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A GENERAL PLASTICITY AND FAILURE CRITERION FOR MATERIALS OF VARIABLE POROSITY

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ABSTRACT

Many criteria have been developed to describe the yielding condition, plastic potential, and failure strength of engineering materials such as metal, concrete, rock, soil, and backfill. In this report, the authors first recall some relatively common criteria used to describe yielding and failure of porous materials. It is then shown that the main features of a large number of these criteria can be represented by a unique set of equations. The ensuing multiaxial criterion is applicable to a wide diversity of materials and loading states. A particularity of the proposed criterion, named $MSDP_u$, is that it includes an explicit porosity-dependency. The validity of this general criterion is demonstrated using experimental results obtained on various types of materials.

Key words: criterion, failure, plasticity, porous material, porosity.

RÉSUMÉ

Plusieurs formulations ont été développées pour décrire les conditions d'écoulement, le potentiel plastique et la résistance à la rupture des matériaux utilisés en ingénierie tel les métaux, les bétons, les roches, les sols et les remblais. Dans ce rapport, les auteurs revoient d'abord quelques critères communément utilisés pour définir la limite élastique et la rupture des matériaux poreux. On montre ensuite que les caractéristiques de plusieurs critères existants peuvent être représentées avec un seul système d'équations. Ce critère multiaxial est applicable à une grande variété de matériaux et de conditions de sollicitation. Une particularité du critère proposé, appelé $MSDP_u$, est le fait qu'il inclut une dépendance explicite sur la porosité. La validité de la formulation générale est démontrée en utilisant des résultats expérimentaux obtenus sur plusieurs types de matériaux.

Mots clés: critère, rupture, plasticité, matériaux poreux, porosité.

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1. INTRODUCTION

In numerous applications requiring analysis of the mechanical behaviour of engineering materials (metal, concrete, soil, rock, backfill, mining waste, etc.), it is necessary to define the conditions associated with various specific states, including the elastic limit, the initiation of crack propagation, and the peak strength. To describe these particular conditions, engineers generally use mathematical functions, called criteria, expressed in the stress space. Over the years numerous criteria have been proposed to represent the limit of the elastic domain (plasticity criterion), the critical state (corresponding to the critical void ratio when no volume change occurs), the plastic potential (which serves to define inelastic deformation), and the failure strength (associated with the maximum peak stress) and residual strengths. For these and various other states, it is possible to utilise a unified function, where the parameters are defined to represent the distinct phenomena.

Over the years, a large number of criteria have been developed for materials used in engineering. In the last two decades, there has been an abundance of literature published where these criteria have been reviewed, analysed, and compared; the following publications includes such reviews: Yong and Ko (1981), Chen and Saleeb (1982), Desai and Siriwardane (1984), Chen and Baladi (1985), Lubliner (1990), Charlez (1991), Chen and Zhang (1991), François et al. (1991, 1995), Lade (1993, 1997), Skrzypek and Hetnarski (1993), Andreev (1995), Sheorey (1997), Potts and Zdravkovic (1999), di Prisco and Pastor (2000), Desai (2001), and Yu et al. (2003). Some of the better known expressions are summarised later in this report to highlight some of their characteristics. The following presentation is limited to the case of isotropic materials.

Before going into the specifics of existing formulations, it is first recalled that function *F* describing the criterion of interest (plastic, failure, etc.) is usually expressed using the principal stresses σ_1, σ_2, and σ_3 (the major, intermediate and minor principal stresses, respectively), or a combination of the invariants of the stress tensor σ_{ij}. One can thus write:

$$F(\sigma_1, \sigma_2, \sigma_3,) = 0 \qquad (1a)$$

or

$$F(I_1, J_2^{1/2}, \theta) = 0 \qquad (1b)$$

where $I_1 = \text{tr}(\sigma_{ij})$ represents the first invariant of the stress tensor; $J_2 = (1/2)\, S_{ij}\, S_{ij}$ is the second invariant of the deviatoric stress tensor S_{ij} (in which $S_{ij} = \sigma_{ij} - (I_1/3)\, \delta_{ij}$; $\delta_{ij} = 0$ if $i \neq j$ and $\delta_{ij} = 1$ if $i = j$); and $\theta = \frac{1}{3}\sin^{-1}\frac{3\sqrt{3}\ J_3}{2\sqrt{J_2^3}}$ is the Lode angle which reflects the position of the stress state in the octahedral (π) plane ($-30° \leq \theta \leq 30°$), in which $J_3 = (1/3)\, S_{ij}\, S_{jk}\, S_{ki}$ is the third invariant of the deviatoric stress tensor.

Some criteria are based on a single principal stress (σ_1 or σ_3) or on the two extreme principal stresses (σ_1 and σ_3). Nevertheless, for general applications, it is usually necessary to use the three principal stresses to adequately represent the behaviour of materials under various regimes of multiaxial loading. Also, when a criterion is expressed from the invariants cited above, it is not uncommon to use only one or two of the invariants; for instance, only J_2 is used with the von Mises criterion, while I_1 and J_2 are included in the Drucker-Prager criterion (see next section). However, a general representation of the behaviour of porous materials should involve the three distinct invariants, including the θ angle proposed by Lode (1926) to provide a better representation of the effect of the intermediate principal stress σ_2 (Nayak and Zienkiewicz 1972; Slater 1977).

It can be practical to express the formulation of a three-dimensional criterion, forming a surface in the stress space (σ_1 - σ_2 - σ_3, or σ_x - σ_y - σ_z where the Cartesian axes are the principal axes), with two expressions giving the position and the form of the surface in the I_1 - $J_2^{1/2}$ plane (function F_0) and in the octahedral (π) plane that is perpendicular to the hydrostatic axes $\sigma_1 = \sigma_2 = \sigma_3$ (function F_π). One can thus write (Aubertin et al. 1994):

$$F = J_2^{1/2} - F_0 \, F_\pi = 0 \qquad (2)$$

This is the basic formulation that will be used for the general model proposed herein.

2. MULTIAXIAL CRITERIA FOR POROUS MATERIALS

2.1 General characteristics

The behaviour of materials beyond the elastic domain depends on several deformation mechanisms. For crystalline materials with a ductile behaviour (such as metals, ice and certain rocks at high temperature), inelastic straining is generally controlled by dislocations motion, which may cause hardening and sometimes damage due to the creation of voids. For materials exhibiting a brittle behaviour, such as consolidated or cemented materials (rock, concrete, plaster, backfill, etc.), the inelastic deformation is principally due to the initiation and propagation of micro cracks, which may eventually become macroscopic fractures that are often associated with the failure condition (Charlez 1991; Aubertin et al. 1998). With loosely consolidated particulate materials (which is usually the case for most soils, powders, and grains), frictional sliding and crushing generate inelastic deformation (under drained conditions) up to a critical state (iso-volumetric) condition (Lade 1977; Chen and Saleeb 1982; Desai and Siriwardane 1984).

In all cases, material loading results in a transitional behaviour, such as the passage from the elastic regime to the inelastic regime or from the pre-peak phase to the post-peak phase. The condition that define the passage from one phase to another may be included in the constitutive laws describing (or predicting) the material response under specific loads. Such conditions are expressed by mathematical functions called criteria. In the great majority of cases, a criterion is formulated with the stress tensor σ_{ij} (or its invariants) and it may be shown graphically in the σ_1 - σ_2 - σ_3 space where it takes the form of a three-dimensional surface.

An important impetus behind the development of 3D plasticity criteria was the need for a generalised representation (in the tridimensional stress space), for yielding observed under uniaxial testing of metals. Criteria used to define the elastic domain are usually expressed by a scalar function of σ_{ij}. Experience has shown that for most isotropic and homogeneous materials, the elastic domain (initial and actual) is convex (Halphen and Salençon 1987). This implies that the criterion used to mathematically define this domain should represent a convex surface in stress space.

In elastoplasticity, a plastic potential is generally introduced in the multiaxial flow law to define the manner in which plastic strain evolves. This potential is often mathematically close to the yield criterion (e.g., Desai and Siriwardane 1984). Such potentials are also used in visco-plasticity (Lemaître and Chaboche 1988; Lubliner 1990).

There are also functions used for describing the peak strength (failure criterion), the threshold for crack initiation (damage criterion), the critical state condition (where a constant void ratio is reached), and the residual strength.

The criteria most often used in engineering generally involve few material parameters that are easily obtained by standard laboratory testing, and have a clear physical meaning. A criterion must satisfactorily describe the important characteristics of the observed behaviour. The expression of the criterion should be unique and form a continuous (and convex) surface in stress space. It should also be reducible to the classic criteria (such as von Mises or Coulomb) for particular cases. The expression should also be capable of reproducing the particularities of more elaborate functions developed over the years, based on observations of fundamental materials behaviour. In geo-engineering, a particular aim is to reproduce some characteristics of the Cambridge critical state models (i.e., Roscoe et al. 1958, 1963; Roscoe and Burland 1968; Schofield and Wroth 1968; Atkinson and Bransby 1978), and the subsequent variations (i.e., DiMaggio and Sandler 1971; Lade

1977; Baladi and Rohani 1979; Desai 1980; Michelis and Brown 1986; Novello and Johnston 1995, 1999), as these were based on experimental observations of the behaviour of soils and rocks.

2.2 Development of existing criteria

The development of most plasticity and failure criteria employed by engineers followed two major axes, one used in mechanical metallurgy and the other in geotechnics (for geomaterials).

For metals, it is common to use a criterion that is independent of the first stress invariant, I_1 (or of the mean stress $\sigma_m = I_1/3$); this is the case with the Tresca (1868) and von Mises (1913) criteria. The frictional component associated with the effect of the spherical (hydrostatic) portion of σ_{ij} is thus neglected. In the case of Tresca, the criterion is based on J_2 and J_3, the second and third invariants of the deviatoric stress. In the case of the (Huber-) Mises criterion, only J_2 is considered; it can be written as $J_2^{1/2} = \sigma_u/\sqrt{3}$ (where σ_u is the uniaxial strength). This last criterion has been judged to be more representative of the general behaviour of metals (Halphen and Salençon 1987).

For the most part, models employed for porous metals and metallic compounds (including metal powders) have kept their roots in the von Mises criterion, or in a version modified by Schleicher (1926) to take into account the difference in strength between uniaxial strength in tension and in compression (e.g., Lee 1988; Lubliner 1990). Furthermore, the von Mises criterion has been extended by Gurson (1977) to describe yielding and the evolution of voids in porous materials; this well known criterion has then given birth to a variety of somewhat similar criteria (e.g., Tvergaard 1981, 1991; Tvergaard and Needleman 1984). There also exist numerous other criteria for porous metals, which can be reduced to the von Mises criterion for particular cases (e.g., Hjelm 1994; Theocaris 1995; Altenbach and Tushtev 2001a, b; Altenbach et al. 2001).

On the other hand, the Coulomb criterion has served as a starting point for the great majority of criteria used for geomaterials (soil, rock, concrete, backfill, etc). In its basic version, the Coulomb criterion (or Mohr-Coulomb criterion) only uses two principal stresses, σ_1 and σ_3, with two basic material parameters, c (cohesion) and ϕ (internal friction angle). This criterion is represented by a line in the Mohr plane σ - τ (where σ and τ are the normal and shear stresses on the given plane, respectively), or in the σ_1 - σ_3 plane. When generalised in three dimensions, the Coulomb criterion is linear in the I_1-$J_2^{1/2}$ plane and it takes the form of an irregular hexagon in the plane of the octahedral stresses, with the axes of symmetry corresponding to the six summits of the Tresca criterion to which it is closely related in the π plane. Drucker and Prager (1952) proposed a circular version of the Coulomb criterion in the octahedral plane (similar to von Mises criterion), while maintaining the linear relationship between I_1 and $J_2^{1/2}$ (without a contribution from θ or J_3). Zienkiewicz and collaborators (Nayak and Zienkiewicz 1972; Zienkiewicz et al. 1972; Zienkiewicz and Pande 1977) have later proposed a modified version of the Coulomb criterion, represented by a rounded triangle in the π plane, with the major axes oriented at a Lode angle of 30° (θ corresponding to conventional triaxial compression testing, CTC).

Starting with the Coulomb criterion, a more general representation of soils behaviour was proposed with the Cambridge model (Roscoe et al. 1958, 1963; Schofield and Wroth 1968), which in turn inspired numerous other expressions, several of which are identified in Table 1 and in Figure 1. These include the "Cap" type criteria elaborated by Desai and collaborators (e.g., Desai 1980; Desai and Faruque 1982; Desai and Salami 1987; Desai 2001), and several more as noted by Ehlers (1995).

Despite these numerous developments, the simplified criterion of Drucker and Prager (1952) continues to be used regularly for numerous frictional materials because of its simplicity (e.g., Bousshine et al. 2001 for soils; Radi et al. 2002 and Liu et al. 2003 for rocks; Hsu et al. 1999 for other porous materials). Nevertheless, it is known that this simplified criterion is not representative of many aspects of porous materials behaviour, considering its linearity in the I_1 - $J_2^{1/2}$ plane (a well known limitation) and also because it neglects the effect of J_3 (or of θ). This latter aspect was again made obvious recently by Peric and Ayari (2002a, b) who demonstrated that the generation of pore pressure during geomaterial loading depends on the Lode angle.

Among the existing multiaxial criteria, some include a dependence on the initial (or actual) porosity of the material. This is the case with the Gurson (1977) model and it's many variations (e.g., Tvergaard 1981; Tvergaard and Needleman 1984; Ponte-Castaneda and Zaidman 1994; da Silva and Ramesh 1997; Mahnken 1999, 2002; Ragab and Saleh 1999; Khan and Zhang 2000; Li et al. 2000; Perrin and Leblond 2000). It is also the case with those of Shima and Oyane (1976) and Rousselier (1987).

Comparative summaries (with some critical reviews) on various existing criteria have been presented by Chen and Saleeb (1982), Desai and Siriwardane (1984), Lade (1993), Olevsky and Molinari (2000), Mahnken (2002), Yu et al. (2002). Over the last several years, there have been some more general functions developed in a convergent manner to reproduce in a unified framework the characteristics of the main criteria developed for various porous materials (e.g., Haggblad and Oldenburg 1994; Ehlers 1995; Desai 2001; Lewis and Khoei 2001; Mahnken 2002). This is also the case for the criterion proposed in Section 3.

Table 1 presents the core equations of many multiaxial criteria developed and utilised for engineering materials with a frictional (pressure dependent) component (i.e. with an influence of I_1). A schematic presentation of the resulting surfaces of these various criteria is shown in Figure 1.

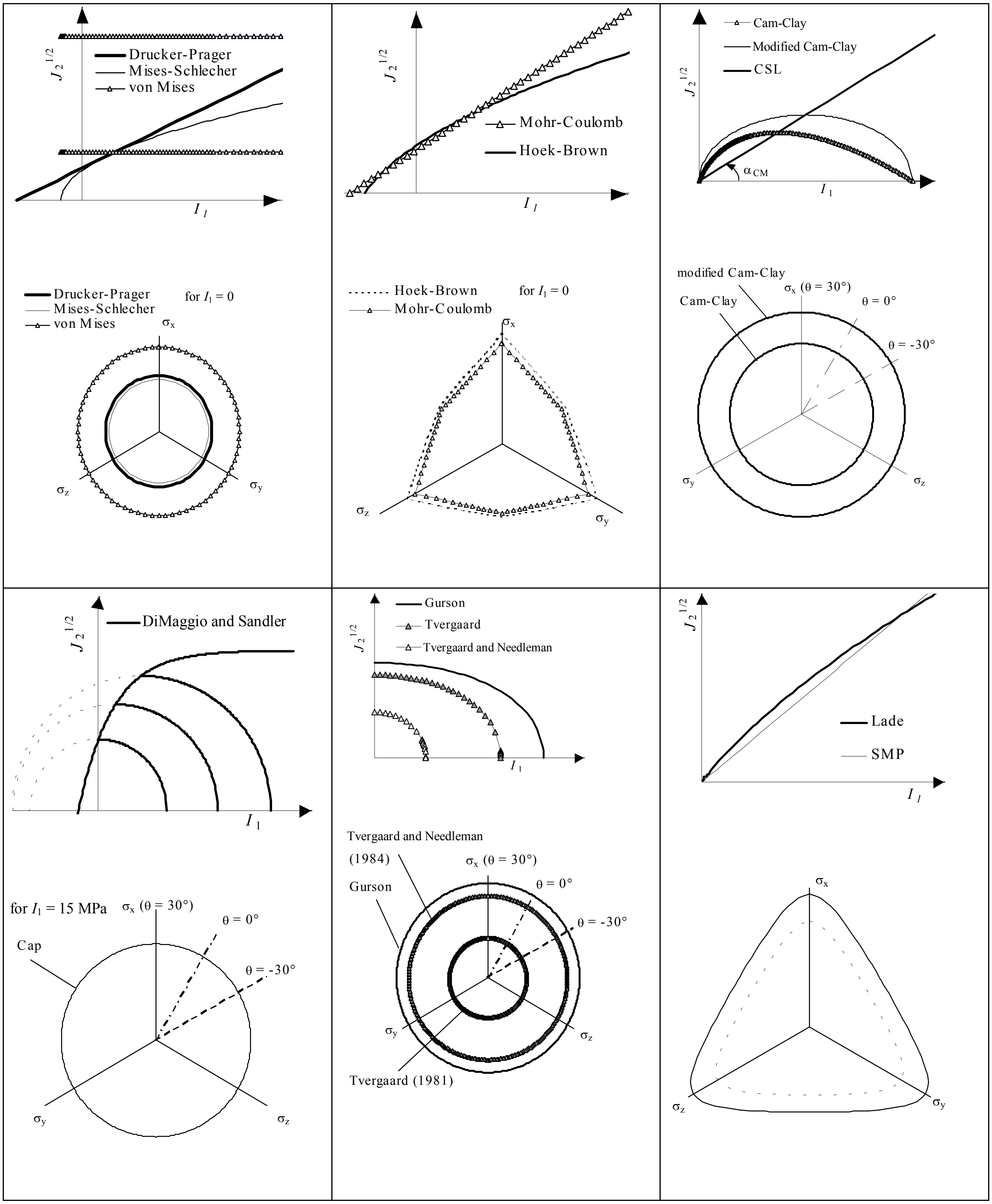
Table 1. Three-dimensional criteria used for frictional porous materials.

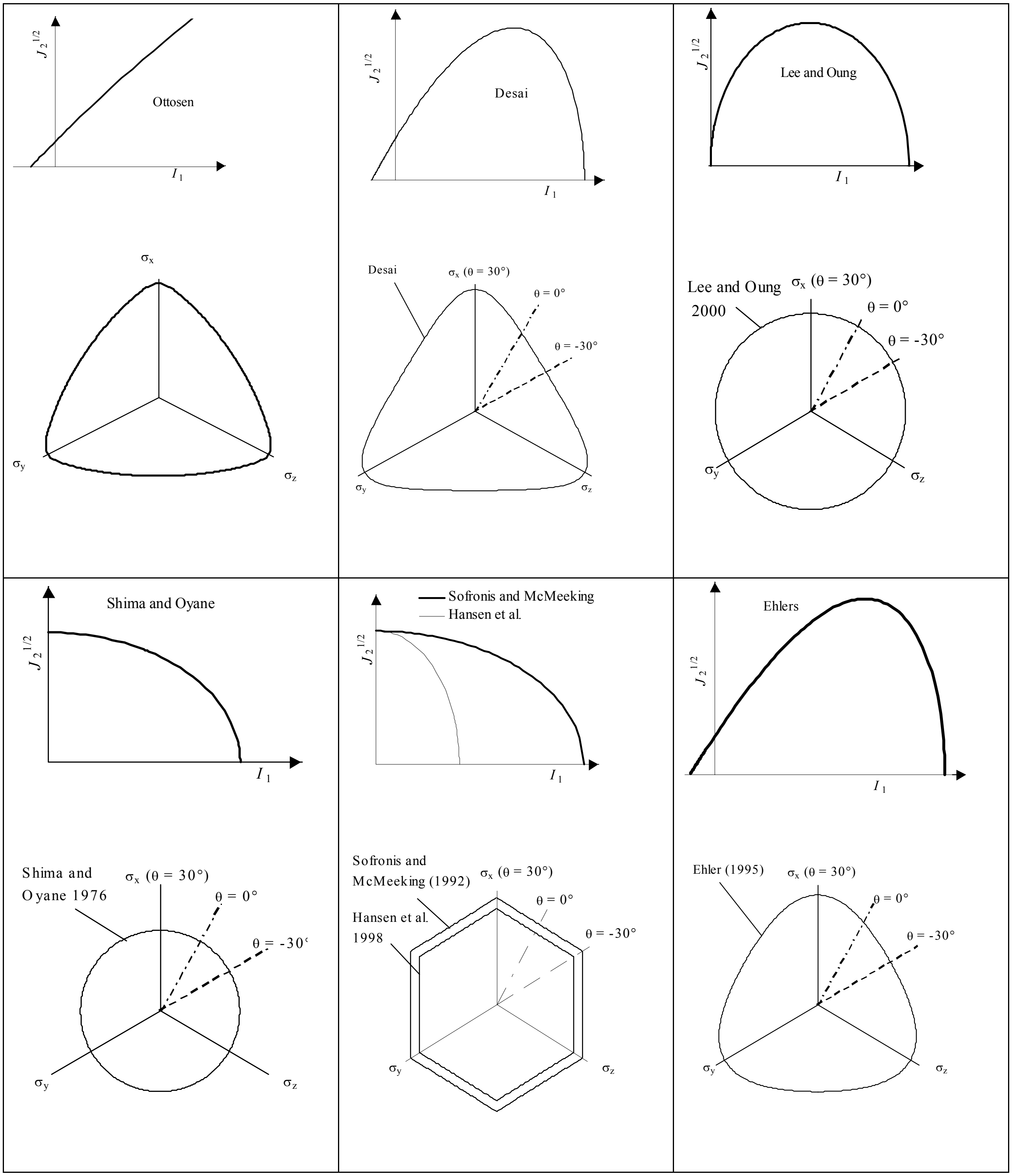
Identification	Equations and Parameters	References
Mises-Schleicher	$\sqrt{J_2}=\sqrt{\left((\sigma_c-\sigma_t)I_1+\sigma_c\sigma_t\right)/3}$	Schleicher (1926)
Mohr-Coulomb	$\sqrt{J_2}=\left((I_1/3)\sin\phi+c\cos\phi\right)/\left(\cos\theta-\sin\theta\sin\phi/\sqrt{3}\right)$ c and ϕ, cohesion and friction angle of material, respectively.	Chen and Saleeb (1982)
Drucker-Prager	$\sqrt{J_2}=\alpha I_1+k$ $k=(\sigma_c-\sigma_t)/(12\alpha)+\alpha(\sigma_c\sigma_t)/(\sigma_c-\sigma_t)$; $\alpha=2\sin\phi/\left(\sqrt{3}(3-\sin\phi)\right)$	Drucker and Prager (1952)
Cam-Clay	$\sqrt{J_2}=-\alpha_{CM}I_1\ln(I_1/I_{10})$	Roscoe et al. (1958, 1963)
Cam-Clay modified	$\sqrt{J_2}=\alpha_{CM}\sqrt{I_1(I_{10}-I_1)}$	Roscoe and Burland (1968)
DiMaggio-Sandler	fixed surface: $f_1=\sqrt{J_2}+\gamma\exp(-\beta I_1)-\alpha=0$ "Cap": $f_2=R^2J_2+(I_1-C)^2=R^2b^2$ R, ratio between the major axis a and the minor axis b of the ellipse; α, β, γ, C, material parameters	DiMaggio and Sandler (1971)
SMP	$2/\sqrt{3}\left(\sqrt{J_2}/I_1\right)^3\sin3\theta+(3/k-1)\left(\sqrt{J_2}/I_1\right)^2+(1/9-1/k)=0$ k, material parameter	Matsuoka and Nakai (1974)
Shima-Oyane	$3J_2/\sigma_M^2+a_1n^{a_2}\left(I_1/(3\sigma_M)\right)^2-(1-n)^5=0$ a_1, a_2, material parameters	Shima and Oyane (1976)
Gurson	$3J_2/\sigma_M^2+2n\cosh\left(I_1/(2\sigma_M)\right)-\left(1+n^2\right)=0$	Gurson (1977)
Lade	$\left(I_1^3/I_3-27\right)\left(I_1/p_a\right)^m-k=0$ p_a, atmospheric pressure; m and k, material parameters	Lade (1977)
Ottosen	$a\left(\sqrt{J_2}/\sigma_c\right)^2+\lambda\left(\sqrt{J_2}/\sigma_c\right)-b\left(I_1/\sigma_c\right)-1=0$ a and b, constants; λ, function of the Lode angle θ: $\lambda=k_1\cos\left((1/3)\cos^{-1}(-k_2\sin3\theta)\right)$, for $30° \geq \theta \geq 0°$ $\lambda=k_1\cos\left(60°-(1/3)\cos^{-1}(k_2\sin3\theta)\right)$, for $0° \geq \theta \geq -30°$ k_1 and k_2, constants	Ottosen (1977)
Desai	$J_2=\left(-\alpha(I_1+I_{1s})^m p_a^{2-m}+\gamma(I_1+I_{1s})^2\right)\left(1-\beta\sin3\theta\right)^{-1/2}$ I_{1s}, interval of axis I_1 due to the uniaxial tensile strength; m, parameter due to the change in phase (contractive to dilatant); γ and β, material parameters; α, tightening function.	Desai (1980)
Modified Gurson	$3J_2/\sigma_M^2+2q_1n\cosh\left(q_2I_1/(2\sigma_M)\right)-\left(1+(q_1n)^2\right)=0$ $3J_2/\sigma_M^2+2q_1n^*\cosh\left(q_2I_1/(2\sigma_M)\right)-\left(1+(q_1n^*)^2\right)=0$ q_1 and q_2, material parameters; n^*, function of the porosity: $n^*=n$ pour $n \leq n'$ $n^*=n'+(1/q_1-n')(n-n')/(n_C-n')$ pour $n>n'$	Tvergaard (1981, 1990) Tvergaard and Needleman (1984)

	n_C, critical porosity at rupture; n' ($< n_C$), threshold linked to the closing of the voids	
Hoek-Brown	$2\sqrt{J_2}\cos\theta - \left[\frac{m\sigma_c}{\sqrt{3}}(\sin\theta - \sqrt{3}\cos\theta)\sqrt{J_2} + \frac{1}{3}I_1 m\sigma_c + s\sigma_c^2\right]^{1/2} = 0$ m and s, parameters	Pan and Hudson (1988)
Sofronis-McMeeking	$2\sqrt{J_2}\cos\theta = \left(1-\left(\frac{mn}{(1-n^{1/m})^m}\right)^{\frac{2}{m+1}}\left(\frac{I_1}{2m}\right)^2\right)^{1/2}\left(\frac{1+n}{1-n}\right)^{\frac{-m}{m+1}}$ m, material parameters.	Sofronis and McMeeking (1992)
Ehlers	$\sqrt{J_2\left(1-2/(3\sqrt{3})\gamma\sin 3\theta\right)^m + \alpha I_1^2/2 + \delta^2 I_1^4} - \beta I_1 + \varepsilon I_1^2 - \kappa = 0$ α, β, δ, ε, γ, κ, and m, material parameters	Ehlers (1995)
Crushed Rock Salt	$2\sqrt{J_2}\cos\theta = \left(1+n^2-\kappa_0\Omega^{\kappa_1}I_1^2/9\right)^{1/2}\kappa_2^{-1/2}\left((1+n)/(1-n)\right)^{\frac{-m}{m+1}}$ $\Omega = \left(n_v m(1-n_v^{1/m})^{-m}\right)^{\frac{2}{m+1}}$; $n_v = n$, for $n \geq n_t$ $n_v = n$, for $n < n_t$ and κ_0, κ_1, κ_2, m and n_t, material parameters	Hansen et al. (1998)
Lee-Oung	$3J_2 + \frac{n}{4}I_1^2 + (1-n)(C-T)(-I_1) - (1-n)^2 CT = 0$ C and T, absolute values of the uniaxial strengths in compression and in tension of equivalent non-porous material.	Lee and Oung (2000)

Notes: I_{10}, value of I_1 at the crushing of the material under hydrostatic pressure; I_3, third invariant of the stress tensor σ_{ij}; n, porosity; σ_c and σ_t, uniaxial compressive and tensile strengths, respectively; σ_M, flow stress of an equivalent non-porous material; α_{CM}, slope of the critical state line (CSL) in the I_1 - $J_2^{1/2}$ plane.

Figure 1 Schematic representation of the surfaces associated with some of the criteria used for porous materials (see Table 1); the σ_x, σ_y, σ_z axes represent principal axes.




Ottosen
Desai
Lee and Oung
Lee and Oung 2000
Shima and Oyane
Shima and Oyane 1976
Sofronis and McMeeking
Hansen et al.
Sofronis and McMeeking (1992)
Hansen et al. 1998
Ehlers
Ehler (1995)
σx (θ = 30°)
θ = 0°
θ = -30°
σx
σy
σz

A few observations can be made from Figure 1:

- When observed in the I_1 - $J_2^{1/2}$ plane, a few criteria are linear (like Drucker-Prager and Matsuoka-Nakai), but the majority are curved downward. All the surfaces of existing criteria are closed on the I_1 axis in tension (for $I_1 \leq 0$), except for the criteria from von Mises and Tresa (not shown here).

  On the positive I_1 axis (compression), some of the surfaces are open (criteria of Schleicher, Drucker–Prager, Hoek-Brown 3D, Matsuoka-Nakai, Ottosen), but many others are closed to reflect possible collapse of the porous matrix under high mean stresses. The models that include a closed portion, referred to as a "Cap", ensue from the work of Roscoe et al. (1958, 1963) for soils and of Gurson (1977) for metals. Other closed criteria on the positive side of I_1 include those of DiMaggio and Sandler (1971), Shima and Oyane (1976), Desai (1980, 2001), Tvergaard and Needleman (1984), Ehlers (1995), Hansen et al. (1998), and Lee and Oung (2000).

- There are a few criteria, like those of Gurson (1977), Shima and Oyane (1976), Sofronis and McMeeking (1992), and Hansen et al. (1998), that do not distinguish between positive and negative values of I_1, as they are symmetric about the $J_2^{1/2}$ axis.

- The vast majority of existing criteria show a singularity at the intersection point of the minimum value of I_1 ($J_2^{1/2}$), while few others tend to be rounded near this value (e.g. Mises-Schleicher and modified Cam-Clay).

- In the π plane, some models have adopted a circular form, with a deviatoric strength $J_2^{1/2}$ independent of the Lode angle θ (as with the von Mises criterion). This is the case, for example, with the criteria of Schleicher (1926), Drucker and Prager (1952), Roscoe et al. (1958, 1963;

also Roscoe and Burland 1968), DiMaggio and Sandler (1971), Gurson (1977), and Tvergaard (1981). Other criteria have adopted an asymmetric hexagonal form, such as the Tresca criterion. Examples include the general 3D version of the criteria from Mohr-Coulomb, Hoek-Brown (Pan and Hudson 1988), and Hansen et al. (1998). There are also those that use the form of a rounded triangle, such as proposed by Nayak and Zienkiewicz (1972), and Zienkiewicz et al. (1972); these include Lade and Duncan (1973, 1975), Matsuoka and Nakai (1974), Lade (1977, 1997), Ottosen (1977), Desai (1980), Ehlers (1995) and Jrad et al. (1995).

3. THE MSDP$_u$ CRITERION FOR POROUS MATERIALS

3.1 General description

The MSDP$_u$ criterion was first elaborated to describe the behaviour of hard rocks and other brittle materials with a low porosity. Its basic characteristics are described in Aubertin et Simon (1996, 1998) and Aubertin et al. (1999). The criterion is represented by a surface in the I_1 - $J_2^{1/2}$ plane that reduces to the non linear Mises-Schleicher MS criterion at low mean stresses and which tends progressively towards the Drucker-Prager DP (or Coulomb) criterion at higher mean stresses. In the π plane, the surface generally takes the form of a rounded triangle. The rounded MS portion at low I_1 can be seen as an embedded tension cut-off in the DP failure surface.

The parameters used to define the position and the form of the surface are easily obtained. They include the uniaxial tensile and compressive strengths, σ_t and σ_c, and the friction angle on smooth surfaces ϕ_b. For brittle materials at low porosity, such as hard rocks and certain types of plaster and concrete, the MSDP$_u$ criterion constitutes a generalised three-dimensional version of the Griffith (1924) criterion as modified by Brace (1960) and McClintock and Walsh (1962). It therefore considers that for such materials, the initiation of the propagation of crack and peak strength are controlled, at high confining stresses (or high values of the mean stress), by friction mobilised between the faces of closed cracks. This criterion has later been extended to the case of porous rocks and rock masses (Aubertin et al. 2000), by adding (among other components) a term which closes the surface on the positive axis of I_1 (to form a "Cap").

Complementary work has also permitted the recent development of a relationship between the uniaxial failure strength in compression (σ_c) and in tension (σ_t), and the material porosity (Li et Aubertin 2003). Porosity (n) constitutes a simple and practical parameter for defining the main features of isotropic materials microstructure. For this reason, the porosity is often related to effective properties of materials (such as the elastic moduli *E*, *G* and *K*, the rate of propagation of voids, permeability, electrical resistance, and uniaxial strengths). A detailed review of the relationships that exist between porosity *n* and various characteristics of porous materials is presented by Chen and Nur (1994). These authors also present various approaches used for introducing particular characteristics associated with porosity (distribution, form, concentration of voids) into constitutive equations.

The results presented by Chen and Nur (1994) reveal the existence of a critical porosity (or transitional void concentration) beyond which certain properties change in a marked manner, because of the non-uniformity in the internal distribution of contact area. This critical porosity n_C is generally much less than 100% (or $n_C < 1$). For instance, the uniaxial compressive strength of rocks may become nil when *n* is greater than 40 to 70%. The existence of a critical porosity n_C was also introduced by Tvergaard et Needleman (1984) to describe the behaviour of porous materials. The value of such critical porosity depends on the shape of the pores and surrounding grain, as shown by Logan (1987) and Chen and Nur (1994).

It is useful to introduce the influence of the initial porosity (in relation to the critical porosity) in the formulation of inelastic criteria for engineering materials. This concept is included in the approach below.

3.2 Equations of the MSDP$_u$ criterion

The general multiaxial MSDP$_u$ criterion is expressed in terms of the stress invariants based on equation (2). The mean stress function is then formulated as follow:

$$F_0 = \left[\alpha^2\left(I_1^2 - 2a_1 I_1\right) + a_2^2 - a_3\left\langle I_1 - I_c\right\rangle^2\right]^{1/2} \quad (3)$$

where α, a_1, a_2, a_3 et I_c are material parameters, defined from basic properties. Parameter α is related to the friction angle ϕ:

$$\alpha = \frac{2\sin\phi}{\sqrt{3}(3 - \sin\phi)} \quad (4)$$

For brittle materials of relatively low porosity, ϕ can be approximated by the residual frictional angle ($\phi \cong \phi_r$). Parameters a_1, a_2 are defined as follows:

$$a_1 = \left(\frac{\sigma_c - \sigma_t}{2}\right) - \left(\frac{\sigma_c^2 - (\sigma_t/b)^2}{6\alpha^2(\sigma_c + \sigma_t)}\right) \quad (5)$$

$$a_2 = \left\{\left(\frac{\sigma_c + (\sigma_t/b^2)}{3(\sigma_c + \sigma_t)} - \alpha^2\right)\sigma_c\sigma_t\right\}^{1/2} \quad (6)$$

where σ_t and σ_c are uniaxial strength in tension and in compression. The relationship of Li and Aubertin (2003) is used to describe the uniaxial strength as a function of porosity:

$$\sigma_{un} = \left\{\sigma_{u0}\left(1 - \sin^{x_1}\left(\frac{\pi}{2}\frac{n}{n_C}\right)\right) + \left\langle\sigma_{u0}\right\rangle\cos^{x_2}\left(\frac{\pi}{2}\frac{n}{n_C}\right)\right\}\left\{1 - \frac{\left\langle\sigma_{u0}\right\rangle}{2\sigma_{u0}}\right\} \quad (7)$$

where σ_u is the uniaxial strength, which may be used for compression ($\sigma_{un} = \sigma_{cn}$) or tension ($\sigma_{un} = \sigma_{tn}$). In this equation, n_C is the critical porosity for which σ_{un} becomes nil, in tension ($n_C = n_{Ct}$) and in compression ($n_C = n_{Cc}$); parameter σ_{u0} represents the theoretical (extrapolated) value of σ_{un} for $n = 0$. Finally, x_1 and x_2 are material properties; $\langle\ \rangle$ are MacCauley brackets (defined as $\langle x \rangle = (x + |x|)/2$).

Parameters a_3 and I_c serve to represent the porous materials behaviour under high hydrostatic compression, when the surface (yield, failure) closes on the positive side of I_1 (for $I_1 > I_{cn}$). The dependence of these parameters on porosity (i.e. a_{3n} and I_{cn}) is discussed in section 4.1. It is noted that for dense materials (low porosity), I_{cn} is very large so this portion of the criterion disappears (i.e. the surface remains open along the positive I_1 axis).

The surface in the octahedral (π) plane, which is perpendicular to the $\sigma_1 = \sigma_2 = \sigma_3$ axis, is represented by the following function of the Lode angle (Aubertin et al. 1994).

$$F_\pi = \left[\frac{b}{\left[b^2 + (1-b^2)\sin^2(45^\circ - 1.5\theta) \right]^{1/2}} \right]^v \quad (8)$$

with

$$v = \exp(-v_1 I_1) \quad (9)$$

Here v is an exponent which reflects the influence of the hydrostatic pressure on the evolution of the surface in the π plane; v_1 is another material parameter. For a shape that does not change with I_1, $v_1 = 0$ (or $v = 1$). Parameter b controls the size of the asymmetric surface, at -30° in the π plane (compared to $\theta = 30°$).

Equations (2) through (9) constitute the $MSDP_u$ criterion for porous materials.

3.3 Schematical representations

Figure 2 presents schematical views of the MSDP$_u$ criterion in the I_1 - $J_2^{1/2}$ plane (Fig. 2a), in the case of conventional triaxial compression (CTC, θ = 30°), for increasing values of porosity; also shown is the surface in the π plane for a value of $v = 1$ (Fig. 2b). Figure 2 corresponds to a condition where $I_1 < I_{cn}$. Note on the figure that an increase in porosity (from n_1 to n_2 to n_3) reduces the size of the surface (or the extent of $J_2^{1/2}$ for the given values of I_1 and θ), because of lower strengths (σ_{cn} and σ_{tn}) when porosity increases.

Figure 2. Schematic representation of the MSDP$_u$ criterion for low porosity materials, with a_{3n} = 0 and ν = 1.

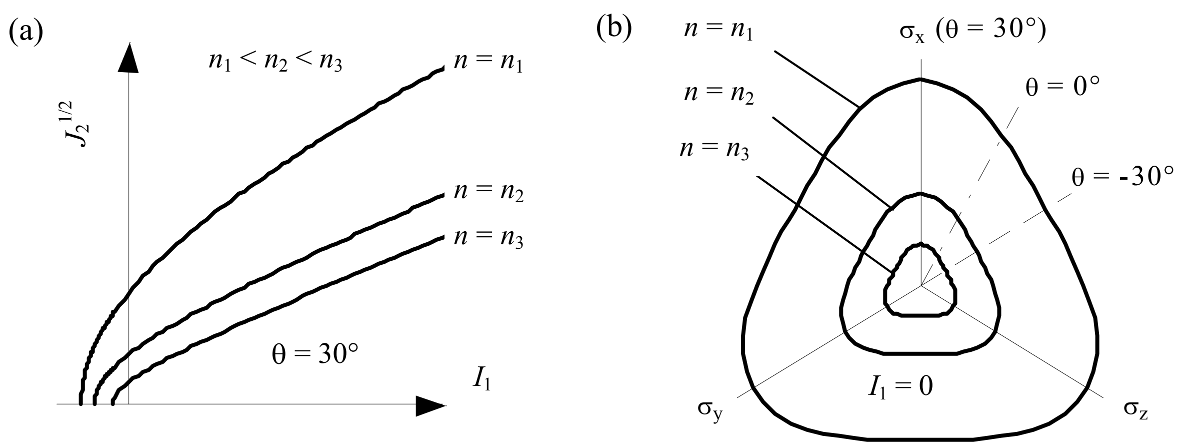


Figure 3 demonstrates the effect of the mean stress I_1 on the shape of the surface in the π plane, when v tends towards 0 (or F_π tends towards 1) at an elevated I_1. This situation is associated with a change in the physical mechanisms described by the surface, from a purely frictional behaviour

($F_\pi < 1$ at $\theta = -30°$) to a ductile/plastic behaviour (at $F_\pi \cong 1$ at $\theta = -30°$) which may arise in the case of crystalline materials (Aubertin et al. 1994, 1998).

Figure 3. Illustration of the evolution of the MSDP$_u$ surface in the π plane, when I_1 increases; $b = 0.75$, $v_1 \neq 0$ (v varies with I_1).

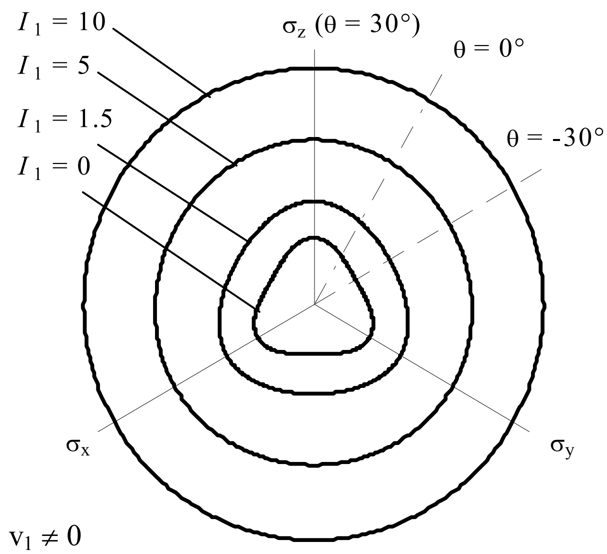
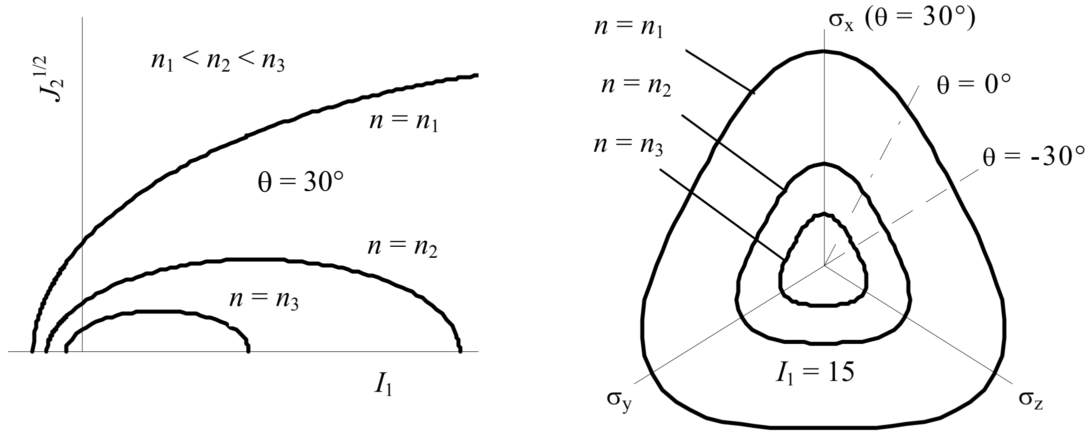


Figure 4 presents a schematical view of the MSDP$_u$ surface for a material with a relatively low value of I_{cn} (for I_1 exceeding I_{cn}). In this case, the closed portion ("Cap") is apparent in the I_1 - $J_2^{1/2}$ plane (shown in CTC), and the surface closes on the positive axis of I_1. This component of the criterion again varies with porosity.

Figure 4. Schematical view of the $MSDP_u$ criterion for porous materials with $I_c \cong 0$, $a_{3n} \neq 0$, $v_1 = 0$ ($v = 1$), $b = 0.75$.



When the frictional component is considered negligible (as in the case of ductile metals), $\alpha = 0$ (or $\phi = 0$). Equation (3) thus becomes:

$$F_0 = \left(\frac{\sigma_{cn}^2 (I_1 + \sigma_{tn}) - (\sigma_{tn}/b)^2 (I_1 - \sigma_{cn})}{3(\sigma_{cn} + \sigma_{tn})} - a_{3n} \langle I_1 - I_{cn} \rangle^2 \right)^{1/2} \tag{10}$$

This version of the criterion resembles a Gurson-Tvergaard surface, as seen on Figure 5 (also refer to Section 3.4). Again, the position of the surface depends on the porosity. For $n = 0$, a von Mises type of surface is recovered (for $\sigma_t = \sigma_c$ and $I_c >> 0$).

Figure 5. Representation of the MSDP$_u$ criterion for ductile materials, with $a_{3n} \neq 0$, $\alpha = 0$, $v_1 = 0$ ($v = 1$).

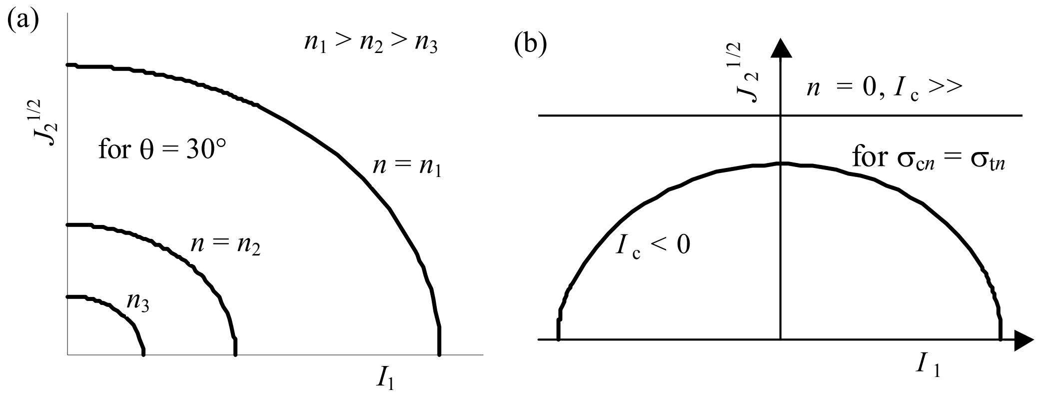


Figure 6 shows the form of the surface in the I_1 - $J_2^{1/2}$ plane (in CTC) for a consolidated frictional material response. This illustrates the effect of porosity (Fig. 6a), friction angle (Fig. 6b), uniaxial compressive strength (Fig. 6c), and uniaxial tensile strength (Fig. 6d), threshold I_{cn} value (Fig. 6e), and parameter a_{3n} (Fig. 6f).

A tri-dimensional view of the complete surface in the σ_1 - σ_2 - σ_3 space is shown on Figure 7 (for the case $v = 1$ and $b = 0.75$), for a range of I_1 extending beyond I_{cn}.

Figure 6. MSDP$_u$ representation in the $I_1 - J_2^{1/2}$ plane, showing the influence of porosity n (a), angle α (or ϕ) (b), uniaxial compressive strength σ_{cn} (c), uniaxial tensile strength σ_{tn} (d), I_{cn} (e), and a_{3n} (f).

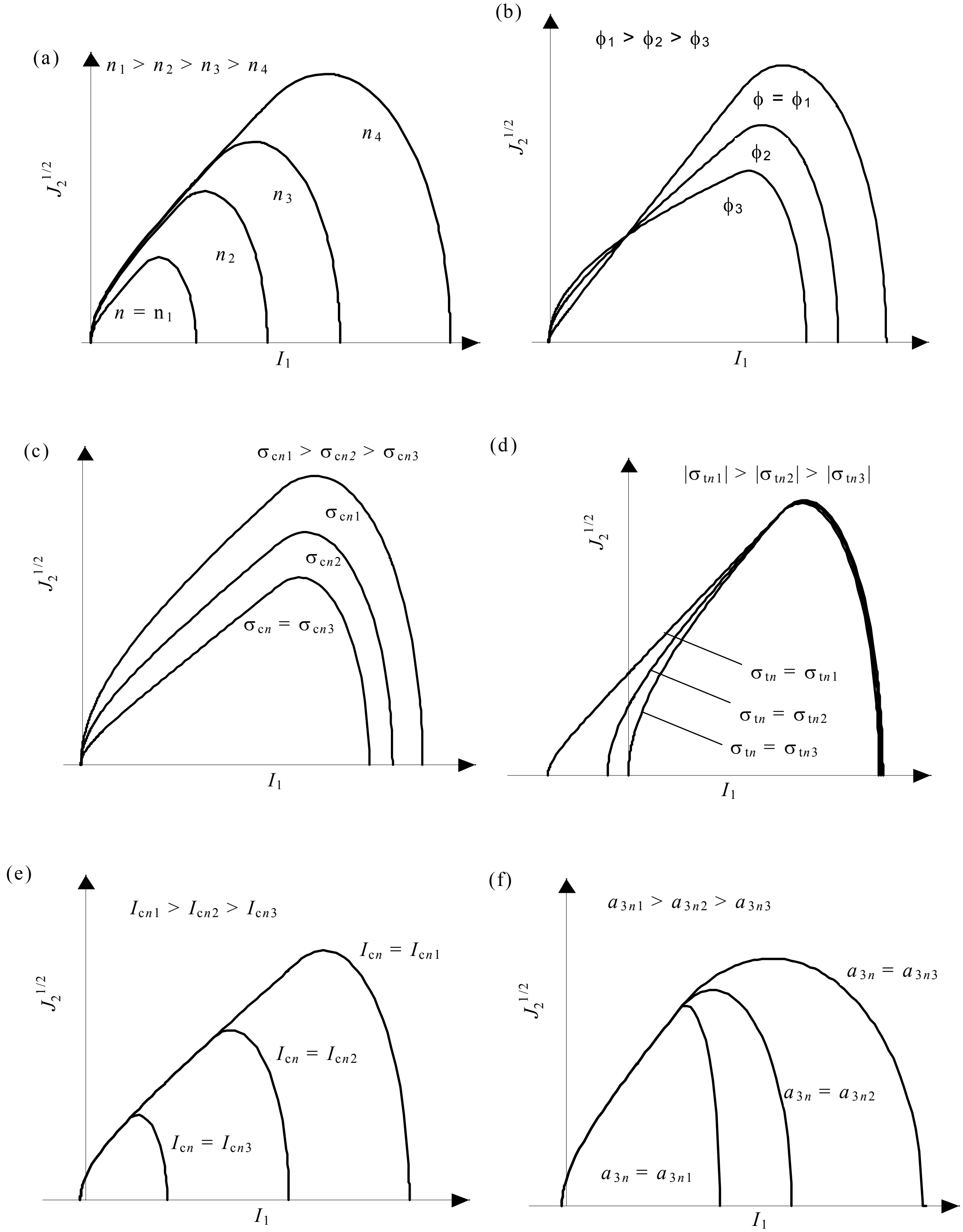
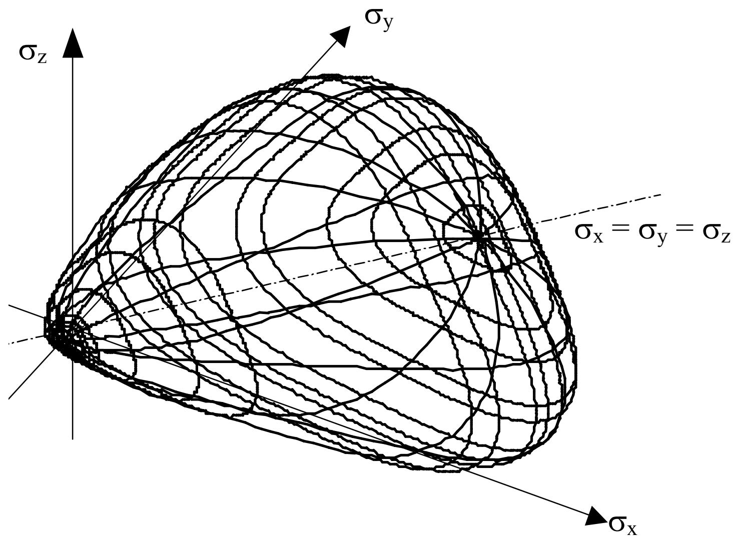


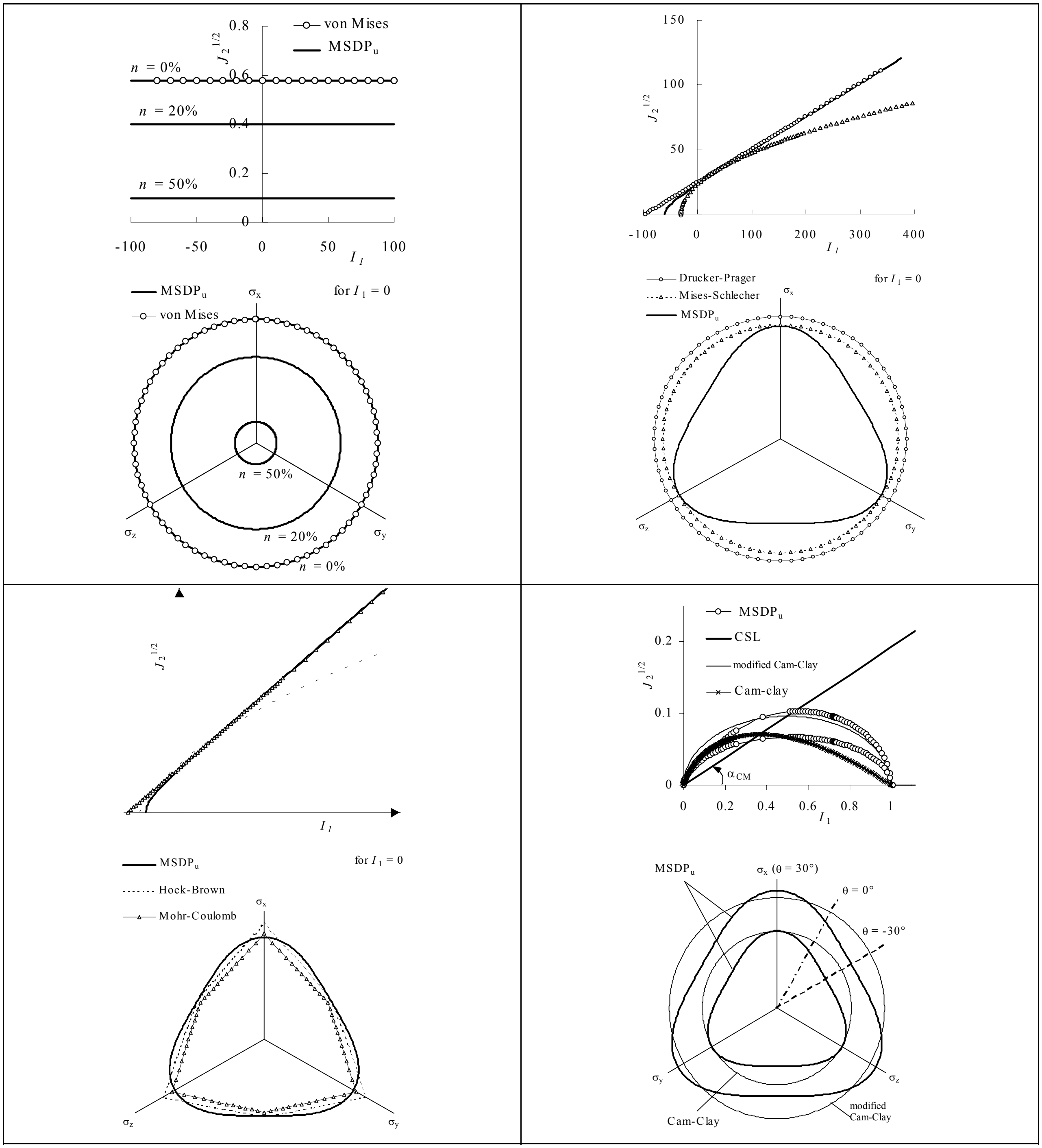
Figure 7. Schematical representation of the $MSDP_u$ criterion in a three-dimensional stress space.

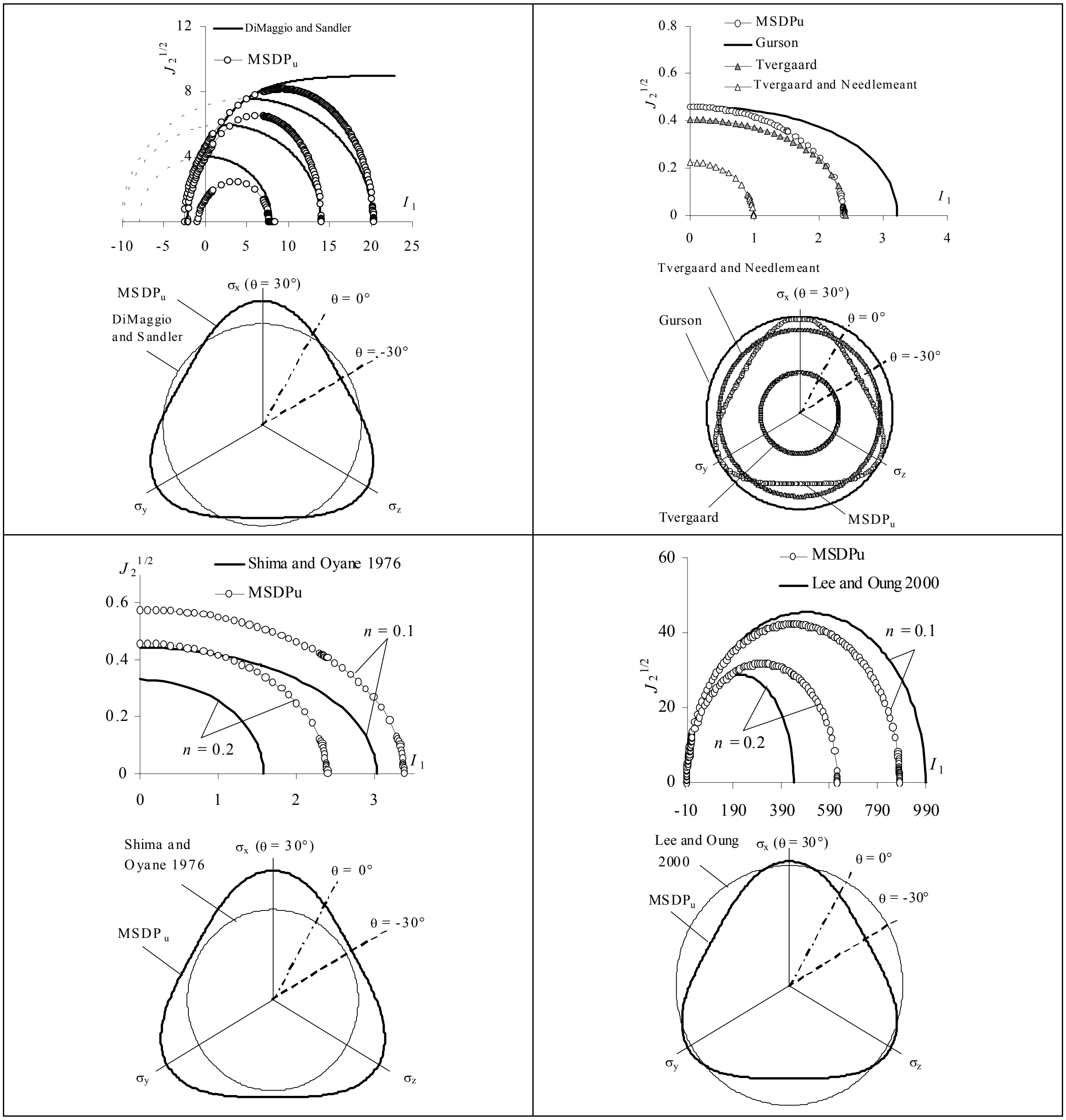


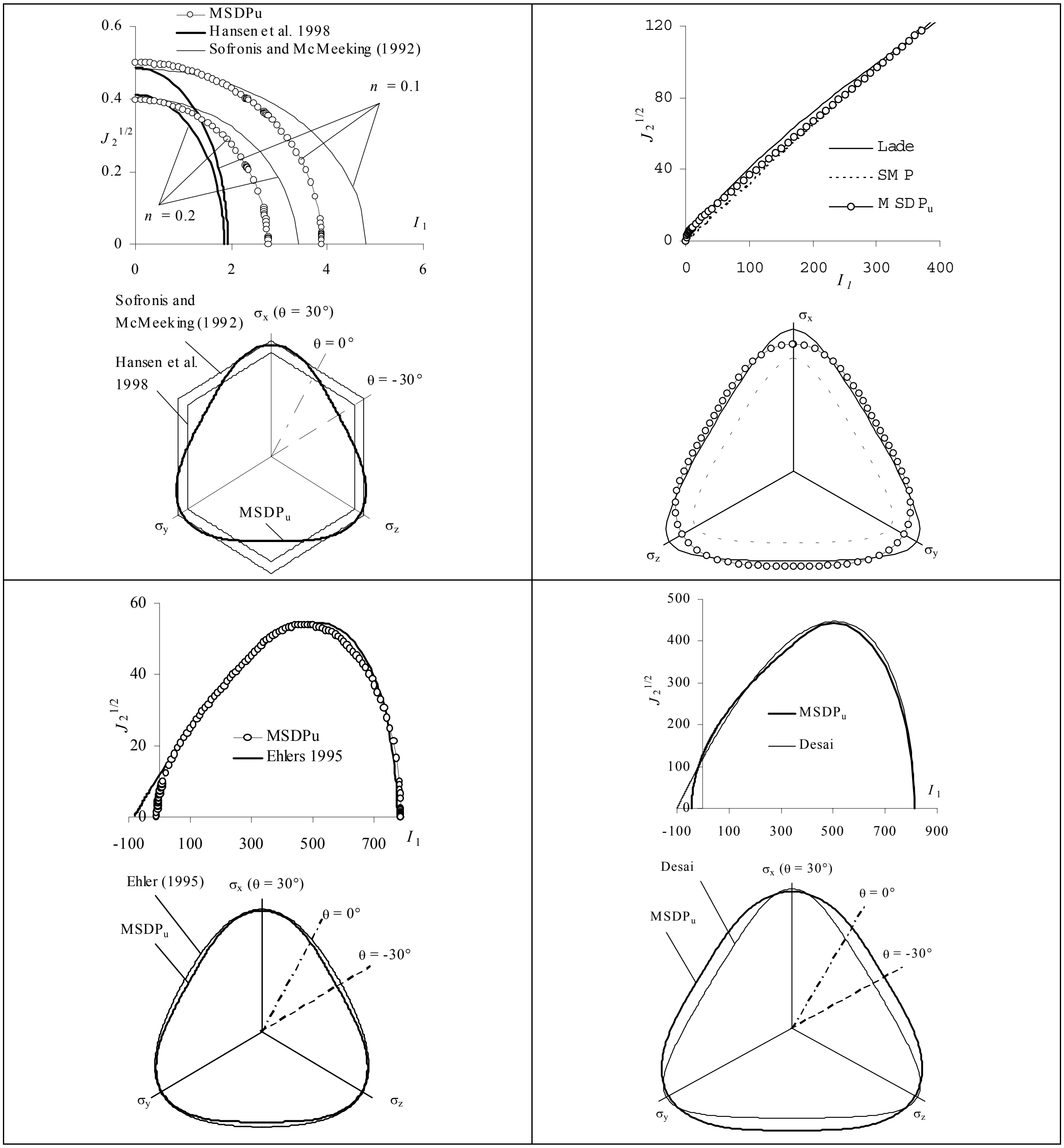
3.4 Comparison with other criteria

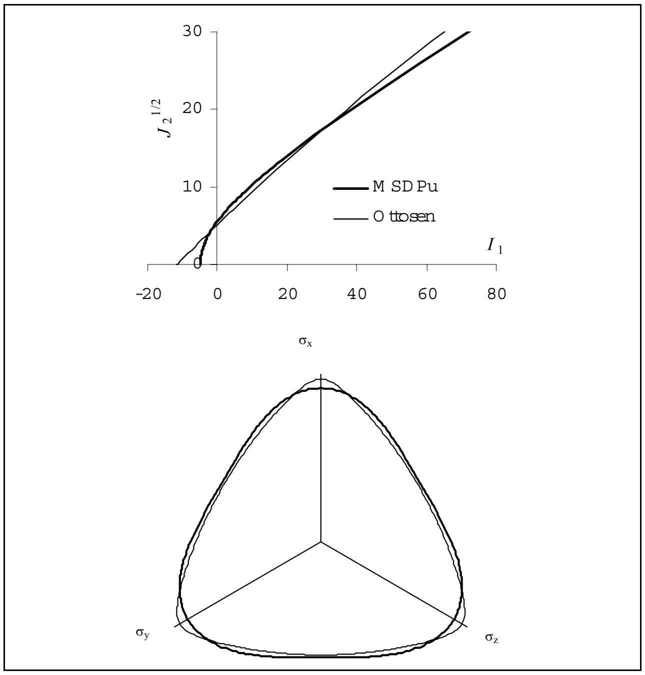
The figures shown above indicate that the proposed criterion can take various forms, depending on the values of the parameters used. It can also be shown that the $MSDP_u$ criterion closely reproduces the characteristics of surfaces associated with other criteria referenced above. Comparisons between $MSDP_u$ and some criteria used for porous materials are shown in Figure 8. In this regard, the proposed criterion may constitute a generalised version of criteria for plasticity and failure developed over the years, for ductile and brittle materials (with variable porosity). Some specific applications of the proposed criterion are presented in the following sections.

Figure 8. Graphical comparison between the $MSDP_u$ criterion and various existing criteria (shown for normalised parameters).




DiMaggio and Sandler
MSDP_u
J_2^1/2
I_1
MSDP_u
σ_x (θ = 30°)
θ = 0°
DiMaggio and Sandler
θ = -30°
σ_y
σ_z
MSDPu
Gurson
Tvergaard
Tvergaard and Needlemeant
Tvergaard and Needlemeant
σ_x (θ = 30°)
θ = 0°
Gurson
θ = -30°
σ_y
σ_z
Tvergaard
MSDP_u
Shima and Oyane 1976
MSDPu
n = 0.1
n = 0.2
Shima and Oyane 1976
σ_x (θ = 30°)
θ = 0°
MSDP_u
θ = -30°
σ_y
σ_z
MSDPu
Lee and Oung 2000
n = 0.1
n = 0.2
Lee and Oung 2000
σ_x (θ = 30°)
θ = 0°
MSDP_u
θ = -30°
σ_y
σ_z


MSDPu
Hansen et al. 1998
Sofronis and McMeeking (1992)
n = 0.1
n = 0.2
J2 1/2
I1
Sofronis and McMeeking (1992)
Hansen et al. 1998
σx (θ = 30°)
θ = 0°
θ = -30°
MSDPu
σy
σz
Lade
SMP
MSDPu
σx
σz
σy
MSDPu
Ehlers 1995
Ehler (1995)
MSDPu
Desai


30
20
10
0
$J_2^{1/2}$
MSDPu
Ottosen
I_1
-20
0
20
40
60
80
σ_x
σ_y
σ_z

4. APPLICATION OF THE MSDP$_u$ CRITERION

4.1 Identification of MSDP$_u$ parameters

Like other criteria, the MSDP$_u$ criterion contains parameters determined experimentally. A general approach for obtaining the optimised parameters is presented by Li et al. (2000). Li and Aubertin (2003) also discuss a method for defining the values of σ_{un} (eq.7) for σ_{cn} and σ_{tn} (used in parameters a_{1n} and a_{2n}).

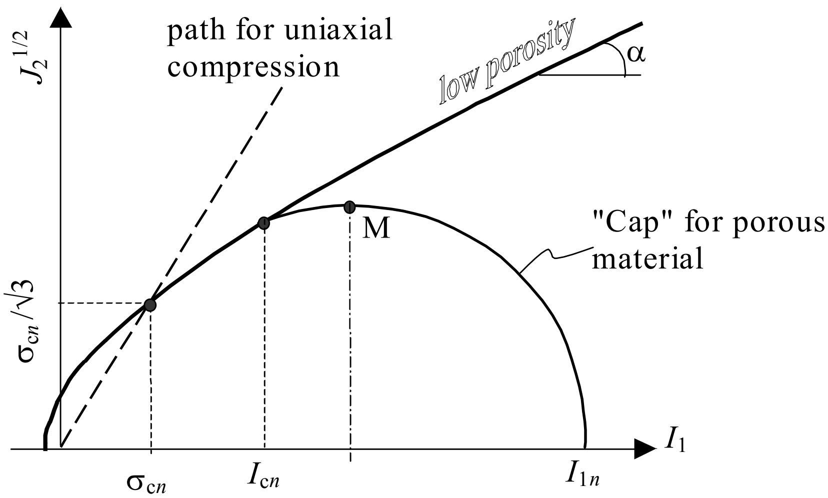
As shown in Figure 9, the surface of the MSDP$_u$ criterion deviates from the surface defined for low porosity materials when $I_1 \geq I_{cn}$. The surface closes at I_{1n} where the deviator is nil. Parameter a_{3n} is also utilised to define the surface as a function of porosity n when the material is under hydrostatic pressure (see Fig. 6f). For a given value of I_{1n} (for porosity n), one can obtain a_{3n} from equations (2) – (6), as follow:

$$a_{3n} = \frac{\alpha^2\left(I_{1n}^2 - 2a_{1n}I_{1n}\right) + a_{2n}^2}{\left(I_{1n} - I_{cn}\right)^2} \tag{11}$$

The value of parameter I_{1n} may be obtained from test results under hydrostatic compression, or deduced from tests under conventional triaxial compression (CTC, θ = 30°) or reduced triaxial extension (RTE, θ = -30°), for $I_1 > I_{cn}$.

To define the relationship between porosity and the values of I_{1n} and I_{cn}, equation (7) may be used or alternative relationships can be proposed. For instance, in soil mechanics, a logarithmic relationship between the void ratio e and the effective stress is often applied (e.g., Roscoe and Burland 1968; Wood 1990). In rock mechanics, exponential laws and power functions are often used (e.g., Li and Aubertin 2003).

Figure 9. Schematical representation of the $MSDP_u$ criterion for dense and porous materials (with "Cap") in CTC conditions (θ = 30°); the effect of porosity on the surface starts from I_{cn}; the surface closes on the I_1 axis at I_{1n}; the maximum value of $J_2^{1/2}$ corresponds to point M. Other parameters are also shown on the figure.



Physically, I_{1n} and I_{cn} tend towards infinity when porosity tends towards zero. This explains the absence of a Cap on the inelastic surface (in the I_1-$J_2^{1/2}$ plane) for materials with a very low porosity (such as hard rocks and certain types of concrete). When porosity n tends towards a critical value n_C (<1), such as defined in equation (7), the material loses its uniaxial strength and I_{1n} and I_{cn} will then reach their minimum value.

Based on these considerations and analysis of available results, the authors have considered the following expressions for I_{1n}:

$$I_{1n} = \left\{ I'_{1n}\left(1-\sin^{x_1}(\frac{\pi}{2}\frac{n}{n_C})\right) + \langle I'_{1n}\rangle \cos^{x_2}(\frac{\pi}{2}\frac{n}{n_C})\right\}\left\{1-\frac{\langle I'_{1n}\rangle}{2I'_{1n}}\right\} \quad \text{(adapted from eq. 7)} \qquad (7')$$

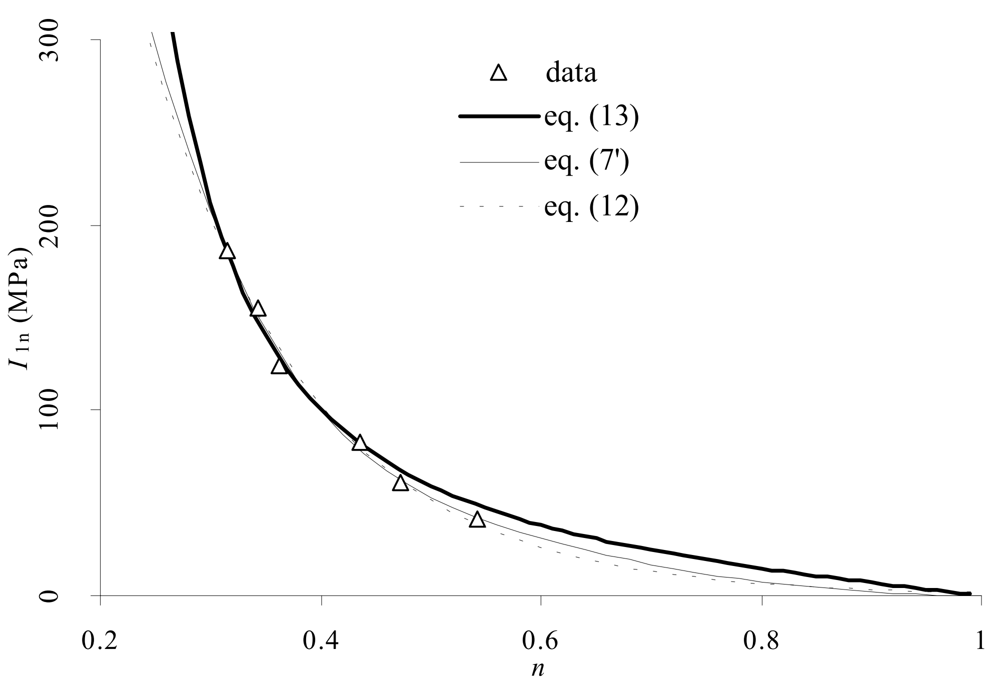
$$I_{1n} = I'_{1n}\exp(-q_1\, n) \qquad (12)$$

$$I_{1n} = I'_{1n} \sinh\left[\left(\frac{n_C}{n} - 1\right)^{p_1}\right] \qquad \text{(newly proposed equation)} \qquad (13)$$

where I'_{1n}, q_1 and p_1 are material parameters.

Figure 10 shows the relationship between I_{1n} and n for plaster under relatively high hydrostatic pressure. The three equations considered above are well correlated to the experimental results. Subsequently, in this report, equation (13) is retained because of its versatility and relative simplicity.

Figure 10. Variation of I_{1n} with porosity of plaster; tests on cylindrical samples under hydrostatic compression (ratio water/plaster = 70%) (data from Nguyen 1972); I'_{1n} = 1052.9 MPa, n_{Cc} = 100%, x_1 = 0.2847 and x_2 = 14.225 with equation (7'); I'_{1n} = 1604.5 MPa and q_1 = 6.876 with equation (12); I_{1n}' = 50.6 MPa, n_{Cc} = 100%, and p_1 = 0.898 with equation (13).



For I_{cn}, it is postulated that the same functions may be employed. The relationship then becomes:

$$I_{cn} = I'_{cn} \sinh\left[\left(\frac{n_C}{n} - 1\right)^{p_2}\right] \tag{14}$$

In equation (14) I'_{cn} and p_2 are additional material properties. As there is little data available to experimentally define parameter I_{cn}, it is postulated that parameters p_1 and p_2 take the same value (i.e. $p_1 = p_2 = p$). It is understood that the validity of this starting hypothesis must be confirmed by experimentation.

4.2 Graphical representation of experimental results

The following comparisons illustrate how the $MSDP_u$ criterion may be applied to describe the characteristic surfaces of materials with different porosity.

In the case of rocks and other brittle materials with very low porosity ($n < 1 – 3\%$), the applicability of $MSDP_u$ has been well documented (Aubertin and Simon 1996, 1997, 1998; Aubertin et al. 1999, 2000). In this case the criterion can describe the condition of failure (see Fig. 11a) and the damage initiation threshold (onset of crack propagation, see Figs. 11b, 11c and 11d), without the closed portion on the positive I_1 axis (i.e. I_{cn} is very large).

Figure 12 presents the application of the criterion (with b=0.75, for $I_1 < I_{cn}$) to describe the strength of crushed rock that behaves in a manner typical of a granular soil. Application are also shown for a metallic powder (Fig. 13), a clay (Fig. 14), and sands (Figs. 15 and 16). In all of these cases, the criterion is in good agreement with the experimental data, whether the envelope is linear or curved.

Figures 17 to 20 show the use of MSDP$_u$ (with b = 0.75), for $I_1 > I_{cn}$, in the case of a rock (Fig. 17), plaster (Fig. 18), clay (Fig. 19), and residual soil (Fig. 20). In these cases, the surface closes on the I_1 axis in compression.

Figures 21 through 23 show how the criterion may be used to describe the elastic limit and the failure condition for various materials, such as a porous rock (Fig. 21), an agglomerated residual soil (Fig. 22), and a paste cemented backfill (Fig. 23).

Finally, Figures 24 and 25 present the application of MSDP$_u$ to describe the strength of sand under different loading geometries.

These few illustrations demonstrate the ability of the MSDP$_u$ criterion to adequately describe the failure strength and the elastic limit of a wide variety of porous materials. Other examples of application have been presented by the authors in the publications cited above.

Figure 11a. MSDP$_u$ applied to the failure of sandstone, with σ_{cn} = 85 MPa, σ_{tn} = 2 MPa, $\phi \approx 28°$, b = 0.75, I_{cn} >>, v_1 = 0 (data from Takahashi and Koide 1989).

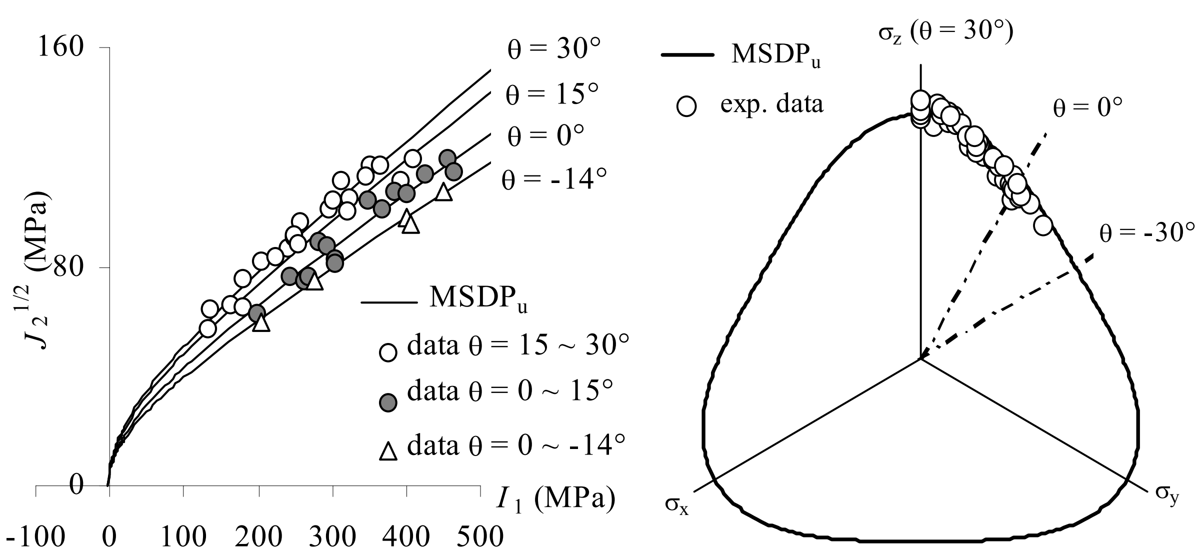


Figure 11b. Application of the $MSDP_u$ criterion to describe the damage initiation threshold of a rock salt submitted to CTC and RTE stress conditions (data from Thorel 1994); $b = 0.75$, $\phi = 0°$, $\sigma_c = 15$ MPa, $\sigma_t = 1.5$ MPa (after Aubertin and Simon 1997).

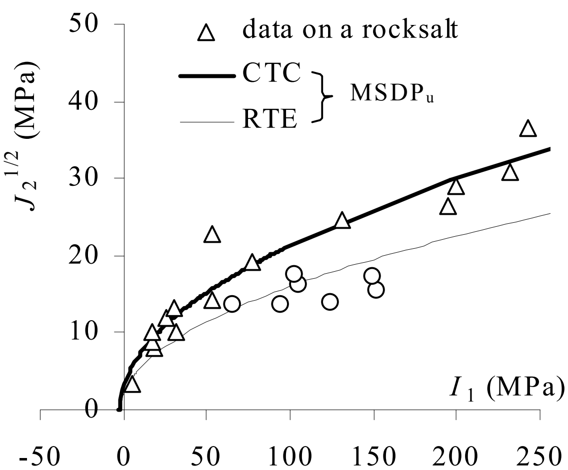


Figure 11c. Application of the $MSDP_u$ criterion to describe the damage initiation threshold of a man-made salt submitted to CTC stress conditions (data from Sgaoula 1997); $b = 0.75$, $\phi = 0°$, $\sigma_c = 37$ MPa, $\sigma_t = 3$ MPa.

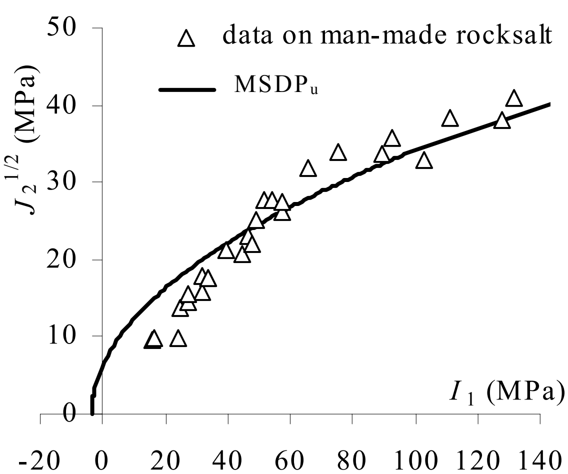


Figure 11d. Application of the MSDP$_u$ criterion to describe the damage initiation threshold of Lac du Bonnet grey granite submitted to CTC stress conditions (data from Lau and Gorski 1991); $b = 0.75$, $\phi = 47°$, $\sigma_c = 70$ MPa, $\sigma_t = 3$ MPa (after Aubertin and Simon 1997).

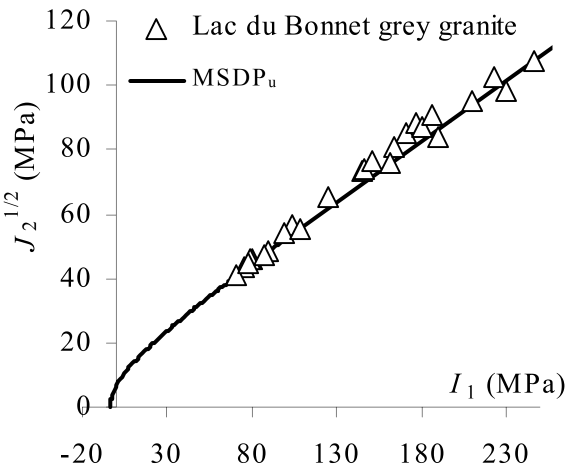


Figure 12. MSDP$_u$ and the failure strength (CTC) of crushed Westerly granite (data from Zoback and Byerlee 1976) with $b = 0.75$, $\phi = 33.8°$, $\sigma_{cn} = 3.1$ MPa, $\sigma_{tn} = 0$ MPa.

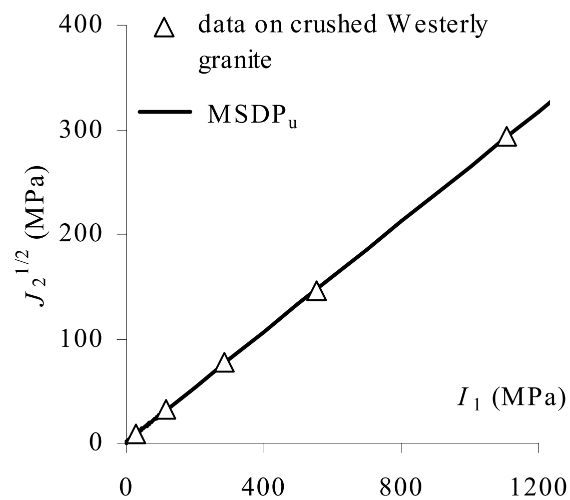


Figure 13. Description of the failure strength (CTC) of powdered aluminum with the $MSDP_u$ criterion (data from Cristescu et al. 1996): a) aluminum A10 with $\phi = 35.5°$, $\sigma_{cn} = 27.4$ kPa, $\sigma_{tn} = 0$ kPa; b) aluminum A16-SG, with $\phi = 30°$, $\sigma_{cn} = 50$ kPa, $\sigma_{tn} = 0$ kPa.

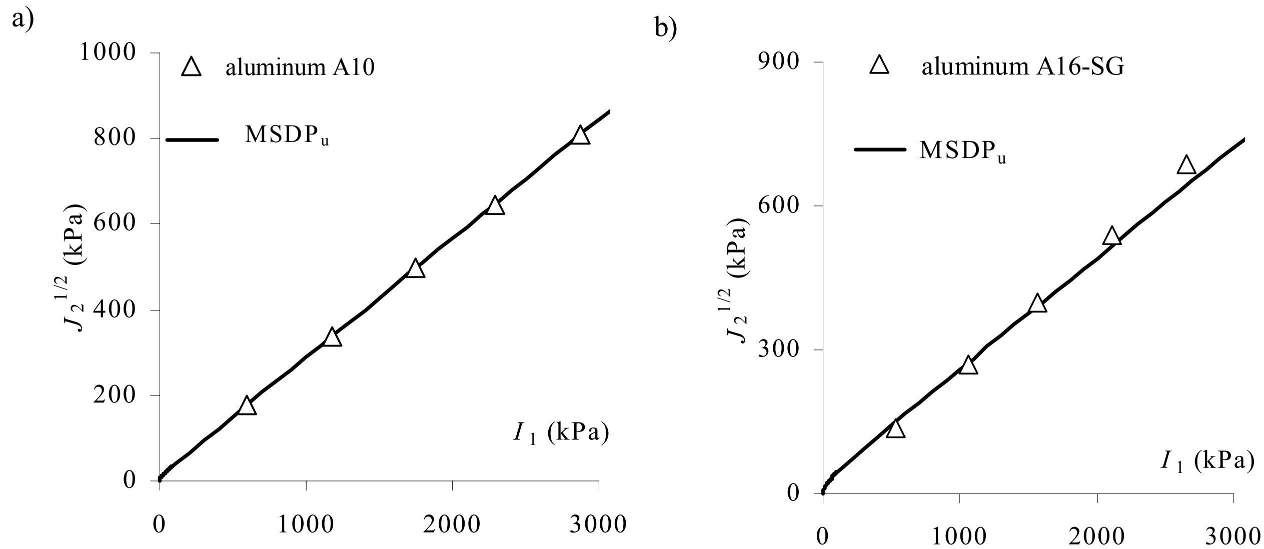


Figure 14. Failure strength of stiff Todi clay (CTC) and the $MSDP_u$ criterion (data from Rampello 1991): a) for samples allowed to swell with $\phi = 51.5°$, $\sigma_{cn} = 0.093$ MPa, $\sigma_{tn} = 0$ MPa; b) for undisturbed samples with $\phi = 61.2°$, $\sigma_{cn} = 0.54$ MPa, $\sigma_{tn} = 0.037$ MPa.

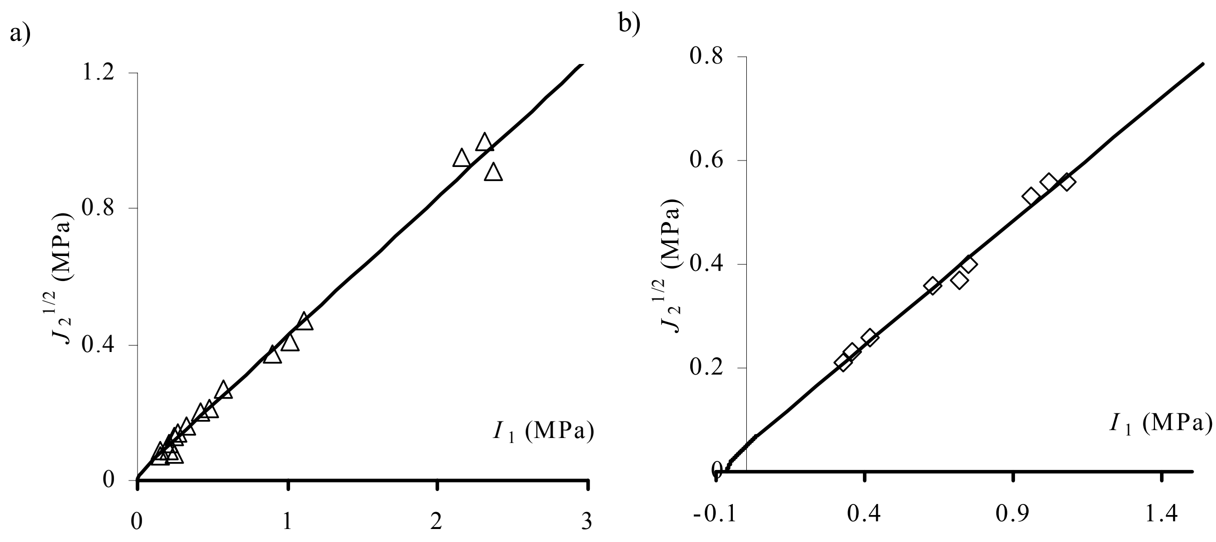


Figure 15. Failure strength (CTC) of Ottawa sand and the $MSDP_u$ criterion (data from Wan and Guo 2001); $b = 0.75$, $\phi = 26.6°$ (estimated), $\sigma_{cn} = 1.9$ MPa (estimated), $\sigma_{tn} = 0$ MPa, $a_{3n} = 0.0482$ (estimated), and $I_{cn} = 1156.6$ MPa (estimated).

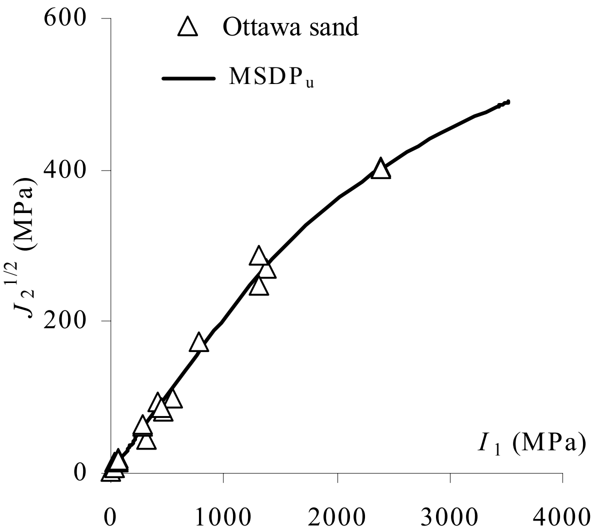


Figure 16. Failure strength (CTC) of Sacramento River sand and the $MSDP_u$ criterion (data from Wan and Guo 1998): a) for dense samples with $\phi = 35.8°$, $\sigma_{cn} = 196.67$ kPa, $\sigma_{tn} = 0$ kPa, $I_{cn} > 12000$ kPa; b) for looser samples with $\phi = 29.1°$, $\sigma_{cn} = 43.67$ kPa, $\sigma_{tn} = 0$ kPa.

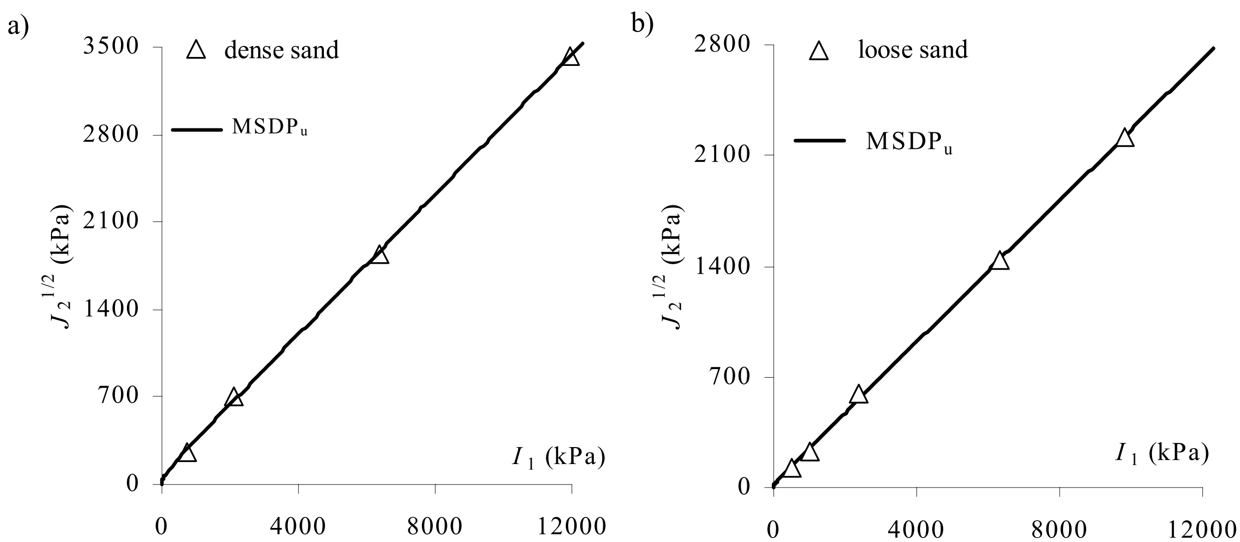


Figure 17a. Failure strength (CTC) of Indiana limestone and the $MSDP_u$ criterion (data from Schwartz 1964) with $\phi = 35°$ (estimated), $\sigma_{cn} = 38$ MPa (measured), $\sigma_{tn} = 3$ MPa (estimated), $a_{3n} = 0.105$ (estimated), and $I_{cn} = 40$ MPa (estimated).

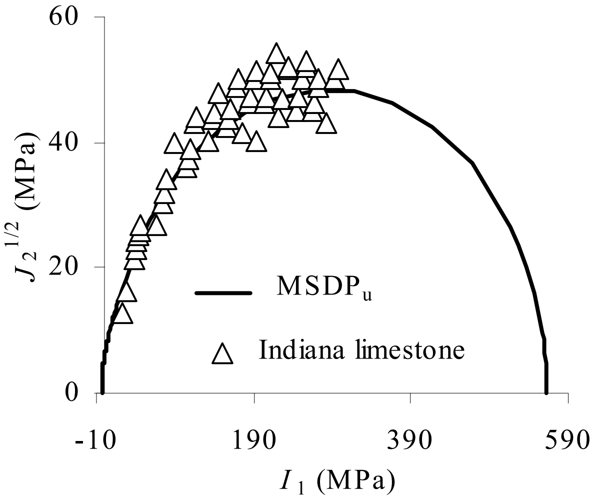


Figure 17b. Failure strength (CTC) of Weald shale and the $MSDP_u$ criterion (data from Madsen et al. 1989); $\phi = 38°$ (estimated), $\sigma_{cn} = 5$ MPa (estimated), $\sigma_{tn} = 0.1$ MPa (estimated), $a_{3n} = 0.21$ (estimated), and $I_{cn} = 45$ MPa (estimated).

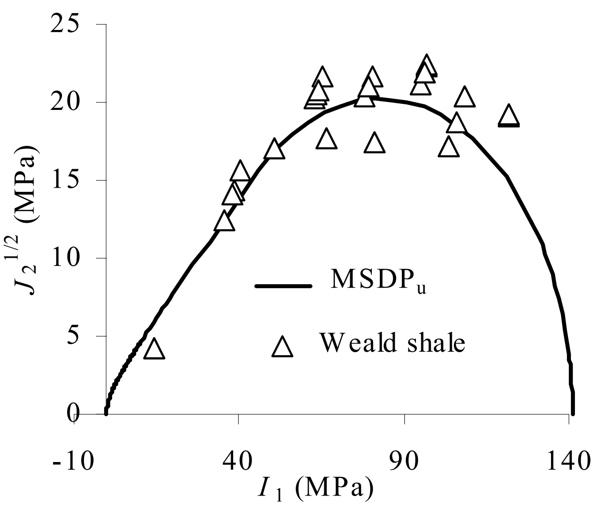


Figure 17c. Failure strength of normally consolidated (at 10 MPa) Trenton limestone and the $MSDP_u$ criterion (data from Nguyen 1972); $\phi = 33°$ (estimated), $\sigma_{cn} = 10$ MPa (estimated), $\sigma_{tn} = 0.5$ MPa (estimated), $a_{3n} = 0.134$ (estimated), and $I_{cn} = 18$ MPa (estimated).

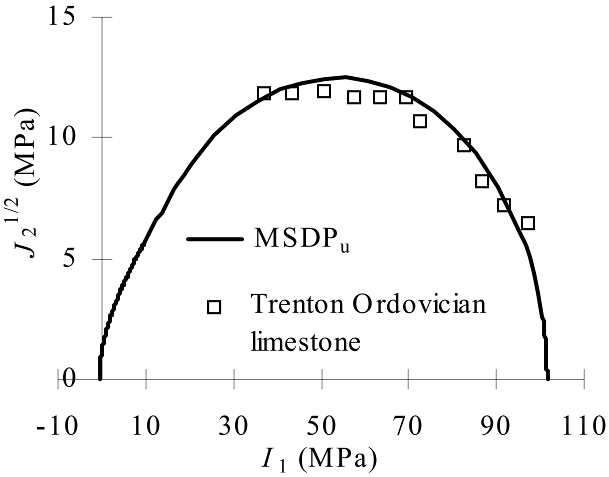


Figure 17d. Failure strength (CTC) of chalk and the $MSDP_u$ criterion (data from Elliott and Brown 1985); $\phi = 28°$ (estimated), $\sigma_{cn} = 8$ MPa (estimated), $\sigma_{tn} = 0.1$ MPa (estimated), $a_{3n} = 0.125$ (estimated), and $I_{cn} = 11$ MPa (estimated).

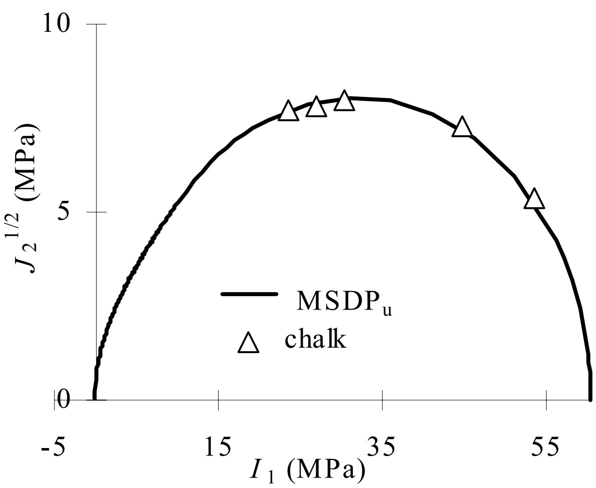


Figure 17e. Failure strength (in CTC) of limestone and the $MSDP_u$ criterion (data from Cheatham 1967); I_{cn} = 0 MPa, σ_{cn} = 20 MPa, σ_{tn} = 0.5 MPa, ϕ = 28°, a_{3n} = 0.102 for preconsolidated (at 34.5 MPa) limestone; I_{cn} = 0 MPa, σ_{cn} = 12 MPa, σ_{tn} = 0.5 MPa, ϕ = 28°, a_{3n} = 0.09 for intact (unconsolidated) limestone.

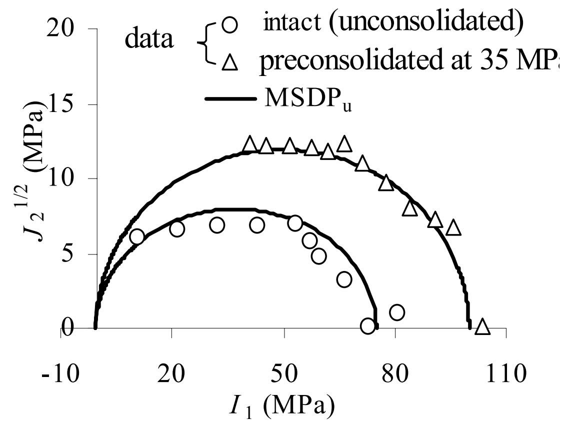


Figure 18. Failure strength (in CTC) of plaster samples (water/plaster = 50%) and the $MSDP_u$ criterion (data from Nguyen 1972); description of intact plaster (n = 44.3%) with ϕ = 30° (estimated), σ_{cn} = 13.6 MPa (measured), σ_{tn} = 2.6 MPa (measured), I_{1n} = 79.6 MPa (measured) and I_{cn} = 8 MPa (estimated); for preconsolidated plaster at 51.7 MPa (n = 32.25%) with ϕ = 30° (estimated), σ_{cn} = 13.3 MPa (measured), σ_{tn} = 2 MPa (estimated), I_{1n} = 154.9 MPa (measured) and I_{cn} = 15 MPa (calculated).

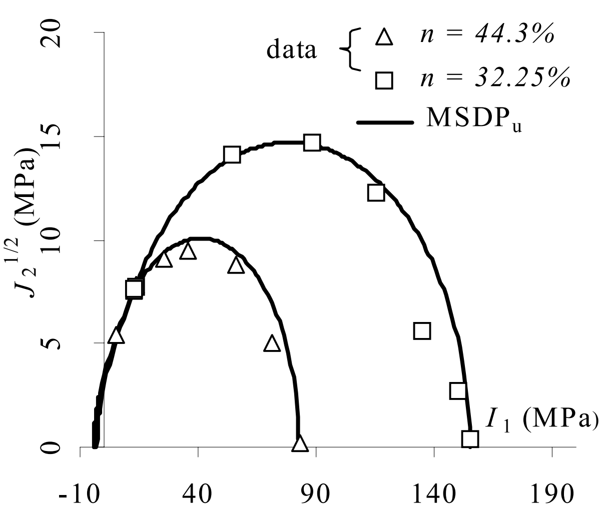


Figure 19a. Failure strength (in CTC) of Matagami clay and the $MSDP_u$ criterion (data from Nguyen 1972); $\phi = 30°$ (estimated), $\sigma_{cn} = 48$ kPa (measured), $\sigma_{tn} = 1$ kPa (estimated), $a_{3n} = 0.9$ (estimated) and $I_{cn} = 180$ kPa (estimated).

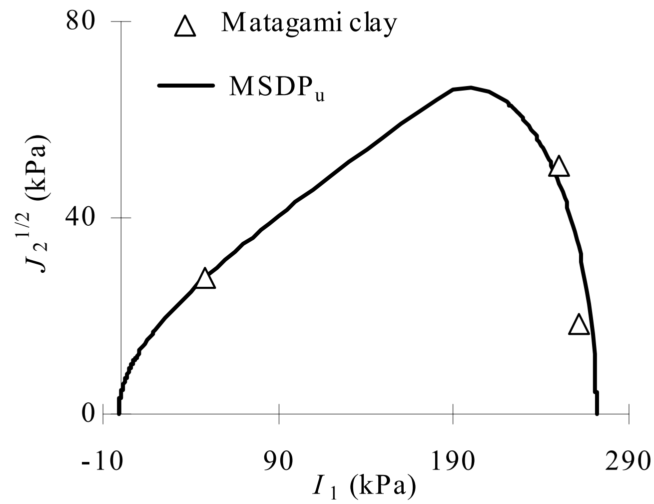


Figure 19b. Failure strength (in CTC) of Leda clay and the $MSDP_u$ criterion (data from Nguyen 1972); $\phi = 10°$ (estimated), $\sigma_{cn} = 107.8$ kPa (measured), $\sigma_{tn} = 15$ kPa (estimated), $a_{3n} = 0.9$ (estimated) and $I_{cn} = 530$ kPa (estimated).

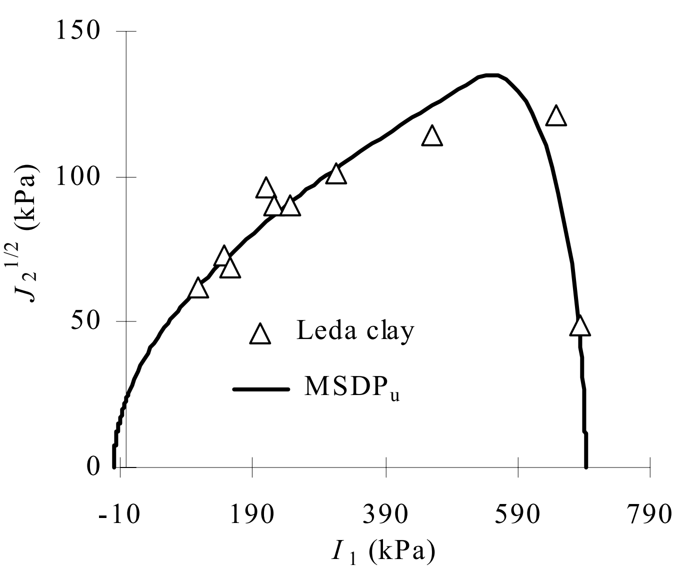


Figure 20a. Failure strength (in CTC) of basalt residual soil and the $MSDP_u$ criterion (data from Maccarini 1987); $\phi = 22.7°$, $\sigma_{cn} = 914.1$ kPa, $\sigma_{tn} = 127.4$ kPa, $a_{3n} = 0.10$ and $I_{cn} = 13.6$ kPa.

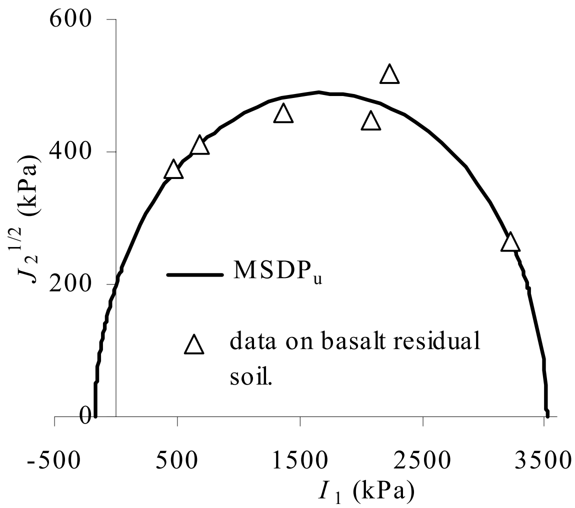


Figure 20b. Failure strength (in CTC) of a gneiss residual soil and the $MSDP_u$ criterion (data from Sandroni 1981); $\phi = 24.5°$, $\sigma_{cn} = 119.9$ kPa, $\sigma_{tn} = 0.9$ kPa, $a_{3n} = 0.09$ and $I_{cn} = 45.4$ kPa.

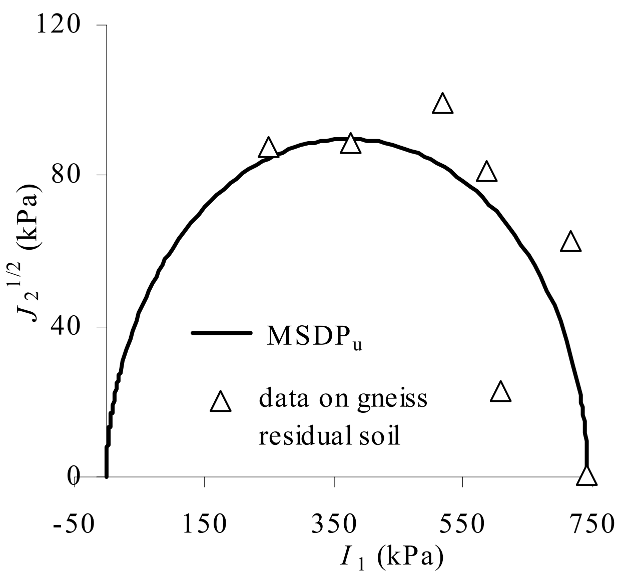


Figure 21a. Failure strength and elastic limit of Kayenta sandstone (in CTC) and the MSDP$_u$ criterion (data from Wong et al. 1992); for failure: $\phi = 30°$ (estimated), $\sigma_{cn} = 30$ MPa (measured), $\sigma_{tn} = 2$ MPa (estimated), $a_{3n} = 0$ (or $I_{cn} >>$); for yield: $\phi = 30°$ (estimated), $\sigma_{cn} = 30$ MPa (measured), $\sigma_{tn} = 2$ MPa (estimated), $a_{3n} = 0.115$ (estimated) and $I_{cn} = 250$ MPa (estimated).

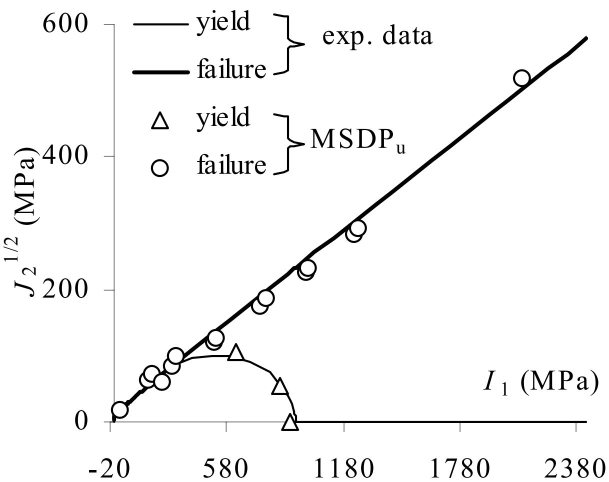


Figure 21b. Failure strength and elastic limit of Bath stone samples (in CTC) and the MSDP$_u$ criterion (data from Elliott and Brown 1985); for failure: $\phi = 30°$ (estimated), $\sigma_{cn} = 15$ MPa (measured), $\sigma_{tn} = 1$ MPa (estimated), $a_{3n} = 0$ (or $I_{cn} >>$); for yield: $\phi = 30°$ (estimated), $\sigma_{cn} = 15$ MPa (measured), $\sigma_{tn} = 1$ MPa (estimated), $a_{3n} = 0.095$ (estimated) and $I_{cn} = 0$ MPa (estimated).

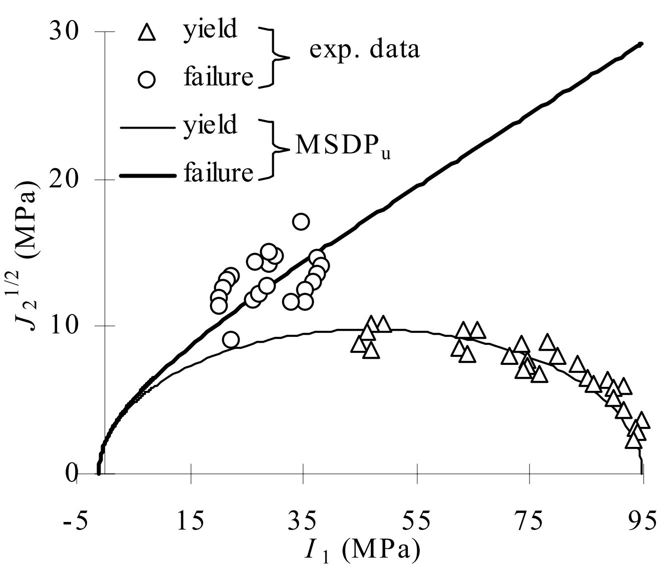


Figure 21c. Failure strength and elastic limit of a tuff (in CTC) with the $MSDP_u$ criterion (data from Pellegrino 1970); for failure: $\phi = 20°$ (estimated), $\sigma_{cn} = 3.8$ MPa (measured), $\sigma_{tn} = 0.5$ MPa (estimated), $a_{3n} = 0$ (or $I_{cn} >>$); for yield $\phi = 20°$ (estimated), $\sigma_{cn} = 3.8$ MPa (measured), $\sigma_{tn} = 0.5$ MPa (estimated), $a_{3n} = 0.115$ (estimated) and $I_{cn} = 6.5$ MPa (estimated).

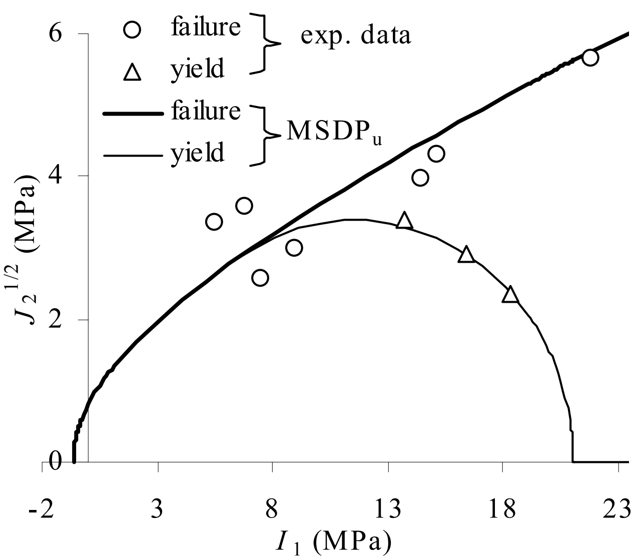


Figure 21d. Failure strength and elastic limit of Epernay chalk in CTC with the $MSDP_u$ criterion (data from Nguyen 1972); for failure: $\phi = 30°$ (estimated), $\sigma_{cn} = 8$ MPa (measured), $\sigma_{tn} = 0.1$ MPa (estimated), $a_{3n} = 0$ (or $I_{cn} >>$); for yield: $\phi = 30°$ (estimated), $\sigma_{cn} = 8$ MPa (measured), $\sigma_{tn} = 0.1$ MPa (estimated), $a_{3n} = 0.55$ (estimated) and $I_{cn} = 30$ MPa (estimated).

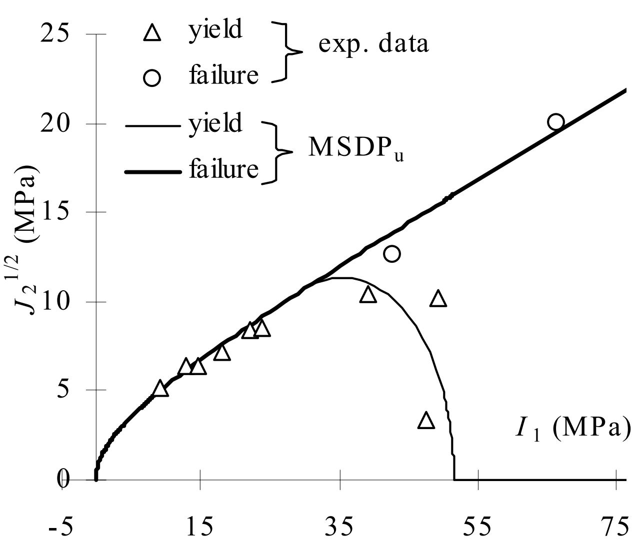


Figure 22. Failure strength and elastic limit of a residual (volanic conglomerate) soil (in CTC) and the $MSDP_u$ criterion (data from Uriel and Serrano 1973); for failure: $\phi = 25°$ (estimated), $\sigma_{cn} = 300$ kPa (measured), $\sigma_{tn} = 5$ kPa (estimated), $a_{3n} = 0$ (or I_{cn} >>); for yield: $\phi = 25°$ (estimated), $\sigma_{cn} = 300$ kPa (measured), $\sigma_{tn} = 5$ kPa (estimated), $a_{3n} = 0.063$ (estimated) and $I_{cn} = 100$ kPa (estimated).

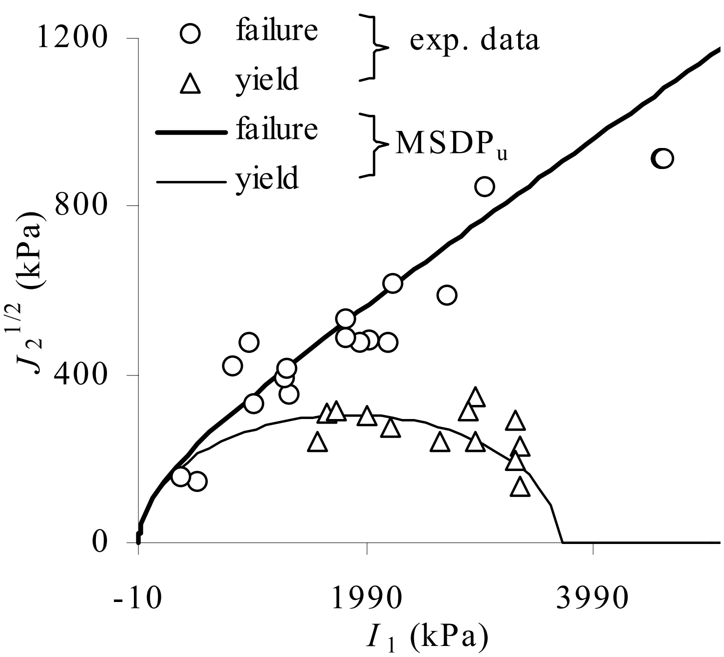


Figure 23. Failure strength and elastic limit of paste fills (in CTC) and the $MSDP_u$ criterion (data from Ouellet and Servant 2000); 6.5% cement tested at 28 days with $\phi = 23°$, $\sigma_{cn} = 580$ kPa, $\sigma_{tn} = 50$ kPa (for failure with $a_{3n} = 0$ or $I_{cn} >>$; for yield with $a_{3n} = 0.14$, and $I_{cn} = 100$ kPa); 6.5% cement at 3 days with $\phi = 32°$, $\sigma_{cn} = 200$ kPa, $\sigma_{tn} = 0.5$ kPa (for failure with $a_{3n} = 0$ or $I_{cn} >>$; for yield with $a_{3n} = 0.14$, and $I_{cn} = 100$ kPa); 3% cement at 15 days with $\phi = 37°$, $\sigma_{cn} = 10$ kPa, $\sigma_{tn} = 0$ kPa (for failure with $a_{3n} = 0$ or $I_{cn} >>$; for yield with $a_{3n} = 0.14$, and $I_{cn} = 150$ kPa).

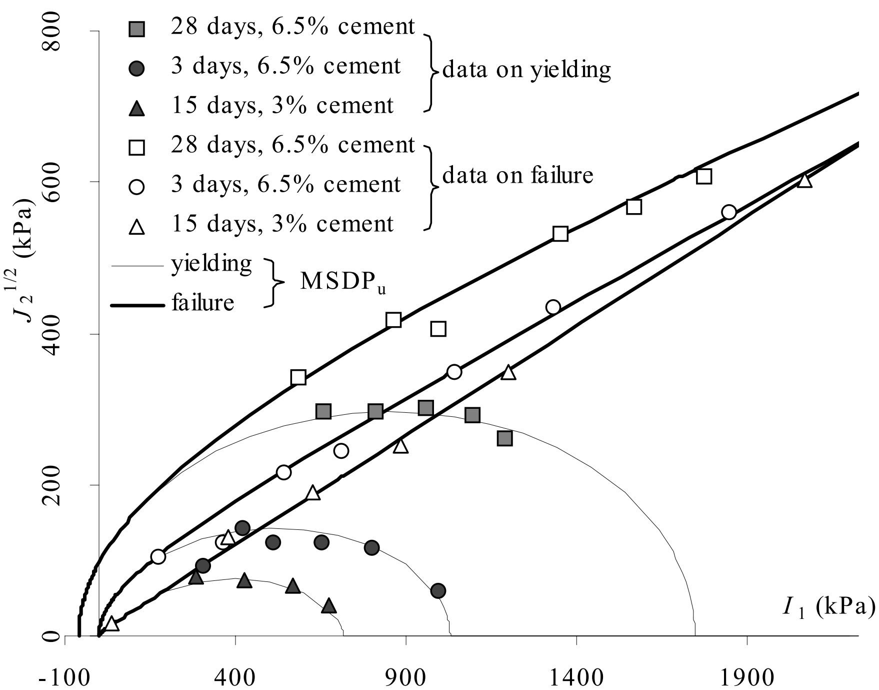


Figure 24. Failure strength of loose Monterey sand and the $MSDP_u$ criterion (n = 43.8%); σ_{cn} = 3 kPa, σ_{tn} = 0, $\phi \approx 38°$, b = 0.75, I_{cn} >> (or a_{3n} = 0), v_1 = 0 (data from Lade and Duncan 1973).

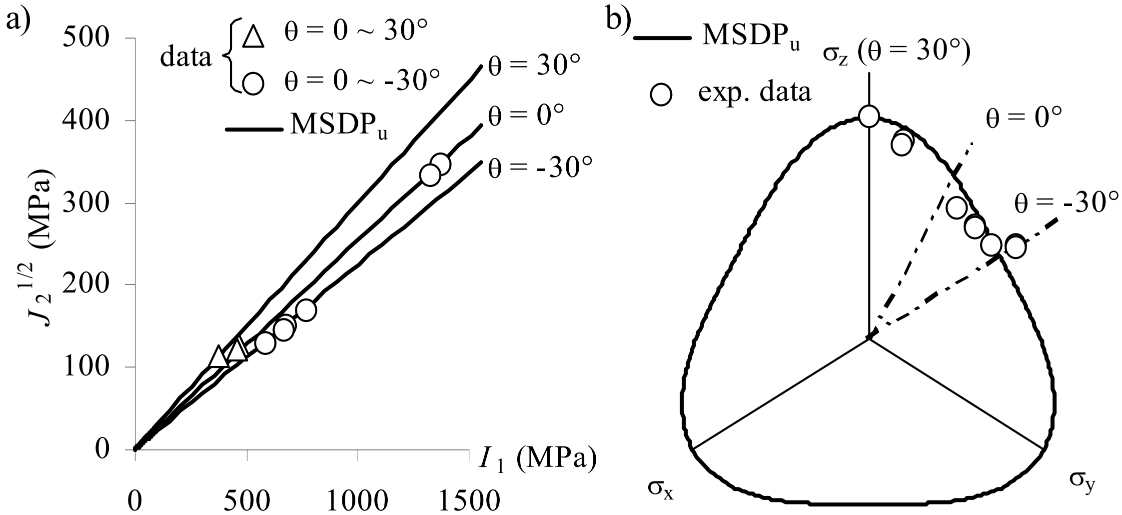
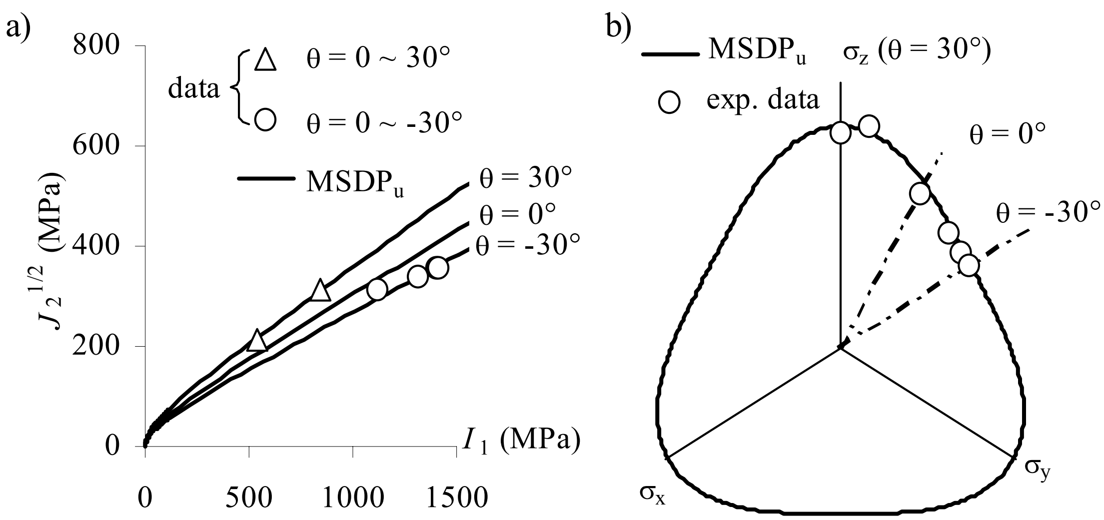


Figure 25. Failure strength of dense Monterey sand and the $MSDP_u$ criterion (n = 36.3%); σ_{cn} = 160 kPa, σ_{tn} = 0, $\phi \approx 38°$, b = 0.75, I_{cn} >> (or a_{3n} = 0), v_1 = 0 (data from Lade and Duncan 1973).



4.3 Description and prediction with the $MSDP_u$ criterion

In the preceding sections, the $MSDP_u$ criterion was used to describe yielding or failure conditions of various materials. Such application typically consists in identifying the material parameters while minimising errors between the available experimental data and the calculated results (Li et al. 2000). Such a descriptive approach is relatively straightforward and easy to use when the functions are adequately formulated; it simply becomes a regression problem. This is the case with $MSDP_u$, in which each parameter has a clear physical meaning.

In certain cases, it may be useful to predict an estimated strength value as a function of the influence parameters. For instance, when the basic parameters necessary for a descriptive application have been obtained, some of these parameters can be used to predict the material response under different loading conditions (e.g., from CTC to RTE) or at a different porosity. In the case of $MSDP_u$, there is an explicit dependency of parameters a_{1n}, a_{2n}, a_{3n} and I_{cn} (or I_{1n}) with respect to porosity n (see eqs. 7, 7', 12 and 13).

It may be useful to illustrate, with a few examples, the method to obtain the necessary parameters to describe (and sometimes predict) the behaviour of materials with variable porosity using $MSDP_u$.

Figure 26a shows a description of uniaxial compressive strength variation for plaster as a function of porosity. The parameters obtained by regression from equation (7) are: x_1 = 1.334, x_2 = 16.013, σ_{c0} = 27.35 MPa, n_{Cc} = 100%. The uniaxial tensile strength, σ_{t0} = -6.3 MPa, is also estimated by regression, from the triaxial compression tests results on “intact” material (at n = 54.2%). The same parameter values obtained for uniaxial compression tests were used to describe the variation of uniaxial tensile strength σ_{tn} (x_1 = 1.334 and n_{Ct} = 100%).

To evaluate the parameters for I_{cn} as a function of porosity n, tests conducted at a given porosity (i.e. n = 44%) are used (with eq. 13); $I_{1\ 44\%}$ = 82.4 MPa is then obtained. Parameters $\sigma_{c44\%}$ (= 16 MPa) and $\sigma_{t44\%}$ (= 2.7 MPa) can be calculated using equation (7), as shown in Figure 26. A regression on the series of data (at n = 44%) gives $I_{c\ 44\%}$ = 45 MPa, $a_{3\ 44\%}$ = 0.480 (based on eq. 11) and ϕ = 30°. Using equation (14), one can also determine that I'_{cn} = 27.64 MPa. All of the parameters required to predict the strength of this material at different porosity are now available. In Figure 26b, it is shown how the $MSDP_u$ criterion predicts the strength for the plaster at different n values. The correspondence between the predicted and measured strength is not perfect, but the procedure provides preliminary values that can be quite useful.

The same procedure has been used to describe (and predict) the behaviour of a sandstone. Figure 27a presents a description of the uniaxial compressive strength (eq. 7), with x_1 = 1.21, x_2 = 25.39, σ_{c0} = 193.04 MPa, and n_{Cc} = 52%. The parameters have been obtained from data provided by Farquhar et al. (1993, 1994). Regression on data for Berea sandstone at n =10.5% in CTC gives $\sigma_{t10.5\%}$ = 3.8 MPa (the compressive values of x_1 and n_C obtained for σ_{cn} are also used for σ_{tn}), ϕ = 32°, $I_{c10.5\%}$ = 380 MPa. Equation (13) and the two measured parameters, $I_{1\ 11\%}$ = 1619.6 MPa and $I_{1\ 13\%}$ = 1299.3 MPa, are then used to obtain p_1 = 0.4365 and I'_{1n} = 538.6 MPa. Parameter I'_{cn} may be deduced from equation (14) with $I_{c\ 10.5\%}$ = 380 MPa (see Fig. 27b); I'_{cn} = 126.4 MPa is then obtained. With these parameters, the $MSDP_u$ criterion may be used to predict other experimental results (see Fig. 27b). These predictions are fairly good, given the significant dispersion of the experimental data.

When I_{cn} is very large or when the range of mean stress is too small ($I_1 < I_{cn}$), the closed portion of the failure or yield surface is not apparent in the I_1-$J_2^{1/2}$ plane. In this case, the number of parameters used to describe the behaviour of the material using $MSDP_u$ is reduced. For example, Figure 28 (a and b) shows that the necessary parameters can be estimated (roughly) from a series of quasi-uniaxial compression tests (Fig. 28a) and a series of triaxial compression tests on crushed basalt which has a porosity n of 33.55% (Fig. 28b). These parameters may then be used to describe and "predict" the failure of basalt (Fig. 28c). As can be seen, these predictions are far from being conclusive; this was expected because the parameters describing the variation of the uniaxial compressive strength as a function of porosity were obtained indirectly from triaxial compression tests crushed basalt. This shows that a representative set of data is required to obtain representative predictions.

A last example, giving better results, is shown by applying the same procedure to crushed rock salt (data from Liedtke and Bleich 1985). The parameters analysis for the description and prediction of failure is illustrated on Figure 29 (a and b). From these, it can be concluded that the $MSDP_u$ criterion may be used to make predictions on how the failure surface is affected by porosity; however, the quality of these predictions largely depend on the quality of the testing data available.

Figure 26a. Variation of the uniaxial compressive strength of plaster as a function of porosity (data from Nguyen 1972): regression with x_1 = 1.334, x_2 = 16.013, σ_{c0} = 27.35 MPa, and n_{Cc} = 100%.

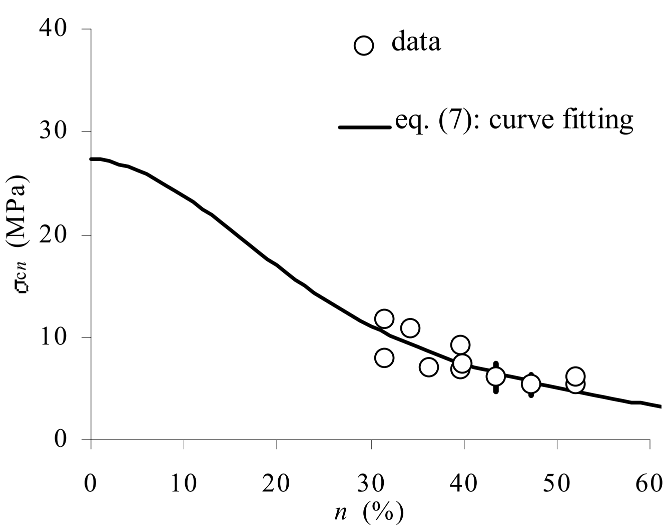


Figure 26b. Description and prediction of the strength in CTC of plaster (water/plaster =70%) with the $MSDP_u$ criterion (data from Nguyen 1972); description (dark line) for n = 43.25% with $\phi = 30°$ (estimated), $\sigma_{cn} = 16$ MPa (calculated), $\sigma_{tn} = 2.7$ MPa (calculated), $I_{1n} = 82.4$ MPa (measured), $I_{cn} = 45$ MPa (estimated), and $a_{3n} = 0.482$ (calculated); predictions (fine lines) with $I'_{1n} = 50.6$ MPa, $I'_{cn} = 27.6$ MPa, and $p_1 = 0.898$.

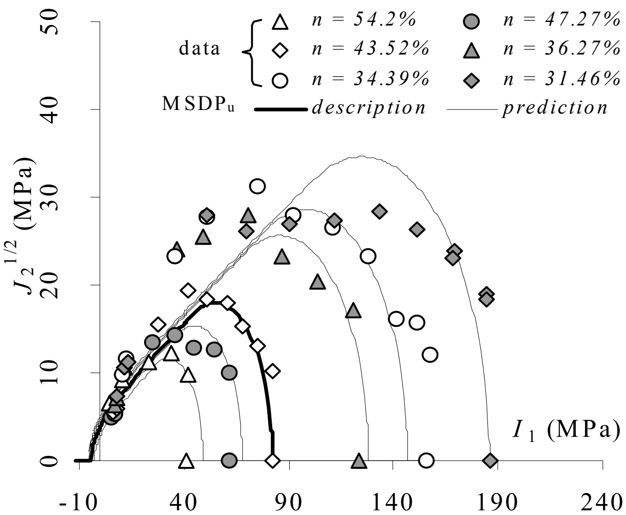


Figure 27a. Description of the uniaxial compressive strength of sandstone, with x_1 = 1.21, x_2 = 25.39, σ_{c0} = 193.04 MPa, and n_{Cc} = 51.94% (data from Farquhar et al. 1993, 1994).

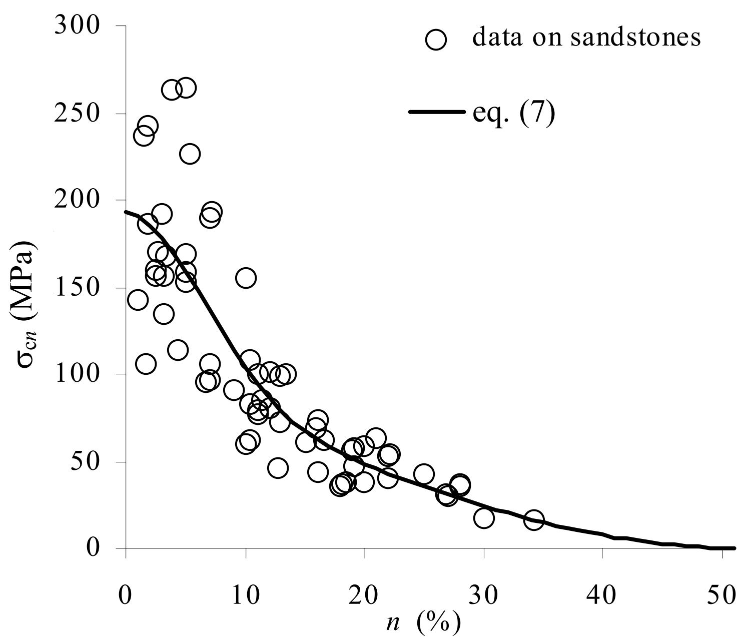


Figure 27b. Description and prediction of the strength (in CTC) of Berea sandstone with the $MSDP_u$ criterion (data from Wong et al. 1992); description (dark line) for n = 10.5% with $\phi = 32°$ (estimated), $\sigma_{cn} = 163.6$ MPa (estimated), $\sigma_{tn} = 3.8$ MPa (calculated), $I_{1n} = 1619.8$ MPa (measured), $I_{cn} = 380$ MPa (estimated), and $a_{3n} = 0.1507$ (calculated); predictions (fine lines) with $I_{1n}' = 538.6$ MPa, $I_{cn}' = 126.4$ MPa, and $p = 0.436$.

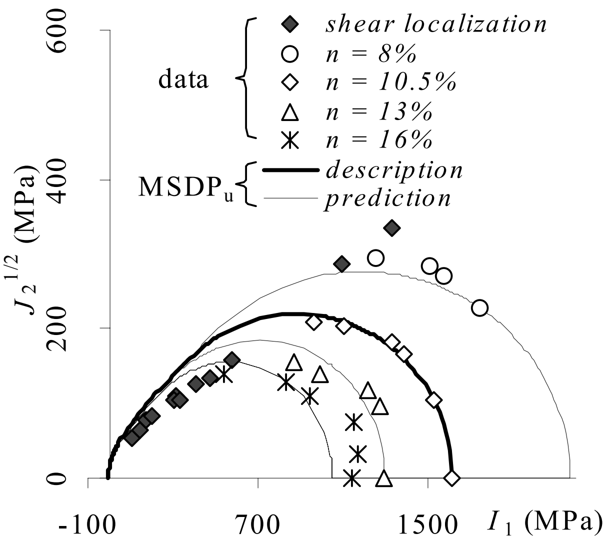


Figure 28a. Parameters determination for the uniaxial compressive strength of crushed basalt for a series of CTC tests at low confinement ($\sigma_3 = 0.413$ MPa) (data from Al-Hussaini 1983): $x_1 = 1.261$, $x_2 = 1.553$, $\sigma_{c0} = 5.42$ MPa (σ_{c0} is the uniaxial compressive strength at $n = 0$), and $n_{Cc} = 80\%$.

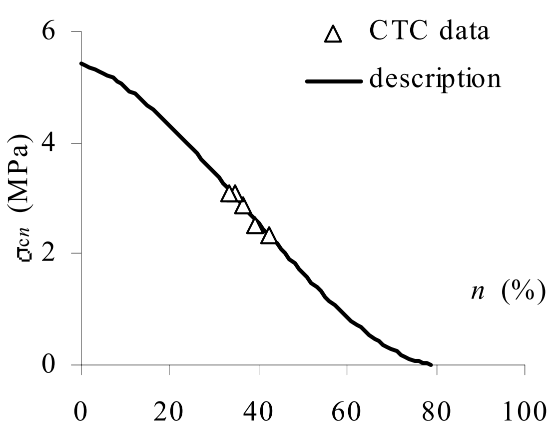


Figure 28b. Parameters determination from a series of CTC tests on crushed basalt, with n = 33.55%: σ_{cn} = 1.384 MPa, σ_{tn} = 0 MPa, and ϕ = 35.64° (data from Al-Hussaini 1983).

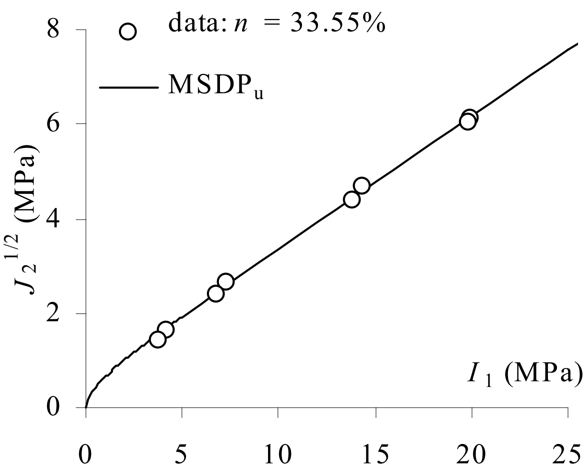


Figure 28c. Description and prediction of the strength (in CTC) of crushed basalt with the $MSDP_u$ criterion (data from Al-Hussaini 1983) with the parameters deduced from Figure 29b: σ_{c0} = 2.396 MPa, and σ_{tn} = 0 MPa.

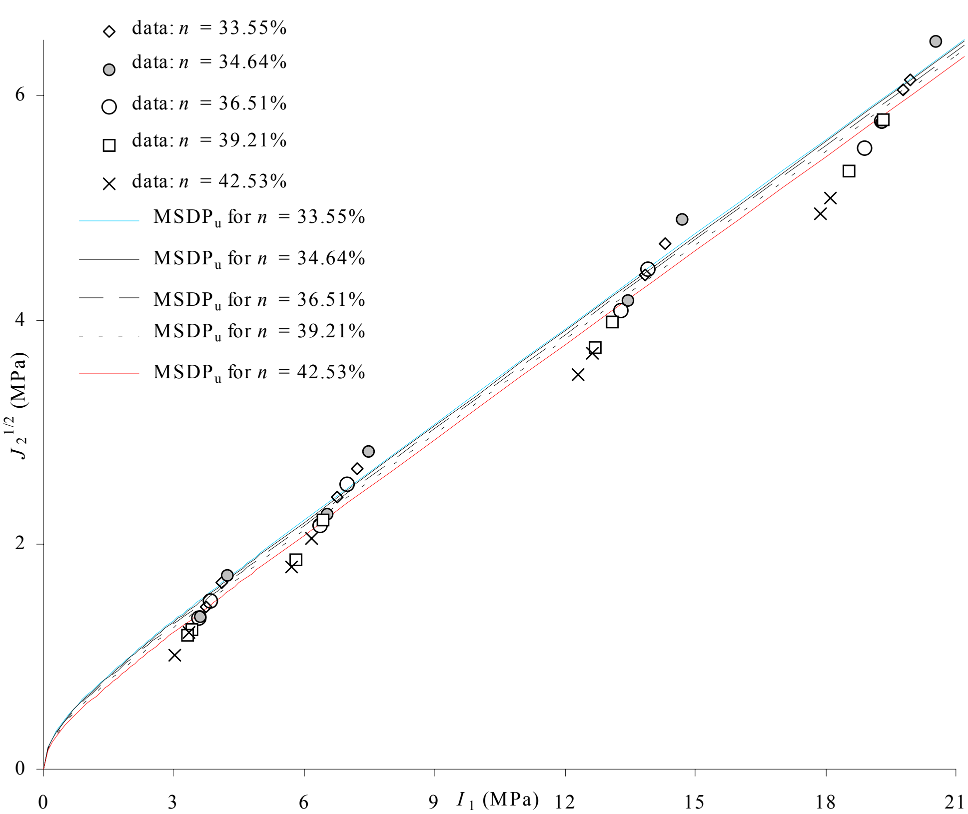


Figure 29a. Determination of the strength parameters of crushed rock salt (data from Liedtke and Bleich 1985) with $x_1 = 0.861$, $x_2 = 30.285$, $\sigma_{c0} = 27.27$ MPa and $n_{Cc} = 81.91\%$.

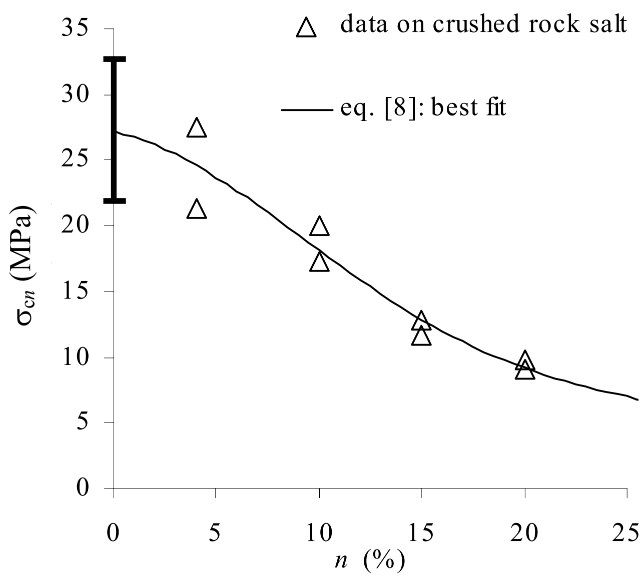
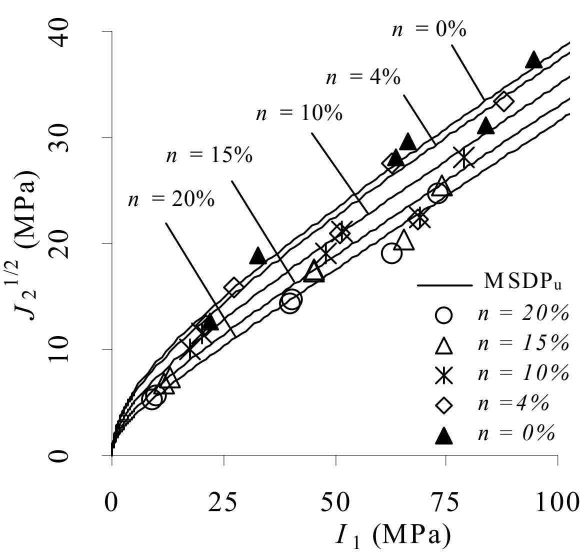


Figure 29b. Description and prediction of the strength of crushed rock salt in CTC with the parameters deduced from Figure 30a; the parameters $\phi = 35.2°$, and $\sigma_{t0} = 0$ MPa have been estimated from the data at $n = 0\%$ (data from Liedtke and Bleich 1985).



5. DISCUSSION

5.1 Application to large size rock mass strength

Among the possible applications of $MSDP_u$, there is the possibility of using this criterion to "predict" the failure strength of rock masses for large scale engineering works such as underground mining openings. In this case, the passage from small scale (in the laboratory) to large scale in the field implies that some of the parameters be adjusted. With $MSDP_u$, this is done in two steps. The first one is a scale effect correction for intact rock, which implies a continuous reduction of strength from the laboratory size samples to the unit block size (Aubertin et al. 2000, 2001, 2002). This reduced large scale intact rock strength then becomes the upper bound value for the rock mass strength in the field, which is further reduced by the addition of new, large scale flaw (not seen in the intact rock) – associated to joints and discontinuities.

To make this further strength adjusted, the authors have proposed the use of continuity parameter Γ, inspired by the work of Kachanov-Rabotnov. Here, Γ is a parameter of continuity which may considered to be an isotropic damage variable D (= 1 - Γ), such as that defined in the approach of Kachanov-Rabotnov to the basic mechanics of damage (Lemaître 1992; Krajcinovic 1996). This parameter is introduced to describe the influence of continuous faults on a grand scale in the structure, such as a network of fractures in massive rock (Aubertin et al. 2000). In this document, it has been assumed that $\Gamma = 1$ (no large scale damage)

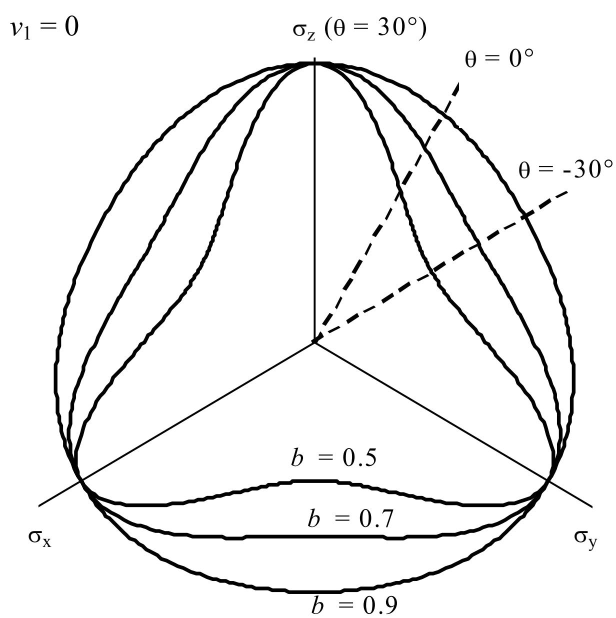
5.2 Modification of function F_π

The preceding sections have shown the great versatility of the $MSDP_u$ criterion, which may be used to describe the surface of various porous materials under different geometry of loading.

As with other criteria, there are nevertheless limits to the applicability of $MSDP_u$. One of these limitations is associated with the value of b (seen in the π plane), which should not be less than about 0.7 (in eq. 8). When b is smaller than 0.7, the surface becomes concave in the octahedral plane, as illustrated in Figure 30. This would violate a premise of the theory of plasticity, which implies that a yield surface must be convex in the π plane.

When this limitation is judged to be important, the authors propose the use of an alternative function, which is based on a relationship developed by Argyris et al. (1974a, b) and William and Warnke (1975). This last formulation was recently used by Peric and Ayari (2002a, b) to extend the Cam-Clay model.

Figure 30. $MSDP_u$ surface (function F_π, eqs. 8 and 9) in the octahedral (π) plane (the figure is normalised to obtain $F_\pi = 1$ at $\theta = 30°$).



Based on this existing relationship, function F_π can be reformulated in a framework compatible with the formulation of $MSDP_u$ to obtain:

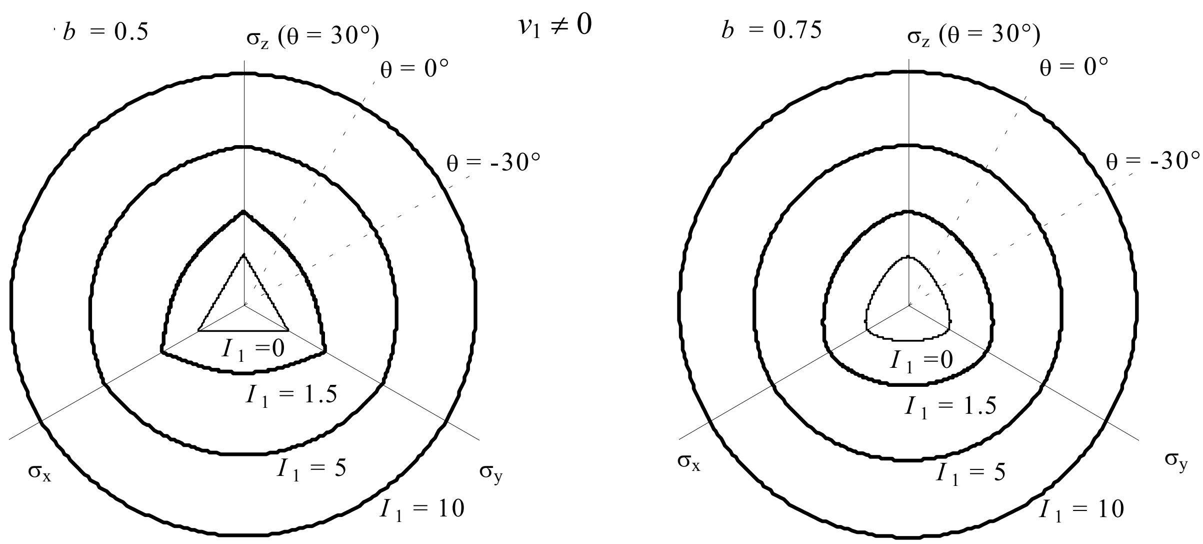
$$F_\pi = \left(\frac{(1-b^2) f_\theta + (2b-1)\left\{(1-b^2) f_\theta^2 + 5b^2 - 4b\right\}^{1/2}}{(1-b^2) f_\theta^2 + (1-2b)^2} \right)^v \tag{15}$$

where

$$f_\theta = \sqrt{3}\cos\theta - \sin\theta \tag{16}$$

The variation of the surface shape as a function of parameters b and v (and of the hydrostatic pressure) is shown schematically in Figure 31. The surface in the π plane may vary from a triangle to a circle. With eq. (15), the value of b may vary from 0.5 to 1.0 without any problem of concavity; this range of values may be required for some geological materials. In this case, it is nevertheless noted that points of singularity appear at θ = 30° (Fig. 31), a characteristic judged to be undesirable from a numerical point of view. There is thus a compromise to be made between these two aspects (i.e. convexity or singular points on the surface).

Figure 31. The $MSDP_u$ surfaces with the alternative function F_π (eqs. (15) and (16)) in the octahedral plane (π).



6. CONCLUSION

In this report, the authors have reviewed the main features of numerous criteria that have been developed to represent the yield condition, plastic potential, and failure strength of engineering materials. Then, it is shown that the characteristics of several existing criteria may be represented by a single system of equations forming the multiaxial criterion known as $MSDP_u$. This criterion explicitly includes a dependency on the porosity. By using experimental results obtained on a wide variety of materials, it has been shown that the $MSDP_u$ criterion is of wide applicability. The implementation of this criterion into a numerical code is underway and will be the object of future communications.

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