BEAM DYNAMICS OF QI STORAGE RINGS

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Very short electron bunches, e.g. sub-millimeter in bunch length, can enhance applications such as time-resolved experiments, next-generation light sources, rings that produce coherent synchrotron radiation, and damping rings for the next generation of linear colliders. A possible method to produce short bunches is to reduce the phase slip factor, or the momentum compaction factor for electron storage rings. Because of its potential benefit, the physics of particle dynamics in low α_c lattices is important.

Recently, we have studied the single particle dynamics and the stability of the QI dynamical system, where the particle motion satisfies the universal Weierstrass equation.[1] The particle motion in a QI storage ring can be described by the universal Weierstrass Hamiltonian,

$$H = \frac{1}{2}p^2 + \frac{1}{2}x^2 - \frac{1}{3}x^2,$$

where

$$x = -\frac{\eta_1}{\eta_0}\frac{\Delta p}{p_0}, \quad p = \frac{dx}{dt}$$

are conjugate phase space variables with the time coordinate $t = \nu_s\theta$, the small amplitude synchrotron tune, $\nu_s = \sqrt{\frac{heV|\eta_0 \cos\phi_s|}{2\pi\beta^2 E}}$, and the orbiting angle $\theta = s/R_0$. The particle motion of the Weierstrass Hamiltonian can be expressed in terms of the Jacobian elliptic function. The synchrotron tune can be expressed in term of the complete elliptic integral of the first kind.

In the presence of synchrotron radiation damping, the damping parameter is enhanced by $1/\nu_s$. However, the RF phase noise is also highly enhanced. The QI dynamics system can become chaotic even with small RF phase modulation. The equation of motion becomes

$$x'' + Ax' + x - x^2 = B\cos\omega_m t.$$

Numerical simulations show a sequence of period-two bifurcations evolving toward global chaos. The stability of particle motion has been throughly studied. We found that the QI dynamical system is not sensitive to the RF voltage modulation provided that the modulation amplitude is less than 20%. On the other hand, we showed that the QI dynamical system exhibited chaotic behavior at a relatively weak RF phase modulation. Due to synchrotron radiation damping, stable fixed points (SFPs) of parametric resonances become attractors. As the amplitude of the applied phase modulation increases, the system exhibits a sequence of period-two bifurcations enroute towards global chaos for the modulation tune $\omega_m \in (0, 2)$. The sequence of period-two bifurcations has been attributed to parametric resonances of the Hamiltonian system. The critical phase modulation amplitude vs. the modulation tune shows a cusp,[2] which is caused by the transition between the 2:1 and the 1:1 parametric resonances.

Electrons in a storage ring emit synchrotron radiations. The synchrotron light frequency spectrum is continuous up to a critical energy given by $\hbar\omega_c = 3\hbar c\gamma^3/2\rho$, where $\hbar$ is the Planck's constant, γ is the relativistic Lorentz factor of the electron, c is the speed of light, and ρ is the bending radius. Synchrotron radiation by an electron is a quantum mechanical process. Since an electron normally emits hundreds to thousands of photons per revolution and the average energy of each emitted photon is small, the effect of photon emission can be simulated by white noise. Thus electrons, in the presence of quantum fluctuations, are influenced by a Langevin force with a white noise spectrum. Including the harmonic RF noise, the equation of motion for the electron in a QI storage ring is similar to a class of physical problems such as the current-biased Josephson junction, stochastic resonances, etc. The Langevin equation of motion is given by

$$x'' + Ax' + x - x^2 = B\cos\omega_m t + D\xi(t)$$

with $\langle\xi\rangle = 0$ and $\langle\xi(t)\xi(t')\rangle = \delta(t-t')$. The Fokker-Planck equation for the distribution function Ψ associated with the Langevin equation is given by

$$\frac{\partial\Psi}{\partial t} = \left[-p\frac{\partial}{\partial x} + A\frac{\partial}{\partial p}p + (x - x^2)\frac{\partial}{\partial p} + \frac{D^2}{2}\frac{\partial^2}{\partial p^2} + B\omega_m\sin(\omega_m t + \phi)\frac{\partial}{\partial p}\right]\Psi.$$

In the case of zero harmonic modulation with $B = 0$, the normalized steady-state distribution function for the Langevin equation is given by

$$\Psi(E) = \frac{1}{E_{\rm th}}e^{-H_0/E_{\rm th}} = \frac{1}{E_{\rm th}}e^{-E/E_{\rm th}},$$

where the *energy* E is a Hamiltonian value, and the "thermal" energy $E_{\rm th}$ is given by

$$E_{\rm th} = \frac{D^2}{2A}\ .$$

It is worth noting that the iso-density contour of the distribution function follows the equi-energy line of the unperturbed Hamiltonian. We are studying the effects of quantum fluctuations by solving the Fokker-Planck equation. We are also studying the equilibrium distribution due to potential-well distortion and Coulomb scattering leading to the Touschek effect.[3]

1. A. Riabko *et al.*, Phys. Rev. E **54**, 815 (1996).
2. D. Jeon *et al.*, to be published in Phys. Rev. E.
3. M. Bai *et al.*, to be submitted to Phys. Rev. E.