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THE INVESTIGATION OF SPACE CHARGE DOMINATED BEAMS IN A SYNCHROTRON

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Knowledge of the longitudinal momentum spread of an electron-cooled proton beam is important for experiments which rely upon the unique properties of this beam. Since the RF cavity produces a conservative force, it cannot change the beam longitudinal phase density. Consequently, for a known RF voltage, the beam time spread normally provides a direct measurement of the beam momentum spread. An electron-cooling system,[1] however, can reduce the ion beam emittance to extremely small values. In this regime the ion beam, interacting with its surrounding, generates longitudinal self-fields which significantly modify, and in fact to first order determine, the bunch shape.[2] It can be shown that for the ion beams below transition, the most significant part of this interaction comes from the electrostatic repulsion between the ions within the bunch. Since in all cases of interest the bunch lengths are much greater than the radius of the surrounding vacuum chamber, one can treat the beam as a thin thread and reduce the electrostatic problem to one dimension. The Vlasov technique can then be used to find the self-consistent longitudinal particle distribution function.[3]

For electron-cooled ion beams, however, one needs to modify the Vlasov equation to include both damping and diffusion.[4] Although the cooling force is generally a complex function of proton momentum,[5] it can be approximated well as a linear function of relative proton momentum, δ, for δ smaller than the longitudinal relative momentum spread of the electron beam. The latter is determined by longitudinal-longitudinal intrabeam scattering, the electron beam space charge depression, and by the high-voltage power supply ripple; the resulting relative momentum spread is typically $> 1 \times 10^{-4}$. Consequently the linear approximation is valid for $\sigma < 1 \times 10^{-4}$, where σ is a relative rms proton momentum spread. The modified Vlasov equation can then be written in Fokker-Planck form:

$$\frac{\partial \Psi}{\partial t} + \dot{s}\frac{\partial \Psi}{\partial s} + \dot{\delta}\frac{\partial \Psi}{\partial \delta} = \frac{\partial}{\partial \delta}\left(\lambda \delta \Psi + \frac{d}{2}\frac{\partial \Psi}{\partial \delta}\right) \tag{1}$$

where $\Psi = \Psi(t, s, \delta)$ is the longitudinal distribution function, t is time, s is the longitudinal coordinate, λ is the cooling rate and d is the diffusion rate. For a stationary distribution, the time dependence of Ψ vanishes and the solution of Eq. (1) for the particles experiencing synchrotron oscillations in the presence of space-charge force can be written as:[6]

$$\Psi(s, \delta) = \frac{1}{(2\pi)^{1/2}\sigma} e^{-\delta^2/2\sigma^2} \rho(s) \tag{2}$$

where $\sigma^2 = d/2\lambda$ and $\rho(s)$ is the longitudinal linear charge density given by the expression:

$$\rho(s)e^{-\alpha\rho(s)} = \rho(0)e^{-\alpha\rho(0)} \exp\left[-\frac{R^2}{h^2\sigma_s^2}\left(1 - cos\left(\frac{hs}{R}\right)\right)\right] \tag{3}$$

The value of $\rho(0)$ must be chosen so that $\rho(s)$ is normalized to qN, where q is the ion charge and N is a total number of particles per bunch. The constants α and σ_s are given by the expressions:

$$\alpha = \frac{g}{\beta^2\gamma^3\sigma^2\eta}\frac{e^2 f_0 Z_0}{Mc^2}\frac{R}{e} \tag{4}$$

$$\sigma_x = \left(\frac{2\pi\beta^2\gamma\sigma^2|\eta|}{h}\frac{Mc^2}{eV_{rf}}\right)^{1/2} R \tag{5}$$

where γ, β are the usual relativistic parameters, $Z_0 = 377\Omega, g$ is a geometrical factor (= ln(pipe radius/beam radius) + 1/2 for a cylindrical perfectly-conductive vacuum pipe and a round uniform density beam), M is the particle mass, V_{rf} is the RF voltage, h is the harmonic number, η is the phase slip factor (≈ -0.86 for the IUCF Cooler ring), f_0 is revolution frequency of the synchronous particle, and R is the radius of the synchrotron storage ring (≈ 13.8 m for the IUCF Cooler ring).

Measurements of the longitudinal bunch profile and longitudinal bunch oscillation frequencies of an electron-cooled 45-MeV proton beam were performed in the IUCF cooler ring as a function of the RF voltage and beam current. The bunch shape was measured using a high bandwidth longitudinal electrostatic pick-up electrode and a 10-bit full scale, 1 GSample/s digitizing scope. The data were transferred to a PC computer for offline analysis. This analysis consisted of fitting the experimental longitudinal bunch density to the theoretical prediction (Eq. 3) using two parameters: σ and g. A comparison between the predicted and measured bunch shapes is presented in Fig. 1: the dashed line "2" is the

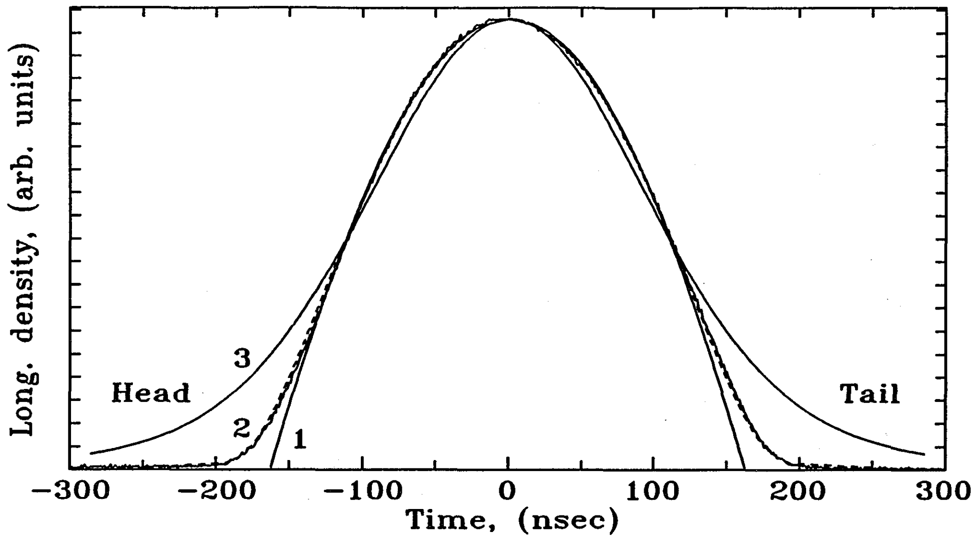

Figure 1. Measured (solid, 2) and theoretical (dashed, 2) linear density. Cosine (1) and Gaussian (3) fits are also presented. $I_p = 400\ \mu\text{A}, \text{V}_{\text{rf}} = 10$ V.

best theoretical fit ($g = 2.1$ and $\sigma = 3.5 \times 10^{-5}$); the solid line "1" is a truncated cosine fit ($g = 2.4, \sigma = 0$) and the solid line "3" a Gaussian fit ($g = 0, \sigma = 8.8 \times 10^{-5}$).

The area under the linear density curve (Fig. 1, curve 2) was used to determine the proton beam current. There is a 5% normalization error due to the uncertainty in the capacitance of the longitudinal pick-up electrode.

The RF cavity voltage was measured both with an RF probe capacitively coupled to the cavity, and by measuring the frequency, ω_s, of a 10°-amplitude dipole synchrotron oscillation at low beam currents. These oscillations were excited by nonadiabaticaly shifting the phase of the cavity voltage by 10° in less than 40 μs.

The fitted values for g and σ can be used to calculate the ratio of the beam rest frame kinetic and electrostatic potential energy spreads, or alternately, the ratio V_{eff}/V_{rf} where V_{eff} is the effective RF voltage amplitude inside the bunch due to the presence of space-charge. This ratio is plotted in Fig. 2 as a function of beam current.

A measurement of the bunch length ("quadrupole") oscillation frequency, ω_q, can also be used to determines the degree by which the space charge force within the bunch compensates the external RF focusing force. For short bunches, where the linear RF force and parabolic bunch distribution is a good approximation to the sinusoidal RF waveform and truncated cosine bunch distribution, there exists a simple analytic expression[7] relating the ratio ω_s/ω_q to the ratio V_{eff}/V_{rf}:

$$\left(\frac{\omega_q}{\omega_s}\right)^2 = \frac{V_{eff}}{V_{rf}} + 3 \qquad (6)$$

In the absence of space charge, $V_{eff} = V_{rf}$ and $\omega_q = 2\omega_s$, which simply corresponds to the rotation of the bunch in the longitudinal phase space; in the case of a zero momentum spread beam, $V_{eff} = 0$, and $\omega_q = \sqrt{3}\omega_s$.

The amplitude of the RF voltage was incremented by a fixed value (≈ 6 V) to excite the bunch length oscillations. The frequency of the bunch length oscillations was measured

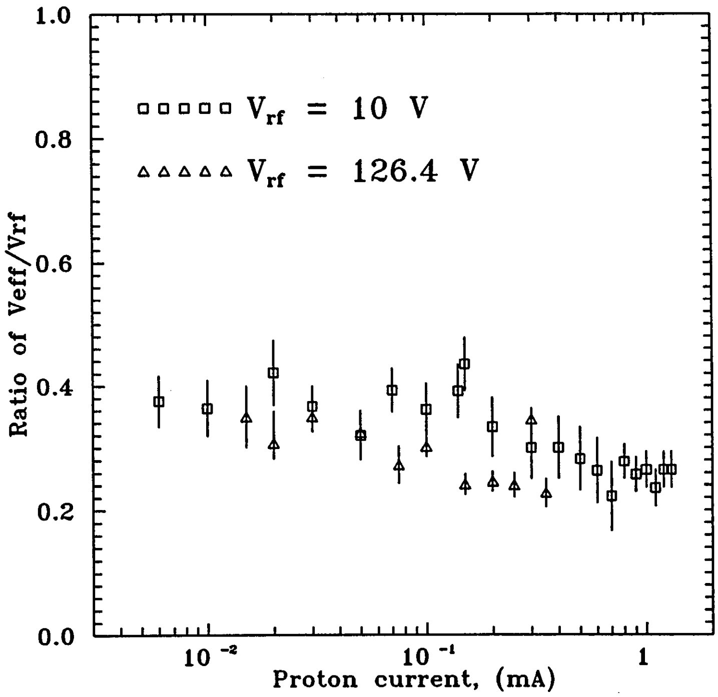

Figure 2. Ratio of V_{eff} to V_{rf} derived by fitting the measured bunch profiles using Eq. 3.

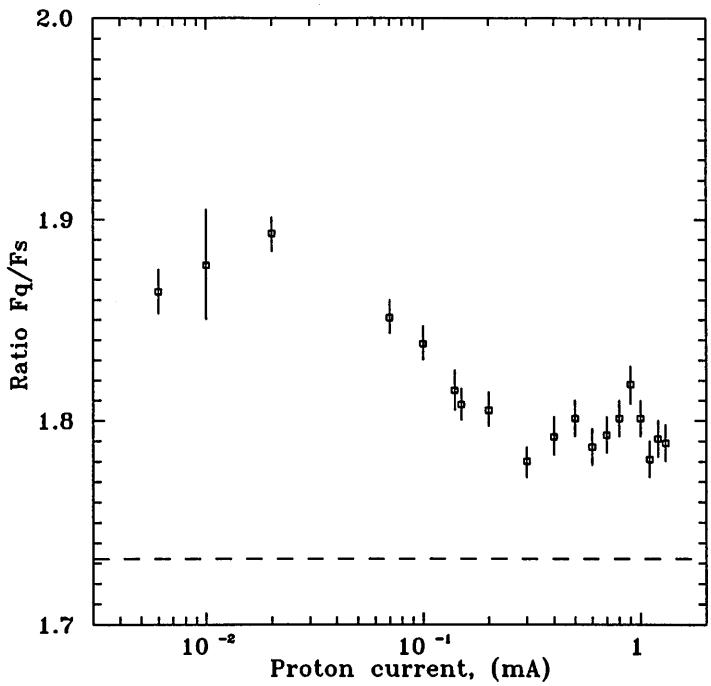

Figure 3. Ratio of quadrupole frequency to synchrotron frequency vs. beam current. Dashed line shows $\sqrt{3}$ value. $V_{rf} = 10$ V.

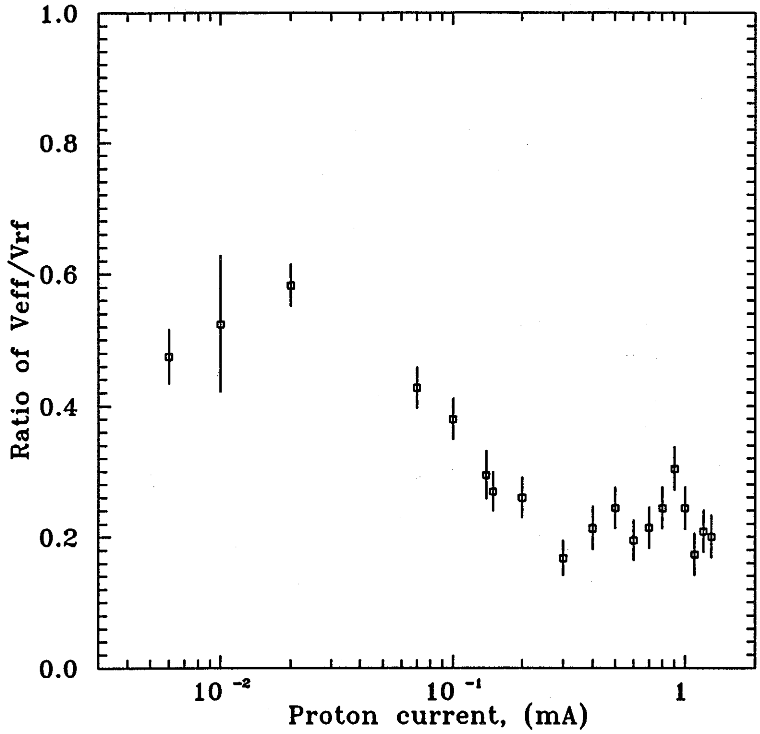

Figure 4. Ratio of effective RF voltage to applied RF voltage derived from the ratio ω_q/ω_s. $V_{rf} = 10$ V.

by monitoring the power in one of the higher revolution frequency harmonics as a function of time. The harmonic number, typically 20-40, was chosen to be sensitive to the bunch length changes.

The measured ratio ω_q/ω_s, shown in Fig. 3, was much closer to $\sqrt{3}$ than to 2, indicating the high degree of space charge compensation. The ratio V_{eff}/V_{rf} calculated from the bunch oscillation frequencies using Eq. 6 is plotted in Fig. 4 as a function of beam current.

We conclude that the monitoring of the bunch modes has the potential to be used on-line in the future experiments which require knowledge of the momentum spread. We shall also continue the measurements of the longitudinal properties of the beam as a function of beam energy and internal target thickness.

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