected analytic calculations were made by hand for the exact values and derivatives of the field at various levels. The computer derived values compared favorably with the predicted results, demonstrating the validity of the program and supporting the reliability of Henderson's basic algorithm.

Special acknowledgments are extended to Indiana University's Wrubel Computing Center for the generous use of its facilities and expertise of its personnel.

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1951 - Interpretation of aeromagnetic maps: Geol. Soc. America Mem. 47, 151 p.

Appendix 1. Fortran IV Program HNDRSN2

The program, as written, contains numerous comment cards identifying the purpose of each section. No subroutines are used.

```
      PROGRAM HNDRSN2   (INPUT,OUTPUT,TAPE1=INPUT,TAPE2=OUTPUT)
C
C     ******************************************************************
C                 THIS PROGRAM FOLLOWS HENDERSONS TECHNIQUE FOR UPWARD
C                 AND DOWNWARD CONTINUATION AND FIRST AND SECOND
C                 DERIVATIVES (SEE HENDERSON,1960,GEOPHYSICS,VOL.25,
C                 NOS.3,P.569-585.). WRITTEN FOR FORTRAN IV USE ON
C                 CDC 6600 SERIES COMPUTER. OUTPUT MAPS ARE THE SIZE OF
C                 INPUT MAPS, UP TO A MAXIMUM OF 25 X 25.
C     ******************************************************************
C
C
C     ******************************************************************
C                 DIMENSION STATEMENTS FOR DATA UP TO 25 X 25
C     ******************************************************************
C
      DIMENSION HEADING (10)
      DIMENSION ISELECT (20), P(75,75), C(11,19),R(25,25,11)
      COMMON P, C, R
C
C     ******************************************************************
C                 READ IN IDENTIFICATION CARD(HEADING) AND ISELECT
C                 CARD WHICH CONTAINS CODES FOR LIST OF MAPS DESIRED.
C                 CODES ARE IDENTIFIED LATER IN THIS PROGRAM.
C     ******************************************************************
C
      READ(1,1)(HEADING(I),I=1,10)
    1 FORMAT (10A8)
      READ(1,2)(ISELECT(L),L=1,19)
    2 FORMAT (19I1)
C
C     ******************************************************************
C                 READ IN MAXIMUM VALUE OF I(=IMAX). I BEGINS AT 26 AND
C                 IMAX MUST BE 50 OR LESS. SIMILARLY FOR JMAX. READ IN
C                 ON SAME CARD THE VALUE (BASE) TO BE SUBTRACTED FROM
C                 MAP VALUES P(I,J).NEXT READ  A SCALE VALUE TO MODIFY
C                 DATA TO FIT THE LIMITED SIZE MAP FORMAT OF F4.0.
C                 P(I,J) DATA IS MULTIPLIED BY THIS SCALE FACTOR.
C     ******************************************************************
C
      READ(1,3) IMAX, JMAX, BASE
    3 FORMAT (2I2,F6.2)
      READ(1,501) SCALE
  501 FORMAT(F10.4)
C
C     ******************************************************************
C                 READ IN P(I,J) DATA.SUBTRACT BASE, MULTIPLY BY SCALE,
C                 PRINT HEADING AND PLOT ON MAP TYPE OUTPUT.
C                 MAP IS PRINTED FROM P(26,26) TO P(IMAX,JMAX)
C                 IF DATA IS LESS THAN 25 X25 , A BLANK
C                 IS PRINTED IN THE SPACES TO FILL OUT THE MAP
```

```
C      *******************************************************************
C
       READ(1,4) ((P(I,J), I=26, IMAX),J=26, JMAX)
     4 FORMAT(17X,F7.2)
       DO 6 I=26, IMAX
       DO 5 J = 26, JMAX
     5 P(I,J)=(P(I,J)-BASE)*SCALE
     6 CONTINUE
       WRITE(2,7)
     7 FORMAT (1H1)
       WRITE(2,101)(HEADING(I),I=1,10)
   101 FORMAT(20X,10A8/)
       WRITE(2,8) BASE,SCALE
     8 FORMAT( 20X,*INPUT DATA LESS BASE OF *F6.2,3X,*MULTIPLIED BY SCALE
      1 OF *F5.2/)
       DO 9 J=26,JMAX
       WRITE(2,222)
   222 FORMAT(4X,24(1H*,4X),1H*)
       WRITE(2,10)(P(I,J),I=26,IMAX)
    10 FORMAT(2X,25(F4.0,1X)/)
       IF(IMAX.LT.50) 600, 9
   600 WRITE(2,601)
   601 FORMAT(1H )
     9 CONTINUE
       IF(JMAX.LT.50) 602, 605
   602 LMAX=(50-JMAX)
       DO 603 LL=1,LMAX
   603 WRITE(2,604)
   604 FORMAT(4X,24(1H*,4X),1H*//)
   605 CONTINUE
C
C      *******************************************************************
C                  NEXT SECTION PREPARES REGIONS BEYOND EDGE OF MAP TO
C                  BE USED IN ANALYSIS. APPROACH IS TO FILL THE SURROUNDING
C                  SPACE BY EXTENDING EACH EDGE VALUE NORMAL TO THE MAP
C                  FOR 25 UNITS.
C      *******************************************************************
C
       IMAX1 = IMAX + 1
       IMAX25 = IMAX + 25
       JMAX1 = JMAX + 1
       JMAX25 = JMAX + 25
       DO 14 J=26,JMAX
       DO 15 I=1,25
    15 P(I,J)=P(26,J)
       DO 16 I = IMAX1, IMAX25
    16 P(I,J)=P(IMAX,J)
    14 CONTINUE
C
C
       DO 17 I=26,IMAX
       DO 18 J=1,25
    18 P(I,J)=P(I,26)
       DO 19 J = JMAX1, JMAX25
    19 P(I,J)=P(I,JMAX)
    17 CONTINUE
```

```
C
C
      DO 20 I=1,25
      DO 21 J=1,25
   21 P(I,J)=P(26,26)
   20 CONTINUE
C
C
      DO 22 I = IMAX1, IMAX25
      DO 23 J=1,25
   23 P(I,J)=P(IMAX,26)
   22 CONTINUE
C
C
      DO 24 I=1,25
      DO 25 J = JMAX1, JMAX25
   25 P(I,J)=P(26,JMAX)
   24 CONTINUE
C
C
      DO 26 I = IMAX1,IMAX25
      DO 27  J = JMAX1,.JMAX25
   27 P(I,J)=P(IMAX,JMAX)
   26 CONTINUE
C
C     ******************************************************************
C                CALCULATION OF AVERAGE VALUE OF DATA ON RINGS CENTERED
C                AT EACH MAP POINT. CALL THESE R (I,J,K), WHERE
C                K=1 TO 11
C     ******************************************************************
C
      M=0
      DO 28 I=26,IMAX
      M=M+1
      N=0
      DO 29 J=26,JMAX
      N=N+1
      R(M,N,1)=P(I,J)
      R(M,N,2)=(P(I,J+1)+P(I,J-1)+P(I+1,J)+P(I-1,J))/4.0
      R(M,N,3)=(P(I+1,J+1)+P(I+1,J-1)+P(I-1,J+1)+P(I-1,J-1))/4.0
      R(M,N,4)=(P(I+2,J+1)+P(I+2,J-1)+P(I-2,J+1)+P(I-2,J-1)
     1+P(I+1,J+1)+P(I+1,J-1)+P(I-1,J+1)+P(I-1,J-1))/8.0
      R(M,N,5)=(P(I+2,J+2)+P(I+2,J-2)+P(I-2,J+2)+P(I-2,J-2))/4.0
       R(M,N,6)=(P(I+2,J+3) +P(I+2,J-3)+P(I-2,J+3)+P(I-2,J-3)
     1+P(I+3,J+2)+P(I+3,J-2)+P(I-3,J+2)+P(I-3,J-2))/8.0
       R(M,N,7)=(P(I+5,J)+P(I-5,J)+P(I,J+5)+P(I,J-5)
     1+P(I+3,J+4)+P(I+3,J-4)+P(I+4,J+3)+P(I+4,J-3)
     2+P(I-3,J+4)+P(I-4,J+3)+P(I-3,J-4)+P(I-4,J-3))/12.0
       R(M,N,8)=(P(I+7,J+1)+P(I+1,J+7)+P(I+7,J-1)+P(I+1,J-7)
     1+P(I-7,J+1)+P(I-1,J+7)+P(I-7,J-1)+P(I-1,J-7)
     2+P(I+5,J+5)+P(I+5,J-5)+P(I-5,J+5)+P(I-5,J-5))/12.0
       R(M,N,9)=(P(I+10,J+6)+P(I+6,J+10)+P(I+10,J-6)+P(I+6,J-10)
     1+P(I-10,J+6)+P(I-6,J+10)+P(I-10,J-6)+P(I-6,J-10))/8.0
       R(M,N,10)=(P(I+7,J+15)+P(I+15,J+7)+P(I-7,J+15)+P(I-15,J+7)
     1+P(I+7,J-15)+P(I+15,J-7)+P(I-7,J-15)+P(I-15,J-7))/8.0
       R(M,N,11)=(P(I,J+25)+P(I,J-25)+P(I-20,J+15)+P(I-15,J+20) +
     1P(I-20,J-15)+P(I-15,J-15)+P(I+20,J+15)+P(I+15,J+20)
     2+P(I+20,J-15)+P(I+15,J-20)+P(I+25,J)+P(I-25,J))/12.0
   29 CONTINUE
   28 CONTINUE
```

```
C
C      ***************************************************************
C                FOR EACH MAP THERE IS A SET OF COEFFICIENTS C(K,L),
C                WHERE K IS THE RING NOS AND L IS THE CODED INTEGER
C                FOR THE DESIRED MAP.CODES ARE A 'ONE' IF THE MAP IS
C                DESIRED, OR A 'ZERO' IF IT IS NOT.  CODES ARE ENTERED
C                ON THE ISELECT CARD FROM COLUMNS 1 TO 19. THE
C                FOLLOWING LIST GIVES THE CODES. THUS, A 'ONE' IN
C                COLUMN 1 REQUESTS A MAP CONTINUED UPWARD ONE
C                GRID UNIT. A 'ZERO' IN COLUMN 2 WILL SUPPRESS THE
C                MAP THAT IS CONTINUED UPWARD 2 GRID UNITS, ETC.
C      ***************************************************************
C
C
C
C
C      ***************************************************************
C                COEFFICIENTS FOR UPWARD CONTINUATION 1. CODE L=1
C      ***************************************************************
C
       C(1,1)=.11193
       C(2,1)=.32193
       C(3,1)=.06062
       C(4,1)=.15206
       C(5,1)=.05335
       C(6,1)=.06586
       C(7,1)=.06650
       C(8,1)=.05635
       C(9,1)=.03855
       C(10,1)=.02273
       C(11,1)=.03015
C
C      ***************************************************************
C                COEFFICIENTS FOR UPWARD CONTINUATION 2. CODE L=2
C      ***************************************************************
C
       C(1,2)=.04034
       C(2,2)=.12988
       C(3,2)=.07588
       C(4,2)=.14559
       C(5,2)=.07651
       C(6,2)=.09902
       C(7,2)=.11100
       C(8,2)=.10351
       C(9,2)=.07379
       C(10,2)=.04464
       C(11,2)=.05998
```

```
C
C     ******************************************************************
C                COEFFICIENTS FOR UPWARD CONTINUATION 3. CODE L=3
C     ******************************************************************
C
      C(1,3)=.01961
      C(2,3)=.06592
      C(3,3)=.05260
      C(4,3)=.10563
      C(5,3)=.07146
      C(6,3)=.10226
      C(7,3)=.12921
      C(8,3)=.13635
      C(9,3)=.10322
      C(10,3)=.06500
      C(11,3)=.08917
C
C     ******************************************************************
C                COEFFICIENTS FOR UPWARD CONTINUATION 4. CODE L=4
C     ******************************************************************
C
      C(1,4)=.01141
      C(2,4)=.03908
      C(3,4)=.03566
      C(4,4)=.07450
      C(5,4)=.05841
      C(6,4)=.09173
      C(7,4)=.12915
      C(8,4)=.15474
      C(9,4)=.12565
      C(10,4)=.08323
      C(11,4)=.11744
C
C     ******************************************************************
C                COEFFICIENTS FOR UPWARD CONTINUATION 5. CODE L=5
C     ******************************************************************
C
      C(1,5)=.00742
      C(2,5)=.02566
      C(3,5)=.02509
      C(4,5)=.05377
      C(5,5)=.04611
      C(6,5)=.07784
      C(7,5)=.11986
      C(8,5)=.16159
      C(9,5)=.14106
      C(10,5)=.09897
      C(11,5)=.14458
```

```
C
C      ***********************************************************************
C                COEFFICIENTS FOR DOWNWARD CONTINUATION 1. CODE L=6
C      ***********************************************************************
C
       C(1,6)= 4.8948
       C(2,6)=-3.0113
       C(3,6)= 0.0081
       C(4,6)=-0.5604
       C(5,6)=-0.0376
       C(6,6)=-0.0689
       C(7,6)=-0.0605
       C(8,6)=-0.0534
       C(9,6)=-0.0380
       C(10,6)=-0.0227
       C(11,6)=-0.0302
C
C      ***********************************************************************
C                COEFFICIENTS FOR DOWNWARD CONTINUATION 2. CODE L=7
C      ***********************************************************************
C
       C(1,7)= 16.1087
       C(2,7)=-13.2209
       C(3,7)=  0.4027
       C(4,7)=-01.9459
       C(5,7)= 00.0644
       C(6,7)=-00.0596
       C(7,7)=-00.0522
       C(8,7)=-00.0828
       C(9,7)=-00.0703
       C(10,7)=-00.0443
       C(11,7)=-00.0600
C
C      ***********************************************************************
C                COEFFICIENTS FOR DOWNWARD CONTINUATION 3. CODE L=8
C      ***********************************************************************
C
       C(1,8)= 41.7731
       C(2,8)=-38.2716
       C(3,8)= 01.7883
       C(4,8)=-04.7820
       C(5,8)= 00.5367
       C(6,8)= 00.1798
       C(7,8)= 00.1342
       C(8,8)=-00.0560
       C(9,8)=-00.0900
       C(10,8)=-00.0639
       C(11,8)=-00.0891
```

```
C
C     ******************************************************************
C                COEFFICIENTS FOR DOWNWARD CONTINUATION 4. CODE L=9
C     ******************************************************************
C
      C(1,9)= 92.5362
      C(2,9)=-89.7403
      C(3,9)= 05.1388
      C(4,9)=-09.9452
      C(5,9)= 01.7478
      C(6,9)= 00.8908
      C(7,9)= 00.6656
      C(8,9)= 00.0718
      C(9,9)=-00.0890
      C(10,9)=-00.0802
      C(11,9)=-00.1173
C
C     ******************************************************************
C                COEFFICIENTS FOR DOWNWARD CONTINUATION 5. CODE L=10
C     ******************************************************************
C
      C(1,10)= 183.2600
      C(2,10)=-183.9380
      C(3,10)= 011.8804
      C(4,10)=-018.6049
      C(5,10)= 004.2324
      C(6,10)= 002.4237
      C(7,10)= 001.7777
      C(8,10)= 000.3606
      C(9,10)=-000.0571
      C(10,10)=-000.0921
      C(11,10)=-000.1444
C
C     ******************************************************************
C                COEFFICIENTS FOR FIRST DERIVATIVE ON SURFACE.CODE L=11
C     ******************************************************************
C
      C(1,11)= 1.87282
      C(2,11)=-1.13625
      C(3,11)=-0.05949
      C(4,11)=-0.30210
      C(5,11)=-0.05857
      C(6,11)=-0.07597
      C(7,11)=-0.07072
      C(8,11)=-0.05758
      C(9,11)=-0.03905
      C(10,11)=-0.02286
      C(11,11)=-0.05020
```

```
C
C     ******************************************************************
C              COEFFICIENTS FOR FIRST DERIVATIVE DOWN 1. CODE L=12
C     ******************************************************************
C
      C(6,12)=-0.04007
      C(6,12)=-0.04856
      C(5,12)= 0.00361
      C(3,12)= 0.12727
      C(4,12)=-0.88750
      C(1,12)= 6.62394
      C(2,12)=-5.62446
      C(7,12)=-0.04007
      C(8,12)=-0.04575
      C(9,12)=-0.03615
      C(10,12)=-0.02233
      C(11,12)=-0.05000
C
C     ******************************************************************
C              COEFFICIENTS FOR FIRST DERIVATIVE DOWN 2. CODE L=13
C     ******************************************************************
C
      C(1,13)= 16.98074
      C(2,13)=-16.05517
      C(3,13)= 00.76135
      C(4,13)=-01.98701
      C(5,13)= 00.23820
      C(6,13)= 00.09219
      C(7,13)= 00.07475
      C(8,13)=-00.00768
      C(9,13)=-00.02726
      C(10,13)=-00.02077
      C(11,13)=-00.04934
C
C     ******************************************************************
C              COEFFICIENTS FOR FIRST DERIVATIVE DOWN 3. CODE L=14
C     ******************************************************************
C
      C(1,14)= 36.11116
      C(2,14)=-35.96237
      C(3,14)= 02.17080
      C(4,14)=-03.83054
      C(5,14)= 00.76745
      C(6,14)= 00.42646
      C(7,14)= 00.32573
      C(8,14)= 00.06859
      C(9,14)=-00.01084
      C(10,14)=-00.01812
      C(11,14)=-00.04832
```

```
C
C     ******************************************************************
C                COEFFICIENTS FOR FIRST DERIVATIVE DOWN 4. CODE L=15
C     ******************************************************************
C
      C(1,15)= 67.88049
      C(2,15)=-69.68033
      C(3,15)= 04.76651
      C(4,15)=-06.69004
      C(5,15)= 01.74330
      C(6,15)= 01.05352
      C(7,15)= 00.77613
      C(8,15)= 00.19699
      C(9,15)= 00.01469
      C(10,15)=-00.01433
      C(11,15)=-00.04693
C
C     ******************************************************************
C                COEFFICIENTS FOR 2ND DERIVATIVE ON THE SURFACE.
C                CODE L=16.
C     ******************************************************************
C
      C(1,16)= 2.82994
      C(2,16)=-2.49489
      C(3,16)= 0.05173
      C(4,16)=-0.39446
      C(5,16)= 0.00932
      C(6,16)=-.00732
      C(7,16)=.00304
      C(8,16)= 0.00219
      C(9,16)= 0.00040
      C(10,16)= 0.00004
      C(11,16)= 0.00000
C
C     ******************************************************************
C                COEFFICIENTS FOR 2ND DERIVATIVE DOWN 1. CODE L=17.
C     ******************************************************************
C
      C(1,17)= 7.08408
      C(2,17)=-6.93715
      C(3,17)= 0.36265
      C(4,17)=-0.80764
      C(5,17)= 0.13050
      C(6,17)= 0.07231
      C(7,17)= 0.06502
      C(8,17)= 0.02312
      C(9,17)= 0.00565
      C(10,17)= 0.00103
      C(11,17)= 0.00043
```

```
C
C     ******************************************************************
C               COEFFICIENTS FOR 2ND DERIVATIVE DOWN 2. CODE L=18
C     ******************************************************************
C
      C(1,18)= 14.15751
      C(2,18)=-14.51327
      C(3,18)= 00.96018
      C(4,18)=-01.42970
      C(5,18)= 00.35907
      C(6,18)= 00.22256
      C(7,18)= 00.17330
      C(8,18)= 00.05501
      C(9,18)= 00.01239
      C(10,18)= 00.00210
      C(11,18)= 00.00085
C
C     ******************************************************************
C               COEFFICIENTS FOR 2ND DERIVATIVE DOWN 3. CODE L=19
C     ******************************************************************
C
      C(1,19)= 24.74755
      C(2,19)=-26.02351
      C(3,19)= 01.92719
      C(4,19)=-02.30269
      C(5,19)= 00.72474
      C(6,19)= 00.46253
      C(7,19)= 00.33920
      C(8,19)= 00.09985
      C(9,19)= 00.02070
      C(10,19)= 00.00322
      C(11,19)= 00.00122
C
C     ******************************************************************
C               THIS SECTION MAKES THE FINAL CALCULATIONS FOR THOSE
C               MAPS SELECTED BY USER IN HIS ISELECT CODE.
C     ******************************************************************
C
      DO 30 L=1,19
      LEVEL=L
      IF (ISELECT(L).LT.1) 30,31
   31 WRITE(2,7)
      WRITE(2,101)(HEADING(I),I=1,10)
      IF(L.EQ. 1)71,52
   52 IF(L.EQ. 2)72,53
   53 IF(L.EQ. 3)73,54
   54 IF(L.EQ. 4)74,55
   55 IF(L.EQ. 5)75,56
   56 IF(L.EQ. 6)76,57
   57 IF(L.EQ. 7)77,58
   58 IF(L.EQ. 8)78,59
   59 IF(L.EQ. 9)79,60
   60 IF(L.EQ.10)80,61
   61 IF(L.EQ.11)81,62
   62 IF(L.EQ.12)82,63
   63 IF(L.EQ.13)83,64
   64 IF(L.EQ.14)84,65
```

```
 65 IF(L.EQ.15)85,66
 66 IF(L.EQ.16)86,67
 67 IF(L.EQ.17)87,68
 68 IF(L.EQ.18)88,69
 69 IF(L.EQ.19)89,90
 71 WRITE(2,171)
171 FORMAT (20X *MAP CONTINUED UPWARD 1 GRID UNIT*//)
    GO TO 320
 72 WRITE(2,172)
172 FORMAT (20X,*MAP CONTINUED UPWARD 2 GRID UNIT*//)
    GO TO 320
 73 WRITE(2,173)
173 FORMAT (20X,*MAP CONTINUED UPWARD 3 GRID UNIT*//)
    GO TO 320
 74 WRITE(2,174)
174 FORMAT (20X,*MAP CONTINUED UPWARD 4 GRID UNIT*//)
    GO TO 320
 75 WRITE(2,175)
175 FORMAT (20X,*MAP CONTINUED UPWARD 5 GRID UNIT*//)
    GO TO 320
 76 WRITE(2,176)
176 FORMAT (20X,*MAP CONTINUED DOWNWARD 1 GRID UNIT*//)
    GO TO 320
 77 WRITE(2,177)
177 FORMAT (20X,*MAP CONTINUED DOWNWARD 2 GRID UNIT*//)
    GO TO 320
 78 WRITE(2,178)
178 FORMAT (20X,*MAP CONTINUED DOWNWARD 3 GRID UNIT*//)
    GO TO 320
 79 WRITE(2,179)
179 FORMAT (20X,*MAP CONTINUED DOWNWARD 4 GRID UNIT*//)
    GO TO 320
 80 WRITE(2,180)
180 FORMAT (20X,*MAP CONTINUED DOWNWARD 5 GRID UNIT*//)
    GO TO 320
 81 WRITE(2,181)
181 FORMAT (20X,*MAP OF FIRST DERIVATIVE ON SURFACE*//)
    GO TO 320
 82 WRITE(2,182)
182 FORMAT (20X,*MAP OF FIRST DERIVATIVE DOWN 1 GRID UNIT*//)
    GO TO 320
 83 WRITE(2,183)
183 FORMAT (20X,*MAP OF FIRST DERIVATIVE DOWN 2 GRID UNIT*//)
    GO TO 320
 84 WRITE(2,184)
184 FORMAT (20X,*MAP OF FIRST DERIVATIVE DOWN 3 GRID UNIT*//)
    GO TO 320
 85 WRITE(2,185)
185 FORMAT (20X,*MAP OF FIRST DERIVATIVE. DOWN 4 GRID UNIT*//)
    GO TO 320
 86 WRITE(2,186)
186 FORMAT (20X,*MAP OF SECOND DERIVATIVE ON SURFACE*//)
    GO TO 320
 87 WRITE(2,187)
187 FORMAT (20X,*MAP OF SECOND DERIVATIVE DOWN 1 GRID UNIT*//)
    GO TO 320
```

```
   88 WRITE(2,188)
  188 FORMAT (20X,*MAP OF SECOND DERIVATIVE DOWN 2 GRID UNIT*//)
      GO TO 320
   89 WRITE(2,189)
  189 FORMAT (20X,*MAP OF SECOND DERIVATIVE DOWN 3 GRID UNIT*//)
      GO TO 320
  320 DO 33 I=26,IMAX
      DO 34 J=26,JMAX
      P(I,J)=0.0
      DO 35 K=1,11
   35 P(I,J)=C(K,L)*R(I-25,J-25,K)+P(I,J)
   34 CONTINUE
   33 CONTINUE
C
C     ******************************************************************
C                 NEXT SECTION PRINTS ALL MAPS IN SAME FORMAT AS
C                 INPUT MAP (SEE PREVIOUS COMMENTS).
C     ******************************************************************
C
      DO 36 J=26,JMAX
      WRITE(2,223)
 223  FORMAT (4X ,24(1H*,4X)1H* )
      WRITE(2,37)(P(I,J),I=26,IMAX )
   37 FORMAT(2X,25(F4.0,1X)/)
      IF(IMAX.LT.50) 700, 36
  700 WRITE(2,701)
  701 FORMAT(1H )
   36 CONTINUE
      IF(JMAX.LT.50) 702, 705
  702 LMAX=(50-JMAX)
      DO 703 LL=1,LMAX
  703 WRITE(2,704)
  704 FORMAT(4X,24(1H*,4X),1H*//)
  705 CONTINUE
   30 CONTINUE
      CALL EXIT
   90 WRITE(2,91)
   91 FORMAT(1X,*ERROR, TOO LARGE L VALUE*)
      CALL EXIT
C
C     ******************************************************************
C                                 JULY, 1974
C     ANY QUESTIONS REGARDING THIS PROGRAM SHOULD BE DIRECTED TO
C                               GEOPHYSICS SECTION
C                               INDIANA GEOLOGICAL SURVEY
C                               BLOOMINGTON, INDIANA 47401
C
C
C     NO RESPONSIBILITY IS ASSUMED BY THE INDIANA GEOLOGICAL SURVEY
C     NOR THE INDIANA UNIVERSITY DEPARTMENT OF GEOLOGY FOR ANY ERRORS,
C     MISTAKES OR MISREPRESENTATIONS THAT MAY OCCUR WHEN USING THIS
C     PROGRAM.  NOR IS RESPONSIBILITY ASSUMED BY THE INDIANA UNIVERSITY
C     RESEARCH COMPUTING CENTER FOR ITS CORRECT REPRODUCTION.
C     ******************************************************************
C
      END
```

Appendix 2. Input Cards for Test Case

Card 1. 80-column identification card. Information from this card is printed as a heading for each of the maps.

Card 2. Codes to select certain map data as output. Cols. 1-19 are either punched with 1 or zero (Format 19 I1). Codes are described in the program comment cards; for example, if col. 1 is punched with a 1, a map continued upward one grid unit (a = .5 km) is printed. For the test case, cols. 7 and 16 were punched to yield two maps: (1) input map continued down two grid units and (2) second derivative of input map (appendices 4 and 5).

Card 3. Gives coordinates of maximum I and J values (I and J begin at 26). For a maximum x coordinate of 25 data points, a value of 50 is entered. For a 25 X 25 map input IMAX and JMAX are read in as 50, 50 (Format 2 I2). A base value (F6.2) is also read in on this card. The base value is subtracted from the original data input.

Card 4. A scale value is read in to multiply the output data. In this test case, the output gravity values are expected to be small and a scale value of 10.0 is used (F10.4).

Card 5. Input data for the maps, one item of data per card with Format (17X, F7.2), are in row and column sequence; for example, for a 25 X 25 data array, 625 cards are now read in.

```
Card 1 →   25X25 GRAVITY FIELD OF VERTICAL CYLINDER,2 KM RADIUS,TOP 2 KM DOWN,BOTTOM 50 KM
Card 2 →   0000001000000001000
Card 3 →   5050 0.0
Card 4 →   10.
Card 5 →                       2.14  ┐
Card 6 →                       2.25  │
   .                           2.35  │
   .                           2.46  │
   .                           2.57  │
                               2.68  │
                               2.79  │
                               2.88  │
                               2.97  │
                               3.04  │
                               3.10  │
                               3.13  │  First 25 data cards are stored as first row
                               3.14  ├  in 25 X 25 map array (Appendix 3)
                               3.13  │
                               3.10  │
                               3.04  │
                               2.97  │
                               2.88  │
                               2.79  │
                               2.68  │
                               2.57  │
                               2.46  │
                               2.35  │
                               2.24  │
                               2.14  ┘
                               2.25  ┐
                               2.36  │
                               2.49  │
                               2.62  │
                               2.75  ├  Beginning of second row
                                 .   │
                                 .   ┘
                                 .
                                 .
                                 .
                                 .
   .                             .   ┐
   .                             .   ├  Final data values of last row
   .                             .   │  of 25 X 25 map array
Card 627 →                     2.35  │
Card 628 →                     2.24  │
Card 629 →                     2.14  ┘
```

Appendices 3-5. Selected Output Maps for Test Case

A gravitational field over a vertical prism was digitized at a grid interval of 0.5 km and used as a test case for program HNDRSN2. The input data were generated from a program following an algorithm from Talwani and Ewing (1960) and is displayed in appendix 3 as a 25 X 25 map. Map values are in milligals multiplied by a scale factor of 10 and then rounded off. For example, the first input value of 2.14 (appendix 2) is printed on the map in the northwest corner as 21. The solid heavy line outlines the prism. Contours show that the input field is a smooth function with a maximum value of 8.8 milligals.

In this test case only two computed maps were specified. The first was a map continued downward two grid units toward the source of the anomaly (appendix 4). The maximum value of the field is greater than the original map data (13.1 milligals), with an overall increase in contour gradient. Contours along the edges of the map are no longer a smooth function, a phenomenon associated with a field as it is continued close to the source and with the inherent limitations imposed on all edge values as discussed previously.

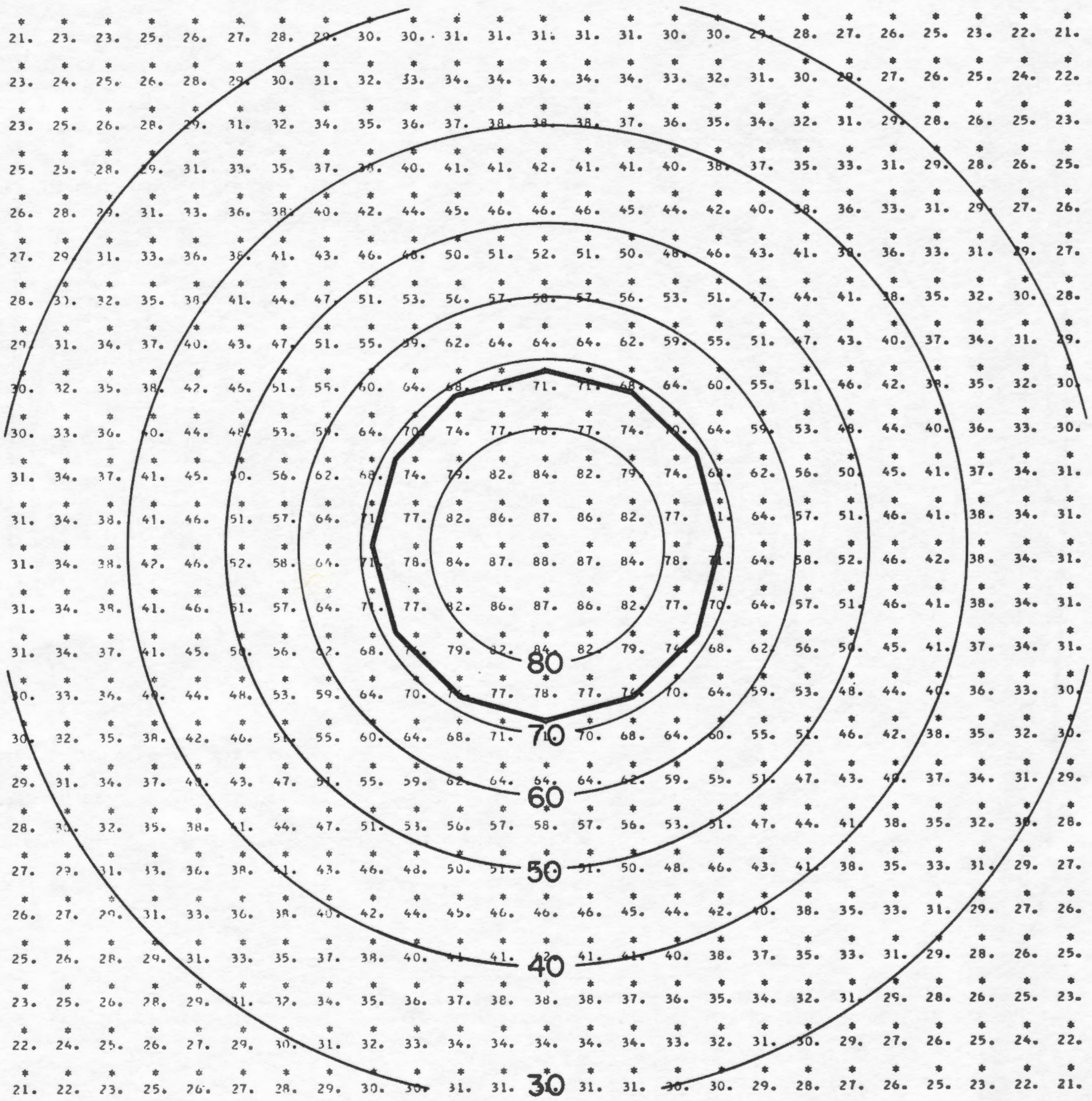
The final map is the second derivative of the input gravity data (appendix 5). Note that the contours display an increased gradient over the center of the anomaly, thus effectively isolating a local source from a regional gradient. In the case of a magnetic field over a vertical prism, the zero contour closely approximates the edges of the source.

Appendix 3. Input Data

[Contours in milligals X 10]

[Gravity field calculated over vertical prism, vertically sided with outline shown by 12-sided figure in center of map]

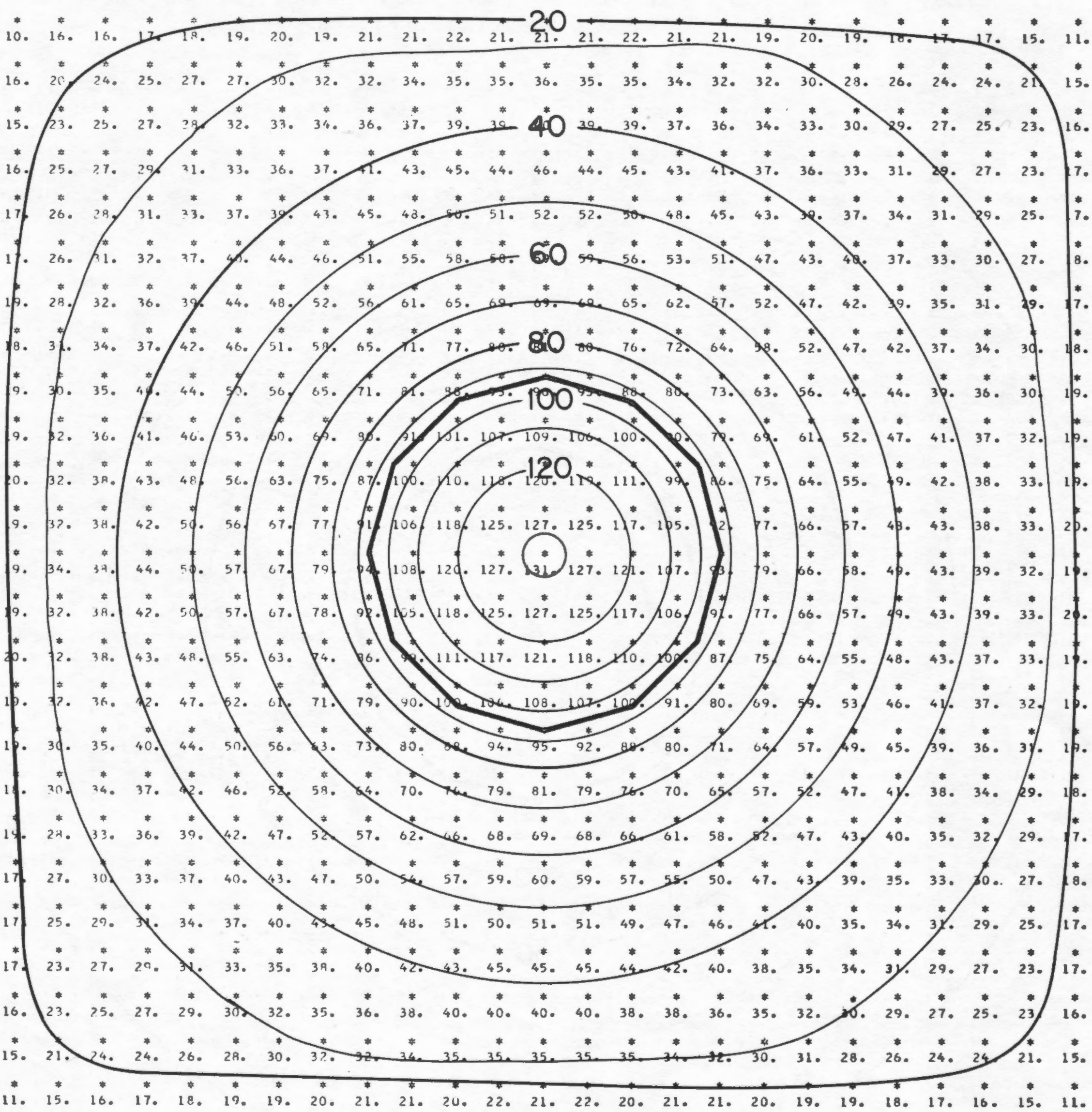
25 X 25 GRAVITY FIELD OF VERTICAL CYLINDER, 2 KM RADIUS, TOP 2 KM DOWN, BOTTOM 50 KM
INPUT DATA LESS BASE OF 0.00 MULTIPLIED BY SCALE OF 10.00



Appendix 4. Input Data Continued Down Two Grid Units

[Contours in milligals X 10]

25 X 25 GRAVITY FIELD OF VERTICAL CYLINDER, 2 KM RADIUS, TOP 2 KM DOWN, BOTTOM 50 KM
MAP CONTINUED DOWNWARD 2 GRID UNIT

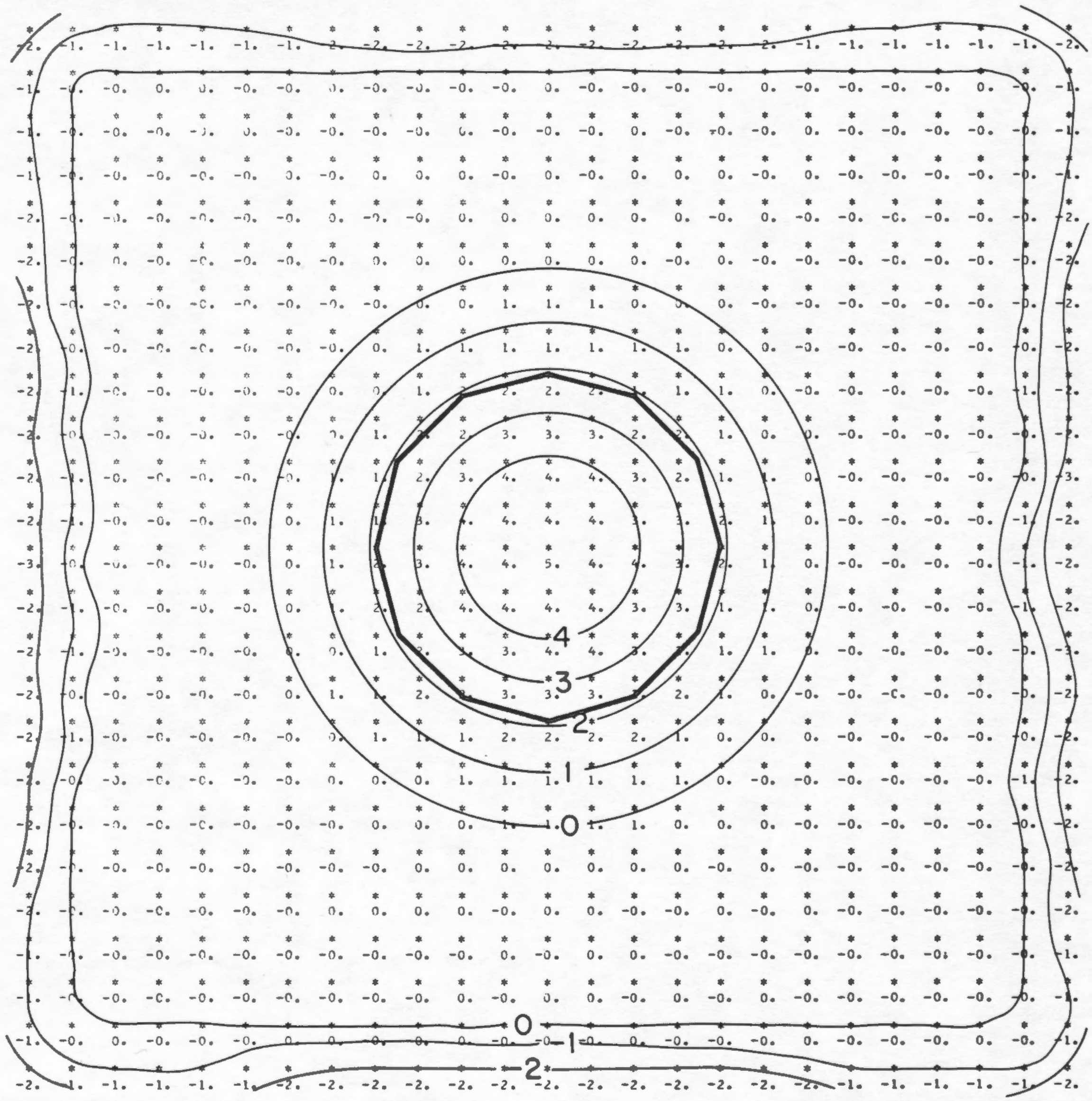


Appendix 5. Second Derivative of Input Data

[Contours in milligals/.5km/.5km]

[Note additional significant figures can be printed out if the "scale" factor is increased]

25 X 25 GRAVITY FIELD OF VERTICAL CYLINDER, 2 KM RADIUS, TOP 2 KM DOWN, BOTTOM 50 KM
MAP OF SECOND DERIVATIVE ON SURFACE



Summary

Program HNDRSN2 implements Henderson's (1960) algorithm to calculate derivative and continued fields. The space required to create 19 derivative and continuation maps from an input array of 25 X 25 (625 data points) was 46,000 octal or 20,000 words. Running time required 26 seconds on a C.D.C. 6600. Single precision arithmetic (14 significant digits) was used in the program.