Evaluation of Seismometer Arrays for Earthquake Location

By **BARRY R. LIENERT, L. NEIL FRAZER,** *and* **ALBERT J. RUDMAN**

DEPARTMENT OF NATURAL RESOURCES
GEOLOGICAL SURVEY OCCASIONAL PAPER 52

GEOPHYSICAL COMPUTER PROGRAMS
11

SCIENTIFIC AND TECHNICAL STAFF OF THE GEOLOGICAL SURVEY

AUTHORS OF THIS REPORT: Barry R. Lienert and L. Neil Frazer, Hawaii Institute of Geophysics, University of Hawaii at Manoa, 2525 Correa Road, Honolulu, HI 96822; Albert J. Rudman, Department of Geology, Indiana University, Bloomington, IN 47405.

Evaluation of Seismometer Arrays for Earthquake Location

By BARRY R. LIENERT, L. NEIL FRAZER, *and* ALBERT J. RUDMAN

GEOPHYSICAL COMPUTER PROGRAM 11

DEPARTMENT OF NATURAL RESOURCES
GEOLOGICAL SURVEY OCCASIONAL PAPER 52

PRINTED BY AUTHORITY OF THE STATE OF INDIANA
BLOOMINGTON, INDIANA: 1986

STATE OF INDIANA
Robert D. Orr, *Governor*
DEPARTMENT OF NATURAL RESOURCES
James M. Ridenour, *Director*
GEOLOGICAL SURVEY
John B. Patton, *State Geologist*

For sale by Publications Section, Geological Survey, 611 North Walnut Grove,
Bloomington, IN 47405
Price $2.50

To the Geophysics Community

This report is one of a series of Geophysical Computer Programs that are being published in the Indiana Geological Survey Occasional Paper Series. Members of the Geophysics Section of the Indiana Geological Survey, with the advice and counsel of an advisory board,* select and edit submitted papers. Readers are invited to submit programs and manuscripts to the Geophysics Section. The primary purpose of this series is to make readily available those programs that deal with established geophysical computations.

Although the editors of some journals solicit only new approaches, we seek to publish programs that also deal with standard and classic problems. Our experience has shown that geophysicists, working alone or at relatively small laboratories, do not always have access to such programs. We also solicit programs implementing new geophysical procedures, but we anticipate that such material will be made available only rarely. Nevertheless, even large laboratories with extensive computer libraries may welcome a study of the other fellow's approach. In the same spirit, we hope that geophysicists will share both their new and standard programs.

The format for this series is intentionally kept simple to encourage others to submit manuscripts. It should contain: (1) a statement to establish the purpose of the program and some discussion of applications; (2) a brief summary of the theory that underlies the algorithm; (3) a discussion of the programs, perhaps with the aid of a flow diagram; and (4) presentation of a test case.

Responsibility for distribution of the program cards or furnished tapes will be assumed by the Indiana Geological Survey.

—Albert J. Rudman and Robert F. Blakely, editors

*Norman S. Neidell, Zenith Exploration Co., Inc.; Sigmund Hammer, University of Wisconsin; Judson Mead, Indiana University; Franklin P. Prosser, Indiana University; and Joseph E. Robinson, Syracuse University.

INDIANA GEOLOGICAL SURVEY GEOPHYSICAL COMPUTER PROGRAMS

No. 1 "Fortran Program for the Upward and Downward Continuation and Derivatives of Potential Fields" (Occasional Paper 10)

No. 2 "Fortran Program for Generation of Synthetic Seismograms" (Occasional Paper 13)

No. 3 "Fortran Program for Correlation of Stratigraphic Time Series" (Occasional Paper 14)

No. 4 "Fortran Program for Generation of Earth Tide Gravity Values" (Occasional Paper 22)

No. 5 "Fortran Program for Reduction of Gravimeter Observations to Bouguer Anomaly" (Occasional Paper 23)

No. 6 "Fortran Program for Correlation of Stratigraphic Time Series. Part 2. Power Spectral Analysis" (Occasional Paper 26)

No. 7 "Application of Finite-Element Analysis to Terrestrial Heat Flow" (Occasional Paper 29)

No. 8 "Generation of Synthetic Seismograms for an Acoustic Layer over an Acoustic Half Space" (Occasional Paper 35)

No. 9 "Computer Calculation of Two-Dimensional Gravity Fields" (Occasional Paper 40)

No. 10 "Generation of Vertically Incident Seismograms" (Occasional Paper 49)

No. 11 "Evaluation of Seismometer Arrays for Earthquake Location" (Occasional Paper 52)

No. 1 is out of print.

Cost of Nos. 2 through 5 is $1.00 each + 50-cent mailing fee (third and fourth class) or 80-cent mailing fee (first class).

Cost of Nos. 6, 7, and 9 is $1.50 each + 50-cent mailing fee (third and fourth class) or 80-cent mailing fee (first class).

Cost of No. 8 is $2.00 + 50-cent mailing fee (third and fourth class) or 80-cent mailing fee (first class).

Cost of Nos. 10 and 11 is $2.50 each + 50-cent mailing fee (third and fourth class) or 80-cent mailing fee (first class).

Contents

Illustrations

Tables

Evaluation of Seismometer Arrays for Earthquake Location

By BARRY R. LIENERT, L. NEIL FRAZER, *and* ALBERT J. RUDMAN

Abstract

Program HYPOERR evaluates the performance of a small network of arbitrary seismic arrays in determining coordinates and times of seismic events. A linearized inversion following the method of Uhrhammer (1980) is performed for a layered velocity structure by determining the eigenvalues and eigenvectors of the partial derivatives of travel time for P and/or S phases with respect to hypocenter position and origin time for each station in the array. A series of covariance matrices is then obtained to evaluate statistical errors for a specified grid of hypocenter locations at any given depth. Contour plots can then be made of the matrix elements by using standard contouring software. Examples are given for (1) the case of a hypothetical quadrapartite array and (2) an actual eight-station ocean-bottom seismometer array deployed around the 95.5° W. Galapagos propagating-rift zone.

Introduction

The performance of a seismic array can be assessed in terms of the accuracy with which it is able to locate seismic events within a given volume. Also of interest is the relative importance of data recorded by individual stations in the array. Formally, the location problem is an inverse problem. The method used to solve this inverse problem has been described by Jackson (1972) and Wiggins (1972). We wish to determine four parameters: the spatial coordinates x, y, and z and the origin time, t, of the hypocenter by using the arrival times of P and/or S phases, t_i, at an array of seismic stations. The performance of the array can then be assessed in terms of the errors in each of these four parameters.

In general, we need at least four separate arrival times with user-specified standard errors for P and/or S to determine the four hypocentral parameters. Ideally, we wish the error in these parameters to be approximately constant throughout the area of interest. We also wish to ensure that the data from each seismic station is of approximately equal value in constraining the solution. Calculation of earthquake-location errors has been described by Flinn (1965) and Peters and Crosson (1972). But they did not address the problems of parameter resolution and data importance that arise from analysis in terms of linearized inverse theory. The method used here to analyze the performance of the array in terms of linearized inverse theory is that described by Uhrhammer (1980) and is implemented in program HYPOERR (appendix 2).

The inverse theory developed for program HYPOERR may be valuable in determining the location and the number of seismic stations necessary to locate hypocenters within some specified area. For example, earthquake studies of the New Madrid Fault Zone are partly based on data from about 32 stations in the Central Mississippi Valley seismic network (Stauder and others, 1984). There are eight of these stations in the Wabash Valley of Illinois and Indiana. The recent emphasis on earthquake-prediction studies of the New Madrid area support the need for objectively evaluating the geometrical configuration of the present network and perhaps the location of future stations in the area.

Theory

The problem in earthquake location is to find the hypocentral coordinates and origin time (x, y, z, t) of the event that minimizes the differences, Δt_i, between the N predicted and observed P and/or S arrival times at a given set of seismic stations, that is, to solve a set of N equations having the form

$$\Delta t_i = G_{ij} \Delta x_j \qquad (1)$$

$$\text{where} \quad \Delta x_j = (\Delta x, \Delta y, \Delta z, \Delta t)$$

$$\text{and} \quad G_{ij} = \partial t_i / \partial x_j$$

Note that if P and S are both used, there will be 2N observations. Equations (1) can be solved for Δx_j by inverting the N x 4 partial derivative matrix G. This is most easily accomplished by decomposing the matrix G by using the singular value decomposition theorem (for example, Noble and Daniel, 1977), so that

$$\underset{(4 \times N)}{(G)^{-1}} = \underset{(4 \times 4)}{V} \quad \underset{(4 \times 4)}{\Lambda^{-1}} \quad \underset{(4 \times N)}{U^T} \qquad (2)$$

where U and V are the matrices of eigenvectors of GG^T and G^TG respectively, and Λ is a diagonal matrix containing the singular values of G. The singular values are equal to the square roots of the common eigenvalues of both GG^T and G^TG.

For an excellent discussion of the geometrical significance of these eigenvectors and eigenvalues, the reader is referred to Mandel (1982). Rather than finding the eigenvectors U and V separately, it is more efficient to obtain the eigenvectors V and then to use the relation (Aki and Richards, 1980, p. 683)

$$U = G V \Lambda^{-1} \qquad (3)$$

This avoids a problem with signs of the eigenvectors and their corresponding eigenvalues (Aki and Richards, 1980, p. 679). The hypocentral-parameter covariance matrix, Υ^2, is usually given by

$$\Upsilon^2 = V \frac{\sigma^2}{\Lambda^2} V^T \qquad (4)$$

where σ^2 is the arrival-time variance. (Uhrhammer, 1980, refers to Υ as the "uncertainty" matrix.) If the variances of the arrival times are not equal or if they covary, then

$$\Upsilon^2 = V \Lambda^{-1} U^T CO\, U \Lambda^{-1} V^T \qquad (5)$$

$$= V C V^T \qquad (6)$$

where CO is the observation-data covariance matrix, and

$$C = \Lambda^{-1} U^T CO U \Lambda^{-1} \quad (7)$$

We define the parameter-variance correlation matrix, ρ_{ij}, by the equation

$$\rho_{ij} = \frac{\Upsilon_{ij}^2}{|\Upsilon_{ii}| |\Upsilon_{jj}|} \quad (8)$$

If ρ_{ij} is equal to the identity matrix, the parameter variances are uncorrelated, and the size of the off-diagonal elements represent the degree of correlation between the appropriate parameter variances (1 = perfect correlation, 0 = no correlation). For example, if the partial derivatives of arrival times with respect to depth are almost equal, there will be a high correlation between origin time and depth. This undesirable situation can be remedied by including an extra arrival time, for example, an S phase.

Similarly the ignorance matrix (covariance matrix for the data-space observations) is given by

$$\psi^2 = U \frac{\sigma^2}{\Lambda^2} U^T \quad (9)$$

or

$$\psi^2 = U \Lambda^{-1} U^T CO U \Lambda^{-1} U^T \quad (10)$$

$$= U C U^T \quad (11)$$

The ignorance matrix (or "lack-of-information" matrix), ψ^2, indicates the error with which we are able to predict the arrival times at individual stations in the array. A value of ψ^2 that is much larger than the variance of a particular arrival time indicates that the arrival time is of little value in constraining the solution. The ratio of ignorance to actual variance is termed the variance-inflation factor (Marquardt and Snee, 1975).

Another useful measure of the value of each arrival time is given by the (NxN) information density matrix, S (called INF in HYPOERR), defined by

$$S = U U^T \quad (12)$$

This matrix is a measure of the independence of the data (Wiggins, 1972). For example, if only four arrival times were used to determine the hypocentral coordinates, S would be an identity matrix (assuming that the observations were capable of fully determining the solution). When there are more observations than parameters, the rows of S represent combinations of the data that maximize, in the least squares sense, the contribution of the diagonal element, that is, the arrival time corresponding to that row. The maximum value of each diagonal element of S is one, so the diagonal elements provide a useful measure of the relative value of each observation; that is, a 1 means that the observation is essential, and a 0 means that the observation is useless.

We can also define an ignorance correlation matrix, Γ_{ij}, (called IGN in HYPOERR), similar to Uhrhammer's linear correlation-coefficient matrix

$$\Gamma_{ij} = \frac{\psi_{ij}^2}{|\psi_{ii}|\,|\psi_{jj}|} \quad (13)$$

which indicates the degree of correlation between the predicted arrival-time data variances. For example, if two stations in an array are very close together, there will be a high correlation between the arrival times for these two stations. Note that equation (13) implies that $\Gamma_{ij} = 1$ when $i = j$.

Two other quantities that are useful in assessing the performance of a seismic array are (1) the condition number CND (ratio of maximum to minimum eigenvalues of G) and (2) the trace SMA (the sum of the principal axes lengths for the uncertainty and ignorance matrices; the trace is termed the "semi-major axis" by Uhrhammer). The trace and the condition number are large when hypocentral parameters are poorly constrained by the data. For example, Buland (1976) has observed that the condition number may exceed 10^9 at modest distances from the center of a four-station array when only P arrivals are used.

Description of the Main Program

A listing of the main program, HYPOERR, appears in appendix 2. The principal constants and variables are listed in appendix 1A. A separate glossary for the subroutine DTDX1 is given in appendix 1B. The program can be divided into three sections, which are described below.

1. *Read input parameters:* Input parameters are read off Fortran unit 5. A brief description of the input is given in comments in program HYPOERR, and details of the input are listed in appendix 4. Included in the input are special TYPE parameters defined in table 1. TYPE1 specifies the seismic phases P and/or S used in the evaluation. Note that the coordinates of the stations (TYPE3) and the output-map data limits are defined relative to an origin at the lower left-hand corner of the XY plane. Coordinates can be given in units of distance (km, miles, feet, etc.) or as latitude/longitude depending on TYPE3 (DST or LAT). In the latter case, the coordinates are converted to kilometers; that is, velocities must be given in km/sec. The variances (standard deviations squared) of the P and S wave arrival times must be specified (TYPE4), either as single values for all stations or as a complete NxN covariance matrix, where N is the total number of observations. The layered-model velocities and depths of boundaries are read in as described in the program listing (appendix 2), depending on the user's selection of TYPE1. The maximum number of elements that can be put on a single line is eight because of format limitations in the output file. The elements that can be evaluated (selected) are given under TYPE2 in table 1.

The CPU time used by HYPOERR on a Harris 800 computer was 16.3 seconds for the calculation of seven elements of the quadrapartite array plotted in figures 2-8 and 14.98 seconds on the CDC 850 for the calculation of eight elements of the Galapagos array. The total storage required for program HYPOERR and its subroutines was 31,696 24-bit words (95,088 bytes) on the Harris 800 and 60,300 60-bit words on the CDC 850.

Table 1. Element definitions for program HYPOERR

TYPE1	USER SELECTS ONE OF THE LISTED (SEISMIC) PHASES
S	S arrivals only are used.
P	P arrivals only are used.
SR	S and P arrivals are used. P velocities are read and the P:S velocity ratio is specified when reading in velocity-model data.
SP	S and P arrivals are used. P and S velocities are read in separately.

TYPE2	USER SELECTS UP TO EIGHT POSSIBLE MATRICES (PRINTED OR PLOTTED) TO EVALUATE ARRAY PERFORMANCE
UNC i j	$= \sigma_{ij}$: uncertainty matrix; standard deviation of ith parameter (i = 1 is x, 2 is y, 3 is z, and 4 is origin time.) If i=j, output is standard deviation of the i^{th} quantity.
UCR i j	$= \rho_{ij}$: parameter-variance correlation matrix; correlation between ith and jth elements of hypocentral-parameter covariance matrix. Note that UCR ij = 1 when i = j.
IGN i j	$= \psi_{ij}$: ignorance matrix; covariance for data-space observations. Consider if P and S arrivals are used, then the ith observation at the kth station has i = 2k for the kth station's S arrival and i = 2k - 1 for its P arrival. For a single arrival (P or S), i = k.

Table 1—Continued

ICR i j	= Γ_{ij}: ignorance correlation matrix; correlation between predicted arrival-time data variances of the ith and jth elements. Note that ICR ij = 1 when i = j.
UXY	= $(\sigma_{11} + \sigma_{22})^{1/2}$: horizontal error as defined by Lee and Lahr (1975).
SMA	= trace $(\Lambda^{-1}\ U^T\ CO\ U\ \Lambda^{-1})$: the sum of the principal axes of the ignorance (or uncertainty) matrix (Uhrhammer, 1980).
INF i j	= S_{ij}: information density matrix; (i,j)th element of information or data importance matrix (S).
CND	= $\log_{10}(\lambda_1/\lambda_4)$: logarithm of condition number. For well-constrained hypocenter locations, CND is usually less than about 20.
TYPE3	USER SPECIFIES VARIANCE OR COVARIANCE MATRIX OF S/P TIMES
S	Single variances specified for the S and P arrival times at all stations
C	The complete (NxN) data covariance matrix is specified
TYPE4	USER SELECTS UNITS OF DISTANCE OR LATITUDE/COGNITIVE
DST	Coordinates are given in units of distance
LAT	Coordinates are given as latitude/longitude in the format (degrees x 100) + minutes

2. *Get time-distance parameters and form G^TG:* The partial derivatives of the travel time for each station with respect to epicenter position are evaluated by subroutine DTDX1 as described in the following section. The order of these derivatives in the matrix is the same as the order in which the station coordinates are specified, with P preceding S for each station when both arrivals are used. The matrix G^TG is then formed and stored in the matrix, AA. The storage format in AA for an M x M symmetric matrix A(J,K) is A(J,K) = AA(J + K) where K = 1, ...J for each J = 1,...M (only $M^2/2$ elements are stored). This is the storage format used by the standard mathematical subroutine EIGEN (listed in appendix 2), which is used to obtain the eigenvalues and eigenvectors of G^TG.

3. *Form singular value decomposition (S.V.D.) inverse, calculate covariance and ignorance matrices, and output elements in a form ready for contouring:* The four eigenvalues λ_i and the matrix V of eigenvectors of G^TG are obtained by using the subroutine EIGEN. The eigenvectors U are then obtained by using equation (3). The next step is the evaluation of the transformed covariance matrix, C, defined by equation (7). The covariance matrix τ^2 and ignorance matrix ψ^2 can now be evaluated by using equations (5) and (9) respectively. The required elements are then determined as described in the previous section. Finally, the user-requested elements (TYPE2) are written on Fortran unit 25 in the format 10F8.3. The first two format fields are taken up by the X and Y coordinates, and the rest are used for the required elements (a maximum of eight). Before the completion of the grid-calculation loop, the elements are searched for their minimum and maximum values. These values are useful when contouring the data as described in a following section. Note that a transformation X = XL - X, where XL is the length of the X grid, is applied to the grid coordinates before calculation of the elements. This is necessary for a right-handed XYZ coordinate system with Z positive down. Before outputting the elements it is therefore necessary to transform back to a coordinate system with X positive to the right by using X = X - XL.

4. *Travel times and their derivatives for a layered velocity model:* Subroutine DTDX1 of program HYPOERR (appendix 2) calculates minimum travel times and their partial derivatives for layered velocity models (fig. 1). The calculation method in DTDX1 follows Lee and Stewart (1981, p. 96-104). Another similar subroutine commonly used for derivative calculations was published by Eaton (1969).

Since refracted arrivals frequently have the minimum travel times and also since calculation of these travel times is faster, subroutine DTDX1 first calculates the critical distances and travel times for all refracted waves. If no refractions are possible, the direct-wave travel times and their derivatives are evaluated. The minimum refracted travel time is then compared with the direct-wave travel time calculated for a ray leaving the hypocenter at a minimum angle ϕ (fig. 1). If the refracted travel time is greater, the direct-wave travel time is calculated by using an iterative procedure. The direct-wave travel time is then compared with the minimum refracted travel time to see if it is less. Subroutine DTDX1 was checked by evaluating travel time versus distance values for three-layer models and comparing them with values given by Knox (1967, fig. 5). It was also checked by comparing the results with those from the subroutine of Eaton (Klein, 1978). Excellent agreement was found in both cases.

5. *Contouring the output values:* The output values can be contoured by using standard contouring software, which is not included because it is usually hardware specific.

Summary

Program HYPOERR (appendix 2) is able to evaluate the performance of a seismic array by calculating errors in the locations and origin times of hypothetical hypocenters near the array. The relatively modest amounts of computer time used by the algorithm allow detailed evaluation of important quantities related to the performance of the array, such as the relative value of arrival times at different stations, as well as the errors and the correlation in the hypocentral parameters themselves. The algorithm follows theory

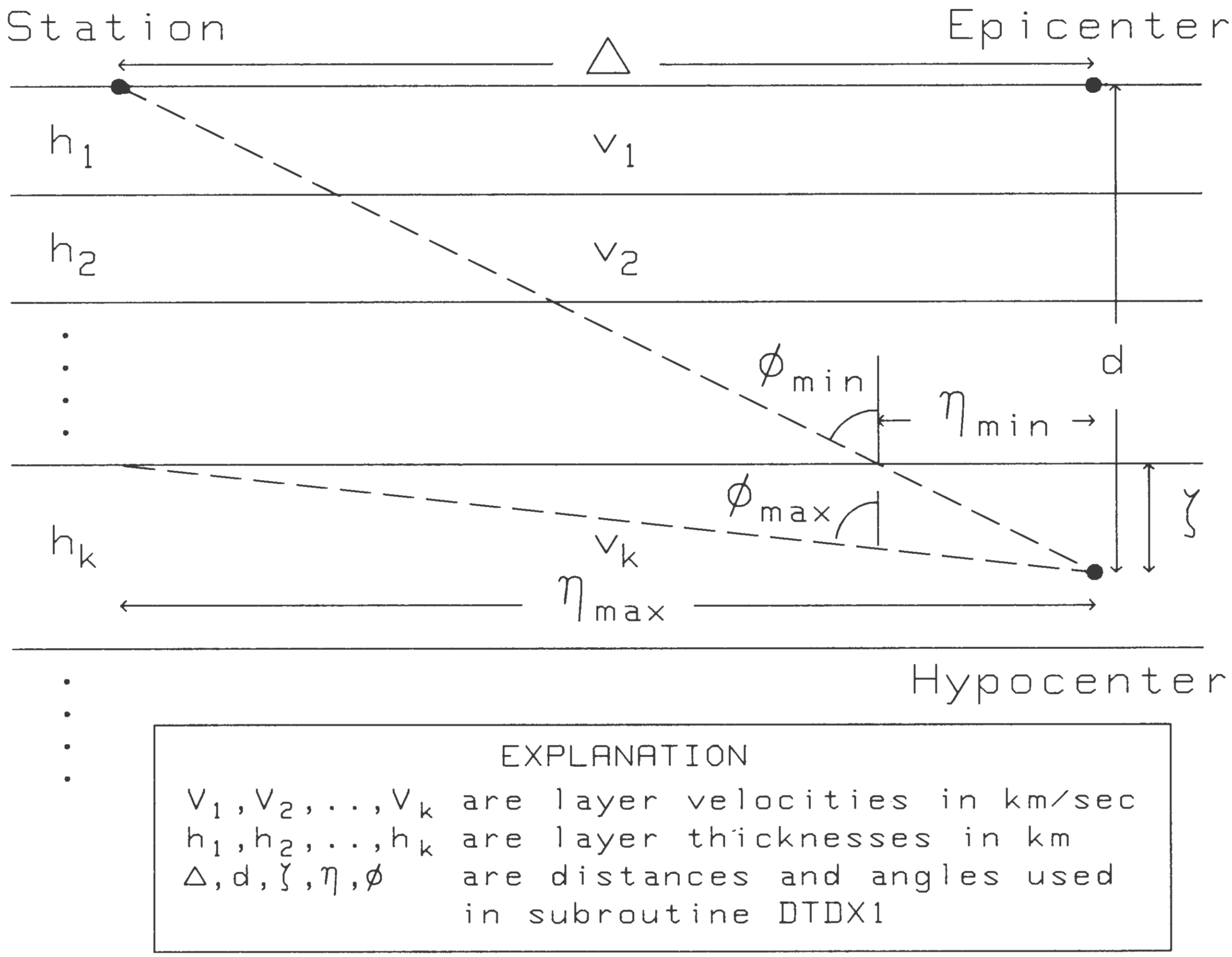


Figure 1. Layered velocity structure used by subroutine DTDX1 to calculate travel times and their partial derivatives. Also shown are the parameters used by the subroutine in its calculations.

developed by Uhrhammer (1980). The structure of the program is summarized in a flow chart (appendix 3). Because the program uses a large number of constants and variables, a glossary of terms is included (appendix 1). Two models were used as test cases to demonstrate the effectiveness of program HYPOERR. Appendix 4 describes the two models and the input data. Appendix 5 describes the results as a printout of a two-dimensional matrix of values and as a series of contour maps (figs. 2-9). Discussion of the models includes a brief analysis of the statistical significance of the results.

Acknowledgments

This work was supported by the National Science Foundation (Grant EAR82-13745), by the Office of Naval Research, and Texaco, USA. Computing time at Indiana University was supported by the Bloomington Academic Computing Center. R. B. Hermann and an anonymous reader (St. Louis University) critically read the manuscript.

Literature Cited

Aki, Keiiti, and Richards, P. G.
1980 - Quantitative seismology theory and methods, v. 2: San Francisco, W. H. Freeman and Co., 932 p.

Buland, Ray
1976 - The mechanics of locating earthquakes: Seismol. Soc. America Bull., v. 66, p. 173-187.

Eaton, J. P.
1969 - HYPOLAYR, a computer program for determining hypocenters of local earthquakes in an earth consisting of uniform flat layers over a half space: U.S. Geol. Survey open-file report.

Flinn, E. A.
1965 - Confidence regions and error determination for seismic event location: Rev. Geophysics, v. 3, p. 157-185.

Jackson, D. D.
1972 - Interpretation of inaccurate, insufficient and inconsistent data: Royal Astron. Soc. Geophys. Jour., v. 28, p. 97-110.

Klein, F. W.
1978 - Hypocenter location program HYPOINVERSE — Pt. 1, Users guide to versions 1, 2, 3, and 4: U.S. Geol. Survey open-file report 78-6914, 113 p.

Knox, W. A.
1967 - Multilayer near-surface refraction computations, *in* Seismic refraction prospecting: Tulsa, Okla., Soc. Exploration Geophysicists, p. 197-216.

Lee, W. H. K., and Lahr, J. C.
1972 - HYPO71 — A computer program for determining hypocenter, magnitude, and first motion pattern of local earthquakes: U.S. Geol. Survey open-file report 75-311.

Lee, W. H. K., and Stewart, S. W.
1981 - Principles and applications of microearthquake networks — Advances in geophysics, supp. 2: New York, Academic Press, 293 p.

Mandel, J. L.
1982 - Use of the singular values decomposition in regression analysis: Am. Statistician, v. 36, p. 15-24.

Marquardt, D. W., and Snee, R. D.
1975 - Ridge regression in practice: Am. Statistician, v. 29, p. 3-20.

Noble, Ben, and Daniel, J. W.
1977 - Applied linear algebra: Englewood Cliffs, N.J., Prentice-Hall, Inc.

Peters, D. C., and Crosson, R. S.
1972 - Application of prediction analysis to hypocenter determination using a local array: Seismol. Soc. America Bull., v. 62, p. 775-788.

Stauder, William, and others
1984 - Central Mississippi Valley earthquake bulletin: Quart. Bull. 39, First Quarter, 1984, 90 p. [mimeo.]

Uhrhammer, R. A.
1980 - Analysis of small seismographic station networks: Seismol. Soc. America Bull., v. 70, p. 1369-1379.

Wiggins, R. A.
1972 - The general linear inverse problem — Implication of surface waves and free oscillations for earth structure: Rev. Geophysics and Space Physics, v. 10, p. 251-285.

Appendix 1. Glossary of Constants and Variables in HYPOERR

A. In the Main Program

AA	Matrix used by subroutine EIGEN
BB	Information matrix (= UU^T)
C	Transformed covariance matrix (equation (7))
CO	Observation-data covariance matrix
CV	Parameter covariance matrix
CVV	Ouput elements
CVMAX	Maximum values of elements
CVMIN	Minimum values of elements
D	Scratch matrix = CV^T
DD	Scratch matrix = CO U/EIG(J)
DX	Partial derivatives of travel time
EIG	Singular values of partial derivative matrix
EL	Scratch matrix used in calculating ignorance-variance correlation matrices
G	Partial derivative matrix
I	Index
I1	Index
IC	Index
IF1	=0 if parameter covariance matrix has not been calculated
IF2	=0 if ignorance matrix has not been calculated
IF3	=0 if information matrix has not been calculated
II	Index
IN	Index
IP	Rows of covariance/ignorance matrices for each element to be calculated

IX	X coordinate index
IY	Y coordinate index
J	Index
J1	Index
JJ	Index
JP	Columns of covariance/ignorance matrices for elements
K	Index
KK	Index
KKK	Index
N	Number of observations
NE1	Layer containing hypocenter
NEL	Number of elements to be calculated
NLAYER	Number of layers in velocity model
NN	Number of single parameters in layered model (2*NLAYER-1)
NPARM	Total number of parameters
NSTAT	Number of stations
NX	Number of X points in grid
NY	Number of y points in grid
PARM	Parameters
PRM	Current parameters for DTDX1
REFLAT	Reference latitude for DIST
REFLONG	Reference longitude for DIST
SUM	Used as summing variable
TITLE	Title used for plotting (maximum of 40 characters)
TMIN	Minimum travel time
TYPE1	Input parameter to determine if S and/or P arrivals are to be used

TYPE2	Type of matrix element to be calculated
TYPE3	Format of input coordinates: 'DST' = distance; 'LAT' = latitude/longitude
TYPE4	Format of variances: 'S' is single S and P time variances; 'C' is complete covariance matrix
U	Eigenvectors of GG^T
V	Eigenvectors of G^TG
VAR	Variance of S or P arrival times
VAR1	Variance of S arrival times if both are used
X	Coordinates of event
XO	Coordinates of stations
X1	X grid lower limit
X2	X grid upper limit
XINC	X grid increment.
XL	Width of grid in X direction = XU-XS
XS	X grid lower limit in distance units
XSS	Used to save XS in left-handed coordinates
XSO	Station coordinates in distance units
XU	X grid upper limit in distance units
XXO	Station coordinates
Y1	Y grid lower limit
Y2	Y grid upper limit
YINC	Y grid increment
YS	Y grid lower limit in distance units
YU	Y grid upper limit in distance units

B. In Subroutine DTDX1

CTHI	Cosine of THI
DCRIT	Critical distances for refracted waves
DELTA	Horizontal offset of hypocenter from station
DIST	Distance of hypocenter from station
DL	Trial offset (Δ^*)
DL1	Offset lower bound (Δ_{min})
DL2	Offset upper bound (Δ_{max})
DX	Partial derivatives of travel time
E	= FACT*PARM(NL+I)
ETA1	Depth of event below top of layer it is in
FACT	1 or 2 depending on part of integration being performed
I	Index
II	Index
K	Index
KK	Index
KMIN	Index of layer in which minimum time refracted wave occurs
NE1	Index of layer containing the hypocenter
NE2	= NE1 + 1
NL	Number of layers in velocity model
NN	Number of parameters (= 2*NL-1)
P	Trial raypath (p^*)
P1	Lower limit of trial p (p_1)
P2	Upper limit of trial p (p_2)
PARM	Velocity-model parameters (velocities, then thicknesses)
PM	= PARM(NL+I)

SS	Total depth below surface of hypocenter
STHI	Sine of THI
SUM	Summing argument
T	tan ϕ^* (ϕ^* = takeoff angle)
T1	tan ϕ_{min} (defined in fig. 1)
T2	tan ϕ_{max} (defined in fig. 1)
TD	Direct-path travel time
THI	Takeoff angle (ϕ^*)
TMIN	Minimum travel time
U1	η_{min} (defined in fig. 1)
U2	η_{max} (defined in fig. 1)
U	η^* ($\eta_{min} < \eta^* < \eta_{max}$)
X	Event coordinates
X0	Station coordinates

Appendix 2. Listing of Fortran Program HYPOERR and Subroutines

Program HYPOERR contains comment cards identifying the purpose for each of the major sections. Besides the main program, there are three subroutines and one function. Principal variables and constants are listed in appendix 1. A flow diagram of HYPOERR is given in appendix 3. Input and output examples are developed in appendixes 4 and 5.

```
C*******************************************************************
C            PROGRAM NAME   HYPOERR
C  PURPOSE   TO CALCULATE COVARIANCE AND IGNORANCE MATRICES FOR
C  AN N-STATION SEISMIC ARRAY BASED ON S AND/OR P ARRIVAL TIMES.
C
C   SEE UHRHAMMER, B.S.S.A. 70 P 1369-1379, 1980, FOR DETAILS
C
C        AUTHOR         BARRY R. LIENERT   SEPT. 1982
C
C      SUBROUTINES CALLED
C         1. DIST  CONVERTS LAT/LONG TO KM
C         2. DTDX1  CALCULATES PARTIAL DERIVATIVES OF TRAVEL TIME
C                   FOR LAYERED MODELS
C         3. EIGEN  GETS EIGENVALUES AND EIGENVECTORS OF A REAL
C                   SYMMETRIC MATRIX
C
C      LOGICAL UNIT ASSIGNMENTS
C        5       INPUT DATA FILE
C        6       OUTPUT LISTING
C       25       GRIDDED OUTPUT DATA FOR CONTOURING
C
C*****************************************************************
C
C
C   DIMENSIONS ARE SET FOR A MAXIMUM OF 50 OBSERVATIONS AND
C   100 PARAMETERS
C
      PROGRAM HYPOERR(TAPE5,TAPE6,TAPE25,OUTPUT)
      DIMENSION XX0(50,3),X(3),X0(3),G(50,4),AA(200),EIG(4),U(2500)
     1,V(16),DX(4),BB(50,50),C(4,4),D(4,4),CV(4,4),DD(4,50),CO(50,50)
      DIMENSION PARM(100),PRM(100),XS0(50,3),TYPE2(20),IP(20),JP(20)
     +,CVV(20),EL(4),CVMAX(8),CVMIN(8),TITLE(8)
      COMMON/REF/REFLAT,REFLONG
C
C   READ INPUT FILE
C
C   PARAMETER FORMAT
C   (1) TYPE1 = "S" OR "P"   - NO PARMS = 2*NLAYER -1 WHERE PARM(I)
C       I=1,NLAYER ARE P OR S VELOCITIES AND PARM(I),I=NLAYER+1,
C       2*NLAYER-1 ARE LAYER THICKNESSES
C
C   (2) TYPE1 = "SR"          - NO PARMS = 2*NLAYER WHERE PARM(I),I=1,NLAYER
C        ARE P VELOCITIES AND PARM(2*NLAYER) IS THE P TO S VELOCITY RATIO
C
C   (3) TYPE1 = "SP"          - NO PARMS = 3*NLAYER -1 WHERE PARM(I),
C       I=2*NLAYER,3*NLAYER-1 ARE THE S VELOCITIES
C
C*************  SECTION 1  READ INPUT PARAMETERS  ************************
C
100   FORMAT(A3,2I2)
      READ(5,400)TITLE
400   FORMAT(8A5)
      WRITE(6,401)TITLE
401   FORMAT(1X,8A5,/)
      READ(5,100)TYPE1
```

```
      WRITE(6,115)TYPE1
115   FORMAT(1X,"TYPE1  ",A3)
      READ(5,*)NEL
         IF (NEL.GT.8) GO TO 153
      WRITE(6,116)NEL
116   FORMAT(1X,"NO OF ELEMENTS =",I3)
      DO 17 I=1,NEL
17    READ(5,100)TYPE2(I),IP(I),JP(I)
      DO 18 I=1,NEL
18    WRITE(6,100)TYPE2(I),IP(I),JP(I)
      READ(5,100)TYPE3
      READ(5,100)TYPE4
C
C   (1) TYPE4 = "DST"   -   COORDINATES IN DISTANCE UNITS USED FOR VELOCITIES
C
C   (2) TYPE4 = "LAT"   -   COORDINATES ARE LONGITUDE/LATITUDE IN FORMAT
C                           100*DEGREES+MINUTES
C
      READ(5,*)NLAYER
      READ(5,*)NSTAT
      WRITE(6,120)TYPE3,TYPE4,NLAYER,NSTAT
120   FORMAT(1X,"TYPE3  ",A3,
     +" TYPE4  ",A3,//,"NO OF LAYERS =",I3,5X,"NO OF STATIONS =",
     +I3,//,"PARAMETERS ")
      IF(TYPE1.EQ."SR ")GO TO 1
      IF(TYPE1.EQ."SP ")GO TO 2
      NPARM=2*NLAYER-1
C
C   N = NO OF OBSERVATIONS
C
      N=NSTAT
      GO TO 3
1     N=2*NSTAT
      NPARM=2*NLAYER
      GO TO 3
2     NPARM=3*NLAYER-1
      N=2*NSTAT
3     CONTINUE
      READ(5,*)(PARM(I),I=1,NPARM)
      WRITE(6,121)(PARM(I),I=1,NPARM)
121   FORMAT(1X,8E10.3)
C
C   VARIANCES
C
C   (1) TYPE3 = "S"   -   P AND/OR S ARRIVAL TIME VARIANCES ARE READ
C                         INDIVIDUALLY (ASSUMED THE SAME FOR ALL STATIONS)
C   (2) TYPE3 = "C"   -   COMPLETE COVARIANCE MATRIX IS READ IN
C
      DO 4 I=1,N
      DO 4 J=1,N
4     CO(I,J)=0.0
      IF(TYPE3.EQ."C  ")GO TO 6
      READ(5,*)VAR
      IF(N.GT.NSTAT)READ(5,*)VAR1
```

```
C
C       VAR = P OR S VARIANCE (DEPENDING ON TYPE1)
C           = P VARIANCE IF TYPE1="SR" OR "SP"
C
C       VAR1 = S VARIANCE (ONLY READ IF TYPE1="SR" OR "SP")
C
      IF(N.EQ.NSTAT)WRITE(6,123)TYPE1,VAR
123   FORMAT(1X,/,A3,"VARIANCE =",E10.3)
      IF(N.GT.NSTAT)WRITE(6,124)VAR1,VAR
124   FORMAT(1X,/,"S VARIANCE =",E10.3," P VARIANCE =",E10.3)
      DO 5 I=1,N
      CO(I,I)=VAR
C
C   I IS ODD FOR P ARRIVALS, EVEN FOR S
C
      IF(N.GT.NSTAT.AND.(I/2)*2.EQ.I)CO(I,I)=VAR1
5     CONTINUE
      GO TO 8
6     CONTINUE
      DO 7 I=1,N
7     READ(5,*)(CO(I,J),J=1,N)
      WRITE(6,122)
122   FORMAT(1X,/,10X,"COVARIANCE MATRIX",/)
      DO 15 I=1,N
      WRITE(6,121)(CO(I,J),J=1,N)
15    WRITE(6,100)
8     CONTINUE
      DO 9 I=1,NSTAT
9      READ(5,*)XS0(I,1),XS0(I,2)
      WRITE(6,125)
125   FORMAT(1X,/,"STATION COORDINATES ")
      DO 19 I=1,NSTAT
19    WRITE(6,126)XS0(I,1),XS0(I,2)
126   FORMAT(1X,2F12.3)
      READ(5,*)X1
      READ(5,*)X2
      READ(5,*)Y1
      READ(5,*)Y2
      READ(5,*)NX
      READ(5,*)NY
      WRITE(6,127)X1,X2,Y1,Y2,NX,NY
127   FORMAT(1X,/,"CONTOUR GRID PARAMETERS ",/,"X =",F12.3,"  TO",
     +F12.3,/,"Y =",F12.3,"  TO",F12.3,//,"NO OF X PTS =",I4,
     +5X,"NO OF Y PTS =",I4,/)
      READ(5,*)X(3)
      WRITE(6,130)X(3)
130   FORMAT(1X,"HYPOCENTER DEPTH =",F12.1," KM",/)
      IF(TYPE4.EQ."DST")GO TO 11
      REFLAT=Y1
      REFLONG=X1
      XS=0.0
      YS=0.0
      CALL DIST(X2,Y2,XU,YU)
      DO 10 I=1,NSTAT
10    CALL DIST(XS0(I,1),XS0(I,2),XX0(I,1),XX0(I,2))
```

```
      WRITE(6,128)
128   FORMAT(1X,"GRID PARAMETERS  IN KILOMETERS  ",/)
      WRITE(6,125)
      DO 20 I=1,NSTAT
20    WRITE(6,126)XX0(I,1),XX0(I,2)
      WRITE(6,100)
      WRITE(6,127)XS,XU,YS,YU,NX,NY
      GO TO 13
11    CONTINUE
      DO 12 I=1,NSTAT
      DO 12 J=1,2
12    XX0(I,J)=XS0(I,J)
      XS=X1
      YS=Y1
      XU=X2
      YU=Y2
13    CONTINUE
      XINC=(XU-XS)/FLOAT(NX-1)
      YINC=(YU-YS)/FLOAT(NY-1)
      WRITE(6,129)XINC,YINC
      WRITE(6,104)
104   FORMAT(1X,/,12X,"VELOCITY MODEL",/)
      WRITE(6,105)
105   FORMAT(8X,"LAYER",7X,"DEPTH",2X,"P VELOCITY",2X,
     +"S VELOCITY")
      SUM=0.0
      M2=NLAYER-1
      DO 21 I=1,M2
      IF(TYPE1.EQ."S   ")WRITE(6,107)I,SUM,PARM(I)
      IF(TYPE1.EQ."P   ")WRITE(6,106)I,SUM,PARM(I)
      IF(TYPE1.EQ."SR  ")WRITE(6,106)I,SUM,
     +PARM(I),PARM(I)/PARM(2*NLAYER)
      IF(TYPE1.EQ."SP  ")WRITE(6,106)I,SUM,PARM(I),
     +PARM(2*NLAYER+I-1)
      SUM=SUM+PARM(NLAYER+I)
21    CONTINUE
106   FORMAT(1X,I12,3F12.3)
107   FORMAT(1X,I12,F12.3,12X,F12.3)
      IF(TYPE1.EQ."P   ")WRITE(6,108)NLAYER,PARM(NLAYER)
      IF(TYPE1.EQ."S   ")WRITE(6,109)NLAYER,PARM(NLAYER)
      IF(TYPE1.EQ."SR  ")WRITE(6,108)NLAYER,PARM(NLAYER),
     +PARM(NLAYER)/PARM(2*NLAYER)
      IF(TYPE1.EQ."SP  ")WRITE(6,108)NLAYER,PARM(NLAYER),
     +PARM(3*NLAYER-1)
108   FORMAT(1X,I12,12X,2F12.3)
109   FORMAT(1X,I12,24X,F12.3)
C
C   CHANGE TO RIGHT-HANDED COORDINATES (X POSITIVE TO THE LEFT)
C
      XL=XU-XS
      XSS=XS
      XS=XL-XU
      XU=XL-XSS
```

```
C
C   FIND LAYER NE1 CONTAINING HYPOCENTER
C
      SUM=0.0
      M2=NLAYER-1
      DO 35 I=1,M2
      SUM=SUM+PARM(NLAYER+I)
      IF(SUM.GT.X(3))GO TO 36
35    CONTINUE
      I=NLAYER
36    NE1=I
      DO 16 I=1,NSTAT
16    XX0(I,1)=XL-XX0(I,1)
129   FORMAT(1X,"XINC =",F8.3,5X,"YINC =",F8.3,/)
      KKK=1
      IF(N.GT.NSTAT)KKK=2
C
C     WRITE HEADINGS FOR GRIDDED OUTPUT ON FILE 25
C
      WRITE(25,902)(TYPE2(K),K=1,NEL)
 902  FORMAT(5X,"X",7X,"Y",7X,10(A3,5X))
      WRITE(25,903)(IP(K),JP(K),K=1,NEL)
 903  FORMAT(20X,10(2I2,4X))
      WRITE(25,904)
  904 FORMAT(1H )
C
C
C******************************************************************
C
C      START OF THE GRID CALCULATION LOOP
C
      DO 93 IX=1,NX
      DO 93 IY=1,NY
C
C*********  SECTION 2  GET TIME/DIST DERIVATIVES AND FORM GT * G  *******
C
      II=1
      X(1)=XS+FLOAT(IX-1)*XINC
      X(2)=YS+FLOAT(IY-1)*YINC
      DO 50 I=1,NSTAT
      DO 23 K=1,3
23    X0(K)=XX0(I,K)
C
C  GET TIME/DIST DERIVATIVES USING DTDX AND STORE THEM IN G(I,J)
C
C   JJ = 1   -   P OR S ARRIVALS
C   JJ = 2   -   S ARRIVALS
C
      DO 50 JJ=1,KKK
      NN=2*NLAYER-1
      DO 27 K=1,NN
      PRM(K)=PARM(K)
27    CONTINUE
      IF(JJ.EQ.1)GO TO 30
```

```
      DO 25 K=1,NLAYER
      IF(TYPE1.EQ."SR ")GO TO 24
      PRM(K)=PARM(2*NLAYER+K-1)
      GO TO 25
24    PRM(K)=PARM(K)/PARM(2*NLAYER)
25    CONTINUE
30    CALL DTDX1(NN,PRM,X,X0,DX,TMIN)
      DX(4)=1.0
      DO 40 J=1,4
40    G(II,J)=DX(J)
      II=II+1
50    CONTINUE
C
C    FORM AA = GT * G
C
C
      IC=1
      DO 58 J=1,4
      DO 58 K=1,J
      SUM=0.0
      DO 57 KK=1,N
57     SUM=SUM+G(KK,J)*G(KK,K)
      AA(IC)=SUM
58     IC=IC+1
C
C*********   SECTION 3  FORM SVD INVERSE AND CALCULATE MATRIX      *******
C                        ELEMENTS
C
C
C  GET EIGENVECTORS V, AND EIGENVALUES (STORED IN DIAGONAL OF AA)
C
      CALL EIGEN(AA,V,4,0)
      II=1
      DO 56 I=1,4
      IF(AA(II).GT.0.0)GO TO 53
      DO 51 J=1,4
      JJ=(I-1)*4+J
51    V(JJ)=-V(JJ)
53    EIG(I)=SQRT(ABS(AA(II)))
56     II=II+I+1
C
C  CALCULATE EIGENVECTORS U(I,J) = V(I,K) * G(J,K) / EIG(I).....EQ. (3)
C
      DO 89 I=1,4
      DO 89 J=1,N
      SUM=0.0
      DO 79 K=1,4
      II=(I-1)*4+K
79    SUM=SUM+V(II)*G(J,K)
      JJ=(I-1)*N+J
89    U(JJ)=SUM/EIG(I)
```

```
C
C   CALCULATE COVARIANCE AND/OR IGNORANCE MATRICES
C
      DO 71 I=1,N
      DO 71 J=1,4
      DD(J,I)=0.0
      DO 71 K=1,N
      JJ=(J-1)*N+K
71    DD(J,I)=DD(J,I)+CO(I,K)*U(JJ)/EIG(J)
C
C   C(I,J) = UT / EIG(I) * CO(I,J) * U / EIG(J).......EQ. (7)
C
      DO 73 I=1,4
      DO 73 J=1,4
      C(I,J)=0.0
      DO 73 K=1,N
      II=(I-1)*N+K
73    C(I,J)=C(I,J)+U(II)*DD(J,K)/EIG(I)
      IF1=0
      IF2=0
      IF3=0
      DO 92 IN=1,NEL
      IF(IN.GT.8)GO TO 92
      IF(TYPE2(IN).EQ."IGN")GO TO 85
      IF(TYPE2(IN).EQ."INF")GO TO 81
      IF1=IF1+1
      IF(IF1.GT.1)GO TO 75
      DO 76 I=1,4
      DO 76 J=1,4
      D(I,J)=0.0
      DO 76 K=1,4
      JJ=(K-1)*4+J
76    D(I,J)=D(I,J)+C(I,K)*V(JJ)
      DO 74 I=1,4
      DO 74 J=1,4
      CV(I,J)=0.0
      DO 74 K=1,4
      II=(K-1)*4+I
C
C   COVARIANCE MATRIX
C  CV(I,J) = V * C(I,J) * VT.......EQ. (6)
C
74    CV(I,J)=CV(I,J)+V(II)*D(K,J)
75    CONTINUE
      I1=IP(IN)
      J1=JP(IN)
      KK=0
      GO TO 66
64    J1=IP(IN)
      GO TO 66
65    I1=JP(IN)
      J1=JP(IN)
66    KK=KK+1
```

```
      IF(TYPE2(IN).EQ."UNC".OR.TYPE2(IN).EQ."UCR")EL(KK)=CV(I1,J1)
      IF(TYPE2(IN).EQ."IGN".OR.TYPE2(IN).EQ."ICR")EL(KK)=BB(I1,J1)
      IF(TYPE2(IN).EQ."UXY")EL(KK)=CV(1,1)+CV(2,2)
      IF(TYPE2(IN).EQ."SMA")EL(KK)=C(1,1)+C(2,2)+C(3,3)+C(4,4)
      IF(TYPE2(IN).EQ."CND")CVV(IN)=ALOG10(EIG(1)/EIG(4))
      IF(TYPE2(IN).EQ."INF")CVV(IN)=BB(I1,J1)
      IF(TYPE2(IN).EQ."INF")GO TO 69
      IF(TYPE2(IN).EQ."CND")GO TO 69
      IF(IP(IN).EQ.JP(IN).AND.TYPE2(IN).NE."ICR".AND.
     +    TYPE2(IN).NE."UCR")GO TO 68
      GO TO (64,65,67)KK
67    CVV(IN)=EL(1)/(SQRT(EL(2)*EL(3)))
      GO TO 69
68    CVV(IN)=SQRT(EL(1))
200   FORMAT(10F8.3)
69    CONTINUE
      GO TO 92
85    CONTINUE
C
C  CALCULATE IGNORANCE MATRIX BB(I,J)
C
      IF2=IF2+1
      IF(IF2.GT.1)GO TO 75
      DO 83 I=1,4
      DO 83 J=1,N
      DD(I,J)=0.0
      DO 83 K=1,4
      JJ=(K-1)*N+J
83    DD(I,J)=DD(I,J)+C(I,K)*U(JJ)
      DO 84 I=1,N
      DO 84 J=1,N
      BB(I,J)=0.0
      DO 84 K=1,4
      II=(K-1)*N+I
C
C       IGNORANCE MATRIX
C      BB(I,J) = U * C(I,J) * UT..........EQ. (11)
C
84    BB(I,J)=BB(I,J)+U(II)*DD(K,J)
      IF3=0
      GO TO 75
C
C     INFORMATION MATRIX = U * UT
C
81    IF3=IF3+1
      IF(IF3.GT.1)GO TO 75
      DO 82 I=1,N
      DO 82 J=1,N
      BB(I,J)=0.0
      DO 82 K=1,4
      II=(K-1)*N+I
      JJ=(K-1)*N+J
82    BB(I,J)=BB(I,J)+U(II)*U(JJ)
      IF2=0
      GO TO 75
```

```
92     CONTINUE
C
C   MAXIMUM ALLOWABLE OUTPUT VALUE IS 9999.999
C
       DO 91 K=1,NEL
91     IF(CVV(K).GT.9999.999)CVV(I)=9999.999
C
C   OUTPUT RESULTS IN LEFT-HANDED COORDINATES
C
       X(1)=XL-X(1)
       WRITE(25,200)X(1),X(2),(CVV(K),K=1,NEL)
       DO 86 I=1,NEL
       IF(IX.NE.1.OR.IY.NE.1)GO TO 87
       CVMIN(I)=CVV(I)
       CVMAX(I)=CVV(I)
87     IF(CVV(I).GT.CVMAX(I))CVMAX(I)=CVV(I)
       IF(CVV(I).LT.CVMIN(I))CVMIN(I)=CVV(I)
86     CONTINUE
93     CONTINUE
C
C***************  END OF GRID CALCULATION LOOP  *******************
C
       WRITE(6,150)
150    FORMAT(1X,//,"ELEMENT",6X,"MAXIMUM",3X,"MINIMUM",/)
       DO 88 I=1,NEL
88     WRITE(6,151) TYPE2(I),IP(I),JP(I),CVMAX(I),CVMIN(I)
151    FORMAT(1X,A3,2I2,2F10.3)
14     CONTINUE
       WRITE(6,152)
 152   FORMAT(//)
       GO TO 155
 153   WRITE(6,154) NEL
 154   FORMAT(1X,"NOS OF ELEMENTS =",I3,3X,"IS TOO MANY  FOR  FORMAT")
 155   STOP
       END
C*********************************************************************
C
C   SUBROUTINE TO CALCULATE TRAVEL TIME AND DERIVATIVES FOR A ONE-
C   DIMENSIONAL NL-LAYERED MODEL. FOLLOWS THE METHOD DESCRIBED
C   IN LEE AND STEWART (1981).
C
C       BARRY R. LIENERT    OCTOBER,1982
C
C   INPUTS      X    EVENT COORDINATES
C               X0   STATION COORDINATES-ASSUMES X0(3)=0
C               NN   NUMBER OF PARAMETERS (=2*NL-1)
C               TMIN MINIMUM TRAVEL TIME
C               DX   SPATIAL DERIVATIVES OF TMIN
C               PARM(I),I=1,NL   LAYER VELOCITIES
C               PARM(I),I=NL+1,NN  LAYER THICKNESSES
C
C          SUBROUTINES CALLED
C               DEL  CALCULATES HORIZONTAL OFFSET FOR DIRECT PATH
C
C*********************************************************************
```

```
C
      SUBROUTINE DTDX1(NN,PARM,X,X0,DX,TMIN)
      DIMENSION X(*),X0(*),DX(*),PARM(*),DCRIT(50)
      IF(ABS(X(3)).LT.0.001)X(3)=0.0
      STHI=0.0
      CTHI=1.0
      DX(1)=0.0
      DX(2)=0.0
C
C     NL=NO OF LAYERS
C
      NL=(NN+1)/2
C
C    FIND LAYER, NE1, THAT EVENT IS IN
C
      SUM=0.0
      M2=NL-1
      DO 1 I=1,M2
      SS=PARM(NL+I)
      SUM=SUM+SS
      IF(X(3)-SUM)2,2,1
1     CONTINUE
      SS=0.0
      I=NL
2     NE1=I
C
C     ETA1 IS THE DEPTH TO THE EVENT FROM THE TOP OF THIS LAYER
C
      ETA1=X(3)-SUM+SS
      IF(ETA1.GE.0.05)GO TO 18
      IF(NE1.EQ.1)GO TO 18
      NE1=NE1-1
      ETA1=PARM(NL+NE1)+ETA1*PARM(NE1+1)/PARM(NE1)
18    CONTINUE
      DELTA=SQRT((X(1)-X0(1))**2+(X(2)-X0(2))**2)
      DIST=SQRT(DELTA**2+X(3)**2)
      IF(NL.GT.1)GO TO 15
      TMIN=DIST/PARM(1)
      IF(DIST.EQ.0.0)GO TO 16
      DX(1)=(X(1)-X0(1))/(DIST*PARM(1))
      DX(2)=(X(2)-X0(2))/(DIST*PARM(1))
      DX(3)=X(3)/(DIST*PARM(1))
      RETURN
16    DO 17 I=1,3
17    DX(I)=1.0/PARM(1)
      RETURN
15    CONTINUE
      KMIN=0
      TMIN=1.E22
      NE2=NE1+1
      IF(DELTA.EQ.0.0)GO TO 24
      IF(NE1.EQ.NL)GO TO 65
```

```
C
C    FIND CRITICAL DISTANCES, DCRIT(K), FOR REFRACTED WAVES. IF NO
C    REFRACTED WAVE IS POSSIBLE, SET DCRIT(K)=-1.0
C
      DO 50 KK=NE2,NL
      K=NL-KK+NE2
      SUM=0.0
      M2=K-1
      DO 40 I=1,M2
      IF(PARM(K).LE.PARM(I))GO TO 49
      FACT=1.0
      IF(I.GE.NE1)FACT=2.0
40    SUM=SUM+PARM(NL+I)*FACT/SQRT((PARM(K)/PARM(I))**2-1.)
      IF(PARM(K).LE.PARM(NE1))GO TO 49
      DCRIT(K)=SUM-ETA1/SQRT((PARM(K)/PARM(NE1))**2-1.)
      IF(DCRIT(K).GT.DELTA)GO TO 49
      GO TO 50
49    DCRIT(K)=-1.0
50    CONTINUE
C
C    FIND THE TRAVEL TIMES, T, FOR ALL POSSIBLE REFRACTED WAVES
C
      DO 60 K=NE2,NL
      IF(DCRIT(K).LT.0.0)GO TO 60
      SUM=0.0
      M2=K-1
      DO 52 I=1,M2
      FACT=1.0
      IF(I.GE.NE1)FACT=2.0
      E=PARM(NL+I)*FACT
52    SUM=SUM+E*SQRT(1./(PARM(I)*PARM(I))-1./(PARM(K)*PARM(K)))
      IF(PARM(NE1).GT.PARM(K))GO TO 60
      T=DELTA/PARM(K)-ETA1*SQRT(1./(PARM(NE1)*PARM(NE1))
     1-1./(PARM(K)*PARM(K)))+SUM
C
C    IF T ¼ TMIN, SET TMIN = T
C
      IF(T.GE.TMIN)GO TO 60
      KMIN=K
      TMIN=T
60    CONTINUE
      IF(X(3).EQ.0.0)GO TO 36
      IF(KMIN.EQ.0)GO TO 65
      K=KMIN
      T1=X(3)/DELTA
      P=1./(PARM(NE1)*SQRT(1.+T1*T1))
      TD=0.0
      DO 61 I=1,NE1
      PM=PARM(NL+I)
      IF(I.EQ.NE1)PM=ETA1
61    TD=TD+PM/(PARM(I)*SQRT(1.-P*P*PARM(I)*PARM(I)))
      IF(TD.LT.TMIN)GO TO 65
```

```
C
C   FIND SPATIAL DERIVATIVES OF TMIN FOR REFRACTED PATH
C
62    K=KMIN
      DX(1)=(X(1)-X0(1))/(DELTA*PARM(K))
      DX(2)=(X(2)-X0(2))/(DELTA*PARM(K))
      IF(PARM(K).LT.PARM(NE1))WRITE(6,500)NE1,K,X(3),PARM(NE1),PARM(K)
500   FORMAT(2I8,3F12.4)
      DX(3)=-SQRT(1./(PARM(NE1)*PARM(NE1))-1./(PARM(K)*PARM(K)))
      RETURN
65    CONTINUE
      IF(DELTA.LT.0.01)GO TO 24
C
C   NOW FIND THE DIRECT PATH TRAVEL TIME, TD
C
C
C    FIND TANGENTS OF MAXIMUM AND MINIMUM TAKEOFF ANGLES,T1 AND T2
C
      IF(NE1.NE.1)GO TO 39
      IF(X(3).EQ.0.0)GO TO 36
      TD=DIST/PARM(NE1)
      THI=ATAN(DELTA/X(3))
      CTHI=COS(THI)
      STHI=SIN(THI)
      GO TO 37
39    CONTINUE
      T1=DELTA/X(3)
      T2=DELTA/ETA1
      U1=ETA1*T1
      U2=DELTA
C
C   FIND RAYPATH PARAMETERS, P1 AND P2 FOR DIRECT WAVES
C
      P1=1./(SQRT(1.+1./(T1*T1))*PARM(NE1))
      P2=1./(SQRT(1.+1./(T2*T2))*PARM(NE1))
      P=P1
      STHI=P1*PARM(NE1)
      CTHI=SQRT(1.-STHI*STHI)
C
C   FIND CORRESPONDING DISTANCES, DL1 AND DL2 FOR THE 2 RAYS
C
      DL1=DEL(P1,ETA1,NE1,NN,PARM)
      IF(ABS(DL1-DELTA).LE.0.01)GO TO 30
41    IF(DL1.LE.DELTA)GO TO 43
      T1=TAN(2.*ATAN(T1)-ATAN(DL1/X(3)))
      P1=1./(SQRT(1.+1./(T1*T1))*PARM(NE1))
      DL1=DEL(P1,ETA1,NE1,NN,PARM)
      GO TO 41
43    CONTINUE
      DL2=DEL(P2,ETA1,NE1,NN,PARM)
      IF(DL1.EQ.DL2)GO TO 30
      U1=ETA1*T1
      U2=DELTA
```

```
C
C    NOW FIND TAKEOFF ANGLE THI AND DISTANCE DELTA1 OF A RAY P WHICH
C    LIES BETWEEN P1 AND P2
C
      II=0
10    U=U1+(U2-U1)*(DELTA-DL1)/(DL2-DL1)
      II=II+1
      P=U/(PARM(NE1)*SQRT(U*U+ETA1*ETA1))
      STHI=P*PARM(NE1)
      CTHI=SQRT(1.0-STHI*STHI)
      DL=DEL(P,ETA1,NE1,NN,PARM)
C
C    ITERATE UNTIL ABS(DELTA-DL)¼0.01
C
      IF(ABS(DL-DELTA).LE.0.01)GO TO 30
      IF(II.GT.50)GO TO 26
      IF(DL.GT.DELTA)GO TO 20
      DL1=DL
      U1=U
      GO TO 10
20    DL2=DL
      U2=U
      GO TO 10
26    WRITE(6,100)
      WRITE(6,101)DELTA,DL
100   FORMAT(" NO DIRECT PATH CONVERGENCE AFTER 50 ITERATIONS")
101   FORMAT(" DELTA = ",F12.4," DL = ",F12.4)
      GO TO 30
24    P=0.0
30    CONTINUE
C
C    CALCULATE THE TRAVEL TIME,TD,FOR THE DIRECT PATH
C
      TD=0.0
      DO 35 I=1,NE1
      PM=PARM(NL+I)
      IF(I.EQ.NE1)PM=ETA1
      IF(PARM(I)*P.GE.1.0)GO TO 38
35    TD=TD+PM/(PARM(I)*SQRT(1.-P*P*PARM(I)*PARM(I)))
      GO TO 37
38    TD=9999.99
      GO TO 37
36    TD=DELTA/PARM(1)
      CTHI=0.0
      STHI=1.0
C
C     SET MINIMUM TRAVEL TIME TO TD
C
37    IF(TD.GT.TMIN)GO TO 62
      TMIN=TD
      CTH2=ABS(1.-(P*PARM(1))**2)
```

```
C
C   DIRECT PATH DERIVATIVES
C
      IF(DELTA.EQ.0.0)GO TO 69
      DX(1)=(X(1)-X0(1))*STHI/(DELTA*PARM(NE1))
      DX(2)=(X(2)-X0(2))*STHI/(DELTA*PARM(NE1))
69    DX(3)=CTHI/PARM(NE1)
      IF(X(3).EQ.0.0.AND.DELTA.EQ.0.0) GO TO 70
      RETURN
70    DX(1)=DX(3)
      DX(2)=DX(3)
      RETURN
      END
C*******************************************************************
C
C   SUBROUTINE TO CALCULATE THE DIRECT PATH HORIZONTAL DISPLACEMENT FOR AN
C   N-LAYERED ONE-DIMENSIONAL MODEL
C
C      BARRY R. LIENERT    OCTOBER, 1982
C
C   INPUTS     P        RAYPATH PARAMETER = SIN(THETA)/VELOCITY
C              ETA1     DEPTH TO EVENT FROM THE TOP OF THE LAYER
C                       IT IS IN
C              N        LAYER EVENT IS IN
C              NN       NO OF PARAMETERS
C              PARM(I) I=1,NN   LAYER VELOCITIES, THEN THICKNESSES
C
C*******************************************************************
      FUNCTION DEL(P,ETA,N,NN,PARM)
      DIMENSION PARM(1)
      NL=(NN+1)/2
      DEL=0.0
      DO 10 I=1,N
      E=PARM(NL+I)
      IF(I.EQ.N) E=ETA
      IF(PARM(I)*P.GE.1.0) GO TO 11
10    DEL=DEL+E/SQRT(1.0/((PARM(I))*P)**2-1.0)
      RETURN
11    DEL=1000
      RETURN
      END
C*************************************************************************
C
C   SUBROUTINE TO CONVERT LATITUDE/LONGITUDE TO DISTANCE IN KM.
C   FROM A REFERENCE POINT (REFLONG,REFLAT)
C
C      INPUT FORMAT IS (DEGREES * 100 + MINUTES)
C      E.G.  22 DEG 45.46 MIN = 2245.46
C
C      X = LONGITUDE (INPUT)
C      XD= X COORDINATE IN KM (OUTPUT)
C      Y = LATITUDE (INPUT)
C      YD= Y COORDINATE IN KM (OUTPUT)
C
C            BARRY R. LIENERT   NOV 1982
```

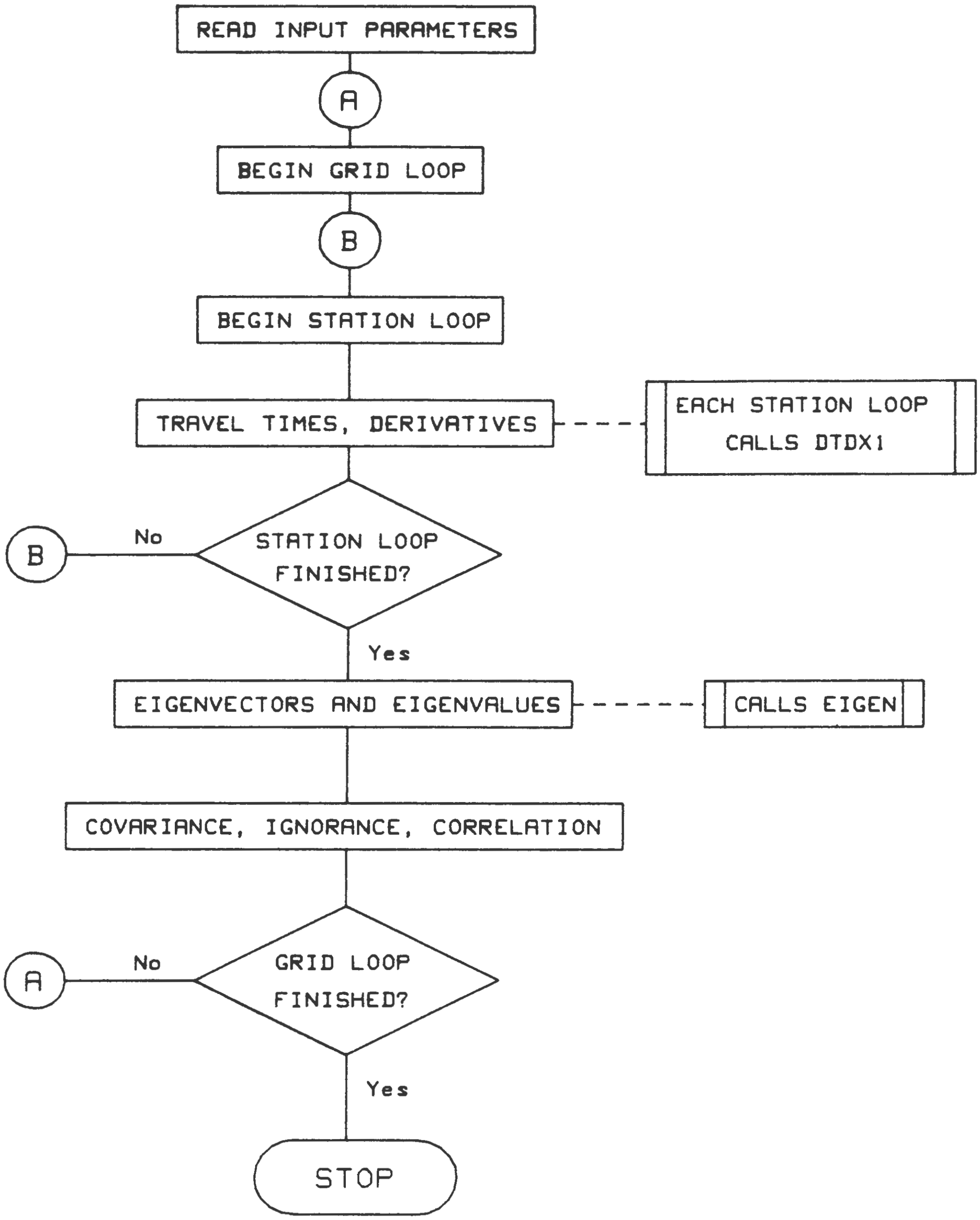
```
C
C***********************************************************************
      SUBROUTINE DIST(X,Y,XD,YD)
      COMMON/REF/REFLAT,REFLONG
      CV(ARG)=ARG/60.-2.*FLOAT(INT(ARG/100.+0.00001))/3.
      AVLAT=(CV(REFLAT)+CV(Y))/2.
      XD=-111.324*(CV(X)-CV(REFLONG))*COS(0.0174533*AVLAT)
      YD=110.949*(CV(Y)-CV(REFLAT))
      RETURN
      END
C
C
C        SUBROUTINE EIGEN
C
C        PURPOSE
C           COMPUTE EIGENVALUES AND EIGENVECTORS OF A REAL SYMMETRIC
C           MATRIX
C
C        USAGE
C           CALL EIGEN(A,R,N,MV)
C
C        DESCRIPTION OF PARAMETERS
C           A - ORIGINAL MATRIX (SYMMETRIC), DESTROYED IN COMPUTATION.
C               RESULTANT EIGENVALUES ARE DEVELOPED IN DIAGONAL OF
C               MATRIX A IN DESCENDING ORDER.
C           R - RESULTANT MATRIX OF EIGENVECTORS (STORED COLUMNWISE,
C               IN SAME SEQUENCE AS EIGENVALUES)
C           N - ORDER OF MATRICES A AND R
C           MV- INPUT CODE
C                   0   COMPUTE EIGENVALUES AND EIGENVECTORS
C                   1   COMPUTE EIGENVALUES ONLY (R NEED NOT BE
C                       DIMENSIONED BUT MUST STILL APPEAR IN CALLING
C                       SEQUENCE)
C
C        REMARKS
C           ORIGINAL MATRIX A MUST BE REAL SYMMETRIC (STORAGE MODE=1)
C           MATRIX A CANNOT BE IN THE SAME LOCATION AS MATRIX R
C
C        SUBROUTINES AND FUNCTION SUBPROGRAMS REQUIRED
C           NONE
C
C        METHOD
C           DIAGONALIZATION METHOD ORIGINATED BY JACOBI AND ADAPTED
C           BY VON NEUMANN FOR LARGE COMPUTERS AS FOUND IN "MATHEMATICAL
C           METHODS FOR DIGITAL COMPUTERS", EDITED BY A. RALSTON AND
C           H.S. WILF, JOHN WILEY AND SONS, NEW YORK, 1962, CHAPTER 7
C
C     ..................................................................
C
      SUBROUTINE EIGEN(A,R,N,MV)
      DIMENSION A(1),R(1)
C
C        ..............................................................
```

```
C
C        IF A DOUBLE PRECISION VERSION OF THIS ROUTINE IS DESIRED, THE
C        C IN COLUMN 1 SHOULD BE REMOVED FROM THE DOUBLE PRECISION
C        STATEMENT WHICH FOLLOWS.
C
C     DOUBLE PRECISION A,R,ANORM,ANRMX,THR,X,Y,SINX,SINX2,COSX,
C    1                 COSX2,SINCS,RANGE
C
C        THE C MUST ALSO BE REMOVED FROM DOUBLE PRECISION STATEMENTS
C        APPEARING IN OTHER ROUTINES USED IN CONJUNCTION WITH THIS
C        ROUTINE.
C
C        THE DOUBLE PRECISION VERSION OF THIS SUBROUTINE MUST ALSO
C        CONTAIN DOUBLE PRECISION FORTRAN FUNCTIONS.  SQRT IN STATEMENTS
C        40, 68, 75, AND 78 MUST BE CHANGED TO DSQRT.  ABS IN STATEMENT
C        62 MUST BE CHANGED TO DABS. THE CONSTANT IN STATEMENT 5 SHOULD
C        BE CHANGED TO 1.0D-12.
C
C        ..............................................................
C
C        GENERATE IDENTITY MATRIX
C
    5 RANGE=1.0E-7
      IF(MV-1) 10,25,10
   10 IQ=-N
      DO 20 J=1,N
      IQ=IQ+N
      DO 20 I=1,N
      IJ=IQ+I
      R(IJ)=0.0
      IF(I-J) 20,15,20
   15 R(IJ)=1.0
   20 CONTINUE
C
C        COMPUTE INITIAL AND FINAL NORMS (ANORM AND ANORMX)
C
   25 ANORM=0.0
      DO 35 I=1,N
      DO 35 J=I,N
      IF(I-J) 30,35,30
   30 IA=I+(J*J-J)/2
      ANORM=ANORM+A(IA)*A(IA)
   35 CONTINUE
      IF(ANORM) 165,165,40
40    ANORM=1.414*SQRT(ANORM)
      ANRMX=ANORM*RANGE/FLOAT(N)
C
C        INITIALIZE INDICATORS AND COMPUTE THRESHOLD, THR
C
      IND=0
      THR=ANORM
   45 THR=THR/FLOAT(N)
   50 L=1
   55 M=L+1
```

```
C
C         COMPUTE SIN AND COS
C
   60 MQ=(M*M-M)/2
      LQ=(L*L-L)/2
      LM=L+MQ
   62 IF( ABS(A(LM))-THR) 130,65,65
   65 IND=1
      LL=L+LQ
      MM=M+MQ
      X=0.5*(A(LL)-A(MM))
   68 Y=-A(LM)/ SQRT(A(LM)*A(LM)+X*X)
      IF(X) 70,75,75
   70 Y=-Y
75    SINX=Y/ SQRT(2.0*(1.0+( SQRT(ABS(1.0-Y*Y)))))
      SINX2=SINX*SINX
   78 COSX= SQRT(1.0-SINX2)
      COSX2=COSX*COSX
      SINCS =SINX*COSX
C
C         ROTATE L AND M COLUMNS
C
      ILQ=N*(L-1)
      IMQ=N*(M-1)
      DO 125 I=1,N
      IQ=(I*I-I)/2
      IF(I-L) 80,115,80
   80 IF(I-M) 85,115,90
   85 IM=I+MQ
      GO TO 95
   90 IM=M+IQ
   95 IF(I-L) 100,105,105
  100 IL=I+LQ
      GO TO 110
  105 IL=L+IQ
  110 X=A(IL)*COSX-A(IM)*SINX
      A(IM)=A(IL)*SINX+A(IM)*COSX
      A(IL)=X
  115 IF(MV-1) 120,125,120
  120 ILR=ILQ+I
      IMR=IMQ+I
      X=R(ILR)*COSX-R(IMR)*SINX
      R(IMR)=R(ILR)*SINX+R(IMR)*COSX
      R(ILR)=X
  125 CONTINUE
      X=2.0*A(LM)*SINCS
      Y=A(LL)*COSX2+A(MM)*SINX2-X
      X=A(LL)*SINX2+A(MM)*COSX2+X
      A(LM)=(A(LL)-A(MM))*SINCS+A(LM)*(COSX2-SINX2)
      A(LL)=Y
      A(MM)=X
```

```
C
C         TESTS FOR COMPLETION
C
C         TEST FOR M = LAST COLUMN
C
  130 IF(M-N) 135,140,135
  135 M=M+1
      GO TO 60
C
C         TEST FOR L = SECOND FROM LAST COLUMN
C
  140 IF(L-(N-1)) 145,150,145
  145 L=L+1
      GO TO 55
  150 IF(IND-1) 160,155,160
  155 IND=0
      GO TO 50
C
C         COMPARE THRESHOLD WITH FINAL NORM
C
  160 IF(THR-ANRMX) 165,165,45
C
C         SORT EIGENVALUES AND EIGENVECTORS
C
  165 IQ=-N
      DO 185 I=1,N
      IQ=IQ+N
      LL=I+(I*I-I)/2
      JQ=N*(I-2)
      DO 185 J=I,N
      JQ=JQ+N
      MM=J+(J*J-J)/2
      IF(A(LL)-A(MM)) 170,185,185
  170 X=A(LL)
      A(LL)=A(MM)
      A(MM)=X
      IF(MV-1) 175,185,175
  175 DO 180 K=1,N
      ILR=IQ+K
      IMR=JQ+K
      X=R(ILR)
      R(ILR)=R(IMR)
180   R(IMR)=X
185   CONTINUE
      RETURN
      END
```

Appendix 3. Generalized Flow Diagram of Program HYPOERR



Appendix 4. Input Records and Descriptions for Two Models

Format for Input Cards for Program HYPOERR
[See appendix 1 for further identification of constants and variables]

Card			
1	TITLE		FORMAT(8A5)
2	TYPE1	Seismic Phase (P and/or S)	FORMAT(A3)
3	NEL	Number of elements to be plotted	FREE FORMAT
4 . . .	TYPE2(I), IP(I), JP(J)	Element identifiers (see table 1)	FORMAT(A3,2I2)
5	TYPE3	Choice of variances or covariance matrix	FORMAT(A3)
6	TYPE4	Coordinate units	FORMAT(A3)
7	NLAYER	Number of velocity layers	FREE FORMAT
8	NSTAT	Number of seismic stations	FREE FORMAT
9 . . .	PARM(I)	P and/or S velocities; velocity model thicknesses; P:S velocity ratio(if required)	FREE FORMAT
10	VAR	Arrival time P variance	FREE FORMAT
11	VAR or CO(I,J)	Arrival time S variance Arrival-time covariance matrix (where I=J=2*NSTAT)	FREE FORMAT FREE FORMAT
12	XSO(I,1), XSO(I,2)	X and Y coordinates of station (where I is station number)	FREE FORMAT
13	X1	W coordinate of hypocenters (output grid)	FREE FORMAT
14	X2	E coordinate of hypocenters (output grid)	FREE FORMAT
15	Y1	S coordinate of hypocenters (output grid)	FREE FORMAT
16	Y2	N coordinate of hypocenters (output grid)	FREE FORMAT
17	NX	Number of x divisions of grid	FREE FORMAT
18	NY	Number of y divisions of grid	FREE FORMAT
19	X(3)	Hypocenter depth	FREE FORMAT

Two models were run for program HYPOERR to illustrate the variety of options available and to test their reliability. Input records are listed below. Corresponding output and discussion are given in appendix 5.

Model 1. Study of Quadrapartite Network (10-km radius)

The first test case is a quadrapartite array presented by Uhrhammer (1980). Three stations are at the vertexes of an equilateral triangle, each 10 km from the centrally located fourth station. (See crossed symbols in figs. 2-6.) Location coordinates are given in units of km (TYPE4 = DST). The velocity model is a simple half space with P and S velocity read in separately (TYPE1 = SP). P velocity = 5.6 km/sec and S velocity = 3.3 km/sec (fig. 1 illustrates the general velocity model with layered structure). Hypocenters for this model are assumed to be at a depth of 10 km extending over a 50-km square grid at x and y increments of 2.5-km spacing. The variance for P and S arrival times is specified (TYPE3 = S) with the variance of a single observation assumed to be .0025.

The first six elements listed below were computed to match Uhrhammer's test cases and an extra seventh element (condition number) was computed as a special case:

(1) TYPE2=UNC 1 1. Uncertainty of the x coordinate of the epicenter (fig. 2).

(2) TYPE2=UNC 3 3. Uncertainty of the z coordinate (depth) of the hypocenter (fig. 3).

(3) TYPE2=UCR 1 3. Linear correlation coefficient (ρ_{xz}) between the x and z values (fig. 4).

(4) TYPE2=IGN 3 3. Ignorance of the P observation at station 2 (fig. 5).

(5) TYPE2=IGN 4 4. Ignorance of the S observation at station 2 (fig. 6).

(6) TYPE2=ICR 4 3. Linear correlation between the P and S observations at station 2 (fig. 7).

(7) TYPE2=CND. Logarithm of the ratio of maximum to minimum eigenvalues of the matrix G (fig. 8).

Table 2 is a copy of the input records used to generate the quadrapartite array (model 1) discussed above. Appendix 5 discusses the output for this model.

Table 2. Input records used in generating the output of model 1
[Descriptions on the right are not part of the input]

QUAD ARRAY	TITLE
SP	TYPE1
7	NOS OF ELEMENTS
UNC 1 1	
UNC 3 3	
UCR 1 3	
IGN 3 3	ELEMENTS (TYPE2)
IGN 4 4	
ICR 4 3	
CND	
S	TYPE3
DST	TYPE4
1	NOS OF LAYERS IN VELOCITY MODEL
4	NOS OF STATIONS
5.6 3.3	MODEL PARAMETERS (P AND S VELOCITIES)
.0025	P ARRIVAL VARIANCE
.0025	S ARRIVAL VARIANCE
25.0 25.0	
25.0 35.0	(X,Y) STATION COORDINATES IN KM
33.66 20.00	
16.34 20.00	
0.0	X GRID LOWER LIMIT
50.0	X GRID UPPER LIMIT
0.0	Y GRID LOWER LIMIT
50.0	Y GRID UPPER LIMIT
21	NOS OF X INCREMENTS
21	NOS OF Y INCREMENTS
10.	HYPOCENTER DEPTH

Model 2. Study of Galapagos Array

A seismic array of eight stations was deployed by the Hawaii Institute of Geophysics in the Galapagos Islands. (See crossed symbols in fig. 9 for station locations.) This array (model 2) was used to demonstrate some of the options available in program HYPOERR and to display a more complex velocity model than the simple half space used in model 1. An eight-layer velocity model (fig. 1) was used with only P velocities read in (TYPE1 = SR) and a P:S ratio of 1.78 designated to compute corresponding S velocities for the eight layers. P velocities varied from 4.4 to 8.1 km/sec, and layer thicknesses ranged from 0.4 to 4.8 km. Hypocenters were assigned to a depth of 5.0 km. The standard errors for P and S arrivals were specified (TYPE3 = S) with a P error of .0025 and an S error of .01. Locations of stations and observation-grid boundaries were given in latitude and longitude (TYPE4 = LAT).

Although the following eight elements were calculated and plotted, only the horizontal uncertainty (TYPE2 = UXY) is reproduced here (fig. 9) to demonstrate application of HYPOERR to a real seismic array with a multilayer velocity model.

(1) TYPE2=UXY. Horizontal error in epicenter location.

(2) TYPE2=UNC 3 3. Uncertainty of the z coordinate (depth) of the hypocenter.

Table 3. Input records used in generating the output of model 2
[Descriptions on the right are not part of the input]

```
GALAPAGOS ARRAY                        TITLE
SR                                     TYPE1
8                                      NOS OF ELEMENTS
UXY
UNC 3 3
UCR 1 3
SMA                                    ELEMENTS (TYPE2)
IGN 1 1
ICR 1 2
INF 2 2
CND
S                                      TYPE3
LAT                                    TYPE4
8                                      NOS OF LAYERS IN VELOCITY MODEL
8                                      NOS OF STATIONS
4.4 5.3 6.9 7.6 7.7 7.85 8 8.1                           P VELOCITIES (KM/SEC)
.8 .4 4.8 1 1 2 1                      MODEL PARAMETERS:  THICKNESSES (KM)
1.78                                                      P:S RATIO
0.0025                                 P ARRIVAL VARIANCE
0.01                                   S ARRIVAL VARIANCE
9539.94 243.57
9540.47 235.16
9535.75 233.43
9534.15 236.82                         STATION COORDINATES
9534.57 245.65                          (IN DEGREES*100+MINUTES)
9527.01 244.63
9527.91 240.90
9528.05 237.04
9600                                   X GRID LOWER LIMIT
9500                                   X GRID UPPER LIMIT
210                                    Y GRID LOWER LIMIT
310                                    Y GRID UPPER LIMIT
21                                     NO OF X INCREMENTS
21                                     NO OF Y INCREMENTS
5.0                                    HYPOCENTER DEPTH
```

(3) TYPE2=UCR 1 3. Linear correlation coefficient (ρ_{XZ}) between the x and z values.

(4) TYPE2=SMA. Semi-major axis magnitude.

(5) TYPE2=IGN 1 1. Ignorance of the P observation at station 1.

(6) TYPE2-ICR 1 2. Linear correlation between the P and S observations at station 1.

(7) TYPE2=INF 2 2. Information (importance) of the S arrival at station 1.

(8) TYPE2=CND. Logarithm of the condition number.

Table 3 is a copy of the input records used to generate the Galapagos array (model 2) discussed above. Appendix 5 discusses the output for this model.

Appendix 5. Output of Two Models

The output of the two models described in appendix 4 is listed below. Output from program HYPOERR consists of (a) line-printer output of the input data (generated in file 6), (b) line-printer output of gridded data for user-selected elements (TYPE2) (generated in file 25), and (c) contour maps of the gridded data (generated by program CONTOUR *not* included in this report).

Model 1. Quadrapartite Array

Discussion of computer output: The input parameters for a four-station array given in table 2 (appendix 4) are part of the output and are reprinted here (table 4).

The gridded output for model 1 is given in table 5 with column headings identifying the coordinates (X, Y) and the seven elements chosen by the user. For this case program HYPOERR outputs 21 values for each element by columns beginning at the southeast corner of the gridded map area. Only a small part of the total (441) values are shown here (21 x 21 grid).

The purpose of this report is to demonstrate the reliability and versatility of program HYPOERR output. Model 1 duplicates Uhrhammer's test cases and is illustrated best by the contour maps in figures 2-8. We have adopted Uhrhammer's coordinate system with x to the left, y up, and z into the page. All of the contours are identical in shape to Uhrhammer's maps, but the absolute values differed. Although not given by Uhrhammer, figure 8 is a map of the logarithm of the condition numbers.

Figure 2 is a plot of the uncertainty in the X coordinate of the epicenter (the square root of one of the three diagonal elements of Υ, the covariance matrix). Values in our plots are half that shown in Uhrhammer's figure.[1] For our purposes it is sufficient to note that the uncertainty (km) increases uniformly outside of the array and that the best values are centered about stations 1-3-4.

Figure 3, the plot of the uncertainty in the Z coordinate of the hypocenter (10 km), demonstrates a strong dependence on the station locations. Again, the contours agree in shape with Uhrhammer's figure, but our values are one-half. The plot is obtained from one of the principal diagonals of the Υ matrix.

Figure 4 is a contour plot of the linear correlation coefficient between x and z as obtained from an off-diagonal element of the covariance matrix. Contour shapes match Uhrhammer's results. This time our values are the square of his values. There is no significant correlation between the x and the z near the stations. Correlations involving x are antisymmetric about the y axis. (Note + values centered about stations 3 and 4.)

Figures 5 and 6 are plots of the principal diagonal elements of the ignorance matrix (ψ) for P and S observations respectively at station 2. For the P observation at station 2 (fig. 5) there are low ignorance values even beyond the array locations, which indicates the importance of P observations in locating hypocenters. For the S observation at station 2 the ignorance values are somewhat higher, but this observation supplies considerable information near the arrays. Both maps are identical in shape to Uhrhammer's plots, and again ours are one-half in value.

Figure 7 plots the linear correlation (redundancy) supplied by the P and S observation at station 2. There is a low redundancy around station 2 (no significant correlation between P and S), which indicates that both P and S values are meaningful at that station in locating hypocenters. Our contours are again similar to Uhrhammer's, although our values again correspond to the square of his values. In addition, there is a sign difference; our results show a low (negative correlation) around station 2, but Uhrhammer's results show a low positive correlation.

---

[1] The discrepancy was discussed with Uhrhammer, who did not offer any explanation for it. Since we solve the problem manually at one grid location and our solution agrees with that given by HYPOERR, we conclude that Uhrhammer's standard errors are not standard deviations, but they may instead be 95-percent confidence limits.

Table 4. Output of quadrapartite array from program HYPOERR listing user's selection of model parameters and computer-calculated data used in generating the final gridded output (table 5)

```
1 QUAD ARRAY
  TYPE1   SP
  NO OF ELEMENTS =   7
 UNC 1 1
 UNC 3 3
 UCR 1 3
 IGN 3 3
 IGN 4 4
 ICR 4 3
 CND 0 0
  TYPE3   S    TYPE4   DST
 NO OF LAYERS =  1       NO OF STATIONS =   4
 PARAMETERS
    .560E+01   .330E+01
 S VARIANCE =   .250E-02 P VARIANCE =   .250E-02
 STATION COORDINATES
        25.000        25.000
        25.000        35.000
        33.660        20.000
        16.340        20.000

 CONTOUR GRID PARAMETERS
 X =          .000   TO        50.000
 Y =          .000   TO        50.000
 NO OF X PTS =  21       NO OF Y PTS =  21
  HYPOCENTER DEPTH =          10.0 KM
  XINC =    2.500       YINC =    2.500
             VELOCITY MODEL
         LAYER          DEPTH   P VELOCITY   S VELOCITY
             1                      5.600         3.300
 ELEMENT          MAXIMUM    MINIMUM
  UNC 1 1           .504       .154
  UNC 3 3          1.623       .283
  UCR 1 3           .730      -.730
  IGN 3 3           .464       .128
  IGN 4 4           .846       .126
  ICR 4 3           .937      -.112
  CND 0 0          1.991      1.252
```

As a check on our algorithm, we calculated the ignorance correlation at a single grid point, as described in appendix 1. The result agreed with that given by HYPOERR. It should also be pointed out that the ignorance correlation is a measure of the covariance of the data resulting from an error in the hypocenter. At large hypocentral distances, a hypocentral error must cause errors of the same sign in both P and S. Therefore, the correlation between S and P arrival time errors must be positive at large distances from the array (fig. 7).

Figure 8, the final output for the quadrapartite array, plots the logarithm of the condition number CND. This parameter is large at modest distances outside the boundary of the array.

Table 5. Output of quadrapartite array from program HYPOERR listing a part of the gridded output for each element (including its x and y coordinates)

1 X	Y	UNC 1 1	UNC 3 3	UCR 1 3	IGN 3 3	IGN 4 4	ICR 4 3	CND 0 0
50.000	.000	.446	1.331	.450	.461	.776	.934	1.907
50.000	2.500	.435	1.180	.474	.397	.669	.911	1.856
50.000	5.000	.426	1.046	.502	.341	.573	.878	1.806
50.000	7.500	.419	.933	.533	.292	.488	.832	1.758
50.000	10.000	.414	.842	.565	.251	.415	.770	1.715
50.000	12.500	.410	.774	.594	.219	.354	.693	1.680
50.000	15.000	.408	.730	.618	.197	.309	.609	1.657
50.000	17.500	.406	.711	.632	.182	.281	.537	1.648
50.000	20.000	.405	.715	.639	.176	.271	.497	1.652
50.000	22.500	.405	.737	.641	.175	.277	.497	1.665
50.000	25.000	.406	.775	.643	.179	.296	.529	1.685
50.000	27.500	.408	.825	.649	.186	.325	.578	1.710
50.000	30.000	.411	.883	.660	.197	.361	.635	1.737
50.000	32.500	.417	.949	.674	.212	.401	.692	1.767
50.000	35.000	.424	1.021	.689	.232	.447	.746	1.797
50.000	37.500	.433	1.100	.705	.257	.497	.795	1.828
50.000	40.000	.443	1.186	.718	.287	.552	.837	1.860
50.000	42.500	.456	1.281	.727	.322	.614	.871	1.892
50.000	45.000	.470	1.384	.730	.364	.683	.899	1.924
50.000	47.500	.486	1.498	.729	.411	.761	.920	1.958
50.000	50.000	.504	1.623	.723	.464	.846	.937	1.991
47.500	.000	.423	1.228	.388	.452	.751	.930	1.874
47.500	2.500	.412	1.073	.407	.389	.645	.906	1.818
47.500	5.000	.403	.937	.431	.332	.551	.870	1.761
.	.	.	.	.	.	.	.	.
.	.	.	.	.	.	.	.	.
.	.	.	.	.	.	.	.	.
.	.	.	.	.	.	.	.	.
2.500	45.000	.449	1.227	-.709	.338	.634	.881	1.874
2.500	47.500	.466	1.337	-.709	.384	.708	.908	1.910
2.500	50.000	.484	1.458	-.702	.437	.792	.929	1.946
.000	.000	.446	1.331	-.450	.461	.776	.934	1.907
.000	2.500	.435	1.180	-.474	.397	.669	.911	1.856
.000	5.000	.426	1.046	-.502	.341	.573	.878	1.806
.000	7.500	.419	.933	-.533	.292	.488	.832	1.758
.000	10.000	.414	.842	-.565	.251	.415	.770	1.715
.000	12.500	.410	.774	-.594	.219	.354	.693	1.680
.000	15.000	.408	.730	-.618	.197	.309	.609	1.657
.000	17.500	.406	.711	-.632	.182	.281	.537	1.648
.000	20.000	.405	.715	-.639	.176	.271	.497	1.652
.000	22.500	.405	.737	-.641	.175	.277	.497	1.665
.000	25.000	.406	.775	-.643	.179	.296	.529	1.685
.000	27.500	.408	.825	-.649	.186	.325	.578	1.710
.000	30.000	.411	.883	-.660	.197	.361	.635	1.737
.000	32.500	.417	.949	-.674	.212	.401	.692	1.767
.000	35.000	.424	1.021	-.689	.232	.447	.746	1.797
.000	37.500	.433	1.100	-.705	.257	.497	.795	1.828
.000	40.000	.443	1.186	-.718	.287	.552	.837	1.860
.000	42.500	.456	1.281	-.727	.322	.614	.871	1.892
.000	45.000	.470	1.384	-.730	.364	.683	.899	1.924
.000	47.500	.486	1.498	-.729	.411	.761	.920	1.958
.000	50.000	.504	1.623	-.723	.464	.846	.937	1.991

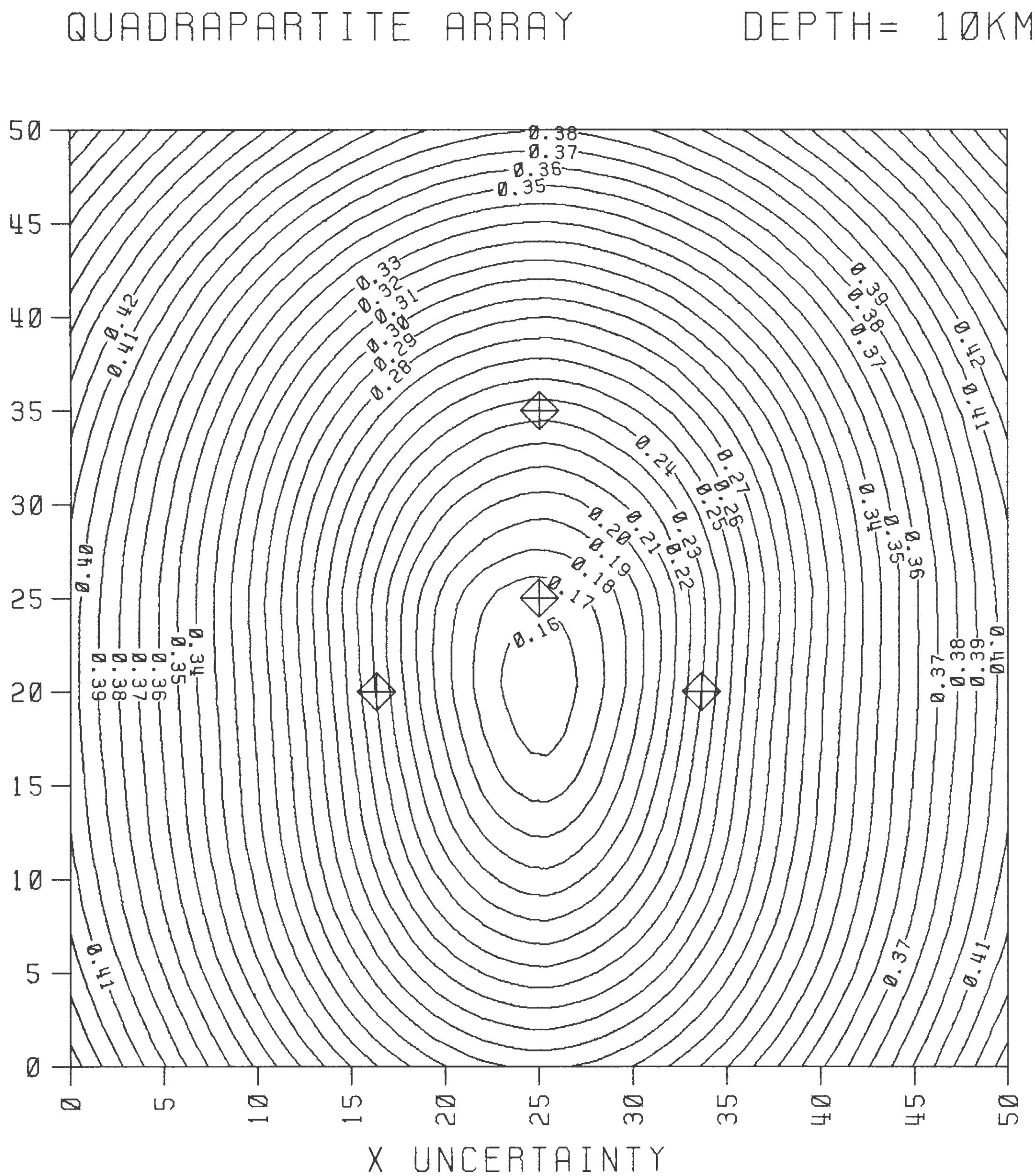


Figure 2. Uncertainty in the X coordinates of hypocenters at a depth of 10 km located by using a quadrapartite station array (crossed symbols). Contours and axes values are in kilometers.

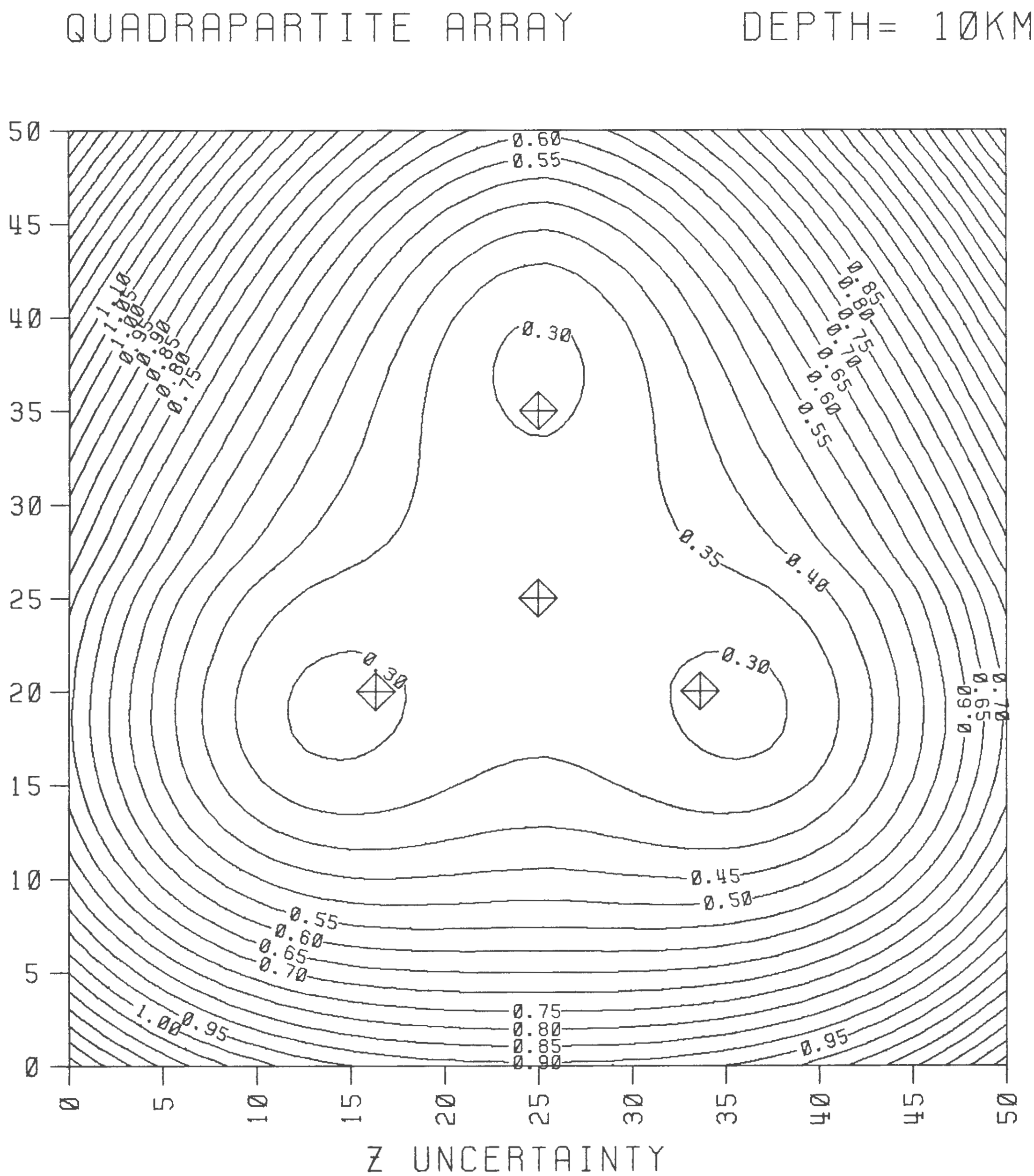


Figure 3. Uncertainty in the Z (depth) coordinates of hypocenters at a depth of 10 km located by using a quadrapartite station array (crossed symbols). Contours and axes values are in kilometers.

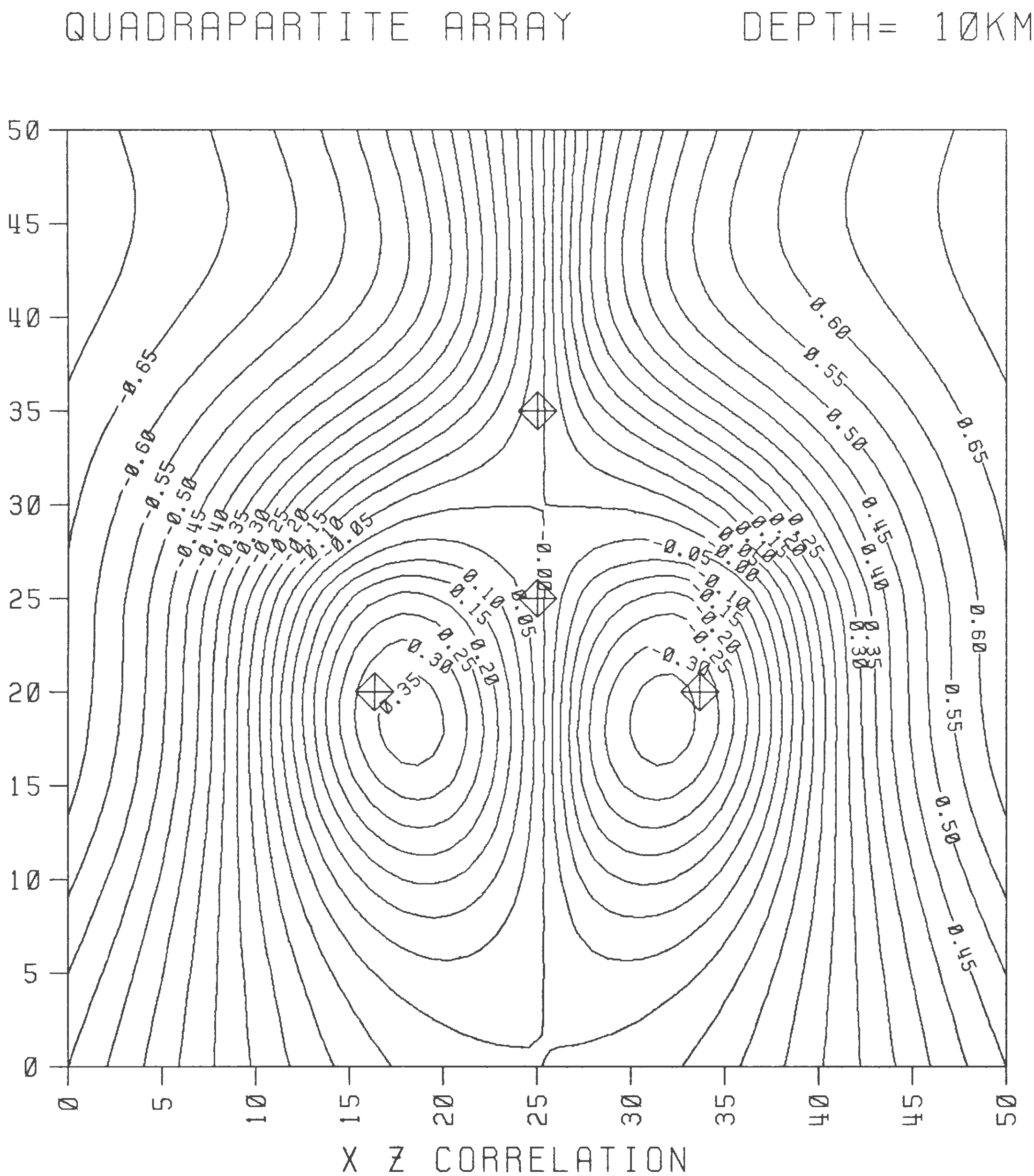


Figure 4. X-Z correlation of hypocenters at a depth of 10 km located by using a quadrapartite station array (crossed symbols). Axes are labeled in kilometers and contours are dimensionless.

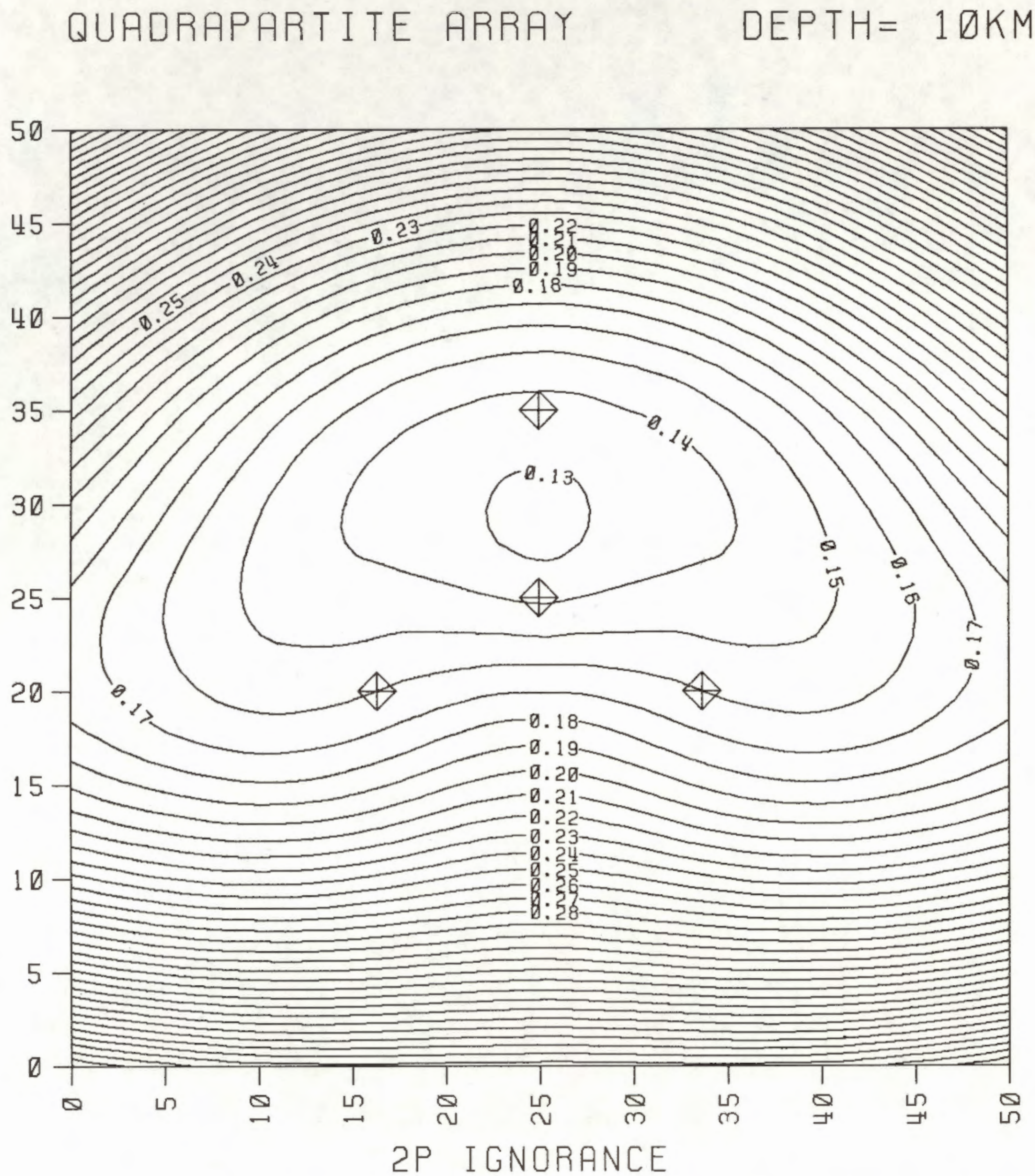


Figure 5. Ignorance of the P observations at station 2. Hypocenters are all at a depth of 10 km and have been located by using a quadrapartite station array (crossed symbols). Axes are labeled in kilometers and contours are in seconds^2.

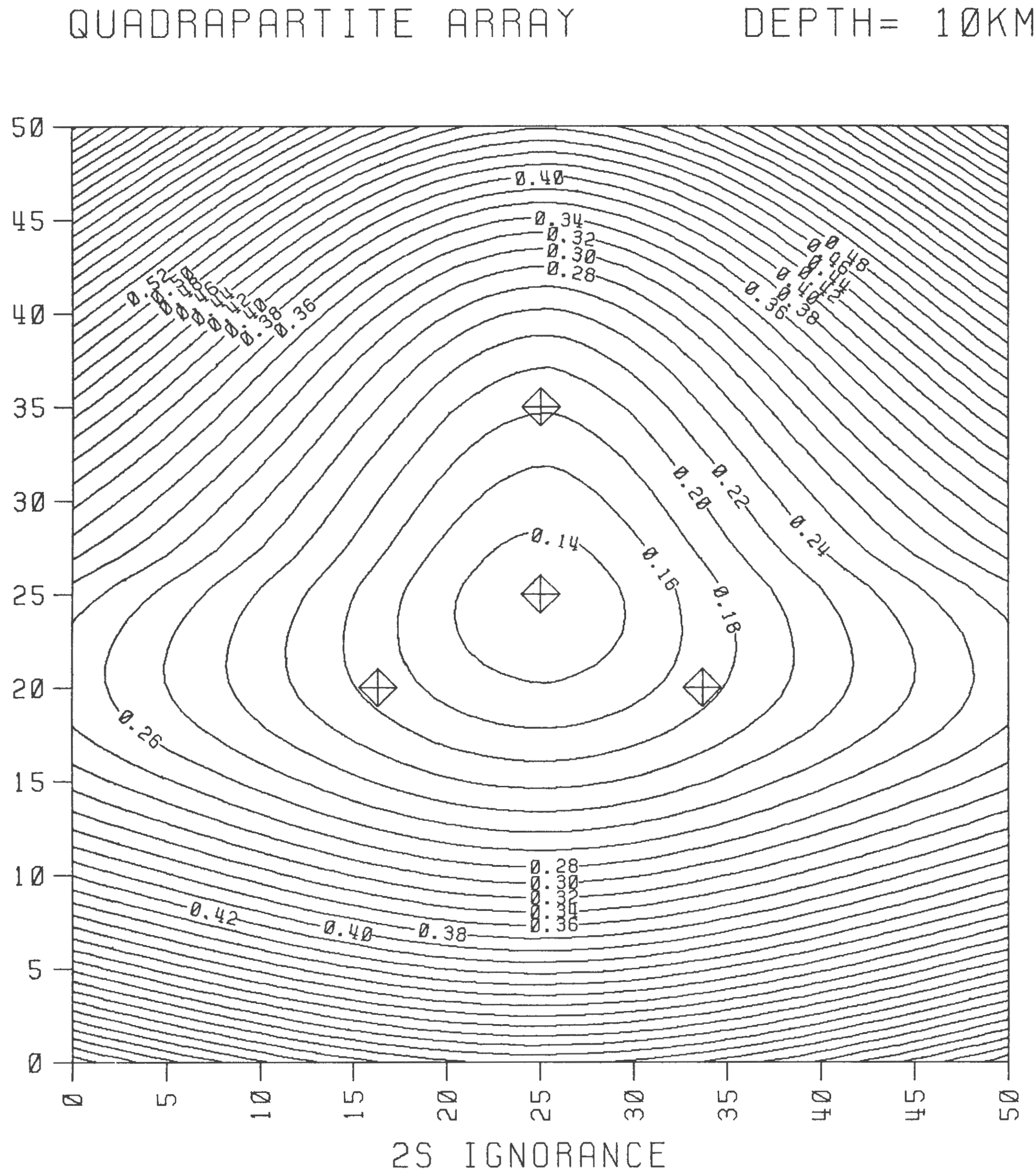


Figure 6. Ignorance of the S observations at station 2. Hypocenters are all at a depth of 10 km and have been located by using a quadrapartite station array (crossed symbols). Axes are labeled in kilometers and contours are in seconds^2.

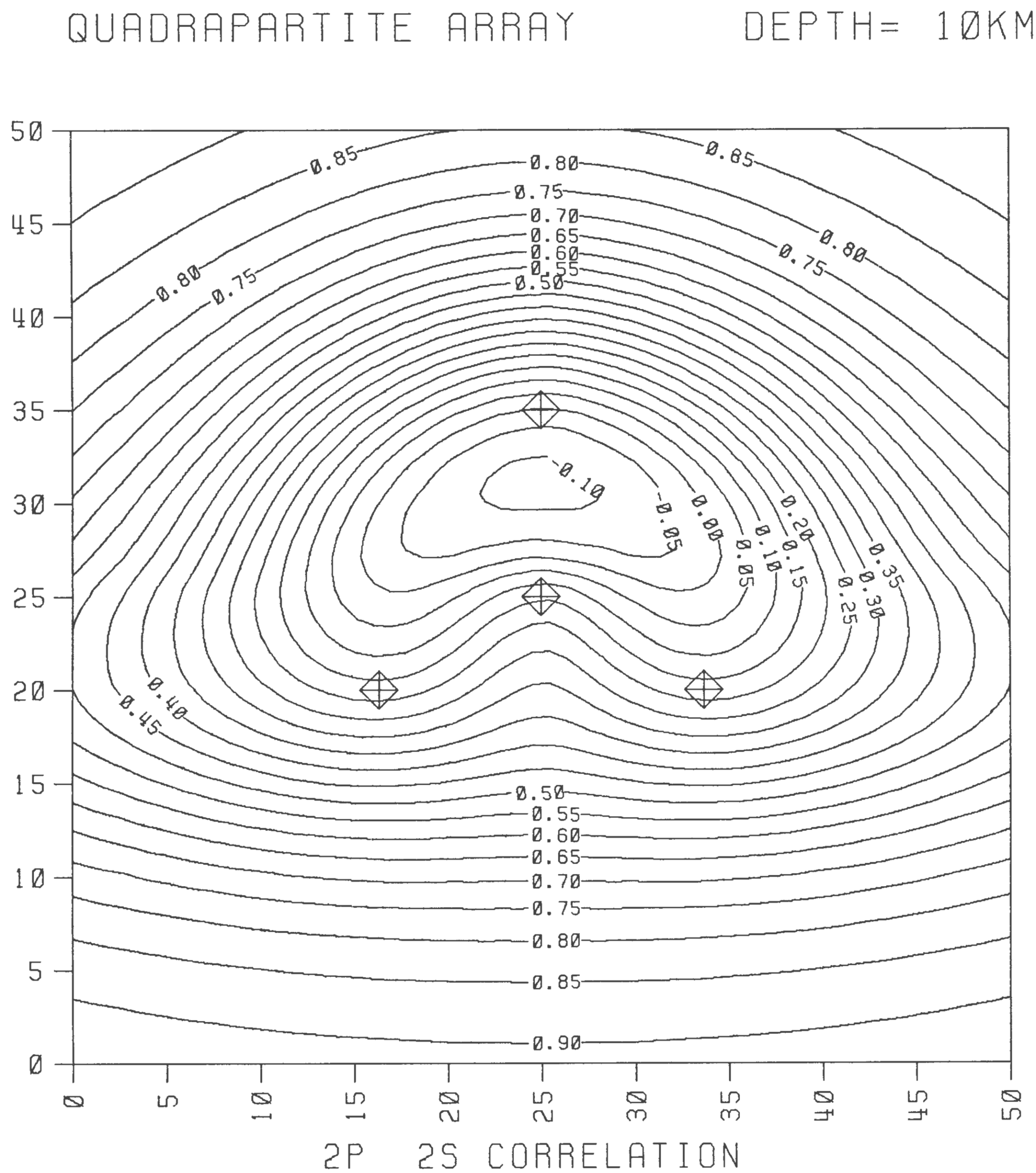


Figure 7. Linear correlation (redundancy) between P and S observations at station 2. Hypocenters are all at a depth of 10 km and have been located by using a quadrapartite station array (crossed symbols). Axes are labeled in kilometers and contours are dimensionless.

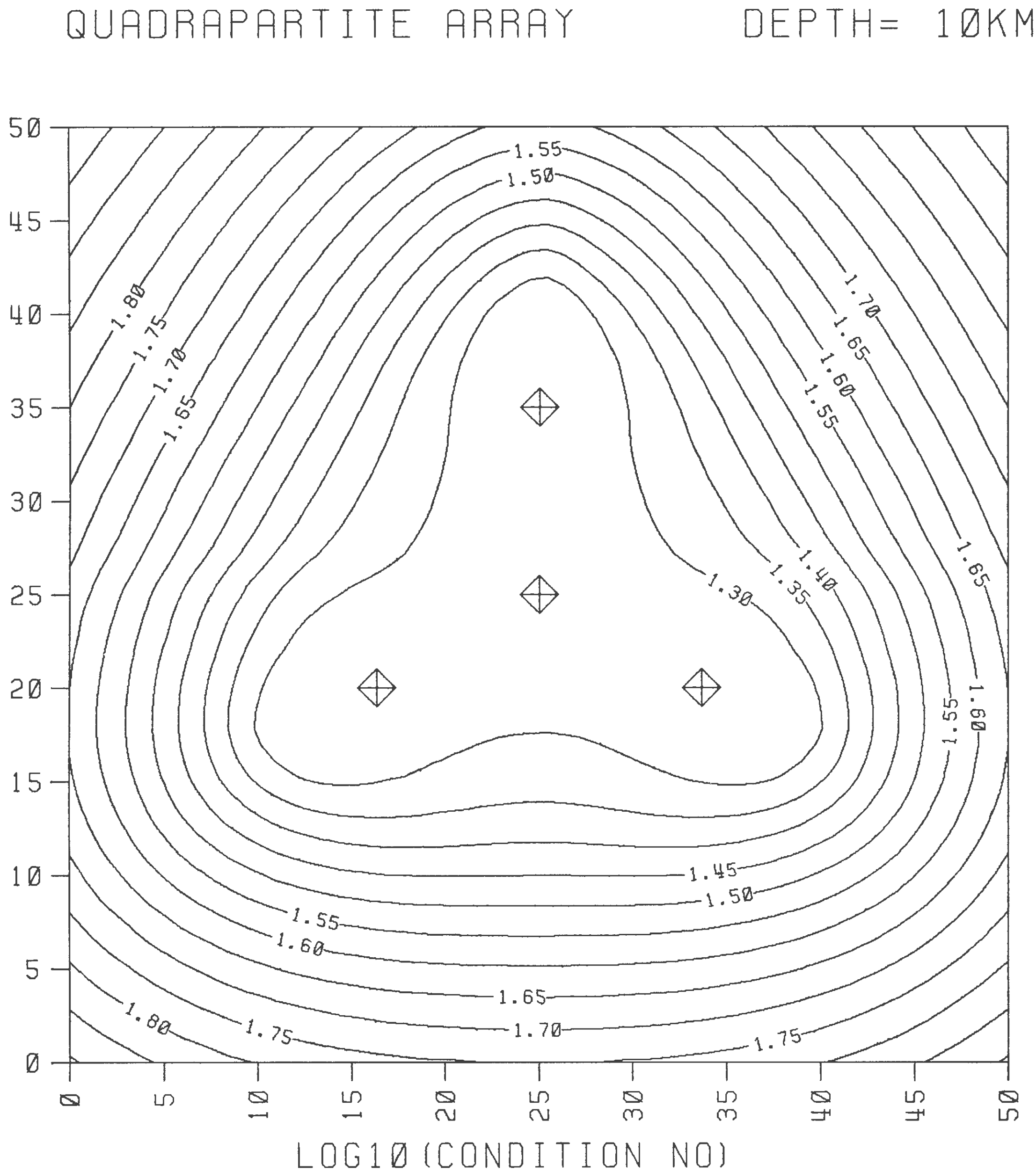


Figure 8. Logarithm of condition number (CND) for hypocenters at a depth of 10 km located by using a quadrapartite station array (crossed symbols). Axes are labeled in kilometers and contours are dimensionless.

Manual computation: As a check on the program, model 1 was solved manually for a hypocenter at the point midway between stations 3 and 4 at a depth of 10 km. Evaluating all the partial derivatives for both S and P in a constant velocity medium with V_p = 5.6 km/sec and V_s = 3.3 km/sec gives the partial derivative matrix as

$$G = \begin{bmatrix} \partial t_i/\partial x & \partial t_i/\partial y & \partial t_i/\partial z & \partial t_i/\partial t_o & \\ 0.000 & 0.080 & 0.160 & 1.000 & 1P \\ 0.000 & 0.135 & 0.271 & 1.000 & 1S \\ 0.000 & 0.149 & 0.099 & 1.000 & 2P \\ 0.000 & 0.252 & 0.168 & 1.000 & 2S \\ 0.117 & 0.000 & 0.135 & 1.000 & 3P \\ 0.198 & 0.000 & 0.229 & 1.000 & 3S \\ -0.117 & 0.000 & 0.135 & 1.000 & 4P \\ -0.198 & 0.000 & 0.229 & 1.000 & 4S \end{bmatrix} \qquad (14)$$

where $\partial/\partial t_o$ represents the partial derivative with respect to origin time, t_o, and the remaining partials are with respect to the hypocentral position (x,y,z). 1P and 1S are the P and S arrival times at station 1, and 2P, 2S, 3P, 3S, 4P, and 4S are arrival times at stations 2, 3, and 4. This gives

$$G^TG = \begin{bmatrix} 0.1058 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.1103 & 0.1065 & 0.6160 \\ 0.0000 & 0.1065 & 0.2784 & 1.4260 \\ 0.0000 & 0.6160 & 1.4260 & 8.0000 \end{bmatrix} \qquad (15)$$

The eigenvalues of G^TG are then given by the equation

$$\det (G^TG - \lambda I) = 0 \qquad (16)$$

where I is the identifying matrix, giving

$$(\lambda - 0.1058)(\lambda^3 - 8.3887 \lambda^2 + 0.716 \lambda - 0.7966) = 0 \qquad (17)$$

Solving this equation for λ gives us the four squared eigenvalues

$$\lambda_1^2 = 8.3026$$

$$\lambda_2^2 = 0.1058$$

$$\lambda_3^3 = 0.0630$$

$$\lambda_4^4 = 0.0231 \qquad (18)$$

These values correspond to the squared eigenvalues of the matrix G; that is, they are the diagonal elements of Λ^2 as defined in equation (2). The eigenvectors, V_{ij}, of G^TG can now be obtained by solving

$$G^TG\ V_i = \lambda_i V_i \qquad (19)$$

After normalization, we get

$$V_1 = [0.0000 \quad 0.0761 \quad 0.1754 \quad 0.9815]$$

$$V_2 = [1.0000 \quad 0.0000 \quad 0.0000 \quad 0.0000]$$

$$V_3 = [0.0000 \quad 0.9930 \quad -0.1024 \quad -0.0587]$$

$$V_4 = [0.0000 \quad 0.0902 \quad 0.9792 \quad -0.1820] \qquad (20)$$

We can now use equation (4) to find the covariance matrix that is

$$\gamma^2 = \sigma^2 \begin{bmatrix} 9.47 & 0.02 & -0.01 & -0.01 \\ 0.02 & 16.03 & 2.19 & -1.65 \\ -0.01 & 2.19 & 41.16 & -7.59 \\ -0.01 & -1.65 & -7.59 & 1.50 \end{bmatrix} \qquad (21)$$

Taking $\sigma^2 = 0.0025$ gives

$$\sigma_{xx} = 0.154$$

$$\sigma_{zz} = 0.324 \quad (22)$$

which agree with the values shown in figures 2 and 3 at the corresponding point. The values given by Uhrhammer (1980) appear to be in error by a factor or two.

In summary, plots made of all the elements plotted by Uhrhammer (1980) agree in the shapes of the contours. But the standard deviations in x and z were a factor of two lower than Uhrhammer's, as were the diagonal elements of the ignorance matrix. Also, the correlation plots were found to correspond to the square of Uhrhammer's correlations. Figure 4 shows a plot of the XZ parameter correlation calculated by HYPOERR. The discrepancy is due to an unusual definition of correlation by Uhrhammer, namely

$$\rho^2_{ij} = \Upsilon^2_{ij} / (\Upsilon_{ii} \Upsilon_{jj}) \quad (23)$$

compared to the definition given by equation (8). Uhrhammer's equation does not give a real value of ρ_{ij} when $\Upsilon^2{}_{ij} < 0$. He apparently takes the square root of $|\Upsilon^2{}_{ij}|$, although he keeps its sign.

Model 2. Galapagos Array

As a final demonstration of the versatility of program HYPOERR, a test was made of selected elements for a seismic array deployed by the Hawaii Institute of Geophysics in the Galapagos Islands. Although the quadrapartite array (model 1) was based on a simple half-space model, the Galapagos velocity model was an eight-layer case. The input file used is given in appendix 4. Table 6 lists the output of model parameters, and table 7 lists a part of the gridded data for eight elements. Only one of the elements, UXY, is plotted (fig. 9). Because the Galapagos data are real, it is difficult to assess the results in terms of reliability. Detailed discussion of the plot (fig. 9) is therefore not given. But uncertainties calculated for this array were in good agreement with location errors obtained from program HYPO71 (Lee and Lahr, 1972), a program used to locate the earthquakes recorded by this array. It is also interesting to observe that reasonable errors can be obtained by using an eight-station array over about 10 times the area actually covered by the array, provided both P and S arrivals are available at every station.

Table 6. Output of the Galapagos array from program HYPOERR listing user's selection of model parameters and computer-calculated data used in generating the final gridded output (table 7)

```
1 GALAPAGOS ARRAY
  TYPE1   SR
  NO OF ELEMENTS =  8
 UXY 0 0
 UNC 3 3
 UCR 1 3
 SMA 0 0
 IGN 1 1
 ICR 1 2
 INF 2 2
 CND 0 0
  TYPE3  S   TYPE4  LAT
 NO CF LAYERS =  8      NO OF STATIONS =  8
 PARAMETERS
    .440E+01  .530E+01  .690E+01  .760E+01  .770E+01  .785E+01  .800E+01
    .800E+00  .400E+00  .480E+01  .100E+01  .100E+01  .200E+01  .100E+01
 S VARIANCE =  .100E-01 P VARIANCE =  .250E-02
 STATION COORDINATES
      9539.940     243.570
      9540.470     235.160
      9535.750     233.430
      9534.150     236.820
      9534.570     245.650
      9527.010     244.630
      9527.910     240.900
      9528.050     237.040
 CONTOUR GRID PARAMETERS
 X =    9600.000  TO    9500.000
 Y =     210.000  TO     310.000
 NO OF X PTS =  21      NO OF Y PTS =  21
  HYPOCENTER DEPTH =          5.0 KM
  GRID PARAMETERS IN KILOMETERS
 STATION COORDINATES
        37.185      62.076
        36.205      46.525
        44.955      43.326
        47.920      49.594
        47.139      65.922
        61.153      64.036
        59.487      57.139
        59.228      50.001
 CONTOUR GRID PARAMETERS
 X =        .000  TO     111.203
 Y =        .000  TO     110.949
 NO OF X PTS =  21      NO OF Y PTS =  21
  XINC =   5.560     YINC =   5.547
```

Table 7. Output of the Galapagos array from program HYPOERR listing a part of the gridded output for each element (including its x and y coordinates)

1 X	Y	UXY	UNC	UCR	SMA	IGN	ICR	INF	CND
		0 0	3 3	1 3	0 0	1 1	1 2	2 2	0 0
111.203	.000	21.292	34.357	-.999	40.419	7.620	1.000	.238	3.260
111.203	5.547	19.750	31.850	-.999	37.476	7.205	1.000	.240	3.227
111.203	11.095	3.112	4.794	-.962	5.716	.176	.691	.132	2.403
111.203	16.642	2.566	3.879	-.958	4.651	.320	.908	.145	2.310
111.203	22.190	3.000	4.582	-.978	5.477	.792	.985	.197	2.385
111.203	27.737	2.851	4.362	-.979	5.211	.772	.984	.202	2.362
111.203	33.285	2.332	3.581	-.972	4.273	.707	.981	.226	2.272
111.203	38.832	2.301	3.625	-.974	4.294	.837	.986	.265	2.274
111.203	44.380	2.318	3.646	-.978	4.321	.826	.986	.269	2.277
111.203	49.927	2.543	4.138	-.985	4.857	1.436	.995	.446	2.330
111.203	55.474	2.440	3.805	-.983	4.521	1.226	.994	.418	2.297
111.203	61.022	2.392	3.712	-.981	4.417	1.183	.993	.432	2.286
111.203	66.569	2.360	3.639	-.976	4.338	1.132	.993	.444	2.276
111.203	72.117	2.414	3.633	-.970	4.363	1.071	.992	.439	2.278
111.203	77.664	2.379	3.551	-.960	4.275	.987	.990	.442	2.267
111.203	83.212	2.368	3.505	-.947	4.231	.907	.988	.444	2.261
111.203	88.759	2.620	3.795	-.954	4.612	.376	.932	.342	2.301
111.203	94.307	2.636	3.816	-.933	4.638	.498	.962	.378	2.302
111.203	99.854	3.283	4.923	-.944	5.917	.622	.976	.397	2.414
111.203	105.402	3.316	4.950	-.933	5.958	.576	.971	.403	2.416
111.203	110.949	34.754	56.235	-.999	66.107	13.214	1.000	.527	3.474
105.643	.000	19.155	30.880	-.998	36.338	6.742	1.000	.237	3.214
105.643	5.547	3.120	4.820	-.953	5.742	.159	.619	.136	2.405
.	.	.	.	.	.	.	.	.	.
.	.	.	.	.	.	.	.	.	.
.	.	.	.	.	.	.	.	.	.
.	.	.	.	.	.	.	.	.	.
.	.	.	.	.	.	.	.	.	.
.	.	.	.	.	.	.	.	.	.
5.560	105.402	2.485	3.842	.923	4.576	1.013	.991	.320	2.305
5.560	110.949	2.436	3.709	.864	4.437	1.307	.994	.447	2.289
.000	.000	2.621	3.837	.837	4.647	.635	.976	.477	2.304
.000	5.547	2.390	3.474	.819	4.218	.523	.965	.471	2.260
.000	11.095	2.355	3.540	.901	4.252	.480	.958	.472	2.266
.000	16.642	2.315	3.520	.918	4.213	.539	.967	.485	2.263
.000	22.190	2.638	4.311	.955	5.055	.372	.931	.441	2.348
.000	27.737	2.499	4.181	.960	4.871	.302	.893	.422	2.332
.000	33.285	2.540	4.282	.972	4.979	.337	.915	.403	2.343
.000	38.832	2.548	4.432	.979	5.113	.274	.870	.367	2.356
.000	44.380	2.413	4.259	.982	4.896	.803	.985	.412	2.337
.000	49.927	2.470	4.380	.985	5.029	.992	.990	.410	2.349
.000	55.474	2.057	3.600	.978	4.147	1.268	.994	.554	2.263
.000	61.022	2.390	4.238	.984	4.866	1.883	.997	.693	2.335
.000	66.569	2.333	4.129	.981	4.743	1.931	.997	.709	2.324
.000	72.117	3.662	6.574	.991	7.525	3.224	.999	.717	2.528
.000	77.664	3.561	6.387	.988	7.312	3.192	.999	.725	2.515
.000	83.212	3.310	5.737	.983	6.624	.849	.987	.185	2.471
.000	88.759	2.673	4.452	.968	5.193	.762	.984	.203	2.363
.000	94.307	2.285	3.568	.944	4.238	.820	.985	.255	2.270
.000	99.854	2.414	3.730	.935	4.444	1.063	.991	.330	2.291
.000	105.402	2.449	3.727	.895	4.460	1.377	.995	.456	2.292
.000	110.949	2.470	3.736	.873	4.480	1.351	.995	.451	2.293

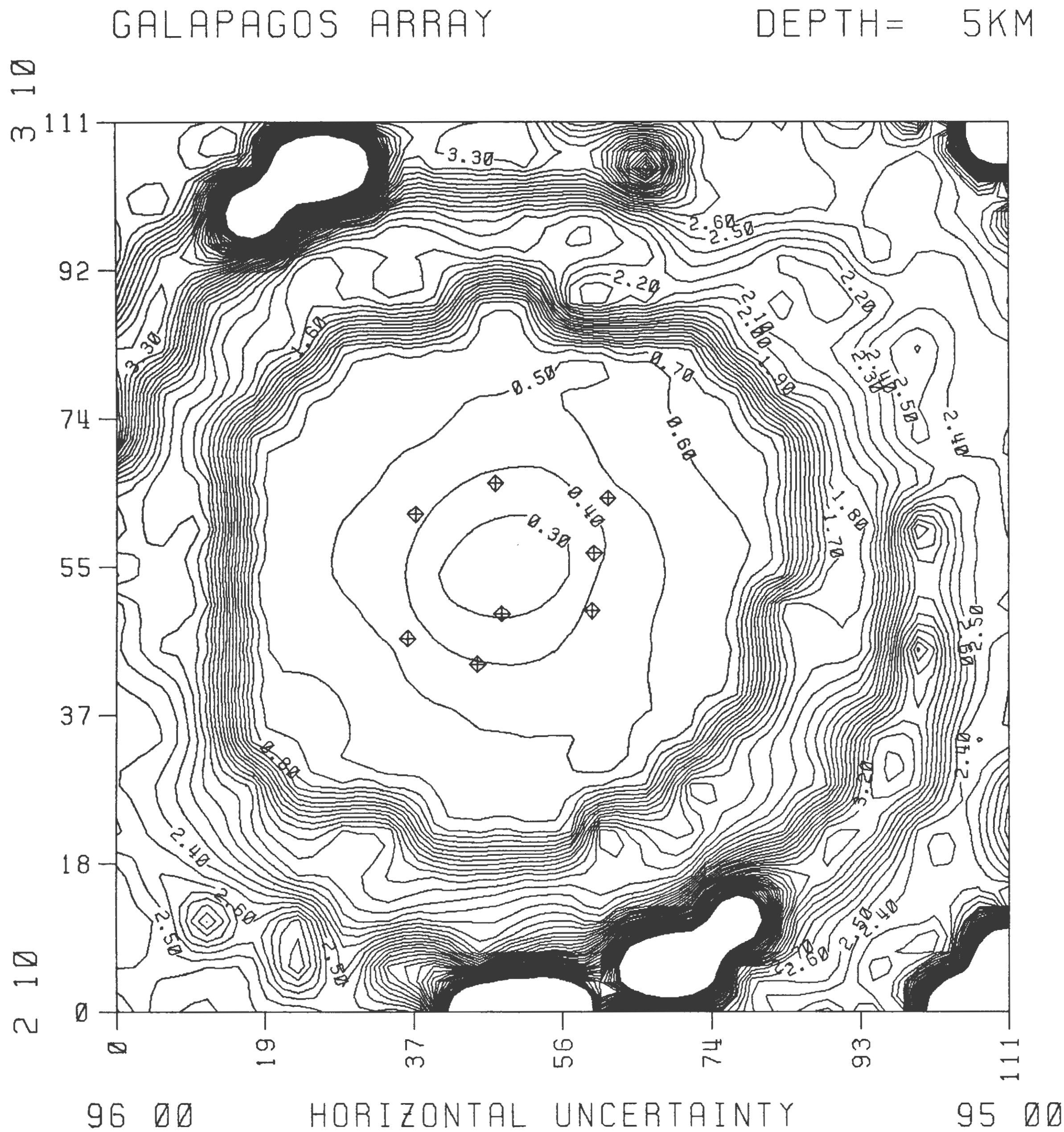


Figure 9. Horizontal uncertainty for hypocenters at a depth of 5 km located by using an eight-station seismic array (crossed symbols). Stations were deployed by the Hawaii Institute of Geophysics near the Galapagos propagating-rift zone. Axes are labeled in kilometers and in degrees latitude/longitude and contours are also in kilometers. Blank areas correspond to maximum values too dense to contour.

INDIANA GEOLOGICAL SURVEY GEOPHYSICAL COMPUTER PROGRAMS
ERRATA

Geophysical Computer Program 1 (Occasional Paper 10)

Page 9, 19 lines from the bottom of the page:

Second line of R(M,N,4) now reads 1+P(I+1,J+1)+P(I+1,J-1)+P(I-1,J+1)+P(I-1,J-1))/8.0

Second line of R(M,N,4) should read 1+P(I+1,J+2)+P(I+1,J-2)+P(I-1,J+2)+P(I-1,J-2))/8.0

Page 9, 4 lines from the bottom of the page:

Second line of R(M,N,11) now reads 1P(I-20,J-15)+P(I-15,J-15)+P(I+20,J+15)+P(I+15,J+20)

Second line of R(M,N,11) should read 1P(I-20,J-15)+P(I-15,J-20)+P(I+20,J+15)+P(I+15,J+20)

Page 14, line 6, which reads C(6,12)=-0.04007, may be deleted.

Geophysical Computer Program 2 (Occasional Paper 13)

Page 11, line 18:

Now reads: (1,170)ITYPE,Z(I),XI(I)

Should read: (2,230)ITYPE,Z(I),XI(I)

Page 12, after line 18:

Insert: 230 FORMAT (I1,F4.0,F4.1)

Geophysical Computer Program 3 (Occasional Paper 14)

Page 12, line 11:

Now reads: 10 A(I+MN)=A(I)

Should read: 10 A(M+K-I)=A(N+K-I)

Geophysical Computer Programs 4 and 5 (Occasional Papers 22 and 23)

Geophysical Computer Programs 4 and 5 require many significant figures. Double precision may be needed on some computers. Indiana University computers use 60-bit words.

Geophysical Computer Program 7 (Occasional Paper 29)

Subroutine MYLINE2 has been removed from the program. Delete all references to this subroutine and read all references to "11 subroutines" as "10 subroutines."

Page 38:

Now reads: 30 × 27 km region

Should read: 31 × 27 km region

Page 39:

Now reads: 6 × 40 km region

Should read: 10 × 40 km region

Page 44:

Now reads: distance 200 km

Should read: distance 20 km

Page 52:

Now reads: as a function time

Should read: as a function of time

Geophysical Computer Program 9 (Occasional Paper 40)

Page 13, line 16:

Now reads: 110 THETA=PI/2.0

Should read: 110 THETA1=PI/2.0