Quantitative Parameters of Spatial Dynamics in Musical Space

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Introduction

Upon Edgard Varèse's death in 1965, Pierre Boulez correctly predicted the future importance of the former's work when he remarked, "Farewell, Varèse, Farewell! Your time is finished and now it begins!"[1] Varèse was a man of great originality, and modern developments attest to the view that his music is indeed one of the main blocks that make up the foundation of twentieth century music. Earle Brown states that "there could not have been a Ligeti or a Penderecki without a Varèse."[2] We may add to the list Xenakis, Lutosławski, and Husa, among others. Indeed, there seem to be many composers today working in a post-Varèse idiom in which the spatial aspects of the music lend themselves easily to sometimes over-used metaphorical descriptions, many of which (somewhat ironically) have precise meanings and are physically measurable quantities in other disciplines. Even certain electronic/ambient artists have successfully applied various

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[1]Joan Peyser, *The New Music: The Sense Behind the Sound* (New York: Delacorte Press, 1970), 182.

[2]Ruth Julius, "Edgard Varèse: An Oral History Project," in *Breaking the Sound Barrier: A Critical Anthology of the New Music*, ed. Gregory Battcock (New York: E. P. Dutton, 1981), 281.

musical processes that occur frequently in this compositional style to recent works (for example, Steve Roach's *The Magnificent Void*). Efforts to develop a comprehensive analytical basis for Varèse's music include work by Robert P. Morgan, Chou Wen-chung, and Jonathan Bernard.[3] These analyses utilize either traditional concepts such as intervallic relationships, trichords, and thematic development, or more modern concepts such as sets, constellations, and aggregates, and their methods can be applied to much music of the post-Varèse idiom. These and other similar analytical methods provide valuable insight and are in general either qualitative or semi-quantitative in nature. (A fully quantitative method is assumed here to use quantities that are directly measurable and thus to yield a body of analytically useful numerical data.)

This study is based on the author's opinion that new analytical tools containing concepts that lend themselves easily to a more quantitative analysis would provide additional insight into the geometrical aspects of musical space in the works of many of these composers. On what shall these tools be based? The answer lies in certain words that are usually used in the previously mentioned metaphorical descriptions of music in the post-Varèse idiom (and even in a few of Varèse's titles as well). Those (including Varèse himself) who talk about this music in terms of volumes, densities, beams, masses, bands, and planes are hitting the target conceptually, for these terms truly are individual parameters that are adjusted continually to shape the spatial dynamics of musical space in these compositions. Indeed, Varèse's initial education in mathematics and engineering greatly influenced his carefully sculpted spatial music as well as his choices for titles: *Hyperprism, Octandre, Intégrales, Ionisation,* and *Density 21.5* are examples.

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[3]Robert P. Morgan, "Notes on Varèse's Rhythm," in *The New Worlds of Edgard Varèse*, ed. Sherman Van Solkema (New York: Institute for Studies in American Music, 1979); Chou Wen-chung, "*Ionisation*: The Function of Timbre in its Formal and Temporal Organization," in *The New Worlds of Edgard Varèse*, ed. Sherman Van Solkema, and "Varèse: A Sketch of the Man and His Music," *Musical Quarterly* 52 (1966): 151–70; and Jonathan W. Bernard, *The Music of Edgard Varèse* (New Haven: Yale University Press, 1987), and "A Theory of Pitch and Register for the Music of Edgard Varèse" (Ph.D. diss., Yale University, 1977).

Rather than limiting Varèse's own terms for the various techniques of combining and opposing sound materials to those of metaphorical indications of the general effect of the music or relying on approximate methods, as some analysts have done, the author believes that these concepts can and should be precisely defined and utilized to describe and compare this music not only qualitatively but quantitatively as well.[4] Indeed, Wen-chung and Bernard have successfully used some of Varèse's own terminology as actual forms of measurement, and this essay attempts to build on their successes by discussing some measurable parameters of musical space that combine concepts from physics with the inherent spatial nature of the music itself. This is accomplished by making analogies to certain physical systems governed by similar mathematical models. The result of this approach is that the majority of what follows defines measurable properties such that the interesting dynamics of a piece may be compared and contrasted quantitatively as well as qualitatively. When applied to portions of music in the style for which this analytical method was conceived, it is found that this approach yields analytically useful data that can shed light into the spatial dynamics of these compositions. While this study is devoted mainly to the presentation and discussion of these parameters, illustrative examples are provided from works by Krzysztof Penderecki, whose personal and highly unique graphic notation seems to invite the type of quantitative, geometric analysis used herein.

The Structure of Musical Space

There are exactly five parameters that characterize sound: pitch, duration, timbre, intensity, and direction in space. All other musical elements, such as attack, release, and rhythm, are combinations and

[4]For an example of the use of metaphorical indications, see Sherman Van Solkema, introduction to *The New Worlds of Edgard Varèse*, vii. For examples of the use of approximate methods, see Martin Gumbel, "Versuch an Varèse *Density 21.5*," *Zeitschrift für Musiktheorie* 1 (1970): 31–38; Robert Erickson, *Sound Structure in Music* (Berkeley: University of California Press, 1975), 47–57; and John Strawn, "The *Intégrales* of Edgard Varèse: Space, Mass, Element, and Form," *Perspectives of New Music* 17 (1978): 138–60.

variations on these five. In this study *musical space* is an abstraction that deals only with the parameters of pitch and duration. (The intensity parameter will be incorporated while discussing *dynamic musical space*, but this should not be considered now.)

Much has been written on the philosophy of space-time in music.[5] However, musical space is treated herein as a purely two-dimensional mathematical space with axes defined by the musical parameters of pitch (vertical) and duration (horizontal). (This musical space, then, has nothing to do with the parameter *direction in space*, which simply refers to the three-dimensional direction vector from the sound source to the receiver.) Since we can move and juxtapose objects in physical space, we can likewise do the same to tones, or *sound objects*, in musical space. This causes changes in their pitch and duration relative to a universal scale and/or to each other. The horizontal dimension is restricted to left-to-right motion at a constant speed since time progresses forward only at a set rate. This limitation is helpful, however, because it means that musical space is a functional coordinate system in which time is the independent variable and pitch is the dependent variable. This system, then, is a *noninjective* mapping of time to an ensemble of pitches, since neither the *existence* of a pitch nor its *uniqueness* can be guaranteed at any given time within the domain of a composition. It is possible (as will be demonstrated), however, to describe this system alternately with a small collection of *injective* functions, which are one-to-one mappings that exist for all time within this domain. These are the functions that have natural counterparts in physics and which may be used to quantify locally a given passage of music. Whether or not this yields useful information depends on how much the style of the composition conforms to this system of measurement.

The type of post-Varèse music in which the *geometrical* character (that is, the parameter-based generalized sonic character) of musical space is evident from the score typically employs masses of sound instead of conventional melodies and harmonies. Whole configurations

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[5]See, for instance, Joseph Vincent McDermott, "The Articulation of Musical Space in the Twentieth Century" (Ph.D. diss., University of Pennsylvania, 1966).

of notes work together to produce a generalized sonic character in which multiple planes of musical activity interact with one another in varying combinations. Upon close inspection, much of the music in this vein seems to invite the use of musical analogies to the physical quantities of measurement already alluded to and thus makes the application of a parameter-based analysis that focuses on the geometrical aspects of a score quite intuitive. This analysis is centered on the concept of the *sound band,* which is defined to be the ensemble of pitches present at a given moment in time within a composition. This vertical sonority can be collectively labeled as a single sound band or can be broken down into stacks of sound bands, each with top and bottom limits. The context of the pitch relationships in time determines which approach is most logical.

Internal Parameters of Sound Bands

A sound band is composed of tones stacked at varying intervals. It could consist of fifty tones, all at quarter-tone intervals, or just a single tone. The *mass* μ of the band is defined as the number of tones that comprise it in a "short score." It should be stressed that the mass (like all other parameters of sound bands to be introduced) is measured instantaneously. The mass of the sound band in example 1 is $\mu = 5$. The *volume* γ of a sound band is the amount of musical space it takes up, or the width of the band. A single tone has zero volume as it takes up no vertical space. The semitone is arbitrarily chosen here as the unit for measuring volume since it is the smallest interval in the traditional tempered scale. Thus, the volume is the number of semitones from the

Example 1. Sound band with $\mu = 5$, $\gamma = 10$, $\rho = 0.4$, $\varphi = 15$, UB = 15, LB = 5, CB = 10, CM = 10.6, *cm* = 5.6, and *cb* = 5

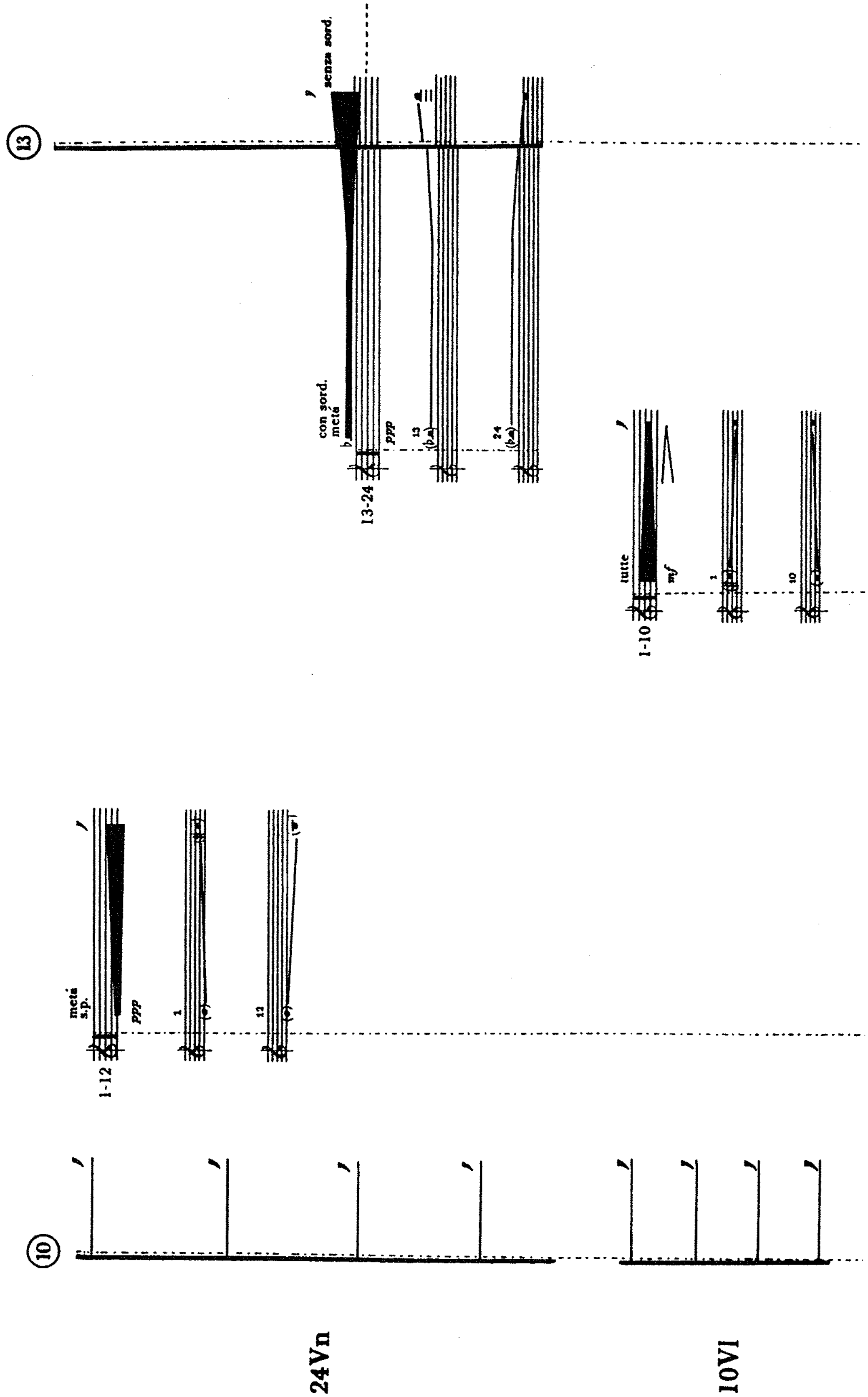
bottom to the top of the band. The volume of the band in example 1 is $\gamma = 10$.

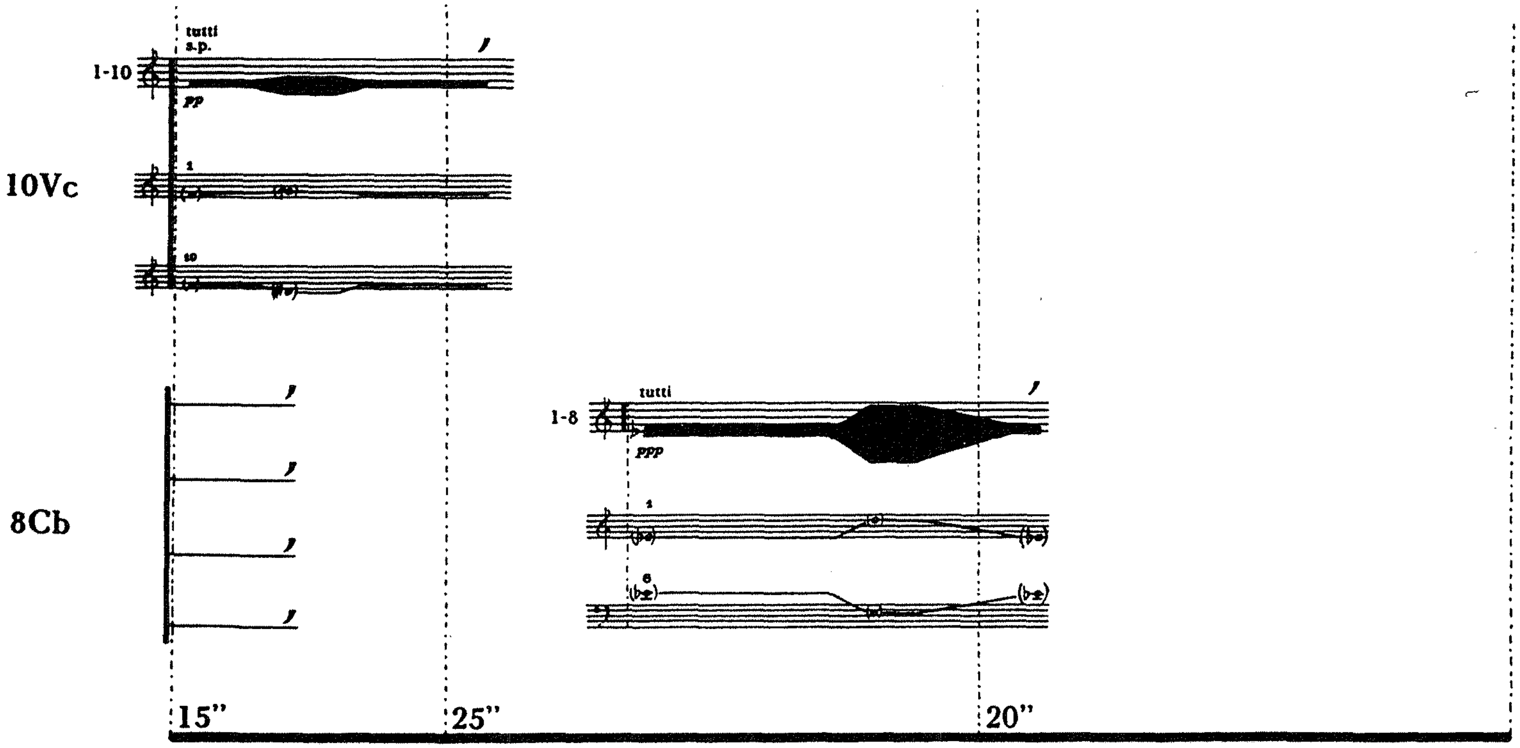
The density of any physical substance is its mass per unit of volume. The *density* ρ of a sound band is defined in the same way, where the semitone is the unit of volume. In this context a band consisting of a single tone (with mass $\mu = 1$) has zero volume and hence represents a singularity in the system, since the density is undefined. (An analogy to this odd situation in the world of physics is a cosmic singularity.) However, the inherent problem, that a *homogeneous* band with semitone intervals always has one more pitch than the number of semitones from its bottom to top, brings out a fundamental difference between the mass distribution in homogeneous physical bodies (in which the mass is spread out evenly on the macroscopic scale) and that in similarly homogeneous sound bands (in which the discrete masses are lumped where the tones occur). This phenomenon seems to invite two separate interpretations of the density. One is simply to divide the mass by the volume as $\rho = \mu/\gamma$, so that an increase in mass (with volume held constant) results in a proportional increase in density, as occurs in physical bodies. However, it is clear that the density of a homogeneous sound band of semitones using this definition would depend on the number of separate tones, yielding $\rho = \mu/\gamma = (\gamma + 1)/\gamma = 2, 1.5, 1.33, \ldots$ for $\mu = 2, 3, 4, \ldots$, respectively. While this sequence does approach 1 in the limit as $\mu \rightarrow \infty$, such a situation is qualitatively *unlike* that which occurs in physics, as it makes it difficult to compare meaningfully the densities of different sound bands that have equal volume but which differ in *both* the number of tones that comprise each one *and* the average intervallic distance between these tones (see example 3). Instead, it is advantageous analytically to specify that a homogeneous band of semitones has a density of 1 *regardless of the actual number of tones,* representing a single pitch added for each additional semitone of volume. The second alternative, then, which (in contrast to the first approach) results in a density that accurately indicates the average intervallic distance between successive pitches in a sound band, is to subtract one from the mass as $\rho = (\mu - 1)/\gamma$. In this way, $\rho < 1$ implies that the average interval is greater than a semitone, while $\rho > 1$ *must always* indicate the existence of microtones, since the average interval is less than a semitone. This is the approach chosen here. Thus, the density of the band in example 1 is $\rho = (5-1)/10 = 0.4$.

Example 2, from page 8 of Penderecki's *Threnody for the Victims of Hiroshima* for 52 strings, presents five bands that change in time. The first one, played by ten celli, expands from a single pitch (F4) to a band in which $\mu = 10$, $\gamma = 4.5$, and $\rho = 2.0$, thus indicating an average interval of a quarter-tone. A contraction back to the original single pitch immediately follows. The next band, by twelve violins, expands from a single pitch (E4) to a band with a volume of 11 and a density of 1.0, thus indicating an average interval of a semitone. In the next band, eight basses expand from E♭3 and then contract again; in the middle portion $\gamma = 22$ and $\rho = 0.318$, which is slightly less dense than stacked minor thirds. Ten violas then contract from a band with $\gamma = 8$ and $\rho = 1.125$, slightly greater than stacked semitones. Finally, twelve violins expand from a single pitch (B♭5) to a band with $\gamma = 13$ and $\rho = 0.846$, which is slightly less than stacked semitones. This example illustrates the dynamic roles of mass, volume, and density on a "local" scale just as chords, keys, and progressions fulfill this role traditionally, since in either case different sonic textures result as these parameters are changed. The parameter measurements obtained in this example will be utilized in testing conjectures regarding symmetries in the section entitled "Spatial Dynamics of Sound Bands" below.

The concept of flux is important in at least two different areas of physics: electromagnetism and fluid mechanics. In the former, an electric field that is uniform in magnitude and direction over some region has an *electric flux* $\varphi = EA$, defined as the product of the field E and the surface of area A, which is perpendicular to the field. In nonturbulent fluid mechanics the fluid particles move along an enclosed tube of lines called streamlines. The density and velocity of the fluid at any point are constant in time. If at a certain point in the tube the density of the fluid is ρ , its velocity is V, and the cross-sectional area of the tube is A, then the *mass flow rate* Q is defined as $Q = \rho VA$. This quantity represents the amount of fluid material flowing past a point during a certain amount of time and is similar in concept to the electric flux.

Example 2. Penderecki, *Threnody to the Victims of Hiroshima*, p. 8





THRENODY FOR THE VICTIMS OF HIROSHIMA, by Kryzystof Penderecki

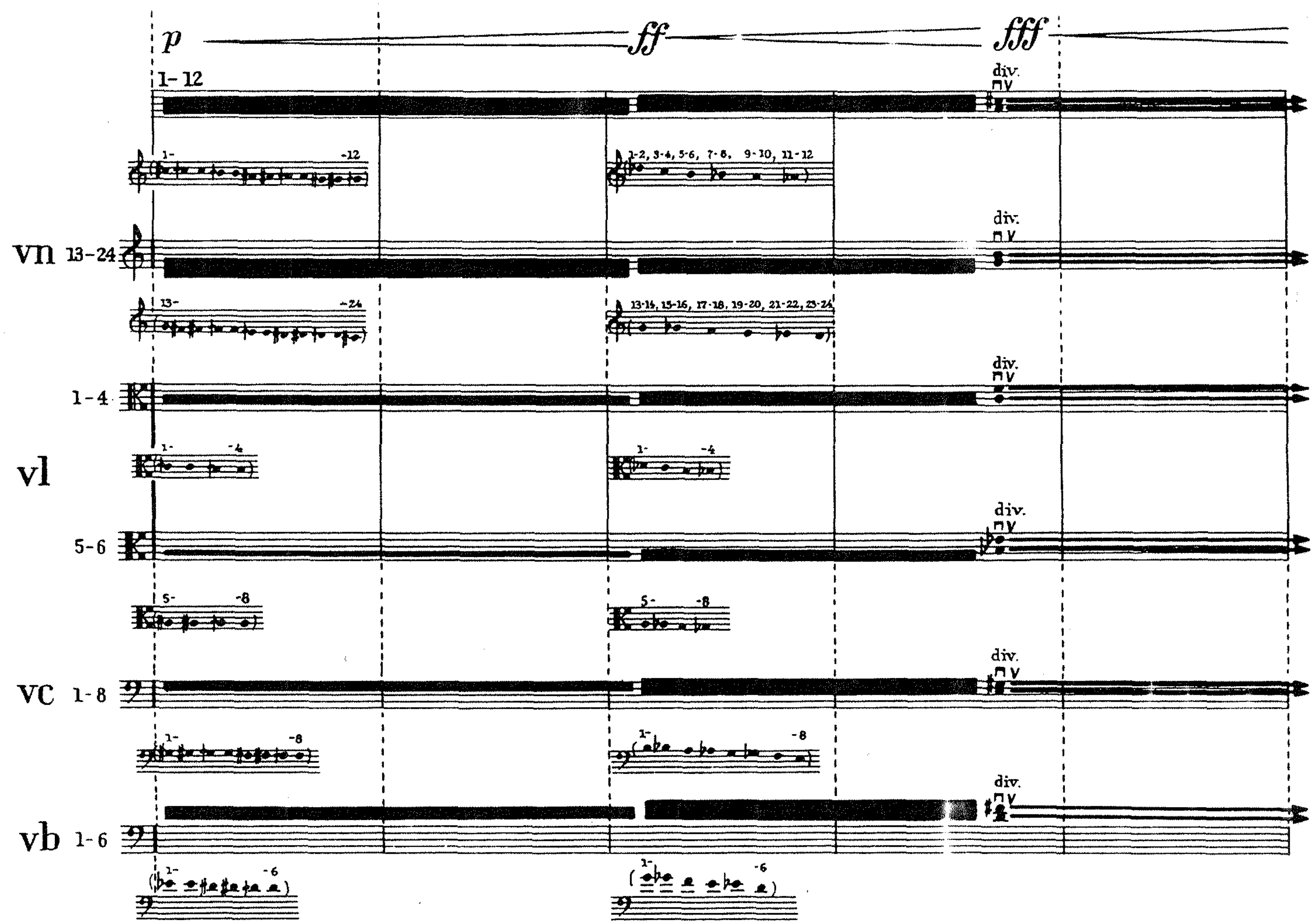
An analogy in musical space may be drawn to the electric flux and mass flow rate. These quantities are concerned with how much of some field or fluid is passing a certain point per unit time. The concern in musical space is how much "sound material" is passing a point per unit of time. Since time is what is moving, of course, we are rather concerned with how much of this stationary sound material is passing through an imaginary vertical line (perpendicular to the time axis) corresponding to a given instant of time that uniformly moves across the score. This amount of sound material can be computed in the same manner as the mass flow rate, according to the same formula. The fluid density is the density of the sound band, while the cross-sectional area is exactly its volume, since the vertical line mentioned above slices an infinitesimal "cross-sectional area," or volume, of the sound band.

The velocity factor is less intuitive. In the fluid mechanics analogy it describes the speed of the fluid, which is represented by the magnitude of the velocity vector at a given point, and the electromagnetic analogy has *lines of force* that have a similar function. We are thus interested in a measurable quantity that describes the strength or amount of sound per point on the pitch axis along the vertical line of an instant in time. Since a single note is represented by a point on the pitch axis, one wants to know the "strength" of this note, or its intensity dynamic. Fortunately, dynamics are traditionally notated only in discrete units for which numerical equivalents can be assigned. The *dynamic parameter* is given a value of 2 if the instantaneous dynamic is ***mf***, 3 if it is ***f***, 4 for ***ff***, and so forth. In the other direction the reciprocals of these values are used, and the dynamic parameter is 1/2, 1/3, 1/4, etc. for dynamics of ***mp***, ***p***, ***pp***, and so forth. These specific incremental values are chosen for two reasons. First, there is no *widely used* "central dynamic" between ***mp*** and ***mf*** (although *poco **f*** might be considered to qualify), and a dynamic parameter of 1 (or its equivalent reciprocal) would have no effect when multiplied in the formula. Second, although an infinite number of values is desired in both directions (corresponding to infinitely loud or infinitely soft), negative values cannot be allowed for the dynamic parameter since this would lead to a meaningless negative amount of sound material at a given point in time. Instead, a reciprocal sequence, which approaches zero while never becoming negative, is chosen for the soft dynamics.

An alternative approach, which may or may not be practical, could theoretically be useful, especially when multiple instrumental groups (e.g., strings and brass) are involved. In these cases, when, for example, a passage marked mezzoforte played by the strings is softer than one played by the trombones, the benefits of an absolute formulation that uses decibels as the dynamic parameter may overcome the weakness of a relative formulation based on the dynamic indication. The variables of acoustic environment and/or recording procedures and apparatus, however, would produce contaminated results, since the object under analysis is the composition *as it is notated in the score*, not as it is heard in performance. Whichever formulation is used, the inclusion of the intensity parameter results in a three-dimensional *dynamic musical space*.

Thus, the amount of sound material at a given point in time is called the *dynamic flux* and is defined as $\varphi = (\rho\gamma + 1)\delta$, where δ is the dynamic parameter. Since the product of the density and volume is the mass minus one, one must be added to this product in the formula. Therefore the dynamic flux is simply the product $\varphi = \mu\delta$ of the mass of the sound band and its dynamic parameter at a certain point in time. This is expected, since the amount of sound material, like the mass flow rate, is the mass flow per unit time. However, since our measurement in musical space is made within an infinitesimal amount of time, the dynamic flux (henceforth called just the *flux)* is simply the product of the mass of the sound band and its "strength" at this point in time. The flux φ of the sound band in example 1 is $5 \times 3 = 15$.

The flux serves as a good example of a geometric parameter which, when measured, may not correspond to immediately-visible elements in the score or even with what is intuitively "obvious." Example 3, from page 15 of Penderecki's *De Natura Sonoris*, initially consists of the 46 strings playing one large band in which each instrument has its own pitch. The volume is 23 and, although the band consists of stacked quarter-tones, the density is slightly less than 2.0 ($\rho = 1.96$) because the C♯4 is missing. The band is then thinned out by dropping every other pitch and, in general, placing two instruments on each single note for the remaining pitches. The result is a band of stacked semitones, Example 3. Penderecki, *De Natura Sonoris*, p. 15, strings



with $\mu = 24$ and $\rho = 1.0$. The band is then thinned further by letting the mass drop to 12, the density drop to 0.48, and by using further doublings. The band now consists of stacked whole-tones with the exception of the minor third between C4 and E♭4. Although the band thins twice, the dynamic notation counters this process in dynamic musical space. Initially $\delta = 1/3$ which gives a flux of 15.33. Then after the first thinning $\delta = 4$ and $\varphi = 96$. Finally after the second thinning $\delta = 5$ and φ drops to 60. Thus, if the climax of the passage is identified with the point at which the flux is a maximum, then this point occurs approximately midway through the entire process. This result is in contrast to the "obvious" choice of the final (loudest) moments in the example and is due in part to the highly unsteady change in intensity, in which most of the increase in intensity occurs around the initial thinning rather than being distributed more uniformly in time. From this example, it is seen that the flux seems to measure the cumulative effect of more elements than simply the dynamic marking, which is not the only factor in deciding where one hears a climax.

External Parameters of Sound Bands and Minimal Sets

Thus far the quantities that have been discussed are *internal parameters* of sound bands. Although these bands can be described in terms of their mass, volume, density, and flux both instantaneously and over time, these values do not specify the position of the bands on the pitch axis of musical space. To locate points of the band on the pitch axis, we must have a point of reference from which to make measurements. In computing centers of mass, centroids, and fluid pressures in physics, an arbitrary *datum*, or reference point, is used. The choice of location is usually aided by the physical nature of the system so as to make the calculations as convenient as possible. *External parameters* of sound bands can likewise be determined by making measurements from a datum, which may be chosen anywhere on the pitch axis. Since this axis can in principle be extended infinitely far in both directions, it is best to locate the datum at the center of the vertical region that is the most utilized. Thus, the datum is set at C4, or middle C. Since this is the zero point, locations above it (treble) are positive while those below (bass) are negative.

Three locations of a band on the pitch axis that can be measured with respect to the datum are its top, bottom, and middle. The top limit of the band is its *upper bound* (UB), and the bottom is its *lower bound* (LB). This means that the volume can be written as $\gamma = \text{UB} - \text{LB}$. The *median*, or center point of the band, is CB = (LB + UB)/2. All measurements are made in half steps from C4 with higher pitches being positive and lower ones negative. The upper bound UB of the sound band in example 1 is 15 and its lower bound LB is 5. Thus, its volume γ is 15 − 5 = 10 as stated earlier, and its median CB is (15 + 5)/2 = 10, or B♭4.

In physical space the center of mass *c.m.* of a system of point masses is found by summing the distance *r* to each mass particle after multiplying each by its weighted point mass *m*, then dividing the result by the total mass *M* of the system:

$$c.m. = \frac{1}{M}\sum_{i=1}^{N_m} r_i m_i \ , \qquad M = \sum_{j=1}^{N_m} m_j \qquad \textbf{(1)}$$

where N_m is the number of mass particles. In musical space the *center of mass* CM of a band can be found in the same manner, in which the number of mass points N_m is simply the mass μ of the band. Thus the center of mass is

$$CM = \frac{1}{\mu}\sum_{i=1}^{\mu} R_i \qquad \textbf{(2)}$$

where R_i is the distance in half steps from the datum to the *i*th pitch. A comparison of a band's center of mass CM to its median CB is useful in showing by how much the tones are weighted towards the top or bottom of the band. The center of mass CM of the sound band in example 1 is (5 + 8 + 12 + 13 + 15)/5 = 10.6. Thus, the placement of the pitches in this example is weighted slightly above the median because of the 0.6 difference. A necessary (but not sufficient) condition for vertical symmetry in a sound band is that the median and center of mass are equal (CB = CM). This is one property of homogeneous sound bands with evenly distributed pitches that may account for their wide use in the repertoire.

The external properties that have been discussed are dependent on *space-fixed* coordinates, since the datum is fixed in musical space regardless of the position of the band. This is why these properties have been indicated with capital letters. Not all internal properties depend on coordinates, but those that do are said to depend on *body-fixed* coordinates. Thus they are measured from points inside the band itself, and hence are indicated with lower-case letters. Volume is such a property, since it is measured from the lower bound. (If it were measured from the upper bound, it would be negative.) The center of mass CM that has been discussed is space-fixed, but another center of mass *cm* that is body-fixed can also be measured from the lower bound. This is expressed as

$$cm = \frac{1}{\mu}\sum_{i=1}^{\mu} r_i \tag{3}$$

where r_i is the distance in half steps from the lower bound to the *i*th pitch. The median can also be measured likewise, and its value is simply $cb = \gamma/2$. In example 1, *cm* is $(0 + 3 + 7 + 8 + 10)/5 = 5.6$ and *cb* is $10/2 = 5$. Thus, the 0.6 difference discussed above still remains in the body-fixed system.

Still another way to calculate the center of mass is to weight each pitch with its dynamic parameter in equations (2) and (3). This *dynamic center of mass* can be either a space-fixed quantity:

$$dCM = \left(\sum_{i=1}^{\mu} \delta_i\right)^{-1} \sum_{j=1}^{\mu} R_j \delta_j \tag{4}$$

or a body-fixed quantity:

$$dcm = \left(\sum_{i=1}^{\mu} \delta_i\right)^{-1} \sum_{j=1}^{\mu} r_j \delta_j \tag{5}$$

These versions are more useful than the previous ones only when pitches in a sound band have different dynamics at the same instant in time. In this situation equations (4) and (5) account for the intensity differences among the constituent pitches. When this is not the case (as in example 1), however, they reduce to equations (2) and (3).

From all the parameters that have been introduced to describe a sound band, only a few are necessary for a complete description at any given time, while all others can be derived from these. It is therefore desirable to find *minimal sets* of these parameters in order to eliminate redundancy in an analysis. Indeed, these represent the minimum number of injective functions (discussed in section 2) that are required to describe completely a sound band in two-dimensional musical space, as opposed to the noninjective mapping of time to aggregates of pitches. Hence, we are concerned not with the dynamic parameter but only with the relevant pitches. It can be easily found that a minimal set contains four parameters, one of which is the mass μ of the band, with different possible combinations of the other three. One such possibility includes the space-fixed parameters LB, UB, and CM, plus the mass μ. All other quantities, including γ, *cm*, *cb*, and CB, may be derived from these. Another possibility is μ, γ, *cm*, and CM, while still another is μ, CB, *cb*, and *cm*. An example illustrating the use of external parameters is given in the next section.

Spatial Dynamics of Sound Bands

Now that all of the parameters needed to describe a *static* sound band have been developed, the *spatial dynamics* (i.e., the change over time, *not* the intensity dynamic such as ***mf***, which is really static) of single and multiple sound bands can be investigated. Since the quantities discussed above can now change with time, the purpose here is simply to mention a few of the possible dynamic phenomena that may arise and to give some brief examples of how they may be analyzed quantitatively.

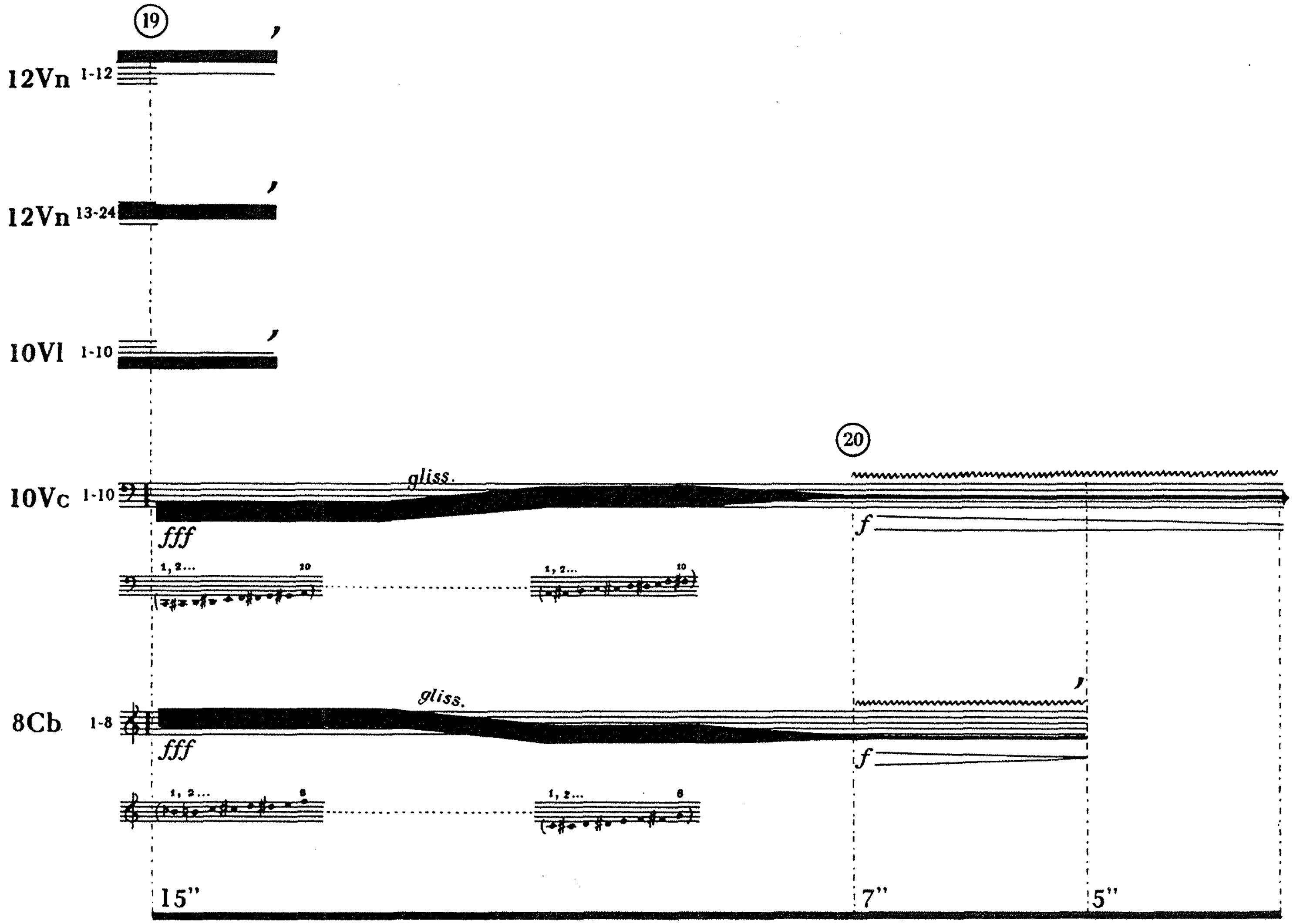
A sound band may maintain a constant volume or it may *expand* (increase in volume) or *contract* (decrease in volume). Both of these occur in example 2. Bands that maintain constant volume over a definite time interval are called *planes* for that space of time. Planes may go up or down (called *translation*) or stay between constant points on the pitch axis. Planes that have constant upper and lower bounds over a certain space of time are called *beams* over that time interval. Beams may vary in density over time. This process is called *thinning* (decreasing density) or *thickening* (increasing density). Example 3, discussed previously, is a wonderful example of thinning in a beam with large volume.

Two planes may occur simultaneously in five different ways. If both of the planes are translating up or down with identical slopes, they are *parallel*. If they are translating the same direction but with unequal slopes, they are *skew*. If one is translating while the other is not, they are *oblique*. If the planes are translating in opposite directions, they are either *converging* (moving closer together) or *diverging* (moving farther apart).

Intersections may occur for planes that are oblique, skew, or converging. Many plane intersections result in *mergence*, in which the planes combine into one (with or without the combined volume of the merging planes). The opposite of mergence is called *division*. Several planes can merge or divide at once, and these processes occur frequently in the repertoire. For example, the dynamics of the two planes in example 4, page 12 from *Threnody,* result in mergence as the celli translate upward and the basses (sounding an octave lower than written) translate downward, after which the resulting band contracts to a single pitch. Mergence and division are essentially the same as Varèse's "penetration" and "repulsion," respectively, as he discussed in 1936, the former of which is graphically highlighted in this example due to Penderecki's unique notation.[6] It is also possible for planes to *cross*, although this occurs less frequently, probably because it is difficult for the ear to follow crossing planes even when their slopes remain constant. Example 5 from Penderecki's *Sinfonie*, however, does contain crossing *bands* (not planes, since the descending one is simultaneously expanding and the ascending one is simultaneously contracting).

[6]Edgard Varèse, "The Liberation of Sound," compiled and annotated by Chou Wen-chung, in *Contemporary Composers on Contemporary Music*, ed. Elliott Schwartz and Barney Childs (New York: Holt, Rinehart, and Winston, 1967), 195–208.

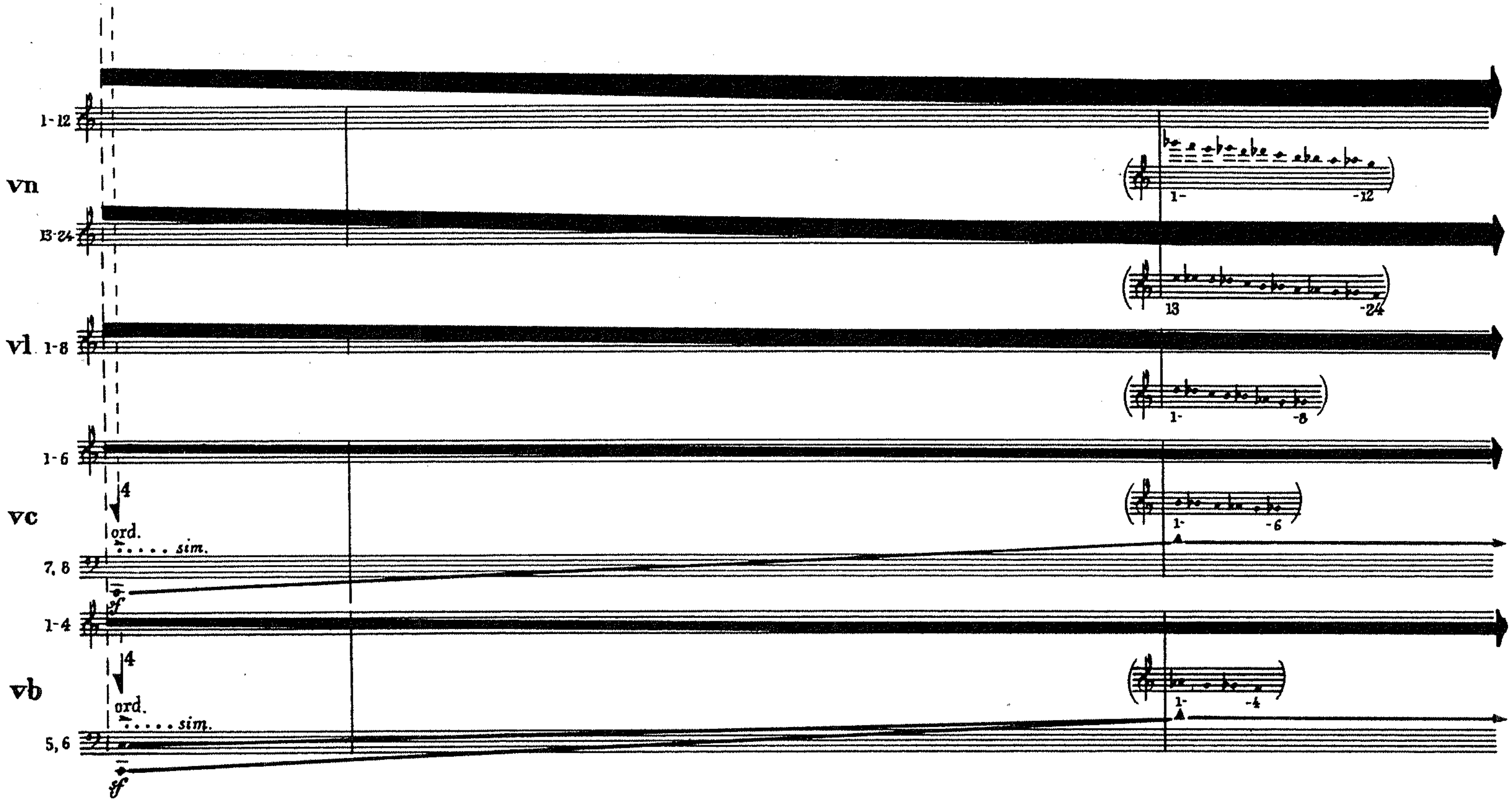
Example 4. Penderecki, *Threnody*, from p. 12



THRENODY FOR THE VICTIMS OF HIROSHIMA, by Kryzystof Penderecki

All of these dynamic processes may certainly be investigated quantitatively using the same parameters developed previously for static sound bands. Here, however, we are concerned with rates of change of these quantities. For any given parameter p (either internal or external), its average rate of change over the time interval Δt is given by $\Delta p/\Delta t$, where Δp is the amount of change in the parameter over this interval. In some cases (especially in some of Penderecki's scores) the precise time interval allotted for various processes is given in seconds. Of course, this enables dynamic processes to be measured in units per second. In example 2 there are three time intervals with specific duration given for the five dynamic processes discussed previously. Even so, the time interval for each expansion/contraction must be interpolated intuitively based on the amount of horizontal space allotted to that process within one of the specified time intervals. Thus, the following measurements for this particular example are not exact but leave some room for "guesswork." The first celli band expands and contracts within a time interval of 15 seconds, with the band being held constant before, after, and in between these events. Assuming that the actual expansion and contraction processes require 3 seconds each, then the rates of change of the band's volume are $\frac{\pm 4.5}{3} = \pm 1.5$ semitones per second. (The positive value is for expansion and the negative value is for contraction.) The expansion in the next band (violins) takes about half of the 25 second interval, or 12.5 seconds. Thus, its volume expansion rate is $\frac{11}{12.5} = 0.88$ semitones per second. The times required for expansion and contraction in the next band (basses) are unequal by visual inspection and were arrived at by interpolation based on measurements for the horizontal lengths in the score. This yields rates of change in the volume of $\frac{22}{1.4} = 15.7$ and $\frac{-22}{4.2} = -5.2$ semitones per second for the expansion and contraction, respectively. Similarly, the subsequent viola contraction and violin expansion occur at volume rates of $\frac{-8}{7.8} = -1.0$ and $\frac{13}{4.4} = 2.9$ semitones per second, respectively. The

Example 5. Penderecki, *Sinfonie*, p. 8, strings



dynamic processes of mergence and contraction in the other excerpt from *Threnody* (example 4) may be similarly quantified.

Some dynamic processes (such as expansion and contraction) may not be perfectly symmetrical vertically, although the instantaneous parameters of the bands might be symmetrical vertically at any given time. In such cases, the use of external parameters measured at several instances in the dynamic process may help to shed light on this issue. Returning to example 2, the obvious qualitative conjecture achieved by a brief examination of the score would be that "the sound bands expand and contract symmetrically about a fixed pitch." However, quantitative analyses of the median and center of mass parameters in each case reveal that this qualitative conclusion is incorrect in most of the cases. For example, the middle of the first band has CM = CB = 5.25 at maximum volume, which is an eighth-tone higher than the starting and ending F4. Since this beam consists of ten equally-spaced quarter-tones, the median and center of mass could have remained exactly F4 if an odd number of celli had been used. However, the goal of having stacked quarter-tones for the number of specified instruments seems to have priority in this case over maintaining a perfectly symmetrical expansion and contraction. The same can be said of the next band, which at maximum volume has CM = CB = 4.5 (a quarter-tone higher than the starting E4). In this case the priority was to have equally-spaced semitones. However, the middle portion of the next band (basses) has CM = CB = −9, which is exactly the starting E♭3, and then the violas contract from CM = CB = 9.0 (A4) to the exact center. Thus, in these cases the initial qualitative conjecture is true since it seems that equal spacings of a "nice" interval (minor thirds and semitones, respectively) has been sacrificed for symmetry. Finally, the second violin band expands to CM = CB = 22.5, which is a quarter-tone higher than the starting B♭5. Although symmetry is again sacrificed, the gain of stacked "nice" intervals is still not achieved since the density is slightly less than a semitone. Hence, the medians and centers of mass remain constant and the conjecture of dynamic symmetry in these processes remains true for only two of these five bands, as can be seen from table 1, which summarizes these results.

Whether these symmetries are detectable by the listener is arguable;

however, as is the case with use of the Golden Section and other formalistic procedures, the motivation results from the aesthetic appeal of the symmetrical proportions in such dynamic processes.[7] Instead of temporal symmetries, however, the focus here is on vertical (pitch) symmetries in musical space. In such spatially dynamic processes as those in example 2, precise vertical symmetry (as measured by the associated external parameters) can help to pinpoint where the climax occurs. For example, the initial band in the previously-discussed example 3 has a median of CB = 1.5, which is a quarter-tone higher than C♯4. Because it initially consists of stacked quarter-tones with a missing C♯4, however, the center of mass (CM = 1.51) is almost, but not exactly, equal to the median. After the first thinning, however, the arrangement of stacked semitones now allows the center of mass to be equal to the median (CM = CB = 1.5). *The achievement of perfect vertical symmetry thus coincides with the climax suggested by the maximum dynamic flux as calculated above.* The second thinning produces a band with stacked whole-tones except for the minor third between C4 and E♭4, thus allowing the center of mass to remain equal to the median, which remains unchanged throughout the entire process. These results are summarized in table 2, in which the climax is indicated by the data in boldface type.

Conclusion

It can be seen from the preceding examples that application of the quantitative parameters of musical space to the dynamic processes discussed above can be potentially valuable, since they can often be used to prove or disprove conjectures regarding symmetry and location of climaxes, among others, which may or may not be intuitively obvious from the score. While many of the parameters introduced were readily visible in the Penderecki examples, they may actually be even more useful in the analysis of some traditionally-notated scores (by Ligeti and Lutosławski, for example), in which the spatially dynamic

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[7]Michael R. Rogers, "The Golden Section in Musical Time: Speculations on Temporal Proportion" (Ph.D. diss., University of Iowa, 1977).

Table 1. Results for the dynamic processes in example 2

Instrument	Vc	Vn 1	Cb	Vl	Vn 2
process (expand/contract)	expand/ contract	expand	expand/ contract	contract	expand
start/end pitch	F4	E4	E♭3	A4	B♭5
μ	10	12	8	10	12
γ	4.5	11	22	8	13
ρ	2.0	1.0	0.318	1.125	0.846
CB=CM	5.25	4.5	−9.0	9.0	22.5
dynamically symmetrical?	no	no	yes	yes	no
approx. rate of exp./cont. (semitones/sec.)	±1.5	0.88	15.7 −5.2	−1.0	2.9

Table 2. Spatial parameters in example 3

Position	initial	after 1st thinning	after 2nd thinning
μ	46	24	12
γ	23	23	23
ρ	1.96	1.0	0.48
δ	1/3	4	5
φ	15.33	**96**	60
CB	1.50	**1.50**	1.50
CM	1.51	**1.50**	1.50

processes discussed here are less apparent visually and where the conjectures may in fact demand quantitative proof. In presenting these quantitative parameters of musical space, many of Varèse's (and others') terms for his own musical style were employed or renamed. Effort was made to define precisely these quantities by borrowing similar concepts from physics and mathematics, an approach which was shown to be significantly more valuable in contributing toward a deeper understanding of the various dynamic processes of these examples than is the reduction of these terms to vague metaphorical descriptions of the music. While the purpose of this study was to introduce these parameters and to provide brief examples of their usefulness, additional analytical tools might involve graphing one or more of these parameters over time for large sections of the score to see if familiar mathematical functions, hidden proportional relationships, or other aspects of symmetry that may not be initially obvious are uncovered. In fact, it is anticipated that further work along these lines will provide many examples of yet more dynamic phenomena of sound bands that went unmentioned here and thus will yield more insight into the dynamics of musical space in many modern compositions in the post-Varèse idiom.