

## NONLINEAR BEAM DYNAMICS EXPERIMENTS: CE-22

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Many nonlinear beam dynamics experiments have been performed in the past year.<sup>1</sup> We studied motion near the 2-D nonlinear resonances given by  $\nu_x - 2\nu_z = -6$  and  $\nu_x + 2\nu_z = 13$ . The Hamiltonian for the accelerator operating near these nonlinear resonances can be expressed as

$$H(J_x, J_z, \phi_x, \phi_z) = H_0(J_x, J_z) + g_{\pm} J_x^{1/2} J_z \cos(\phi_x \pm 2\phi_z - \ell_{\pm}\theta + \chi_{\pm}), \quad (1)$$

where  $H_0 = \nu_x J_x + \nu_z J_z + \frac{1}{2}\alpha_{xx} J_x^2 + \alpha_{xz} J_x J_z + \frac{1}{2}\alpha_{zz} J_z^2$  is the unperturbed non-resonant Hamiltonian,  $J_x$  and  $J_z$  are actions of the horizontal and the vertical betatron oscillations, and  $\phi_x$  and  $\phi_z$  are their respective conjugate variables. In the single resonance approximation, the Hamiltonian of Eq. (1) is integrable. The Poincaré map in the resonance rotating frame will trace out a torus, which is characterized by a Courant-Snyder invariant circle and a resonance circle. Particle motion near resonance is permanently perturbed by the resonance circles. For a difference resonance,  $\nu_x - 2\nu_z = \ell_-$ , the motion is bounded, i.e.,  $2J_x + J_z = \text{constant}$ . Figure 1 shows an example of invariant tori at the difference resonance.<sup>1</sup>

On the other hand, the sum resonance,  $\nu_x + 2\nu_z = \ell_+$ , is intrinsically unstable because the invariant of the Hamiltonian is  $2J_x - J_z = \text{constant}$ . Therefore, both  $J_x$  and  $J_z$  can become "unbounded". Particles at the sum resonance are lost to the vacuum chamber walls in less than 100 turns. Thus the study of a sum resonance is very difficult. At IUCF, we have built a ferrite Panofsky quadrupole in order to change the betatron tune by about 0.03 within 1  $\mu\text{s}$ .<sup>2</sup> In this way, the accelerator can be placed at the resonance condition suddenly. The centroid of the beam motion can be tracked to observe the effect of a nonlinear resonance in action. Figure 2 shows  $J_x$  vs.  $J_z$ ; and in Fig. 3 the Poincaré map of the data is plotted in the resonance frame.<sup>3</sup> Note here that the beam loss occurred within

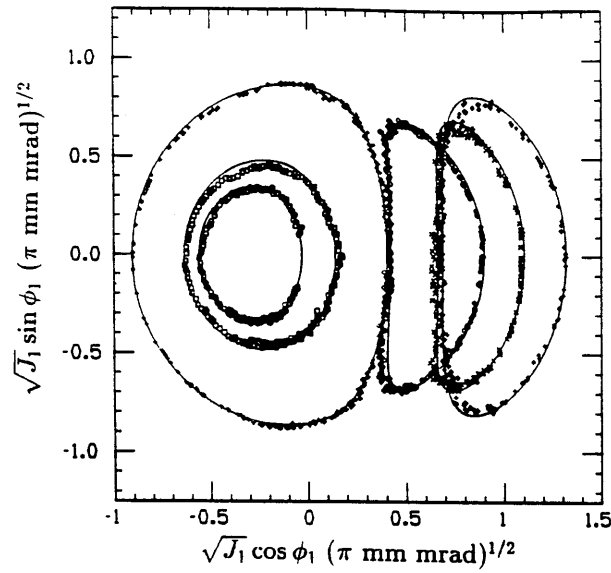


Figure 1. Invariant tori of the Hamiltonian at the difference resonance  $\nu_x - 2\nu_z = -6$  in the resonance rotating reference frame.

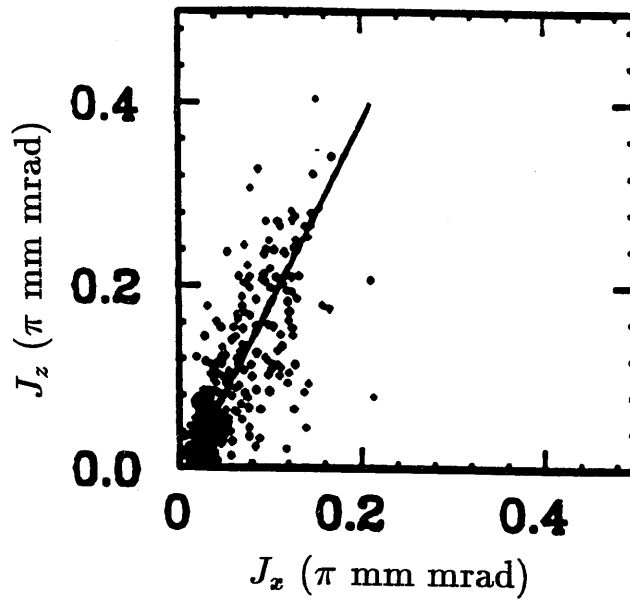


Figure 2. Plot of  $J_z$  vs.  $J_x$  from turn-by-turn data for the sum resonance  $\nu_x + 2\nu_z = 13$ . The slope of the solid line is about 2.

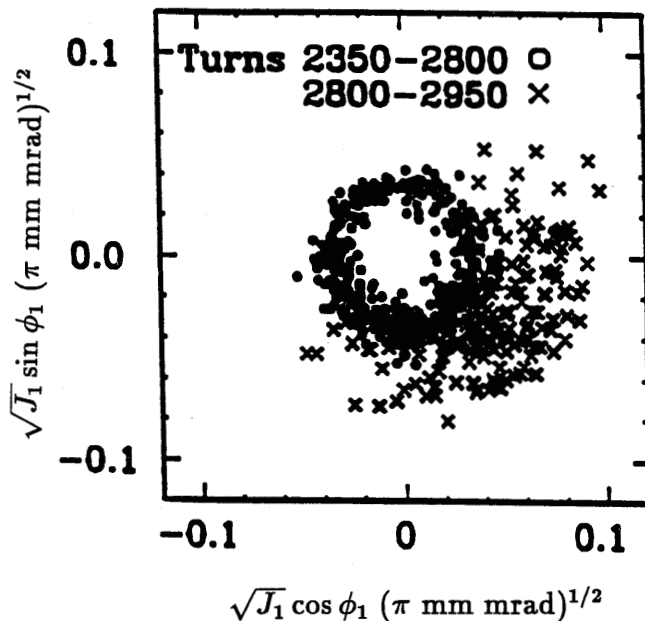


Figure 3. Poincaré map of data taken at the sum resonance  $\nu_x + 2\nu_z = 13$  in the resonance rotating reference frame.

about 100 turns. Although our preliminary experimental data on the  $\nu_x + 2\nu_z$  resonance can not be used yet to derive the nonlinear Hamiltonian, we are confident that the Panofsky quadrupole works. Our future studies will concentrate on deriving the strength of the nonlinear resonance and its corrections.

Besides the transverse nonlinear beam dynamics experiments, we have performed a series of longitudinal beam dynamics experiments, where (1) the RF phase was modulated sinusoidally, (2) the RF voltage was modulated sinusoidally, and (3) a dipole field was modulated sinusoidally.

In both case (1) and (2), Poincaré maps of the data revealed tori of the resonant Hamiltonian. The longitudinal particle motion is confined to resonance islands and, with electron cooling, is damped to the basins of these islands within a few damping periods. Bunch splitting into separate beamlets in different islands due to this effect was clearly observed. In the case (3), the effect was equivalent to the RF phase modulation due to synchro-betatron coupling.

These experiments can shed new light on RF noise problems, ground vibration, beam diffusion due to RF sinusoidal modulation for a very slow beam extraction, and the possibility of multi-bunch instability feedback through parametric resonances. Our fruitful studies can be summarized by several papers which have been prepared for publication.<sup>4-9</sup>

1. S.Y. Lee, Proc. of the IEEE Particle Acc. Conf., (1993); M. Ellison, *et al.*, *ibid.*
2. J. Budnick, *et al.*, Proc. of the IEEE Part. Acc. Conf., (1993).
3. Y. Wang, *et al.*, Proc. of the IEEE Part. Acc. Conf., (1993).
4. D.D. Caussyn, *et al.*, Proc. of the IEEE Part. Acc. Conf., (1993).
5. M. Ellison, *et al.*, Phys. Rev. Lett. **70**, 591 (1993).
6. M. Syphers, *et al.*, submitted to the Phys. Rev. Lett.; and Proc. of the IEEE Part. Acc. Conf., (1993).
7. D. Li, *et al.*, submitted to Phys. Rev. E. Rapid Communications.
8. H. Huang, *et al.*, to be submitted to Phys. Rev. E.
9. Y. Wang, *et al.*, to be published.