

Theory Ruled by Practice: Zarlino's Reversal of the Classical System of Proportions¹

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*Thus two things most of all should be dealt with in this treatise, namely, consonances, which are natural, out of which songs are made, and songs themselves, which are artificial.*²

Gioseffo Zarlino's ivory-tower pronouncements on music, such as the above, provided ample fuel for the scathing polemics subsequently directed against him. Of all his critics, his former pupil Vincenzo Galilei was by far the most merciless. "Nature has neither hands nor mouth," Galilei was quick to remind him. "It cannot play music or

¹It is a pleasure to thank my colleagues Walter Robert, Lewis E. Rowell, and Allen C. Winold for reading an earlier draft of this paper and sharing with me their valuable observations.

²"... però havendosi principalmente in esso a trattare due cose: cioè le Consonanze, che sono cose naturali, di che si fanno le Cantilene: et esse Cantilene, che sono artificiali." Gioseffo Zarlino, *Le istituzioni harmoniche*, 3rd ed. (Venice: Francesco Senese, 1573; facs. ed., Ridgewood, NJ: Gregg Press, 1966), proemium, 3.

sing, and our playing and singing is totally artificial.”³ And concerning the older man’s frequent references to *numero sonoro*, sonorous number, as the norm for what is harmonious in music,⁴ Galilei had a ready quip: “Since number has no body, it cannot be sonorous.”⁵

The quarrel between these two antagonists has been recounted in detail by modern historians.⁶ Galilei emerges as the more down-to-earth and the more attuned to new developments in sixteenth-century musical practice. In contrast, Zarlino is depicted as the proponent of a compositional method laden with abstract speculation. Reviewing Zarlino’s list of “nature-ordained” cadential pitches for each of the twelve modes, Carl Dahlhaus pronounces a verdict:

Zarlino’s I-V-III schema, the rule that in every mode the *clausula primaria* should be on degree I, the *clausula secundaria* on degree V, and the *clausula tertiaria* on degree III, is speculative and not based on empirical evidence.⁷

³“La natura poi per non havere ne mani ne bocca non gli è concedutto ch’ella suoni ne ch’ella canti, & il sonare & il cantar’ nostro è tutt’arte.” Vincenzo Galilei, *Discorso intorno alle opere di Gioseffo Zarlino* (Florence: G. Marescotti, 1589; facs. ed., Milan: Bollettino Bibliographico Musicale, 1933), 20. The preceding quotation from Zarlino’s *Istitutioni* (see n. 1) does not appear in the proemium of the first edition (1558). That Zarlino should deliberately insert it in the revised proemium indicates the emphasis that he wished to give to it. Henceforth references to the *Istitutioni* (*Ist. harm.*) will pertain to the 1573 edition.

⁴Zarlino, *Ist. harm.*, pt. 1, chaps. 13, 15, 19, etc.

⁵“ . . . il numero come quello che non ha corpo, non puo esse sonoro.” V. Galilei, *Il primo libro della pratica de Contrapunto* (MS 1589; mod. ed. in Frieder Rempp, *Die Kontrapunkttraktate Vincenzo Galileis* [Cologne: Arno Volk, 1980]), 15.

⁶See especially the excellent accounts in Claude V. Palisca, *Humanism in Italian Renaissance Musical Thought* (New Haven and London: Yale University Press, 1985), 244-50; 265-79; idem, *Studies in the History of Italian Music and Music Theory* (Oxford: Clarendon Press, 1994), 30-53.

⁷Carl Dahlhaus, *Studies on the Origin of Harmonic Tonality*, trans. Robert O. Gjerdingen (Princeton: Princeton University Press, 1990), 219-20.

As will be shown later in this paper, Dahlhaus does not take into account the full context of Zarlino's schema. But his negative critique brings to the fore a fundamental question: Does Zarlino's speculative bias effectively undermine the reliability of his testimony on matters pertaining to sixteenth-century musical practice? I would argue that it does not. Zarlino's writings, properly read, can still inform us about the way sixteenth-century musicians actually practiced their craft. Having said that, I must warn the reader that this paper does not center on musical practice. The focus will be on Zarlino's speculative theory and the question of its relationship to his view of musical practice.

To be sure, not everything that Zarlino wrote or taught could have been free of error. Each tenet should be evaluated according to its own merit. But to accomplish that task of weighing and evaluating, we must first rectify a preexisting imbalance. From the outset the scale is tilted toward the assumption, inspired by Galilei's clever argumentation, that Zarlino's teachings are unsoundly based on a priori reasoning. Claude Palisca offers a succinct summary of that point of view:

Galilei was skeptical of a process of thought that began with nature as the occasion for some idea but left it to elaborate theories never further subjected to verification by controlled observation of nature.

The laws of composition set down by Zarlino constituted such a theory, evolved from an ideal view of nature, leaving the immediate facts in the distance.⁸

We can trace the deductive aspect of Zarlino's theory by drawing a series of propositions that lead to some of his more well-known compositional rules:⁹

⁸Claude V. Palisca, "Vincenzo Galilei's Counterpoint Treatise: A Code for the *Seconda Pratica*," *Journal of the American Musicological Society* 9 (1956): 85; reprint in *idem*, *Studies in the History of Italian Music and Music Theory*, 36-37.

⁹Propositions a to f are derived from *Ist. harm.*, pt. 1, proemium and chaps. 1-4, 11, 13, and 16; g and h are derived from pt. 3, chaps. 10 and 59, and pt. 4, chaps. 18-32.

- (a) The highest kind of knowledge is that which grasps ultimate causes or reasons.
- (b) Therefore composers truly know their art only if they can explain the reasons for their choices.
- (c) But artistic reasons should be based on nature, the nature of music consists of proportion, and proportion consists of number.
- (d) Therefore artistic reasons should be based on the nature of number.
- (e) But six (*senario*) is the first perfect number.
- (f) Therefore the *senario* is the primary source of harmony in music.
- (g) Therefore the art of musical composition must be guided by the harmony of the *senario*.
- (h) Therefore, in accordance with the nature of the *senario*, polyphonic works should employ complete harmonies; major five-three sonorities should express joy, and minor sonorities sorrow; cadences should occur on the I, V, and III degrees of a given mode.

The pattern of discourse outlined above proceeds along the path of Aristotelian deduction. Zarlino, after all, welcomed every opportunity of displaying his familiarity with the Stagirite's methods.¹⁰ But it is important to understand the nature of this one method in particular. Modern interpreters have shown that deduction, as conceived by Aristotle, serves a specific purpose. It is an instrument of teaching

¹⁰The proemium and pt. 1, chap. 1, of *Ist. harm.* echo several ideas from the beginning of Aristotle's *Metaphysica*. Both works expound on the attributes of scientific knowledge, stating that it should culminate in the understanding of ultimate causes.

rather than of research. One who reasons through a syllogism is not really in search of new knowledge. Through a syllogism one aims, rather, to demonstrate to others the rationality of what one already knows.¹¹ Zarlino's mode of discourse should thus be viewed in that light. It hardly needs mentioning that, prior to writing his treatise, he had already learned composition through actual practice, not through speculative deduction, under the guidance of Adrian Willaert. It was this practical legacy of Willaert that Zarlino aimed to transmit to his readers.¹² The deductive route that he took to rationalize certain rules of composition may have been completely wrong. But that does not necessarily bear on the authenticity and empirical basis of the rules themselves.

Zarlino's critics will argue, however, that he misused the deductive method. From their vantage point Zarlino appears as a system builder so enamored of number theory that he willfully misrepresents the commonly accepted practice of the day to make it conform to his ideal order. They ignore the fact that whenever he presents a rule of composition, he almost always cites examples from the existing repertory, or appeals to the judgment of the senses, or even allows room for many exceptions. This is hardly characteristic of one who spins arbitrary precepts out of abstract generalities. The rule concerning cadential pitches, which Dahlhaus dismisses as speculative and not based on empirical evidence, is in fact abundantly supplemented with reminders that alternative choices are possible and indeed sanctioned by convention.¹³ For the rest of this paper, however, I shall follow a different tack to show the strongly empirical foundation of Zarlino's teachings. I will argue that, far from misrepresenting the commonly

¹¹John Herman Randal, *Aristotle* (New York: Columbia University Press, 1960), 40; G. E. L. Owen, "Aristotle: Method, Physics, and Cosmology," in *Dictionary of Scientific Biography*, ed. Charles Coulston Gillispie (New York: Charles Scribner's Sons, 1970-80), 1:252; Jonathan Barnes (trans.), *Aristotle's Posterior Analytics* (Oxford: Clarendon Press, 1975), translator's introduction, x-xi.

¹²Zarlino, *Ist. harm.*, proemium, 2.

¹³See *ibid.*, pt. 4, chaps. 18-30.

accepted practice of his time in order to satisfy a speculative bias, he actually devised an intricate reversal of the ancient theory of proportions in order to reconcile it with his own era's newly developing harmonic consciousness.

On the surface, Zarlino's number theory may seem to be a mere display of classical learning. Most of his conceptual tools can indeed be traced back to the ancients. But he has an ulterior design: he will deftly transform the old theory to make it applicable to the music of his own day. He begins by recalling nine categories of number that have a bearing on music:¹⁴ (1) even, (2) odd, (3) manifoldly even,¹⁵ (4) prime or non-composite, (5) composite, (6) mutually prime,¹⁶ (7) square, (8) cube, and (9) perfect. The first eight of these do not require extensive comment. They simply represent the various types that a student would encounter when computing the numerical values of pitch intervals. (From the outset it should be noted that, in early music theory, numbers signify string lengths. Theorists had as yet no tools for measuring frequencies of vibration. Thus a larger number represents a lower pitch, a smaller number a higher pitch.)

It is the ninth category, the perfect number, which fascinates Zarlino to the point of obsession. A perfect number is that whose parts not only add up to itself but are also each capable of dividing it into equal, integral segments. For example, 6 contains the smaller members 1, 2, and 3 which, added together, produce 6 also. Moreover, 6 is divisible by 1, by 2, and by 3. Six, *senario*, is thus the first perfect number.¹⁷ But what Zarlino finds particularly remarkable is that the *senario* also happens to comprise what he calls the *harmonic* or

¹⁴Ibid., pt. 1, chap. 13.

¹⁵*Parimente pari*, that is, a number divisible by two equal parts, which in turn are likewise divisible.

¹⁶*Contra se primi*, that is, two numbers which cannot be divided by the same divisor except 1; for example, 9 and 10.

¹⁷*Senario* can be translated "that which contains six," or "the sixfold." After *senario*, the next perfect number is 28, whose parts are 1, 2, 4, 7, and 14; then 496, whose parts are 1, 2, 4, 8, 16, 31, 62, 124, and 248.

sonorous number. All the simple consonances that could ever occur in harmony or in song are contained within that number. The octave is produced by two string lengths related by the proportion 2:1, the perfect fifth by 3:2, the perfect fourth by 4:3, the major third by 5:4, and the minor third by 6:5. All other consonances are mere composites of those basic pillars. The major sixth, 5:3, is a composite of 5:4 and 4:3; the minor sixth, 8:5, is a composite of 6:5 and 4:3; and so on for consonances larger than the octave. And now the theorist makes a poetic leap. Behold, he says, how the wonderful perfection of the *senario* is manifest not only in art but also in nature itself.¹⁸ Of the twelve signs of the zodiac, six lie above the hemisphere and six lie below. Six planets move along the zodiac. There are six circles in the heavens. There are six qualities of substance, six natural states, six types of motion, and so on. Prodigies such as these show that with nature's blessing the *senario* supersedes the *quaternario* (the fourfold) as the favored repository of musical consonance.¹⁹

As the *senario* governs the nature of harmony, it extends its influence to the way musical pieces are composed. This Zarlino attempts to show in Part Three, the practical section of the *Istitutioni*, where he establishes a mathematical norm for determining the affective power of the musical modes. Certain modes are lively and joyful, "because in them we frequently hear the consonances arranged according to the nature of the sonorous number."²⁰ Here the theorist goes beyond the traditional awareness of individual consonant intervals and enters the realm of three-note chordal sonorities. By

¹⁸*Ist. harm.*, pt. 1, chap. 14.

¹⁹Pythagoras and his followers recognized as consonant only those intervals whose proportions are expressed by numbers within the *quaternario*. For them thirds and sixths were dissonant. The difference between the Pythagoreans' and Zarlino's systems of tuning has been sufficiently explained in earlier studies. See Claude V. Palisca, "Theory," in *New Grove Dictionary of Music and Musicians*, ed. Stanley Sadie (London: Macmillan & Co., 1980), 18:754-56; Cecil Adkins, "Monochord," *New Grove*, 12:495.

²⁰"... conciosia che in essi udimo spesse fiata le Consonanze collocate secondo la natura del Numero sonoro." *Ist. harm.*, pt. 3, chap. 10, 182.

“arrangement” of consonances he means the stacking together of two intervals, producing a more complex harmonic structure. Specifically, he calls attention to the five-three chord and points out a crucial distinction between two arrangements of the imperfect thirds which divide the perfect fifth. The major third can be positioned below the minor third, producing a major five-three chord; or the minor third can lie below the major, producing a minor chord.

As Zarlino has pointed out, consonances arranged “according to the nature of the sonorous number” evoke lively and joyful feelings. He is referring to the major five-three chord. To understand his reasoning, we must review the notion of “proportionality.”²¹ Just as a two-note interval consists of a proportion between two string lengths expressed in numbers, so a three-note chord creates a compound relationship called a proportionality. The latter results when a proportion, such as 2:1 (or 4:2), is divided or “mediated” by a middle term, namely, 3, producing the formula 4:3:2. This row of numbers signifies that the large proportion, 4:2, is broken into smaller ones, namely, 4:3 and 3:2.²² Thus proportionality is sometimes referred to as a kind of mediation. The term itself, proportionality, denotes a proportion between two proportions.²³

In classical theory, there are three types of proportionality in music: (1) the geometric, (2) the arithmetic, (3) and the harmonic. In geometric proportionality, such as 4:2:1, the first proportion (4:2) is in a proportional relationship of equality with the second (2:1). In arithmetic proportionality, such as 4:3:2, the difference between the terms of the first proportion ($4 - 3 = 1$) is equal to the difference between the terms of the second ($3 - 2 = 1$). In harmonic proportionality, such as 6:4:3, the proportion between the two differences (2:1) is equal to the proportion between the outer terms

²¹*Ist. harm.*, pt. 1, chaps. 35-39.

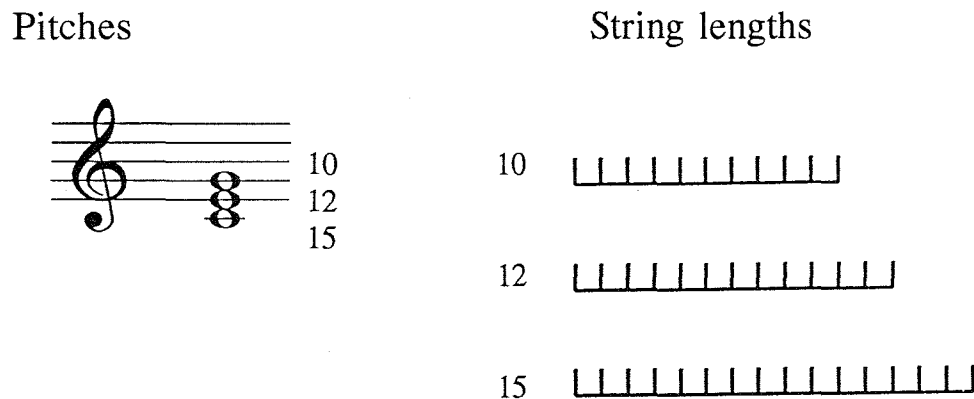
²²The size of a proportion is reckoned by dividing the larger term by the smaller.

²³The Greek mathematicians used the term *logos* for proportion and *analogia* for proportionality. In English, sometimes *ratio* and *proportion* are used in place of proportion and proportionality respectively.

(6:3).

Zarlino follows convention when he represents the harmonic proportionality of the major five-three chord with the formula 15:12:10. That is to say, the perfect fifth (15:10)²⁴ is mediated by a major third at the bottom (15:12)²⁵ and a minor third on top (12:10).²⁶ See Example 1.

Example 1. Harmonic Proportionality




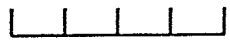


The minor five-three chord, on the other hand, is arranged in an arithmetic proportionality of 6:5:4. Here the perfect fifth (6:4) is mediated by a minor third at the bottom (6:5) and a major third on top (5:4). See Example 2.

²⁴15:10 is a compound form of 3:2.

²⁵15:12 = 5:4.

²⁶12:10 = 6:5. Since the numbers signify string lengths, the corresponding pitches become higher as the numbers become smaller. See Example 1. In the harmonic proportionality of 15:12:10, the proportions of the constituent imperfect thirds (15:12 and 12:10), if taken separately, can be expressed in smaller numbers within the *senario* (5:4 and 6:5 respectively). But to express the proportionality in a continuous row of three numbers, the larger numbers (15:12:10) have to be used.

Example 2. Arithmetic Proportionality

Pitches		String lengths
	4 5 6	4 
		5 
		6 

The difficult question that now arises is why Zarlino considers the arrangement of numbers in harmonic proportionality to be more “natural” than the arrangement in arithmetic proportionality. His exact words in Part Three of the *Istitutioni* seem to imply that the reason is too obvious to be stated explicitly, and he is caught arguing in circles:

These modes are very gay and lively, because in them we frequently hear the consonances arranged according to the nature of the sonorous number [le Consonanze collocate secondo la natura del Numero sonoro], that is, the fifth harmonically divided into a major and minor third; this greatly pleases the ear. I say that in these [modes] the consonances are arranged according to the nature of the sonorous number, because the consonances are placed in their natural positions [le Consonanze sono poste ne i loro luoghi naturali]. . . . In the other modes, then . . . the fifth is arranged otherwise. It is arithmetically divided by a middle note, in such a way that often we hear the consonances arranged contrary to the nature of the said number [molte volte udimo le Consonanze poste contra la natura del

nominato Numero].²⁷

Offhand a reader might think that Zarlino's idea of an arrangement "according to the nature of the sonorous number" is not all that problematic. The reader might reason that in a major five-three chord, the major third (5:4) is positioned below, and is therefore prior to, the minor third (6:5). Thus, seemingly in accordance with nature, the numbers do not exceed the *senario*, and they appear to be arranged from smaller to larger. But such is not the case. One should be reminded again that numbers represent string lengths. If the major third is to be positioned at the bottom of the chord, it should have longer strings than the minor third on top. The 5:4 proportion of the major third has to be changed to 15:12, and the 6:5 of the minor third should be changed to 12:10. The major five-three chord cannot be represented by numbers smaller than 15:12:10. 6:5:4 can refer only to the minor five-three chord. Hence, from a purely mathematical perspective, the numerical arrangement of the major chord, contrary to Zarlino's assertion, seems to be less in accord with the nature of the sonorous number.

Indeed, in the original tradition from which Zarlino learned the theory of numbers, arithmetic proportionality is accorded "natural" precedence over harmonic. The second-century Neopythagorean Nicomachus of Gerasa reports that three types of proportionality—arithmetic, geometric, and harmonic—were "acknowledged by all the ancients, Pythagoras, Plato, and Aristotle."²⁸ He then explains the order in which he will discuss them:

What, then, is the reason that we shall treat of this [arithmetic proportionality] first, and not another? Is it not clear that Nature [ἡ φύσις] shows it forth first before the rest? For in the natural series of simple numbers [ἐν γὰρ τῇ τοῦ ἀπλοῦ ἀριθμοῦ

²⁷*Ist. harm.*, pt. 3, chap. 10, 182.

²⁸Nicomachus of Gerasa, *Introduction to Arithmetic*, trans. Martin Luther D'Ooge (New York: Macmillan Co., 1926), bk. 2, chap. 22, 266.

φυσικῆ], beginning with 1, with no term passed over or omitted, the definition of this proportion[ality] alone is preserved. . . . Thus it is natural²⁹ that the mean which shares the name of arithmetic will not unreasonably take precedence of the means which are named for the other sciences, the geometric and harmonic. . . . As the first and original, therefore, since it is most deserving of the honor, let the arithmetic proportion[ality] have its discussion at our hands before the others.³⁰

The author implies that what is prior in logic is also prior in nature. Arithmetic proportionality has the advantage of an orderly, “natural” succession of numerical terms. Boethius’ sixth-century treatise on arithmetic, being in large part a translation of Nicomachus’ work, repeats the same idea quoted above.³¹ Zarlino certainly read Boethius’ version.

The original, classical, concept of proportionality, when applied to music, included no reference whatever to chord structures. Classical theorists had as yet no established notion of three-note chords, much less of major or minor five-three chords. They viewed proportionality—that is, the mediation of proportions—simply as a means of deriving smaller musical intervals from larger ones. Furthermore, according to the then prevailing Pythagorean tuning, thirds and sixths were considered dissonant because of their complex numerical proportions: 81:64 for the ditone (major third), 96:81 for the semiditone (minor third), and so on. It was only in the late fifteenth and early sixteenth centuries, in the writings of Ramos de Pareia, Giovanni Spataro, and

²⁹The word here is *εἰκότως*. The phrase is better translated: “Thus it is fitting that. . . .”

³⁰*Intro. to Arithmetic*, bk. 2, chap. 22, 267-68.

³¹“Quae igitur causa est, huiusmodi terminorum habitudinem, id est arithmetica, cunctis aliis proportionalitatibus antepone? Primum, quod hanc nobis in principio *ipsa numerorum natura et vis naturalis quantitatis* obponit. Huiusmodi enim proportiones quaeque ad terminorum differentias pertinent, ut paulo post demonstrabitur, in *naturalis primum numeri dispositione* cognoscimus.” Boethius, *De institutione arithmetica libri duo*, ed. Gottfried Friedlein (Leipzig: Teubner, 1867), bk. 2, chap. 43. Italics mine.

Lodovico Fogliano, that a tuning akin to Ptolemy's syntonic diatonic was reintroduced into music theory.³² To reflect the modern status of thirds and sixths as imperfect consonances, these intervals were now assigned much simpler proportions, which Zarlino eventually adopted. Spataro affirmed that the more euphonious effect of the new tuning was indeed audible,³³ and Fogliano dismissed the Pythagorean version as out of touch with modern practice.³⁴ Here is thus an instance of theory being guided, in fact altered, by practice. Zarlino will go even further in that direction.

Fogliano's teaching is of particular interest, because he makes some surprising remarks concerning proportionality, which depart from the classical doctrine. It is almost certain that Zarlino was influenced by him, because Zarlino singles him out as (in his mistaken opinion) the only recent author who has gone beyond Boethius to investigate the mathematical foundations of modern harmony. Fogliano, nevertheless, has his own peculiar point of view. He openly declares harmonic proportionality to be the only type worth studying in music:

Although in mathematics there are more proportionalities, such as arithmetic proportionality, geometric proportionality, harmonic proportionality, and many others, nevertheless, because I strive to be brief, I shall bypass all those that have no relevance whatsoever to the musician and speak here only about harmonic proportionality, according to which, as will be seen below, all harmony is arranged.³⁵

Fogliano focuses solely on harmonic proportionality, because in

³²See Palisca, "Theory," *New Grove*, 18:752-54.

³³Giovanni Spataro, *Errori de Franchino Gafurio da Lodi* (Bologna: Benedictus Hectoris, 1521), fols. 21v-22r.

³⁴Fogliano, *Musica theorica* (Venice: Io. Antonius & Fratres de Sabio, 1529; facs. ed., New York: Broude Brothers, 1969), sec. 2, chap. 1, fol. 11v.

³⁵*Ibid.*, sec. 1, chap. 9, fol. 9v.

his view it provides the most musically feasible route for obtaining the pitches of the diatonic scale. He observes that the successive harmonic mediation of various octaves and fifths in the monochord produces pitches that are most conducive to harmony. In the course of his demonstration, he alludes to the possibility of an opposite arrangement, namely, through arithmetic division. He quickly lets it be known which arrangement he considers to be better, naming the authority on which he bases his judgment—aural perception:

As we have shown in the first section, a duple proportion divided in harmonic proportionality [6:4:3] clearly yields a sesquialter proportion [6:4] between the larger numbers, and a sesquitercian proportion [4:3] between the smaller numbers. Consequently, the fifth [6:4] will occupy the lower position and the fourth [4:3] will occupy the higher. . . . Such a division produces the most perfect harmony. For if the fifth and fourth were arranged in an opposite manner within the octave, namely, the fourth at the bottom and the fifth on top [4:3:2], they will not produce as perfect a harmony. This is manifest to the ears [*ut patet ad sensum*].³⁶

Fogliano proceeds to explain the harmonically mediated fifth [15:12:10] and to compare it with its opposite counterpart [6:5:4]:

[A middle note] distant from the lower by a major third and from the higher by a minor tone [is] necessary for the completion of the harmony. For when the major third and minor third are arranged other than harmonically within the span of the fifth, they will not produce such a perfect harmony, as will be apparent to anyone with good perception [*ut cuilibet bene sensato patere potest*].³⁷

³⁶Ibid., sec. 3, chap. 1, fols. 33r-33v.

³⁷Ibid., fol. 33v.

Clearly then, well before Zarlino's own work, the major and minor five-three chords had already been subjected to mathematical speculation and linked to the system of harmonic and arithmetic proportionality. To Fogliano it is obvious that the major five-three chord is the more euphonious arrangement and therefore "better" than the minor. He may have been aware that the classical theory of numbers poses an obstacle by ranking arithmetic proportionality above harmonic. This does not faze him, and he sees no need to reconcile the two views. Cognizant of the fact that evolving musical practice can render classical theory partly obsolete, he is content to preserve aspects of the theory that are useful and ignore those that are not.

Zarlino is not one to let such a glaring theoretical anomaly remain unattended. Just as he sees the need to eulogize the *senario* in order to make it a worthy replacement for Pythagoras' *quaternario*, so now he has to show that harmonic proportionality is numerically more simple and more natural than the contrary arithmetic. He betrays his initial discomfort when he introduces the topic of proportionality for the first time, in Part One, chapter fifteen, of the *Istitutioni*. To demonstrate the orderly harmonic and numerical patterns that are produced within the limits of the *senario*, he singles out the harmonic mediation of the perfect octave and of the perfect fifth. When the proportion of the perfect octave is converted from 2:1 to 4:2, it becomes capable of being divided into two smaller proportions, which, he says, are in "harmonic proportionality" with each other.³⁸ We must assume that he conceives the resulting numerical sequence to be 4:3:2, because, according to him, the octave (4:2) is "divided into two parts, namely, into two consonances: first the diatessaron, which is found between 4 and 3, and [then] the diapente located between 3 and 2." (The problem with this, of course, is that 4:3:2 is an arithmetic, not harmonic, proportionality. But let us not get ahead of our author.) In turn, when the proportion of the perfect fifth is converted from 3:2 to 6:4, it becomes divisible into two parts, the middle term being the number 5. Zarlino lists the resulting two consonances as follows: ". . . a ditone

³⁸ ". . . sono divise in due parti in harmonica proportionalità." *Ist. harm.*, pt. 1, chap. 15, 32.

contained between 5 and 4 and a semiditone contained between 6 and 5.” In the original (1558) edition of the *Istitutioni*, he seems to take it for granted that his readers will readily accept all this as corresponding to harmonic proportionality. But in the third edition (1573), he inserts a comment which amounts to an acknowledgment of a logical problem:

I said that they are divided into two parts in harmonic proportionality; not in terms of the order of proportions, which in fact is arithmetical, but [simply] in terms of the proportion of the parts through the mediation of a middle term.

(Ho detto, che sono divise in due parti in Harmonica proportionalità: non già quanto all'ordine delle Proportioni: che veramente è Arithmetico: ma si bene quanto alla proportione delle parti, mediante il termine mezano.)³⁹

But how can that distinction solve the problem? The continuation of the above passage offers a still opaque explanation:

For they [namely, the 6:5 and 5:4 proportions, which are the ‘parts’ of 6:4] consist of quantities and proportions the same as those produced by a harmonic middle term or divisor, albeit in reverse order, as we shall see later.

(Percioche sono di tanta quantità & proportione: di quanta sono quelle, che da un mezano termine o divisore harmonico sono fatte: a ben che con ordine contrario: come più a basso vederemo al suo luogo.)⁴⁰

The promise to clear up the anomaly “later” is carried out in Chapter Forty. Here Zarlino observes again that when a perfect fifth is mediated harmonically to form a major five-three chord, the resulting

³⁹Ibid.

⁴⁰Ibid.

two imperfect thirds (6:5 and 5:4) will be no different from those that would result in the case of a minor five-three chord. The only variance will be the order in which the perfect thirds are positioned. In the case of the minor chord, the minor third will be below, and the major third will be above: 6:5:4. In the major chord, the positions will be reversed.⁴¹ The crucial question, which the author does not raise explicitly, but which he is obviously trying to solve, is: How can the numbers 6, 5, and 4 be rearranged to express the major chord? The reverse order, 4:5:6, will not do, because the numbers still represent string lengths, and therefore the lowest interval would still be the minor third (5:6 or 6:5). No juggling of numbers will make a difference.

Zarlino escapes the dilemma by sleight of hand. He points to a crucial distinction between harmonic and arithmetic numbers. Arithmetic numbers progress cumulatively by addition, such as 1, 2, 3, 4, etc. Harmonic numbers, because they represent a sonorous body, proceed by diminution through the division of a string into progressively smaller sections.⁴² Zarlino's line of argument is convoluted and his accompanying illustrations somewhat puzzling. What follows is my interpretation with my own illustrations. Zarlino's own words and illustrations are provided in the appendix to this article. My interpretation draws inferences that Zarlino did not explicitly make.

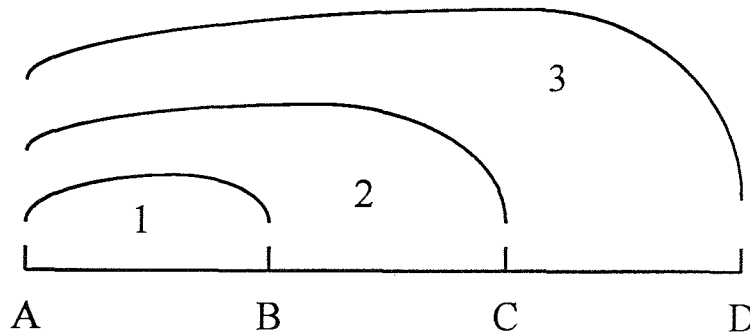
In an arithmetic progression (see Example 3), the starting point is the indivisible number 1, represented by the line AB. By doubling 1 we obtain 2, represented by AC. AB and AC are in duple proportion,

⁴¹“Et perche le parti delle Quantità sonore, dalle quali nascono le Consonanze, sono ordinate & divise dal Musico secondo la ragione de i numeri; i quali sono le loro forme; & in loro progressi sono senza dubbio aritmetici; de qui nasce, che non si vede alcuna divisione, ovvero Proportionalità harmonica, che appartenga a i concetti musicali, che non si ritrovi medesimamente nell'Arithmetica: percioche quelle proportioni, che ne dà l'Harmonica, l'istesse l'Arithmetica ne concede: ancora che in diverso modo.” *Ist. harm.*, pt. 1, chap. 40, 61.

⁴²“ . . . imperoche l'Arithmetica non attende ad altro, che alla moltiplicatione della Unità, ponendola nell'ordine naturale de numeri nel primo luogo, nel secondo il Binario; dal quale nasce immediatamente la Dupla proportione, il Ternario nel terzo, & così gli altri per ordine: ma l'Harmonica all'incontro attende alla sua diminutione: cioè alla diminutione, o divisione del corpo sonoro.” *Ist. harm.*, pt. 1, chap. 40, 61.

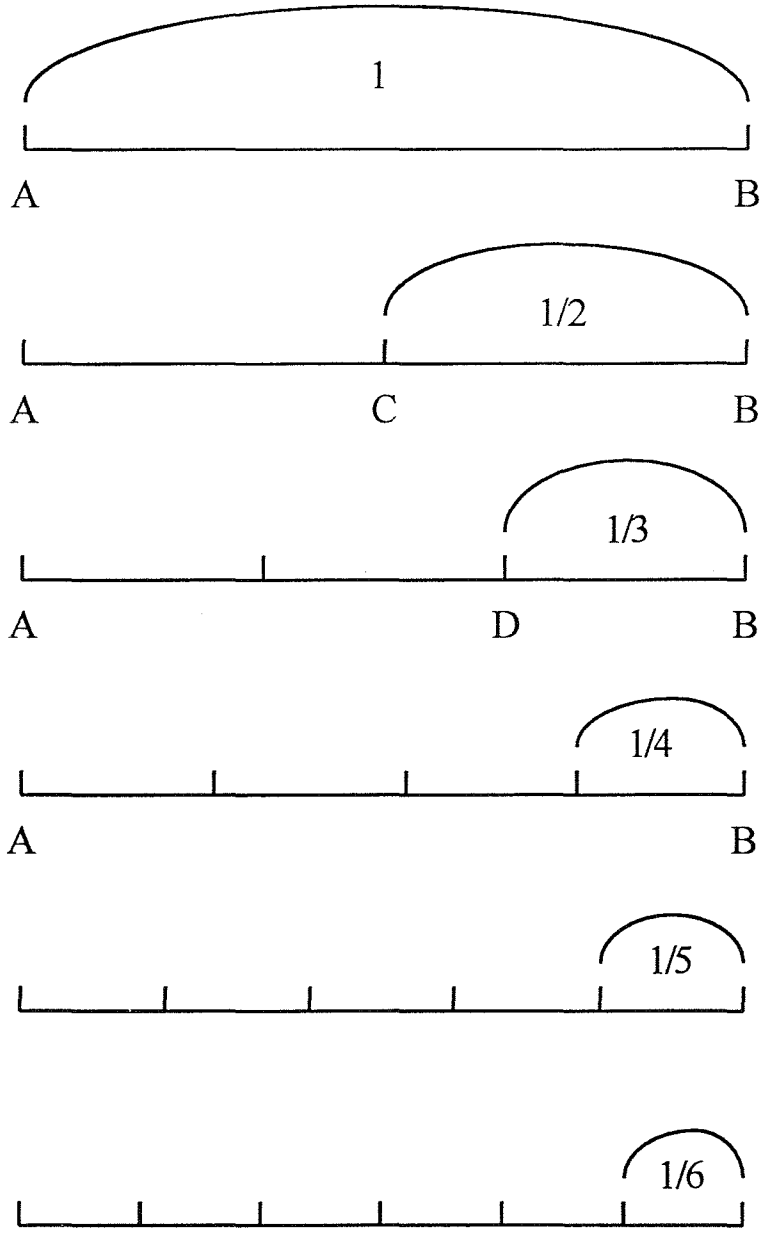
which is “first in the natural order of proportions.” Progressing to the next number, 3, represented by AD, we get a triple proportion between AB and AD. This proportion is mediated by AC, producing 3:2:1. Zarlino does not say so, but he implies that 3:2 is second in the “natural order of proportions.” The series can continue on through 4:3, 5:4, 6:5, and so forth.

Example 3. Arithmetical Numbers



In the case of a harmonic number progression (see Example 4), the starting point is again the line AB, but this time it undergoes progressive diminution rather than augmentation. By dividing it in half at the point C, we obtain CB, which is half the length of AB. Between AB and CB there is a duple proportion, which, as we recall, is first in the natural order of proportions. Next, AB is divided into three, yielding DB, which is one third of AB. Between CB (1/2) and DB (1/3) there is a 3:2 proportion, second in the natural order of proportions. Moreover, the triple proportion between AB (1) and DB (1/3) is mediated by CB (1/2), yielding two proportions in harmonic proportionality: $1 : 1/2 : 1/3$. Continuing this process of dividing the line AB, we obtain the proportions of the major five-three chord by

Example 4. Harmonic Numbers



taking the 3:2 proportion between $1/4$ and $1/6$ ⁴³ and mediating it by $1/5$. This yields 5:4 (= major third) between $1/4$ and $1/5$, which is fourth in the natural order of proportions, and 6:5 (= minor third) between $1/5$ and $1/6$, which is fifth in the natural order.

What comes out from all this is that there are two ways of representing the dichotomy between harmonic and arithmetic proportionality. The first way, which was commonly accepted before Zarlino's treatise, used exclusively integral numbers:

Harmonic Proportionality
(Major Five-Three Chord)
15:12:10

Arithmetic Proportionality
(Minor Five-Three Chord)
6:5:4

On the basis of the above formulation, arithmetic proportionality was obviously more sequentially ordered and therefore the more "natural" of the two. The second way, worked out by Zarlino, brings out the contrast between string divisions in music and integral numbers in arithmetic:

Harmonic Proportionality
(Major Five-Three Chord)
 $1/4 : 1/5 : 1/6$

Arithmetic Proportionality
(Minor Five-Three Chord)
6:5:4

Here the "natural order of the sonorous number" in harmonic proportionality becomes evident, although the terms consist of fractions instead of integers.⁴⁴ It should be pointed out, however, that Zarlino never explicitly used these fractions to represent harmonic proportionality. They are only implied in his discourse.

Having labored through this complicated rationalization in Part One of the *Istitutioni*, Zarlino takes it for granted in Part Three. He uses the more familiar 15:12:10 formula for the major five-three chord:

⁴³ $1/4 : 1/6$ is a 3:2 proportion, as can be seen from the fact that it is equal to $1/2 : 1/3$. Also, $1/4$ and $1/6$ of a string, sounded together, will produce a perfect fifth.

⁴⁴If a string is sixty inches long, then $1/4 = 15$, $1/5 = 12$, and $1/6 = 10$. Therefore $1/4 : 1/5 : 1/6 = 15:12:10$.

It is indeed wonderful that nature has ordered one consonance after another in such a manner that the form of the arithmetically divided fifth is found within the parts of the *senario*, in these terms: 6:5:4; in the opposite direction [con uno ordine contrario], the musician finds the two parts of the harmonically divided fifth in these terms: 15:12:10.⁴⁵

The wording of that passage, seeming to put arithmetic division ahead of harmonic, comes close to undermining Zarlino's thesis, but he assumes the reader's awareness of an earlier intricate discourse.

Zarlino has taken a convoluted route to show that the major five-three chord is more "natural" than the minor. The proof is clever. But how could he have gotten the idea? To have been able to devise such subtle detours in his argument, he had to have known ahead of time what conclusion he wanted to reach. He is now simply rationalizing what he has already learned from his studies under Willaert and from his experience as a practicing composer. Beneath the ivory-tower speculations lie a solid base of information regarding the actual practice of sixteenth-century composition.

⁴⁵*Ist. harm.*, pt. 3, chap. 15.

Appendix

Ist. harm., pt. 1, chap. 40, 61-63.

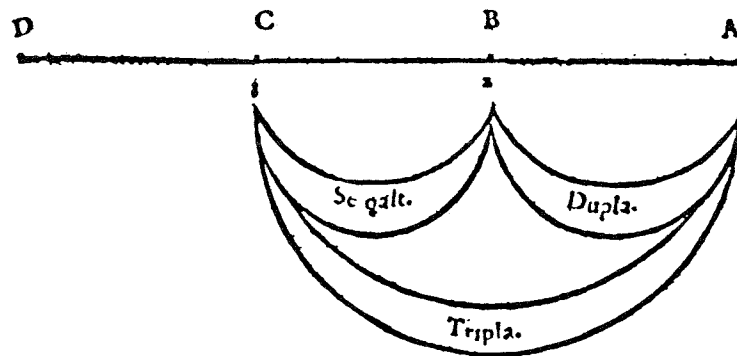
Et accioche queste cose siano meglio intese, verremo à darne uno essemplio. Poniamo la linea AB, la quale all'Arithmetico sia Unità; & al Musico corpo sonoro: cioè una chorda; & sia lunga un piede: dico che volendo dare un progresso arithmetico, sarà necessario lasciarla intera & indivisibile; imperoche procedendo arithmeticamente, non si concede che la Unità si possa dividere. Sia adunque tal progresso contenuto da tre termini in questo modo, che la proportione Tripla sia divisa dal mezano in due parti: onde sarà bisogno di procedere in tal modo: cioè di raddoppiar prima (se fusse possibile) la detta linea, nel modo che veggiamo la Unità esser raddoppiata nel Binario, il quale segue senza mezo alcuno la Unità: il perche havendola raddoppiata, haveremo la linea AC lunga due piedi. Se noi compararemo la linea AC raddoppiata alla linea AB, ritroveremo tra loro la proportione Dupla, che è prima nell'ordine naturale delle proportioni: si come si ritrova anco

And so that these things may be better understood, we will give one example. Let us posit a line AB, which would be the arithmetician's unity and the musician's sonorous body, namely, a string. Let it be one foot long. I say that in order to produce an arithmetical progression, it will be necessary to let it remain whole and indivisible, because in an arithmetical progression unity cannot be divided. Therefore let such a progression consist of three terms, so that a triple proportion is divided at the middle into two parts. From there the procedure will be as follows. First double (if possible) the said line, so that we see the unity doubled in the binary, which follows unity with nothing in between. By such doubling we will have the line AC, two feet long. If we compare the duple line AC with the line AB, we will find a duple proportion between them, which is first in the natural order of proportions. This is seen also in the case

ne i numeri tra il Binario & la Unità. Hora per dare il terzo termine di tal progressione, faremo la linea AC lunga tre piedi, di modo che arrivi in punto D: conciosia che il Ternario segue immediatamente il Binario; & haveremo tra la DA & la BA la proportione Tripla: imperoche la AD è misurata tre volte à punto dalla AB: over la AD contiene tre volte la AB: si come ne i numeri il Ternario contiene tre volte la Unità. Et cosi tal proportione resterà mediata & divisa in due parti dalla AC in una Dupla CA & BA; & in una Sesquialtera DA & CA, in proportionalità arithmetica: si come tra li termini nello esempio manifestamente si può vedere.

of numbers, between binary and unity. Now, to obtain the third term of such a progression, we make the line AC three feet long, so that it extends to point D. For ternary immediately follows binary. Between DA and BA we have a triple proportion, because AD measures exactly three times AB; or AD contains AB three times, just as in the case of numbers ternary contains unity three times. Thus at AC the proportion is mediated and divided into two parts in arithmetic proportionality, namely, duple between CA and BA, and sesquialter [3:2] between DA and CA. This can be seen clearly in the terms of the example. [See Figure 1]⁴⁶

[Figure 1]



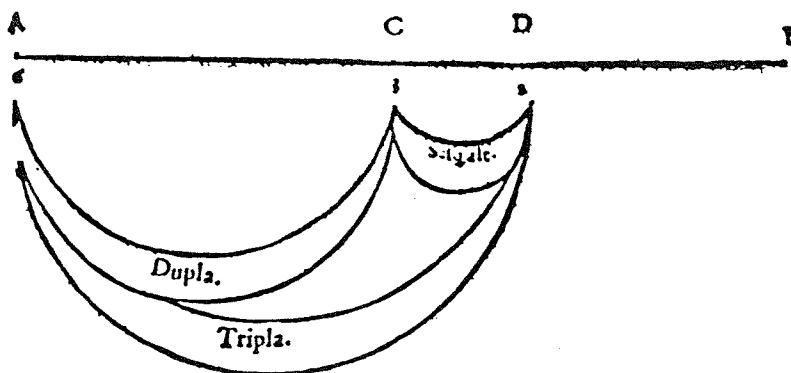
⁴⁶The numbers in the figure, from left to right, are 3, 2, 1. The label for the proportion between 3 and 2 is *sesquialtera*.

Ma se noi vorremo dare un Progresso harmonico, procederemo in questo modo. Diminuiremo prima la detta linea AB della sua metà in punto C; conciosiache la metà sia prima di ogn'altra parte; il che fatto dico, che tra la data chorda, o linea AB & la sua meta, la quale è la CB (per le ragioni, che altrove vederemo) si ritrova la proportione Dupla: che è la prima nell'ordine naturale delle proportioni. Diminuiremo dipoi la detta AB di due terze parti in punto D, & haveremo la proportione Sesquialtera: la quale è nel secondo luogo nell'ordine delle proportioni. La Sesquialtera dico tra CB & DB; & la Tripla ancora tra AB & DB; la quale dalla CB è mediata & divisa in due proportioni in harmonica proportionalità: come nell'esempio si vede.

But if we want a harmonic progression, we will proceed in the following manner. We will first divide the said line AB at the midpoint C. For the half is the first of all parts. Thus I say that between the given string or line AB and its half, which is CB, there is found a duple proportion (for reasons that we will see elsewhere). This proportion is the first in the natural order of proportions. Then we will shorten the said AB to two-thirds at point D, and we will have a sesquialter proportion, which is second in the order of proportions. I say that the sesquialter is between CB and DB, and the triple is between AB and DB. The triple proportion is mediated and divided by CB into two proportions in harmonic proportionality, as can be seen in the example. [See Figure 2]⁴⁷

⁴⁷The numbers from left to right, are 6, 3, 2. The label for the proportion between 3 and 2 is *sesquialtera*.

[Figure 2]



Et si come i termini della Progressione arithmetica sono Unità moltiplicate; così quelli dell'Harmonica sono il numero delle parti numerate nel Corpo sonoro, che nascono dalla sua divisione: essendo che in quella si considera la moltiplicatione della Unità contenuta in questo ordine 3.2.1; & in questa si considera la moltiplicatione delle parti nel soggetto diviso, contenute tra questi termini 6.3.2. Percioche se noi consideraremo il Tutto diviso nelle parti, ritroveremo che la linea CD è la minima parte della linea AB, & misura la AB sei volte intere; la CB tre volte; & la DB due volte. Hora si può vedere, che tra i maggior termini della Progressione harmonica sono contenute le proportioni maggiori & li suoni gravi; & tra li

Just as the terms of the arithmetic progression are unity multiplied, so those of the harmonic are the number of the sonorous body's numbered parts, which result from its division. For in the arithmetic progression one considers the multiplication of unity contained in the following order: 3.2.1. In the harmonic progression one considers the multiplication of the divided subject's parts contained in the terms 6.3.2. For if we consider the whole divided into parts, we will find that the line CD is the smallest part of the line AB; and CD measures AB six times, CB three times, and DB two times. Now it can be seen that among the harmonic progression's larger terms are contained the larger proportions and lower pitches; and among the smaller

minori le minori & li suoni acuti: conciosiache questi sono prodotti dalle chorde di minore estensione; & quelli da quelle di maggiore. Et potiamo anco vedere, che si come nell'Arithmetica (dato che si potesse fare al mostrato modo) si procederebbe dell'acuto al grave moltiplicando la chorda; cosi nell'Harmonica per il contrario si v` dal grave all'acuto diminuendola; & nella progressione, o proportionalità Arithmetica gli intervalli di minor proportione haverebbono luogo nel grave, contra la natura dell'Harmonia; il cui propio è, di havere i Suoni gravi di maggiore intervallo de gli acuti; & questi per il contrario di minore.

terms the smaller proportions and higher pitches. For the latter are produced by strings of shorter extension, and the former by strings of longer extension. And we can also see that while in the arithmetic progression (given that this is possible in the manner just shown) the procedure is from low to high by multiplying the string. In the harmonic progression, on the other hand, the procedure is from low to high by dividing the string. In the arithmetic progression or proportionality the intervals of smaller proportion occur in the low pitches, contrary to the nature of harmony. It is the property of harmony to have low pitches at larger intervals and, in contrast, high pitches at smaller intervals.