TRANSVERSE AND LONGITUDINAL NONLINEAR BEAM DYNAMICS

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In the past year, a great deal of progress has been made in developing the equipment and methods required for studying nonlinear beam dynamics, and many interesting results have been obtained. These studies track the motion of the beam centroid after collectively perturbing the beam. The degree to which the beam centroid motion accurately represents the motion of a single particle depends on the emittance of the beam; the smaller the emittance of the beam, the more accurate its representation of single particle motion. In this respect, the IUCF Cooler Ring provides an ideal laboratory for nonlinear beam dynamics experiments.

In experiments done earlier this year, one dimensional (horizontal) motion of the beam in phase space was determined from turn-by-turn measurements of the beam position, x_1 and x_2 in two capacitive pickup beam position monitors, BPMs, located about 90° apart in betatron phase. The conjugate variable of x_1, namely p_{x1}, is a function of the measured beta functions at the BPMs, the betatron phase advance of the detectors, and the measured positions, x_1 and x_2. When the beam is kicked horizontally, the resulting motion for a linear machine lies on a circle in the p_{x1}–x_1 phase space map. Deviations of the phase space map from this behavior provide information about the strength of the nonlinearities.

While the nonlinear terms in the accelerator Hamiltonian are intrinsically weak, particle motion can be strongly affected when resonance conditions are encountered. A resonance condition is specified by $m\nu_x + n\nu_z = p$, where ν_x and ν_z are the horizontal and vertical betatron tune, and m, n, and p are integers. The order of the nonlinearity is related to the values of m and n. For a resonance in one dimension, either m or n is zero. In this case, particle oscillation around fixed points in a phase space map can be observed and are referred to as resonance islands. In Fig. 1, some recently taken data near the fourth order resonance ($m = 4$ and $n = 0$) are shown for a series of increasingly large horizontal kicks. Here, the motion near the separatrix and around the fixed points is well illustrated. These data confirm the work done earlier this past year in which the Cooler

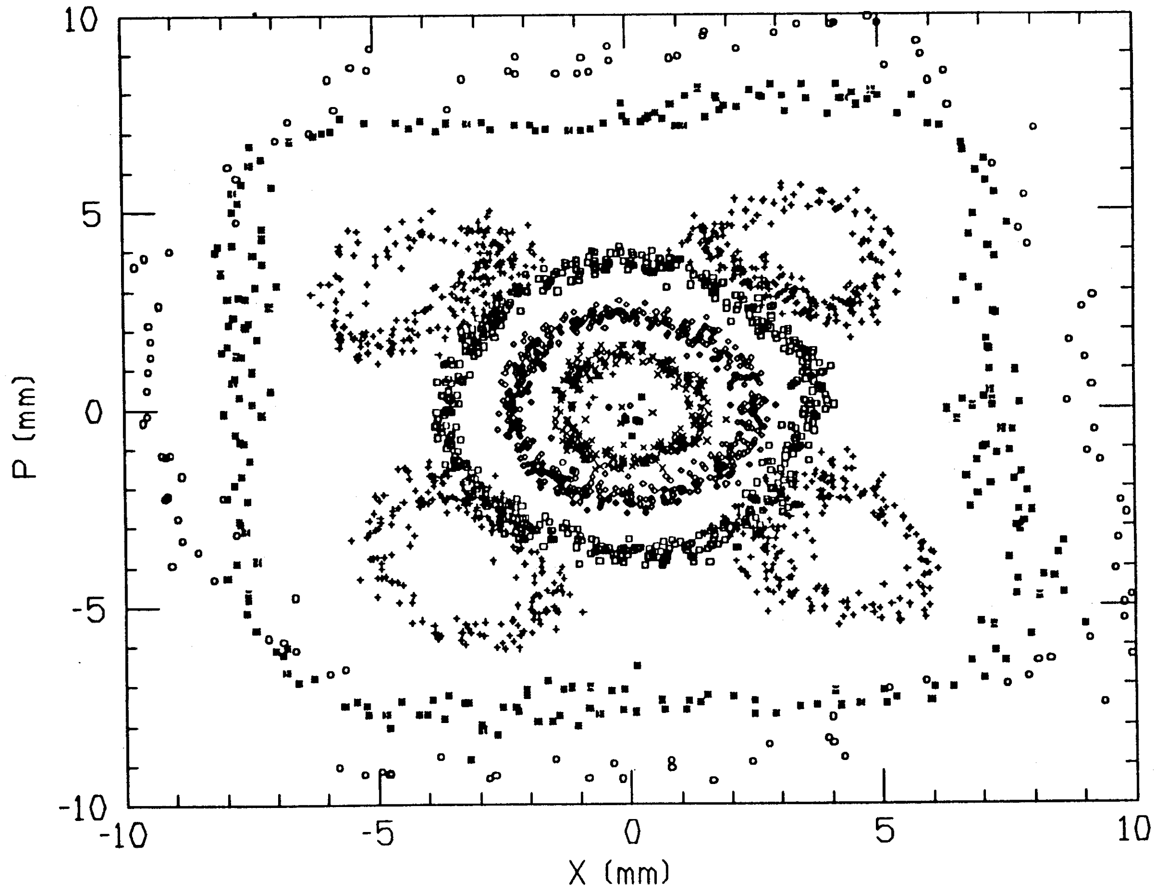

Figure 1. A transverse phase space map for operation near the fourth order resonance ($m = 4$) where $\nu_x = 3.75$.

ring Hamiltonian at $\nu_x = 3.75$ was determined.[1] These data will be used for more detailed study of the motion within resonance islands, a task made possible by the implementation of a method to decouple the vertical and horizontal motion of the beam.

Our most recent experimental work involves the extension of the studies of transverse nonlinear motion from one to two dimensions. Although many numerical studies are available, due to their difficulty there have been few attempts to perform 2D beam dynamics experiments.[2] However, since beam loss arises essentially from 2D coupling resonances, it is especially important to explore these resonances in great detail. To observe the nonlinear coupling resonance, we studied the nonlinear coupling resonance at $\nu_{x0} - 2\nu_{z0} = -6$. In Fig. 2, some data for the horizontal and vertical motion near this resonance are shown. The growth in oscillation amplitude in one dimension with a corresponding decrease in amplitude in the other dimension is a characteristic of the nonlinear difference resonance. With further analysis of these data, the contribution of this resonance to the Hamiltonian will also be determined.

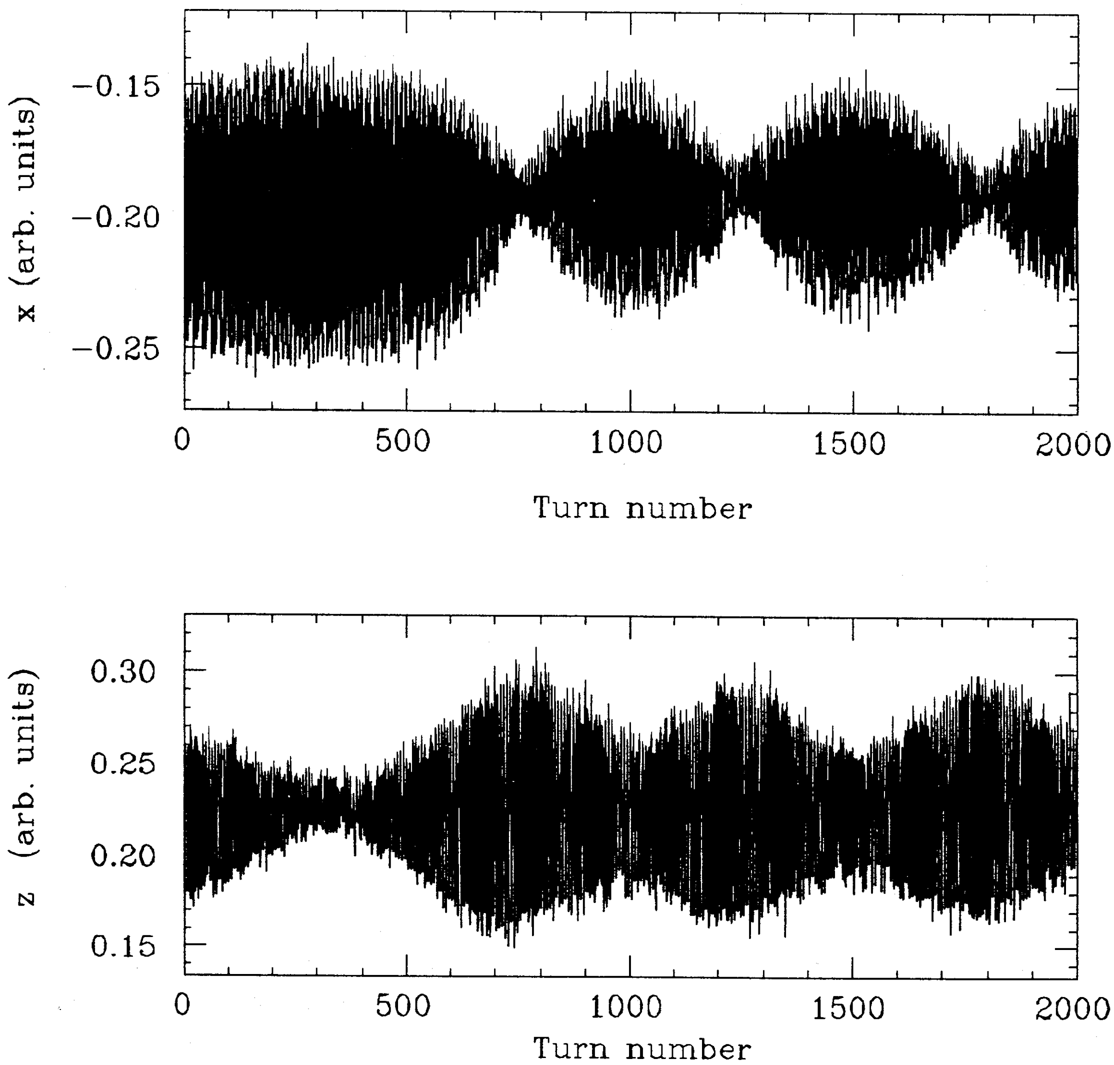

Figure 2. The x and z positions from single BPMs near a third order difference coupling resonance at $\nu_x \approx 3.70,\ \nu_z \approx 4.85$.

The existence of chaotic solutions and transition to chaos in parametric resonant dynamics system are of interest to many disciplines of physics. One such system is the synchrotron motion[3] discovered in 1945 by McMillan and Veksler. For a circular accelerator, the equation of phase oscillation is given by

$$\ddot{\phi} + \omega_s^2 \sin\phi = \omega_s^2 \sin\phi_0,$$

where ω_s is the small amplitude synchrotron angular frequency; ϕ_0 is the synchronous phase angle, ω_0 is the angular frequency, and $\delta = \Delta p/p$ and $\phi = -h\theta$ are conjugate variables with θ being the orbital angle and h the harmonic number.

Recently, we have tracked the longitudinal motion of a single particle and used it to study synchrotron motion for the first time. The beam was kicked longitudinally by phase shifting the rf. The resulting beam-centroid displacement in momentum was tracked by digitizing and recording the horizontal transverse position of the beam at a high dispersion location; the beam-centroid displacement in phase was tracked by digitizing the phase of the signal from a longitudinal pickup relative to the rf. The momentum deviation, $\Delta p/p$, is related to the off momentum closed orbit, Δx_{co}, by $\Delta x_{co} = D_x \Delta p/p$, where D_x is the horizontal dispersion function.

To study the effect of a driven motion, the phase of the rf was modulated producing a modulation of the synchronous phase. Fig. 3 shows that the numerical simulations of this system agree well with the measured responses at small driven amplitudes, a. When the modulation amplitude is larger than about 5.7°, the numerical simulations predict that the the particles can jump outside the rf bucket and can be recaptured into the same bucket ($h = 1$ rf system).

In conclusion, our experimental studies of nonlinear beam dynamics have produced interesting results from studying both transverse and longitudinal motion. The ground work for possible six dimensional phase space tracking in nonlinear beam dynamics experiments has been layed. Further experimental studies using these techniques may contribute to a better understanding of the bunch diffusion process caused by phase and amplitude noises. Another avenue of experimental exploration, now available, is the effect of amplitude modulation. Amplitude modulation can create islands within the dominant rf bucket. Such manipulations may offer a possible superslow extraction for fixed target experiments at SSC energies. Also, for highly damped parametric resonance physical systems,[4,5] many interesting phenomena, such as strange attractors, period doubling, etc., can be observed. Using a rf phase feedback loop, highly damped parametric resonances can also be studied in a synchrotron.

1. S.Y. Lee, et al., Phys. Rev. Lett. **67**, 3768 (1991).
2. See for example the Fifth Advanced Beam Dynamics Workshop.
3. E.M. MacMillan, Phys. Rev., **68**, 143 (1945); V.I. Veksler, Compt. Rend. Acad. Sci. U.R.S.S., **43**, 329 (1944); **44**, 365 (1944).
4. M. Octavio, Phys. Rev. **B29**, 1231 (1984).
5. D. D'Humieres, M.R. Beasley, B.A. Huberman and Libchaber, Phys. Rev. **A26**, 3483 (1982).

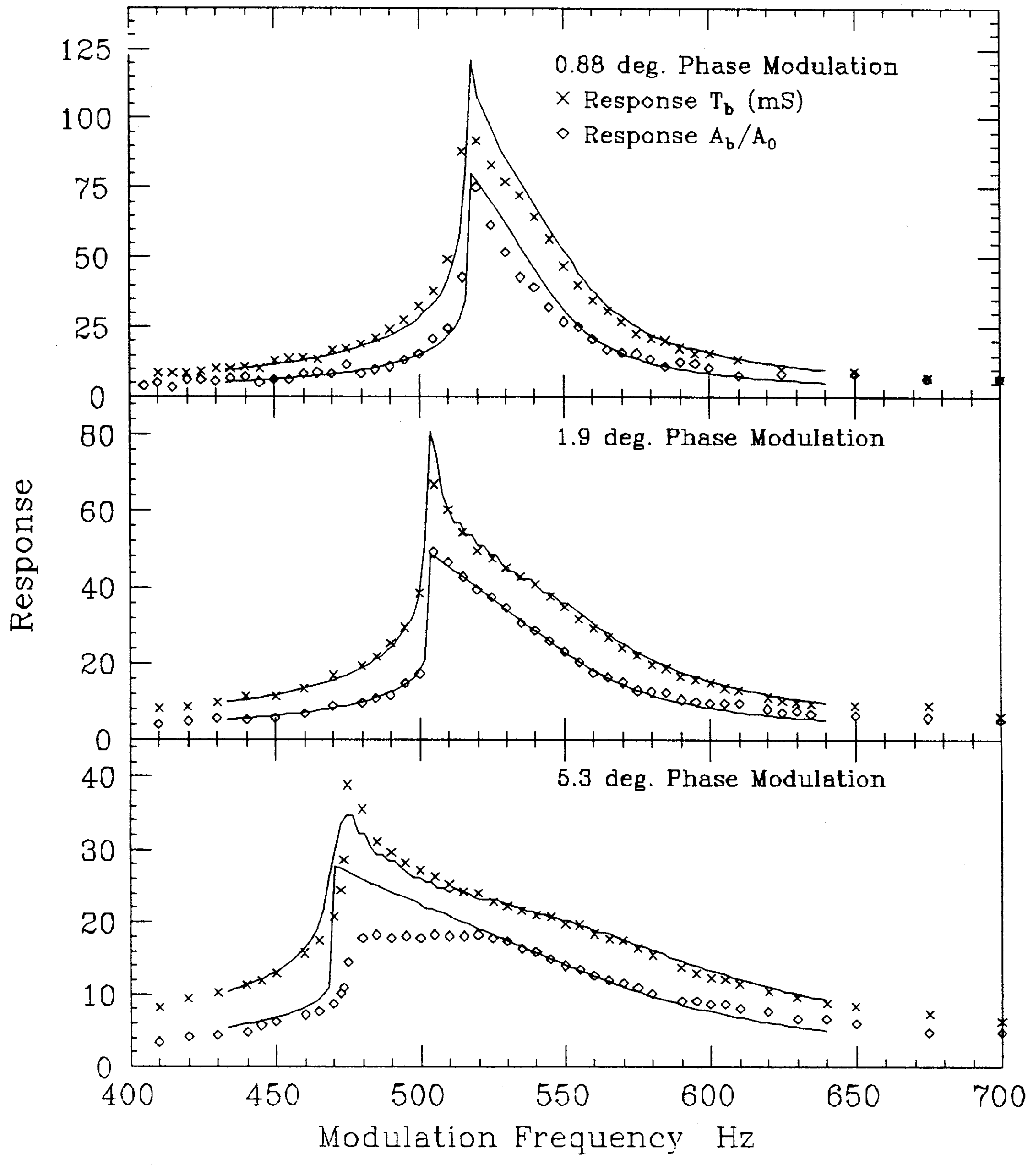

Figure 3. The measured amplitude and period of amplitude modulation as a function of modulation frequency for various phase modulation amplitudes, $a = 0.88°, 1.9°, 5.3°$.