

# Stretto by Direct and Contrary Motion in Three Fugues by J. S. Bach

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Fugue literature contains many examples of stretto by direct motion but few by direct and contrary motion on the same theme. In the latter case, in the overlapping segments of such progressions, voices in stretto taken together may express only a tonic triad. In other cases, one of the voices may contain at least one melodic interval altered in size. A particularly effective method involves a leading voice augmented while a follower carries the subject in contrary motion, but in its original rhythm. In such an event, it is likely that an interval will be altered in the following voice. In any of these forms a stretto by contrary motion can add excitement to the performance of a well written fugue. That Bach employed these forms is illustrated by examples 1, 2a, and 2b below. (To clarify procedures and relationships described in the discussion to follow, all examples of stretto will show only the voices directly involved.)<sup>1</sup>

Example 1. Counterpoint VII from *The Art of Fugue, rectus-inversus*

45 Alto Rectus

Tenor Inversus

iv: G minor                      i

<sup>1</sup> Analysis of stretto out of context is appropriate because a subject to be used in stretto “is not constructed as a whole. The S(ubject) and its overlapping imitation is constructed step by step . . . as in canon.” Maurice Lieberman, *Creative Counterpoint* (Boston: Allyn and Bacon, 1966), 310.

In measures 45-48 the upper voices, alike in both rhythm and intervals, carry a stretto with the subject both in its original form (*rectus*) and inverted (*inversus*). In the overlapping segment the two voices together stress the notes of a G minor triad as a temporary tonic (m. 46, beat 2-m. 47, beat 1).

Example 2a. Subject, Fugue VIII from *WTC 1*



Example 2b. Fugue VIII from *WTC 1*, stretto by augmentation and inversion

Fugue VIII contains only one stretto by contrary motion (mm. 62-67). In example 2b, the leading voice, *rectus* augmented, appears in the traditional form of a tonal answer (beginning with a skip of a fourth, rather than a fifth). The follower, *inversus*, contains a skip of a third where the leader moves by step. (See the brackets marked “X.”) Compare both voices to example 2a, which shows the subject in its original form.

At least three fugues by Bach are exceptional. In each of the three, overlapping imitation of the subject is faithfully carried out by direct motion in some stretto passages and by contrary motion in others, and at least two harmonic functions are required as the voices overlap. Two of the three fugues are in *The Well-Tempered Clavier*: No. VI in D minor (Vol. I) and No. XXII in B-flat minor (Vol. II). The third is Counterpoint V in *The Art of Fugue*.

Apparently no explanation exists of a method to create such stretto treatments, nor is there a description of elements in a fugue subject that would permit such a method to be successfully applied. The eminent theorist

Ebenezer Prout, in his *Fugal Analysis* (1892), included an analysis of Contrapunctus V from *The Art of Fugue*, which he called a “fugue by inversion.” His analysis included the following remarks: “Though inversion is often employed incidentally in fugues . . . it is rarely used systematically throughout an entire fugue. . . . As a matter of fact, there are so few existing specimens of this form, at least by composers of eminence, that we have not sufficient material from which to deduce general rules.”<sup>2</sup> In his excellent book *Creative Counterpoint* (1966), Maurice Lieberman presents examples of stretto by direct and contrary motion from Fugue VI, *WTC* Vol. I. And, considering all types of stretto, he concludes that “working out a stretto passage is generally a matter of *systematic* experimentation.”<sup>3</sup>

Anyone familiar with stretto must agree with this statement. But in the fugues to be discussed here (which include those cited by Prout and Lieberman), similarities with regard to construction of the subjects and the application of melodic inversion suggest that, in these fugues, Bach had less cause to experiment than one might expect.

A stretto on any subject in its original form (*rectus*) will yield an equally good stretto on its inverted form (*inversus*) without difficulty.<sup>4</sup> But in stretto passages such as those cited above, in which the same stretto includes a *rectus* form in one voice and an *inversus* form in another, contrary motion can be acceptable only if certain restrictions are imposed on the structure of the subject and on the voice leading and harmonic intervals present in a stretto by direct motion.

### ***The Fugue Subjects***

The subjects in these three fugues, given in examples 3, 18 and 23, have certain characteristics in common, none of which would be remarkable alone. When taken together, however, they provide for all the stretto treatments cited above.

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<sup>2</sup> Ebenezer Prout, *Fugal Analysis: A Companion to “Fugue”* (London: Augener, Ltd., 1892), 84.

<sup>3</sup> Maurice Lieberman, *Creative Counterpoint* (Boston: Allyn and Bacon, 1966), 310.

<sup>4</sup> This procedure is described as early as 1597 in Thomas Morley’s *Plaine and Easie Introduction to Practicall Musick*, which is cited in H. C. Colles, ed., *Grove’s Dictionary of Music and Musicians*, 3rd ed., vol. 2 (New York: MacMillan, 1948), s.v. “Invertible Counterpoint,” by Sir Henry Walford Davies, 726. The procedure will be discussed and illustrated on pages to follow.

Figure 1. Significant characteristics shared by the three subjects are:

- 1) Each begins on scale step  $\hat{1}$  or  $\hat{5}$ , eventually rises to  $\hat{6}$ , then descends to end on a note of the tonic triad. The lowest scale step is either  $\hat{7}$  or  $\hat{1}$ .
- 2) At a reasonably slow harmonic rhythm, each subject can accept harmonization by the tonic triad and the leading-tone seventh chord. No other chords are required, except where a tied note may imply a third harmony.
- 3) Except in the case of a suspension, all connections of the two harmonies are made directly by chord members. When any member of the leading-tone seventh chord proceeds to a member of the tonic triad it does so by step. Also scale step  $\hat{7}$  always ascends to  $\hat{1}$  and step  $\hat{6}$  always descends to  $\hat{5}$ . However, a member of the tonic triad (scale step  $\hat{1}$ ,  $\hat{3}$ , or  $\hat{5}$ ) may move to a note of the leading-tone chord either by step or leap.
- 4) Within a harmony leaps involve only chord members. Stepwise motion is accomplished by unaccented passing and neighbor notes or by a suspension. Motion between the root and seventh of a leading-tone seventh chord, which would form an augmented second, does not occur.

A fugue subject that exhibits all of these characteristics may provide stretto treatments by both direct and contrary motion if melodic inversion is employed at the “common third.”

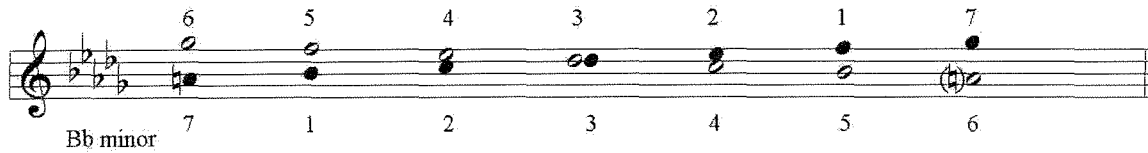
### ***Inversion at the Common Third***

In example 3, lines proceeding in contrary motion intersect on the third scale degree. Thus, they share a “common third.” Wherever the third scale step appears in a melodic line, the third scale step will reappear in its inversion at the common third.<sup>5</sup>

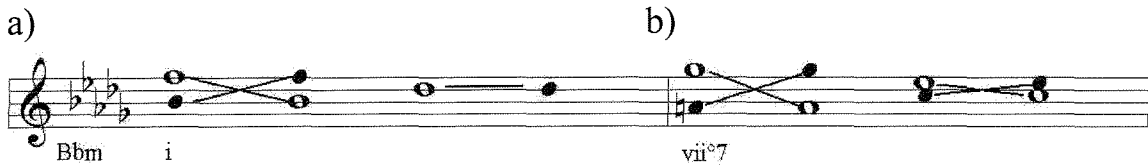
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<sup>5</sup> In this discussion the terms “common third” (or sixth or seventh), “commonality” and “note held in common” refer only to a note that appears at the same point or points in both a melodic line and its inversion.

## Example 3.



## Example 4.



Examples 4a and 4b illustrate the effect of inversion at the common third on harmony. In example 4a, the first and fifth scale degrees are interchanged and the third degree, midway between them, remains in place. Therefore, notes of the tonic triad in *rectus* also appear as members of that harmony in *inversus*.<sup>6</sup> The remaining notes of a major or minor scale make up the leading-tone seventh chord, and members of that harmony in *rectus* are exchanged for other notes of the same chord in *inversus* (example 4b).

Since in stretto one voice enters shortly after another, third scale degrees do not occur simultaneously but may comprise the second note (or third, fourth, etc.) in each line.

An important structural element shared by all three fugue subjects is that, at a reasonably slow harmonic rhythm, they may be harmonized by no more than two chords: a tonic triad and a leading-tone seventh chord (see figure 1, item 2). Ideally, a fugue subject will accommodate harmonic rhythm that in stretto permits vertical correspondence to be maintained between line segments that accept the tonic triad and those that accept the leading-tone seventh chord. In such a case, if an acceptable two-part stretto by direct motion is available, any of four different stretto treatments may be

<sup>6</sup> By inversion at the common third harmonization by the tonic triad is possible at corresponding points in *rectus* and *inversus*. Consequently, tonality can be expressed equally well by both voices. Since that is not true of inversion at any other commonality, it may account for the more frequent use of inversion at the common third in Baroque counterpoint.

derived from any other by inverting one or both voices at the common third. The four treatments are *rectus-rectus*, *inversus-inversus*, *rectus-inversus*, and *inversus-rectus*.

The subjects of Fugues VI and XXII from *The Well-Tempered Clavier* allow such vertical correspondence. Therefore, in the following discussion, the chord symbols “i” and “vii<sup>o7</sup>” will appear frequently.

These chord symbols represent a hypothetical harmonization; they may have little relationship to the harmonies that exist in a multi-voiced fugue. But in Fugues VI and XXII they reveal that voices in stretto *are aligned vertically as described above*, and as a result, *the voices in stretto do not produce harmonic intervals that preclude good counterpoint* (see figure 1, items 3 and 4).

These properties will be the basis for analysis of the simplest possible harmonization for each expression of a subject, whether isolated or in stretto. The analysis will consist of the chord symbols *i* and *vii<sup>o7</sup>* placed to indicate line segments that can be harmonized by one of these chords. In some cases, only one note of a tonic or leading-tone harmony will exist in a line segment designated by a chord symbol. Keep in mind that the chord symbol represents only a *possible* harmonization of that line segment.

In the overlapping portion of a stretto, wherever both voices at once accommodate either *i* or *vii<sup>o7</sup>*, observe that the notes provided by those voices can participate in a number of effective harmonizations. (In the three- or four-part textures of Fugues VI and XXII, added voices and/or accidentals provide a variety of good harmonizations.)

To clarify what is meant by “a reasonably slow harmonic rhythm” involving only tonic and leading-tone harmonies (figure 1, item 2) and to demonstrate how the four stretto treatments mentioned above may be derived from a subject appropriately constructed (and also from each other), the subject of Fugue XXII in B-flat minor (*WTC* Vol. II) will be used as a model (see example 5). It was chosen for two reasons: 1) it clearly exhibits all of the characteristics that enable a subject to produce stretto procedures by direct and contrary motion, and 2) none of the examples of stretto given here appear in Fugue XXII.<sup>7</sup>

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<sup>7</sup> They were composed by the author, who applied inversion at the common third in the simplest possible way. Later it will be shown that Bach applied that procedure in Fugue XXII in a logical but surprising way.

## Example 5. Subject, Fugue XXII

B♭m:      i   vii<sup>°7</sup>   i   vii<sup>°7</sup>   i   vii<sup>°7</sup>   i   vii<sup>°7</sup>   i   vii<sup>°7</sup>   i   vii<sup>°7</sup>   i

The subject starts on tonic, rises to the submediant (m. 4) and ends on the mediant. The leading tone, the lowest note, appears once (m. 2); the submediant, the highest note, occurs twice toward the end. There are no suspensions and no tied notes. All skips may be interpreted as lying between notes of either a tonic triad or a leading-tone seventh chord.

The subject will accept harmonization by tonic and leading-tone chords, neither of which is represented by more than two notes in any occurrence. Interpreted in this way, these chords alternate in beat values through the first two measures and in a syncopated harmonic rhythm at the end. The third measure will accommodate either pattern, which—as we shall see—allows vertical correspondence of like harmonies to be maintained throughout a stretto.

Progressions from one harmony to another are accomplished by chord members. When vii<sup>°7</sup>-i occurs, voice-leading is stepwise, and the leading tone resolves strictly to tonic. All leaps within a harmony lie between chord tones (mm. 2-4). Stepwise motion within a harmony involves nonharmonic tones, which are indicated with “+”. One neighboring tone (m. 4) and three passing tones (m. 3) appear; placement of the latter is determined by the harmonic rhythm in context. All other notes are chord tones.

Examples 6-13 illustrate relationships between *rectus* and *inversus* in stretto. Inversion at the common third is applied to the subject of Fugue XXII in the simplest way: starting with a stretto *rectus-rectus* at the octave with the follower entering after two beats, at the second appearance of a tonic chord member in the leading voice.

In example 6 the third scale step, D<sub>b</sub>, enters as the third note in each voice and is heard several times at corresponding points thereafter. Observe that the continuous eighth notes in the third measure of the subject allow the

harmonic rhythm of the progression  $i-vii^{\circ 7}$  to coincide with metrical beats (m. 3) or to be syncopated (m. 4).<sup>8</sup>

Inverting both voices of *rectus-rectus* at the common third produces the stretto progressions shown in examples 7a and 7b. Note that the third scale step,  $D\flat$ , maintains its placement as the diatonic axis of inversion in all three examples. The subject as *rectus* begins on tonic (ex. 6); therefore, *inversus* starts on the dominant.

Example 6. *Rectus-rectus* at the upper octave

Bbm: i vii<sup>o7</sup> i vii<sup>o7</sup> i vii<sup>o7</sup> i vii<sup>o7</sup> i

vii<sup>o7</sup> i vii<sup>o7</sup> i vii<sup>o7</sup> i

<sup>8</sup> To determine the simplest harmonization possible for each voice in a stretto, examine each leader and each follower separately throughout its entire length as demonstrated below.

In example 6, both voices accept the progression  $i-vii^{\circ 7}$  at the same time. Begin by observing all segments of the leading (lower) voice designated by the symbol “i”. In that voice, m. 1, you will find  $B\flat$  and  $D\flat$  (root and third of the tonic triad);  $B\flat$  again in m. 2;  $B\flat$ ,  $D\flat$ ,  $F$  (root, third, fifth) in m. 3; and  $F$ ,  $D\flat$  (fifth and third) in m. 4. All other notes over the symbol “i” in the leading voice are nonchord tones. Observe that no  $A$  or  $A\flat$  exists that might be a chordal seventh, and no  $G$  or  $G\flat$  that could change the analysis to include a submediant harmony.

Now compare contents of line segments labeled “vii<sup>o7</sup>” to notes that make up that chord. In the leading voice, m. 1, where “vii<sup>o7</sup>” exists you will find  $C$  (the chordal third);  $E\flat$ ,  $A$ ,  $C$  (fifth, root, third) in m. 2;  $E\flat$ ,  $C$  (fifth, third) in m. 3; and  $G\flat$ ,  $E\flat$ ,  $C$  (seventh, fifth, third) in m. 4.

When the leader is examined throughout its length,  $i$  and  $vii^{\circ 7}$  do indeed represent the simplest possible harmonization of that voice. By imitation at the upper octave, the same harmonic analysis applies to the follower.



Example 7a. *Inversus-inversus* at the upper octave

Bm: i vii<sup>o7</sup> i vii<sup>o7</sup> i vii<sup>o7</sup> i vii<sup>o7</sup> i

vii<sup>o7</sup> i vii<sup>o7</sup> i vii<sup>o7</sup> i

Example 7b. *Inversus-inversus* at the lower octave

Bm: i vii<sup>o7</sup> i vii<sup>o7</sup> i vii<sup>o7</sup> i vii<sup>o7</sup> i

vii<sup>o7</sup> i vii<sup>o7</sup> i vii<sup>o7</sup> i

The harmonic intervals of example 6 (*rectus-rectus*) are inverted in example 7a but not in example 7b. When both voices of a texture are melodically inverted at the same commonality (i.e., at their respective common thirds, sixths, sevenths, etc.) harmonic intervals in one passage are inverted in the other. This can be seen by comparing examples 6 and 7a. If double counterpoint at the fifteenth is also applied, the same harmonic intervals in both order and size appear in both passages, as can be seen by comparing examples 6 and 7b.

Drawing a satisfactory stretto *inversus-inversus* from a stretto *rectus-rectus* is a simple procedure. But certain acceptable progressions must be avoided in *rectus-rectus* in order to produce good counterpoint when only one voice is inverted. For example, if *rectus-rectus* displays contrary motion through consecutive harmonic intervals involving scale degrees interchangeable by inversion at the common third, the result in *rectus-inversus* and *inversus-rectus* will be parallel octaves or unisons, as shown in examples 8a and 8b.

### Example 8.

a) Where the stretto *rectus* contains:

b) Inverting one voice at the common third will produce:

The musical notation for Example 8 consists of two staves. The first staff, labeled 'a)', shows a sequence of four chords in the key of Bbm. The upper voice contains the notes 6, 5, 4, 3, and the lower voice contains the notes 7, 1, 2, 3. The second staff, labeled 'b)', shows the result of inverting one voice at the common third, resulting in parallel octaves.

This limitation on contrary motion is offset to some extent by the efficacy of parallel thirds and sixths. When parallel thirds or sixths occur and each interval comprises two members of either the tonic triad or the leading-tone seventh chord, inverting one voice at the common third creates a new set of intervals that will accept the same chord progression (see example 9).

Example 9.

a) Where a stretto contains: or:

Bbm vii°7 i vii°7 i vii°7 vii°7 i vii°7 i vii°7

b) One voice inverted at the common third will produce:

Bbm vii°7 i vii°7 i vii°7 vii°7 i vii°7 i vii°7

However, there are two possible intervals of a third or sixth that suggest neither tonic nor leading-tone harmony. One interval contains the third and sixth scale degrees, the other the fifth and seventh. When one of these intervals appears in isolation in a well-crafted stretto, inversion of one voice at the common third can generate intervals of a seventh with acceptable resolutions (see examples 10a and 10b). If they occur in succession, however, there will be an exchange of notes between intervals of a second or between intervals of a seventh and ninth, as shown in examples 11a and 11b.

Example 10.

a) Where a stretto contains: b) Inverting one part will produce:

Bbm 7 6 6 7 or 7 6 7 6  
5 4 1 2 1 2 1 2

Example 11.

a) But where a stretto contains: b) Inverting one part at the common third will produce:

1 7 6 5 5 6 7 1  
6 7 1 7 6 5 1 7

In all progressions of parallel thirds or sixths in examples 6, 7a, and 7b, each harmonic interval consists either of two members of the tonic triad or two members of the leading-tone seventh chord. Also, contrary motion exists only in the fourth measure (beats 2-3) of each example and does not involve notes exchanged by inversion at the common third. Therefore, neither unwanted parallelism (examples 8a and 8b) nor consecutive dissonances (examples 11a and 11b) can result from inverting one voice of *rectus-rectus* at the common third.

The stretto given in example 12 can be created by inverting at the common third the follower of *rectus-rectus* (example 6), the leader of *inversus-inversus* (example 7a), or the leader of 7b (if the voices are also interchanged). Imitation begins at the upper twelfth, and the third scale step, D $\flat$ , exists at the same points in each line. Here and in example 13, inversion at the common third produces a stretto by contrary motion and also establishes the relationship of the voices within it.

Example 12. *Rectus-inversus* at the common third

Bbm: i vii<sup>o7</sup> i vii<sup>o7</sup> i vii<sup>o7</sup> i vii<sup>o7</sup> i

vii<sup>o7</sup> i vii<sup>o7</sup> i vii<sup>o7</sup> i

The *inversus-rectus* shown in example 13 was taken from *rectus-inversus* (example 12) by inverting both voices at the common third and also inverting them texturally. *Inversus-rectus* could also be derived by inverting

at the common third the follower in example 7b, or by similarly inverting the follower in 7a and interchanging the parts. Finally, *inversus-rectus* could also be drawn from example 6 by inverting the leader at the common third and inverting the texture at the fifteenth.

Example 13. *Inversus-rectus* at the common third

The image shows two systems of musical notation for Example 13. Each system consists of a grand staff (treble and bass clefs) with notes and rests. Below the first system, the figured bass is: Bm: i vii<sup>o7</sup> i vii<sup>o7</sup> i vii<sup>o7</sup> i vii<sup>o7</sup> i. Below the second system, the figured bass is: vii<sup>o7</sup> i vii<sup>o7</sup> i vii<sup>o7</sup> i. The notes in the staves are in B-flat minor, and the rhythm is 3/2. The first system has a treble staff with notes on G4, A4, Bb4, C5, D5, E5, F5, G5, A5, Bb5, C6, D6, E6, F6, G6, A6, Bb6, C7, D7, E7, F7, G7, A7, Bb7, C8, D8, E8, F8, G8, A8, Bb8, C9, D9, E9, F9, G9, A9, Bb9, C10, D10, E10, F10, G10, A10, Bb10, C11, D11, E11, F11, G11, A11, Bb11, C12, D12, E12, F12, G12, A12, Bb12, C13, D13, E13, F13, G13, A13, Bb13, C14, D14, E14, F14, G14, A14, Bb14, C15, D15, E15, F15, G15, A15, Bb15, C16, D16, E16, F16, G16, A16, Bb16, C17, D17, E17, F17, G17, A17, Bb17, C18, D18, E18, F18, G18, A18, Bb18, C19, D19, E19, F19, G19, A19, Bb19, C20, D20, E20, F20, G20, A20, Bb20, C21, D21, E21, F21, G21, A21, Bb21, C22, D22, E22, F22, G22, A22, Bb22, C23, D23, E23, F23, G23, A23, Bb23, C24, D24, E24, F24, G24, A24, Bb24, C25, D25, E25, F25, G25, A25, Bb25, C26, D26, E26, F26, G26, A26, Bb26, C27, D27, E27, F27, G27, A27, Bb27, C28, D28, E28, F28, G28, A28, Bb28, C29, D29, E29, F29, G29, A29, Bb29, C30, D30, E30, F30, G30, A30, Bb30, C31, D31, E31, F31, G31, A31, Bb31, C32, D32, E32, F32, G32, A32, Bb32, C33, D33, E33, F33, G33, A33, Bb33, C34, D34, E34, F34, G34, A34, Bb34, C35, D35, E35, F35, G35, A35, Bb35, C36, D36, E36, F36, G36, A36, Bb36, C37, D37, E37, F37, G37, A37, Bb37, C38, D38, E38, F38, G38, A38, Bb38, C39, D39, E39, F39, G39, A39, Bb39, C40, D40, E40, F40, G40, A40, Bb40, C41, D41, E41, F41, G41, A41, Bb41, C42, D42, E42, F42, G42, A42, Bb42, C43, D43, E43, F43, G43, A43, Bb43, C44, D44, E44, F44, G44, A44, Bb44, C45, D45, E45, F45, G45, A45, Bb45, C46, D46, E46, F46, G46, A46, Bb46, C47, D47, E47, F47, G47, A47, Bb47, C48, D48, E48, F48, G48, A48, Bb48, C49, D49, E49, F49, G49, A49, Bb49, C50, D50, E50, F50, G50, A50, Bb50, C51, D51, E51, F51, G51, A51, Bb51, C52, D52, E52, F52, G52, A52, Bb52, C53, D53, E53, F53, G53, A53, Bb53, C54, D54, E54, F54, G54, A54, Bb54, C55, D55, E55, F55, G55, A55, Bb55, C56, D56, E56, F56, G56, A56, Bb56, C57, D57, E57, F57, G57, A57, Bb57, C58, D58, E58, F58, G58, A58, Bb58, C59, D59, E59, F59, G59, A59, Bb59, C60, D60, E60, F60, G60, A60, Bb60, C61, D61, E61, F61, G61, A61, Bb61, C62, D62, E62, F62, G62, A62, Bb62, C63, D63, E63, F63, G63, A63, Bb63, C64, D64, E64, F64, G64, A64, Bb64, C65, D65, E65, F65, G65, A65, Bb65, C66, D66, E66, F66, G66, A66, Bb66, C67, D67, E67, F67, G67, A67, Bb67, C68, D68, E68, F68, G68, A68, Bb68, C69, D69, E69, F69, G69, A69, Bb69, C70, D70, E70, F70, G70, A70, Bb70, C71, D71, E71, F71, G71, A71, Bb71, C72, D72, E72, F72, G72, A72, Bb72, C73, D73, E73, F73, G73, A73, Bb73, C74, D74, E74, F74, G74, A74, Bb74, C75, D75, E75, F75, G75, A75, Bb75, C76, D76, E76, F76, G76, A76, Bb76, C77, D77, E77, F77, G77, A77, Bb77, C78, D78, E78, F78, G78, A78, Bb78, C79, D79, E79, F79, G79, A79, Bb79, C80, D80, E80, F80, G80, A80, Bb80, C81, D81, E81, F81, G81, A81, Bb81, C82, D82, E82, F82, G82, A82, Bb82, C83, D83, E83, F83, G83, A83, Bb83, C84, D84, E84, F84, G84, A84, Bb84, C85, D85, E85, F85, G85, A85, Bb85, C86, D86, E86, F86, G86, A86, Bb86, C87, D87, E87, F87, G87, A87, Bb87, C88, D88, E88, F88, G88, A88, Bb88, C89, D89, E89, F89, G89, A89, Bb89, C90, D90, E90, F90, G90, A90, Bb90, C91, D91, E91, F91, G91, A91, Bb91, C92, D92, E92, F92, G92, A92, Bb92, C93, D93, E93, F93, G93, A93, Bb93, C94, D94, E94, F94, G94, A94, Bb94, C95, D95, E95, F95, G95, A95, Bb95, C96, D96, E96, F96, G96, A96, Bb96, C97, D97, E97, F97, G97, A97, Bb97, C98, D98, E98, F98, G98, A98, Bb98, C99, D99, E99, F99, G99, A99, Bb99, C100, D100, E100, F100, G100, A100, Bb100, C101, D101, E101, F101, G101, A101, Bb101, C102, D102, E102, F102, G102, A102, Bb102, C103, D103, E103, F103, G103, A103, Bb103, C104, D104, E104, F104, G104, A104, Bb104, C105, D105, E105, F105, G105, A105, Bb105, C106, D106, E106, F106, G106, A106, Bb106, C107, D107, E107, F107, G107, A107, Bb107, C108, D108, E108, F108, G108, A108, Bb108, C109, D109, E109, F109, G109, A109, Bb109, C110, D110, E110, F110, G110, A110, Bb110, C111, D111, E111, F111, G111, A111, Bb111, C112, D112, E112, F112, G112, A112, Bb112, C113, D113, E113, F113, G113, A113, Bb113, C114, D114, E114, F114, G114, A114, Bb114, C115, D115, E115, F115, G115, A115, Bb115, C116, D116, E116, F116, G116, A116, Bb116, C117, D117, E117, F117, G117, A117, Bb117, C118, D118, E118, F118, G118, A118, Bb118, C119, D119, E119, F119, G119, A119, Bb119, C120, D120, E120, F120, G120, A120, Bb120, C121, D121, E121, F121, G121, A121, Bb121, C122, D122, E122, F122, G122, A122, Bb122, C123, D123, E123, F123, G123, A123, Bb123, C124, D124, E124, F124, G124, A124, Bb124, C125, D125, E125, F125, G125, A125, Bb125, C126, D126, E126, F126, G126, A126, Bb126, C127, D127, E127, F127, G127, A127, Bb127, C128, D128, E128, F128, G128, A128, Bb128, C129, D129, E129, F129, G129, A129, Bb129, C130, D130, E130, F130, G130, A130, Bb130, C131, D131, E131, F131, G131, A131, Bb131, C132, D132, E132, F132, G132, A132, Bb132, C133, D133, E133, F133, G133, A133, Bb133, C134, D134, E134, F134, G134, A134, Bb134, C135, D135, E135, F135, G135, A135, Bb135, C136, D136, E136, F136, G136, A136, Bb136, C137, D137, E137, F137, G137, A137, Bb137, C138, D138, E138, F138, G138, A138, Bb138, C139, D139, E139, F139, G139, A139, Bb139, C140, D140, E140, F140, G140, A140, Bb140, C141, D141, E141, F141, G141, A141, Bb141, C142, D142, E142, F142, G142, A142, Bb142, C143, D143, E143, F143, G143, A143, Bb143, C144, D144, E144, F144, G144, A144, Bb144, C145, D145, E145, F145, G145, A145, Bb145, C146, D146, E146, F146, G146, A146, Bb146, C147, D147, E147, F147, G147, A147, Bb147, C148, D148, E148, F148, G148, A148, Bb148, C149, D149, E149, F149, G149, A149, Bb149, C150, D150, E150, F150, G150, A150, Bb150, C151, D151, E151, F151, G151, A151, Bb151, C152, D152, E152, F152, G152, A152, Bb152, C153, D153, E153, F153, G153, A153, Bb153, C154, D154, E154, F154, G154, A154, Bb154, C155, D155, E155, F155, G155, A155, Bb155, C156, D156, E156, F156, G156, A156, Bb156, C157, D157, E157, F157, G157, A157, Bb157, C158, D158, E158, F158, G158, A158, Bb158, C159, D159, E159, F159, G159, A159, Bb159, C160, D160, E160, F160, G160, A160, Bb160, C161, D161, E161, F161, G161, A161, Bb161, C162, D162, E162, F162, G162, A162, Bb162, C163, D163, E163, F163, G163, A163, Bb163, C164, D164, E164, F164, G164, A164, Bb164, C165, D165, E165, F165, G165, A165, Bb165, C166, D166, E166, F166, G166, A166, Bb166, C167, D167, E167, F167, G167, A167, Bb167, C168, D168, E168, F168, G168, A168, Bb168, C169, D169, E169, F169, G169, A169, Bb169, C170, D170, E170, F170, G170, A170, Bb170, C171, D171, E171, F171, G171, A171, Bb171, C172, D172, E172, F172, G172, A172, Bb172, C173, D173, E173, F173, G173, A173, Bb173, C174, D174, E174, F174, G174, A174, Bb174, C175, D175, E175, F175, G175, A175, Bb175, C176, D176, E176, F176, G176, A176, Bb176, C177, D177, E177, F177, G177, A177, Bb177, C178, D178, E178, F178, G178, A178, Bb178, C179, D179, E179, F179, G179, A179, Bb179, C180, D180, E180, F180, G180, A180, Bb180, C181, D181, E181, F181, G181, A181, Bb181, C182, D182, E182, F182, G182, A182, Bb182, C183, D183, E183, F183, G183, A183, Bb183, C184, D184, E184, F184, G184, A184, Bb184, C185, D185, E185, F185, G185, A185, Bb185, C186, D186, E186, F186, G186, A186, Bb186, C187, D187, E187, F187, G187, A187, Bb187, C188, D188, E188, F188, G188, A188, Bb188, C189, D189, E189, F189, G189, A189, Bb189, C190, D190, E190, F190, G190, A190, Bb190, C191, D191, E191, F191, G191, A191, Bb191, C192, D192, E192, F192, G192, A192, Bb192, C193, D193, E193, F193, G193, A193, Bb193, C194, D194, E194, F194, G194, A194, Bb194, C195, D195, E195, F195, G195, A195, Bb195, C196, D196, E196, F196, G196, A196, Bb196, C197, D197, E197, F197, G197, A197, Bb197, C198, D198, E198, F198, G198, A198, Bb198, C199, D199, E199, F199, G199, A199, Bb199, C200, D200, E200, F200, G200, A200, Bb200, C201, D201, E201, F201, G201, A201, Bb201, C202, D202, E202, F202, G202, A202, Bb202, C203, D203, E203, F203, G203, A203, Bb203, C204, D204, E204, F204, G204, A204, Bb204, C205, D205, E205, F205, G205, A205, Bb205, C206, D206, E206, F206, G206, A206, Bb206, C207, D207, E207, F207, G207, A207, Bb207, C208, D208, E208, F208, G208, A208, Bb208, C209, D209, E209, F209, G209, A209, Bb209, C210, D210, E210, F210, G210, A210, Bb210, C211, D211, E211, F211, G211, A211, Bb211, C212, D212, E212, F212, G212, A212, Bb212, C213, D213, E213, F213, G213, A213, Bb213, C214, D214, E214, F214, G214, A214, Bb214, C215, D215, E215, F215, G215, A215, Bb215, C216, D216, E216, F216, G216, A216, Bb216, C217, D217, E217, F217, G217, A217, Bb217, C218, D218, E218, F218, G218, A218, Bb218, C219, D219, E219, F219, G219, A219, Bb219, C220, D220, E220, F220, G220, A220, Bb220, C221, D221, E221, F221, G221, A221, Bb221, C222, D222, E222, F222, G222, A222, Bb222, C223, D223, E223, F223, G223, A223, Bb223, C224, D224, E224, F224, G224, A224, Bb224, C225, D225, E225, F225, G225, A225, Bb225, C226, D226, E226, F226, G226, A226, Bb226, C227, D227, E227, F227, G227, A227, Bb227, C228, D228, E228, F228, G228, A228, Bb228, C229, D229, E229, F229, G229, A229, Bb229, C230, D230, E230, F230, G230, A230, Bb230, C231, D231, E231, F231, G231, A231, Bb231, C232, D232, E232, F232, G232, A232, Bb232, C233, D233, E233, F233, G233, A233, Bb233, C234, D234, E234, F234, G234, A234, Bb234, C235, D235, E235, F235, G235, A235, Bb235, C236, D236, E236, F236, G236, A236, Bb236, C237, D237, E237, F237, G237, A237, Bb237, C238, D238, E238, F238, G238, A238, Bb238, C239, D239, E239, F239, G239, A239, Bb239, C240, D240, E240, F240, G240, A240, Bb240, C241, D241, E241, F241, G241, A241, Bb241, C242, D242, E242, F242, G242, A242, Bb242, C243, D243, E243, F243, G243, A243, Bb243, C244, D244, E244, F244, G244, A244, Bb244, C245, D245, E245, F245, G245, A245, Bb245, C246, D246, E246, F246, G246, A246, Bb246, C247, D247, E247, F247, G247, A247, Bb247, C248, D248, E248, F248, G248, A248, Bb248, C249, D249, E249, F249, G249, A249, Bb249, C250, D250, E250, F250, G250, A250, Bb250, C251, D251, E251, F251, G251, A251, Bb251, C252, D252, E252, F252, G252, A252, Bb252, C253, D253, E253, F253, G253, A253, Bb253, C254, D254, E254, F254, G254, A254, Bb254, C255, D255, E255, F255, G255, A255, Bb255, C256, D256, E256, F256, G256, A256, Bb256, C257, D257, E257, F257, G257, A257, Bb257, C258, D258, E258, F258, G258, A258, Bb258, C259, D259, E259, F259, G259, A259, Bb259, C260, D260, E260, F260, G260, A260, Bb260, C261, D261, E261, F261, G261, A261, Bb261, C262, D262, E262, F262, G262, A262, Bb262, C263, D263, E263, F263, G263, A263, Bb263, C264, D264, E264, F264, G264, A264, Bb264, C265, D265, E265, F265, G265, A265, Bb265, C266, D266, E266, F266, G266, A266, Bb266, C267, D267, E267, F267, G267, A267, Bb267, C268, D268, E268, F268, G268, A268, Bb268, C269, D269, E269, F269, G269, A269, Bb269, C270, D270, E270, F270, G270, A270, Bb270, C271, D271, E271, F271, G271, A271, Bb271, C272, D272, E272, F272, G272, A272, Bb272, C273, D273, E273, F273, G273, A273, Bb273, C274, D274, E274, F274, G274, A274, Bb274, C275, D275, E275, F275, G275, A275, Bb275, C276, D276, E276, F276, G276, A276, Bb276, C277, D277, E277, F277, G277, A277, Bb277, C278, D278, E278, F278, G278, A278, Bb278, C279, D279, E279, F279, G279, A279, Bb279, C280, D280, E280, F280, G280, A280, Bb280, C281, D281, E281, F281, G281, A281, Bb281, C282, D282, E282, F282, G282, A282, Bb282, C283, D283, E283, F283, G283, A283, Bb283, C284, D284, E284, F284, G284, A284, Bb284, C285, D285, E285, F285, G285, A285, Bb285, C286, D286, E286, F286, G286, A286, Bb286, C287, D287, E287, F287, G287, A287, Bb287, C288, D288, E288, F288, G288, A288, Bb288, C289, D289, E289, F289, G289, A289, Bb289, C290, D290, E290, F290, G290, A290, Bb290, C291, D291, E291, F291, G291, A291, Bb291, C292, D292, E292, F292, G292, A292, Bb292, C293, D293, E293, F293, G293, A293, Bb293, C294, D294, E294, F294, G294, A294, Bb294, C295, D295, E295, F295, G295, A295, Bb295, C296, D296, E296, F296, G296, A296, Bb296, C297, D297, E297, F297, G297, A297, Bb297, C298, D298, E298, F298, G298, A298, Bb298, C299, D299, E299, F299, G299, A299, Bb299, C300, D300, E300, F300, G300, A300, Bb300, C301, D301, E301, F301, G301, A301, Bb301, C302, D302, E302, F302, G302, A302, Bb302, C303, D303, E303, F303, G303, A303, Bb303, C304, D304, E304, F304, G304, A304, Bb304, C305, D305, E305, F305, G305, A305, Bb305, C306, D306, E306, F306, G306, A306, Bb306, C307, D307, E307, F307, G307, A307, Bb307, C308, D308, E308, F308, G308, A308, Bb308, C309, D309, E309, F309, G309, A309, Bb309, C310, D310, E310, F310, G310, A310, Bb310, C311, D311, E311, F311, G311, A311, Bb311, C312, D312, E312, F312, G312, A312, Bb312, C313, D313, E313, F313, G313, A313, Bb313, C314, D314, E314, F314, G314, A314, Bb314, C315, D315, E315, F315, G315, A315, Bb315, C316, D316, E316, F316, G316, A316, Bb316, C317, D317, E317, F317, G317, A317, Bb317, C318, D318, E318, F318, G318, A318, Bb318, C319, D319, E319, F319, G319, A319, Bb319, C320, D320, E320, F320, G320, A320, Bb320, C321, D321, E321, F321, G321, A321, Bb321, C322, D322, E322, F322, G322, A322, Bb322, C323, D323, E323, F323, G323, A323, Bb323, C324, D324, E324, F324, G324, A324, Bb324, C325, D325, E325, F325, G325, A325, Bb325, C326, D326, E326, F326, G326, A326, Bb326, C327, D327, E327, F327, G327, A327, Bb327, C328, D328, E328, F328, G328, A328, Bb328, C329, D329, E329, F329, G329, A329, Bb329, C330, D330, E330, F330, G330, A330, Bb330, C331, D331, E331, F331, G331, A331, Bb331, C332, D332, E332, F332, G332, A332, Bb332, C333, D333, E333, F333, G333, A333, Bb333, C334, D334, E334, F334, G334, A334, Bb334, C335, D335, E335, F335, G335, A335, Bb335, C336, D336, E336, F336, G336, A336, Bb336, C337, D337, E337, F337, G337, A337, Bb337, C338, D338, E338, F338, G338, A338, Bb338, C339, D339, E339, F339, G339, A339, Bb339, C340, D340, E340, F340, G340, A340, Bb340, C341, D341, E341, F341, G341, A341, Bb341, C342, D342, E342, F342, G342, A342, Bb342, C343, D343, E343, F343, G343, A343, Bb343, C344, D344, E344, F344, G344, A344, Bb344, C345, D345, E345, F345, G345, A345, Bb345, C346, D346, E346, F346, G346, A346, Bb346, C347, D347, E347, F347, G347, A347, Bb347, C348, D348, E348, F348, G348, A348, Bb348, C349, D349, E349, F349, G349, A349, Bb349, C350, D350, E350, F350, G350, A350, Bb350, C351, D351, E351, F351, G351, A351, Bb351, C352, D352, E352, F352, G352, A352, Bb352, C353, D353, E353, F353, G353, A353, Bb353, C354, D354, E354, F354, G354, A354, Bb354, C355

nowhere to be found in Fugue XXII. Instead, imitation is introduced at the upper seventh rather than the octave, and by contrary motion at the lower thirteenth rather than the twelfth. In every case, the follower enters after one beat, while the leader expresses for the first time a member of the leading-tone seventh chord. The result is a series of superb stretto statements made possible by the progression  $i-vii^{\circ 7}$  as accepted by the subject and by a systematic application of inversion at the common third.

It has been established that when a melodic line begins on the first scale degree its inversion at the common third will begin on the fifth, and that these notes comprise the root and fifth of the tonic triad. But if that melodic line is recast to begin on any other scale step, in inversion at the common third *rectus* will start on the root of one triad and *inversus* on the fifth of another.

In Fugue XXII the leading voice as *rectus* enters on the root of the tonic triad and as *inversus* on the fifth of that harmony. Therefore the leader accepts harmonization by  $i$  and  $vii^{\circ 7}$  in either case. The relationship of  $i$  to  $vii^{\circ 7}$  is that of a triad to a seventh chord built upon the scale step immediately below the root of the triad. In Fugue XXII, that relationship between harmonies is also evident with respect to the following voice in every stretto. The follower, as *rectus*, enters on the root of the leading-tone triad and therefore will accept harmonization simply by  $vii^{\circ}$  and  $vi^{\flat 7}$ . By inversion at the common third the follower as *inversus* begins on the fifth of the supertonic triad, and therefore will accommodate harmonization by  $ii^{\circ}$  and  $i^7$  or  $I^7$ . As a result, in Fugue XXII, when leader and follower overlap in a two-part stretto, harmonic intervals are provided by vertical alignments of a triad with a seventh chord that contains all of the scale steps present in the triad. Examples of stretto taken from Fugue XXII will display chord symbols that identify those harmonies for each voice.

In example 14 the leader starts on tonic, and the following voice enters after one beat on the leading tone at the first occurrence of a member of  $vii^{\circ 7}$  in the leader. The follower accepts harmonization by  $vii^{\circ}$  and  $vi^{\flat 7}$ .<sup>9</sup>

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<sup>9</sup> A stepwise ascent from scale step  $\hat{6}$  to  $\hat{7}$  does not take place when the subject begins on tonic, but it does occur when the subject begins on the leading tone. In the following (upper) voice of example 14, the sixth scale degree is chromatically raised to smooth the ascent to the seventh (m. 28). Consequently, in the follower, certain line segments are labeled  $vi^{\flat 7}$  because, taken all together they present scale steps  $\hat{6}$ ,  $\hat{1}$ ,  $\hat{3}$ , and  $\hat{5}$ , which occur, respectively, only as  $G^{\sharp}$ ,  $B^{\flat}$ ,  $D^{\flat}$ , and  $F$ . In the leader, the sixth scale degree occurs

Example 14. Fugue XXII, *Rectus-rectus* at the upper seventh

27

vii° vi<sup>♯7</sup>    vii° vi<sup>♯7</sup> vii°    vi<sup>♯7</sup> vii° vi<sup>♯7</sup>

B♭m:    i    vii<sup>♭7</sup>    i    vii<sup>♭7</sup>    i    vii<sup>♭7</sup>    i    vii<sup>♭7</sup>    i

30

vii° vi<sup>♯7</sup>    vii° vi<sup>♯7</sup>    vii°

vii<sup>♭7</sup>    i    vii<sup>♭7</sup>    i

The root of the leading-tone harmony ( $A^{\sharp}$ ) occurs only once in the leading voice (m. 28); by imitation at the upper seventh the root of the submediant chord ( $G^{\sharp}$ ) appears but once in the follower, one beat later. Elsewhere, the remaining members of  $vi^{\sharp 7}$  comprise a tonic triad. Except for the second beat of measure 28, the two voices together can accept harmonization simply by tonic and leading-tone chords.

In example 14 the mediant ( $D^{\flat}$ ) is introduced in the leader as the third note, while in the follower it is the fourth. When these voices are inverted at the common third to create other approaches to stretto, all placements of the mediant in leader and follower are, of course, maintained.

Example 15 may be produced by inverting both voices of example 14 at the common third. The harmonic intervals of example 14 are inverted here, because the voices have not been interchanged. As *inversus* the follower starts on the sixth scale degree, the fifth of a triad on the supertonic, and

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only as  $G^{\flat}$ , as the seventh of  $vii^{\circ 7}$ . Therefore, throughout the overlapping portion of the stretto vertical correspondence exists between members of  $vii^{\circ 7}$  and  $vii^{\circ}$  and between members of  $i$  and  $vi^{\sharp 7}$ .

Example 15. Fugue XXII, *Inversus-inversus* at the upper ninth

67

$\text{Bm:}$   $ii^\circ$   $i^7$   $ii^\circ$   $I^7$   $ii^\circ$   $i^7$   $ii^\circ$   $i^7$

$i$   $vii^{o7}$   $i$   $vii^{o7}$   $i$   $vii^{o7}$   $i$   $vii^{o7}$   $i$

70

$ii^\circ$   $i^7$   $ii^\circ$   $i^7$   $ii^\circ$

$vii^{o7}$   $i$   $vii^{o7}$   $i$   $vii^{o7}$

accepts harmonization by  $ii^\circ$  and  $i^7$  or  $I^7$ .<sup>10</sup> Since  $ii^\circ$  in the follower is contained within  $vii^{o7}$  in the leader, this stretto could accept harmonization solely by chords built on tonic and the leading tone.

Where in stretto *rectus-rectus* the following voice contains the root of  $vi^{o7}$  (ex. 14, m. 28), the follower in *inversus-inversus* carries the third of  $I^7$ , a chord which, in context, is altered to serve as  $V^7/iv$  (ex. 15, m. 68). The resolution of that chord, as given in the four-part setting of the stretto, is indicated in parenthesis. In subsequent examples of stretto from Fugue XXII, harmony in context will be shown in parenthesis at this point.

In examples 14 and 15 parallel thirds and sixths prevail, and contrary motion is nowhere to be found. With no contrary motion in a stretto by direct motion, unwanted parallels cannot be produced by the inversion of one voice at the common third. (See examples 8a and 8b.) Also, in the second measure of these examples, where the follower as *rectus* contains the sixth scale degree and as *inversus* the seventh, intervals of a sixth and tenth

<sup>10</sup> In example 15, measure 68, chromatic alteration of the third scale degree results in a symbol " $I^7$ " in the follower where a symbol " $i$ " occurs in the leader. Keep in mind that these symbols do not constitute a harmonic analysis of the example. They serve only to indicate that each voice displays members of a tonic harmony at that point. Therefore, if one or both of these voices is inverted at the common third, a tonic harmony will still be acceptable.



exist in which notes of tonic and nontonic harmonies are combined. Scale degrees  $\hat{1}$  and  $\hat{6}$  (B $\flat$ -G) may be seen in example 14, m. 28, and scale steps  $\hat{5}$  and  $\hat{7}$  (F-A $\flat$ ) appear in example 15, measure 68.

Example 16. Fugue XXII, *Inversus-rectus* at the common sixth

80 Soprano  $i$   $vii^{\#7}$   $i$   $vii^{\#7}$   $i$   $vii^{\#7}$   $i$   $vii^{\#7}$   $i$

Tenor  $(I^6 IV vii^{o6})$

VII: A $\flat$ :  $vii^{\circ}$   $vi^7$   $vii^{\circ}$   $vi^7$   $vii^{\circ}$   $vi^7$   $vii^{\circ}$   $vi^7$

83  $vii^{\circ}$   $vi^7$   $vii^{\circ}$   $vi^7$   $vii^{\circ}$

The first stretto by contrary motion, *inversus-rectus* (ex. 16) may be derived by inverting the leading voice of example 14 at the common third, interchanging the parts, and placing the resulting stretto in the key of the subtonic, A-flat major. Since the leading voice implies the same chord progression in both *rectus* and *inversus*, harmonic symbols given for individual voices are the same in examples 14 and 16. However, because the leader and follower enter on the fifth and seventh scale degrees, respectively—not the fifth and first—the axis of inversion, or note held in common by these voices, is the sixth scale degree, F. That note occurs only once in each voice and is indicated by the symbol **6**. Where F appears in the follower there is a harmonic interval of seventh that includes members of both tonic and leading-tone harmonies: F-E $\flat$ , measure 81. At that point contextual harmonic analysis is shown in parenthesis.

Example 17. Fugue XXII, *Rectus-inversus* at the common seventh

89  $ii^\circ$   $i^7$   $ii^\circ$   $I^7$   $ii^\circ$   $i^7$   $ii^\circ$   $i^7$

$(V^{9-9b}/iv$   $iv$   $vii^\circ{}^7$ )

B $\flat$ m:  $i$   $vii^\circ{}^7$   $i$   $vii^\circ{}^7$   $i$   $vii^\circ{}^7$   $i$   $vii^\circ{}^7$   $i$

92  $ii^\circ$   $i^7$   $ii^\circ$   $I^7$   $ii^\circ$

$(V^7/iv$   $iv$ )

$vii^\circ{}^7$   $i$   $vii^\circ{}^7$   $I$

The *rectus-inversus* in example 17 may be derived from the stretto given in example 14 by inverting at the common third not the leader, but the following voice. Therefore, the harmonies implied by the following voices in the two examples differ. In Fugue XXII this stretto is set in a four-part texture. In that context, where the axis of inversion (the seventh scale degree, 7) appears in the follower as  $A\flat$ , the leader carries tonic as the root of  $V^{9b}/iv$  (m. 90).

The stretto progressions in examples 16 and 17 produce the same harmonic intervals. Either one could be drawn from the other by inverting both voices at the common third and applying double counterpoint at the fifteenth.

Example 18, *rectus-inversus*, is the final stretto in Fugue XXII. It presents imitation in contrary motion by voices paired in parallel thirds or sixths.

The soprano and tenor parts of this example are a textural inversion at the fifteenth of example 17. As usual, *rectus* (soprano) begins on the root of the tonic triad, while *inversus* (tenor) starts on scale degree  $\hat{6}$ , which can be seen as the fifth of a triad on the supertonic. The pairing of soprano and bass

Example 18. Fugue XXII, *Rectus-inversus*

96

i vii<sup>o7</sup> i vii<sup>o7</sup> i vii<sup>o7</sup> i vii<sup>o7</sup> i

III ii<sup>o7</sup> III ii<sup>o7</sup> iii<sup>o</sup> V<sup>7</sup> vi<sup>o</sup> V<sup>7</sup> vi<sup>o</sup>

ii<sup>o</sup> i<sup>7</sup> ii<sup>o</sup> i<sup>7</sup> ii<sup>o</sup> i<sup>7</sup> ii<sup>o</sup> i<sup>7</sup> ii<sup>o</sup> i<sup>7</sup>

Bbm: vii<sup>o</sup> vi<sup>o7</sup> vii<sup>o</sup> vi<sup>o7</sup> vii<sup>o</sup> vi<sup>o7</sup> vii<sup>o</sup> vi<sup>o7</sup>

Harmony in context: ii<sup>o</sup> III iv vii<sup>o</sup> V<sup>7</sup>/iv iv vii<sup>o7</sup> vi<sup>o7</sup> vii<sup>o7</sup> i

99

vi<sup>o7</sup> i vii<sup>o7</sup> i

V<sup>7</sup> vi<sup>o</sup> V<sup>7</sup> vi<sup>o</sup>

ii<sup>o</sup> i<sup>7</sup> ii<sup>o</sup>

vii<sup>o</sup> vi<sup>o7</sup> vii<sup>o</sup> vii<sup>o7</sup> i<sup>7</sup> V<sup>7</sup> i

voices represents invertible counterpoint at the tenth of example 17. *Inversus* in the bass begins on scale step 4̂, the fifth of the leading-tone triad. This form of *inversus* will accept the same harmonization as the *rectus* form of the followers, all of which begin on the root of vii<sup>o</sup> (see examples 14 and 16).

By this arrangement, bass and tenor progress in parallel thirds, and the soprano is imitated in contrary motion at the common seventh and the common sixth simultaneously. The seventh degree, A/A<sup>b</sup>, is heard as the fifth note in both soprano and tenor (m. 97). The sixth degree, G/G<sup>b</sup>, occurs as the twentieth note in both soprano and bass (m. 99). The alto voice, more freely added, reinforces the harmony implied by the leader first at a sixth and later at a third below.

In example 18, the harmonic functions suggested by the fugue theme in each voice are shown above and below the staves. (Harmonic functions in context are given below all others.) The simplest harmonization indicated for

each voice reveals that the only unwanted dissonance that can be produced by all three together is a simultaneous expression by the lower voices of the root of  $vi^{\circ 7}$  (G) and the seventh of  $i^7$  or  $I^7$  ( $A\flat$ ). Since these voices maintain an interval of a third between them, that alignment obviously cannot exist.

As noted above, the relationships between the soprano and the tenor and between the soprano and the bass can be derived from the stretto in example 17 by means of strict invertible counterpoint. The subject in the alto voice is altered, however, to avoid unacceptable dissonance.

Example 19.

a) b)

The musical notation for Example 19 consists of two staves, treble and bass clef, in a key signature of three flats (B-flat major or D-flat minor). The notation is divided into two parts, a) and b). Part a) shows two measures of music. The first measure has a 9th interval between the soprano and tenor voices. The second measure has a 14th interval between the soprano and tenor voices. Part b) shows two measures of music. The first measure has a 9th interval between the soprano and tenor voices. The second measure has an 11th interval between the soprano and tenor voices. The bass line in both parts consists of chords that move in parallel motion with the soprano line.

When four voices paired in parallel thirds or sixths proceed in contrary motion, a chord of four consecutive scale degrees will exist when outer parts are separated by certain intervals. Compare example 19a to measures 96-97 and example 19b to measures 97-100. Harmonies comprising four consecutive scale steps do not occur because of the change in position by the alto voice in measure 96. Such chords would be formed 1) if the alto were a third below the soprano in measure 97 when the tenor reached the seventh scale degree,  $A\flat$ ; and 2) if the alto were to supply a lower sixth each time the soprano reached the sixth degree,  $G\flat$ , in measure 99. The first of these chords corresponds to example 19b, where outer voices are separated by an eleventh (but more likely with  $G\flat$  rather than  $G^{\sharp}$ ), and the second to example 19a, where they are a fourteenth apart.

*Fugue VI in D Minor from "The Well-Tempered Clavier," Vol. I*

## Example 20. Subject, Fugue VI

Dm: i vii<sup>°7</sup> i vii<sup>°7</sup> i

Fugue VI is expressed in triple meter in a predominantly three-voiced texture. The subject begins on  $\hat{1}$ , ends on  $\hat{5}$ , and displays  $\hat{6}$  prominently as its highest note. The sixth degree occurs only once, as does the seventh, which serves only as a neighboring tone embellishing tonic in the second measure. Neither  $i$  or  $vii^{\circ 7}$  is represented by more than two of its members; the two members are a third apart in every case.

The subject permits a follower to enter in stretto during beat one of its second measure and thereby maintain vertical correspondence of line segments that accept  $i$  and those that accept  $vii^{\circ 7}$ . The follower enters at that point in all of the stretto statements in Fugue VI. As in Fugue XXII, the simplest harmonic progression accepted by any expression of the subject is a triad followed by a seventh chord built upon the next scale degree below the root of that triad.

Example 21a. Fugue VI, *Rectus-rectus*

V: Am i vii<sup>°7</sup> I vii<sup>°7</sup> i vii<sup>°7</sup> i

In Fugue VI stretto *rectus-rectus* at the upper octave occurs on the dominant (ex. 21a, mm. 17-20) and the tonic (ex. 21b, mm. 39-42). The two

passages differ only in tonal centers and in their treatment of the seventh degree, which is introduced as the leading tone in one and a subtonic in the other. (See m. 18 and m. 40, beat 1.) In the second measure of each example, the mediant is chromatically raised to lead strongly to subdominant.

Example 21b. Fugue VI, *Rectus-rectus*

39

Dm: i vii<sup>o7</sup> I vii<sup>o7</sup> i vii<sup>o7</sup> i

In example 22 (mm. 27-31), a stretto *inversus-rectus-inversus* exhibits entrances at the lower twelfth and the lower fourth. The third degree, F/F#, held in common, exists as the third (+) and sixth note in all three voices.

Example 22. Fugue VI, *Inversus-rectus-inversus* at the common third

27

Delayed resolution of suspension (S) alters harmony in context as shown in ( )

(iv ii<sup>o7</sup> vii<sup>o</sup>/iv + V/iv V<sup>6-5</sup>/iv iv)

Dm: i vii<sup>o7</sup> i vii<sup>o7</sup> i

Lower parts: vii<sup>o7</sup> I

Except for a suspension (S) in the leading voice, the initial *inversus-rectus* (mm. 27-30) could become the stretto in example 21b by inverting the leader (mm. 27-29) at the common third, interchanging the parts, and applying appropriate accidentals to notes F and C.

When the bass enters in measure 29 with the subject inverted, the stretto *rectus-inversus* is formed by the lower parts. This stretto is equivalent to the preceding *inversus-rectus* with both of its voices inverted at the common third. Note that the harmonic intervals of measure 28 are inverted in the lower voices of measure 29. As the stretto ends, the follower is altered to lead toward subdominant (m. 30). The harmony in context at that point is given in parenthesis between the staves.

There are three more stretto statements by contrary motion. In all of them the axis of inversion is the seventh, not the third, scale degree.

### Example 23.

a) Inversion at the common third

b) Inversion at the common seventh

Dm: vii<sup>°7</sup> i vii<sup>°7</sup> i vii<sup>°7</sup> i vii<sup>°7</sup> Gm: IV<sup>7</sup> V IV<sup>7</sup> V IV<sup>7</sup> V IV<sup>7</sup>

In a minor key, a stretto by contrary motion at the common third may be transformed into a similar stretto at the common seventh by altering scale degrees  $\hat{3}$  and  $\hat{7}$  (see examples 23a and 23b). The mediant maintains its position in the melodic lines but becomes a leading tone held in common in the key of the subdominant. In other words, except for the alteration of scale degrees, the same notes, in order, express a stretto both at the common third in tonic and at the common seventh in subdominant.

In Fugue VI, this property permits a subject that begins on D as the root of *i* to begin on D in the subdominant key as the root of *V*. (See examples 23a, 23b, and 24.) In this fugue, all stretto passages by contrary motion at the common seventh display the subject as *rectus* beginning on  $\hat{5}$ . In all but one case (to be dealt with later) the subject as *inversus* begins on  $\hat{2}$ . This permits harmonization of both *rectus* and *inversus* simply by *V* and *IV*<sup>7</sup>, as shown in example 23b. (Note that the root of the latter is one scale step below the root of the dominant triad.) By that hypothetical harmonization members of the tonic harmony can occur only as the two upper notes of a subdominant seventh chord. Consequently, tonic is not strongly expressed as the stretto ends. To strengthen a feeling of tonic or to approach a new tonal center, such passages are extended, or the follower is altered or abandoned

as the leader ends. In each of the following examples, tonic is heard briefly in the second measure, and is indicated by [i]. (In examples 24, 25, and 26 harmony in context will be indicated in parenthesis by functional symbols without inversions.)

Example 24. Fugue VI, *Rectus-inversus* at the common seventh (F#)

In example 24 a partial stretto *rectus-inversus* at the common seventh is set in the subdominant, G minor. The stretto occurs in the lower two of three voices, and the common seventh (F#) enters as the third note in each lower voice. In measure 35 the leader leaps above the follower, and is embellished so that the two parts proceed in parallel thirds. As a result of the embellishment, the common seventh, introduced as a passing tone (+) in the leading voice, becomes a member of the dominant triad in the follower (m. 35, beat 2). When the leader ends, the follower is abandoned (m. 36).

The third voice mirrors the follower and at the same time imitates the head of the leader at a tenth above. Imitation at the tenth would be impractical if the leader were stated in its original form, for then both voices would reach the final note, A, from a G below in parallel octaves. (See *rectus* in the original form in example 21b.) Except for the embellishment, the leader and its imitation *inversus* could be drawn from the stretto *rectus-rectus* in example 21b simply by adding sharps to any natural version of F and C and inverting the head of the follower at the common third.

Fugue VI contains two more examples of *rectus-inversus* at the common seventh. Both include the two lower voices in stretto given in example 24, but without embellishment.



Example 25. Fugue VI, *Rectus-inversus* at the common seventh (C#)

Dm: (V [i] vi° [i] V)

Example 25 involves all three parts seen in example 24, but it is set in the tonic key and the outer parts are interchanged. In measure 14 the follower is again mirrored, but by the bass, which imitates the leader at the lower thirteenth. The leading voice appears without embellishment, but parallel octaves between outer parts are avoided by a leap in the bass to A in measure 15. The follower, which appears complete, is altered at the end as the passage moves toward a new tonal center.<sup>11</sup>

Example 26. Fugue VI, *Rectus-inversus-inversus*

Dm: V IV7 V IV7 V VI V7 VI V7

Example 26 presents a stretto *rectus-inversus* at the common seventh, C#, (mm. 21-24) It consists of the two upper parts of example 25 interchanged at a distance of three octaves, but without alteration of any intervals in the

<sup>11</sup> Note that in example 25, where a skip of a sixth occurs in the leading voice (m. 14), a skip of a fifth takes place in the follower (m. 15).

following voice.<sup>12</sup> In measure 23 the follower becomes the leader of a second stretto, *inversus-inversus*, with imitation at a surprising interval, the lower fourteenth, rather than the octave. The entire passage (mm. 21-25) is set in a texture of two voices and the tonic key is clear throughout. In each of these two stretto progressions the leader will accept harmonization by a dominant triad and a subdominant seventh chord. As a result, the overlapping portion of *rectus-inversus* displays harmonic intervals drawn from those chords (m. 22).

In the stretto *inversus-inversus* (mm. 22-25), imitation at the lower fourteenth and a final note D, rather than B or B $\flat$ , contribute to a strong expression of tonic. More importantly, the root and third of iv<sup>7</sup> (G and B $\flat$ , upper voice, m. 23) combine with the fifth and third of V<sup>7</sup> (E-C $\sharp$ , lower voice) to create an unmistakable leading-tone seventh chord.<sup>13</sup> Finally, the third scale step of D minor, a neighboring tone in the subject, is stressed briefly in an octave as a chord tone (F, upper voice, m. 23).

Imitation at the lower fourteenth is not the only surprise afforded by the follower *inversus* in example 26. Considering D as tonic, the follower can be derived by inversion at the common fourth from the subject *rectus* of measures 21-23. That commonality occurs (as G $\sharp$  and G, respectively) as the eighth note in each of these theme statements, and nowhere else in Fugue VI, Fugue XXII, or Counterpoint V.

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<sup>12</sup> For the first time in a stretto by contrary motion at the common seventh, *inversus* appears complete with the root of iv<sup>7</sup> in place (G, in the follower, m. 23).

<sup>13</sup> Note that, without the tied B $\flat$ , there would be parallel motion from a diminished to a perfect fifth between mm. 23-24.

**Counterpoint V from “The Art of Fugue”**

## Example 27. Subject, Counterpoint V

Counterpoint V is in  $\frac{4}{4}$  meter and in four voices; its subject is a variant of the theme of *The Art of Fugue* inverted at the common third. The subject begins and ends on  $\hat{5}$ ; its lowest and highest notes are, respectively, 1 and 6, the latter a member of the leading-tone seventh chord. These two notes, which occur only once each, fall on strong beats. The remaining strong beats are occupied by members of the tonic triad. In the fourth measure the subject displays a tied note that accepts tonic harmony throughout, but in a stretto permits a number of different harmonizations.

Except for  $\hat{6}$  at the start of the third measure, members of the leading-tone harmony in the first three bars serve only as passing tones. In the fourth measure nonharmonic tones may be derived either from tonic or leading-tone chords, depending upon which of the two harmonizations given above exists at that point.

In stretto, the structure of this subject permits vertical correspondence to be maintained only where line segments accept tonic harmony. Thus, harmonic intervals comprising members of both  $i$  and  $vii^{\circ 7}$  exist in the third and fourth measures of every stretto by direct motion, and in the third bar of every stretto by contrary motion.

There are six stretto statements, all of which comprise two parts of a four-part texture. There are two involving the subject *rectus*, two involving the subject *inversus*, and two involving contrary motion. In these passages, in contrast to the excerpts from Fugues VI and XXII from *The Well-Tempered Clavier*, both melodic inversion and contrary motion occur only at the common third. Furthermore, imitation by direct motion occurs only at the octave, while imitation by contrary motion takes place only at the fourth or fifth. But the follower does not enter at the same point each time.

In the examples to follow, where members of tonic and leading-tone chords are combined, harmonies expressed by the complete four-part setting in the fugue are indicated in parenthesis below the staff. Elsewhere, “i” or “I” (inversions not indicated) represent the simplest possible harmonization of both voices.

The first stretto *rectus-rectus* (ex. 28a) is in the key of the submediant, B-flat major. The bass leads and is followed by the tenor at the upper octave after one and a half measures.

Example 28a. Counterpoint V, *Rectus-rectus*

47

VI: B $\flat$  I I vi I I (vi<sup>6</sup> ii<sup>6</sup>) I

In example 28b, the soprano and tenor carry the subject as it would appear in tonic. In context, added voices place greater emphasis on the relative major. The chord symbols provided for example 28b relate to the tonic D minor, however.

Example 28b. Counterpoint V, *Rectus-rectus*

69

Dm: i i (VI<sup>6</sup>) i (iv<sup>7</sup>) i i I (V<sup>6</sup>/iv)

The two *inversus-inversus* stretto treatments are in tonic, D minor, with the first note of the leader changed from a D to an E.<sup>14</sup> Except for that change of pitch, each treatment is comparable to a stretto *rectus-rectus* discussed above, with both voices inverted at the common third and interchanged.

Compare example 28a with 29a and example 28b with 29b: harmonic intervals are the same in size, and alignments of notes of tonic triads with notes of leading-tone chords occur at corresponding points.

Example 29a. Counterpoint V, *Inversus-inversus*

Dm: i i V<sup>2</sup> i i V<sup>7</sup> i i

Example 29b. Counterpoint V, *inversus-inversus*

Dm: i i (III<sup>+6</sup> i<sup>6</sup>) (III<sup>+6</sup> i<sup>6</sup>)

In examples 30a and 30b entries are only a half measure apart. Both examples exhibit contrary motion at the common third (3). The stretto in example 30a establishes the key of the mediant, while that of example 30b expresses subdominant. Except for the difference in tonal centers, either

<sup>14</sup> This kind of alteration occurs several times in the fugue; it first appears in a tonal answer (m. 10) and opens the theme with a skip of a fourth rather than a fifth. It may cause some confusion in analysis because in a stretto by direct motion imitation prevailing at the octave begins at the seventh or ninth, and in a stretto by contrary motion inversion at the common third begins not with the first note, but with the second.

stretto could engender the other through inversion of both voices at the common third.

Example 30a. Counterpoint V, *Inversus-rectus*

III: F I I (V<sup>6-5</sup> IV<sub>4</sub> ii<sup>b</sup>) (I<sup>6</sup>) I

Example 30b. Counterpoint V, *Rectus-inversus*

iv: Gm i i (IV<sup>7</sup> V<sub>4</sub> 4 i<sup>6</sup>) i

A progression of scale degrees  $\hat{5}-\hat{6}-\hat{5}$  takes place halfway through the subject *rectus*. By imitation in contrary motion at the common third, that progression creates parallel thirds or sixths made up of scale steps  $\hat{5}$  plus  $\hat{7}$  and  $\hat{1}$  plus  $\hat{6}$ . See m. 43 in example 30b (parallel thirds) and m. 35 in example 30a (parallel sixths). Where these parallel progressions occur, a counterpart by direct motion would entail an exchange of notes between intervals of a second, or between intervals of a seventh and a ninth. (See examples 11a and 11b.) Consequently, for these stretto passages no counterpart by direct motion could be acceptable.

**Conclusion**

To compose stretto statements by both direct and contrary motion on the same theme, one must consider limitations imposed upon the structure of the fugue subject. (See figure 1, items 1-4) One must also be aware of limitations regarding progressions of harmonic intervals resulting from inversion at the common third. (See examples 8a and b, 11a and b, and the table below.)

<b>In Stretto by Direct Motion</b>			<b>In Stretto by Contrary Motion</b>	
CONTRARY MOTION TO	ON SCALE STEPS	BECOMES	SIMILAR OR PARALLEL	ON SCALE STEPS
unison, octave	1 + 1		4 <sup>th</sup> , 5 <sup>th</sup>	1 + 5
	5 + 5		4 <sup>th</sup> , 5 <sup>th</sup>	1 + 5
2 <sup>nd</sup> , 7 <sup>th</sup> , 9 <sup>th</sup>	1 + 2		4 <sup>th</sup> , 5 <sup>th</sup>	1 + 4, 2 + 5
	4 + 5		4 <sup>th</sup> , 5 <sup>th</sup>	1 + 4, 2 + 5
3 <sup>rd</sup> , 6 <sup>th</sup>	2 + 7		4 <sup>th</sup> , 5 <sup>th</sup>	2 + 6, 4 + 7
	4 + 6		4 <sup>th</sup> , 5 <sup>th</sup>	2 + 6, 4 + 7
	5 + 7		2 <sup>nd</sup> , 7 <sup>th</sup> , 9 <sup>th</sup>	1 + 7, 5 + 6
	1 + 6		2 <sup>nd</sup> , 7 <sup>th</sup> , 9 <sup>th</sup>	1 + 7, 5 + 6
4 <sup>th</sup> , 5 <sup>th</sup>	1 + 4		2 <sup>nd</sup> , 7 <sup>th</sup> , 9 <sup>th</sup>	1 + 2, 4 + 5
	2 + 5		2 <sup>nd</sup> , 7 <sup>th</sup> , 9 <sup>th</sup>	1 + 2, 4 + 5
	3 + 6		4 <sup>th</sup> , 5 <sup>th</sup>	3 + 7
	3 + 7		4 <sup>th</sup> , 5 <sup>th</sup>	3 + 6

Working within these limitations, Bach demonstrates both his command of counterpoint and his apparently boundless inspiration. In Fugue VI chromatic alterations change relationships between melodic lines and tonal centers, and *inversus* is derived from *rectus* by inversion at the common fourth rather than the common third. Fugue XXII displays stretto passages with surprising intervals of imitation, and includes an example of a leader and a follower comprising four voices paired in parallel thirds or sixths. In Fugues VI and XXII imitations by direct and contrary motion enter at the same point; in Counterpoint V they enter at different points.

While these elements lend energy and variety to the music, a systematic use of inversion, both of melodic lines and textural, contributes to unity and

balance of structure. Truly, these three fugues must be counted among the finest examples of the art of Baroque counterpoint.

In *Grove's Dictionary of Music and Musicians* (1948), the composer, organist, and teacher Sir H. Walford Davies pays tribute to inspired composers in general and to invertible counterpoint in particular in a passage that might have been written with these fugues in mind:

Exact balance of that which we call construction with that which we call inspiration is rare indeed. . . . It is to the works of men who, like . . . Bach, reconcile both sides of the art that we may well turn for the true examples of such musical mechanism as invertible counterpoints. They bring their best inspiration to the best construction of which they, or rather their age, may be capable. They show us how to combine the joy of freedom with the dignity of restraint. They are servants of laws not less than we are, but they find their service perfect freedom. They . . . instil [*sic*] abounding life into every intellectual device. . . . They obey old laws and silently enact new ones, setting their own particular seal of permanence upon things hitherto only tentatively expressed or scarcely even apprehended, making their own fine attempts to express a perfect thing perfectly. Towards this high end the art of invertible counterpoint is not the least important contributor.<sup>15</sup>

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<sup>15</sup> H. C. Colles, ed., *Grove's Dictionary of Music and Musicians*, s.v. "Invertible Counterpoint," by Sir Henry Walford Davies, 732.