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<b>Auteurs:</b> Authors:	Robert P. Chapuis and Michel Aubertin
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SOILS USING THE KOZENY-CARMAN EQUATION**

Robert P. Chapuis and Michel Aubertin  
Département des génies civil, géologique et des mines  
École Polytechnique de Montréal

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**Robert P. Chapuis and Michel Aubertin**

**Department CGM, École Polytechnique de Montréal  
P.O. Box 6079, Sta. CV, Montreal, QC, Canada, H3C 3A7**

**Tél.: (514) 340 4711, ext. 4427 – Fax: (514) 340 4477**

**E-mail: [robert.chapuis@polymtl.ca](mailto:robert.chapuis@polymtl.ca)**

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## Abstract

The saturated hydraulic conductivity of a soil can be predicted using empirical relationships, capillary models, statistical models and hydraulic radius theories. A well-known relationship between permeability and properties of pores was proposed by Kozeny and later modified by Carman. The resulting equation is largely known under the name of Kozeny-Carman, although these authors never published together. In the geotechnical literature, there is a large consensus that the Kozeny-Carman (KC) equation applies to sands but not to clays. Such opinion, however, is supported only by partial demonstration. This report evaluates the background and the validity of the KC equation with laboratory permeability tests. Considered test results were taken from publications that provided all information needed to make a prediction: void ratio, and either the measured specific surface for cohesive soils, or the gradation curve for non-cohesive soils. This report shows how to estimate the specific surface of a non-cohesive soil from its gradation curve. The results presented here show that, as a general rule, the KC equation predicts fairly well the saturated hydraulic conductivity of most soils. Many of the observed discrepancies can be related to either practical reasons (e.g. inaccurate specific surface value, steady flow not reached, unsaturated specimens, etc.) or theoretical reasons (some water is motionless, and the predictive equation is isotropic whereas hydraulic conductivity is an anisotropic property). These issues are discussed in relation to the predictive capabilities of the KC equation.

*Key words:* permeability, prediction, gradation curve, specific surface

## Résumé

La conductivité hydraulique saturée d'un sol peut être prédite par des relations empiriques, des modèles capillaires, des modèles statistiques et des théories de rayon hydraulique. Une relation bien connue entre perméabilité et propriétés des pores fut proposée par Kozeny et modifiée par Carman. L'équation résultante est largement connue sous le nom Kozeny-Carman (KC), bien que ces auteurs n'aient jamais publié ensemble. Dans la littérature géotechnique, il existe un large consensus à l'effet que l'équation de Kozeny-Carman s'applique aux sables mais pas aux argiles. Cependant, cette opinion n'est appuyée que par une démonstration partielle. Cet article examine les fondements et la validité de l'équation KC à l'aide d'essais de perméabilité en laboratoire. Les résultats d'essais proviennent de diverses publications qui ont fourni toute l'information requise pour faire une prédiction : indice des vides et soit la surface spécifique mesurée pour les sols cohérents, soit la courbe granulométrique pour les sols pulvérulents. L'article montre comment calculer la surface spécifique d'un sol pulvérulent à partir de sa courbe granulométrique. Les résultats présentés ici indiquent qu'en général, l'équation de Kozeny-Carman prédit assez bien la conductivité hydraulique saturée de la plupart des sols. Plusieurs des divergences constatées peuvent être reliées soit à des raisons pratiques (e.g. valeur imprécise de la surface spécifique, régime permanent pas établi, échantillons non saturés, etc.) soit à des raisons théoriques (une partie de l'eau est immobile, et l'équation de prédiction est isotrope alors que la conductivité hydraulique est une propriété anisotrope). Ces aspects sont discutés dans l'article en relation avec la capacité de prédiction de l'équation de Kozeny-Carman.

*Mots clés :* perméabilité, prédiction, granulométrie, surface spécifique

## Introduction

Since Seelheim (1880) wrote that the permeability should be related to the squared value of some characteristic pore diameter, many equations have been proposed to predict the saturated hydraulic conductivity,  $k$ , of porous materials. According to state-of-the-art publications (e.g. Scheidegger 1953, 1954, 1974; Bear 1972; Houpeurt 1974), the  $k$ -value for a single fluid flow can be predicted using empirical relationships, capillary models, statistical models and hydraulic radius theories. The best models include at least three parameters to account for the relationships between the flowrate and the porous space, for example the size of the pores, their tortuosity and their connectivity.

A frequently quoted relation was proposed by Kozeny (1927) and later modified by Carman (1937, 1956). The resulting equation is largely known as the Kozeny-Carman (KC) equation, although the two authors have never published together. This equation was developed after considering a porous material as an assembly of capillary tubes for which the equation of Navier-Stokes can be used. It yielded the hydraulic conductivity  $k$  as a function of the porosity  $n$  (or void ratio  $e$ ), the specific surface  $S$  ( $\text{m}^2/\text{kg}$  of solids), and a factor  $C$  to take into account the shape and tortuosity of channels. Since its first appearance (Carman 1937) to the present, this equation has taken several forms, including the following one that is commonly used:

$$k = C \frac{g}{\mu_w \rho_w} \frac{e^3}{S^2 D_R^2 (1+e)} \quad [1]$$

where  $k$  is the hydraulic conductivity or coefficient of permeability,  $C$  a constant,  $g$  the gravitational constant,  $\mu_w$  the dynamic viscosity of water,  $\rho_w$  the density of water,  $\rho_s$  the density of solids,  $D_R$  the specific weight ( $D_R = \rho_s / \rho_w$ ) of solids,  $S$  the specific surface and,  $e$  the void ratio. This equation predicts that, for a given soil, there should be a linear relationship between  $k$  and  $e^3 / (1+e)$ . It can also be used to predict the intrinsic permeability,  $K$  (unit  $\text{m}^2$ ), knowing that:

$$k = K \gamma_w / \mu_w = K \rho_w g / \mu_w = K \rho_w / \nu_w \quad [2]$$

where  $\gamma_w$  is the unit weight of water ( $\gamma_w = g \rho_w$ ) and  $\nu_w$  the kinematic viscosity of water ( $\mu_w = g \nu_w$ ).

According to classical soil mechanics textbooks (e.g. Taylor 1948, Lambe and Whitman 1969), the Kozeny-Carman equation is approximately valid for sands, and is not valid for clays. The same opinion appears also in classical hydrogeology textbooks (e.g. Freeze and Cherry 1979; Domenico and Schwartz 1990).

In practice, eq. [1] is not frequently used. The reason seems to lie in the difficulty to determine the soil specific surface that can be either measured or estimated. Several methods are available for measuring the specific surface (e.g. Dallavale 1948, Dullien 1979, Lowell and Shields 1991) but they are not commonly used in soil mechanics and hydrogeology. In addition, such methods seem accurate only for granular soils with few non-plastic fine particles. These practical difficulties may explain why the KC predictive equation is not commonly used.

Chapuis and L egar e (1992) proposed a method for estimating the specific surface of a non-cohesive soil from its complete grain size curve. This method is used herein to evaluate the capability of the KC equation to predict the soil  $k$ -value. Many laboratory test results were gathered for the evaluation. They were taken from publications that provided all the information needed for this evaluation: void ratio, and either a measured specific surface for a cohesive soil or the complete gradation curve for a non-cohesive soil. The report presents successively (1)

some background on the Kozeny-Carman equation, (2) the results that various authors presented to validate or invalidate this equation, and an analysis of their argumentation, (3) the test results that are used in the present evaluation, (4) the method to estimate the specific surface from the gradation curve, and (5) the comparison of measured and predicted  $k$ -values.

## Background

### *Original developments*

Kozeny (1927) developed a theory for a series of capillary tubes of equal length and obtained the following equation (quotation with the original notations):

$$v = \gamma(I/\mu)c(p^3/\sigma_1^2) \quad [3]$$

where  $v$  was the Darcy velocity,  $\gamma$  the unit weight of the fluid,  $I$  the hydraulic gradient,  $\mu$  its viscosity,  $c$  a geometric constant,  $p$  the porosity of the material and  $\sigma_1$  its specific surface expressed in squared meters per unit bulk volume of the porous material. Kozeny (1927) gave the values of factor  $c$  for different tube cross-sections: 0.50 (circle), 0.562 (square), 0.597 (equilateral triangle) and 0.66 (thin slot).

Carman (1937, 1938a and b, 1939) verified the Kozeny equation (eq. [3]), introduced the notion of hydraulic radius and expressed the specific surface per unit mass of solid (it does not vary with the porosity as in eq. [3]). Furthermore, Carman (1939) considered that water does not move in straight channels but around irregularly shaped solid particles. He tried to take this into account by introducing angular deviations of  $45^\circ$  from the mean straight trajectory. He proposed an equation similar to eq. [1], with  $C = 0.2$  and  $n^3/(1-n)^2$  where  $n$  is the usual notation for porosity. Note that presently, there is a preference to use  $e^3/(1+e) = n^3/(1-n)^2$  as in eq. [1]. According to Carman (1939) a factor  $C = 0.20$  gave the best fit with experimental results. This value of 0.20 included simultaneously the notions of equivalent capillary channel cross-section and tortuosity. Later, these notions were considered independently by other authors (e.g. Sullivan and Hertel 1942; Rose and Bruce 1949; Wyllie and Rose 1950, etc.).

The Kozeny-Carman equation was also used as a starting point to develop diphasic flow equations (Rose and Bruce 1949; Thornton 1949; Rapoport and Lea 1951; Wyllie and Spangler 1952; Wyllie and Gardner 1958a and 1958b, etc.). Some authors, following Sullivan and Hertel (1942), have replaced the specific surface term,  $S^2$ , by a term  $d_m^2$  where  $d_m$  is the pore diameter of the equivalent capillary. Some textbooks (e.g. Freeze and Cherry 1979; Domenico and Schwartz 1997) present the KC equation with  $d_m^2$  instead of  $S^2$  in eq. [1], sometimes calling  $d_m$  a representative grain size, without any indication of how to calculate this equivalent diameter.

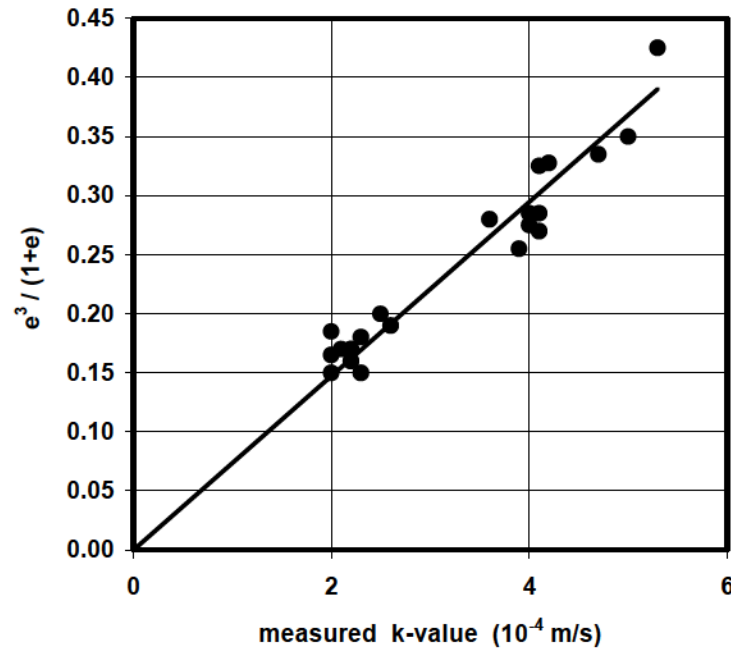
To conclude this brief history of the development of the Kozeny-Carman equation, it is worth mentioning that Kozeny (1927) proposed his equation in German without knowledge of the previous and somewhat similar works by Blake (1922), whereas a few years later, Fair and Hatch (1933) proposed in English a similar equation. It is also interesting to note that if many recent textbooks refer to Carman, Carman himself cited D'Arcy instead of Darcy (1856).

### *Opinion No.1: the KC equation is valid for non-plastic soils*

This opinion is widespread. The test results of Carman (1937, 1938a and b, 1939), and others he reported, clearly established the validity of the equation for materials having the size of gravel and sand, including various industrial materials. In soil mechanics, Taylor (1948) illustrated the relationship between  $k$  and  $e^3/(1+e)$  with the results reproduced in Fig. 1. He registered also a good correlation between  $k$  and  $e^2$ , a relationship previously proposed by Terzaghi (1925) for



clays. However, Taylor (1948) did not use the complete formulation of the KC equation with the specific surface and did not evaluate the value of the constant,  $C$ .



**Figure 1: Relationship between  $k$  and  $e^3 / (1+e)$  for a sand, according to Taylor (1948).**

It should be remembered that Taylor, Kozeny and Carman were not interested in hydraulic conductivity for the same reasons. For Terzaghi (1925, 1943) and Taylor (1948), a relationship between  $k$  and  $e$  enables a passage from a value  $k_1(e_1)$  – measured at a void ratio  $e_1$  or at a given dry density – to another value  $k_2(e_2)$  of that same soil densified at  $e_2$ . For Kozeny (1927) and Carman (1937), the air or water permeability test was used to determine the specific surface,  $S$ , of industrial powders. At a time when the determination of  $S$  by other methods was too slow (over 24 h) and inaccurate, a 30-min air permeability test provided a fast method to control the "quality" of an industrial powder.

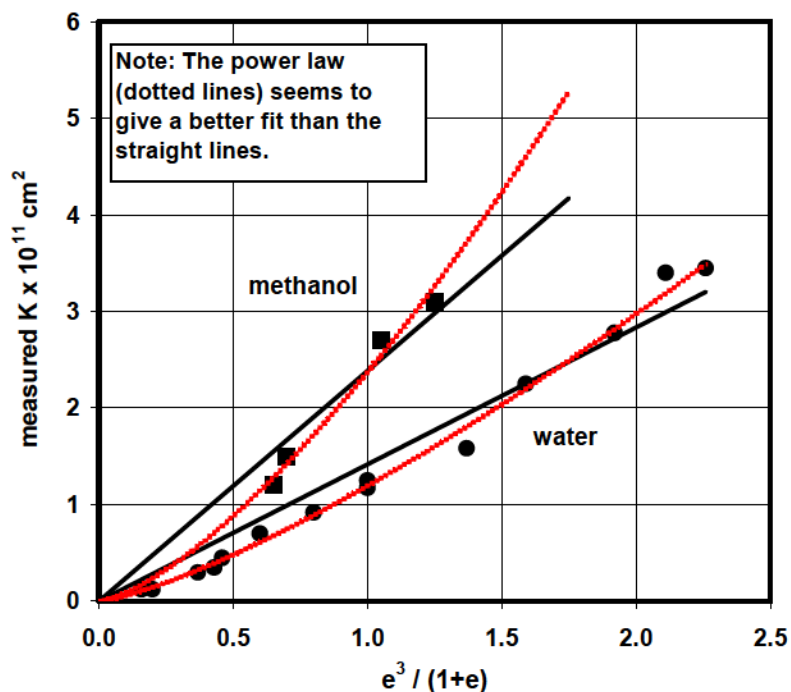
***Opinion No.2: the KC equation is inadequate for clays***

This opinion is also widespread. Using test results from Terzaghi (1925) and Zunker (1932) for natural clays undergoing consolidation, Carman (1939) found that the experimental ratio  $k(1-n)^2 / n^3$  was not a constant but rather a decreasing function of porosity  $n$ . Thus, he concluded that clays do not obey eq. [1]. Carman (1939) ascribed the divergence to a thin water layer that would be immobilized at the surface of clayey particles. He calculated the water thickness that would be required to explain the divergence from the equation. He obtained a thickness of 72 Å for the clay tested by Zunker (1932), and of 103, 110 and 99 Å respectively for the three clays tested by Terzaghi (1925).

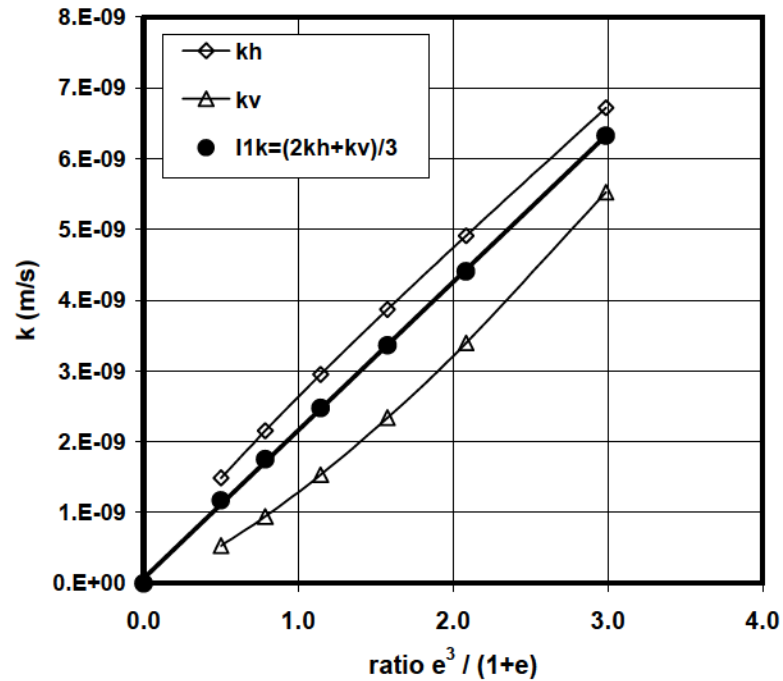
In soil mechanics, Taylor (1948, section 6.13) adopted Carman's opinion (1937) but he also wrote that there was no method to calculate the motionless water film thickness. In addition,

Taylor (1948) gave experimental results to show a linear correlation between  $\log k$  and  $e$  for fine-grained soils and suggested that the dispersion of experimental results probably came from differences in the degree of saturation and internal structure of compacted clays.

Michaels and Lin (1954) showed that most soils give a linear correlation between  $\log k$  and  $e$ . They presented test results for a kaolinite powder and different fluids used to form the clay that was later tested with the same fluid. Parts of their results are given in Lambe and Whitman (1969). Michaels and Lin (1954) tried to verify whether the KC equation was valid for clays, and more specifically whether the intrinsic permeability,  $K$  (eq. [2]), was really a geometric property of pores as assumed in the equation. Their results, for water and ethanol only, are reproduced in Fig. 2. These indicated (1) that the linear relationship between  $K$  and  $e^3/(1+e)$  was not well verified, and (2) that the intrinsic permeability depended on the type of fluid. Consequently, other properties of the fluid, such as its polarity, and characteristics of the solid-fluid interface, should be considered to obtain a more general predictive equation for clays (e.g. Bardon and Jacquin 1968; Goldman et al. 1990). Later, Al-Tabbaa and Wood (1987) used more recent testing equipment and methods to get permeability values for kaolinite percolated by water in the vertical and horizontal directions. According to their results (Fig. 3), the directional  $k$ -values are not linearly correlated to the ratio  $e^3/(1+e)$ , however the first invariant defined as  $I_{1k} = (2k_h + k_v) / 3$  of the  $k$ -matrix (where  $k_h$  and  $k_v$  are the horizontal and vertical hydraulic conductivities respectively) is linearly correlated to  $e^3/(1+e)$  as predicted by the KC equation. The variations of this first invariant and the anisotropy ratio  $k_h / k_v$  were examined by Chapuis et al. (1989b) for sand, and by Chapuis and Gill (1989) for sand, clay and sandstone, as a function of the compaction mode (or stress history) and void ratio.



**Figure 2: Intrinsic permeability of kaolinite to water and methanol (from Michaels and Lin 1954).**



**Figure 3: Hydraulic conductivity of kaolin (from results of Al-Tabbaa and Wood 1987).**

In addition, Lambe and Whitman (1969) demonstrated the influence of micro- and macro-structures on the hydraulic conductivity of fine grained soils, as already mentioned by Terzaghi (1922), after testing specimens compacted either dry or wet of the optimum Proctor. Since the frequently quoted paper of Mitchell et al. (1965), this issue has been widely studied in relation with clay liners for environmental projects (e.g. Chapuis 2002).

#### ***Comments on the usual opinions***

The previously mentioned common opinions about the KC equation have been based on partial verifications, usually without any independent measurement of the specific surface. According to published results, it appears that the KC equation has been only approximately verified, despite having a sound theoretical basis. The problem arises mainly with clayey particles because solid-fluid interactions are not considered in the equation. Furthermore, the  $k$  value predicted by this equation is isotropic because it involves only scalar parameters, whereas permeability is often anisotropic (Chapuis et al. 1989b). This may be sufficient to seriously limit the predictive capacities of the equation, as well as those of other similar equations.

### **Test results and analysis**

#### ***Identification***

About 300 laboratory test results (many of them taken from the literature) were used to evaluate the capacity of the Kozeny-Carman equation to predict the  $k$ -value. The selected references that reported the results usually gave all the required information, i.e. void ratio and either the specific surface as measured for cohesive soils, or the complete grain size curve for non-cohesive soils.

The test results used here (see Table 1) were taken from Mavis and Wilsey (1937) for five

sands (Ottawa, Iowa, pit-run Iowa, uniform Iowa, and non-uniform Iowa); Morris and Johnson (1967) for over twenty soils; Loiselle and Hurtubise (1976) for various non-plastic tills; École Polytechnique and Terratech for the James Bay Corporation before 1983 on various tills; Chapuis et al. (1989b) for a sand ( $k_h$  and  $k_v$ , two compaction modes); Mesri and Olson (1971) for three clays (smectite, illite and kaolinite); Olsen (1960) for three clays (kaolinite, illite, Boston blue clay); Navfac DM7 (1974) for clean sands and gravels; Tavenas et al. (1983b) for Champlain sea clays of St-Zotique, St-Thuribe and St-Alban. In addition, the authors have used several of their own test results on homogenized mine tailings (e.g. Aubertin et al. 1993, Bussière 1993), other unpublished results for sands, silts and tills from Quebec, and also sand-smectite mixes with high percentages of smectite (Chapuis 1990, 2002).

**Table 1: Data examined in this report**

Soil	Reference	method for S
five sands	Mavis & Wilsey (1937)	Chapuis & Légaré
sand and gravel	Navfac DM7 (1974)	Chapuis & Légaré
non cohesive soils	Morris & Johnson (1967)	Chapuis & Légaré
non-plastic tills	Loiselle and Hurtubise (1976)	__ or Fig.4 or BET
sand	Chapuis et al. (1989b)	Chapuis & Légaré
three clays	Mesri and Olson (1971)	provided by authors
three clays	Olsen (1960)	provided by authors
Champlain clays	Tavenas et al. (1983b)	Locat et al.
sand-smectite	Chapuis (1990, 2002)	Olsen
mine tailings	Bussière (1993)	Chapuis & Légaré
sands, silts & tills	authors' data (unpublished)	Chapuis & Légaré

### *Estimates of specific surface for non-plastic soils*

The specific surface  $S$  of a soil is seldom evaluated (and used) in soil mechanics and hydrogeology. However, it is an essential parameter for bituminous mixes, to verify whether the solid particles are adequately coated with bitumen. In such mixes, the filler is the major contributor to the specific surface. Usual methods to evaluate  $S$  are approximate and often based on local experience. Various simple predictive equations are available (e.g. Hveem 1974; Duriez and Arrambide 1962; standard Can/Bnq-2300-900). Craus and Ishai (1977) proposed a relatively complex analytical method. This lengthy method introduces a shape factor that is visually evaluated under a microscope. It depends on the operator and can be seen as a "fudge factor" to obtain a better fit between predicted and measured  $S$  values.

Chapuis and Légaré (1992) proposed an operator-independent method that was compared with four other methods. It assumes that simple geometric considerations can be used to estimate the specific surface of a non-plastic soil. If  $d$  is the diameter of a sphere or the side of a cube, the specific surface  $S$  of a group of spheres or cubes is given by:

$$S(d) = 6 / d \rho_s \quad \text{in m}^2/\text{kg} \quad [4]$$

where  $\rho_s$  is the density ( $\text{kg}/\text{m}^3$ ) of the spheres or cubes. Starting with eq. [4], many theoretical developments have been proposed to better define the  $S$  value of real particles. These introduced

shape factors, roughness factors, or projection factors (e.g. Dallavale 1948; Orr and Dallavale 1959; Gregg and Sing 1967). In the case of fine-grained non-plastic soils, such as fillers used in bituminous mixes, Chapuis and Légaré (1992) have proposed to apply eq. [4] as follows:

$$S = (6/\rho_s) \Sigma [(P_{No D} - P_{No d}) / d] \quad \text{in m}^2/\text{kg} \quad [5]$$

where  $(P_{No D} - P_{No d})$  is the percentage by weight smaller than size  $D$  ( $P_{No D}$ ) and larger than next size  $d$  ( $P_{No d}$ ). Equation [5] was applied to the five fillers used as references by Craus and Ishai (1977) for which the specific surface had been measured according to the standard method (ASTM C 204 2002) based on the work of Blaine (1941) and of Ober and Frederick (1959). Table 2 illustrates how to use the complete grain size curve of the limestone filler to calculate its specific surface,  $S$

The grain size curves always have a minimum measurable particle size,  $D_{min}$ , e.g. 5  $\mu\text{m}$  for the filler of Craus and Ishai (1977) in Table 2. In the method of Chapuis and Légaré (1992), an equivalent size,  $d_{eq.}$ , must be defined for all particles smaller than the minimum size for the curve. This equivalent size corresponds to the mean size with respect of the specific surface. It is given by:

$$d_{eq.}^2 = \frac{1}{D_{min}} \int_0^{D_{min}} y^2 dy = \frac{D_{min}^2}{3} \quad [6]$$

When the minimum size,  $D_{min}$ , is 5  $\mu\text{m}$ , the equivalent diameter,  $d_{eq.}$ , is 2.9  $\mu\text{m}$  (see Table 2).

**TABLE 2 -- Specific surface ( $\text{m}^2/\text{kg}$ ) of a limestone filler  
( $\rho_s = 2880 \text{ kg}/\text{m}^3$ ); gradation curve from Craus and Ishai (1977).**

Size (mm)	Cumulative passing (%)	Difference $X$ ( $P_{No D} - P_{No d}$ )	$S = 6/d\rho_s$ $\text{m}^2/\text{kg}$	$XS$ $\text{m}^2/\text{kg}$
0.074	100	----	-----	----
0.060	94	0.06	34.72	2.08
0.050	89	0.06	41.67	2.08
0.040	83	0.06	52.08	3.13
0.030	76	0.07	69.44	4.86
0.020	65	0.11	104.17	11.46
0.010	45	0.20	208.33	41.67
0.005	24	0.21	416.67	87.50
$d_{eq.} = 0.0029$		0.24	718.39	172.41
			Specific surface $S$ ( $\text{m}^2/\text{kg}$ ) =	325.2

Notes : 1. The value of  $S$  is obtained as  $\Sigma(XS) = 325.2 \text{ m}^2/\text{kg}$ .  
2. The equation giving "d equivalent" is provided in the text.

This method can be applied to fillers because they have no plasticity (Langlois et al. 1991) and their finest particles ( $< 5 \mu\text{m}$ ) are mainly inactive rock flour. Table 3 gives the values of  $S$  as calculated with four different methods. It appears that the proposed method (eqs. [4-6]), without using any visually estimated shape factor, correctly evaluates the specific surface of non-plastic fine powders, except for hydrated lime for which the grain size curve is not easy to obtain by

sedimentation. The method of Eqs. [4-6] was used to estimate the specific surface of non-plastic soils to be used in the KC equation.

**TABLE 3 -- Estimated specific surfaces ( $m^2/kg$ ) for fillers  
(from Chapuis and Légaré 1992)**

Filler type	Predicted values of $S$ ( $m^2/kg$ )				
	S measured ASTM C 204	P-1 1977	P-2 1992	P-3 1962	P-4 1962
Limestone	263	258	325	600	346
Hydrated lime	869	750	615	1104	553
Glass beads	86	78	98	120	72
Dolomite	202	183	206	324	195
Basalt	247	217	247	420	240

Notes: P-1 = Craus and Ishai (1977)

P-2 = Chapuis and Légaré (1992)

P-3 and P-4 = Duriez and Arrambide (1962, tome 1, p.288)  
for fillers classified as very fine or fine

#### *Estimation of specific surface for tills and Champlain clays*

In the case of cohesive soils, authors have usually provided the specific surface, except for a few test results on Champlain clays (Tavenas et al. 1983b). Similarly, several test results for tills (13 reports), provided by the James Bay Corporation, did not include data on specific surface. For applying the KC equation to these soils, the specific surfaces of Champlain clays and Quebec tills were determined as follows. Locat et al. (1984) estimated specific surfaces of several Quebec clays using the methylene blue method (Tran 1977). Figure 4 plots the estimated  $S$  versus the percentage of particles smaller than  $2 \mu m$  as obtained by sedimentation. It appears that tested clays with a low plasticity ( $8 < IP < 15$ , where  $IP$  is the plasticity index) have a specific surface  $S$  between  $23$  and  $30 \times 10^3 m^2/kg$ , independently of the percentage of particles smaller than  $2 \mu m$  (see the two horizontal lines in Fig. 4). This is the case for clays of the Great Whale River, Shawinigan, Chicoutimi and Outardes. This finding was used to estimate the specific surface of several tested tills having a low plasticity or no plasticity: for the coarse fraction down to  $2 \mu m$ ,  $S$  was calculated by the method of Chapuis and Légaré (1992) and, a  $S$  value of  $27 \times 10^3 m^2/kg$  was attributed to the fraction smaller than  $2 \mu m$ . It can be seen also in Fig. 4 that the  $S$  values provided by Locat et al. (1984) for Champlain and North-West Quebec clays fall within sloping lines in Fig. 4. Similar zones may be defined in Fig. 5 where  $S$  is plotted versus the sum [ $IP + (\% < 2 \mu m)$ ]. The  $S$ -values of Champlain clays tested by Tavenas et al. (1983b) were given by Locat et al (1984), for example for St-Alban, or evaluated using Figs. 4 and 5, for example for St-Zotique. In this case, the initial void ratio,  $e$ , was close to 2.5 and the natural water content,  $w$ , was close to 91% whereas  $IP = 36\%$  and  $(\% < 2 \mu m) = 80$ . The specific surface  $S$  was then estimated from Figs. 4-5 as  $S = 61 \pm 5 \times 10^3 m^2/kg$ .

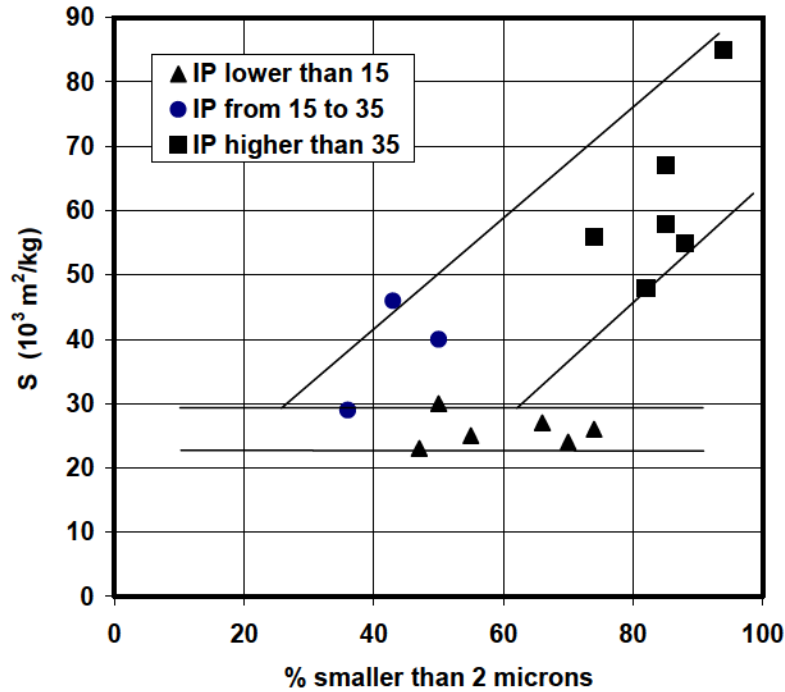


Figure 4: Correlation between specific surface  $S$  and percentage smaller than  $2 \mu\text{m}$  (results of Locat et al. 1984).

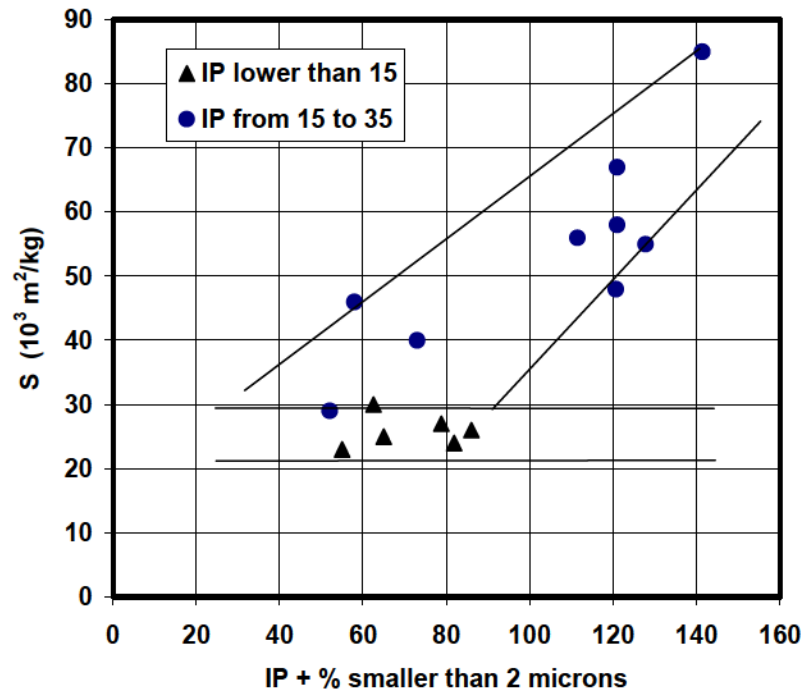
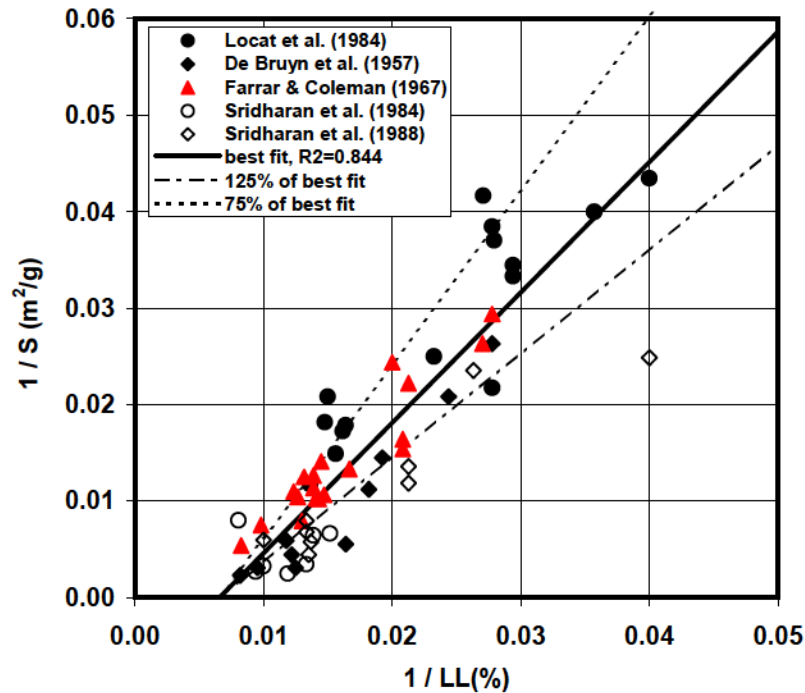


Figure 5: Correlation between measured specific surface and the sum of plasticity index plus the percentage of fines smaller than  $2 \mu\text{m}$ .

More generally, the specific surface of any clay,  $S$ , may be assessed from its liquid limit,  $LL$  (e.g. Muhunthan 1991). The results of De Bruyn et al. (1957), Farrar and Coleman (1967), Locat et al. (1984), and Sridharan et al. (1984, 1988), have been gathered in Fig. 6 to illustrate that there is an approximately linear correlation between  $1/S$  and  $1/LL$ . The best fit straight line ( $R^2 = 0.88$ ) of Fig.6 corresponds to the equation:

$$1/S \text{ (m}^2\text{/g)} = 1.3513 (1/LL) - 0.0089 \quad [7]$$

when the liquid limit,  $LL$ , is lower than 110. A power law function of  $LL$  for  $S$  could also be used (Mbonimpa et al. 2002) but provides basically the same estimate of  $S$  for  $LL$  values. Equation [7] usually predicts an  $S$  value within  $\pm 25\%$  of the measured value when  $1/LL > 0.167$  ( $LL < 60\%$ ) as shown in Fig. 6. Poorer predictions are achieved using eq. [7] for soils with  $LL > 60\%$ , especially clayey soils containing some bentonite. Taking again the example of St-Zotique that had an  $LL = 61\%$ , eq. [7] predicts  $S = 75 \pm 19 \times 10^3 \text{ m}^2\text{/kg}$  which is close to but slightly higher and more inaccurate than the  $S$  values predicted from Figs 4 and 5.



**Figure 6: Correlation between the inverse of the specific surface,  $S$ , and the inverse of the liquid limit,  $LL$ , of clays.**

### Application and evaluation

It was mentioned earlier that previous evaluations of Kozeny-Carman equation were partial and usually limited to the  $k-e$  component of the relationship. A relatively thorough evaluation is presented hereafter. For that purpose, the next figures present  $\log [k / 1\text{m/s}]$  versus  $\log [e^3 / D_R^2 S^2 (1+e)]$ . According to Eq. [1], the experimental data should verify the following relationship:

$$\log [k / (1\text{m/s})] = A + \log [e^3 / D_R^2 S^2 (1+e)] \quad [8]$$

where  $A = 0.29$  to  $0.51$  for a  $C$  value (see eq.1) between  $0.2$  and  $0.5$  as suggested by Carman (1939). The figures presented below illustrate the correlations that have been obtained for



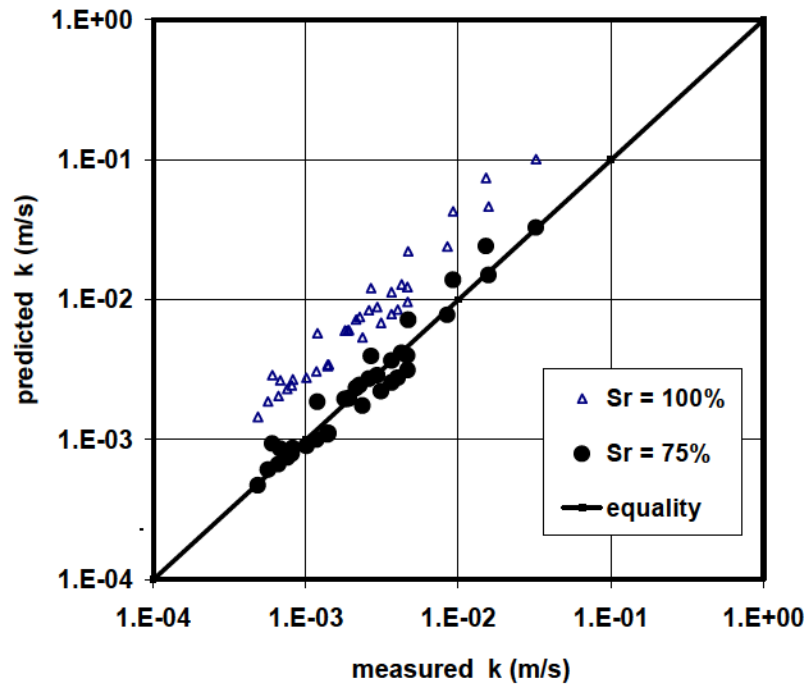
several soil types using a factor  $A = 0.5$ , resulting in:

$$\log [k_{\text{predicted}} / (1\text{m/s})] = 0.5 + \log [e^3 / D_R^2 S^2 (1+e)] \quad [9]$$

where  $k_{\text{predicted}}$  is in m/s,  $D_R$  and  $e$  have no dimension and  $S$  is in  $\text{m}^2/\text{kg}$ .

### *Sand and gravel*

Results for sand and gravel are presented in Figs. 7 to 9. Their  $S$ -values were determined using the method of Chapuis and Légaré (1992). The predicted  $k$ -values were obtained using Eq. 9. Measured  $k$ -values are in the range  $10^{-1}$  to  $10^{-5}$  m/s. Most data for the sands of Mavis and Wilsey (1937) are aligned in a narrow band along Eq. [9] (Fig. 7).



**Figure 7: Predicted versus measured  $k$ -values for the sands of Mavis and Wilsey (1937)**

The position of any point in Fig. 7 depends mainly on two factors: the uncertainty in the values of specific surface,  $S$ , and degree of saturation,  $S_r$ , of the tested specimen. For coarse (non-plastic) soils, the uncertainty due to  $S$  should not exceed 10%, thus 20% for  $S^2$ , which represents an uncertainty  $\Delta y = \pm 0.08$  cycle in Fig. 7. The uncertainty due to  $S_r$  is deemed higher, because the experimental  $k$ -values of Fig. 7 were obtained in rigid-wall permeameters, where usually  $S_r$  is unknown. The current standard for this test is ASTM D2434 (2002), in which "saturation" is supposed to have been obtained after using a vacuum pump. This standard procedure, however, does not provide any means to check whether the soil specimen is fully saturated (degree of saturation  $S_r = 100\%$ ) or not. Such a method was proposed by Chapuis et al. (1989a), who have established its accuracy and shown that if a rigid-wall permeameter is used (without the recently defined precautions),  $S_r$  usually lies between 75 and 85%. Then the measured  $k$  value represents only about 15 to 30% of  $k$  ( $S_r=100\%$ ), which results in an underestimate of  $x$ ,  $\Delta x = -0.5$  to  $-0.8$  in Fig. 7. Such a condition could well apply to the results of Mavis and Wilsey (1937). The measured unsaturated hydraulic conductivity  $k(S_r)$  may be

evaluated by several equations. Here a simplified equation proposed by Mualem (1976) is retained:

$$k(S_r) / k(\text{sat}) = (S_r - S_0)^3 / (1 - S_0)^3 \quad [10]$$

in which  $S_0$  is the degree of saturation corresponding to a residual water content taken as 0.2 for sand and gravel. If a degree of saturation  $S_r = 75\%$  is considered for the tests of Mavis and Wilsey (1937), and Eq. [9] is used to predict  $k(\text{sat})$  with Eq. [10] to predict  $k(S_r)$ , it appears (Fig. 7) that the measured  $k$ -values are well-predicted by the KC equation.

To use the chart of Navfac DM7 (1974) the sand must have a coefficient of uniformity,  $C_u$ , between 2 and 12, a void ratio  $e$  between 0.3 and 0.7, a diameter  $D_{10}$  between 0.1 and 3 mm, and a ratio  $D_{10}/D_5$  lower than 1.4. The last condition means that the grain-size distribution curve of the sand cannot end with a flat portion. A flat final portion may indicate a risk of segregation and particle movement within the soil, or risk of suffosion. Such risks can be evaluated by using the criteria of Kezdi (1969), Sherard (1979) or Kenney and Lau (1985, 1986). These suffosion criteria have been shown to be mathematically similar and they can be replaced by minimum values for the secant slope of the grain-size distribution curve (Chapuis 1992, 1995). Eight sands were defined by straight-line grain size curves using eight values of  $D_{10}$  (0.2-0.3-0.4-0.5-0.6-0.8-1.0-1.5 mm) and a coefficient of uniformity of 7 which represents the means of 2 and 12 that are the limits of the chart. The  $S$ -values of the eight sands were estimated using the method of Eqs. [4-6]. The KC equation predicts saturated  $k$ -values (for  $S_r = 100\%$ ) that are usually higher than the measured  $k$ -values (Fig.8). A better agreement between predicted and measured  $k$ -values is obtained using the Mualem equation (Eq. [10]) and assuming a  $S_r$ -value of about 85% for these tests.

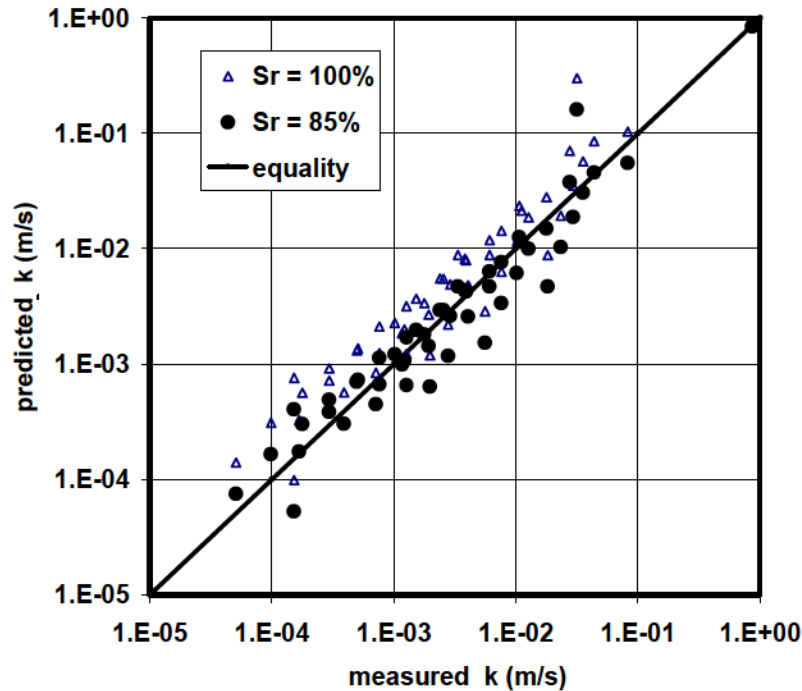
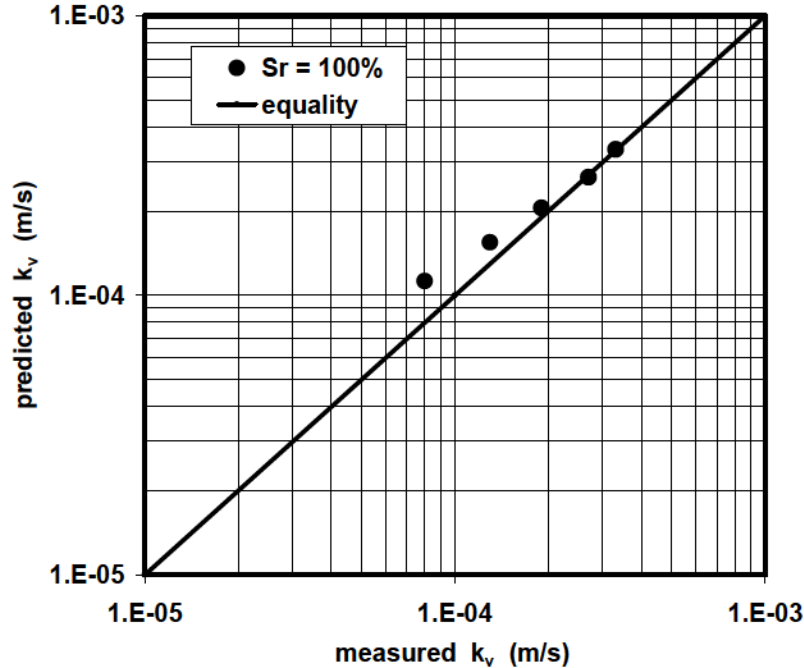


Figure 8: Predicted versus measured  $k$ -values for the sands of Navfac DM7 (1974)

In the tests of Figs 7 and 8, the soil specimens were most probably not fully saturated ( $S_r = 75$  to 85%). The more recent tests of Chapuis et al. (1989a), however, were designed to ensure full saturation of the tested sand as checked by a mass and volume method. Here the KC equation predicts saturated  $k$ -values that are close to the measured  $k$ -values at  $S_r=100\%$  (Fig. 9).



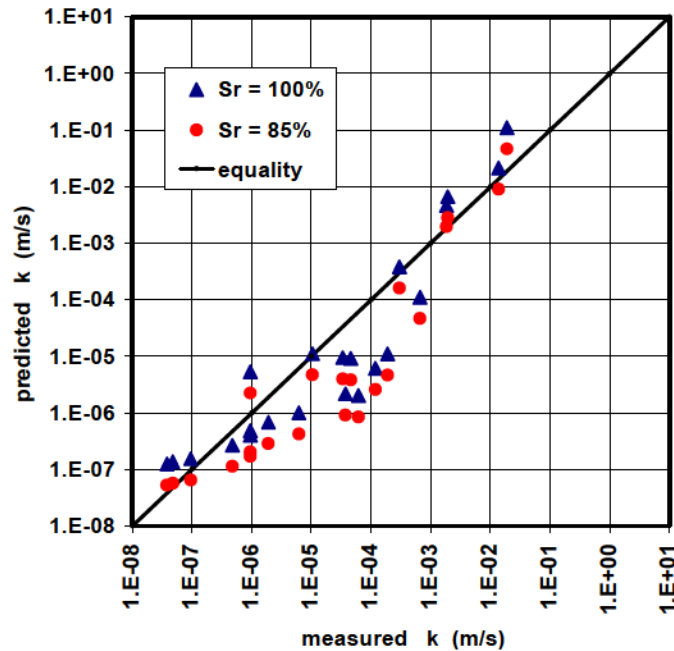
**Figure 9: Predicted versus measured  $k$ -values for the sand tested by Chapuis et al. (1989a) in the vertical direction after static compaction.**

The results in Figs 7 to 9 confirm that the KC equation (Eq. [9]) provides a fair estimate of the vertical hydraulic conductivity,  $k_v$ , of saturated sand and gravel. When the specimens are not fully saturated ( $75\% < S_r < 100\%$ ), Eq. [9] can be used jointly with Eq. [10] to estimate the  $k(S_r)$ -value.

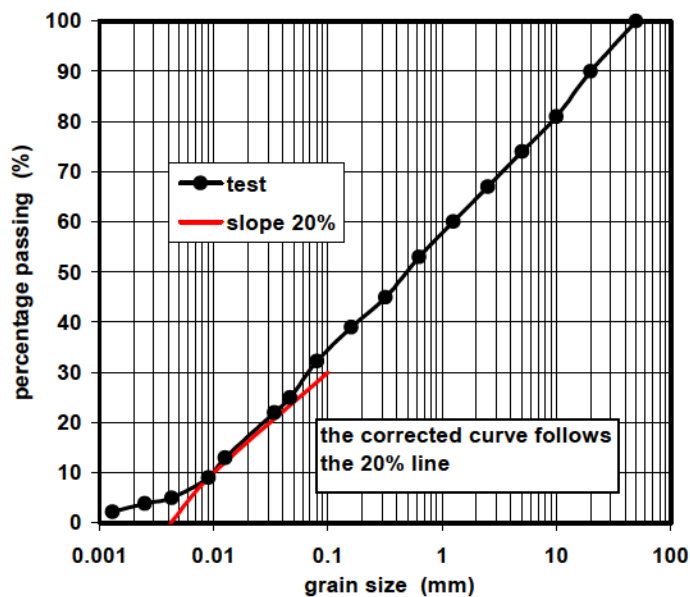
#### ***Other non cohesive soils***

Other data provided by Morris and Johnson (1967) for sands and silty sands are examined hereafter. The specific surface  $S$  of these non cohesive soils was established using the method of Chapuis and Légaré (1992). When the complete grain-size distributions are used to assess  $S$ , then the predicted  $k$ -values (Eq. [9]) are relatively dispersed around the equality line ( $y = x$ ) for  $S_r$  between 85 and 100% (Fig. 10). This dispersion in Fig. 10 may be due to the fact that the grain-size distribution of several soils specimens had a slope lower than 20% in the fine size zone, thus indicating a risk of segregation and particle movement or suffosion (Chapuis 1992, 1995). It also indicates that such specimens were probably formed artificially by mixing several soil layers that may have been naturally adjacent but not mixed. In such cases, the fine particles that can move with water do not really belong to the solid skeleton restraining the water seepage, and thus should not be considered in the  $S$ -value that contributes to the  $k$ -value of such soils. Consequently, the grain-size distribution of these soils was modified to follow a minimum slope of 20% as shown in Fig. 11. New specific surfaces for these soils were obtained from their modified gradation curves and used to predict their new  $k$ -values (Eq. [9]) that are compared with

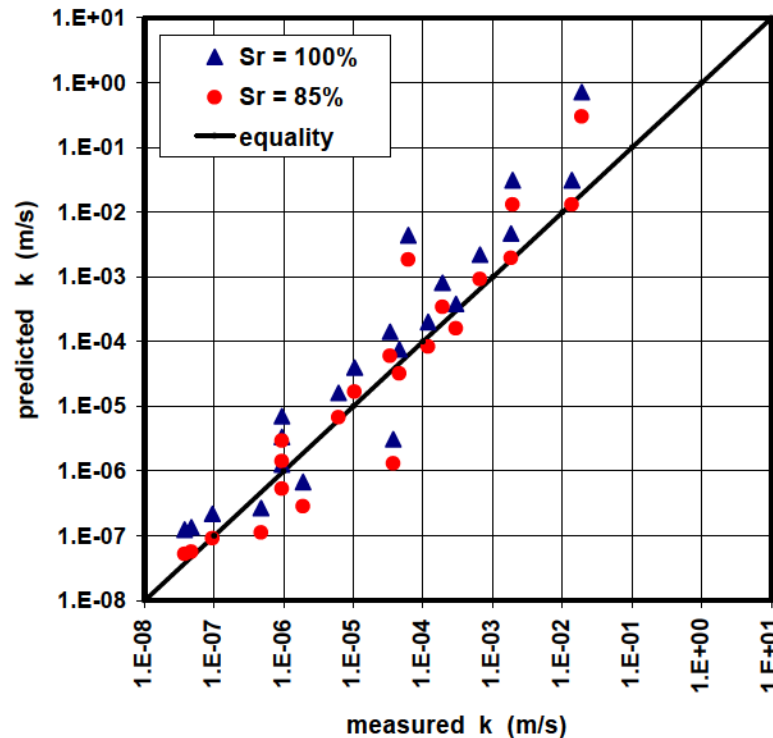
the measured  $k$ -values in Fig. 12. Now the comparison is better, although for several tests the difference between predicted and measured  $k$ -values exceeds one order of magnitude. Again, the discrepancy may be attributed to incomplete saturation in rigid-wall permeameter tests and to gradation curves with risks of suffosion.



**Figure 10: Predicted versus measured  $k$ -values for the non-cohesive soils tested by Morris and Johnson (1967). The complete particle gradation curve was used to calculate the specific surface,  $S$ .**



**Figure 11: Example of a slope flatter than 20% (per cycle) and modification of the gradation curve to take into account the degree of freedom of fine particles.**



**Figure 12: Predicted versus measured  $k$ -values for the non-cohesive soils tested by Morris and Johnson (1967). The modified particle gradation curve (Fig.11) was used to calculate  $S$ .**

### *Tills and silty sands of Quebec*

When the specific surface  $S$  of such soils can be adequately evaluated as discussed before, Eq. [9] gives a good prediction of the  $k$ -value as shown in Fig. 13. When the  $S$ -value is not accurately evaluated, the prediction is not so good as shown below.

For the Quebec tills tested for Hydro-Quebec by Loiselle and Hurtubise (1976), the specific surface  $S$  was assessed by three methods. The first method was to estimate  $S$  using the complete gradation curve and the method of Chapuis and Légaré (1992) for non-plastic soils. In the second method, the 1<sup>st</sup> method was used only for the fraction coarser than 2  $\mu\text{m}$ , and then the fraction smaller than 2  $\mu\text{m}$  was assumed to have a specific surface of  $27 \times 10^3 \text{ m}^2/\text{kg}$ . Thus it was assumed that the fine fraction of tills from James Bay and Outardes was somewhat similar to the low plasticity clays of these regions, for which Locat et al. (1984) measured a specific surface between 23 and  $30 \times 10^3 \text{ m}^2/\text{kg}$  (see Figs. 4 and 5). In the third method, the 1<sup>st</sup> method was used only for the fraction coarser than 0.63 mm, and then the fraction smaller than 0.63 mm was assumed to have a specific surface of  $1.7 \times 10^3 \text{ m}^2/\text{kg}$ . This value was obtained using the BET (Brunauer-Emmett-Teller) method for a few till specimens of the Laurentides area (North of Montreal), having a plasticity index,  $IP$ , lower than 5 and grain size curves similar to those tested by Loiselle and Hurtubise (1976). Only the fraction smaller than 0.63 mm was tested using the BET method (currently considered as the best method to evaluate  $S$  for fine-grained soils), and its  $S$ -value was always close to  $1.7 \times 10^3 \text{ m}^2/\text{kg}$ .

Predicted and measured  $k$ -values for the tills tested by Loiselle and Hurtubise (1976) are gathered in Fig. 14. The 1<sup>st</sup> method, based only on gradation (down to approximately 1.3 microns), predicts a  $k$ -value that is usually 3 to 10 times higher than the measured  $k$ -value. The

2<sup>nd</sup> method, based on a specific surface of  $27 \times 10^3 \text{ m}^2/\text{kg}$  for the fraction smaller than  $2 \mu\text{m}$ , predicts a  $k$ -value that is usually 3 to 10 times lower than the measured  $k$ -value. The 3<sup>rd</sup> method, based on the BET specific surface of  $1.7 \times 10^3 \text{ m}^2/\text{kg}$  for the fraction smaller than  $0.63 \text{ mm}$ , predicts more correctly the  $k$ -value than the two previous methods. Thus the BET method (real measurements) gave better predictions for the  $k$ -value than the 1<sup>st</sup> and 2<sup>nd</sup> methods that are estimates based on assumptions.

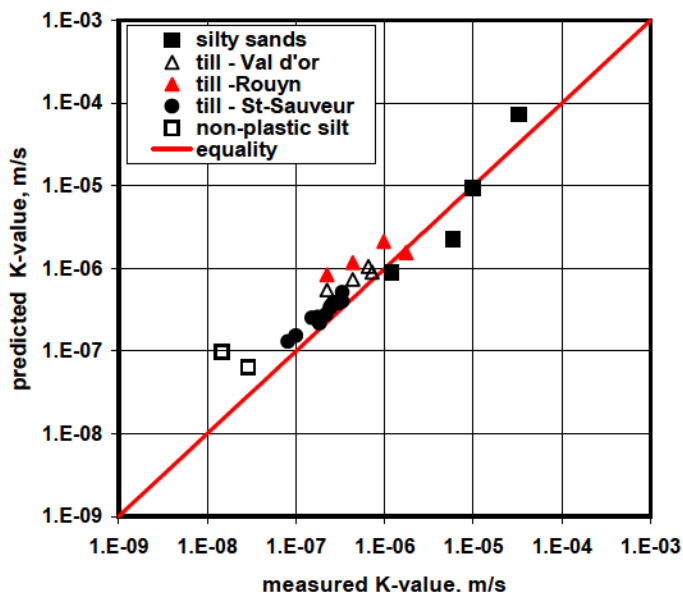


Figure 13: Predicted versus measured  $k$ -values for non-plastic tills and silty sand specimens (authors results).

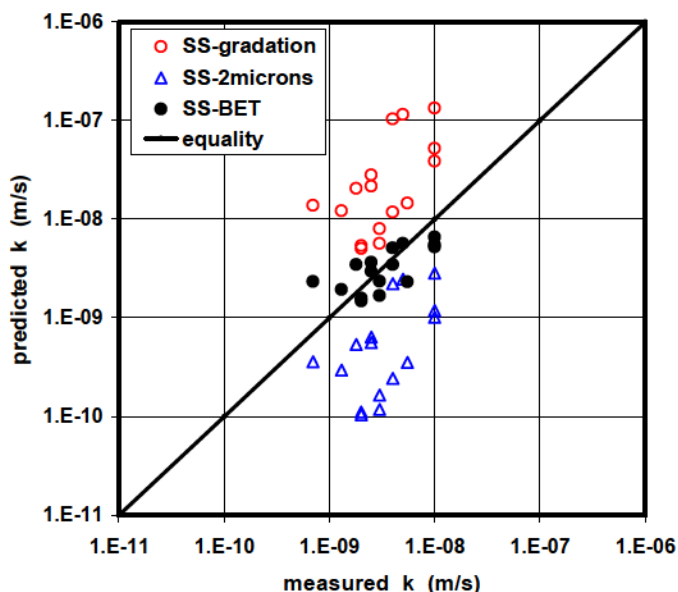


Figure 14: Predicted versus measured  $k$ -values for the tills tested by Loiselle and Hurtubise (1976).

Several other northern tills tested for SEBJ (James Bay Corporation) had fines with a plasticity index  $IP$  higher than 5, but the  $IP$  values of the tested specimens were not reported. Thus it was impossible to estimate the  $S$  value and then to correlate predicted and measured  $k$ -values. When the  $IP$  is higher than 5, these tills have an  $S$  value that is much higher than that of tills tested by Loiselle and Hurtubise (1976). As a result the measured  $k$ -values for these northern tills are lower than those of Fig. 14, and may reach  $10^{-10}$  to  $10^{-11}$  m/s, a range that may be found with the plastic tills of Southern and Eastern Quebec (Appalachian areas). To illustrate the influence of fines in the evaluation of  $S$  and  $k$ , the ratio of measured  $k$ -value over predicted  $k$ -value (1<sup>st</sup> method based on gradation only) is plotted versus the  $D_{10}$  in Fig. 15. This figure indicates that the prediction worsens when the  $D_{10}$  decreases, as anticipated, given the importance of the fine particles for the value of  $S$ . For such plastic soils, a good prediction of  $k$  requires either an independent determination of  $S$ , or the complete gradation curve and the Atterberg limits of the tested specimen. In the latter case, Figs 5 or 6 would then be used to assess  $S$  and then  $k$ .

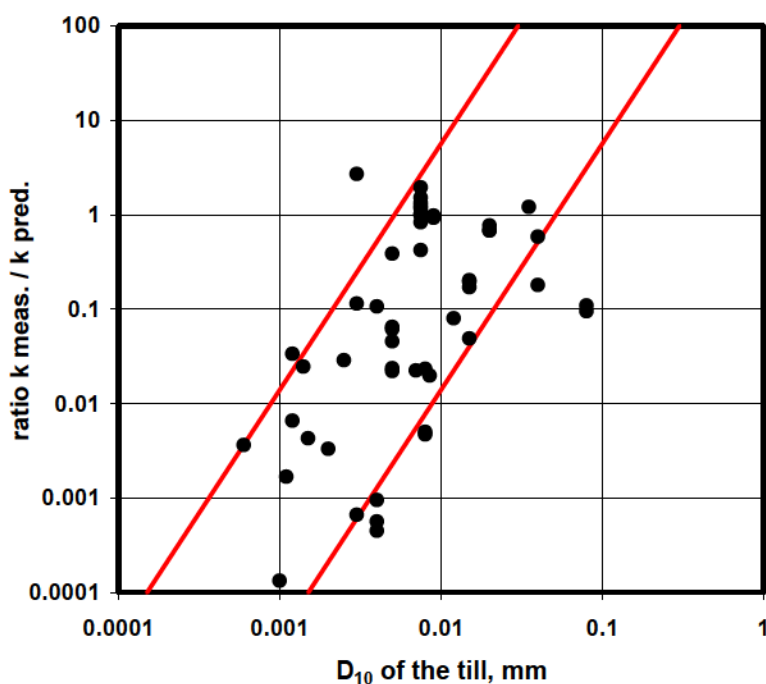


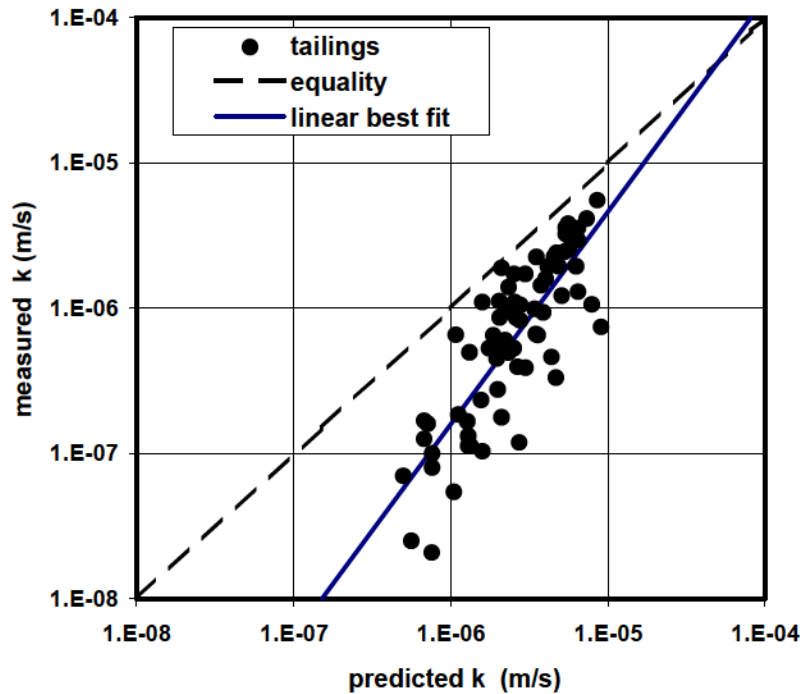
Figure 15: Ratio  $k_{\text{measured}} / k_{\text{predicted}}$  versus the  $D_{10}$  for the tills of Fig.14.

### *Mine tailings*

Many mine tailings have been tested for permeability at Polytechnique (L'Écuyer et al. 1992; Aubertin et al. 1993, 1996; Bussière 1993; Monzon 1998). These tailings are finely crushed hard rock particles, with gradations of silts, and usually no or little plasticity. Here, their specific surface has been estimated from their complete gradation curve using the method of Chapuis and Légaré (1992). A few results (Bussière 1993) are presented here for homogenized samples. Measured  $k$ -values have been obtained for fully saturated specimens tested in either rigid-wall or flexible wall permeameters. It was checked that both permeameters gave similar results.

The predicted  $k$ -values do not match the measured  $k$ -values in the case of mine tailings, as

shown in Fig. 16. The difference may be explained by several factors. First, the fine particles of tailings are angular, sometimes acicular. The void ratio,  $e$ , of tailings is usually much higher than the void ratio of silts having similar grain-size curves. As a result, the void space between the solids is not similar to the void space of a natural soil. According to numerous experiments, the measured  $k$ -value of tailings depends on a ratio  $e^{3+a} / (1+e)$  where  $a$  is positive, and on their liquid limit when it is higher than 40 (Aubertin et al. 1996; Mbonimpa et al. 2002). In addition, tailings are prone to several phenomena (Bussière 1993) such as creation of new fines during compaction (particle breakage) and chemical reactions during permeability testing. Consequently, the predicted  $k$ -value must take these phenomena into account.



**Figure 16: Predicted versus measured  $k$ -values for mine tailings.**

According to the results of Fig. 16, the best-fit linear equation can be expressed as:

$$\log k_{\text{measured}} = 1.46 \log k_{\text{predicted}} + 1.99 \quad [11]$$

Consequently, it is proposed here to predict the  $k$ -value of mine tailings as follows. The  $S$ -value is first determined using the complete gradation curve and the method of Chapuis and L egar e (1992). Then the  $k$ -value is predicted using Eq. [9] modified by Eq. [11] to give Eq. [12]:

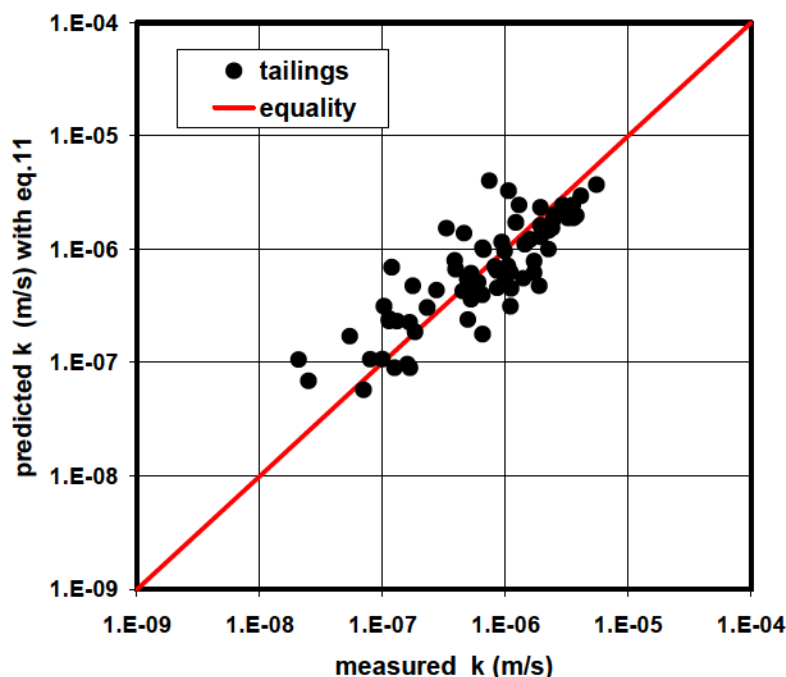
$$\log [k / 1 \text{ m/s}] = 1.46 (0.5 + \log [e^3 / D_R^2 S^2 (1+e)]) + 1.99 \quad [12]$$

The predicted  $k$ -values (using Eq. [12]) versus measured  $k$ -values for tested tailings now are fairly close as shown in Fig. 17.

Here a note of caution must be made: the  $k$ -value that can be predicted using either Eq. [12] or other equations (Aubertin et al. 1996; Mbonimpa et al. 2002) is that of homogenized tailings tested in laboratory saturated conditions. The  $k$ -value of intact saturated samples of tailings cannot be predicted by the KC equation (L ecuyer et al. 1992), because intact tailings are typically finely stratified (cm or mm scale) and have a high anisotropy in  $k$ . The KC equation



assumes that the specimen is homogeneous, and it cannot predict the  $k$ -value of a heterogeneous stratified specimen.

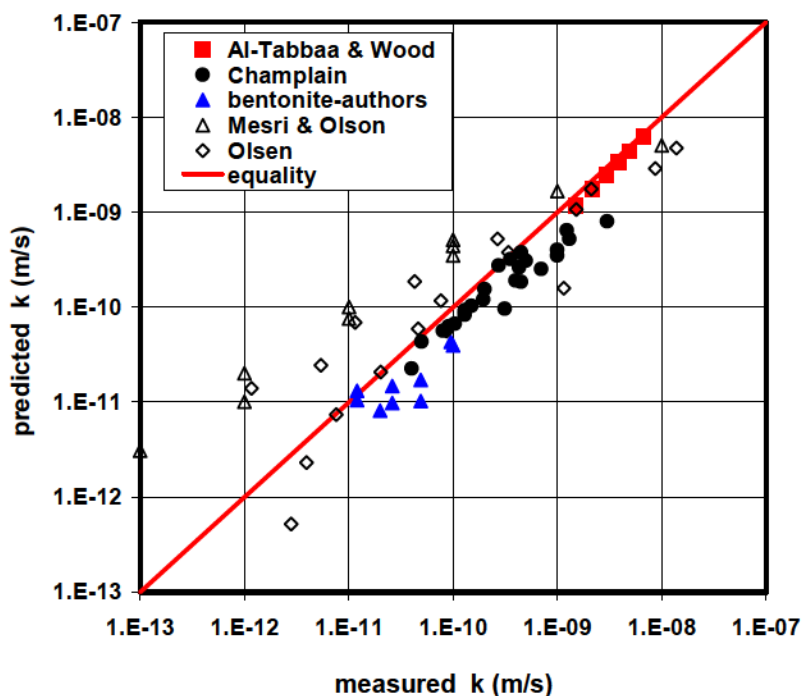


**Figure 17: Predicted versus measured  $k$ -values for mine tailings using Eq.12 with the  $S$ -value estimated from the gradation curve.**

### *Fine-grained plastic soils*

The measured  $k$ -values were provided by Mesri and Olson (1971) for smectite, illite and kaolinite, by Olsen (1960) for kaolinite, illite and Boston blue clay, by Al-Tabbaa and Wood (1987) for kaolinite, and by Tavenas et al. (1983b) for Champlain (Quebec) intact clays. The specific surface of these clays was either provided in the publications or evaluated using Figs. 4 or 5. Other measured  $k$ -values are for several soil-bentonite mixes containing a high content (over 20%) of bentonite that completely fills the pore space (Chapuis 1990, 2002), for which the specific surface of bentonite was taken as  $6 \times 10^5 \text{ m}^2/\text{kg}$ , the average value provided by Mesri and Olson (1971). The predicted versus measured  $k$ -values for all these clays are shown in Fig. 18.

In Fig. 18, the few points that are far from the equality line represent older results for smectite and illite, for which the  $k(e)$  proposed curves fall to the  $10^{-12}$  to  $10^{-13}$  m/s range. The authors obtained these  $k$  values not directly from permeability tests but indirectly from consolidation curves using Terzaghi's theory. Such an indirect method is presently known to provide unreliable  $k$ -values (Tavenas et al. 1983a). Further developments in testing techniques, better understanding of phenomena and improved accuracy (e.g. Haug et al. 1994, Hossain 1995, Tavenas et al. 1983a) as well as duration considerations for clays such as smectite (e.g. Chapuis 1990) have produced  $k(e)$  curves that do not fall below  $10^{-11}$  m/s. In Fig. 18 the more recent test data for smectite are close to the equality line. They were obtained using triaxial equipment and specimens 2-3 cm high that have given values down to 1 to  $5 \times 10^{-11}$  m/s, after very long times (2 to 4 weeks) to insure full hydration (100%) and complete consolidation or swelling of this special clay (Chapuis 1990).



**Figure 18: Predicted versus measured  $k$ -values for clays.**

The uncertainty for data shown in Fig. 18 depends mainly on four factors: the uncertainties in estimated specific surface,  $S$ , and degree of saturation,  $S_r$ , during the test, whether or not enough time was allowed to reach steady-state conditions after consolidation and/or swelling, and the thin rigidified water layer at the surface of clay particles. Several of these issues have already been discussed in the case of sand and gravel. For tested clays, the specific surface was provided by the authors and is relatively well known: it should not produce an error greater than 20%, thus 40% for  $S^2$ , which represents  $\Delta\alpha = \pm 0.15$  in Fig. 18, even for Quebec intact clays. There is little uncertainty related to the degree of saturation of tested clay specimens. It is probably close to 100% because most specimens were tested in triaxial cells with a high back-pressure to increase the saturation (Lowe and Johnson 1960; Black and Lee 1973; Daniel et al. 1984; Rad and Clough 1984, 1986; Camapum de Carvalho et al. 1986; Donaghe et al. 1986).

Considering the small uncertainty related to recent data for long testing duration, the KC equation (Eq. [9]) provides a fair estimate of the  $k$ -value of intact specimens of natural clays and of laboratory-made samples that are prepared with a clay, hydrated, saturated and consolidated before permeability testing.

However, the  $k$ -value of compacted clays (clay liners and covers) cannot be predicted by the KC equation (Eq. [9]). The  $k$ -value of compacted clay does not depend only on its void ratio and specific surface, but also on the preparation and compaction modes (e.g. Terzaghi 1922; Lambe 1954, 1958; Bjerrum and Huder 1957; Peirce et al. 1987; Wright et al. 1997). Mitchell et al. (1965) found that clay specimens compacted wet of optimum may have  $k$ -values 2 or 3 orders of magnitude less than specimens compacted dry of the optimum. Also, they identified a dual porosity in compacted clay. The porosity of the clay mass (equivalent to primary porosity in hydrogeology) corresponds to the fine structure at the micron scale of solid particles. The porosity between clay clods (equivalent to secondary porosity) corresponds to a macrostructure resulting from excavation, transport, handling and remolding by field equipment. The resulting

$k$ -value can be predicted by an equation taking into account the primary and secondary porosities. The observed  $k$ -value is mostly related to the secondary porosity and can be expressed by a power law that is close to a cubic law, the theoretical law of flow in narrow apertures (Chapuis 2002).

Similarly, the  $k$ -value of soil-bentonite mixes in which the bentonite powder does not fill completely the void space cannot be predicted using Eq. [9]. It can, however, be predicted by other methods (Chapuis 1990, 2002) considering the dual porosity of such mixes.

### Discussion

Many of the previous results have been grouped in Fig. 19 to illustrate that the KC equation can be used to predict fairly well the  $k$ -value of different soils when information is available to determine correctly the specific surface and if adequate precautions are taken for the permeability test. These data include sands and gravels (Mavis and Wilsey 1937, Navfac 1974), granular soils (Morris and Johnson 1967), mine tailings (authors), tills (authors), Quebec natural (intact) clays (Tavenas et al. 1983b), kaolinite (Mesri and Olsen 1971; Olsen 1960; Al-Tabbaa and Wood 1987), pure bentonite and soil-bentonite mixes with at least 20% bentonite (Chapuis 2002). Usually Eq. [9] predicts a  $k$ -value that is between 1/3 and 3 times the measured  $k$ -value, which is within the expected margin of variation for laboratory permeability test results.

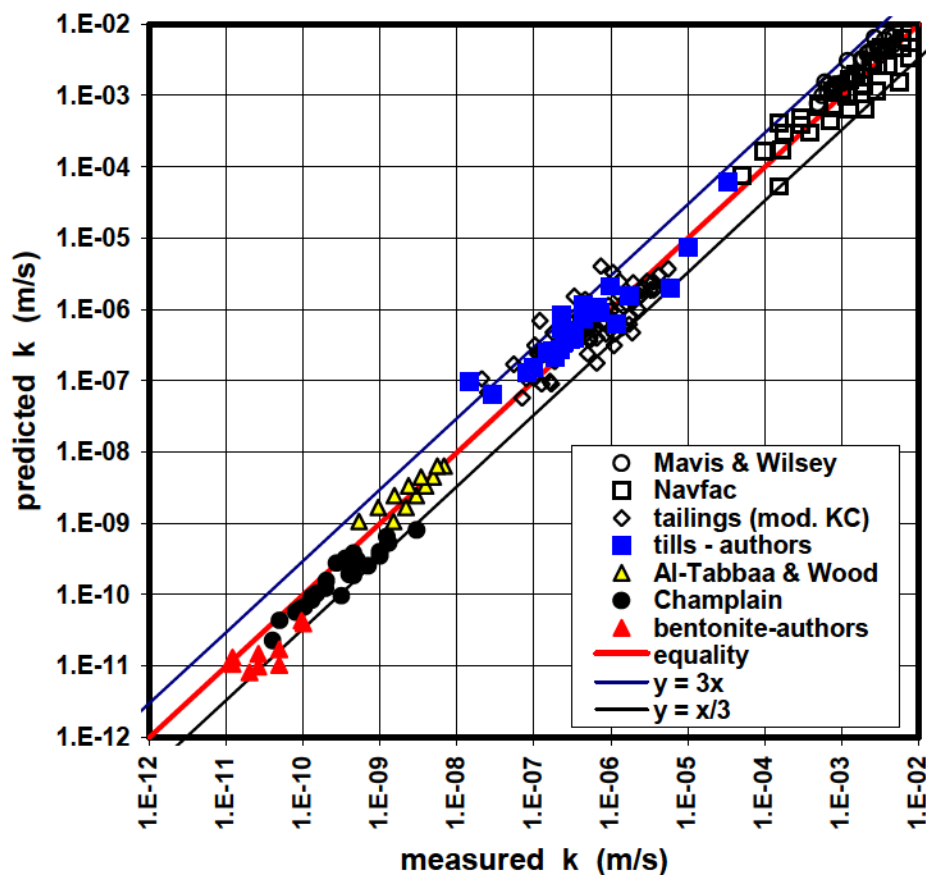


Figure 19: Predicted versus measured  $k$ -values for different soils.

According to present standards for laboratory permeability tests (ASTM D2434, D5084 and D5856, 2002), the real precision of these testing methods seems unknown and therefore their bias cannot be determined. According to the authors tests and literature review, the precision depends on testing procedures and soil intrinsic variability. For example, an excellent precision ( $k$ -value within  $\pm 20\%$  for 3 specimens) can be reached with sand and gravel when two conditions are met. First, a special procedure (using both vacuum and de-aired water) must be followed with an improved permeameter for ensuring full saturation (Chapuis et al. 1989a). Second, the soil gradation must not be prone to internal erosion (Chapuis 1992). It is also important to determine the real gradient using lateral piezometers (ASTM D2434, 2002). The real degree of saturation must be determined using the mass-and-volume method (Chapuis et al. 1989a).

In the case of a low  $k$ -value specimen tested in a rigid-wall or flexible-wall permeameter, it is known that saturating the specimen by back-pressure takes a long time (sometimes several weeks), whereas inflow and outflow rates are very small. It may be assumed that when the ratio of outflow to inflow rate is between 0.75 and 1.25 (e.g. ASTM D5084, 2002), a steady-state condition has been reached. However, the test may be far from a steady-state condition if saturation is not completed. Usually the inflow and outflow rates show some trend, increasing or decreasing with time. Stopping a test too early may lead to an underestimate of the  $k$ -value by up to two orders of magnitude (Chapuis 1990).

The specimen preparation method may also influence the test results. This is true for sand: the directional  $k$ -value depends on the compaction mode (Chapuis et al. 1989b). This is equally true for compacted clay to be used in a liner or a cover. This is true also for compacted till specimens that may exhibit a dual porosity like compacted clays (Watabe et al. 2001). In addition, when a wet till specimen is heavily compacted by impact, compaction may generate high pore pressure and produce either local internal erosion or clogging (inhomogeneous material), resulting in either overestimated or underestimated measured  $k$ -values. Compaction of dry till, on the other hand, may produce micro-fissures and increase the  $k$ -value. Such effects were not documented for the tests on non-plastic tills presented here.

In the case of silty, non-plastic soils, three tests on three specimens of the same sample may give  $k$ -values ranging between half and twice the mean value. This seems to be due to at least two reasons. First, a variation of  $\pm 2\%$  in the 2-microns fines content may induce a large variation in  $S$ -value and in  $k$ -value. Second, the soil gradation may be prone to some segregation of fines during placement and/or percolation. In the case of clays, when a paste is prepared at a water content slightly higher than the liquid limit and then consolidated, an excellent precision can be reached (e.g. kaolin, results by Al-Tabbaa and Wood 1987). In the case of natural homogeneous clays sampled with thin-wall samplers, three specimens taken at elevations  $z$ ,  $z+1\text{m}$  and  $z+2\text{m}$ , may give  $k$ -values ranging between 75 and 125% of the mean value, even when the void ratios and the Atterberg limits are very similar. When the clay properties are more variable, the measured  $k$ -values may range between 1/3 and 3 times the mean value. Here again, special precautions must be taken with natural clays to ensure full saturation. It is also required to wait long enough to complete consolidation (or swelling) and to measure equal inflow and outflow volumes for long periods of time, at least 2-3 days for ordinary clay and 3 weeks or more for a bentonite specimen 2-3 cm-high.

As a result, it is usually admitted that the true  $k$ -value of a soil lies between 1/3 and 3 times the value given by a good laboratory test. In a graph like that of Fig. 19, the resulting inaccuracy along the  $y$ -axis is  $\pm 0.5$  as shown by the two straight lines that run parallel to the equality line.

## Conclusion

By using many permeability test results, the authors show that the KC equation provides good predictions of the vertical hydraulic conductivity,  $k$ , of homogenized soil specimens. Consequently, it may be used with confidence to estimate the  $k$ -value of a soil in the range of  $10^{-1}$  to  $10^{-11}$  m/s. This is a rough prediction, however, usually falling in the range of 1/3 to 3 times the measured  $k$ -value.

Observed differences between predicted and measured  $k$ -values may be due to inaccurate estimates of specific surface,  $S$ , to faulty permeability testing procedures (incomplete saturation, etc.) as discussed in the report, and also to theoretical limitations of the equation. This isotropic equation cannot represent correctly hydraulic conductivity that is in most cases an anisotropic parameter. This is one reason why the predictions are only approximately valid. For example, the  $k_h(e)$  and  $k_v(e)$  functions at full saturation, as determined by a series of directional permeability tests (Chapuis et al. 1989b), do not verify exactly the equation. Similarly, the results of this report are valid only for hydraulic conductivity and cannot be extrapolated to another liquid. For such an extension, other properties of the liquid (e.g. polarity), and of the solid-liquid interface should be considered.

A frequent reason for having dispersed data is inadequate permeability test procedures. Considering difficulty of obtaining excellent laboratory test data, it may be concluded that current laboratory test results are not accurate and precise enough to give the best value of factor  $C$  in the Kozeny-Carman equation (Eq. [8]).

Nevertheless, the authors believe that the KC equation represented by Eq. [9] is a good predictive tool for any natural homogeneous soil. Specialists in geotechnique and hydrogeology should use it more systematically. It can be used for quick estimates of the  $k$ -value of a series of soil specimens (after determination of  $S$ ), and as a check of the quality of permeability tests.

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