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An exploration of a Tibetan lama's study of the Pythagorean theorem in the mid-18th century

L'introduction du théorème de Pythagore au Tibet au milieu du XVIIIe siècle

Lobsang Yongdan

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Background

1 The fact that Akya Lobzang Tenpai Gyaltsen (a kyā blo bzang bstan pa'i rgyal mtshan) studied the Pythagorean theorem and was able to transmit it to Tibet was not a historical anomaly. His work took place within the broader framework of Tibetan astronomical traditions and Tibetans' desire to seek new knowledge. Importantly, this knowledge reached Tibet after the Qing rule had changed its attitude towards Jesuit scholars, who were instrumental in introducing European science to China and, consequently, to Tibet. The Pythagorean theorem was one of large European topics of scientific knowledge studied by Tibetans and transmitted to Tibet by the Qing court during the 18th and 19th centuries.

2 In the 6th century BCE, Pythagoras demonstrated that the square of the hypotenuse of a right triangle is equal to the sum of the squares of the other two sides. This geometric

theorem was included in the first book of Euclid's *Elements* and became known as the Pythagorean theorem*.* The book became one of the most important mathematical and logical texts in the world. In the 4th century CE, a Greek scholar known as Theon of Alexandria produced an edition of Euclid. In 800 CE, the Arabs translated the *Elements,* which they received from the Byzantines, into Arabic. Although it was known in the Byzantine Empire, until 1120, it was virtually unknown to Western Europe. Then an English monk known as Adlard of Bath translated it into Latin from an Arabic translation. The first printed edition appeared in 1482 in Europe and has since been republished many times and translated into all the main European languages.

3 Mathematics was an integral part of Chinese culture. Before Euclidean geometry was introduced in China in the 17th century, the method of measuring the right triangle known as *gougu* (勾股) already existed in China. It was this work known as the *Zhoubi suanjing* 周髀算經 which mentions the *gougu* theorem. The author of the compilation is unknown, but it is believed to be composed sometime between 100 BCE and 100 CE. The next important mathematical text is known as the *Jiuzhang suanshu* 九章算术 (*Nine chapters on the mathematical art*). This work was compiled during the Han dynasty (late 3rd century BCE to early 2nd century CE). According to the preface, it was written by two Han-dynasty scholars, and included ideas that were similar to the Pythagorean theorem (Swetz & Kao 2001). However, official scholars such as Xu Guangqi (1562-1633) considered that Chinese *gougu* was more of a general concept than a theorem; furthermore, it had not been proven in the Euclidean sense (Needham 1959, p. 21).

4 In the 16th century, European Jesuits, driven by religious conviction, came to China to spread Christianity. In their efforts to proselytise their religions, Jesuit scholars translated and disseminated a substantial number of European scientific texts into Chinese, alongside their religious works. Impressed by their scientific expertise, particularly in astronomical and calendrical science, the imperial court often employed them as mathematicians, map-makers, and astronomers (Dunne 1962, Mungello 2009). In 1607, the Jesuit missionary Matteo Ricci (1552-1610), with Xu Guangqi, translated some of Euclid's *Elements* into Chinese, and these sections included the Pythagorean theorem (Engelfriet 1998).

5 During the Qing dynasty, Emperor Kangxi (1654-1722) showed great interest in European science. Between 1670 and 1674, the emperor learned some aspects of Western mathematics from a Flemish Jesuit, Ferdinand Verbiest (1623-1688); when King Louis XIV (1638-1715) sent French Jesuits to Beijing in 1685, the emperor's interest in Western mathematics increased. He appointed French and Portuguese Jesuits as his tutors to teach him astronomy, mathematics, and geometry (Jami 2012), and their lectures resulted in the printing of new versions of Euclid's *Elements* in both Manchu and Chinese (Engelfriet 1998, p. 136).

6 Tibet also had a long history of studying astronomy and mathematics. As early as the 8th or 9th century (Khang 2012, p. 23), Tibetans had translated large numbers of astronomical and mathematical works from India and China. In the 11th century, the Kālacakra system of astronomy was introduced from India. This system provided practical mathematical methods for calculating the distance of the planets and the movements of stars and eclipses. Astronomy studies flourished and many books were written on the subject (Henning 2007). Moreover, Tibetan scholars devised and produced several calendrical systems; these included the best-known Puk (Phug) and Tsur (Tshur) traditions (Schuh 2012, Yamaguchi 1989). Tibetans were one of the earliest peoples to use this Hindu-Arabic

numeral system. For example, the scholar Jikme Rikpe Lodrö ('Jigs med rigs pa'i blo gros) (1910-1985), commonly known as Tseten Zhabdrung [tshe tan zhabs drung]) stated that he had seen a handwritten text by Zhalu Lotsawa Chökyong Zangpo (Zhwa lu lo tsā ba Chos skyong bzang po, 1441-1514), a translator from Zhalu monastery in Central Tibet, in which the numbers were transcribed using this numerical system (tshe tan zhabs drung 2007, p. 353).

7 Desi Sangye Gyatso (1653-1705), the regent of the Fifth Dalai Lama, was a scholarly layman. He established a medical college on a hill-top site called Chakpo Ri (Iron Mountain) where students could study several aspects of astronomy, mathematics and medicine (Gyatso J. 2015, 114). He also wrote extensively on many different topics; among these are influential astronomical works such as *Bai dkar* (*The white beryl*) and *gYa' sel* (*The eradication of rusting*).

8 By the early 18th century, some Tibetan scholars had become aware that their own systems of astronomy and mathematics were not providing sufficiently accurate information, and they began to seek new information. They found that the Jesuits' approaches provided suitable new methods and techniques to reform and update Tibetan time reckoning systems. The Tibetans encountered European mathematical and astronomical traditions at the imperial court in Beijing. In the early 18th century, under Emperor Kangxi, Tibetan and Mongolian scholars translated a large of number of the European astronomical texts into Mongolian and Tibetan, under the title *The Tibetan translation of the Kangxi emperor Chinese astronomical works* (*'Jam dbyangs bde ldan rgyal pos mdzad pa'i rgya rtsis bod skad du bsgyur ba*), or in short *The great Chinese astronomical compendium* (*rGya rtsis chen mo*). Among these translated texts were: *sKud pa brgyad kyi ngos 'dzin* (*The identification of tables of the eight line*), a trigonometric work and tables by Johann Schreck (1576-1630) (Schreck 1715). Moreover, sometimes Buddhists were also involved in surveying and mapping the Qing empire (Yongdan 2015)

9 In the mid-18th century, Tibetans were actively involved in making calendars at the imperial court in Beijing; this involved their studying the Jesuits' modern calendrical science. In subsequent years, some of these lamas secretly transcribed private imperial calendrical manuals into Tibetan. Consequently, in Amdo, the country's northeast region, Tibet adopted the Jesuits' calendrical system (Yongdan 2017). In the 18th and 19th centuries, Tibetans studied and transmitted Jesuit astronomical works into Tibetan. This enabled them to reform their calendar and update their geographical information about the world. This new knowledge led to many new scholarly arguments and debates about various aspects of European mathematical, astronomical, and geographical issues. Within this framework of Tibetan and Jesuit knowledge transfer through the Qing court, the Pythagorean theorem was introduced to Tibet.

A polymath at the imperial court in Beijing

10 The Tibetan who studied and introduced the Pythagorean theorem to Tibet was Akya Lobzang Tenpai Gyaltsen (1708-1768) or simply Akya Loten. He was not only a Tibetan *trulku* (*sprul sku*), or reincarnation, but also a Qing imperial Gongme Tamka Lama, "seal-holder lama". These lamas provided Buddhist teachings, religious rituals, and prayers to emperors and imperial court in exchange for economic, military, and, often, political assistance. They were not servants or officials in the secular sense; according to Tibetan Buddhist traditions, they were viewed as enlightened human beings whose purpose was

to serve people living in *samsāra* (cyclic existence). This relationship was conducted through the rituals, norms, and politics of *chö-yon* (*mchod yon*), which is often translated as a patron and priest relationship. Since the Manchu had had extensive contacts with Mongols, who were Tibetan Buddhists, they adopted this religions tradition. Later on, Qing rulers invited a group of Tibetan lamas to Beijing. Many of them served in this religious capacity at the court, and they also acted as envoys, translators, and map-makers.

11 Akya Loten was one of these imperial seal-holder Lamas in Beijing; he was also an important *trulku* in Tibet. He was born in 1708, in a village near Kumbum monastery, one of the largest monasteries in Tibet and the birthplace of Tsongkhapa (1357-1419), who founded the Gelugpa tradition of Tibetan Buddhism. This area had been a crossroads for several civilisations, and was inhabited by diverse groups of people. This diversity is reflected in his name, Akya Lobzang Tenpai Gyaltsen. There are two opinions about why he was called Akya, or a kyā: the first opinion asserts that he was the reincarnation of Tsongkhapa's father, Lumbum Ge (Klu 'bum dge), and was therefore called "Akya [...] as father is known as Akya in the Amdo dialect" (Rinpoche 2010). The second opinion asserts that Akya is formed from two words: the Tibetan ཨ and Chinese *kya*, Jia 家, meaning that he came from the A, which is a Tibetan syllable family (Mi nyag mgon po 1997, 526). Historically, Amdo had a mix of ethnic groups and languages, so it was common to use Chinese terms to describe family lineages. In any case, he was identified at a young age as the reincarnation of Akya Sherap Zangpo, an influential lama from Kumbum monastery. Accordingly, he was first taken to this monastery for training and then, as was customary, he was sent to study at Sera monastery in Lhasa when he was older. In 1735, on his return from Lhasa, he became the abbot of Kumbum monastery's Tantric College. He also established a retreat center called Senggé Khar Gyi Ritrö (Seng ge mkhar gyi ri khrod [The Lion's Fortress Retreat]), where he spent most of his time.

12 In 1735, driven by filial piety and his belief in Tibetan Buddhism, Emperor Qianlong (r. 1735-1795) established the Tibetan Gelugpa monastery known as Yonghegong (Ganden Jinchak Ling [dga' ldan byin chags gling] in Tibetan). In 1744, Qianlong asked a Tibetan Buddhist polymath at the imperial court, Changkya Rölpé Dorjé (1717-1786), to invite lamas and teachers from Tibet to the monastery to teach monks from Mongolia. Subsequently, he invited eighteen Tibetan scholars with the qualification of *geshe* (*dge bshes*), the highest degree in the Gelugpa tradition of Tibetan Buddhism, to teach in the monastery (Tuken 1989, p. 220). Akya Loten was among the invited Tibetan scholars, and he stayed for three years. It is easy to see why he was appointed. As well as being a Tibetan scholar trained in the Gelugpa monastic tradition, he also appears to have been multilingual, speaking Chinese and Tibetan – it was common for Tibetans born Amdo to speak several languages. While living in Beijing, he also travelled to many parts of China. In 1762, he was sent by imperial order to travel through the Kham regions of Tibet to Lhasa, to enthrone the eighth Dalai Lama. He was known as a great scholar during his lifetime; he not only trained many important Buddhist lamas in Kumbum monastery, but also wrote extensively on many different subjects. His writings include influential astronomical notes and dates. After his death at Kumbum monastery in 1768, his collective works (*sungbum*) was carved as a Tibetan woodblock print and can still be seen at his official residence, Akya Nangchen, in the monastery.

Akya Loten's work in astronomy and mathematics

13 Akya Loten's body of work covers a wide range of topics including Buddhist philosophy, rituals, prayers, and grammatical commentaries as well as letters, poetry, and history. These reflect his life as a scholar and his own interests. While many of his works derived from existing Tibetan knowledge systems, they include some completely new information gained from Chinese sources – including a piece about the dating of Buddha's birth and death. For centuries, there had been extensive debates about this issue, and Akya Loten put forward a new method of dating. Because his method was based on some Chinese sources, it differed from the existing Tibetan chronology. Many scholars, including the modern Tibetan scholar Tséten Zhapdrung consider this information very valuable (tshe tan zhabs drung 2007, pp. 7-9).

14 The most interesting and relevant of his texts is the fifty-page version of the Tibetan work known as *ma hA tsi na'i byang mtha' rgyal khabs chen po pe cing gi gtso bor gyur pa'i byang phyogs kyi yul 'khor la 'os pa'i dus spyor gyi rnam bzhag pad+mo'i tshal rab 'byed pa'i nyi ma gzhon nu* (*Commentaries on the time lines of those centered by the great Chinese city of Beijing and its northern territory known as the Youthful Sun of Lotus Bloom*). This is a collection of personal notes and various commentaries on mathematics, astronomy, and trigonometry that he had studied China during his staying in Beijing, and it includes some European astronomy, mathematics, and trigonometry. However, the title is misleading because it refers to just one commentary in this work; in fact, it contains twenty-six commentaries and notes on various mathematical issues and astronomical calculations, as well as historical issues.

15 While Akya Loten wrote most of these texts himself, at least one of the articles was written by someone else: the timetable chart known as *o'i rod kyi yul du 'os pa'i dus sbyor gyi re'u mig* (*The timetable in the land of Oirat*), produced by Hubila Gen Rabjampa Gun Pandita, also known as Sonam Chojo Hubil Lagen. It is difficult to determine this scholar's ethnicity – he could have been a Mongol or a Tibetan. However, since he was given the honorific titles Rabjampa (*rab 'byams pa*, a degree from a Gelugpa monastery such as Kumbum) and Pandita (one who has mastered five subjects), he must have studied in Tibet. In any case, he was probably one of the Tibetan Buddhists who was trained by the Jesuits to be mathematicians and surveyors. He was one of two main translators of the Jesuits' astronomical works into Tibetan in 1715 (Yongdan 2015, p. 181). The fact that Akya Loten included this time-chart in his work suggests that he considered it an important text.

16 From these mathematical notes and commentaries, it is clear that Akya Loten had been influenced by some of the mathematics and astronomy of the Jesuits when he was in Beijing, although he does not mention them or their country of origin; stating that the new methods of study were Chinese. In fact, they were the European forms of mathematics and astronomy brought to China by the Jesuits. The Jesuits' influence on Akya Loten is particularly evident in his assertion that the Earth was spherical but, instead of mentioning its European provenance, he used this statement to raise questions about certain astronomical views on Naktsi (*nag rtsis*) astrology. After criticising the long-held Tibetan historical account that the elemental divination known as Naktsi came from China, via Mañjuśrī, a bodhisattva associated with *prajñā* in Mahayana Buddhism, he made several points to argue that Naktsi was not Chinese. Instead, he suggested that it

was more likely to be a Tibetan invention. He also gave specific details about the differences in China's calendrical science from that of Tibet. At this point, he introduced the idea of a spherical Earth, without any reference to the Jesuits:

> *'jig rten chags pa'i sa 'di zlum po la nyi zla gza' skar phal che ba zhig steng 'og tu 'khor zhing 'gro bas bgrod tshul gyi dbang gis nyi ma zla bas sgrib pa dang zla ba sa gzhi rnams nyi ma rnams thad kar drang bor bab pa na sa gzhi'i grib pa zla ba phog pa'i dbang gis zla 'dzin byung ba yin zer*
> This physical Earth is spherical and the sun, moon and other planets are orbiting around [it]. Because of the movements, the moon blocks sunlight. When the sun, Earth and moon are aligned together, the Earth's shadow covers the moon, and thus the lunar eclipse occurs. (Blo bzang bstan pa'i rgyal mtshan, p. 42a)

17 He describes the Earth as spherical by using the Tibetan term *zlum po*. Although Tseten Zhabdrung implied that it was a heliocentric (sun-centred) model (tshe tan zhabs drung 2007, pp. 37-72), this is clearly not the case because Akya Loten's version depicted the sun, moon and other planets orbiting around the Earth, rather than the Earth and other planets orbiting around the sun. Thus, Akya Loten probably described the Tychonic system rather than Copernican model. Although some Jesuits introduced the Copernican model to China (Sivin 1973), the Qing court accepted the Tychonic system as its official position. This was not the first Tibetan work to mention that the Earth was spherical; *The Great Chinese Mathematical Compendium,* had made this claim. However, as far as I know, Akya Loten was the first Tibetan to discuss this in his own writings. We also can see from these types of works that he was willing to consider new theories and to advocate change. For example, many Tibetans had previously relied on ancient Indian Tantric works like the Kālacakra to determine the time in specific locations. Khedrub Norsang Gyatso (mkhas grub nor bzang rgya mtsho) (1423-1513), an important astronomer, had used the Kālacakra tantra to assert that there were thirty-six hours in the equivalent of a day in Tibet's longest summers[1], and slightly less in China (Gyatso K. N. 2016, pp. 134-135). However, Akya Loten believed that this system could not measure time and location accurately; instead, he suggested that periods of observation should be used to measure time:

> *da lta dus tshod brtag byed kyi 'khor lo tshad ldan sogs kyis legs par brtags pa na rgya nag gi pe cing du dbyar nyi ring mtha'i tshe nyin mo chu tshod so bdun yod pa mngon sum du grub pa 'di gnyis kyi 'gal ba ji ltar spongs smra dgos so/de'i phyir bod yul du yang dbyar nyi ring mtha'i tshe nam langs lag ris mthong ba nas nyi nub lag ris mthong ba'i bar la dus tshod brtag byed kyi 'khor lo sogs brtag thabs du mas chu tshod ji tsam 'dug legs par brtags pas shes par 'gyur gyi/ rgyud nas gsung sgras zin ji lta ba bzhin du bkod pa'i da lta'i dus spyor gyi yi ge 'di dag la yin brtan dka'am snyam mo*
> Nowadays if we use a standard period of watching to observe and measure time, we find that the longest days in summer in Beijing are thirty-seven hours. This is a directly perceived fact. So how can we solve this contradiction? Instead of relying on the words of Tantra and some contemporary works on measuring time, [we] should be using good measuring practices like watching to observe time from morning to evening, when people can see their hands clearly, even in the longest summers. (Blo bzang bstan pa'i rgyal mtshan 2000, p. 43a)

18 Here he was challenging the authority of the Kālacakra and some contemporary works on astronomy. His work is full of notes and commentaries on mathematical and geometric calculations, including ways to calculate a square root, to measure distances and heights, and so on. It is beyond the scope of this article to cover all the aspects of this book. Still, it is certainly one of the earliest Tibetan texts in which we can find such traces of European

astronomical and mathematical influence, alongside Tibetan reactions and responses to them.

The Pythagorean theorem

19 Without mentioning its historical background or significance, Akya Loten introduces the Pythagorean theorem in this text:

> *mi mnyam pa byung tshe ngos ring shos dang 'bring gru gsum ngos gsum*[2] *dang thung ngu gsum las 'bring dang thung ngu gnyis la brten nas ring shos gzhal na/' bring dang thung ngu gnyis kyi tshad dang sor sogs kyis gzhal te/ tshad gyi grangs gang byung ba de so sor rang nyid kyi grangs kyis bsgyur/ bsgyur pa rnams mnyam du bsres/ de la phing h+phang gis bgos pa'i nor gyis ring shos tshad rnyed par 'gyur ro/ dper na gru gsum ngos ring shos la khru lnga/ 'bring la khri bzhi/ thung ba la khru gsum yod pa zhig gi ring shos kyi ngos gzhal tshe/ 'bring gi khru bzhi bzhi nyid kyis bsgyur/ thung ngu'i khru gsum gsum gyis bsgyur/ snga ma la bcu drug dang phyi ma la dgu byung mnyam du bsres nyer lnga 'byung/ de'i mthar tshon gyi don thig gcig/ de'i mthar phun gyis don du thig gcig te thig le gnyis bgod/ de la phing h+phang gis bgos pa'i nor la lnga 'byung/ de ni khru lnga yod pa'i don te ring shos kyi tshad rnyed pa yin no*
> If a triangle has a long, medium and short side, the size of the longest side can be determined by using the medium and shortest sides. To do this, the medium and short sides need to be measured, and each number then squared. After the two numbers are added together, the square root of the sum will produce the size of the hypotenuse. For example, if a triangle has a long side (hypotenuse) of 5 cubits, middle side of 4 cubits, and short side of 3 cubits, square the middle number 4 to become 16 and the small number 3 to become 9. The sum of these two numbers is 25. After that, in representing *tshon* (*cun*?), a zero needs to be added and after that, in representing *phun* (*fen*), another zero needs to be added. If we take the square root of this (*phing h+phang*), the sum will be 50. This means that the hypotenuse is 5 cubits. (Blo bzang bstan pa'i rgyal mtshan 2000, p. 52a)

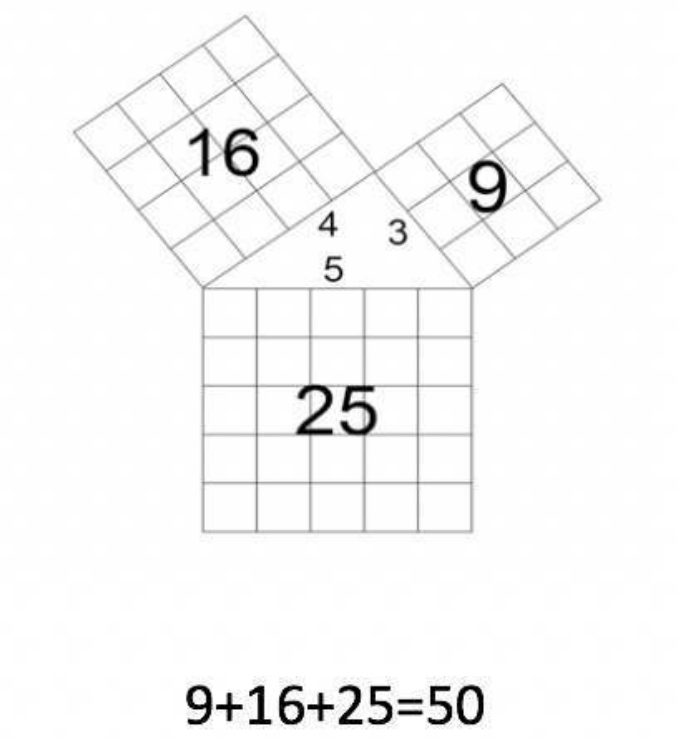
20 The very simple terms he used make it easy to follow his explanation. For example, he used *gru gsum ngos gsum* to represent a right triangle, which literally means three edges and three vertices. He used *rang nyid kyi grangs kyis bsgyur* for "squared", which means to multiply by its own number. Using (*phing h+phang),* which is a Chinese term 平方 (*píngfāng*), he says that the square root needs to be taken. Because of its utilitarian character, it is not surprising that Akya Loten also used this right triangle of 3.4.5 to show how it works. An equation suffices to identify the Pythagorean theorem for a particular case of the right triangle of 3.4.5. Using modern mathematical symbolic notations, this would be written as:

$$c = \sqrt{a^2 + b^2}$$

21 Akya Loten does not mention explicitly that this triangle is a right triangle. However, it must be a right triangle, otherwise the Pythagorean theorem would not apply to it. In *bSnan pa'i ngos 'dzin gyi 'byung khungs ri mo brjod pa* (*The descriptions of sources of the illustration of the adding tables*), a graphic mathematical text in *rGya rtsis chen mo* (*The great*

Chinese astronomical compendium) with graphical illustrations, there is a discussion of *zur gsum* (triangles) (Anonymous 1715, p. 3b). In this discussion, the text describes the measurements of right triangles. The explanation the text provides is different from Akya Loten's use of *gru gsum ngos gsum* (three edges and three vertices) to describe right triangles. Nevertheless, the meaning is essentially the same, as both texts describe a triangle that is a polygon with three edges and three vertices.

22 The clear indication that the triangle is a right triangle is his use of *tshon* (Chinese, *cun* 寸) and *phun* (Chinese *fen* 分) and adding two zeros to 25. According to the unities of Chinese measurements, one *chi* is divided into ten *cun* or Chinese inches, and one *cun* is divided into ten *fen*. Since he does not mention that zeros come after decimal points, after adding two zeros 25, it becomes 2500. Then without mentioning 2500, he says that the square root of this number is 50. This 50 appears to be the total sum of sides of a, b, and c right triangles:



23 As he indicates, the hypotenuse must be 5. If it were a right triangle, the sum of the squares of three sides could not be 50 and the hypotenuse could not be 5. This is the clearest indication that it is a right triangle.

24 After this proposition, he moves on to describe how to measure the circumference of a bundle of incense and how to calculate the units of coins and silver if they are thrown on the ground. Then he comes back to the Pythagorean theorem and describe how to solve this equation:

$$= \sqrt{a^2-b^2}$$

25 He writes the following:

> *sngar gyi gru gsum mi mnyam pa de'i ring shos dang 'bring gnyis rnyed nas thung ngu'i tshad tshol na/ring shos dang 'bring shos gnyis kyi tshar so sor rang gis grangs nyid kyis bsgyur te so sor gnas pa la/nyung bas mang pa la sbyangs pa'i lhag ma gang byung de la phing h+phang gis bgos pa'i nor gyis thung ngu'i tshad rnyed par 'gyur zhing*
> As described earlier, if we want to find out triangle's shortest side after knowing hypotenuse and the medium size, the numbers of hypotenuse and the medium are squared and, subtract small numbers from big number and then the number needs to be square rooted, and its sum is the size of shortest side. (Blo bzang bstan pa'i rgyal mtshan 2000, p. 53a)

26 Following this, he also describes how to find the medium size of a triangle. He writes:

> *de bzhin du ring shos dang thung bu'i tshad rnyed nas 'bring po'i tshad 'tshol na yang ring thung gnyis kyi tshad so sor rang gi grangs nyid kyis bsgyur te nyung bas mang pa la sbyangs pa'i lhag ma gang byung ba de la phing h+phang gis bgos pa'i nor de nyid 'bring po'i tshad yin no*
> Like this, if we want to find out a triangle's medium size after knowing the hypotenuse and the shortest size, the numbers of hypotenuse and the shortest are squared. After subtracting squared numbers from big number and then its number need to be square rooted, its sum is the size of medium size. (Blo bzang bstan pa'i rgyal mtshan 2000, p. 53a-b)

$$b = \sqrt{c^2-a^2}$$

27 As far as I understand, before Akya Loten, no Tibetan scholars mention the square root of the numbers. He did not translate the term "square root" into Tibetan; instead, he uses (*phing h+phang*), which is a Chinese term 平方 (*píngfāng*). This is interesting because it suggests that the Tibetan language might not have included the concept of a square root; thus, he used the phonetic term *píngfāng* as a substitute. Earlier in this text, he had clearly described its meaning through his detailed explanation of how to find the square root of 49 and why this is seven (Blo bzang bstan pa'i rgyal mtshan 2000, pp. 16a-17b). Although the Chinese had known some aspects of the square root for a long time, the term *píngfāng* was a modern term that was first used by Matteo Ricci and the Ming official-scholar Li Zhi Zao (李之藻, 1565-1630) in *Tongwen Suanzhi* 同文算指 (*Rules of arithmetic common to cultures*) (Siu 2015) and expanded on by scholars like Xu Guangqi (Needham 1959, p. 65, Engelfriet & Siu 2001, p. 304).

Where did he learn mathematics?

28 Since Akya Loten did not provide any background to his texts, it is difficult to pinpoint where he learned mathematical concepts. When Akya Loten arrived in Beijing, the glory days of the Jesuits were over. However, it was still the height of cultural activities. Qianlong established one of the largest Tibetan monasteries in Beijing and gave some tasks of making the official calendar to monks at Yonghegong (Yongdan 2017, p. 99); moreover, he was also involved with many translation works from Tibetan into Mongolian and Manchu. As an imperial court lama and the abbot of Yonghegong, as well as close to Changkya Rölpe Dorje (Lcang skya rol pa'i rdo rje) (1717-1786), a close confident of the Qianlong emperor, Akya Loten could have had access to a range of works and scholars. In fact, according to a biographical account of Changkya Rölpé Dorjé, written by his brother, the third Chuzang Ngakwang Tupten Wangchuk (Chu bzang ngag dbang thub bstan dbang phyug) (1725-1796), Rölpe Dorje and Akya Loten studied astronomy together in Beijing (Chu bzang ngag dbang thub bstan dbang phyug 2015, pp. 57b-58a). This is significant because Changkya Rölpe Dorje was more than just a scholar; he was the principal Buddhist teacher at the Qing court and a close confidant of the Qianlong Emperor. This would mean that they would have had access to any teachers they required, including Jesuits and Chinese scholars.

29 This raises questions about where he might have learned about these concepts and from whom. As mentioned earlier, he could have had access to a whole range of European works in Chinese, Manchu, and Tibetan as well as in Mongolian. Importantly, since he could read Chinese, it was certainly possible that he could have read all European mathematical and astronomical works that European Jesuits translated or complied in the 17th and early 18th centuries. Some of these works were not only available in Chinese, but also in Manchu, Mongolian and Tibetan. For example, under Kangxi's order, the French Jesuits known as the King's mathematicians gave lectures on Euclid's *Elements* and translated it into Manchu in the later 17th century, and this effectively replaced Ricci and Xu Guangqi's translations and the French geometry gave different proofs from the Ricci-Xu's translation (Elman 2009, p. 152). In addition to Euclid's *Elements* and *Tongwen Suanzhi,* Xu Guangqi also worked on book called *Gougu yi* 勾股义 (*The principle of gougu*) (Xu Guangqi & Sun Yuanhua 2011. This work was written by Xu Quangqi with his student named Sun Yuanhua in 1612. There are 15 problems on *gougu* in this work, and in the first proposition, the author describes how to find the hypotenuse of a right triangle when the lengths of other sides are known (Engelfriet & Siu 2001, pp. 294-295). It also uses the 3-4-5 right triangle. This is similar to what Akya Loten writes:

30 If a triangle has a long, medium and short side, the size of the longest side can be determined by using the medium and shortest sides. To do this, the medium and short sides need to be measured, and each number then squared. After the two numbers are added together, the square root of the sum will produce the size of the hypotenuse.

31 In particular, in *The Great Chinese Mathematical Compendium,* there is a text known as *sKud pa brgyad kyi ngos 'dzin*[3]. This is a trigonometric work. It was translated from the *Geyuan baxian biao* 割圜八線表, written by Jesuits (Iannaccone 1998). It includes a graphical presentation with descriptions of how to calculate the trigonometric functions of tangent, secant, cotangent, cosecant, sine, cosine, versine, and coversine (Chen 2015, pp. 495-497).

32 However, if we could dive into these questions, there are several possible sources. First of all, it is quite possible that his source could be *Zhoubi suanjing.* This "piling up the rectangles" proceed involved with a triangle having side lengths of 3, 4 and a diagonal length of 5, and it also provides a diagram showing the measurements (Joseph 2010, pp. 248-249). With this description, it might be easy to assume that Akya Loten's usage of numerals such as 3, 4, and 5 perfectly matches the *Zhoubi suanjing.*

33 It is also possible that the work of Minggatu (明安图1692-1763) was the source of Akya Loten's work. Minggatu was a Mongolian mathematician. Historians have regarded him as one of the "outstanding native mathematicians". He expounded on Pierre Jartoux's (1668-1720) geometrical works *Ge Yuan Mi Lü Jie Fa* 割圜密率捷法 (*The Quick Method for Obtaining the Precise Ratio of Division of a Circle Ge*) (Jami & Gernet 1990, Martzloff 2006, pp. 31-32, Elman 2009, p. 152).

34 Similarly, the many texts by Mei Wending (1633-1721), another renowned Chinese mathematician, (Jami 2012, pp. 81-101) could have provided him with information. This scholar compiled ancient mathematical material and discussed a number of almost forgotten topics. In his work *'Gougu juyu* 勾股举隅 (*Illustration of the right triangle*), he talks about the Pythagorean theorem and provides two forms of evidence for it as well as other applications of the theorem (Liu & Dauben 2002, p. 300). Moreover, when he was in Beijing in the mid-18th century, he may have had access to several knowledgeable persons, including European Jesuit missionaries such as Michel Benoist (1715-1774) and Jean Joseph Marie Amiot (1718-1793).

35 However, it is safe to assume that many of Akya Loten's writings, including his work on the Pythagorean theorem, were not based on Chinese scholarship; rather, they draw on European mathematical traditions transmitted to China by Jesuit missionaries. First, if we look at this text as a whole, as exemplified by Tycho Brahe's cosmological models, it is clear that that he had access to some of the new cosmological and mathematical knowledge that the European Jesuits brought to China. So, it can be argued that he had range of options and choices. Secondly, in this particular case, Akya Loten uses terms such as *gru gsum ngos gsum* to describe the right triangle or Sānjiǎo 三角 in Chinese, and the term was introduced first into Chinese by Matteo Ricci and Li Zhizao (李之藻) (Feng 2006, 52). Thirdly, his usage of the square root or *píngfāng* indicates that Akya Loten's source must be one of the Jesuit's works which they translated or written in Chinese. Again, this term was coined and used as a modern mathematical term by Matteo Ricci and Li Zhizao, in *Tongwen Suanzhi* 同文算指.This work was compiled by Matteo Ricci and Li Zhizao in 1613, and it is based on the *Epitome Arithmeticae Practicae* from 1583 by Christopher Clavius (1538-1612) (Li & Ricci [1613] 1993). It was not entirely a work of translation; rather it is considered as a hybrid work in which Li Zhizao attempted to integrate some European mathematics with traditional Chinese mathematics (Siu 2015).

36 However, it is fairly certain that his information might have come from one of these European sources, which were translated or transmitted by European Jesuits. Usage of Western concepts for words like triangle and square root suggests that Akya Loten's source is likely to have been one of the Chinese mathematical works that were translated or derived from European works by European missionaries. Therefore, Akya Loten's method of determining that the square of the length of the hypotenuse of a right triangle equals the sum of the squares of the lengths of the other two sides is probably the Pythagorean theorem. In any case, it is probable that Akya Loten had multiple ways of learning the Jesuits' forms of mathematics and astronomy. First, in addition to Tibetan

translations of Jesuits' texts, he may have had access to some of their original books and commentaries in Beijing. Second, as an important Tibetan *trulku* and an official at the Qing court, Lama Akya Loten lived and worked in the cosmopolitan Buddhist Qing world. Unlike most people, he could have had access to mathematical texts and knowledgeable persons. Having gained this knowledge from multiple sources, he expressed his ability to understand it by writing a series of notes and commentaries about it.

Conclusion

37 While it is not easy to identify Akya Loten's exact sources, he most likely drew on European sources translated into Chinese and contempory discussions of the equation for the Pythagorean theorem. The intellectual history of Tibet, in particular the Euro-Tibetan intellectual encounter, is very much a neglected field. To my knowledge, there are very few works available on the subject. Diving into this uncharted intellectual history is quite a daring experience, and, no doubt, mistakes will be made in the process. In spite of the challenges, looking at the Euro-Tibetan intellectual encounter raises several interesting and important questions about Tibet and the Jesuits' mission to China. First, as I have mentioned elsewhere, historians studying the Jesuits' mission to China have focused on a two-way exchange: that is, how the Jesuits introduced European science to China and how China was then introduced to Europe. However, this two-way model does not fully represent the Jesuits' history in China. Not only the translations of the Jesuits' astronomical and mathematical texts into Mongolian and Tibetan but also the reformation of calendars in the 18th century suggest that impacts and influences of the Jesuits were more extensive and far reaching than previously thought. Thus, academics researching the Jesuits' impacts on the history of science in Asia in the future should also consider their impact on Inner Asia and on Tibet in particular.

38 Secondly, the arrival of the Jesuits' scientific knowledge in Tibet needs to be studied within the broader framework of the history of astronomy during the Qing dynasty. During the Kangxi reign, the Jesuits played an important role in reforming and implementing new forms of calendrical science, and the emperor himself was interested in all aspect of the Jesuits' work, including trigonometry. However, the Jesuits influence declined from the beginning of the 18th century. Scholars who have studied the period suggest that this was largely due to changing imperial attitudes towards the Jesuits and the rise of Chinese scholars such as Mei Wending[4]. It appears, however, that Mongols and Tibetans may have been the greatest beneficiaries of the Jesuits' science. In light of the fact that Tibetans and Mongolians had begun to take on the Jesuits' astronomical responsibilities, did this influence their power in the court?

39 Third, without any deep analysis or understanding of Tibet's intellectual history, it has been widely assumed that Tibetans did not know about – or were not interested in – European science until the British and Chinese brought it to Tibet in the 20th century. The example of Akya Loten's explanation of the Pythagorean theorem clearly demonstrates that Tibetans were already engaging with some aspects of European mathematical and astronomical science two centuries prior to this. The idea that Tibet was the most isolated place on Earth and did not have any modern scientific influence until the British and Chinese introduced them betrays a lack of understanding of Tibetan intellectual history.

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NOTES

1. Before modern watches and clocks were introduced into Tibet, time was measured by water clocks, and the Indian hour is one-fifth of a *khyim*, which is 24 minutes (Das 2004, p. 419, Henning 2007, p. 12).
2. Akya Loten clearly uses *gru gsum ngo gsum* for "triangle" in the cited text. As I mentioned in this article, literally, *gru gsum* means "three edges" and *ngos gsum* means "three vertices" and the expression denotes a triangle; *gru gsum ngos ring shos* here signifies the hypotenuse of the triangle.
3. I plan to engage with this important trigonometric work in due course.
4. Changing imperial attitude towards the Jesuits and the rise of Chinese scholars such as Mei Wending, for more details see Jami 2012.

ABSTRACTS

Although ancient civilisations like India and China had their ways of dealing with the right triangle, one particular method attributed to the Greek mathematician Pythagoras (570-495 BCE) was introduced to China in the 17th century by the European Jesuit Matteo Ricci (1552-1610). In the 18th century, when Tibetans began to take an interest in European astronomical, geographical and medical science, Euclidean geometry was one of the mathematical ideas brought into Tibet. According to most academics, European science did not reached Tibet until the 20th century. In contrast, this article describes how the Tibetan scholar Akya Lobzang Tenpai Gyaltsen (1708-1768) studied the Pythagorean theorem in Beijing and disseminated it in Tibet two centuries earlier.

Bien que les civilisations anciennes comme l'Inde et la Chine aient eu leurs propres façons de mesurer les triangles à angle droit, une méthode particulière attribuée au mathématicien grec Pythagore (570-495 av. J.-C.) fut introduite en Chine au XVI[e] siècle par le jésuite européen Matteo Ricci (1552-1610). Au XVIII[e] siècle, lorsque les Tibétains commencèrent à s'intéresser à l'astronomie, à la géographie et à la médicine européennes, la géométrie euclidienne fut l'une des idées mathématiques apportées au Tibet. Cet article décrit comment le savant tibétain Akya Lobzang Tenpai Gyaltsen (1708-1768) étudia le théorème de Pythagore à Pékin et le diffusa au Tibet, deux siècles avant la date d'introduction de la science européenne au Tibet généralement donnée par les spécialistes.

INDEX

Keywords: Jesuits, China, Tibetans, lama, Beijing, Qing, Pythagorean theorem, science, Europe, mathematics, exchange

Mots-clés: jésuites, Chine, Tibétains, lama, Beijing, Qing, théorème de Pythagore, science, Europe, mathématiques, échange

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