Titre: Title:	Development of a Model to Predict the Water Retention Curve Using Basic Geotechnical Properties
Auteurs: Authors:	Michel Aubertin, M. Mbonimpa, B. Buissière, R.P. Chapuis
Date:	2003
Type:	Rapport / Report
Référence: Citation:	Aubertin, Michel, Mbonimpa, M., Buissière, B. et Chapuis, R. P. (2003). Development of a Model to Predict the Water Retention Curve Using Basic Geotechnical Properties. Rapport technique. EPM-RT-2003-01.

EPM–RT–2003-01

DEVELOPMENT OF A MODEL TO PREDICT THE WATER RETENTION CURVE USING BASIC GEOTECHNICAL PROPERTIES

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January 2003

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EPM-RT-2003-01

DEVELOPMENT OF A MODEL TO PREDICT THE WATER RETENTION CURVE USING BASIC GEOTECHNICAL PROPERTIES.

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JANUARY 2003

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EPM-RT-2003-01

Oxygen diffusion and consumption in unsaturated cover materials

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ABSTRACT

The water retention curve (WRC) has become a key material function to define the unsaturated behavior of soils and of other particulate media. In many instances, it can be very useful to have an estimate of the WRC early in a project, when little or no test results are available. Predictive models, based on easy to obtain geotechnical properties, can also be employed to evaluate how changing parameters (e.g. porosity or grain size) affect the WRC. In this paper, the authors present a general set of equations developed for predicting the relationship between volumetric water content θ (and the corresponding degree of saturation S_r) and suction ψ. The proposed WRC model is a modified version of the Kovács (1981) model, which makes a distinction between water retention due to capillary forces and retention by adhesion. The complete set of equations is given together with complementary relationships developed for specific applications on granular materials and on plastic/cohesive (clayey) soils. It is shown that the model provides a simple and practical means to estimate the water retention curve from basic properties. A discussion follows on the capabilities and limitations of the model.

Keywords: water retention curve, unsaturated soils, prediction, porosity, grain size, liquid limit.

RÉSUMÉ

La courbe de rétention d'eau (CRE) est devenue une fonction clé pour définir le comportement non saturé des sols et d'autres matériaux meubles. Dans beaucoup de cas, il peut être très utile d'avoir une évaluation de la CRE dans les premières phases d'un projet, lorsque peu ou pas de résultats d'essais sont disponibles. Des modèles prédictifs, basés sur les propriétés géotechniques de base, peuvent aussi être utilisés pour évaluer comment le changement des paramètres (en termes de porosité ou de granulométrie) affecte la CRE. Dans cet article, les auteurs présentent un ensemble d'équations développées pour prédire la relation entre la teneur en eau volumique θ (et le degré de saturation S_r correspondant) et la succion ψ. Le modèle proposé pour la prédiction de la CRE est une version modifiée du modèle de Kovács (1981), qui fait une distinction entre la rétention d'eau due aux forces capillaires et celle par adhésion. Ce jeu d'équations est donné avec des relations complémentaires développées pour des applications spécifiques sur des matériaux granulaires et sur des sols (argileux) plastiques/cohérents. Il est montré que le modèle constitue un moyen simple et pratique pour estimer la courbe de rétention d'eau à partir des propriétés géotechniques de base. Une discussion suit sur les capacités et les limitations du modèle.

Mots clés: courbe de rétention d'eau, sols non saturés, prédiction, porosité, granulométrie, limite de liquidité.

ACKNOWLEDGEMENTS

The post-doctoral grant provided to the first author (Mamert Mbonimpa) by the Institut de Recherche en Santé et Sécurité du Travail du Québec (IRSST) is thankfully acknowledged. Special thanks also go to Antonio Gatien and to the graduate students who performed the diffusion tests over the years. The authors also received financial support from NSERC and from a number of industrial participants to the industrial NSERC Polytechnique-UQAT Chair on Environment and Mine Wastes Management.

TABLE OF CONTENTS

1. INTRODUCTION 1

2. BASIC CONCEPTS 4

3. THE MK MODEL 6

3.1 The equivalent capillary rise 7

3.2 The WRC equations 11

4. APPLICATIONS 17

4.1 Residual suction 17

4.2 Parameter a_c and m 22

5. DISCUSSION 28

5.1 Prediction of the residual suction and air entry value 29

5.2 The equivalent capillary rise h_{co} 34

5.3 Parameters with preset values 36

5.4 Final remarks 38

6. CONCLUSION 39

7. REFERENCES 41

8. APPENDIX: SAMPLE CALCULATIONS 50

LIST OF NOTATIONS

a_c	adhesion coefficient (-)
A_v	surface of voids (cm^2)
b	coarse grained material parameter to calculate $h_{co,G}$ (cm^2)
b_1	coarse grained material parameter to estimate ψ_a (cm^{x_1+1})
b_2	coarse grained material parameter to correlate $h_{co,G}$ and ψ_a (-)
C_ψ	corrector factor (-)
C_U	uniformity coefficient (-) (C_U=D_{60}/D_{10})
d	diameter of a tube (cm)
D_{10}	diameter corresponding to 10% passing on the cumulative grain-size distribution curve (cm)
D_{60}	diameter corresponding to 60% passing on the cumulative grain-size distribution curve (cm)
d_{eq}	equivalent pore size diameter (cm)
D_H	equivalent grain size diameter (cm)
e	void ratio (-)
D_r	specific gravity of the solid particles (-)
h_c	capillary rise in a tube (cm)
h_{cmax}	maximum capillary rise [L]
h_{cmin}	minimum capillary rise [L]
h_{co}	equivalent capillary rise in a porous material (cm)
$h_{co,G}$	equivalent capillary rise in a coarse grained material (cm)
$h_{co,P}$	equivalent capillary rise in a plastic/cohesive material (cm)
m	pore size distribution parameter in the MK model (-)
n	porosity (-)
S_a	adhesion component of the degree of saturation (-)
S_a*	truncated adhesion component of the degree of saturation (-)
S_c	capillary component of the degree of saturation (-)
S_m	specific surface area per unit mass of solid (m^2/g)

S_r	degree of saturation (-)
σ_w	surface tension of water (N/m)
u_a	air pressure (cm)
u_w	water pressure (cm)
V_v	volume of the voids (cm^3)
w	gravimetric water content (-)
w_L	liquid limit (%)
w_P	plastic limit (%)
x_1	coarse grained material parameter to estimate ψ_a (-)
x_2	coarse grained material parameter to correlate $h_{co,G}$ and ψ_a (-)
ψ	suction (cm)
ξ	plastic/cohesive material parameter required to calculate $h_{co,P}$ (cm)
ρ_s	solid grain density (kg/m^3)
α	shape factor (-)
χ	material parameter used to estimate S_m (-)
λ	material parameter used to estimate S_m (m^2/g)
θ	volumetric water content (-)
ψ_0	suction at complete dryness ($S_r = 0$) (cm)
ψ_{90}	suction corresponding to a degree of saturation of 90% (cm)
ψ_{95}	suction corresponding to a degree of saturation of 95% (cm)
ψ_a	air entry value or AEV (cm)
$\psi_{a,exp}$	air entry value determined from the experimental data (cm)
$\psi_{a,MK}$	air entry value determined from the WRC predicted with the MK model (cm)
ψ_r	residual suction (corresponding to the residual water content) (cm)
$\psi_{r,exp}$	residual suction determined from the experimental data (cm)
β_w	contact angle (-)
γ_w	unit weight of water (kN/m^3)

LIST OF FIGURES

Figure 1. Illustration of the capillary and adhesion saturation contributions to the total degree of saturation for a non cohesive (low plasticity) soil in the MK model (for D_{10} = 0.006 cm, C_U = 10, e=0.6; ψ_r =190 cm, m=0.1 and a_c=0.01); ψ_a is the pressure (suction) corresponding to the air entry value (AEV), ψ_r is the pressure corresponding to the residual water content (also called water entry value-WEV), and ψ_{90} and ψ_{95} are suctions corresponding to a degree of saturation S_r of 0.90 and 0.95 respectively.--**13**

Figure 2. Illustration of the capillary and adhesion saturation contributions to the total degree of saturation for a cohesive (plastic) soil (for w_L=30%, e=0.6, ρ_s =2700 kg/m^3, ψ_r =9.7x10^5 cm, m=3x10^{-5}, and a_c=7x10^{-4}); ψ_a is the pressure (suction) corresponding to the air entry value (AEV), ψ_{90} and ψ_{95} are suctions corresponding to a degree of saturation S_r of 0.90 and 0.95 respectively. --------------------------------------**14**

Figure 3. Effect of the pore size distribution parameter m on the position and shape of the WRC according to the MK model.---**15**

Figure 4. . Effect of the residual suction ψ_r on the WRC according to the MK model. -----------**16**

Figure 5. Effect of the adhesion coefficient a_c on the WRC according to the MK model. --------**16**

Figure 6. Relationship between the residual suction $\psi_{r,,exp}$ determined from the experimental data (with best fit WRC), the void ratio e, and the equivalent grain size diameter D_H for granular materials identified in Table 1 (D_H is defined by equations 5 and 6).---**20**

Figure 7. Relationship between the residual suction $\psi_{r,,exp}$ determined from the measured data (and best fit WRC), and the equivalent capillary rise $h_{co,G}$ (eq. 7) for granular materials. --**21**

Figure 8. Comparison of the ψ_r values obtained from equations 19 (see Figure 6) and 20 (see Figure 7) for the granular materials identified in Table 1. ---------------------------------**22**

Figure 9 . Application of the MK model to a coarse, uniform, and relatively loose sand (data from Sydor 1992).--**24**

Figure 10. Application of the MK model to a fine, uniform, and dense sand (data from Bruch 1993). ---**25**
Figure 11. Application of the MK model to tailings Sigma (silty material, well-graded, and loose) (data from authors).---**25**
Figure 12. Application of the MK model to tailings Sigma mixed with 10 % bentonite (data from authors).---**26**
Figure 13. Application of the MK model to Guadalix red silty clay (data from Vanapalli et al. 1998) ---**26**
Figure 14. Application of the MK model to a till (data from O'Kane et al. 1998) -----------------**27**
Figure 15. Application of the MK model to Indian Head Till (data from Fredlund 1999) --------**27**
Figure 16. Comparison between the air entry values obtained from the experimental data $\psi_{a,exp}$ (with best fit WRC) and obtained on the WRC predicted by the MK model ($\psi_{a,MK}$) for granular materials.---**31**
Figure 17. Proposed relationships for estimating the air entry value $\psi_{a,exp}$ obtained on the experimental data for granular materials.---**32**
Figure 18. Proposed relationship for estimating the air entry value $\psi_{a,MK}$ obtained on the WRC predicted by the MK model for granular materials.---------------------------------------**33**
Figure 19. Relationships between the predicted air entry value $\psi_{a,MK}$ and suctions $\psi_{90,MK}$ and $\psi_{95,MK}$ corresponding respectively to degrees of saturation of 90% and 95%, (obtained on the WRC predicted by the MK model) for granular materials. ------------------**34**
Figure 20. Relationships between the equivalent capillary rise $h_{co,G}$ and the air entry value for granular materials; $\psi_{a,exp}$ and $\psi_{a,MK}$ are obtained on the experimental and on the MK model predicted WRC respectively. ---**36**

LIST OF TABLES

Table 1. Nature, origin, and basic geotechnical properties of the granular materials.--------------**18**

Table 2. Nature, origin, and basic geotechnical properties of the plastic/cohesive materials. --**19**

Table 3. Parameters *m* and a_c leading to the best fit WRC shown in Figures 9 to 15, and the empirically determined values. --**28**

Table A-1. Typical calculation results to predict the WRC of granular materials: Case of Sigma tailings (see Figure 11- data from Authors). ---**50**

Table A-2. Typical calculation results to predict the WRC of plastic/cohesive soils: Case of Indian Head Till (See Figure 15- Record 3728-data from Fredlund 1999). ---------------**51**

1. INTRODUCTION

In geotechnical engineering, subsurface water is often divided into free water in the saturated zone and moisture retained in the unsaturated (vadose) zone (e.g. Bowles 1984; Smith 1990; McCarthy 1998). Even if the behavior of water and soils under saturated condition can be treated as a special case of the more general partially saturated situation (e.g. Fredlund and Rahardjo 1993), such distinction is justified by the physical differences that characterize the two zones. Owing to the particularities that exist in unsaturated soils, special attention should be paid to adequately define the response of water above the phreatic surface.

Geotechnique for unsaturated media is a rapidly expanding field, which is related to a wide variety of applications in soil mechanics and soil physics including: estimation of field capacity (Meyer and Gee 1999); efficiency analysis of covers with capillary barrier effects (Nicholson et al. 1989; Aubertin et al. 1993, 1995a, 1996, 1997; Khire et al. 1995; O'Kane et al. 1998; Morris and Stormont 1999; Bussière 1999); bearing capacity of unsaturated foundation materials (Rassam and Williams 1999); seepage through dams (Chapuis et al. 1996, 2001; Fredlund 2000a); compressibility and swelling soil response (Fredlund, 1987; Rampino et al. 2000; Hung and Fredlund 2000); shear strength and constitutive behavior (e.g. Alonso et al. 1990; Fredlund et al. 1996, Vanapalli et al. 1996; Cui and Delage 1996); contaminant transport (Badv and Rowe 1996; Lim et al. 1998; Barbour 1998, Esposito et al. 1999); land subsidence (Thu and Fredlund 2000); freeze-thaw response of road structures (Konrad and Roy 2000). Fredlund (2000a) presents an interesting overview of various applications of unsaturated soil mechanics.

The distribution and motion of water in the unsaturated zone are closely related to forces caused by molecular attraction responsible for water adhering to solid surfaces (i.e. hygroscopic or adsorbed water on soil particles) and for surface tension causing capillary retention due to cohesion between water molecules at the interface with air (e.g. Bear 1972; Marshall et al. 1996). In saturated media, the adsorption forces tend to reduce the space available for water flow, hence reducing the effective porosity and hydraulic conductivity, while capillary forces disappear in the absence of a water-air interface. In partly saturated conditions, these two distinct but complementary types of force affect the hydrogeological and mechanical behavior of soils. The corresponding properties, including hydraulic conductivity and shear strength parameters, are no longer material constants but rather depend on the relative amount of water and gas in the pore space.

The quantity of water retained in a soil by capillary and adhesion forces depends on many factors, namely: shape, size and distribution of pore space; mineralogy and surface activity of solid grain particles; chemical composition of interstitial water. Richards (1931) equation, which is a combination of Darcy's law and conservation equation, is commonly used to solve groundwater flow problems for variably saturated conditions. In this equation, the hydraulic conductivity is a function of the state variables, which are typically expressed in terms of volumetric water content θ or of suction ψ (e.g. Leong and Rahardjo 1997a; Leij et al. 1997). For a given porous media, these two parameters, θ and ψ, are related to each other and form what is being considered as a fundamental material characteristic identified here as the water retention curve WRC. Many other names are used for the θ-ψ relationship, including the soil-water characteristic curve (SWCC) which however becomes less appropriate when dealing with other types of porous material (such as ceramic plates or geotextiles) and with man made particulate media (such as mined rock wastes and tailings).

In a porous system, increasing the value of suction ψ (defined by u_a-u_w, where u_a is the air (gas) pressure and u_w is the water pressure) tends to reduce θ. The desaturation is typically more pronounced in coarse grained materials (such as sand and gravel) than in fine grained materials (such as silt and clay). The value of θ at a given ψ also depends on the path, whether it occurs during a wetting or drying phase. Different paths induce somewhat different curves (e.g. Parlange 1976; Mualem 1984), but such hysteresis phenomena will not be addressed directly here, as only the drainage path is considered (to simplify the presentation).

When θ is diminished, the flow of water is more difficult because of the reduced area available in the water phase. The unsaturated hydraulic conductivity k_u thus depends on θ (or ψ). At a sufficiently low water content, the water phase becomes discontinuous and k_u is reduced to a very small (near zero) value.

To perform groundwater flow analyses as required in a number of applications in geotechnique and soil physics, engineers and scientists must be able to quantify the k_u function. Because direct measurement of k_u can be difficult, time consuming and costly, it is customary to use, as a starting point, some relationship between θ and ψ (e.g. Mualem 1976, 1986; Fredlund et al. 1994) because the WRC can be evaluated more easily than k_u in the laboratory or in the field. The resulting θ-ψ data, obtained from test measurements, are plotted and used to derive specific mathematical functions with curves that run through the data points. Such fitting equations have been proposed by Gardner (1956), Brooks and Corey (1964), Brutsaert (1966), Haverkamp et al. (1977), Vauclin et al. (1979), van Genuchten (1980), Bumb et al. (1992), Fredlund and Xing (1994), and Kosugi (1994), to name a few. These mathematical expressions with the corresponding parameter values deduced experimentally or otherwise, can be very useful for particular applications when the necessary data is available. Many of these equations have been compared to each other over the years (e.g. Khire et al. 1995; Leij et al. 1997; Leong and Rahardjo 1997b; Burger and Shackelford, 2001), showing that they each have some advantages and limitations, depending on the technique and data bank used for comparison.

In some cases, it can be very useful to obtain *a priori* an estimate of the WRC from easily and rapidly obtained properties. Unfortunately, most predictive models, sometimes referred to as pedotransfer functions (e.g. Verecken et al. 1992; Schaap and Leij 1998; Elsenbeer 2001), are based on empirically derived expressions that bear little physical meanings, and this limits their significance when trying to extend their use outside the range of properties considered for their development and calibration.

In the following, the authors propose a set of equations ensuing from the physical model originally developed by Kovács (1981). This model, which includes a capillary and an adhesion component, was previously used to predict the WRC of tailings from hard-rock mines (Aubertin et al. 1998). In this paper, the model is generalized and all its components are explicitly defined so that it can be applied to a variety of homogeneous media, including coarse and fine grained soils, tailings, and bentonite mixtures.

2. BASIC CONCEPTS

The water retention curve (WRC) represents the relationship between the volumetric water content θ and suction ψ. The former parameter is related to porosity n and degree of saturation S_r ($\theta = nS_r$). It can also be expressed as a function of the more common massic (gravimetric) water content w (i.e. $\theta = w(1-n)D_r$, where D_r is the relative density of the solid particles). The suction value ψ, on the other hand, is typically expressed as a potential using either pressure units (ex. kPa), pressure head (ex. m of water), or centimetric logarithmic head (known as pF); at 20°C, 9.81kPa ≡ 100cm of water ≡ pF of 2. The total suction ψ is the sum of the matric potential ψ_m and of the osmotic potential π, but the latter is often neglect in geotechnical applications ($\psi = \psi_m$ is assumed here). For convenience, ψ takes a positive value under negative water pressure u_w above the phreatic water surface (where $u_w = u_a$ or $\psi = u_a - u_w = 0$, with u_a taken as the reference air pressure).

A large number of experimental investigations have been devoted to developing and applying techniques to obtain the θ-ψ relationship of soils. Accordingly, various direct and indirect measurement methods have been proposed for the WRC, and many of these have been reviewed in recent textbooks and monographs (e.g. Klute 1986; Carter, 1993; Fredlund and Rahardjo 1993; Marshall et al. 1996). The evolution and understanding of the WRC and of the corresponding measurement techniques have been recently presented by Barbour (1998), who provides an interesting historical perspective.

Measurement of the WRC in the laboratory and in the field, although less demanding than that of k_u, can be relatively time consuming and expensive. In some situations, it may be very useful to have estimates of the WRC, especially during the preliminary stages of a project when the available information is limited. Predictive models for the WRC, which are usually based on simple geotechnical (pedologic) properties, such as grain size and porosity, can also be of help when analyzing and validating test results, and also for evaluating the variations that can be expected in the field within a non homogenous deposit.

Over the last two decades or so, quite a few predictive models have been proposed (see Elsenbeer 2001, and the corresponding Monograph Issue on the topic). These include a number of functional regression methods using some of the above mentioned empirical WRC equations. The fitting parameters are then related to particular textural parameters (e.g. Cosby et al. 1984; Schaap et al. 1998; Fredlund 2000b). Other discrete regression methods make no *a priori* assumption regarding the shape of the WRC, and construct empirical models from the regression equations linking the θ-ψ values and basic properties (e.g. Gupta and Larson 1979; Rawls and Brackensiek 1982; Rawls et al. 1991; Vereecken et al. 1992; Tietje and Tapkenhinrichs 1993). It appears preferable however to use predictive models for which the WRC equation is based on physical characteristics of the media. This latter type of model includes those of Arya and Paris (1981), Haverkamp and Parlange (1986), Tyler and Wheatcraft (1989, 1990), Haverkamp et al. (1999), and Arya et al. (1999).

Such a physically-based model was also proposed by Kovács (1981). Still relatively unknown and seldom used in the geotechnical field, the Kovács (1981) model recognizes the need to distinguish between capillary and adhesive forces. This approach potentially provides a more realistic view, as it can be related to the actual processes involved (e.g. Celia et al. 1995; Nitao and Bear 1996). However, the Kovács (1981) model, in its original version, did not easily lend itself to practical engineering applications because of the relatively poor definition of key parameters needed for its use. The authors have nevertheless been able to apply it, after some modifications (hence the name Modified Kovács or MK model) to the WRC of artificial silts or tailings (Aubertin et al. 1998). In the following, the MK model is extended for general applications to various types of soils, from coarse sand to clayey materials. The basic assumptions and theoretical considerations behind these extensions are briefly presented. The descriptive and predictive capabilities of the MK model are shown using tests results taken from the literature and obtained in the author's own facilities.

3. THE MK MODEL

The Kovács (1981) model is used here as the basis to develop a practical set of equations for predicting the WRC of various types of soils from commonly used geotechnical properties. The original model has been described in details in Kovács' (1981) textbook, and was also reviewed by the authors in a paper where the model has been adapted for tailings from hard rock mines (Aubertin et al., 1998). Hence, only its basic components will be recalled.

The proposed MK model presented in the following retains the sound physical concepts from which the original model was constructed. The non-trivial modifications introduced serve to generalize the statistical function used to describe the pore-size distribution of the media appearing in the capillary component. Some parameters, ill defined originally, are expressed more specifically as a function of basic soil properties. The ensuing set of equations allows the user to obtain an evaluation of the WRC, which can be employed for practical calculations involving problems with unsaturated conditions. As will be shown in the next sections, the MK model is easy to use and generally provides a good estimate of the WRC for various types of granular as well as plastic/cohesive materials. The MK model can also be used in a more classical (descriptive) manner to draw the WRC from a few relevant testing results. Because of their close relationship with material response, the model equations can be employed to evaluate the expected influence of changing properties on the WRC, such as varying grain size, porosity, or liquid limit.

3.1 The equivalent capillary rise

The MK model makes use of a parameter defined as the equivalent capillary rise h_{co} [L] in the porous medium. The role of this parameter is the same as the so-called average capillary rise in the original Kovács (1981) model. The adopted denomination is justified below. This parameter can be derived from the well-known expression used for the capillary rise h_c [L] of water in a tube having a diameter d. The value of h_c is given by (Smith 1990; Chin 2000):

[1] $$h_c = \frac{4\sigma_w \cos\beta_w}{\gamma_w d}$$

where σ_w [MT^{-2}] is the surface tension of water (σ_w =0.073 N/m at 20°C), β_w [-] is the contact angle between water and the tube surface (β_w = 0° for quartz and glass), γ_w [$ML^{-2}T^{-2}$] is the unit weight of water (γ_w=9.8 kN/m^3 at 20°C). This well-known equation indicates that capillary rise is inversely proportional to the diameter of the tube. When applied to pore space in soils above the phreatic surface, this equation helps to understand why there can be a much larger capillary rise in fine grained soils, where the voids (somewhat similar to capillary tubes) are small, than in coarse grained soils where the voids are typically larger.

In soils however, the pore size is not uniform so h_c is not easily defined. This pore system can be substituted by a system of regular channels with a diameter expressed as the equivalent hydraulic pore diameter d_{eq} [L] defined as (e.g. Bear 1972; Kovács 1981):

[2] $$d_{eq} = 4\frac{V_v}{A_v}$$

where V_v [L^3] and A_v [L^2] are respectively the volume and surface of the voids. In practice, A_v approximately corresponds to the surface area A_G [L^2] of the solid grains. By relating A_G to the massic specific surface area S_m [L^2M^{-1}], equation (2) can be transformed as (Scheidegger 1974):

[3] $$d_{eq} = 4\frac{e}{\rho_s S_m}$$

In this equation, e [-] is the void ratio (V_v/V_s, where V_s is the volume of the solid grains) and ρ_s [ML^{-3}] is the solid grain density.

The capillary rise h_{co} of a soil above the water table, obtained by replacing diameter d (in equation 1) by the equivalent hydraulic pore diameter d_{eq}, can therefore be expressed as:

[4] $$h_{co} = \frac{\sigma_w \cos\beta_w}{\gamma_w}\,\frac{\rho_s S_m}{e}$$

Parameter h_{co} defined by equation 4 is the equivalent capillary rise of the soil.

Although S_m can be directly measured by various techniques (e.g. Lowell and Shields 1984; Igwe 1991), in most practical cases, the value of S_m is not readily available. For coarse grained soils, the specific surface area can nevertheless be estimated from the grain size distribution using the following expression (Kovács 1981):

[5] $$S_m = \frac{\alpha}{\rho_s D_H}$$

where α [-] is a shape factor ($6 \le \alpha \le 18$; $\alpha = 10$ is used here - see discussion below), and D_H [L] is an equivalent particle diameter for a heterogeneous mixture. The equivalent diameter D_H for a heterogeneous mix of particles theoretically represents the diameter of a homogeneous mix (with a single size) that has the same specific surface area as the heterogeneous one.
Equation 4 can thus be rewritten to calculate the equivalent capillary rise in a soil above the water table:

[6] $$h_{co,G} = \frac{\sigma_w \cos\beta_w}{\gamma_w} \frac{\alpha}{eD_H}$$

where subscript G stands for granular (low plasticity, low cohesion) materials, as opposed to clayey (plastic/cohesive) materials (which will be discussed below). In this equation, the contact angle β_w will be taken as zero (e.g. Marshall et al. 1996).

In granular soils, D_H can be evaluated by subdividing the grain size curve (e.g. Chapuis and Légaré 1992). For practical geotechnical applications, the value of D_H can also be approximated using a function of a representative size, like the effective diameter D_{10} [L] (diameter corresponding to 10 % passing on the cumulative grain-size distribution curve), and of the uniformity coefficient C_U (i.e. $D_H=[1+1.17\log(C_U)]D_{10}$; Aubertin et al. 1998; Mbonimpa et al. 2000, 2002). For the capillary rise of water in granular soils, equation (6) is expressed as follow:

[7] $$h_{co,G} = \frac{b}{eD_{10}}$$

with

[8] $$b = \frac{10\sigma_w}{[1.17\log(C_U) + 1]\gamma_w}$$

Parameter b [L^2] is expressed in cm^2, while h_{co} and D_{10} are expressed in cm. For the values of σ_w and γ_w given above, equation 8 becomes:

[9] $$b = \frac{0.75}{1.17\log(C_U) + 1}$$

These equations mean that, for a uniform sand with D_{10} = 0.01cm, C_U = 5 and e = 0.6, b = 0.412 cm^2, the equivalent capillary rise $h_{co,G}$ is about 69 cm, while for a silt with D_{10} = 0.0005 cm, C_U=20 and e = 0.6, b = 0.297 cm^2, and $h_{co,G}$ would be 990 cm approximately. These values of

$h_{co,G}$ are much larger than the height of the capillary fringe (as defined in most textbooks) which rather represents more or less the air entry value (AEV) of the material. The physical meaning of these parameters will be further discussed below.

For clayey (plastic/cohesive) soils, the above equations can provide inadequate estimates of S_m and h_{co}, particularly when the liquid limit w_L (%) is above about 30 to 40%. For such fine grained soils, other factors influence their water retention capacity. In this case, S_m (in equation 4) is better estimated using the relationship that exists between the specific surface area and the liquid limit w_L (%). The following expression, recently proposed by Mbonimpa et al. (2002), is used here:

[10] $$S_m = \lambda \; w_L^{\chi}$$

where λ [L^2M^{-1}] and χ (unitless) are material parameters. Using a fairly large number of tests results from various sources, it has been established that $\lambda \approx 0.2$ m^2/g and $\chi = 1.45$ for materials with 22 m^2/g $\leq S_m \leq$ 433 m^2/g and 18 % $\leq w_L \leq$ 127 %.

Combining equations 4 and 10 gives:

[11] $$h_{co,P} = \frac{\xi}{e} \, w_L^{1.45}$$

where subscript P stands for plastic/cohesive materials. From the previous developments, parameter ξ [L] can be expressed as:

[12] $$\xi = \frac{\sigma_w \cos \beta_w}{\gamma_w} \lambda \, \rho_s$$

The units for ξ and $h_{co,P}$ are cm. For the above values of σ_w and γ_w, β_w =0°, $\lambda \approx 0.2$ m^2/g, and ρ_s in kg/m^3, ξ becomes:

[13] $\xi \approx 0.15\, \rho_s$

From equation 11 and 13, one can calculate that for a clayey soil with w_L = 40%, ρ_s = 2700 kg/m^3 and e= 0.8, ξ = 405 cm^2, and $h_{co,P}$ is equal to 106500 cm; for w_L = 80%, ρ_s = 2700 kg/m^3 and e = 0.8, one obtains ξ = 405 cm^2 and $h_{co,P}$ = 290968 cm. Again, these values are much larger than the water rise corresponding to the capillary fringe. It will be seen below that h_{co} should rather be compared to the potential elevation (or suction ψ_r) for the residual water content.

3.2 The WRC equations

The MK model uses h_{co} as a reference parameter to define the relationship between volumetric water content θ and matric suction ψ. As mentioned above, both the original Kovács (1981) model and the MK model consider that water is held by two types of forces, i.e. capillary forces responsible for capillary saturation S_c and adhesive forces causing saturation by adhesion S_a. The S_c component equation is obtained from a cumulative pore size distribution function, while the equation of S_a is given by an interaction law with van der Waals type attraction between grain surface and water dipoles. The S_c component is more important at relatively low suction values, while the S_a component becomes dominant at higher suction when capillary water has been withdrawn.

The proposed set of equations for the MK model is written as follows for the degree of saturation S_r:

[14] $$S_r = \frac{\theta}{n} = S_c + S_a^{*}(1 - S_c)$$

[15] $$S_c = 1 - \left[\left(h_{co}/\psi \right)^2 + 1 \right]^m exp\left[-m \left(h_{co}/\psi \right)^2 \right]$$

[16] $$S_a^{*} = 1 - \langle 1 - S_a \rangle$$

[17] $$S_a = a_c\, C_\psi \frac{\left(h_{co} / \psi_n \right)^{2/3}}{e^{1/3} \left(\psi / \psi_n \right)^{1/6}}$$

with

[18] $$C_\psi = 1 - \frac{ln(1 + \psi / \psi_r)}{ln(1 + \psi_o / \psi_r)}$$

Equation 14 expresses the total degree of saturation by combining the capillary and adhesion components with contributions of S_c and S_a^{*} given by equations 15 to 18. In these equations m and a_c are material parameters, and ψ_r is the pressure (suction) corresponding to the residual water content (ψ_r is also equal to the water entry value - WEV - when no distinction is made between drainage and wetting path). Figures 1 and 2 show typical curves drawn from the MK model in a semi-log S_r-ψ plane, illustrating the contributions of S_c and S_a^{*} for granular (Fig. 1a) and plastic/cohesive (Fig 1b) materials. Hypothetical (but representative) values for D_{10}, C_U, e, w_L, G_s, h_{co}, m, a_c, and ψ_r have been used for these plots. The value ψ_r and the pressure (suction) corresponding to the air entry value ψ_a are also shown on these Figures together with two other parameters ψ_{90} and ψ_{95} defining suctions for preset degrees of saturation (of 90% and 95% respectively).

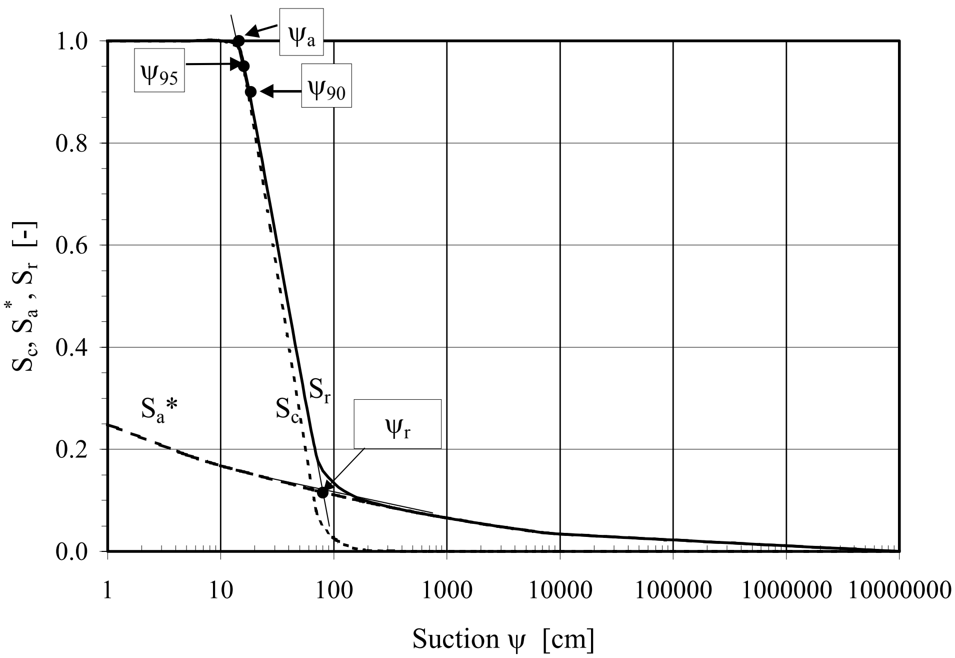

Figure 1. Illustration of the capillary and adhesion saturation contributions to the total degree of saturation for a non cohesive (low plasticity) soil in the MK model (for D_{10} = 0.006 cm, C_U = 10, e=0.6; ψ_r =190 cm, m=0.1 and a_c=0.01); ψ_a is the pressure (suction) corresponding to the air entry value (AEV), ψ_r is the pressure corresponding to the residual water content (also called water entry value-WEV), and ψ_{90} and ψ_{95} are suctions corresponding to a degree of saturation S_r of 0.90 and 0.95 respectively.

Equation 15 provides the expression to evaluate S_c ($0 \leq S_c \leq 1$) as a function of the equivalent capillary rise h_{co} (given by the expressions proposed above) and suction ψ. This equation is a generalization of the one developed by Kovács (1981), in which the statistical exponential function has been expanded to better reflect the influence of pore-size distribution through the distribution parameter m. The statistical distribution expression used for S_c is one that can also be applied for grain size curves, as there is a well-known similarity between the latter and the WRC (e.g. Aubertin et al. 1998; Fredlund et al. 2000).

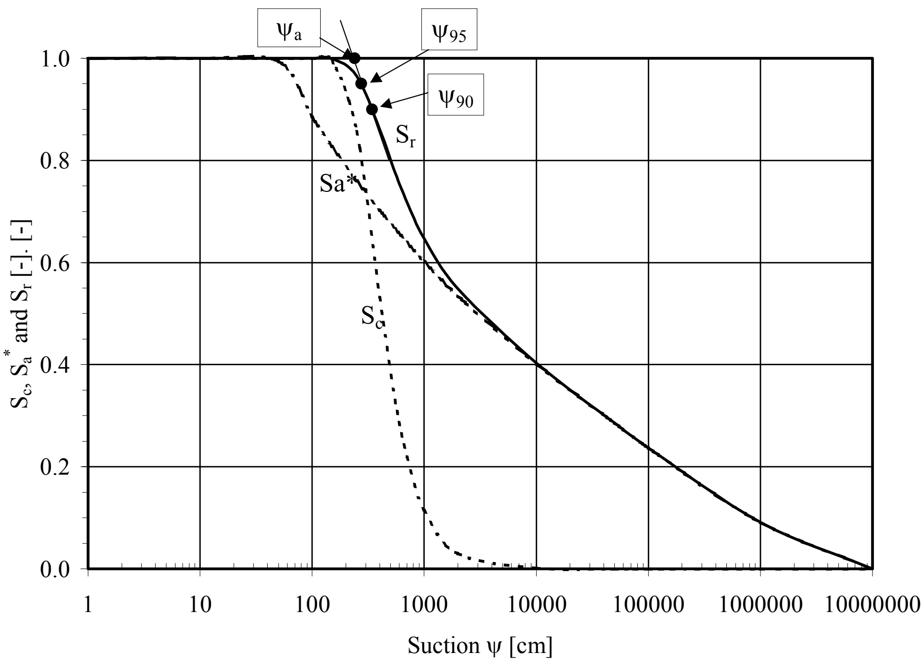

Figure 2. Illustration of the capillary and adhesion saturation contributions to the total degree of saturation for a cohesive (plastic) soil (for w_L=30%, e=0.6, ρ_s =2700 kg/m^3, ψ_r =9.7x10^5 cm, m=3x10^{-5}, and a_c=7x10^{-4}); ψ_a is the pressure (suction) corresponding to the air entry value (AEV), ψ_{90} and ψ_{95} are suctions corresponding to a degree of saturation S_r of 0.90 and 0.95 respectively.

Figure 3 illustrates how the θ-ψ relationship is affected by the value of parameter *m*. As can be seen in this figure, both the position and the slope of the curve are related to *m*. On the WRC, parameter *m* influences the air entry value ψ_a (or AEV), which theoretically correspond to the suction when the largest pores start to drain, and also to the rate of decline beyond ψ_a in the capillary range. For practical applications, the value of *m* will be expressed as a function of some very basic geotechnical properties.

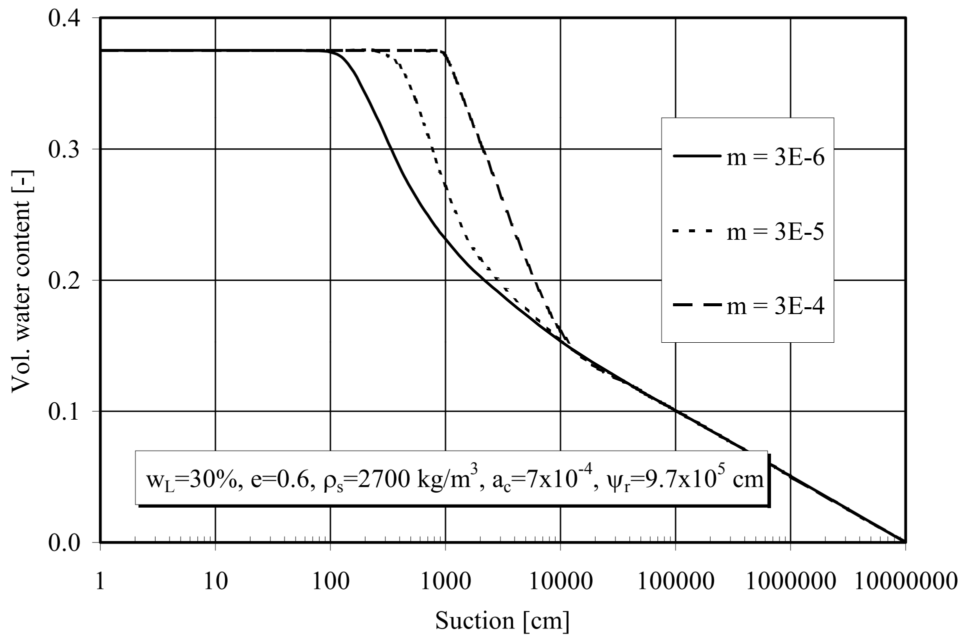

Figure 3. Effect of the pore size distribution parameter *m* on the position and shape of the WRC according to the MK model.

Equations 16 and 17 define the adhesion component, again as a function of h_{co} and ψ. A truncated value of S_a (i.e. S_a^*) is used to make sure that the adhesion component does not exceed unity at low suction ($0 \leq S_a^* \leq 1$). In equation 16, $\langle \ \rangle$ represents the Macauley brackets ($\langle y \rangle = 0.5(y + |y|)$); for $S_a \geq 1$, $S_a^* = 1$, and for $S_a < 1$, $S_a^* = S_a$. Equation 17 is based on Kovács (1981) expression in which a sixth order hyperbola is used to relate the adhesion saturation (due to the film of water adsorbed on grain surface) to suction. In this equation, a_c is the adhesion coefficient (dimensionless) and ψ_n is a normalization parameter introduced for unit consistencies (ψ_n =1 cm when ψ is given in cm, corresponding to $\psi_n = 10^{-3}$ atmosphere). Parameter C_ψ (equation 18), taken from Fredlund and Xing (1994), forces the water content to zero when ψ reaches a limit imposed by thermodynamic equilibrium ($\theta = 0$ at $\psi = \psi_0 = 10^7$ cm of water, corresponding approximately to complete dryness). In equation 18, ψ_r represents the suction at residual water content, which will be shown below to depend on basic soil properties (as is the case with ψ_a and h_{co}). The effect of varying the value ψ_r in the MK model can be seen on Figure 4. Figure 5 shows the influence of the adhesion coefficient a_c on the WRC (for typical clayey soils).

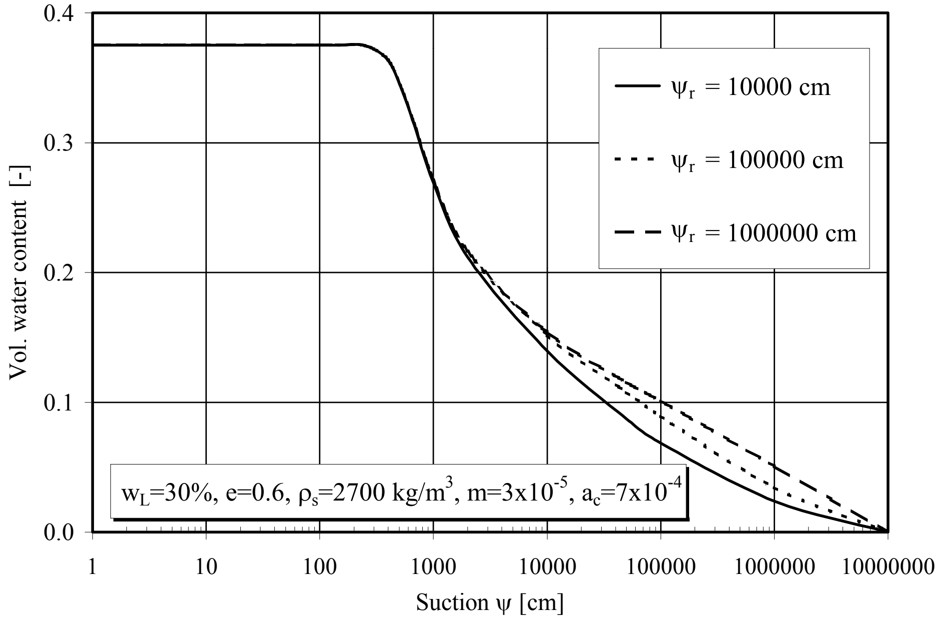

Figure 4. . Effect of the residual suction ψ_r on the WRC according to the MK model.

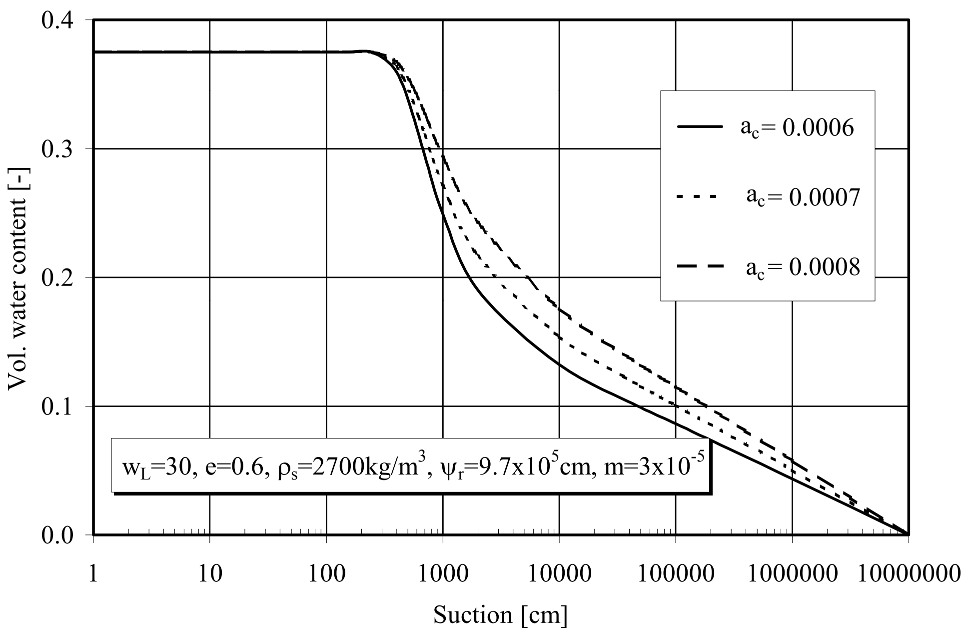

Figure 5. Effect of the adhesion coefficient a_c on the WRC according to the MK model.

4. APPLICATIONS

The MK model presented above includes a set of equations that provides an estimate of the WRC from full saturation ($S_r = 1$, $\theta = n$) to complete dryness ($S_r = 0 = \theta$). To apply the model however, three open parameters in the constitutive equations have to be defined explicitly. These are parameter m in equation 15, a_c in equation 17, and ψ_r in equation 18. Based on investigations carried by the authors on a diversity of soils and particulate media, it has been found that the values of m, a_c and ψ_r can be related to some very basic properties.

The experimental data used here for granular materials have been taken from various investigations performed on the properties of sands and low plasticity silts and tailings. The authors results have been obtained with either plate extractor or Tempe cell according to procedure described in Aubertin et al. (1995b, 1998), while some others have been taken from the literature (see Table 1). All the experimental results on plastic/cohesive materials have been taken from the literature (see Table 2). However, to limit the possibility of excessive shrinkage during testing (see Discussion), the clayey soils retained cover a relatively small range of liquid limit and porosity values.

4.1 Residual suction

To apply the model, the residual suction ψ_r introduced in the expression of factor C_ψ was evaluated first. For granular materials, values of ψ_r were determined using the tangent method described by Fredlund and Xing (1994) (see Figure 1). The curves used to this end were obtained by fitting the experimental data with a descriptive equation (i.e. the van Genuchten (1980) model implemented in the code RETC; van Genuchten et al. 1991); the tangent method was then applied on the corresponding WRC best fit (N.B. the best fit curve could also have been obtained with the MK model).

Table 1. Nature, origin, and basic geotechnical properties of the granular materials.

Source	Material	D_{10} [cm]	C_U [-]	e [-]
Sydor (1992)	Coarse Sand	0.05800	1.3	0.750
	Borden Sand	0.00910	1.7	0.590
	Modified Borden Sand	0.00800	1.8	0.640
Bruch (1993)	B.Creek sand consolidated at 5 kPa	0.00930	2.6	0.269
	B. Creek sand consolidated 10 kPa	0.00930	2.6	0.267
Ricard (1994)	Tailings Bevcon	0.00035	11.4	0.674
		0.00035	11.4	0.710
		0.00035	11.4	0.795
	Tailings Senator	0.00031	11.9	0.798
		0.00031	11.9	0.929
	Tailings Sigma	0.00033	14.6	0.695
		0.00033	14.6	0.746
		0.00033	14.6	0.802
	Tailings Sigma +10% bentonite	0.00010	35.0	0.698
		0.00010	35.0	0.944
Kissiova (1996)	Tailings Bevcon	0.00038	11.1	0.570
		0.00038	11.1	0.680
		0.00038	11.1	0.920
	Tailings Sigma	0.00034	14.7	0.660
		0.00034	14.7	0.720
	Sacrete sand	0.01450	3.5	0.570
		0.01450	3.5	0.630
MacKay (1997)	London Silt	0.00060	5.5	0.634
	Ottawa Sand	0.00937	1.7	0.587
Lim et al. (1998)	B. Creek sand consolidated at 5 kPa	0.00930	2.6	0.618
Rassam and Williams (1999)	Tailings at 50m	0.00600	5.0	0.637
	Tailings at 150m	0.00184	9.5	0.637
Authors own results	Tailings Sigma (fine)	0.00040	9.8	0.720
		0.00040	9.8	0.740
	Tailings Sigma (coarse)	0.00040	8.6	0.640
		0.00040	8.6	0.710
		0.00040	8.6	0.780
	Till	0.00035	4.5	0.620
		0.00035	4.5	0.700
		0.00035	4.5	0.720
		0.00035	4.5	0.790

Table 2. Nature, origin, and basic geotechnical properties of the plastic/cohesive materials.

Source	Material	w_L [%]	G_s [-]	e [-]
Alimi-Ichola and Bentoumi (1995)	Gault clay	40	2.650	0.942
Vanapalli et al. (1996)	Sandy clay till (200 kPa prec.)	35.5	2.730	0.430
	Sandy clay till (25 kPa prec.)	35.5	2.730	0.540
	Sandy clay till (25 kPa prec.)	35.5	2.730	0.545
Huang et al. (1998)	Silty sand PPCT11	22.2	2.680	0.536
	Silty sand PPCT21	22.2	2.680	0.502
	Silty sand PPCT16	22.2	2.680	0.463
	Silty sand PPCT26	22.2	2.680	0.425
O'Kane et al. (1998)	Till Cover	40	2.770	0.493
Vanapalli et al. (1998)	Guadalix Red silty clay	33	2.660	0.480
Fredlund (1999) (in Soilvision)	Record 3713	35.5	2.65	0.545
	Record 3714	35.5	2.65	0.545
	Record 3715	35.5	2.65	0.449
	Record 3716	35.5	2.65	0.474
	Record 3717	35.5	2.65	0.474
	Record 3718	35.5	2.65	0.518
	Record 3720	35.5	2.65	0.546
	Record 3728	35.5	2.65	0.438
	Record 55	35.5	2.65	0.475
	Record 65	35.5	2.73	0.546
	Record 66	35.5	2.73	0.438
	Record 70	35.5	2.73	0.444
	Record 71	35.5	2.73	0.518
	Record 72	35.5	2.73	0.472
	Record 73	35.5	2.73	0.545
	Record 75	35.5	2.73	0.430
	Record 76	35.5	2.73	0.372

After considering various geotechnical parameters (alone or combined) with a physical significance, it was established that the following relationship provides an adequate estimate of the residual suction (see Figure 6).

[19] $$\psi_r = \frac{0.42}{(eD_H)^{1.26}}$$

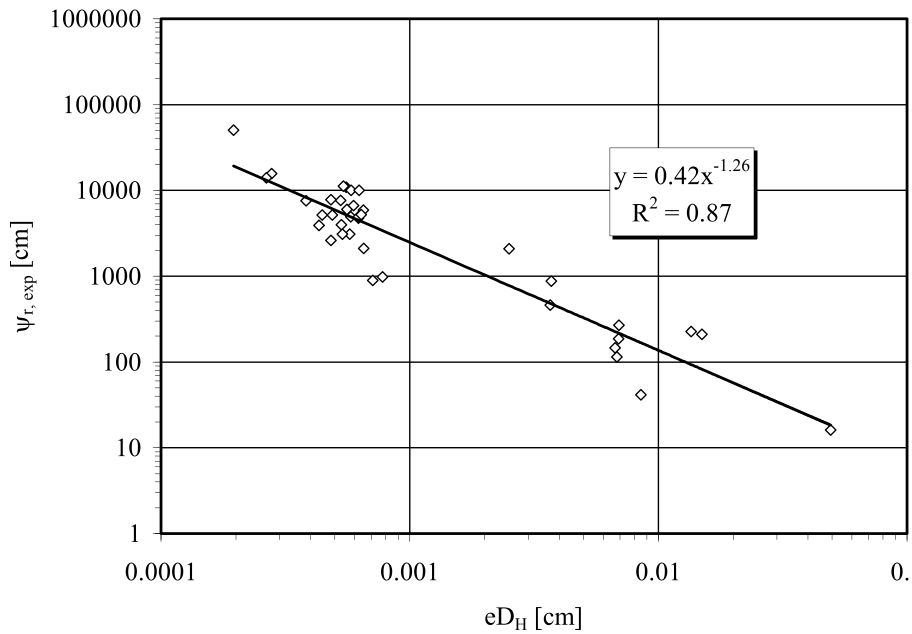

Figure 6. Relationship between the residual suction $\psi_{r,,exp}$ determined from the experimental data (with best fit WRC), the void ratio e, and the equivalent grain size diameter D_H for granular materials identified in Table 1 (D_H is defined by equations 5 and 6).

Based on this expression and on equation 7, the authors also developed a simple relationship between ψ_r and the equivalent capillary rise $h_{co,G}$ for granular materials, which is expressed as follow (see Figure 7):

[20] $$\psi_r = 0.86\ h_{co,G}^{1.2}$$

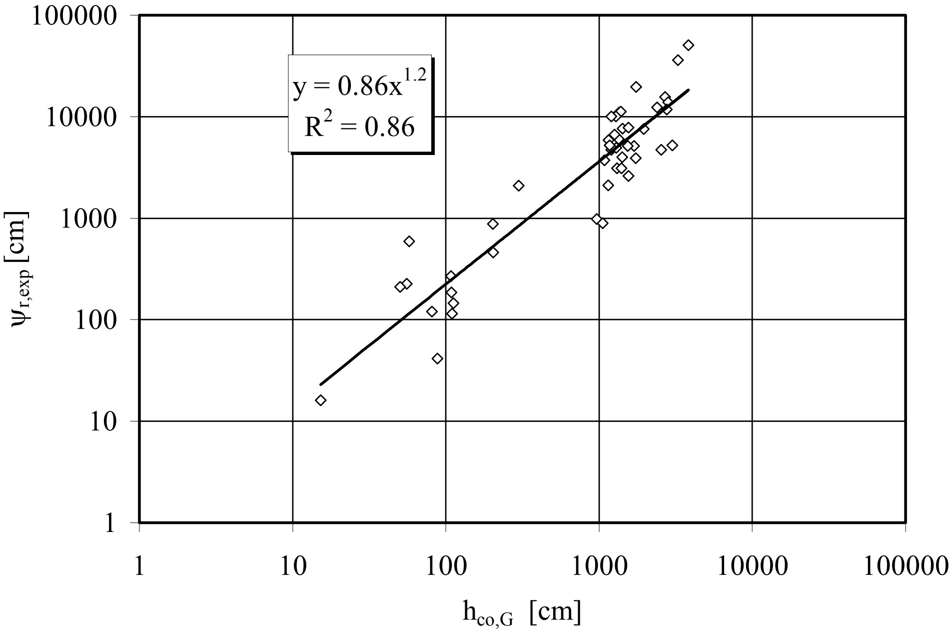

Figure 7. Relationship between the residual suction $\psi_{r,,exp}$ determined from the measured data (and best fit WRC), and the equivalent capillary rise $h_{co,G}$ (eq. 7) for granular materials.

The validity of this equation is confirmed by Figure 8, which shows that equations 19 and 20 give identical residual suction values.

The basic expression obtained for granular soils (eq. 19) is frequently impractical for clayey soils, because D_{10} and C_U are often unknown. Furthermore, the WRC of plastic/cohesive materials is rarely bilinear (around the residual water content) in a semi-log plot, for suction above the air entry value. Considering also that the experimental results are not customarily available at high suctions, it can be argued that the change in the slope on the WRC that indicates the residual suction is generally not well defined (see also discussion by Corey 1994). The graphical determination of the residual suction ψ_r thus becomes very difficult for these soils.

In the following applications of the MK model, it is assumed that the residual suction values ψ_r for plastic/cohesive soils can be related to the equivalent capillary rise $h_{co,P}$ (equation 11) using the same dependency as for granular soils. Hence, the expression used for ψ_r of clayey soils becomes:

[21] $\psi_r = 0.86\left(\frac{\xi}{e}\right)^{1.2} w_L^{1.74}$

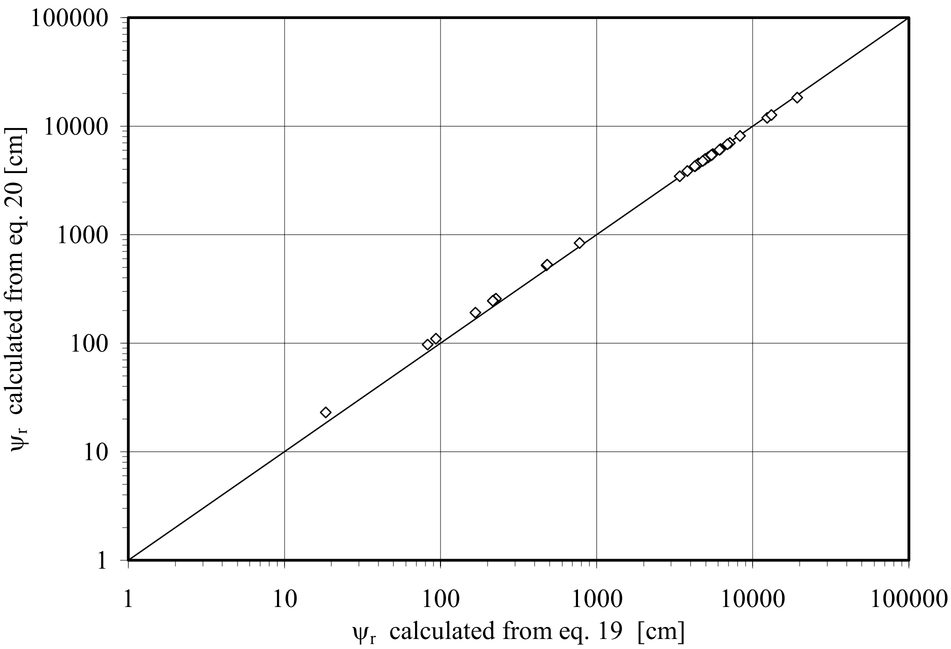

Figure 8. Comparison of the ψ_r values obtained from equations 19 (see Figure 6) and 20 (see Figure 7) for the granular materials identified in Table 1.

4.2 Parameter a_c and m

Once ψ_r was determined for each material in the data base (see Table 1 and 2), the remaining open parameters (a_c and m) value was evaluated from a fitting procedure, so that calculated WRC match experimental data as closely as possible (with the MK model). After repeating the regression calculation for the various results on different materials, further investigations have been conducted to relate the obtained parameter values to basic geotechnical properties. For granular soils, where $h_{co,G}$ is given by equation 7, the analysis of the available results has indicated that the value of the pore-size distribution parameter m can be closely approximated by the inverse of the uniformity coefficient (i.e. $m=1/C_U$). When $C_U = 1$, $m = 1$ and Kovács (1981) original expression for S_c is then recovered from the MK model. This confirms that this newly introduced parameter is closely related to the grain size distribution. For these same granular materials, the analysis indicated that the adhesion coefficient a_c is approximately constant, with a_c= 0.01.

For the plastic/cohesive soils considered here, with $h_{co,P}$ given by equation 11, both m and a_c values can be taken as constants (with m = 3x10^{-5} and a_c = 7x10^{-4}) in the predictive applications. In this case, the influence of grain-size distribution is somewhat superseded by the dominant effect of the surface activity (defined here through the w_L function).

The relationships and parameter values defined for the MK model can now be used to evaluate the WRC of various materials. Tables A-1 and A-2, in Appendix, give sample results to show explicitly how the presented set of equations are used to obtain the WRC. Figures 9 to 15 compare typical fitted (with adjusted a_c and m values) and predicted (with preset a_c and m values) curves using the experimental data obtained for different granular and clayey materials. Identification and basic properties (D_{10}, C_u and e for granular materials; w_L, e and ρ_s for plastic/cohesive soils) are given in each figure. The value of a_c and m that leads to the best fit curves shown in Figure 9 to 15 are given in Table 3. As can be seen on these figures, there is generally a good agreement between the predicted and the measured WRC, despite the differences sometimes observed between the best fit and predicted parameter values (especially with m for loose or clayey materials). Such good agreement is obtained with most of the results identified in Tables 1 and 2. In fact, the predictions made with the MK model compare favorably to those obtained with most other models, which often include more input parameters (and thus require more information for their application), as depicted in recent review publications (e.g. Wösten et al. 2001; Schaap et al. 2001; Rawls et al. 2001; Zhuang et al. 2001). For instance, predicted volumetric water content values, at a given suction, generally approach the measured values within a range of 0.02 to 0.05, which is comparable to the best available pedotransfer functions, that often relies on particular measured points on the WRC (i.e. water content at suctions of 10, 33, or 1500 kPa for instance). It must be said however that the authors have concentrated most of their efforts on the practical development of the model equations for geotechnical applications, than on statistical analyses of the MK model capabilities (and on systematic comparison with other available functions). More work on this aspect will have to be performed in the future. In the meantime, the proposed set of equations has been successfully used by the authors on actual field projects (e.g. Aubertin et al. 1999; Nastev and Aubertin 2000), and also by others on different types of particulate media (outside the authors' data bank). The success obtained so far are due, in part, to the sound physical description of the WRC provided by the MK model.

There are nevertheless some discrepancies between the predicted curves and actual data. Some aspects related to these differences are discussed below, together with other features, capabilities, and limitations of the MK model.

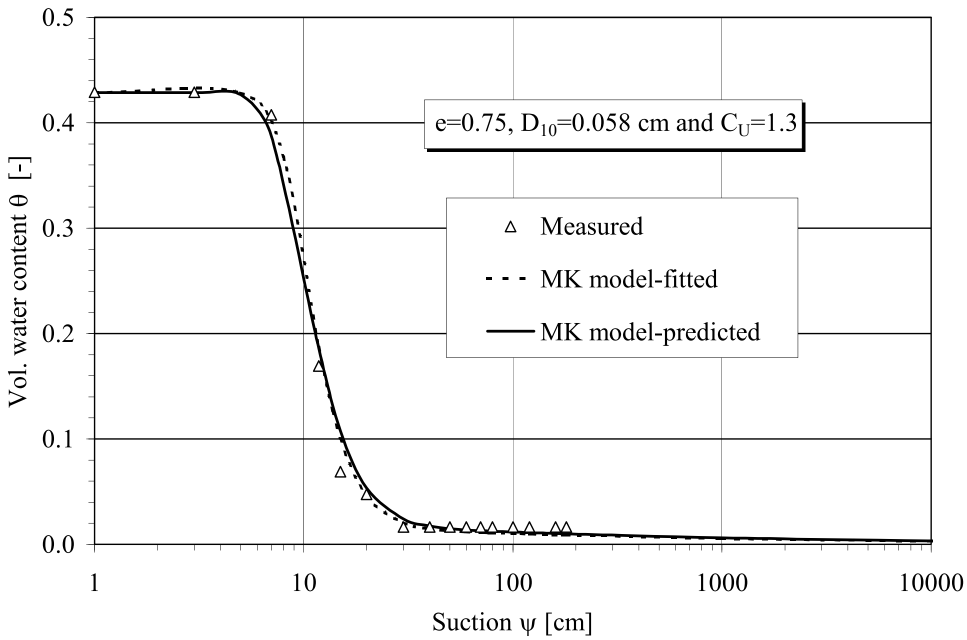

Figure 9 . Application of the MK model to a coarse, uniform, and relatively loose sand (data from Sydor 1992).

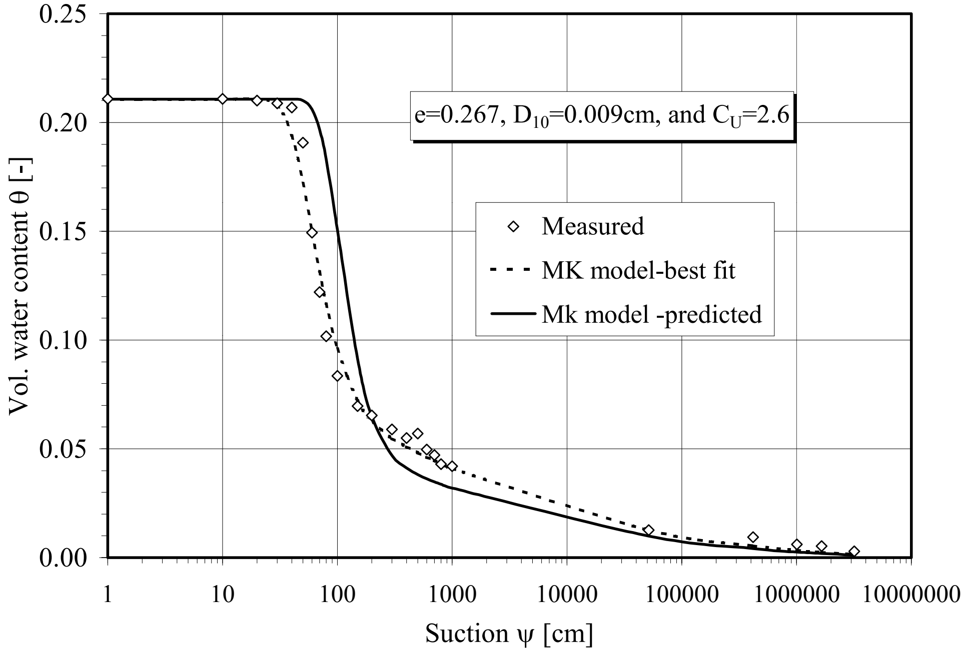

Figure 10. Application of the MK model to a fine, uniform, and dense sand (data from Bruch 1993).

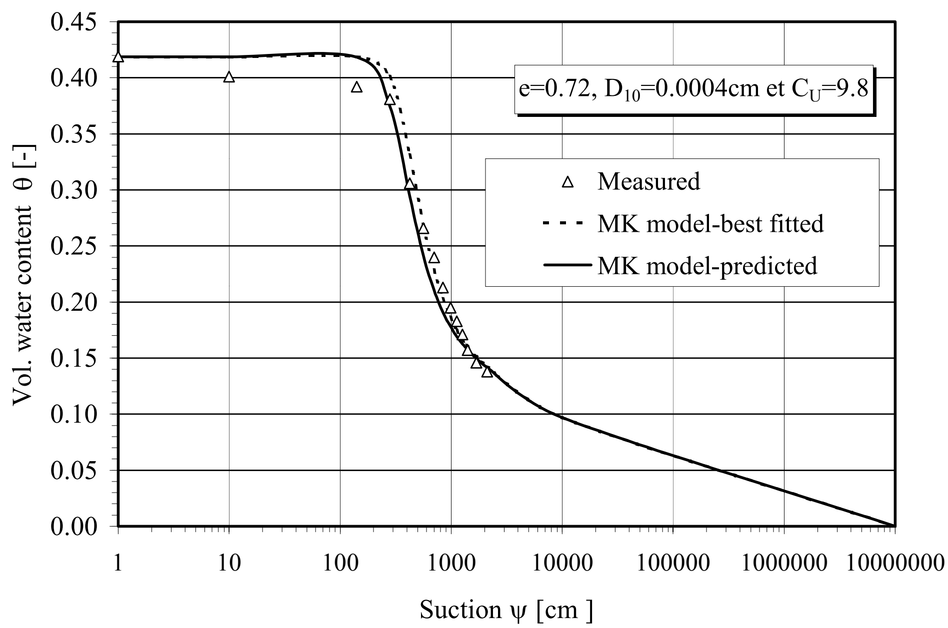

Figure 11. Application of the MK model to tailings Sigma (silty material, well-graded, and loose) (data from authors).

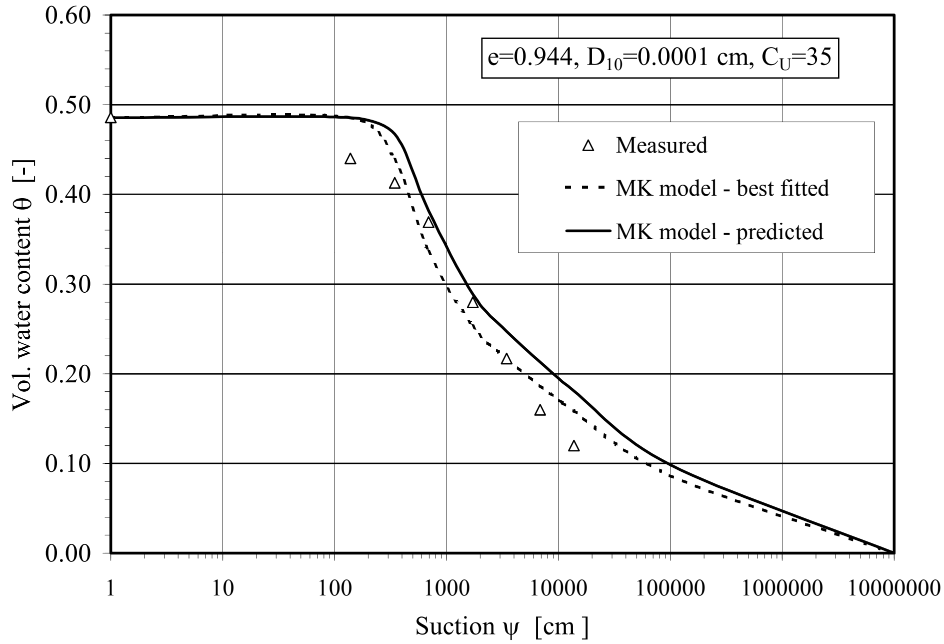

Figure 12. Application of the MK model to tailings Sigma mixed with 10 % bentonite (data from authors).

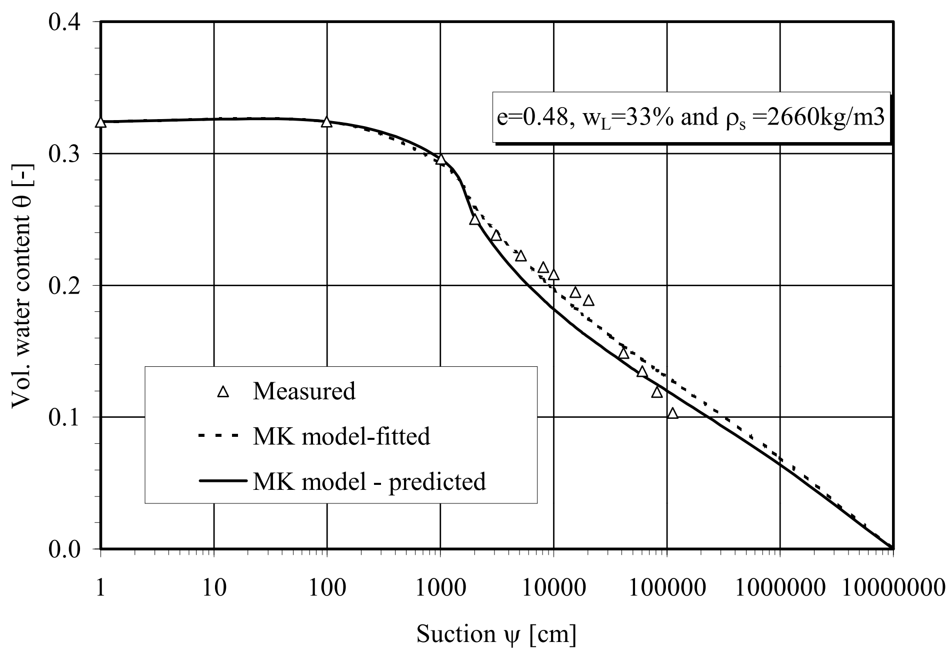

Figure 13. Application of the MK model to Guadalix red silty clay (data from Vanapalli et al. 1998)

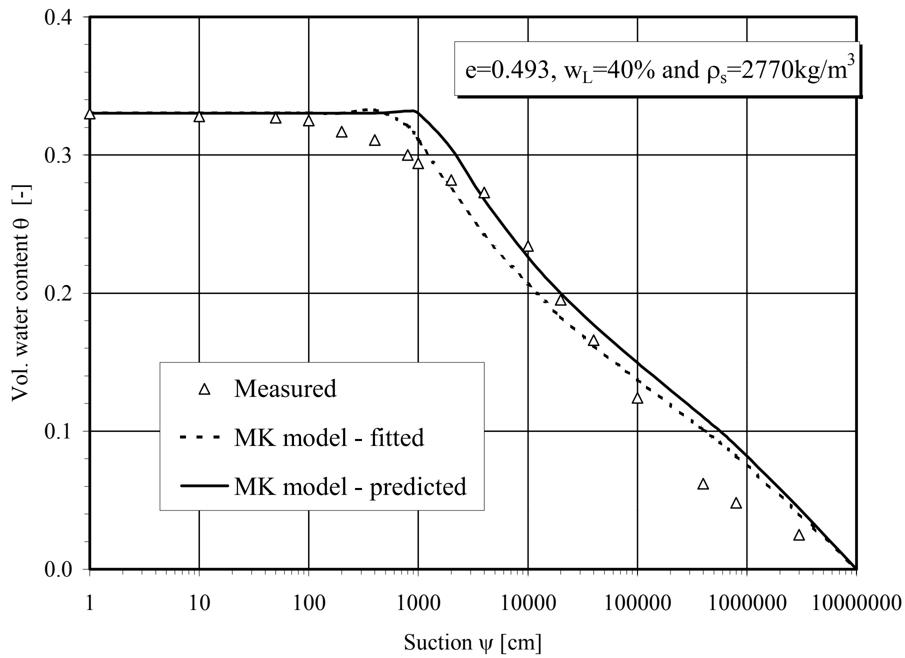

Figure 14. Application of the MK model to a till (data from O'Kane et al. 1998)

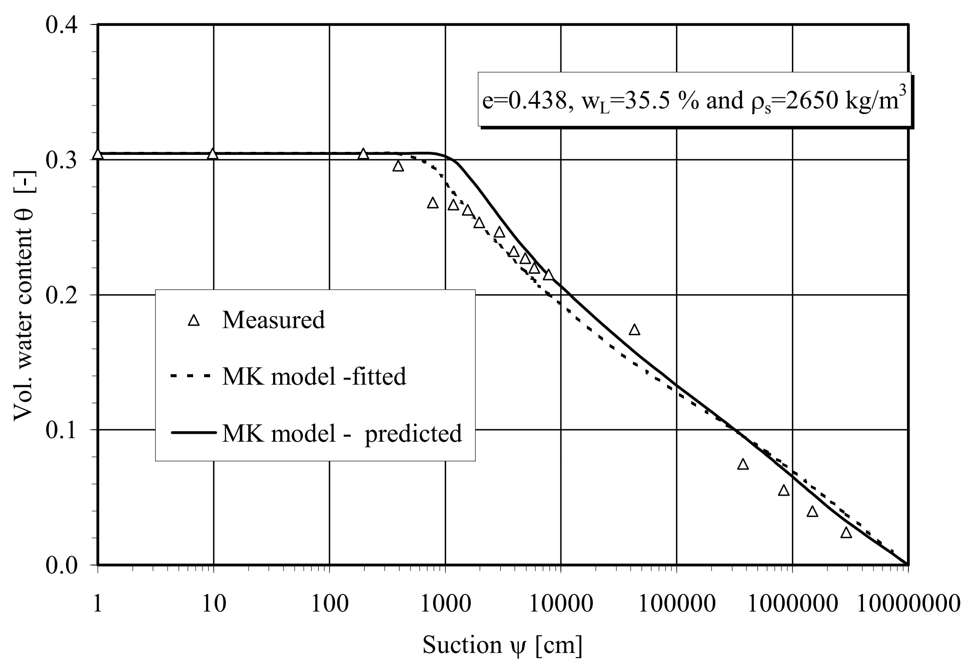

Figure 15. Application of the MK model to Indian Head Till (data from Fredlund 1999)

Table 3. Parameters m and a_c leading to the best fit WRC shown in Figures 9 to 15, and the empirically determined values.

	Fitted WRC		Predicted WRC	
Material	m	a_c	m	a_c
Coarse, uniform and relatively dense sand (data from Sydor 1992), see Figure 9	0.827	0.007	0.769	0.01
Fine and dense sand (data from Bruch 1993), see Figure 10	0.091	0.013	0.388	0.01
Tailings Sigma (silty material, coarse and loose) (Authors data), see Figure 11	0.161	0.010	0.102	0.01
Tailings Sigma mixed with 10% bentonite (Ricard 1994), see Figure 12	0.019	0.009	0.029	0.01
Guadalix Red silty clay (data from Vanapalli et al. 1998), see Figure 13	$3.6x10^{-6}$	$7.6x10^{-4}$	$3.0x10^{-5}$	$7.0x10^{-4}$
Till (data from O'Kane et al. 1998), see Figure 14	$8.1x10^{-6}$	$6.5x10^{-4}$	$3.0x10^{-5}$	$7.0x10^{-4}$
Indian Head Till (Record 728; data from Fredlund 1999), see Figure 15	$1.0x10^{-9}$	$7.0x10^{-4}$	$3.0x10^{-5}$	$7.0x10^{-4}$

5. DISCUSSION

The proposed functions presented above provide a simple means to estimate the WRC for granular and plastic/cohesive soils, as well as for other particulate media. The MK model equations have been developed as an extension of the Kovács' (1981) model, which was selected initially because of its sound physical bases regarding water retention phenomena (Ricard, 1994; Aubertin et al. 1995b, 1998). It is considered important that the model distinguish capillary and adhesion forces as being responsible for moisture retention. The evaluation of these two components for predictive purposes requires only basic geotechnical properties such as the grain size (through D_{10}, C_U), porosity n (or void ratio e), solid grain density (ρ_s), and liquid limit (w_L).

When analyzing the quality of predictions made with the MK model and the discrepancies sometimes observed, one should take into account the variety of the WRC measurement techniques and the associated uncertainties and limitations. These may have a significant effect on the curves and on the estimates for parameters ψ_r, m, and a_c in the MK model. In that regard, it can be expected that the quality of the predictions is influenced by the data themselves. Despite this situation, the accordance between predicted and measured WRC is generally good, and the results can be used for preliminary analysis. The proposed model however shows some limitations in its predictive capabilities. Some of these are discussed in the following.

5.1 Prediction of the residual suction and air entry value

The residual suction ψ_r and the air entry value ψ_a (or AEV) are familiar points defined on the WRC, that are often used for practical applications. The residual suction ψ_r has already been defined above with the MK model. However, the actual value of this suction (and of the corresponding volumetric water content) is difficult to determine precisely, and even its physical meaning has been questioned (e.g. Corey 1994). For the applications shown here, there is more than one approach that can be used to define ψ_r. It can be measured on the fitted curves for the available experimental data (with MK, or with other models such as the van Genuchten 1980 equation used in the preliminary phase of the model development); it corresponds to the intersection point between two tangents (see Fig. 1a). It can also be evaluated on the predicted WRC. When the best fit and predicted curves are close to each other (like in Figure 9), the residual suctions obtained from the two approaches are then almost identical, and correspond well to the value predicted by equation 19. However, when the best fit and predicted curves differ (like in Figure 10 and 12), the residual suction can then take fairly different values depending on the way it is determined.

For the MK model application, equation 19 is used to define suction ψ_r, which is then used in the WRC construction from the set of equations (eq. 14 to 18). This means that the predicted ψ_r (from eq. 19) may be somewhat different than the one deduced from the complete WRC obtained from the model. Furthermore, strictly speaking according to the MK model (for the best fit or predicted WRC), residual suction is reached only when the capillary component S_c goes to (almost) zero, but this condition is not exactly equal to the intersection point between the two tangents. Hence, at this point, one must infer that this specific state can only be approximated with the predictive tools at hand.

A somewhat similar situation can be described for the air entry value. The AEV can also be estimated with the set of equations proposed above. However, more than one method exists to determine the AEV (e.g. Corey, 1994; Fredlund and Xing, 1994; Aubertin et al. 1998), so different values may be obtained for a given information. Furthermore, because the MK model aims at describing the entire WRC, it is not particularly efficient in defining precisely specific points on the curve, such as the AEV. Hence, in this case, one may need to make some adjustments to better reflect the observed ψ_a; the adjustments to the predicted curves may be especially significant for loose (compressible) materials. Fortunately, the ψ_a determined from actual experimental data ($\psi_{a,exp}$), can be related to values obtained from the WRC predicted by the MK model ($\psi_{a,MK}$). A relationship for that purpose is shown in Figure 16. Values $\psi_{a,exp}$ and $\psi_{a,MK}$ were determined using the tangent method (see Figure 1). The curves used to determine $\psi_{a,exp}$ were obtained by fitting the experimental data with a descriptive equation; again the van Genuchten (1980) model was used (N.B. fitted curves with MK would provide essentially the same AEV)). The relationship presented in Figure 16 indicates that the MK model tends to overpredict ψ_a. A correction factor of 0.83 nevertheless allows a fairly good representation of the AEV deduced from experimental data (see best fit line on the figure).

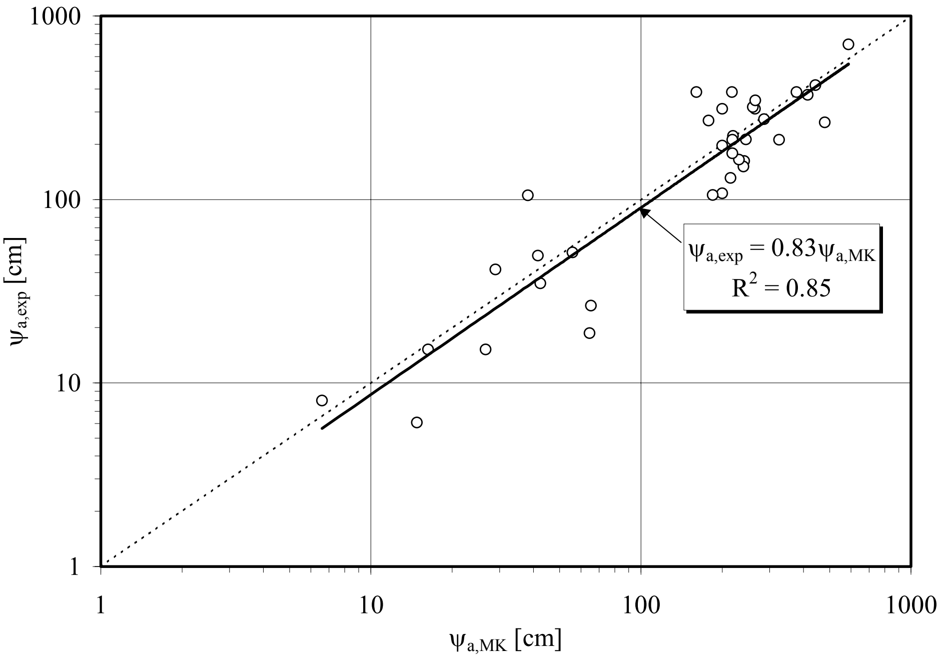

Figure 16. Comparison between the air entry values obtained from the experimental data $\psi_{a,exp}$ (with best fit WRC) and obtained on the WRC predicted by the MK model ($\psi_{a,MK}$) for granular materials.

One can also estimate the experimental AEV using instead some basic geotechnical properties. Various specific relationships have been proposed for that purpose (e.g. Palubarinova-Kochina 1962; Bear 1972; Yanful 1991). According to the authors' own analyses, the AEV obtained with most existing functions tend to agree with each other for coarse grained materials such as sands, but significantly diverge for fine grained materials (Aubertin et al. 1993). A recent investigation by the authors to develop a more representative relationship for granular materials has lead to the following expression (see Figure 17):

[22] $$\psi_{a,est.} = \frac{b_1}{(eD_H)^{x_1}}$$

where b_1 and x_1 are fitting parameters and D_H has been given above in equations 6-8 (i.e. $D_H=[1+1.17\log(C_U)]D_{10}$). As can be seen in Figure 17, almost similar predictions (for the range of data considered here) can be obtained with b_1 = 0.43, x_1 = 0.84 and with b_1 = 0.15, x_1 = 1.0, in equation 22. In this equation, the estimated ψ_a is related explicitly to e, D_{10}, and C_U as was suggested by Aubertin et al. (1998).

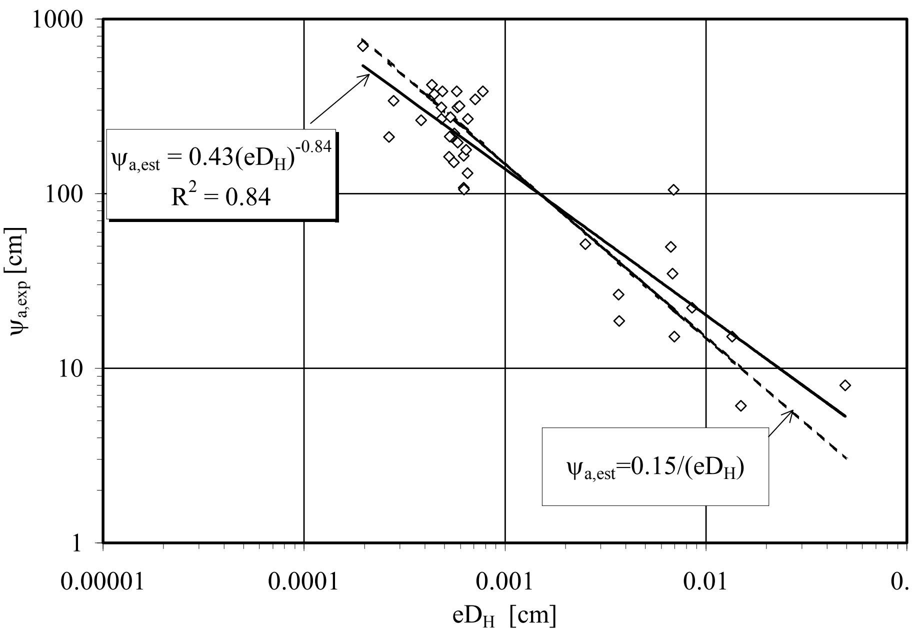

Figure 17. Proposed relationships for estimating the air entry value $\psi_{a,exp}$ obtained on the experimental data for granular materials.

Equation 22 can also be used to estimate the air entry values deduced from the WRC predicted by the MK model (i.e. $\psi_{a,MK}$, with $b_1 = 0.6$, $x_1 = 0.8$ -see Figure 18). These simple relationships can be used to make a correction of the AEV obtained from the MK model, to better reflect the actual measured ψ_a. Hence, this allows the user to improve upon the prediction of this particular condition from the proposed equations.

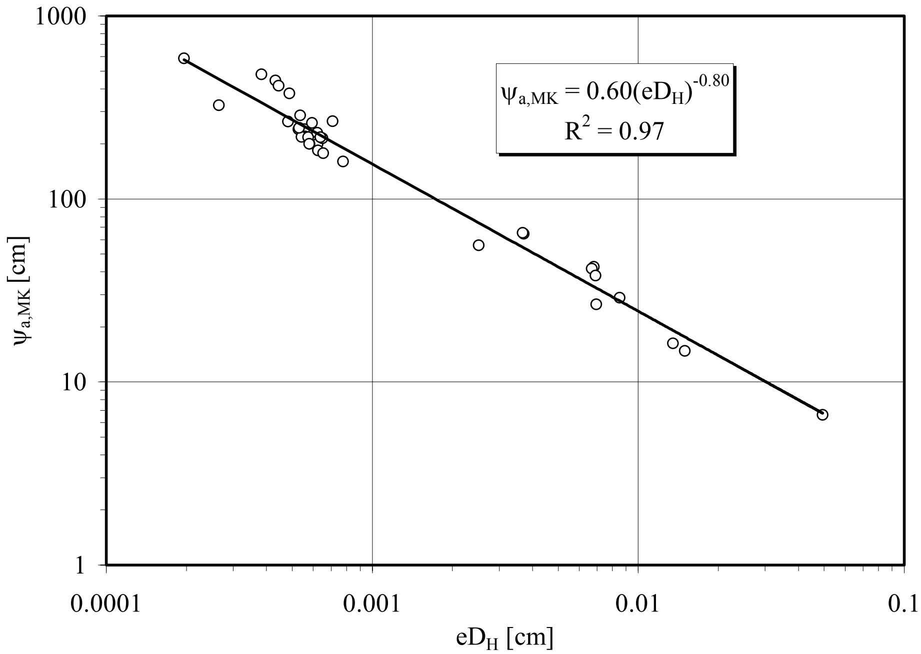

Figure 18. Proposed relationship for estimating the air entry value $\psi_{a,MK}$ obtained on the WRC predicted by the MK model for granular materials.

Also, it was suggested also by the authors (Aubertin et al. 1998) that the AEV could be defined from the suction corresponding to a given degree of saturation S_r. Figure 1 shows two such conditions for S_r equal to 90% and 95%. The predicted air entry value $\psi_{a,MK}$ is compared to calculated suctions corresponding respectively these two degrees of saturation (which do not require extrapolation or any interpretation of the WRC). Figure 19 shows that there is a systematic relationship between $\psi_{a,MK}$ and $\psi_{95,MK}$ and $\psi_{90,MK}$. These results, combined with Figure 16, indicate that the experimental $\psi_{a,exp}$ can be related to the latter two parameters (ex. $\psi_{a,exp}$ = 0.83$\psi_{a,MK}$ =0.83x0.89$\psi_{95,MK}$ =0.74$\psi_{95,MK}$). This in good agreement with previous analyses with other curve fitting equations (of Brooks and Corey 1964 and van Genuchten 1980), which have indicated that the rather steep curvature around the AEV induced by the Kovács and MK models typically tends to overestimate the actual AEV (Aubertin et al. 1998). If a rather conservative approach is required for a given project (i.e. underestimation of the AEV is favored), the authors suggest using ψ_a equal to about 0.6 $\psi_{90,MK,}$ or 0.74 $\psi_{95,MK}$ obtained on the predicted WRC from the MK model.

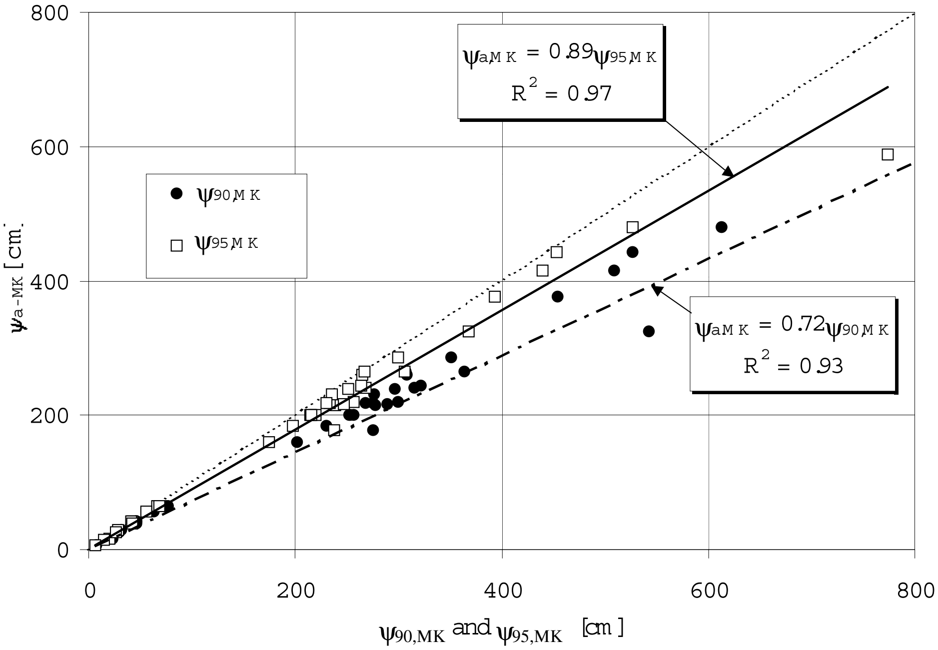

Figure 19. Relationships between the predicted air entry value $\psi_{a,MK}$ and suctions $\psi_{90,MK}$ and $\psi_{95,MK}$ corresponding respectively to degrees of saturation of 90% and 95%, (obtained on the WRC predicted by the MK model) for granular materials.

Finally on this issue, the authors have not yet been successful at developing an acceptable correlation between ψ_a and basic geotechnical properties for plastic/coherent materials (mainly because of the paucity of available results). The potential user should thus be careful when using the MK model for that purpose, in the absence of an independent validation relationship. The approach presented above, associating the AEV and the suction at S_r=0.95 or 0.90 in the MK model, may still be applicable in this case, but too few data have been analyzed at this stage to confirm their validity.

5.2 The equivalent capillary rise h_{co}

The equivalent capillary rise h_{co} constitutes a central parameter in the MK model. This parameter is defined by equations 7 and 9 for granular materials and by equations 11 and 13 for plastic/cohesive soils. This parameter theoretically represents the capillary head in a pore with a diameter equal to the equivalent hydraulic diameter d_{eq} defined according eq. 3. Considering that the pore diameter d_{eq} lies between the limiting values (minimal and maximal diameter) of the pore size distribution, to which a maximum (h_{cmax}) and minimum (h_{cmin}) capillary rise can be associated, it is be expected that h_{co} should lie between h_{cmax} and h_{cmin}.

Because water can only be retained by adhesion forces above h_{cmax}, this elevation could thus be associated to the residual suction ψ_r defined on the drainage water retention curve (e.g. Lambe and Whitman 1979). Therefore, one should observe that $h_{co} \leq \psi_r$. This is in agreement with the relationship between ψ_r and h_{co} given by equation 20. On the other hand, the minimum capillary rise h_{cmin} can be associated the air entry value ψ_a. As shown on Figure 20 for granular materials, the equivalent capillary rise $h_{co,G}$ and the air entry value can be correlated by the following expression:

[23] $$h_{co,G} = b_2 \psi_a^{x_2}$$

From the available data, the authors have obtained $b_2 \approx 5.3$ and $x_2 \approx 1$ when ψ_a is determined from the experimental curve ($\psi_{a,exp}$), and $b_2 \approx 1.7$ and $x_2 \approx 1.2$ when ψ_a is given by the MK predictive model ($\psi_{a,MK}$) at full saturation (see Figure 1). As mentioned before, these relationships indicate that the equivalent capillary rise h_{co} is much larger than the air entry value, which corresponds more or less to the height of the capillary fringe in a homogeneous deposit.

With the MK model, the equivalent capillary rise h_{co} has been defined with a theoretical function of the specific surface area S_m, a property that is rarely evaluated precisely. Because S_m can vary over a wide range (from less than 10 m^2/g to more than 600 m^2/g for various soils; e.g. Marshall et al. 1996), the proportionality between h_{co} and S_m has a major influence on the estimation of h_{co}. In particular, for clayey soils, it has been postulated that the specific surface can be related explicitly to the liquid limit w_L, which only reflects indirectly the surface characteristics of soils. The MK model application presented here thus introduces S_m in a relatively crude (but practical) manner, and this approach can induce a relatively large uncertainty on the value of h_{co}. That is why this parameter should be viewed as a semi-empirical component introduced in an otherwise physically based model; such pragmatic simplification is very useful for engineering applications.

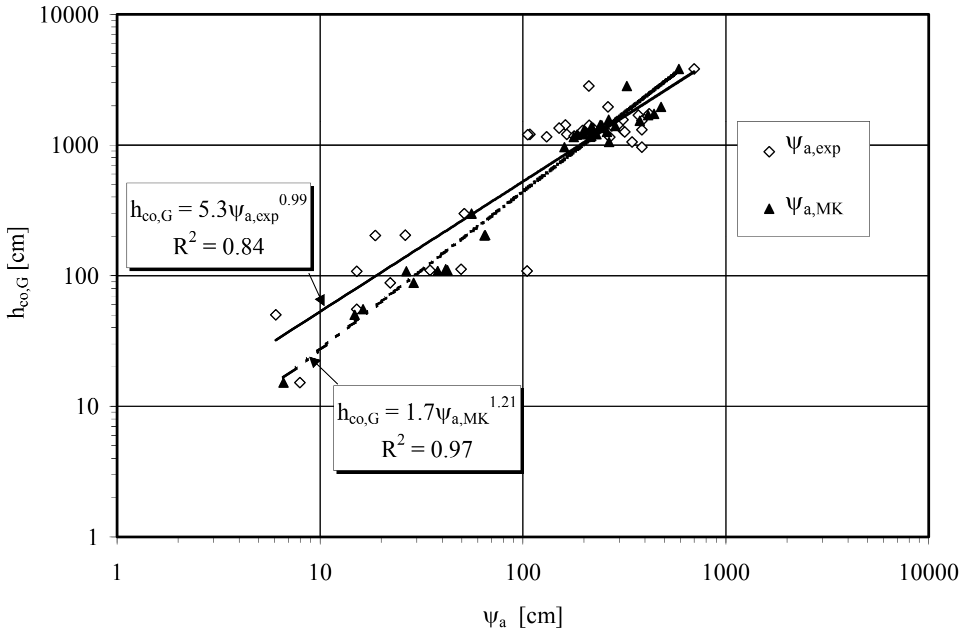

Figure 20. Relationships between the equivalent capillary rise $h_{co,G}$ and the air entry value for granular materials; $\psi_{a,exp}$ and $\psi_{a,MK}$ are obtained on the experimental and on the MK model predicted WRC respectively.

5.3 Parameters with preset values

In the above applications, values have been pre-assigned to the adhesion coefficient a_c (0.01 in the case of granular materials for w_L below about 30 to 40%, and $7x10^{-4}$ in the case of plastic/cohesive materials). When the MK model is adjusted to all the experimental WRC considered in this study, the actual value of a_c leading to best fit curves is not constant but rather varies from 0.001 to 0.024 for granular materials and from $6x10^{-4}$ to $9x10^{-4}$ for clayey soils. Thus, the actual deviation on a_c is fairly high, being above one order of magnitude for granular materials. Equation 17 shows that parameter a_c is linearly related to the adhesion degree of saturation S_a (see Figure 5). In practice however, this variation has an almost negligible effect on the WRC because the water content θ is generally very small in the range of high suction, where the adhesion component S_a prevails. It is also interesting to mention that, even if parameter a_c is greater for granular materials than for plastic/cohesive soils, the adhesion degree of saturation S_a

is more important for the latter than for the former materials, because $h_{co,P}$ (see eq. 7) is much larger than $h_{co,G}$ (see eq. 11). Hence, the value of a_c can not be linked alone to the magnitude of this component of the degree of saturation in the MK model.

A fixed value has been also assigned to the pore-size distribution parameter m (m=3x10^{-5}) in the case of plastic/cohesive materials, while m has been related to the uniformity coefficient C_U (m=1/C_U) for granular materials. For the clayey soils considered here (see Table 2), the value of parameter m leading to best fit curves can actually vary from 5.0x10^{-6} to 1.1x10^{-4}. Figure 3 shows the effect of m on the calculated WRC, for values differing by two orders of magnitude from the preset value of 3x10^{-5}, i.e. m=3x10^{-6} to 3x10^{-4}. One can see on this Figure that such large variation of m induces a much smaller variation of the capillary saturation S_c (defined with equation 15). In fact, it can be shown that the relatively large differences in m values, between the fitted and predicted situations, produce a modest variation on the θ value on the WRC calculated with the MK model. Hence, it has been inferred that this somewhat weak dependency does not, at this point, warrant a more elaborate definition of parameter m.

In the expression (eq. 17) for the adhesion component S_a, a normalization suction ψ_n has been introduced for unit consistencies (to obtain a dimensionless adhesion coefficient a_c). The value of a_c however depends on the selected value of ψ_n. In the presented applications, where ψ and h_{co} are given in cm, ψ_n is fixed at 1 cm (or 10^{-3} atmosphere), so a_c=0.01 for granular soils and a_c=7x10^{-4} for plastic/cohesive materials. If ψ (and h_{co}) is expressed in other units, as it is sometimes done, the value of ψ_n must be converted to the corresponding value. For instance, if ψ and h_{co} are given kPa, one has to use $\psi_n \cong 0.098$ kPa.

In the model development, the specific surface was deemed to depend on the particles shape. This is taken into account here with the shape factor α in equation 5. Such shape can be broadly assessed from the three typical components defined by sphericity, roundness and surface texture (Barret 1980). Sphericity is a measure of how closely the particle shape resembles a sphere, roundness refers to the curvature of the edges, and surface texture relates to local features on the grains surface. The shape factor α introduced here is in fact a sphericity factor (Kovács 1981). Typical values α for some regular geometrical forms are 6 for sphere, 10.4 for cube and octahedron, and 18 for tetrahedron. Generally, the shape factor tends to depend on the grains size for many soils. Kovács (1981) has shown that the shape factor α often becomes higher for finer grained soils. For simplification, a fixed value α of 10 has been used in the MK model for coarse grained materials. This value is considered to be fairly representative of most granular materials, since their sphericity can generally be approximated by more or less rounded or angular cubes and octahedrons. However, most fine grained (clayey) plastic/cohesive soils are made of plate-like particles. Using a constant shape factor α with a fixed value (of 10) is, in this case, not very representative. This flaw is partly overcome in the model, by expressing S_m as a function of the liquid limit w_L, which in a way reflects the interactions between water molecules and the particles. Again, this is a pragmatic choice, like many others that need to be made while constructing pedotransfer functions.

5.4 Final remarks

A fundamental characteristic of the MK model is that the degree of saturation includes a capillary and an adhesion component. The model considers that the adhesive interaction between water and grains is imputable to van der Waals forces alone. This is probably true for most granular materials. For cohesive materials however, it is expected that mineralogical and chemical composition of the grains influence the surface activity and hence, the adhesion of water on the solid particles. As mentioned previously, this aspect is treated indirectly in the MK model through the use of the liquid limit w_L. A somewhat similar approach, that takes into account the influence of surface activity, was proposed by Tinjum et al. (1997) who developed empirical equations relating the van Genuchten parameters to the plasticity index I_P ($I_P = w_L$-w_P, where w_P is the plastic limit) and to compaction conditions (i.e. actual and optimum water contents). The influence of compaction conditions, that may affect the microstructure (fabric) and pore-size distribution is not explicitly taken into account in the MK model and this should be kept in mind by users who may wish to apply it to compacted clay liners for instance.

Various other influence factors have not been considered in the proposed MK model, including hysteresis of the WRC (e.g. Parlange 1976; Mualem 1984), suction induced compressibility (e.g. Vanapalli et al. 1999; Al-Mukhtar et al. 1999; Ng and Pang 2000), stress history (Vanapalli et al.

1999), heterogeneous and/or multimodal pore size distribution (Burger and Shackelford 2001), and presence of very coarse particles (Yazdani et al. 2000). Work is under way to introduce some of these factors into the predictive model.

Additional ongoing work being conducted aims at using MK to evaluate the relative hydraulic conductivity $k_r(\psi)$, by integrating the equations presented above. Function $k_r(\psi)$, with a predictive model of the saturated hydraulic conductivity k_s (Mbonimpa et al. 2000, 2002) will be used to obtain the unsaturated permeability function $k_u(\psi)$.

6. CONCLUSION

The authors have presented a simple set of equations for predicting the water retention curve (WRC). These functions are derived from the Kovács (1981) model, which has been modified to better define the key input parameters, and generalized to extend its application to a variety of porous media, including granular and cohesive soils.

In the proposed model, the degree of saturation includes two components: one created by capillary forces whose contribution is more important at relatively low suction, and one associated to adhesion which mainly contributes to the WRC at large suction. The main advantage of the predictive model is that both components can be evaluated from basic (and generally available) material properties, including the effective diameter D_{10}, uniformity coefficient C_U, liquid limit w_L, void ratio e and solid grain density ρ_s. These properties are used to define the equivalent capillary rise h_{co}, which constitutes the central parameter of the MK model.

The set of equations presented to predict the WRC contains three parameters required for the model application: the residual suction ψ_r, the pore size distribution parameter m, and the adhesion coefficient a_c. In the case of granular materials, a relationship has been developed between the value of ψ_r, determined with the experimental data (and fitted WRC), and their basic geotechnical properties; a simple relationship between ψ_r and h_{co} has also been established Because ψ_r can be very difficult to determine for plastic/cohesive (clayey) soils, the function $\psi_r(h_{co})$ developed for granular materials is also used for clayey soils. The two other parameters, m and a_c, have been obtained first by a curve-fit procedure so that calculated WRC match experimental data as closely as possible with the MK model. Their values were then expressed from basic geotechnical properties.

In all the cases considered here, the MK model allowed a good representation of the experimental WRC for different granular and clayey materials. It can therefore be presented as a useful tool for predicting the WRC during the preliminary phases of a project. The model is not meant to replace the required tests to obtain representative data for particular conditions, but rather provides the potential users with a simple and practical means of foreseeing the possible characteristics of the WRC. The same can be said about the complementary relationships proposed to estimate the air entry value (AEV) from material basic properties. When the experimental data becomes available, these can be used to improve upon the predictive capabilities of the proposed equations, by making the necessary adjustments for specific engineering materials like soils and tailings. The discussion presented at the end of the paper also draws attention to the inherent limitations of the proposed model, especially in regards to the way the specific surface is introduces into the equations.

7. REFERENCES

Alimi-Ichola, I. and Bentoumi, O. 1995. Hydraulic conductivity and diffusivity in vertical and horizontal inflow, In Unsaturated Soils / Sols non saturés. Edited by Alonso, E.E. and Delage, P., Vol. 1, A.A. Balkema: pp. 335-341.

Al-Mukhtar, M., Qi, Y., Alcover, J.-F., and Bergaya, F. 1999. Oedometric and water-retention behavior of highly compacted unsaturated smectites. Canadian Geotechnical Journal, **36**(4): 675-684.

Alonso, E., Gens, A., and Josa, A. 1990. A constitutive model for partially saturated soils, Géotechnique, **40**(3): 405-430.

Arya, L.M., and Paris, J.F. 1981. A physico-empirical model to predict the soil moisture - characteristic from particle size distribution and bulk density data. Soil Science Society of America Journal, **45**:1023-1030.

Arya, L.M., Leij, F.J., van Genuchten, M.T., and Shouse, P.J. 1999. Scaling parameter to predict the soil water characteristic from particle-size distribution data. Soil Science Society of America Journal, **63**:510-519.

Aubertin, M., Chapuis, R.P., Bussière, B. et Aachib, M. 1993. Propriétés des résidus miniers utilisés comme matériaux de recouvrement pour limiter le DMA. In Symposium International Géologie et Confinement des Déchets Toxiques, Géoconfine'93, Edited by Arnould, Barrès & Cômes, Balkema, 299-308.

Aubertin, M., Chapuis, R.P., Aachib, M., Bussière, B., Ricard, J.F, and Tremblay, L. 1995a. Évaluation en laboratoire de barrières sèches construites à partir de résidus miniers. École Polytechnique de Montréal, MEND Project 2.22.2a, CANMET, Ottawa, 164 pp.

Aubertin, M, Ricard, J.F. and Chapuis, R.P. 1995b. A study of capillary properties of mine tailings: Measurements and modeling. Proc. 48th Canadian Geotechnical Conference, Vacouver, pp. 17-24.

Aubertin, M., Bussière, B., Aachib, M., Chapuis, R.P, and Crespo, J.R. 1996. Une modélisation numérique des écoulements non saturés dans des couvertures multicouches en sols. Hydrogéologie, **1**: 3-13.

Aubertin, M., Aachib, M., Monzon, M., Joanes, A.M., Bussière, B., Chapuis, R.P. 1997. Étude en laboratoire sur l'efficacité des barrières de recouvrement construites à partir de résidus miniers. Rapport de recherche. Projet CDT 1899.3 (version préliminaire décembre 1997 – version finale mars 1999). Rapport NEDEM/MEND 2.22.2b, 110 pp.

Aubertin, M., Ricard, J.-F., and Chapuis, R.P. 1998. A predictive model for the water retention curve: application to tailings from hard-rock mines, Canadian Geotechnical Journal, **35**: 55-69.

Aubertin, M., Bussière, B. Monzon, M., Joanes, A.-M., Gagnon, D., Barbera, J.-M., Aachib, M.,Bédard, C. Chapuis, R.P. and Bernier, L. 1999. Étude sur les barrières sèches construites à partir des résidus miniers. – Phase II : Essais en place. Rapport de recherche Projet CDT P1899. Rapport NEDEM/MEND 2.22.2c, 331 pp.

Badv, K., and Rowe, R.K. 1996. Contaminant transport through a soil liner underlain by an unsaturated stone collection layer. Canadian Geotechnical Journal, **33**(3) : 416-430.

Barbour, S.L. 1998. Nineteenth Canadian Geotechnical Colloqium : The soil-water characteristic curve : a historical perspective. Canadian Geotechnical Journal, **35**(5) : 873-894.

Barret, P. J. 1980. The shape of rock, a critical review. Sedimentology, **27**: 291–303.

Bear, J. 1972. Dynamics of Fluids in Porous Media, Dover Publications Inc., New York.

Bowles, J.E. 1984. Physical and Geotechnical Properties of Soils. 2nd Edition, McGraw-Hill Book Company, New York.

Brooks, R.H., and Corey, A.T. 1964. Hydraulic properties of porous media. Colorado State University, Fort Collins, Co. Hydrology Paper N° 3.

Bruch, P.G. 1993. A laboratory study of evaporative fluxes in homogeneous and layered soils. Thesis of the degree of Master Science. Department of Civil Engineering, University of Saskatchewan.

Brutsaert. W. 1966. Probability laws for pore size distributions. Soil Science, **101**: 85-92.

Bumb, A.C., Murphy, C.L, and Everett, L.G. 1992. A comparison of three functional forms to representing soil moisture characteristics. Ground Water, **3**: 177-185.

Burger, C.A., and Shackelford, C.D. 2001. Evaluating dual porosity of pelletized diatomaceous earth using bimodal soil water characteristic curve functions. Canadian Geotechnical Journal, **38**(1) : 53-66.

Bussière, B. 1999. Étude du comportement hydrique de couvertures avec effet de barrière capillaire inclinées à l'aide de modélisations physiques et numériques. Thèse de doctorat, École Polytechnique de Montréal.

Carter, M.R. 1993. Soil sampling and methods of analysis, Lewis, Boca Raton, Fla.

Celia, M.A., Reeves, P.C., and Ferrand, L.A. 1995. Pore scale models for multiphase flow in porous media. U.S. Natl. Rep. Int. Union Geod. Geophys. 1991-1994, Rev. Geophys., **33**: 1049-1057.

Chapuis, R.P., and Aubertin, M. 2001. A simplified method to estimate of saturated and unsaturated seepage through dikes under steady state conditions. Canadian Geotechnical Journal, **38**(6): 1321-1328.

Chapuis, R.P., and Légaré, P.P. 1992. A simple method for determining the surface area of fine aggregates and fillers in bituminous mixtures. In Effects of aggregates and mineral filler on asphalt mixture performance, ASTM STP, **1147** : 177-186.

Chapuis, R.P., Aubertin, M., and Crespo, R. 1996. Débits de fuite saturés et non saturés à travers des digues homogènes, des digues à noyau, et des tapis d'étanchéité. *In* Proceedings of the 49th Canadian Geotechnical Conference, St-John's, pp. 887-894.

Chapuis, R.P., Chenaf, D., Bussière, B., Aubertin, M. and Crespo, R. 2001. A user's approach to assess numerical codes for saturated and unsaturated seepage conditions. Canadian Geotechnical Journal, **38**(5): 1113-1126.

Chin, D.A. 2000. Water-Resources Engineering, Prentice Hill, Upper Saddle River, N.J.

Corey, A.T. 1994. Mechanics of Immiscible Fluids in Porous Media. Water Resources Publications, Colorado.

Cosby, B.J., Hornberger,G.M., Clapp, R.B., and Ginn, T.R. 1984. A statistical exploration of the relationships of soil moisture characteristics to the physical properties of soils. Water Resources Research, **20**: 682-690.

Cui, Y.J., and Delage, P. 1996.. Yielding and plastic behaviour of unsaturated compacted silt. Géotechnique, **46**(2): 291:311.

Elsenbeer, H. 2001. Editorial: Pedotransfer fuctions in hydrology. Journal of Hydrology (Special Issue), **251**(3-4): 121-122.

Esposito, G., Allersma, H.G.B., and Selvadurai, A.P.S. 1999. Centrifuge modeling of LNAPL transport in partially saturated sand. Journal of Geotechnical and Geoenvironmental Engineering, ASCE, **125**(12): 066-1071.

Fredlund, D.G. 1987. The prediction and performance of structures on expansive soils. Proceedings, International Symposium on Prediction and Performance in Geotechnical Engineering, Calgary, 51-60.

Fredlund, M.D. 1999. Soilvision 2.0, A knowledge-based database system for unsaturated-saturated soil properties, version 2.0n. Soilvision Systems Ltd.

Fredlund, D.G. 2000a. The 1999 R.M. Hardy Lecture: The implementation of unsaturated soil mechanics into geotechnical engineering. Canadian Geotechnical Journal, **37**: 963-986.

Fredlund, M. D. 2000b. The role of unsaturated soil properties functions in the practice of unsaturated soil mechanics, Ph.D Dissertation, University of Saskatchewan, Saskatoon, Saskatchewan, Canada.

Fredlund, D.G., and Rahardjo, H. 1993. Soil Mechanics for Unsaturated Soils. John Wiley & Sons, Inc., New York, N.Y.

Fredlund, D.G., and Xing, A. 1994. Equations for the soil-water characteristic curve. Canadian Geotechnical Journal, **31**(4) : 521-532.

Fredlund, D.G., Xing, A., and Huang, S. 1994. Predicting the permeability function for unsaturated soils using the soil-water characteristic curve. Canadian Geotechnical Journal, **31**: 533-546.

Fredlund, D.G., Xing, A., Fredlund, M.D., and Barbour, S.L. 1996. The relationship of the unsaturated soil shear strength to the soil-water characteristic curve. Canadian Geotechnical Journal, **33**: 440-448.

Fredlund, M.D., Fredlund, D.G., and Wilson, G.W. 2000. An equation to represent grain-size distribution. Canadian Geotechnical Journal, **37**: 817-827.

Gardner W.R. 1956. Calculation of capillary conductivity from pressure plate outflow data. Soil Science Society of America Journal, **3**: 317-320.

Gupta, S.C., and Larson, W.E. 1979. Estimating soil-water retention characteristics from particle size distribution, organic matter percent, and bulk density. Water Resources Research, **15**(6): 1633-1635.

Haverkamp, R., and Parlange, J.-Y. 1986. Predicting the water retention curve from particle size distribution: 1. Sandy soils without organic matter. Soil Science, **142**: 325-339.

Haverkamp, R., Vauclin, M., Touma, J., Wierenga, P, and Vauchaud, G. 1977. A comparison of numerical simulation models for one-dimensional infiltration. Soil Science Society of America Journal, **41**: 285-294.

Haverkamp, R., Bouraoui, F., Zammit, C., and Angulo-Jaramillo, R. 1999. Soil properties and moisture movement in the unsaturated zone, In The Handbook of Groundwater Engineering, Edited by J.W. Delleur, CRC Press New York: 5.1-5.47.

Huang, S., Barbour, S.L., and Fredlund, D.G. 1998. Development and verification of a coefficient of permeability function for a deformable unsaturated soil. Canadian Geotechnical Journal, **35**(3) : 411-425.

Hung, V.Q., and Fredlund, D.G. 2000. Volume change predictions in expansive soils using a two dimensional finite element method. Proceedings of the Asian Conference on Unsaturated Soils, UNSAT-ASIA 2000, pp.231-236.

Igwe, G.J.I. 1991 Powder Technology and Multiphase systems. Gas Permeametry and Surface Area Measurement. Ellis Horwood, New York.

Khire, M., Meerdink, J., Benson, C., and Bosscher, P. 1995. Unsaturated hydraulic conductivity and water balance predictions for earthen landfill final covers. In Soil suction applications in geotechnical engineering practice, Edited by W. Wray and S. Houston, eds. ASCE, New York, N.Y., 38-57.

Kissiova, M. 1996. Étude des modèles de prédiction de la conductivité hydraulique des matériaux meubles non-saturés. Master of Engineering Thesis (M. Ing.), Dept. Mineral Engng, École Polytechnique de Montréal, 271 pp.

Klute, A. 1986. Water retention: laboratory methods. In Methods of soil analysis, part I. Physical and Mineralogical Methods, 2nd ed. Edited by A. Klute. Agronomy Monograph No 9, American Society of Agronomy, Soil Science Society of America, Madison Wis.: 635-662.

Konrad, J.-M., and Roy, M. 2000. Flexible pavements in cold regions: a geotechnical perspective. Canadian Geotechnical Journal, 37(3) : 689-699.

Kosugi, K. 1994. Three-parameter lognormal distribution model for soil water retention. Water Resources Research, **30**(4): 891-901.

Kovács, G. 1981. Seepage Hydraulics. Elsevier Science Publishers, Amsterdam.

Lambe, T.W. and Whitman, R.V. 1979. Soil mechanics, SI Version. John Wiley & Sons, New York.

Leij, F.J., Russel, W.B., and Lesch, S.M. 1997. Closed-form expressions for water retention and conductivity data. Ground Water, **35**(5): 848-858.

Leong, E.C. and Rahardjo, H. 1997a. Permeability functions for unsaturated soils. Journal of Geotechnical and Geoenvironmental Engineering, ASCE, **123**(12): 1118-1126.

Leong, E.C. and Rahardjo, H. 1997b. Review of soil-water characteristic curve functions, Journal of Geotechnical and Geoenvironmental Engineering, ASCE, **123**(12): 1106-1117.

Lim, P.C., Barbour, S.L., and Fredlund, D.G. 1998. The influence of degree of saturation on the coefficient of aqueous diffusion. Canadian Geotechnical Journal, **35**(5) : 811-827.

Lowell, S., and Shields, J.E. 1984 Powder Surface Area and Porosity. 2nd Edition. Chapman and Hall, London.

MacKay, P.C. 1997. Evaluation of oxygen diffusion in unsaturated soils. Thesis of the degree of Master of Engineering Science, University of Western Ontario, New York.

Marshall, T.J., Holmes, J.W. and Rose, C.W. 1996. Soil Physics, Third Edition, Cambridge University Press.

Mbonimpa, M., Aubertin, M., Chapuis, R.P., and Bussière, B. 2000. Développement de fonctions hydriques utilisant les propriétés géotechniques de base. Proceedings, 1st Joint IAH-CNC and CGS Groundwater Specialty Conference, 53rd Canadian Geotechnical Conference, Montreal, Quebec, Canada, pp. 343-350.

Mbonimpa, M., Aubertin, M., Chapuis, R.P., and Bussière, B. 2002. Practical pedotransfer functions for estimating the saturated hydraulic conductivity. Geotechnical and Geological Engineering, **20**: 235-259.

McCarthy, D.F. 1998. Essentials of Soil Mechanics and Foundations, Basic Geotechnics. 5th Edition, Prentice Hall, Upper Saddle River, New Jersy.

Meyer, P.D., and Gee, G.W. 1999. Advective-diffusion contaminant migration in unsaturated sand and gravel. Journal of Geotechnical and Geoenvironmental Engineering, ASCE, **122**(12): 965-975.

Morris, C.E., and Stormont, J.C. 1999. Parametric study of unsaturated drainage layers in a capillary barrier. Journal of Geotechnical and Geoenvironmental Engineering, ASCE, **125**(12): 1057-1065.

Mualem, Y. 1976. A new model for predicting the hydraulic conductivity of unsaturated porous media. Water Resources Research, **12**: 513-522.

Mualem, Y. 1984. A modified dependent-domain theory of hysteresis. Soil Science, **137**: 283-291.

Mualem, Y. 1986 Hydraulic conductivity of unsaturated soil: Prediction and formulas. In: Klute (ed.), Methods of soil analysis, Part I. ASA and SSSA, Madison, WI. Agronomy Mongr., **9**: 799-823.

Nastev, M. and Aubertin M. 2000. Hydrogeological modelling for the reclamation work at the Lorraine Mine Site, Québec. Proceedings, 1st Joint IAH-CNC-CGS Groundwater Specialty Conference, Montreal, Quebec, Canada, pp. 311-318.

Ng, C.W.W. and Pang, Y.W. 2000. Experimental investigations of the soil-water characteristics of a volcanic soil. Canadian Geotechnical Journal, **37**: 1252-1264

Nicholson, R.V., Gillham, R.W., Cherry, J.A., and Reardon, E.J. 1989. Reduction of acid generation in mine tailings through the use of moisture-retaining layers as oxygen barriers. Canadian Geotechnical Journal, **26**: 1-8.

Nitao, J.J., and Bear, J. 1996. Potentials and their role in transport in porous media, Water Resources Research **32**(2): 225-250.

O'Kane, M. Wilson, G.W. and Barbaour S.L. 1998. Instrumentation and monitoring of an engineered soil cover system for mine waste rock. Canadian Geotechnical Journal, **35**(5) : 828-846.

Palubarinova-Kochina, P.Y. 1962. Theory of Ground water Movement (in Russian). English transl. By R.J.M. De Wiest, Princeton University Press, Princeton, N.J.

Parlange, J.-Y. 1976. Capillary hysteresis and the relationship between drying and wetting curves. Water Resources Research, **12**(2) : 224-228.

Rampino, C., Mancuso, C., and Vinale, F. 2000. Experimental behaviour and modeling of an unsaturated compacted soil. Canadian Geotechnical Journal, 37(4) : 748-763.

Rassam, D.W. and Williams, D.J. 1999. A relationship describing the shear strength of unsaturated soils. Canadian Geotechnical Journal, **36**(2) : 363-368

Rawls, W.J., Brackensiek, D.L. 1982. Estimating soil water retention from soil properties. Journal of Irrigation and Drainage Division, **108**: 166-171.

Rawls, W.J., Gish, T.J., and Brakensiek, D.L. 1991. Estimating soil water retention from soil physical properties and characteristics. Advances in Soil Science, **16**: 213-234.

Rawls, W.J., Pachepsky Y.A., and Shen, M.H. 2001. Testing soil water retention estimation with the MUUF pedotransfer model using data from the southern United States. Journal of Hydrology, **251**: 177-185.

Ricard, J.F. 1994. Étude en laboratoire de la relation capillaire et de la conductivité hydraulique des résidus miniers. Mémoire de Maîtrise, Ecole Polytechnique.

Richards, L.A. 1931. Capillary conduction of liquids in porous mediums. Physics, **1**:318-333.

Schaap,M.G., and Leij, J.F. 1998. Database related accuracy and uncertainty of pedotransfer functions, Soil Science, **163**: 765-779.

Schaap, M.G., Leij, F.J., van Genuchten, M.Th. 1998. Neural network analysis for hierarchical prediction of soil water retention and saturated hydraulic conductivity. Soil Science Society of America Journal, **64**: 843-855.

Schaap, M.G., Leij, F.J., van Genuchten, M.Th. 2001. ROSETTA: a computer program for estimating soil hydraulic parameters with hierarchical pedotransfer functions. Journal of Hydrology, **251**: 163-176.

Scheidegger, A.E. 1974. The Physics of Flow Through Porous Media, 3rd Ed., University of Toronto Press.

Smith, G.N. 1990. Elements of Soil Mechanics, 6th edition, BSP Professional Books, Oxford.

Sydor, R.C. 1992. Engineered mine tailings cover : verification of drainage behaviour and investigations of design. Thesis of the degree of Master Science in Earth Science, University of Waaterloo.

Thu, T.M, and Fredlund, D.G. 2000 Modelling subsidence in Hanoi City area, Vietnam. Canadian Geotechnical Journal, **37**(7) : 621-637.

Tietje,O., and Tapkenhinrichs, M. 1993. Evaluation of pedotransfer functions. Soil Science Society of America Journal, 57:1088-1095

Tinjum, J.M., Benson, C.H., and Blotz ,L.R. 1997. Soil-water characteristic curves for compacted clays. Journal of Geotechnical and Geoenvironmental Engineering, **123**(11). 1060-1069.

Tyler, S.W., and Wheatcraft, S.W. 1989. Application of fractal mathematics to soil water retention estimation. Soil Science Society of America Journal, **53**(4): 987-996.

Tyler, S.W., and Wheatcraft, S.W. 1990. Fractal processes in soil water retention. Water Resources Research, **26**(5): 1047-1054.

van Genuchten, M.Th. 1980. A closed-form equation for predicting the hydraulic conductivity of unsaturated soils. Soil Science Society of America Journal, **44**: 892-898.

van Genuchten, M.TH., Leij, F.J., and Yates, S.R. 1991. The RETC code for quantifying the hydraulic functions of unsaturated soils. EPA/600/2-91/065.

Vanapalli, S.K., Fredlund, D.G., Pufahl, D.E., and Clifton, A.W. 1996. Model for the prediction of shear strength with respect to soil suction. Canadian Geotechnical Journal, **33**: 379-392.

Vanapalli, S.K., Fredlund , D.G., and Pufahl, D.E. 1999. The influence of soil structure and stress history on the soil water characteristics of a compacted till. Geotechnique, **49**: 143-159.

Vauclin, M., Khanji, D., and Vachaud, G. 1979. Experimental and numerical study of a transient two-dimensional unsaturated-saturated watertable recharge problem. Water Resources Research, **15**: 1089-1101.

Vereecken, H., Diels, J., van Orshoven, J., Feyen, J., and Bouma, J. 1992. Functional evaluation of pedotransfer functions for the estimation of soil hydraulic properties. Soil Science Society of America Journal, **56**: 1371-1378.

Wösten, J.H.M., Pachepsky, Ya. A., and Rawls, W.J. 2001. Pedotransfer functions: bridging the gap between available basic soil data and missing soil hydraulic characteristics. Journal of Hydrology, **251**: 123-150.

Yanful, E.K. 1991. Engineered soil covers for reactive tailings management: Theoretical concepts and laboratory development. Proceedings of the 2nd International Conference on the Abatement of Acidic Drainage. Montréal, Québec, Tome 1: 461-485

Zhuang, J., Jin, Y., and Miyazaki, T. 2001. Estimating water retention characteristic from soil particle-size distribution using a non-similar media concept. Journal of Hydrology, **251**: 308-321.

8. APPENDIX: SAMPLE CALCULATIONS

Table A-1 Typical calculation results to predict the WRC of granular materials: Case of Sigma tailings (see Figure 11- data from Authors).

D_{10} [cm]	e [-]	C_U [-]	b (eq. 8) [cm^2]	$h_{co,G}$ (eq. 7) [cm]
0.0004	0.72	9.8	0.347	1206

ψ_r (eq. 19) [cm]	ψ_0 (fixed) [cm]	ψ_n (fixed) [cm]	a_c (fixed) [-]	$m = 1/C_U$ [-]
40603	$1x10^7$	1	0.01	0.102

Suction [cm]	S_c (eq. 15) [-]	C_ψ (eq. 18) [-]	S_a (eq. 17) [-]	S_a* (eq. 16) [-]	S_r (eq. 14) [-]	Predicted θ (eq. 14) [-]	Measured θ [-]
1	1.000	1.000	1.264	1.000	1.000	0.419	0.419
10	1.000	1.000	0.861	0.861	1.000	0.419	0.401
141	0.999	0.996	0.552	0.552	1.000	0.418	0.392
281	0.793	0.992	0.490	0.490	0.895	0.374	0.381
422	0.455	0.989	0.456	0.456	0.704	0.295	0.306
563	0.254	0.985	0.433	0.433	0.577	0.242	0.266
703	0.148	0.982	0.416	0.416	0.502	0.210	0.24
844	0.090	0.978	0.402	0.402	0.456	0.191	0.213
984	0.058	0.975	0.391	0.391	0.426	0.178	0.195
1125	0.038	0.972	0.381	0.381	0.405	0.169	0.183
1266	0.026	0.968	0.372	0.372	0.389	0.163	0.171
1406	0.019	0.965	0.365	0.365	0.376	0.158	0.157
1688	0.010	0.959	0.351	0.351	0.358	0.150	0.146
2110	0.004	0.951	0.336	0.336	0.339	0.142	0.138
10000	0.000	0.850	0.231	0.231	0.231	0.097	
10000000	0.000	0.000	0.000	0.000	0.000	0.000	

Table A-2 Typical calculation results to predict the WRC of plastic/cohesive soils: Case of Indian Head Till (See Figure 15- Record 3728-data from Fredlund 1999).

w_L [%]	e [-]	ρ_s [kg/m^3]	ξ (eq. 12) [cm^2]	$h_{co,P}$ (eq. 11) [cm]
35.5	0.438	2650	394.8	159489

ψ_r (eq. 21) [cm]	ψ_0 [cm]	ψ_n (fixed) [cm]	a_c (fixed) [-]	m (fixed) [-]
1505828	$1x10^7$	1	$7x10^{-4}$	$3x10^{-5}$

Suction [cm]	S_c (eq. 15) [-]	C_ψ (eq. 18) [-]	S_a (eq. 17) [-]	S_a* (eq. 16) [-]	S_r (eq. 14) [-]	Predicted θ (eq. 14) [-]	Measured θ [-]
1	1.000	1.000	2.778	1.00	1.000	0.305	0.305
9.8	1.000	1.000	1.899	1.00	1.000	0.305	0.305
196	1.000	1.000	1.152	1.00	1.000	0.305	0.305
392	1.000	1.000	1.027	1.00	1.000	0.305	0.295
784	0.916	1.000	0.914	0.91	0.993	0.302	0.268
1176	0.668	0.999	0.855	0.85	0.952	0.290	0.267
1568	0.462	0.999	0.814	0.81	0.900	0.274	0.263
1960	0.328	0.999	0.784	0.78	0.855	0.260	0.254
2940	0.161	0.999	0.733	0.73	0.776	0.236	0.247
3920	0.094	0.998	0.698	0.70	0.727	0.221	0.232
4900	0.061	0.998	0.672	0.67	0.692	0.211	0.227
5880	0.043	0.997	0.652	0.65	0.667	0.203	0.220
7840	0.024	0.996	0.621	0.62	0.630	0.192	0.215
43120	0.001	0.979	0.459	0.46	0.460	0.140	0.175
372400	0.000	0.850	0.278	0.28	0.278	0.085	0.075
838880	0.000	0.720	0.206	0.21	0.206	0.063	0.055
1486660	0.000	0.592	0.154	0.15	0.154	0.047	0.040
2916480	0.000	0.406	0.094	0.09	0.094	0.029	0.024
10000000	0.000	0.000	0.000	0.00	0.000	0.000	0.000

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