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STOCHASTIC BILEVEL MODELS FOR REVENUE MANAGEMENT IN THE HOTEL INDUSTRY

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STOCHASTIC BILEVEL MODELS FOR REVENUE MANAGEMENT IN THE HOTEL INDUSTRY

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This thesis is dedicated to Paulina & Francisca,
You are always in my heart ...

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RÉSUMÉ

La Gestion du Revenu consiste à maximiser le revenu des compagnies. Cette technique est pratiquée, entre autres, dans les secteurs de l'aéronautique, des télécommunications et de l'hôtellerie. Dans cette thèse, nous développons et résolvons un modèle stochastique biniveau pour l'industrie hôtelière qui est considérée, de nos jours, comme une industrie mûre caractérisée par une forte compétition et une gestion des inventaires compliquée. Nous avons remarqué que durant ces trente dernières années, la recherche dans le domaine de la gestion du revenu dans l'industrie hôtelière n'a pas proposé ou résolu de modèles qui considèrent simultanément l'affectation des inventaires, le prix, la longueur du séjour, la qualité de service et l'incertitude. Par conséquent, le but de cette thèse est de développer un nouveau modèle de gestion du revenu dans l'industrie hôtelière qui permet aux gestionnaires d'hôtels de prendre en compte certaines données pertinentes pour la prise des décisions relatives à la tarification et l'affectation des inventaires en se basant sur une meilleure compréhension du comportement des clients et de l'incertitude du marché. Nous nous inspirons pour cela des modèles biniveau de tarification et des modèles stochastiques à deux étapes.

Dans le cas déterministe, le meneur (leader) de l'industrie essaie, au niveau supérieur, de fixer les prix de ses inventaires de façon à maximiser ses revenus. Puis, les usagers essaient, au niveau inférieur, de minimiser leurs dépenses en fonction des différentes alternatives. Dans le but d'introduire le facteur de l'incertitude, nous avons développé un modèle stochastique à deux étapes : à la première étape, le meneur, comme dans le cas déterministe, fixe ses prix en maximisant ses profits. Puis, chaque groupe d'utilisateurs choisit, au niveau inférieur, les inventaires les moins chers tout en considérant les attributs qu'ils ont préalablement définis (distance et qualité de service). À la seconde étape, nous introduisons de l'incertitude sur le prix fixé par les concurrents ainsi que sur la demande. En réaction, le meneur doit ajuster ses prix et ses affectations d'inventaires, ce qui implique des changements dans les distributions des groupes d'usagers aussi. Ces deux étapes sont liées par des contraintes absolues et proportionnelles relatives à la variation du prix de chaque inventaire. Comme ce modèle est un modèle stochastique biniveau à deux étapes, il hérite la propriété NP-Difficile du modèle biniveau déterministe.

Dans ce modèle, nous considérons que l'incertitude peut être modélisée en utilisant des vecteurs aléatoires qui suivent une certaine distribution de probabilité connue. Cette information peut provenir des données historiques ou d'une connaissance empirique de la fonction de masse qui représente fidèlement la vraie distribution. Nous supposons que les vecteurs aléatoires ont un nombre fini de réalisations qui, dans notre cas, correspondent aux scénarios.

Afin de résoudre notre modèle, nous avons développé non seulement des stratégies exactes, mais aussi des heuristiques. La stratégie exacte consiste à transformer le problème de base en un problème MIP (Mixed Integer Program), qui est standard pour ce type de problème. La principale réussite en termes d'heuristiques est le développement d'une heuristique gloutonne capable de résoudre le problème de manière efficace. Cette heuristique consiste à copier les prix des concurrents et à ré-optimiser en faveur du meneur. Pour continuer avec une recherche globale, le processus d'exploration a été suivi par un problème MIP restreint qui se base sur la solution fournie par notre heuristique. Finalement, la stratégie exacte supportée par les heuristiques consiste à ajouter au problème MIP original une heuristique qui cherche les solutions entières, par la procédure d'évaluation et séparation progressive (B&B), et qui permet d'ajuster directement la borne inférieure.

Une fois que les heuristiques et le modèle ont été développés, nous avons créé un processus de génération de données. Ce processus cherche non seulement à générer des instances réalistes pour l'industrie, mais aussi à éviter les situations atypiques. Pour cela, nous avons modélisé la fluctuation du prix et de la demande en utilisant des variables aléatoires uniformes, et nous avons développé un processus analytique qui permet d'ignorer rapidement les situations atypiques. Les résultats numériques sont présentés pour les trois stratégies précédentes. Le résultat le plus satisfaisant est celui basé sur notre heuristique complétée par un problème MIP restreint. De plus, les résultats obtenus sont en accord avec le comportement économique. Selon que le meneur a ou n'a pas d'avantage compétitif en ce qui concerne la localisation des hôtels, il aura un comportement plus ou moins prédateur face à ses concurrents. Dans le cas où il a un avantage compétitif, le meneur cherchera à imiter le prix de ses concurrents afin d'attirer les groupes d'usagers offrant les revenus les plus importants. Lorsque le meneur n'est pas dans une position avantageuse, il fixera ses prix plus bas que ses concurrents pour attirer les groupes d'utilisateurs qui sont sensibles à la distance, mais aussi ceux qui sont plus sensibles à la qualité du service. Pour cela, il devra relocaliser ses inventaires en ignorant les groupes d'usagers qui lui procureront de faibles revenus.

Finalement, un certain nombre d'analyses de sensibilité ont été réalisées pour évaluer la performance du modèle. Premièrement, nous avons introduit la stochasticité simultanément sur le prix et la demande. Ensuite, nous avons complexifié le modèle en variant la capacité de l'industrie. Notre heuristique a permis d'obtenir un résultat conforme au comportement économique espéré.

Par conséquent, les principales contributions de cette recherche sont : l'élaboration d'un modèle complexe pour la gestion des revenus hôteliers, la résolution de grands et de petits exemples en un temps de calcul raisonnable, l'obtention de bons résultats grâce à l'utilisation de notre heuristique (même si nous ne pouvons pas garantir qu'il s'agit de la solution optimale), et l'offre de résultats utiles pour la prise de décision dans l'industrie hôtelière.

ABSTRACT

Revenue Management consists in maximizing a company's revenue. This technique is applied in the airline, telecommunications, and hospitality industry, among others. In this thesis, we develop and solve a stochastic bilevel model for the hotel industry, which is nowadays considered as a mature industry marked by an intense competition and by a complex inventory management. We noticed that over the last 30 years, Hotel Revenue Management research has not proposed and solved models that consider simultaneously inventory assignments, price, length of stay, quality of service and uncertainty. Therefore, the purpose of this doctoral research is to develop a new model for Hotel Revenue Management that is inspired from bilevel pricing models and from the Two-stage Stochastic Models and that allows hotel's managers to account with useful data for pricing decision and assignment allocation, based on a better understanding of consumers' behavior and market uncertainty.

In a deterministic model, the leader of the industry tries to set prices to its inventories, maximizing its revenue in the upper level, and users choose the lowest cumulative expenditures among available alternatives, at the lower level. In order to introduce uncertainty information, we have developed a two-stage model: in the first stage the leader set its prices with the goal of maximizing profits in the upper level, and each users' group chooses the least expensive inventory considering the attributes previously defined by them (distance and quality of service), at the lower level. In the second stage, we introduce uncertain information about competitors' prices and demand, and thus the leader must set again its prices and inventory allocations, which also implies changes in users' group distributions. The stages are tied by price variation in each inventory through an absolute and proportional constraint.

It is difficult to solve the bilevel programming problem. The non-convexity usually present in bilevel programming results in the complexity of the solution algorithm. Even a very simple bilevel problem is still a NP-hard problem The NP-hard property of deteministic bilevel programs is also present in our two-stage stochastic bilevel model.

We consider that uncertainty can be modeled with the support of random vectors that follow a known distribution function. This information might come from historical data or from the empirical knowledge of the distribution function, and that is close to the true unknown uncertainty. We assume that the random vectors have a finite number of realizations, which in our case corresponds to the scenarios.

In order to solve our model, we developed not only exact strategies but also heuristics. The exact strategy consisted in transforming the basic problem into a Mixed Integer Program problem using the Karush–Kuhn–Tucker Conditions conditions (or optimality conditions),

through the use of big constants and auxiliary binary variables. The main achievement in terms of heuristics is the development of our greedy heuristic, which was able to solve the problem efficiently. This heuristic consisted in copying competitors' prices and re-optimizing in favor of the leader. To keep a global search, the exploration process was followed by a Mixed Integer Program restricted problem that took as origin the solution provided by our heuristic. Finally, the exact strategy supported by heuristics consisted in adding to the Mixed Integer Program original problem a heuristic that looks for integer solutions directly in the branch and bound (B&B) tree.

Once the model and the heuristics were developed, a data generation process was designed. The procedure sought not only to generate realistic instances for the industry but also to avoid unfeasible situations. To do this, we modeled price and demand fluctuations through the use of uniform random variables and we developed an analytical process that allowed us to disregard quickly atypical situations. The numerical results are presented for the two previous strategies, being the most performing the one based on our heuristic complemented with the Mixed Integer Program restricted problem. Moreover, the obtained results performed as expected in terms of its economic behavior. Depending on having or not a competitive advantage with respect to the location of its hotels, the leader has a more or less predatory behavior with its competition. In a situation under a competitive advantage, the leader seeks to imitate the price of its competitors in order to attract users' groups that provide the highest revenue. If the leader is not in an advantageous position, it set lower prices than the competition to compensate users' groups more sensible to distance. At the same time, it set competitive prices to attract users' groups that are more sensitive to quality of service than to distance, which implies that the leader reallocates its inventories and disregards users' groups providing lower revenues.

Finally, a certain number of sensitivity analyzes were conducted to evaluate the performance of the model. First, we introduced stochasticity on price and demand simultaneously and then, we added more complexity by varying the capacity of the industry. The heuristic was able to obtain a result, which was again behaving economically as expected.

Therefore, the main contributions of this research are to provide a elaborated model for Hotel Revenue Management, to solve small and large instances in a reasonable computing time, to obtain good results through the use of our heuristic (although we cannot assure it is the optimal solution), and to provide very useful results such as: pricing information, users group distribution in inventories, users group revenue contributions, sensitivity to capacity parameters, for decision making in the hotel industry.

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ABS	Absolute Constraints Case
ATI	Air Transportation Industry
BLP	Bilevel Programming
BLSCCP	Bilevel Chance-Constrained Programming
EEV	Expected solution of the Expected Value
EMSR	Expected Marginal Seat Revenue
EV	Expected Value
EVPI	Expected Value of Perfect Information
HBP	Multipath-based Heuristic
HRM	Hotel Revenue Management
KKT	Karush–Kuhn–Tucker Conditions
LOS	Length-of-stay
MIP	Mixed Integer Program
MPEC	Mathematical Programs with Equilibrium Constrains
PMS	Property Management System
PROP	Proportional Constraints Case
QoS	Quality of Service
RM	Revenue Management
RP	Recourse Problem
SBLP	Stochastic Bilevel Programming
RSSBP	Recourse Stochastic Bilevel Problem
SCCP	Chance-Constrained Program
SHBP	Stochastic Hotel Bilevel Program
SLP	Stochastic Linear Programming
SP	Stochastic Programming
TBLP	Two-stage Bilevel Programming
TSLP	Two-stage Stochastic Linear Program
TSP	Two-stage Stochastic Programming
VSS	Value of the Stochastic Solution
WS	Wait and See (W&S)

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CHAPTER 1

INTRODUCTION

In this Chapter, we introduce the main features and current practice of the hotel industry and cast them within the general framework of Revenue Management Revenue Management (RM). In particular, we survey various RM models for the Hotel Industry that have been developed over the past thirty years. Toward the end of the chapter, we introduce the model which is the topic of this thesis, and whose aim is to address issues yet unresolved in Hotel Revenue Management.

1.1 The Hotel Industry

Prior to the 20^{th} century, the hotel industry was mainly focused on renting space for travelers. This was the case in France at the beginning of the fifteenth century, or yet in Japan with Ryokan guest houses or England with coaching inns. Catering to different customers were medical facilities for long-term illness, such as sanitariums, or health-oriented resorts that could be put in parallel with modern spas. Sanitariums were famous not only for combining comfortable and even luxurious lodges for resting, with fresh air, good food and mineral spring water, but also for including thermal bathing, heliotherapy, and other services. However, these types of facilities tended to disappear with the advent of improved health standards and modern medicine, paritucarly so after the discovery of antibiotics.

The industrial revolution facilitated the development of hotels in Europe and in the USA. Actually, the modern concept of dedicated building for a hotel derives from the City Hotel in New York. In this very city, the Tremont House is considered as the first luxury class service hotel with inside toilets, locks and restaurants. By 1908, another hotel, the "Buffalo Statler", included many current services such as amenities, private baths, ice water and newspapers. In the 1920's, several large luxurious hotels opened their doors, the Waldorf Astoria in New York City being a quintessential example.

During the 20^{th} century, the holiday and Hotel Industry grew in parallel, and at high pace. New services were developed both to increase hotel revenues and to deal with competition from different hotel brands. Investments in this sector have been constant and have taken into account the preferences and needs of very diverse customers. From an investor perspective, occupancy rate, *i.e.,* , the ration of the number of rooms solve over a specified period of time over the tota supply, has been one of the most important measures of hotel performance.

Nowadays, the Hotel Industry is considered as a mature industry marked by an intense competition and in which any market share increase/decrease comes typically at a competitor's expense/gain. Although the North American Hotel Industry is still considered very important in size, the industry-wide growth is now occurring outside the USA, particularly in Asia. Using as a reference the hotel classification from the North American market, we can consider the following hotel categories :

Conference Centers : Mainly oriented towards conferences or meetings. Space is assigned to large groups, and provide all services and equipment required for handling conventions or expositions.

Commercial Hotels : Oriented towards business clients. They have the most common services as room-service, coffee, dining-room, bar, pressing, valet service, parking, internet, computer desk, printing and fax services.

Economy Hotels : Low price hotels. They usually provide basic services for tourist class guests.

Suite or All-Suite Hotels : Well-known as hotels offering beautiful design and spacious layout. They have big spaces for people that need to work but also some entertainments in addition to the room.

Residential Hotels : Oriented to long term accommodations and sometimes family accommodations.

Airport Hotels : Located close to airports, they provide a wide but not always defined category of services such as room-service, rent-a-car, parking, bus or limousine to the airport.

Highway Hotel : Located close to major highways but not in suburban areas.

In the Hotel Industry, some new trends can also be observed such as :

Casino Hotels : Usually very luxurious, they provide entertainments facilities such as theaters, bars and restaurants, togethr with all kind of gambling experiences. Following into the footsteps of Las Vega, Macau developed some of the most luxurious casino hotels in the world, catering to internatial customers.

Resort Hotels : Located in picturesque cities or nature environments (seaside, mountain), they cater to leisure travelers, providing them with various forms of entertainement, indoors and outdoors, such tennis, sailing, diving, trekking, etc.

Theme Hotels : Mainly dedicated to offer a new "experience", they have evolved symbiotically with parks and resorts, but are not limited to them. There are a proliferation of hotels dedicated to Eco-and Social-responsible experience (huts and tribal experiences), medical tourism, and unconventional accommodations such as palace heritage accommodations.

Cruise liner : Cruise liners have been involved in the Hotel Industry, through their vessels increasingly becoming floating hotels. They share features wiht casino hotels and resorts.

Cabin Hotels : A new trend in the industry, they have many resemblances with Motels and VIP airport lounges. These hotels, such as "Yotel" offer a reduced room space for individuals, and some entertainments oriented to etime-conscious travelers or young professionals.

Boutique Hotels : Very stylish and fashionable small to medium size hotels, usually located in "trending" or "expensive" neighborhoods. They follow a non-standard aproach to satisfy different guest preferences in terms of entertainment and relaxation, such as themes or dramatic atmospheres with luxury furniture combined with antiquities. Other particular characteristics of these hotels are their design and architecture, music, lighting and artistic objects, but they can also include technological gadgets. They focused on middle-to-upper income young clientele.

The contemporary Hotel Industry picture is dominated by franchised chains and brands, which are nowadays considered as commodities, with property management companies playing an increasing role. The companies are frequently detached from the real estate business. In this evolving environment, only lodging, bar and restaurant services represent the core "classical" services.

Recently, the market has witnessed the advent of websites that promote a direct contact between customers looking for short term bookings and private owners of houses or flats. In some markets, these new entrants threaten the business model of traditional channels, since they are not subject to the same legal or fiscal regulations. In certain tourist hubs, they have increased the supply of available rooms significantly. Among the most well-known websites are : Airbnb, Hostelbookers and Localo.

An alternative classification for the Hotel Industry, this one based on physical location, can also be considered. Indeed, hotels can be categorized as : Urban – Only located in densely populated areas. Suburban – Only located in suburbs of main cities. These are moderately priced. Their size and their proximity from downtown vary widely. Airport – Distance may vary to downtown, but they are close to airports. Highway – Hotels in main highways or major roads that are not considered as suburban. Resort/Leisure – Located in leisure destination for travelling. Small-town – Located in urban areas but distant to main cities, these hotels are close to mining areas, refineries, industrial complexes, plants, etc. Their sizes are irrelevant.

It also possible to categorize hotels according to price segments which are frequently used in the hotel industry to reflect different qualities of service. These may be regrouped into five main categories, according to the average price, and are correlaated to the star system used in many conuntries : Luxury and Upper Upscale (top 15th percentile) ; Upscale (next 15th percentile) ; Mid-Price (middle 30th percentile) ; Economy (next 20th percentile) and Budget (lowest 20th percentile). We note that this feature is not shared by the airline industry, where most flights have one or two comfort categories (economy, business(, although quality is included artificially through the advantages related to the product, such as full reimbursement, fidelity programs, etc.

As the hotel industry becomes more competitive, inventory management also becomes more complex. In order to manage its inventories [1] firms must asssess the impact of disruptive technologies, such as mobile internet, as well as unexpected events. This has led the industry to increasingly rely on logistic systems that make use of complex simulations for decision making. In order to optimize profits, these systems must explicitly take into account customer behavior, and thus fit the general framework of revenue management. These techniques, which will be described in the following section of this chapter, are referred as revenue management, which is the topic of the next section.

1.2 Revenue Management (RM)

Revenue management refers to a set of tools for optimizing the revenue associated with a given set of available resources. It applies mostly to businesses characterized by high fixed costs and relatively stable inventories. The reader is referred toMcGill and Van Ryzin (1999), where the main aspects of RM are introduced, together with a comprehensive literature review.

RM was initiated in the 70s, with an article published by Rothstein (1971) for the airline industry, whilst the first efforts to apply this technique to other industries arose in 1974 (Rothstein, 1974). In the early 70's, the main concern of the airlines was the widespread price decline associated with a sluggish demand. Companies responded by adopting "overbooking" [2] strategies to offset the large number of last minute cancellations.

Throughout the the 70s, the introduction of restrictions began to be part of the current practices of airlines. The advent of automated reservation that gather statistics about their reservations allowed to adjust prices to customers' expectiations, thus protecting seats for high-fare passengers, as pointed out by Weatherford and Bodily (1992); Weatherford et al. (1993).

1. An inventory is a set of rooms with equivalent price/quality features.
2. Overbooking consists in selling more units than the available capacity.

In 1978, Straszheim (1978) showed that business trips were much less sensitive to tariff changes than leisure trips. Therefore, it was argued that a way to make more profits would be to use RM in a systematic way, always trying to protect seats for the most profitable passengers. This practice, known at first as "protection levels" triggered a the concept of "Booking limit", whereby access to cheaper fare classes is dynamically forbidden. Subsequent studies showed another interesting phenomenon : demand is more sensitive to individual income and economic growth than to fares. Therefore, it is more important to prevent business travelers to find the cheapest seats than to limit tourist access to cheap tickets. This phenomenon of unequal elasticity is still present in many industries. Consequently, the main effective restriction (known as "fences") for selling seats arises at booking time, and can be considered as a measure of risk willingness of the manager regarding customer unfairness perception.

During the 80's, after the deregulation of the U.S. Air Transportation Industry (ATI), competition brought new actors and pricing began to play an important role in RM. One of the practices implemented was that of "bid-price", which is a set of threshold prices for selling resource units. A bid-price is an estimate of the marginal cost of consuming the next unit of the resource capacity. Therefore, when a booking comes in, the revenue of that request is compared to the bid-price of the resource unit. If the revenue exceeds the bid-price, the request is accepted. Otherwise, it is declined.

Moreover, as a result of the deregulation, a large number of small airlines categorized as "low-cost" (as People Express) appeared in the industry with half the operating costs of large companies (American, Delta, United and TWA at that moment).

From the 90's on, various researchers studied the impact of fare on revenue. As a copnsequence of competition, it has become important for companies to send a signal to the market in order to influence it, but also to offer different products for each type of consumer at different prices. According to Weatherford and Bodily (1992); Weatherford et al. (1993); Weatherford (1995); Weatherford and Kimes (2003), RM has been successful for doing reservations and allocations among several different price categories ("booking limits"). Nowadays, RM takes advantage from highly complex Global Distribution Systems [3] which facilitate segmentation and price differentiation.

The most common approach utilized in RM for setting price or booking limits, and largely used in the ATI, is the Expected Marginal Seat Revenue (EMSR) Belobaba (1987b, 1998a,b). This method does not consider competitors, only the aggregated demand and assumes that clients will not be influenced by them. For the success of this model, it is assumed that fares are known to clients and that information is controlled.

3. A network system that automates transactions between booking agents, like Amadeus, Sabre, Travelport and System One for the Air Transportation Industry (ATI) and Pegasus Solutions for the Hotel Industry, but also for car rentals and activities services.

According to Kimes and Mutkoski (1989); Kimes (1989b,a), RM is characterized by a set of features listed below :

Rigid capacity : Inventory capacity subject to RM cannot be modified in the short term in an intended way.

High fixed cost : Adding an additional inventory unit is very expensive. Inventories are costly and cannot be created easily.

Perishable inventory : The inventory units are sold at a fixed date. If the units are not used, they are lost.

Products sold in advance : Services are booked in advance. There is a delay between the booking and the purchase of the product and the time of consumption. The product may consist of a promise to sell, as seen in the model of Quan (2002).

Low variable costs : The consumption of an additional unit of product causes only negligible additional costs.

A variable demand in time : Demand is variable or uncertain, thus stochastic. However, simplified models are satisfied with using fixed seasonal demands.

Market segmentation : The model must differentiate customers according to their willingness to pay or risk aversion by establishing difficult barriers to circumvent such as anticipated reservations, age, sex, method of payment, membership, etc. It is necessary that the quality service-net income-consumption level trio allows the company to increase revenue, but not necessarily by always selling to customers who bring the most.

Based on the literature in this field, Gigli (2001) described a number of success factors for RM :

Fixed price : Find the point where supply and demand are in harmony, in a context where it is very difficult (expensive) to increase production capacity. In the presence of uncertainty the problem becomes even more complex because of the investment risk.

Link price to market and not to production costs : Selected prices should allow the company to survive in the long term.

Take account of market segmentation : It is necessary that the product is adapted to consumer needs.

Assign the product to the consumer who is most profitable : Companies must identify customers' types. To do this, they must choose effective barriers to differentiate.

Base decisions on facts, not assumptions : Use information from previous data and statistics well established, consider clients behavior, and use proper algorithms.

Know the true value of each product : The value of a product depends on selling date and consumers' willingness to pay. This is a consequence of market segmentation. The concept of customer perceived value is different from selling price.

Reassess sales strategies continuously : All price packages and segments are valid within a time slot. Management must follow customer expectations and reassess its strategies. It must also continually assess competitors' actions (offer and price).

In terms of the most popular resolution techniques in RM, Kimes has pointed out the following :

Methods based on mathematical programming : Linear programming and Mixed Integer Program (MIP), linear probabilistic programming and dynamic programming.

Methods based on economic theory : Models based on marginal revenue are still the most popular and widely used in the ATI and Hotel Industry.

Methods based on threshold chart analysis : According to Kimes (1989b) and Belobaba (1987a), these methods are practical methods, but not always optimal.

Methods of artificial intelligence : Expert and neural network systems like Relihan III (1989).

The first models (mathematical programming and economic theory models) set the rules for assessing various situations. The second models (artificial intelligence or data mining) are better suited to forecasting. According to Kimes (2003), research in RM can be divided into three main areas, one of them descriptive (studying the application of RM concepts in different industries), and the other two prescriptive : pricing and inventory management.

Revenue Management is today commonly observed in the air transportation, hospitality, parcels, telecommunication, car rental industries, as well as internet retail, among others. Although most industries have developed "In-House" systems, all of them rely on the same concepts : Demand curve, client-price elasticity of demand [4], and capacity displacement cost (shadow-prices). Critics to classic RM consider that most RM practices neglect the main market's components : competitors' changes such as new entrants and new clients stratifications or groups. Moreover, current RM practice focuses excessively on "price" to influence clients decisions and neglect companies' financial stretch or indebtedness. Sometimes, there is also an excessive focus on operational decisions, forgetting business development strategies, and some lack of interest to integrate "Data Mining" over expert judgment.

While the advance of RM is often credited to the airline industry, pioneer studies for the hotel industry were also conducted in the early 70's. In fact, it is sometimes claimed that RM

4. This elasticity somehow incorporates competition.

practices in the Hotel Industry occurred prior to those in the ATI, as pointed out by Tritsch (1989), although the approaches were very naive. Afterwards, as we will see in the following section, the hotel industry borrowed many of the concepts and tools from the airline revenue management to develop their own revenue management systems.

1.3 Hotel Revenue Management (HRM)

Economic and political uncertainties have pushed the hotel industry to evaluate different scenarios and competitors' behavior. Revenue Management became a practice for maximizing revenue by selling products or services to the right customers at the right time and the right price, as described by Kimes and Mutkoski (1989); Kimes et al. (1998). In HRM there are very few models based on economical theory like Gu (1997); Tony S.M. and Poon (2012). In the most common form, the praxis consider a fine forecasting of the market demand, and then an intense effort to assign limited resources to customers, playing with the allowance of these customers for over-paying such products when they are available. Everything is done with the support of different marketing techniques such as focus advertising, customer loyalty and bonus, etc.

In the Hotel Industry, the basic product is a single/double room offered for a single night, and other services are added as surcharges : Bar, Gym, Parking, Internet, Restaurant, Pool, Press, On Demand TV, etc. This is similar to a Single Leg Trip in the ATI. Because of the complexity of HRM, inventory "Booking Limits" and "Bid Prices" were the only practices retained by the hotel industry. From the perspective of the Hotel Industry, booking limits is the amount of capacity (rooms) that can be sold at a given price and point in time, and room prices are equivalent to air fares or "fare classes". However, a deluxe room and a standard room will always have different attributes, rather than price, such as space, furniture, decoration, services, etc. In some cases, the room price list is just a mean to charge different prices for non-differentiable products. Differentiation may correspond to natural flowers, juice, bathrobe or unrelated bedroom characteristics such as Wifi, room service, pressing, free parking, etc. All these features enable hotels to offer "upgrades" for value-added products. In the hotel industry, a third element that does not exist in the ATI is the Length-of-stay (LOS) [5]. However the LOS represents the same difficult part as for the airline problem is the "round trip ticket".

For HRM, it is very common to price sets of identical quality rooms, considering the LOS and the day of the week. Nevertheless, it is also very frequent that the services included in the room's price can change. To take into account the LOS, Weatherford and Bodily (1992) introduce the concept of "displacement", which means the impact of one stay spread

5. LOS is the the total room nights in a hotel in a booking.

over several nights on the revenue (or costs) associated to other customers. He states that this displacement effect is related to the residual capacity of the hotel. The analysis of this phenomenon is crucial in the calculation of "no-show" per day of the week, by price or by category of users. According to his study, it is possible to increase revenue by nearly 3% taking into account the LOS. However, overbooking effect on the LOS was not reported in these studies.

In terms of revenue management practices, inventory and LOS controls are widely used in the Hotel Industry, where we can find "overbooking" and "forecasting" strategies. Several RM systems also control the LOS for a range of price categories. In that sense, an integrated Property Management System (PMS) allows to one hotel chain to control a big territory keeping the inventory together, like a "City-Wide Revenue Management" or a province (Disney and Marriott in Florida). The dynamic of a PMS consists on accepting or refusing bookings for a future date at a determined price. After each new booking, an inventory update is conducted. The process of queries is random, so is the interest of clients for matching the current hotel's room price. As a consequence, any early bad decision means lower revenue for the season.

Inventories are generally controlled in three ways in the hotel industry :

Divided Inventory (small hotels) : The inventory is physically divided into rooms according to service quality. Each Quality of Service (QoS) has a number of quality rooms with identical services, and it is not allowed to change the category of any room.

Nested inventories : The inventory is shared by all qualities of service. Certain rooms are more luxurious but most of them are standard. Higher revenue reservations have access to the entire inventory, while low revenue reservations are limited to standard rooms.

Virtual Inventory : The inventory is shared according to quality of service. As room characteristics are similar, the difference is made by adding services. This system is obviously the most flexible and widely used in major hotels.

Some of the currently systems in practice to control LOS Allocation, by using several price categories and inventories, are single inventories (inflexible), nested inventories, and virtual inventories (very flexible).

In terms of techniques, the most common paradigm in the HRM is the expected marginal room revenue, which is a derivation from the EMSR as described by Katul (1995). Marginal estimated values allow accepting, rejecting, upgrading, and downgrading a reservation based on price and not on reservation constraints. In general, hotels stratify clients by their willingness to pay from low to high, which is not always a valid assumption. In fact, in order to maintain that system running and to reduce its errors, many adjustments are needed. To update booking limits or prices, a new optimization of the EMSR and/or simulations with the new forecasted scenarios of demand is needed. The most difficult problem remains in determining the preferences and choices of customers groups as well as the reaction of competitors (mainly in terms of prices), and particular problems appear in the EMSR models when the LOS is considered.

In conjunction with client price-elasticity, client preferences and competitor's price changes, it is also important to address the issue of demand stochasticity, through the introduction of scenarios. Althought this in not the only source of randomness in HRM, it is certainly the most important.

In terms of RM models and resolution methods, we can refer to Kimes and Mutkoski (1989); Kimes (1989b,a, 2003), and to Gigli (2001), Baker and Collier (1999, 2003); Baker et al. (2002) for mathematical programming hotel models.

Baker and Collier (1999) contains a very good summary of the mathematical tools used in the field, which are :

1. Dynamic system for forecasting demand.
2. Allocation model among several classes.
3. Decision method for overbooking, *i.e.,* when and how many rooms to overbook.
4. Pricing method for each users class

From the standpoint of models, these authors compare the "Bid Prices" method (BPM) of Talluri and van Ryzin (2001) to the PSM method of Gallego and Ryzin (1997) :

BPM : In this method the opportunity cost of re-allocating a room is estimated from the inventory using a nonlinear program that estimates the parameters of the function mass of the random variable "number of reservations" by minimizing the error relative to the observed demand. The PSM method assumes that competitors' prices and packages available on a planning horizon are fixed.

PSM : it is similar to BPM, but the price varies according to demand and available capacity. The values provided by the method are used to calibrate the demand, under the assumption that arrivals follow a Poisson distribution.

Most of the optimization models use a complex forecasting process over a long planning horizon to determine the marginal room value. For example, Weatherford (1995); Weatherford and Bodily (1992); Weatherford et al. (1993); Weatherford and Kimes (2003); Aziz et al. (2011) present models with some simulations as a way to test their efficacy and several scenarios of demand. In these simulations, a price sample of a large hotel was used and assumed to adequately represent the elasticity of the demand [6]. It was observed that customers are more willing to honor their reservations when prices are high. These simulations allowed these authors to validate heuristic methods in an environment with several parameters.

6. This elasticity incorporates somehow competitors.

Research development is still ongoing in Forecasting and Overbooking (real service demand), reservations, cancellations, LOS and price variation. For example, Quan (2002) proposes a model based on option valuation theory (European option), in which the booking represents an insurance over the room's price for the arrival date and clients are free to cancel, make other bookings in the same hotel, or switch to the competition. According to Quan (2002), reservation has customer value and creates a cost to the issuer. For price-sensitive customers, a hotel reservation can be seen as a call European-style option on a hotel room price. Models are used to determine the value of the option and to determine the value of a reservation for the client and the costs incurred by the hotel. The Hotel Industry does not charge for booking, but the author demonstrates that such a practice could be useful in conjunction with a cancellation and refund policy. He considers two different forms of booking, each with a different level of price commitment. Customer choice would indicate the type of customer (price-conscious or "yieldable"[7]).

Another interesting approach is that of Liu (2011) using a "general equilibrium approach". In this approach, a maximization problem models the provider and the guest interests based on the probability that a booking will be realized, using information from the client about the certainty of a trip. This model, with some important differences, may be the closest to our bilevel programming approach.

RM problems can be also formulated as mathematical dynamic programs, but including overbooking in the formulation is still considered as a challenge in the HRM literature (Rothstein (1974, 1971), Hersh and Ladany (1978); Ladany (2001); Toh and Dekay (2002); Aziz et al. (2011) and Alstrup et al. (1986)). Bitran and Mondschein (2003, 1995) proposed dynamic programming for the LOS, and the optimal policy was obtained in function of the capacity and the delay at the day of arrival. Generally speaking, all stochastic dynamic models assume a Poisson distribution parameterized with respect to clients' arrival rates, but ignore the order of arrival. Another very close aproach is consider HRM as special case for network revenue management as in Gallego and Ryzin (1997) or Zhang and Weatherford (2012); Aziz et al. (2011).

When demand is static, Chen (2000) illustrates how to solve the room allocation problem using the maximum flow model in a network. He also presents an extension for group bookings, which is akin to the models developed in Montecinos (2007).

During the past years, new stochastic linear models have appeared such as those proposed by Goldman et al. (2002); Lai and Ng (2005); Liu and Lu (2005); Liu et al. (2006, 2008); Guadix et al. (2010). Most of the recent models solve simultaneously allocation and overbooking and penalize the lack of capacity for a unsatisfied demand. The recent models are

7. Capable to pay a higher tariff by persuasion or force.

mathematical programming models that take into account the LOS, such as Chen (2000), and Hormby et al. (2010); Koushik et al. (2012); Pekgün et al. (2013), and that include both demand and overbooking scenarios. The complexity of the resulting nonlinear models is very high. Therefore, Liu and Lu (2005), who criticize the model of Lai and Ng (2005) for its complexity, treat forecasting and allocation problems separately, solving the allocation problems as MIPs, where a penalty for each unmet demand is imposed. Note that, in their model, there is a single price category. According to the authors, the model corresponds to a knapsack problem and can be solved using a greedy heuristic algorithm.

In summary, HRM has been focused on the collection and analysis of data, the development of prediction/estimation models, and the optimization and control of inventories, all of that in an environment supported by information technology.

1.4 New Trends in Hotel Revenue Management

For practitioners, several books have consolidated the main trends in hotel revenue management, such as Phillips (2005); Talluri and Van Ryzin (2005); Yeoman and McMahon-Beattie (2011); Rouse and Maguire (2011); Forgács (2010); Mauri (2013) and Huefner (2011). The Hotel Industry has also been studied in numerous new journals and academic publishing. Chiang et al. (2007); Koushik et al. (2012), for instance, expose the interesting case of "Intercontinental Hotels Group", one of the growing franchises in the Hotel Industry : internationally, it reaches more rooms than any other hotel chain, 672,000 in almost 100 countries [8]. Other franchising chain experiences can be seen in Hormby et al. (2010); Pekgün et al. (2013).

In recent years, HRM has evolved to group booking (tourist operators, meetings and conferences). Indeed, block reservations count for almost 50% of the bookings, so they deserve special attention. In Choi (2006), a model to account for acceptable minimum rates for accepting or refusing block groups is presented. In Guadix et al. (2010), a different approach was proposed, which uses individual and group forecasting among stochastic programming and simulation to test four models and their corresponding heuristics for classical property control (no control, booking limits and nested inventories) but applied to individuals (not to block-booking). In the counterpart, Anderson and Xie (2009) analyze a tour operator that wants to minimize the number of rooms that could not be sold but that have to be paid to the hotel.

An interesting point to highlight is the public acceptance of HRM practices, particularly in daily hotel rates, as stated by Rohlfs and Kimes (2007). This is an important issue, in the view that the advent of internet has made tariffs more transparent. In that sense, numerous articles

8. According to Forbes ™.

consider fairness perception, asymmetry information, distribution channels Choi and Kimes (2002), and the role of tour operators, travel agents and hotel brands in their examinations as in Chen and Schwartz (2006); Thakran and Verma (2013); Kwortnik and Han (2011); Wilson (2011). Another important aspect in the Hotel Industry is the increasing interest on hotel "function space"[9] , as a source of revenue, as stated by Kimes and McGuire (2001), which is also relevant in other business, such as spa centers.

There are also new models mainly related to small hotels. These try to bring back simplicity into HRM, by assuming only two classes but several periods with increasing prices, as proposed by Hadi and Rohollah (2012). Instead of complex calculations "off-the-shelf", they use past PMS information and statistical regressions to deal information : temperature, competitors prices or bookings, seasonality, services and especial customers preferences data as described by Planagumà and Julve (2012) and Bayoumi et al. (2012). These models include manager adjustable weights to combine different estimators for price or to adjust discount factors over fixed tariffs. These models can also estimate customers' price-elasticity using not only historical data but also market conditions in combination with simple simulations, in order to offer managers a decision choice. Those models improve the methods presented in Relihan III (1989), as threshold curves models still in use, but they are much more flexible, parameterized, and largely supported by computer databases, although data remain under manager's control. These models adapt very well in situation where marginal pricing does not reflect correctly seasonality, or when demand is much lower than capacity. In particular, the article presented by Kachitvichyanukul et al. (2012) is interesting because the problem is re-framed as a duopoly pricing economic model, including overbooking, and solved through the use of game theory techniques. In the MSc thesis of Montecinos (2007), a deterministic bilevel model for HRM was proposed, which comprised the allocation of several units of hotel inventory (hotels) to various groups of clients (users), and the simultaneous pricing of the same allocated units. In this study, the planning horizon equivalent to a season was considered. Among the characteristics included for the groups of clients were the proximity to an attracting place, the price, and the quality of service. In that model, the resolution of numerical models was obtained following a transformation from a bilevel problem to a mixed integer problem by its Karush–Kuhn–Tucker Conditions (KKT).

Low budget small hotels have also introduced HRM, but switching directly to variable pricing, sometimes called real-time pricing, which differs from traditional pricing where a fixed tariff discount or an offer is applied and replaced by a constantly in-line price change or auctions. Recent examples can be seen in Aziz et al. (2011); Rasyanti (2013). Large chains

9. Rooms, floors or terraces that can be used for meetings, banquets, exhibitions, etc. This space usually is setup on purpose with minimum turnover-time in amphitheaters, round tables, stands, expositions, etc.

have also addressed similar issues using "opaque selling"[10] to maintain anonymity but still addressing the last-minute buyers segment, according to Jerath et al. (2010).

1.5 The Challenges of Hotel Revenue Management

The new tendencies in HRM have pointed out to a more critical view of the almost 30 years of practice, as stated by Anderson and Xie (2010) but also to an even deeper critic of the vision of the models that do not consider LOS, multiple inventory share, conjoint overbooking, room availability guarantee, special events, and "Total RM" over mulitple properties and services, as presented by Ivanov and Zhechev (2012). In particular, those authors account not only for the complexity of effectively controlling the downward spiral of revenue and the current fencing, but also the case of long-stay booking, as Ling et al. (2012) describe, or multiply-day booking in periods of demand shortage. Indeed, in a world with excess of offers in lodging, rooms become a simple commodity. On the other hand, the challenges behind managing a more individualized demand, as highlighted by Cross et al. (2009), lead to an urgent re-specialization of HRM focused on choice-driven-pricing and service-pricing, and require a close collaboration with marketing as suggested by Kwortnik and Vosburgh (2007); Victorino et al. (2009); Verma (2010); Sengupta and Dev (2011).

In the forecasting aspect of HRM, common models have always relied on past occupancy and room rates records, and some efforts have been made to cope with the unconstrained demand estimation like in Fiori et al. (2013). Deep segregation and fencing have also provided a better understanding of group behavior. But these efforts do not always include local or global effects, such as travel trends and fashion. While the proximity to points of interest has been recognized for airport hotels Lee and Jang (2011), it is still frequently ignored. HRM will also take advantage of technological advances, like the internet and GIS[11], which make much easier the task of estimating room rates and even predicting temporal prices by monitoring competitors' prices. Another approach of interest is that of "Spatial Modelling", a form of modelling by disaggregation, in which an area is divided into geographical units. Several algorithms simulate or analyze real-world conditions using the spatial relationships of the geographical units. These geographical models are linked to a GIS for Geovisualization (GVis) or data input. In "Geographic knowledge discovery (GKD)" several algorithms can explore large spatial databases using geographic data mining techniques to unveil human patterns. Nevertheless, these new tools (GIS, spatial models and GKD) are in the middle point of being part of the Hotel Industry, but also in tours operators, travel agencies, or

10. It is the practice of using intermediaries to hide descriptive attributes of the hotel, like address and room characteristics. Examples are Hotwire ™and Priceline ™.
11. Geographic Information System.

intermediate agencies that maintain control over several large databases. These new tools establish very different data mining algorithms that are capable to correlate geographical data (proximity to amenities, business centers, airports, other hotels, waterfront, hairdressers, etc.) and websites or internet data to make accurate estimations on pricing. In these systems, operators and experts or practitioners can form educated guesses about the most appropriate model. We refer the reader to the article by Kisilevich et al. (2013) for a detailed analysis of these novel concepts.

As several authors have emphasized, there is a periodic, sequential and probabilistic reservation dynamic in the hotel industry. It is thus possible to formulate the problem as a stochastic dynamic program that takes into account simultaneously overbooking and RM rules (Rothstein (1971), Hersh and Ladany (1978), Alstrup et al. (1986)).

As previously discussed, stochastic pricing is one of the most common sources of uncertainty faced by hotels, together with demand. This uncertainty is usually caused by fluctuations in the offer but is also related to the asymmetrical information game that is taking place in this industry, in which suppliers try to hide information and to speculate. Other secondary sources of uncertainty in the hotel industry are processing information errors (models and software errors, quantization for discrete categories, sampling errors), lack of understanding of the problem, partial information accessibility, and distribution channels bias.

From the point of view of the suppliers, the purpose of stochastic pricing is to shield themselves from the spiral-down price effect, which occurs when incorrect assumptions about customers' and competitors' behavior make tariffs and revenues to decrease over time, and lead to misleading demand forecasts. Alternatively, this should prevent companies to enter into price-wars, or simply being to much "price-driven".

Demand uncertainty is usually a consequence of external factors that can have an impact on the tourism industry and on international relations. These may be related to weather, disasters, or yet unforeseen changes in the offer in different geographical zones. Tour operators and travel agents also add some quotes of speculation in their relationships with main hotel brands, thus playing a partial information game. Since hotels are the last boundary in money flow, they must dilute these losses caused by demand uncertainty.

1.6 Purpose of this thesis

The issues of HRM highlighted in the previous section motivate us to propose models for the Hotel Industry that considers not only customer's preferences but also demand stochasticity. It follows into the footsteps of the model proposed by Montecinos (2007), who argued that scenarios could be embedded within a bilevel model, although this added complexity would call for more sophisticated resolution techniques. The stochastic model that we analyze and solve has the following features :

— A leader of the Hotel Industry maximizes its revenue while facing multiple competitors.

— The objective function of the leader involves the revenue expectation and risk aversion.

— Customer population takes the form of user groups, each one endowed with several preferences that follow a random distribution over different price classes, with deterministic arrival day and LOS.

— The optimization occurs over a limited planning horizon.

— "Booking limits" [12] or "Protection levels" [13] estimation.

— Prices are positive and can take any possible value.

Our objective is to maximize a firm's revenue through the price adjustment and the management of its inventory (setting optimal booking limits), while taking into account customer behaviour and prices of the competition. To this end, we investigate several bilevel models, where demand is random, that fit the generic framework of two-stage stochastic programming. To solve this problem, we develop mixed integer programming formulations. Since solving them to global optimality is not possible (note that the deterministic version is already NP-hard), we have developed novel heuristic procedures that are capable of addressing nontrivial instances.

The main contributions of this thesis are :

— Mathematical Models that considers several elements : Pricing on several days, uncertainty, length-of-stay, quality of service, inventory capacity and groups preferences.

— A data generation process to overcome atypical and/or irrelevant cases

— Exact and heuristic resolution methods

— To Provide useful results for decision making in the hotel industry

This doctoral thesis is organized as follow : In Chapter 2, a deterministic bilevel model for hotel revenue management is introduced. In Chapter 3, we briefly review the literature on stochastic programming and stochastic bilevel programming that lies at the heart of our models. In Chapter 4, a Stochastic Bilevel Programming (BLP) is introduced, followed by models that include expectation in the first level objective. In Chapter 5, we present diverse

12. Booking Limits are the maximum number of reservations per weekday considering business rules for LOS.

13. Protection Levels are the number of rooms to reserve per weekday for a particular class.

heuristics used for solving our model and a performance analysis of our algorithm with respect to MIP+. In the next chapter we present the numerical results of several tested instances , and the results of different sensitivity analysis. Some instances are also analyzed from an economic point of view. Finally, in the conclusion, we summarize our research, we point out the main contributions of this thesis and its main limits, as well as some avenues for research.

CHAPTER 2

A DETERMINISTIC BILEVEL MODEL OF HOTEL REVENUE MANAGEMENT

In this chapter, we will describe a bilevel deterministic model of HRM. Before doing that, we will present a review of Bilevel Programming (BLP) and its economical applications particularly to RM.

2.1 Bilevel Programming (BLP)

Bilevel programs BLP are hierarchical optimization problems in which an objective function is minimized (maximized) over a set of solutions from a second problem parameterized by the decisions taken in the first level. Bilevel programming allows the modelling of hierarchical situations in which a subset of decision variables is not controlled by the leader (at the first level) but by a follower (at the second level), who rationally optimizes its own objective with respect to the variables set by the leader. This framework is very well adapted to our problem : An hotel manager integrates into its booking and pricing policies the behavior of its clients. We present some of its properties.

2.1.1 Bases of BLP

Bilevel programming allows us to model problems in which not all the decision's variables are controlled by the main agent (a leader) but controlled by another agent (the follower) that optimizes its own objective over a subset of variables. The variables controlled by the leader are constant for the second agent, who reacts looking for its own optimality and its strategy is closely tied to the decisions of the leader (they never oppose the leader). Generally, the existence of upper level constraints depends on the application under consideration. Problems with upper level constraints can arise if the leader does not want certain optimal solutions of the follower. This is conditonated to the existence of multiple optimal lower level solutions. Also, a bilevel programming model can change its optimal solution if constraints are switched from one level to the other (Mersha and Dempe, 2006). Necessary and sufficient optimality conditions can bed derived from the Karush–Kuhn–Tucker Conditions (KKT). If the leader controls a vector of variables x, the follower controls a vector of variables y, and we have the following formulation,

$$\text{(BPP)} \qquad \begin{aligned} \min_{x,y} \quad & F(x,y) \\ \text{s.t.} \quad & g(x,y) \leq 0 \\ & y \in \operatorname*{argmin}_{y'} f(x,y') \\ & \qquad \text{s.t.} \quad h(x,y') \leq 0, \end{aligned}$$

with $x \in \mathbb{R}^{n_x}$, $y \in \mathbb{R}^{n_y}$, F,$f : \mathbb{R}^{n_x+n_y} \to \mathbb{R}$, $g : \mathbb{R}^{n_x+n_y} \to \mathbb{R}^{n_u}$ and $h : \mathbb{R}^{n_x+n_y} \to \mathbb{R}^{n_l}$. The relaxed feasible set,

$$F = \{(x,y) \mid g(x,y) \leq 0, h(x,y) \leq 0\},$$

for every fixed x, the feasible domain for the follower is,

$$F_s(x) = \{y \mid h(x,y) \leq 0\},$$

and its optimal subset is, for x constant, the follower reaction set,

$$R_s(x) = \{y \in \operatorname{argmin} f(x,y) \mid y \in F_s(x)\}$$

The induced or inducible region is,

$$R_I(x) = \{(x,y) \mid g(x,y) \leq 0, y \in R_s(x)\}.$$

This subset is generally non convex, and possibly disconnected. When g and h are differentiable convex functions, and the sets X and $Y(x) = \{(x,y) : h_i(x,y) \leq 0, i = 1, \ldots, n.\}$ are both convex and regulars [1], the problem can be stated as,

$$\begin{aligned} \min_{x,y} \quad & F(x,y) \\ \text{s.t.} \quad & (x,y) \in X \\ & y \in R_s(x). \end{aligned}$$

When R_s is not a singleton and the leader is free to select any element from R_s, we have an optimistic formulation. The pessimistic formulation refers to the contrary case : the leader

1. A set is regular when satisfy a constraint qualification, like the Mangasarian-Fromovitz.

fights against the worst possible situation. It is possible to formulate this as,

$$\begin{aligned} \min_x \max_y \quad & F(x,y) \\ \text{s.t.} \quad & (x,y) \in X \\ & y \in R_s(x). \end{aligned}$$

Another common formulation related to bilevel programs is Mathematical Programs with Equilibrium Constrains (MPEC). In MPECs, the lower level corresponds to an equilibrium problem. The MPEC formulation is the following,

$$\text{(MPEC)} \qquad \begin{aligned} \min_{x,y} \quad & F(x,y) \\ \text{s.t.} \quad & (x,y) \in X \\ & y \in Y(x) \\ & -G(x,y) \in N_{Y(x)}(y), \end{aligned}$$

where $Y(x) = \{y \mid (x,y) \in Y\}$, $N_{Y(x)}(y)$ is the normal cone to Y_x at the point y. If $G = \nabla_y g(x,y)$, g is convex in y, and the set Y is convex, we can also formulate the bilevel problem as,

$$\text{(BP-VI)} \quad \begin{aligned} \min_{x,y} \quad & F(x,y) \\ \text{s.t.} \quad & (x,y) \in X, \\ & \langle \nabla_{y2} g(x,y), y - y' \rangle \leq 0, \\ & y, y' \in Y(x). \end{aligned}$$

A deep examination on this subject is presented in Luo et al. (1996) and Shimizu et al. (1997).

Bilevel problems are difficult to solve, and in particular, Vicente et al. (1994) have shown that a simple verification of local optimality is a NP-hard problem (Roch et al., 2005). When the lower level is parametrically convex, it is possible to transform a bilevel problem to a single level by using the Karush, Kuhn & Tucker conditions (KKT) over the lower level. But this is only possible under an optimistic position [2] and when the lower level respects the regular conditions.

2. We have an optimistic position when the lower level solution favorizes the leader's interest.

2.1.2 Bilevel Linear Programs

When F, f, g and h are either linear or affine functions, we have a Bilevel Linear Program[3] :

$$
\begin{aligned}
\text{(BLP)} \quad \min_{x} \quad & F(x,y) = c_1 x + d_1 y \\
\text{s.t.} \quad & \min_{y} \quad f(x,y) = c_2 x + d_2 y \\
& \quad \text{s.t.} \quad h(x,y) = Ax + By - b \leq 0 \\
& \qquad x, y \geq 0,
\end{aligned}
$$

where,

$$c_1, c_2, x \in \mathbb{R}^{n_x}, d_1, d_2, y \in \mathbb{R}^{n_y}, A \in \mathbb{R}^{m_1\, n_x}, A^2 \in \mathbb{R}^{m_2\, n_y}, B \in \mathbb{R}^{m_1\, n_y}, B^2 \in \mathbb{R}^{m_2\, n_y}, b \in \mathbb{R}^{m_2}.$$

A simple polynomial transformation allows the conversion of BLP into Linear Mixed 0-1 integer programs (MIP_{0-1}) and vice versa, as we can see in Marcotte and Savard (2005). The transformation also introduces auxiliary constants.

Bilinear Bilevel Programs

The Disjoint Bilinear[4] Program (BILP) was introduced by Konno (1971) to compute Nash equilibra on bimatrix games based on Mills' works in Mills (1960). Audet et al. (1997); Alarie et al. (2001) were able to construct a branch-and-cut algorithm for the BILP using the separability of its variables. The main aspect of their works is the construction of an auxiliary bilevel problem (a "max-min" problem) to solve the original separable bilinear program. As presented in Audet (1997); Audet et al. (1997); Nahapetyan (2009), there is a coupled interest in Bilinear Disjoint Programs, Bilevel Programming and MIP0–1, which are related to the possibility to transform problems, to their ability to construct useful algorithms, and to their complexity (hardiness).

2.1.3 Bilevel Programming and Some Applications

In the literature, several Bilevel programming models can be found, which consider either a network structure or an assignment problem. In some problems, revenue maximisation is

3. For an easy reading, we can delete the operator "argmin" and the vector y' from the lower level.
4. A function of two variables is bilinear if it is linear with respect to each of its variables, *e.g.,* the quadratic function $xQy + cx + by$ is bilinear respect to (x, y).

hidden under other objectives, such as quality improvement or any other valuable (priceable) objective for companies.

In particular to RM, bilevel models have been adopted in the following applications :

1. Hazardous Material Transport Mitigation : In Marcotte et al. (2009a), it was presented that toll policies can be very efficient to minimize hazardous materials transportation.
2. Healthcare Facility Network : A bilevel non-linear optimization model was presented in Zhang et al. (2010).
3. Product Design : Piedras et al. (2006) optimize a product development process.

The bilevel programming paradigm has been adapted to Revenue Management and Pricing in various fields, such as

1. Supply Chain Management : Shouping and Baozhuang (2007) and Gao et al. (2011) and also Sun and Gao (2004).
2. Network Transportation and Highway Toll setting : Numerous articles in this field, such as Labbé et al. (1998); Brotcorne et al. (2001, 2000a,b, 2011, 2003); Heilporn et al. (2009, 2010, 2011); Brotcorne et al. (2012).
3. Airline Revenue Management : Côté (2001); Côté et al. (2003); Xu and Xu (2012). The first article settled a turning point in the classical Airline Revenue Management. Moreover, the article of Marcotte et al. (2009b) offers an interesting example of a strategic decision problem.
4. Telecommunications : Bouhtou et al. (2002); Altman and Wynter (2004); Bouhtou et al. (2006, 2007)
5. Network pricing : Bouhtou et al. (2003); Heilporn et al. (2010)
6. Rail Freight Transportation : In Crevier et al. (2012) a bilevel formulation that tied pricing and network planning policies was presented.
7. Reverse auction problem : Cheng et al. (2011).

In Labbé et al. (1998), a bilevel network pricing problem was introduced, consisting in maximizing the leader tariffs' revenue, knowing that user flows are assigned to the cheapest paths. The leader just sets tariffs on a subset of the transportation network's arcs, and the follower sets the flows to the cheapest paths of a multi-commodity transportation network. Too high tariffs will push users towards tariff-free paths, and too low tariffs will attract users with a consequence lost in revenue.

For a given vector T controlled by the leader, who wants to maximize its revenue, the follower minimizes its operating costs.

As previously denoted, F corresponds to the leader's objective function, f to the follower's function, and y to the follower's vector corresponding to goods (or activities) in which y_1 represents the vector of goods (or activities) subject to taxation, y_2 the vector of goods not subject to taxes, and T_i the taxation level for the good (or activity) i. We note that this model is a bilinear model that has interesting properties, which will be reviewed in the next section.

2.2 A Bilevel Deterministic Model for HRM

In a general taxation problem over a network, the leader adjusts the price over the arcs T (those that it controls) for optimizing its revenue. The competitors control some arcs with fixed prices P_a. This situation is illustrated in Figure 2.1. The followers, *i.e.,* the clients, react to the leader arc prices by looking for the service that maximizes their utilities (like in MaxToll).

For the hotel industry, we aim to determine the tariffs for the inventories [5] of rooms and the "*booking limits*" [6]. By defining "room inventory" as a set of similar rooms but with QoS that can be modified, we can create complex paradigms such as hotels in different places and with different prices. The followers are clients that book for a fixed date and for a LOS predetermined. Competitors are not players in this game.

2.2.1 A Basic Bilevel Deterministic Model for HRM

As presented in Montecinos (2007), it is possible to use a bilevel deterministic problem to model an industry leader that seeks to maximize its revenues, facing competition and many groups of clients with different preferences (clients can choose among prices, hotel proximities to tourist attractions and qualities of services). These models are also suitable for setting price and quantity at the same time and for dealing with group preferences for clients. The models also perform extremely well with many LOS. These models can be set under Labbé et al. (1999) framework, in which the network in the lower level minimum cost flow problem is simplified to a generalized assignment problem in several dimensions, one for each day (very much like in a multidimensional knapsack). There are, however, important differences with respect to capacity restrictions. Indeed, in transportation problems capacity is related to arcs congestion in a origin-destination path, whilst in the hotel industry capacity restriction occurs within each hotel inventory in every day of the rolling horizon.

The LOS can be seen as an origin-destination problem within a network, which means that

5. A set of rooms.
6. The number of rooms sold at a fixed price.

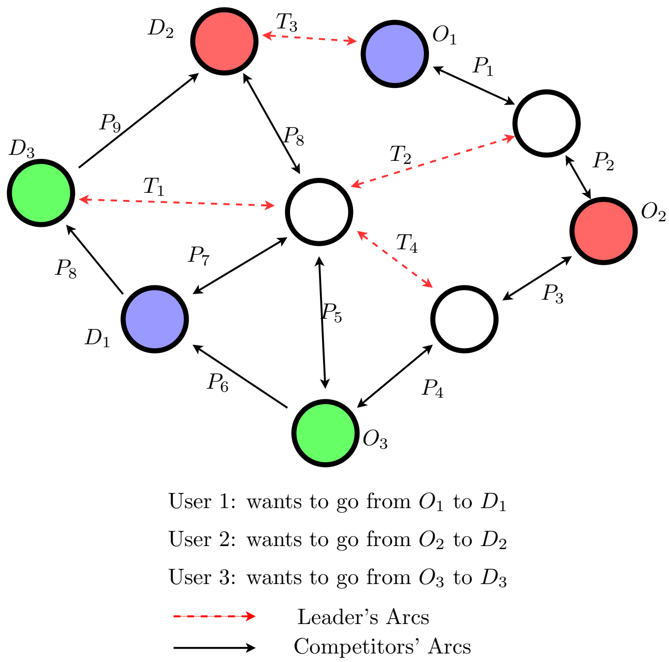


Figure 2.1 Taxation Model.

for every day in a planning horizon (a season) we create a node (an origin and a destination node) and the arcs linking the origin “O” to the destination “D” correspond to one stay. Every demand is a flow in this O–D network.

The following hypotheses are used for setting the model :

— A deterministic demand.

— Competitors' prices are known for every day in the planning horizon.

— The information related to the preferences of clients : length of stay (LOS), QoS, price, etc., is known for every day in the planning horizon. We suppose that the PMS supports and integrates forecasting.

In the lower level, clients make their decisions only based on room prices. The following notation is used :

j, j' : Indices denoting arrival dates ($j = 1, \ldots, W$).

l : Indices denoting length of stay ($l = 1, \ldots, L$).

$x_{j,l}$: "*Leader Booking limit (number of rooms)*"; $j + l$ is the departure date.

$y_{j,l}$: "*Competitors' Booking limit*" (number of rooms).

P : Denotes competitor's price.

T : Leader's price variable.

K^1 : Capacity of the leader inventory.

K^2 : Capacity of competitors.

W : Planning horizon Length.

L : Maximal length of stay.

The model is,

$$\max_{T} \quad \sum_{j=1}^{|W|} \sum_{l=1}^{|L|} l\, T\, x_{j,l}$$

s.t.

$$\min_{x,y} \quad \sum_{j=1}^{|W|} \sum_{l=1}^{|L|} l\, T\, x_{j,l} + \sum_{j=1}^{|W|} \sum_{l=1}^{|L|} l\, P\, y_{j,l}$$

s.t.

$$\forall j, l : \quad x_{j,l} + y_{j,l} = d_{j,l} \tag{2.1}$$

$$\forall j : \quad \sum_{l=1}^{|L|} x_{j,l} + \sum_{j'<j} \sum_{l \geq j-j'} x_{j',l} - \sum_{l=1}^{|L|} x_{(j-l),l} \leq K^1 \tag{2.2}$$

$$\forall j : \quad \sum_{l=1}^{|L|} y_{j,l} + \sum_{j'<j} \sum_{l \geq j-j'} y_{j',l} - \sum_{l=1}^{|L|} y_{(j-l),l} \leq K^2 \tag{2.3}$$

$$x, y \geq 0.$$

In the objective function, we have the revenue contribution $T\, x_{j,l}$ and $P\, y_{j,l}$ by day. At the lower level, followers are undistinguished and they can choose a price room either T or P. We have imposed that the maximum number of rooms allocated cannot exceed the maximum inventory, *i.e.,* there is not overbooking. The constraints (2.1) denote demand's constraints. Constraints (2.2) denote capacity constraint in the leader's inventories and (2.3) in the competitors' inventories. Figure 2.2 illustrates the lower level problem for day j. The clients staying in an inventory in day j are equal to new comers (a) plus clients already in the hotel (b) less departures (c), and they must not surpass the hotel capacity. We do not ask for integer solutions. Even for a very simple case, the number of variables is be high.

Chen (2000), Lai and Ng (2005) and Liu and Lu (2005) have proposed similar models for booking assignment. Chen (2000) is inspired on the original work from Glover et al. (1992) and Hersh and Ladany (1978), in the aerial industry.

2.2.2 A Bilevel Model Including User Groups, Multiples Hotels and Three Criteria

We can modify the basic model to consider the case in which there is a hotel network settled in a city (in several towns). In this case, the leader is managing several inventories in different places. We consider a common point of interest for clients and every hotel (Leader and competitor's) is at a known distance to this point. Each hotel has its own price list. In this case, we have that clients are segmented in several groups (G). The members of the same group share their willingness to pay for the Quality of Service (QoS) (C) and for the proximity to the interest point. We also use a circular rolling horizon with size W for a stationary solution.

G : Set of client groups.

S : Set of Length-of-Stay, (S = {1,. . .,L}).

$Q_{\max}$: Upper Quality of Service .

C : Set of Quality of Service, (C = {1,. . .,$Q_{\max}$}).

W : Rolling horizon Length.

$d_{j,g,l}$: Demand for day $j \in W$ and $l \in S$, for group $g \in G$.

α_g : Value of the proximity to attractions, for group $g \in G$.

$\beta_{q,g}$: Value for the quality $q \in C$, for group $g \in G$.

A : Set of leader's inventories.

B : Set of competitor's inventories.

λ_i : Distance to attractions of inventory $i \in \{A \cup B\}$

T^a_q : Price of the inventory $a \in A$, for the quality of service $q \in C$.

T^b_q : Price of the inventory $b \in B$, for the quality of service $q \in C$.

K^a : Maximal capacity of the inventory $a \in A$, for the quality of service.

K^b : Maximal capacity of the inventory $b \in B$, for the quality of service.

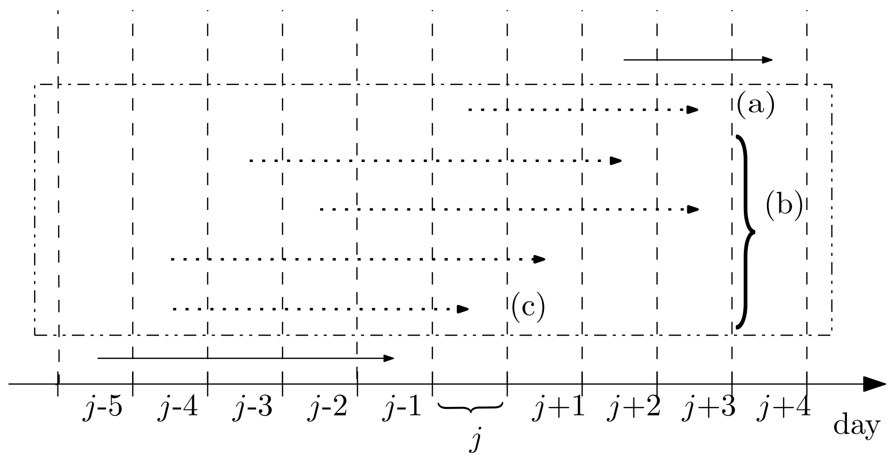


Figure 2.2 Second level problem, showing day j.

$\pi^{i}_{g,q}$: Perceived price of the inventory $i \in \{A \cup B\}$.

The perceived price for the client of group $g \in G$ is :

$$\pi^{i}_{g,q} = \left\{ T^{i}_{q} + \alpha_g \lambda_i + \beta_g \frac{1}{q} \right\}, \forall q \in C, \forall q \in C, i \in \{A \cup B\}.$$

In this model, we have,

— **First Level Objective :** Maximize the revenue from all inventories for the rolling horizon

— **Second Level Objective :** Minimize the sum of the “perceived costs” for every stay in the rolling horizon.

— **Constraints of the lower level :**

1. Respect to the maximal capacity for every inventory (no overbooking).
2. The whole demand should be satisfied for every day and length of stay.
3. The *Booking Limits* for every group of clients and for every inventory should be positive.

The model, as proposed in (Montecinos, 2007) is then formulated as follows :

$$\max_{T^a} \sum_{j=1}^{|W|} \sum_{a\in A} \sum_{g\in G} \sum_{l\in S} \sum_{q\in C} l\, T^a_q\, x^a_{j,g,l,q}$$

$$\min_{x} \sum_{j=1}^{|W|} \left\{ \sum_{a\in A} \sum_{g\in G} \sum_{l\in S} \sum_{q\in C} l\, \pi^a_{g,q}\, x^a_{j,g,l,q} + \sum_{b\in B} \sum_{g\in G} \sum_{l\in S} \sum_{q\in C} l\, \pi^b_{g,q}\, x^b_{j,g,l,q} \right\}$$

s.t.

$$\sum_{q\in C} \sum_{g\in G} \left\{ \sum_{l\in S} x^i_{j,g,l,q} + \sum_{k>1} \sum_{l>k} x^i_{(j-l+1) \mod W,g,l,q} \right\} \leq K^i \qquad \forall j \in W, \forall i \in \{A \cup B\}$$

$$\sum_{a\in A} \sum_{q\in C} x^a_{j,g,l,q} + \sum_{b\in B} \sum_{q\in C} x^b_{j,g,l,q} = d_{j,g,l} \qquad \forall j \in W, \forall g \in G, \forall l \in S$$

$$x^i_{j,g,l,q} \geq 0. \qquad \forall j \in W, \forall g \in G, \forall q \in C, \forall l \in S, i \in \{A \cup B\}$$

For presenting this model, it is better to use a matrix notation,

$\pi^i_{g,q} \in \mathbb{R}^{1\,(|S|\,|W|)}$: Line vector of perceived cost for the leader's (competitor) rooms for the clients of group $g \in G$, and quality of service $q \in C$, $\forall i \in \{A \cup B\}$.

$x^i_{g,q} \in \mathbb{R}^{(|S|\,|W|)}$: Column vector with the booking limits of leader's (competitor) rooms for the clients of group $g \in G$, and quality of service $q \in C$,$\forall i \in \{A \cup B\}$.

$K^a \in \mathbb{R}^{(|W|\,|C|)}$: Column vector with the capacities of the leader for the quality of service q, for day j in the rolling horizon.

$K^b \in \mathbb{R}^{(|W|\,|C|)}$: Column vector with the capacities of the competitors for the quality of service q, for day j in the rolling horizon.

$d \in \mathbb{R}^{(|S|\,|W|)}$: Column vector of client's demands.

$G^a \in \{0,\ 1\}^{(|W|\,|C|)\,(|S|)}$: Binary matrix showing which rooms of quality q belong to the leader, for a length of stay l in day j, with a length of stay l.

$$[G^a]_{(j,q,l)} \in \{0,\ 1\} : \begin{cases} 1 & \text{If the room is in the leader's inventory for day } j\text{, quality } q\text{, a length of stay } l. \\ 0 & \text{None} \end{cases}$$

$G^b \in \{0,\ 1\}^{(|W|\,|C|)\,(|S|)}$: Binary matrix showing which rooms belong to competitors for the length of stay l day j, and quality q.

$$[G^b]_{(j,q,l)} \in \{0,\ 1\} : \begin{cases} 1 & \text{If the room is in the inventory of competitors for day } j\text{, with length of stay } l. \\ 0 & \text{None} \end{cases}$$

$A^a \in \{0,\ 1\}^{(|W|\,(|S|)\,|C|}$: Binary matrix representing the Leader's rooms for the length of stay l in day j.

$$[A^a]_{(j,l,q)} \in \{0,\ 1\} : \begin{cases} 1 & \text{If the leader's room can be used for allocating demand for day } j\text{, and length of stay } l. \\ 0 & \text{None.} \end{cases}$$

$A^b \in \{0,\ 1\}^{(|W|\,(|S|)\,|C|}$: Binary matrix representing the competitors' rooms for the length of stay l in day j over the total demand.

$$[A^b]_{(j,l,q)} \in \{0,\ 1\} : \begin{cases} 1 & \text{If the competitor's room can be used for allocating demand for day } j\text{, and length of stay } l. \\ 0 & \text{None.} \end{cases}$$

$\xi \in \mathbb{R}^{1\,(|W|\,|C|)}$: Line vector representing the dual variables associated to the leader capacity constraints.

$\nu \in \mathbb{R}^{1\,(|W|\,|C|)}$: Line vector representing the dual variables associated to the competitor capacity constraints.

$\eta \in \mathbb{R}^{1\,(|W|\,|S|)}$: Line vector representing the dual variables associated to the demand constraints.

and we re-write the model as HRM (2.4),

$$(\text{HRM}) \quad \max_{T} \quad \sum_{\substack{g\in G\\ q\in C\\ a\in A}} T^a x^a_{g,q} \tag{2.4}$$

$$\text{s.t.} \tag{2.5}$$

$$\min_{x^a,x^b} \quad \sum_{\substack{g\in G\\ q\in C\\ a\in A}} \pi^a_{g,q} x^a_{g,q} + \sum_{\substack{g\in G\\ q\in C\\ b\in B}} \pi^b_{g,q} x^b_{g,q} \tag{2.6}$$

$$\text{s.t.} \tag{2.7}$$

$$\sum_{\substack{q\in C\\ a\in A}} A^a x^a_{g,q} + \sum_{\substack{q\in C\\ b\in B}} A^b x^b_{g,q} = d_g, \qquad g\in G \qquad\qquad (\eta_g) \tag{2.8}$$

$$\sum_{\substack{g\in G\\ q\in C}} G^a x^a_{g,q} \le K^a, \qquad \forall a\in A \tag{2.9}$$

$$\sum_{\substack{g\in G\\ q\in C}} G^b x^b_{g,q} \le K^b, \qquad \forall b\in B \qquad\qquad (\nu_b) \tag{2.10}$$

$$x^a_{g,q}, x^b_{g,q} \ge 0, \qquad g\in G, a\in A, b\in B \text{ et } q\in Q. \tag{2.11}$$

In most industries, it is generally accepted that better qualities must correspond to higher prices. Therefore, the corresponding "Service-Quality Order" constraints are introduced into the upper level as a manner to obtain a suitable order in prices for different quality services,

$$T_{a,q} \le T_{a,q+1}, \forall a\in A, \forall q\in \{1 \le, \ldots, q, \ldots, |C|-1.\} \tag{2.12}$$

2.2.3 Moving Constraints to the First Level

Generally, bilevel problems are very difficult, having a feasible set usually disconnected in the presence of first level Constraints. Constraints cannot be moved freely from the lower level to the first level and vice versa, because the solution (and the feasible set) can be altered. First level constraint are enforced by the leader through the choise of appropriate values for the price vector, but the lower level constraints must be satisfied by the followers. In our model, the leader inventory capacity constraints (2.9) can be moved to the first level without affecting the optimal solution. This can help the resolution by reducing the numbers of variables needed. It is evident that this action does not hold for the inventory capacity of competitors (2.10), and in fact, it is easy to construct a counterexample.

Proposition : Assuming that HRM is feasible, bounded from the above, with a non-negative matrix of capacities A^a, $\forall a \in A$, then the leader inventory capacity constraints can be moved to the first level without affecting the optimal solution.

Proof : The proof is identical to the proof presented in Brotcorne et al. (2003) for the “Joint Design and Pricing in a Network (JDP)” problem. That means in practice that dual variables ξ_a, $\forall a \in A$ can be set to zero.

Without capacity constraints in the lower level, the corresponding problem (in x) results in an independent minimal cost assignment problem (an easy problem). Fortunately, there are very few competitors’ capacity constraints (2.10). Economically speaking, the presence of the competitor’s capacity contraints, implied that dual variables ν_b, $b \in B$ can take any negative value (when competitors inventories are at full capacity). This affect (negatively) the leader’s revenue modulated by the size of these competitors inventories, *i.e.,* the leader should put more “attention” to its main competitors with big inventories.

CHAPTER 3

STOCHASTIC BILEVEL PROGRAMMING

3.1 Stochastic Programming (SP)

In this section, we briefly introduce some concepts and common definitions from theory as presented in Kall and Wallace (2003); Birge and Louveaux (1997); Kall and Mayer (2005), and particularly, we focus on what is useful for the development of this thesis.

3.1.1 General Definitions

Let's define a general formulation of stochastic programs (SGP),

$$
\begin{aligned}
\text{(SGP)} \qquad \min_{x} \quad & g_0(x,\xi) && (3.1)\\
\text{s.t.} \quad & g_i(x,\xi) \leq 0 \qquad i = \{1,\dots,m.\},\\
& x \in X \subset \mathbb{R}^n,\\
& \xi \in \Xi \subset \mathbb{R}^k,\\
& k,n,m \in \mathbb{N},
\end{aligned}
$$

where ξ is a random vector varying over a support set $\Xi \subset \mathbb{R}^k$, in the probability space $(\Xi, \mathcal{F}, P)$, with a family $\mathcal{F}$ of measurable events and whose probability distribution P on $\mathcal{F}$ is given. We also have that for any subset $A \in \Xi, A \subset \mathcal{F}$ we know $P(A)$. The functions $g_i(x,\xi) : \Xi \to \mathbb{R},\ \forall x,\ \forall \xi,\ i = 0,\dots,m.$ are random variables.

After introducing this general formulation, we can formulate the Stochastic Linear Program (SLP) :

$$
\begin{aligned}
\text{(SLP)} \qquad \min_{x} \quad & c^T x && (3.2)\\
\text{s.t.} \quad & Ax = b,\\
& T(\xi)x = h(\xi), && (3.3)\\
& x \geq 0,\\
& x \in X \subset \mathbb{R}^n,\\
& \xi \in \Xi \subset \mathbb{R}^k,
\end{aligned}
$$

with $A \in \mathbb{R}^{m\times n}$, $b \in \mathbb{R}^m$, $c \in \mathbb{R}^n$. We should also assume $T(\xi) \in \mathbb{R}^{s\times n}$, $h(\xi) \in \mathbb{R}^s$ as random functions depending on ξ,

$$T(\xi) = T^0 + \xi_1 T^1 + \ldots + \xi_k T^k,$$
$$h(\xi) = h^0 + \xi_1 h^1 + \ldots + \xi_k h^k,$$

with matrices $T^0, \ldots, T^k$ and vectors $h^0, \ldots, h^k$ as deterministics. It is important to notice that these models (3.1), (3.2) are not well defined : the minimization and the constraints are not clear. In effect, the revision of this model can be done in several ways to obtain a deterministic equivalent. Stochastic linear program SLP (3.2) will be discussed in the next sections.

3.1.2 Single-stage SP Models

We consider that the problem needs a single decision "here and now"[1]. We suppose that any decision taken on the x variables does not have any effect on the random variables. The constraints (3.3) can be re-stated as a random function $\zeta(x, \xi)$, which is included in the objective. We have then the following definition,

$$\zeta(x, \xi) := T(\xi)x - h(\xi),$$

where $\xi : \Xi \to \mathbb{R}^r$ is a random vector, $T(\xi) \in \mathbb{R}^{s \times n}$ is a random matrix and $h(\xi) \in \mathbb{R}^s$ is a random vector. We can suppose a joint probability distribution for $(T(\xi), h(\xi))$, which is generally nonlinear.

3.1.3 Two-stage SP Models

It is important to observe that (3.2) is formulated with equality constraints. The definition presented in (3.1) has strict inequalities. In order to introduce more flexibility, the restrictions (3.3) must be replaced by a "recourse function". We define the set $Y = \{y \geq 0, y \in \mathbb{R}^n\}$ and the following constrains will replace (3.3),

$$Wy = T(\xi)x - h(\xi),$$
$$y \in Y,$$

with $W \in \mathbb{R}^{n \times n}$ deterministic. We assume a vector of cost $q \in \mathbb{R}^n$ for y, and we define the total cost as $q^T y$, which is linear. Obviously, we want to achieve any compensation with the minimal cost, $Q(x, T, h) = \min_y q^T y$.

1. There is no "corrective action" after the realization of the random variables.

The recourse function is defined as,

$$\begin{aligned} Q(x, T(\xi), h(\xi)) = \min_{y} \; & q^T y \\ \text{s.t.} \quad & Wy = T(\xi)x - h(\xi), \\ & y \in Y. \end{aligned}$$

It is convenient to consider the average of the cost on x and y and set the problem into the Two-stage Stochastic Programming (TSP) (3.4) form,

$$\begin{aligned} \text{(TSP)} \qquad \min_{x \in X} \quad & c^T x + \mathbb{E}_{\xi \in \Xi}\{Q(x, T(\xi), h(\xi))\} \\ \text{s.t.} \quad & Ax = b, \\ & x \geq 0, \\ & y \in Y. \end{aligned} \tag{3.4}$$

If the random variables in $T(\xi)$ and $h(\xi)$ have a continuous joint distribution, this problem is also nonlinear. Nevertheless, the most common paradigm in the literature is the Two-stage Stochastic Linear Program (TSLP) :

$$\begin{aligned} \text{(TSLP)} \quad \min_{x} \quad & c^T x + \mathcal{Q}(x) \\ \text{s.t.} \quad & Ax = b \\ & \mathcal{Q}(x) = \sum_j p^j Q(x, \xi^j) \\ & x \geq 0, \end{aligned}$$

where $Q(x, \xi) = \min\{q(\xi)^T y \mid W(\xi)y = h(\xi) - T(\xi)x, y \geq 0\}$, is the "Recourse function" and p^j is the probability of $\xi = \xi^j$, a particular realization of the random vector. The vector $\mathcal{Q}(x)$ is called the "expected recourse vector". This assumes that an early decision can be compensated by one later decision. Four specials cases arise from the definition of the constraints $Wy = T(\xi)x - h(\xi)$ in the recourse function :

1. Fixed Recourse : If $W(\xi) = W$ fixed (not random),
2. Simple Recourse : If $W(\xi) = [I, -I]$
3. Complete Recourse : If $\{t | t = W(\xi)y, \forall y \geq 0, \forall \xi\} = \mathbb{R}^m, m = rank(W)$
4. Relatively Complete Recourse : If $\{Ax = b, \forall x \geq 0\} \Rightarrow \{t | t = W(\xi)y, \forall y \geq 0, \forall \xi\} = \mathbb{R}^m$

3.1.4 Multi-stage Stochastic Linear Programs

A Multi-stage Stochastic linear program is a dynamic decision model where decisions are taken sequentially in stages (more than one). We define then feasibility sets for every stage as :

$$B_t = (x_1, \ldots, x_{t-1}; \xi_2, \ldots, \xi_t), \quad t = \{1, \ldots, T.\}.$$

In this case, after a first stage vector decision $x_1 \in B_1$, we observe the realization of vector random variables $\xi_2 \in B_1$. A new stage begins at this point, a decision $x_2(x_1, \xi_2) \in B_2$ is taken, and following to this decision an observation $\xi_3 \in B_2$ is done. We can continue for any stage needed. The Multi-stage Stochastic linear program (MSLP) is stated as :

$$\begin{aligned}
\text{(MSLP)} \quad \min_{x} \quad & \{c_1^T x_1 + \mathbb{E}\{\sum_{t=2}^{T} c_t^T(\xi_t) x_t(\xi_t)\}\} \\
\text{s.t.} \quad & A_{11} x_1 = b_1 \\
& A_{t1}(\xi_\tau) x_1 + \sum_{\tau=2}^{t} A_{t\tau}(\xi_\tau) x_\tau(\xi_\tau) = b_t(\xi_\tau), \quad t = 2, \ldots, T., \\
& x_1 \geq 0, \\
& x_t(\xi_\tau) \geq 0, \quad t = 2, \ldots, T.
\end{aligned}$$

Here the constraints $Wy = T(\xi)x - h(\xi), y \in Y$ are equivalent to $A_{t1}(x_1) + \sum_{\tau=2}^{t} A_{t\tau} x_\tau = b_t(\xi_\tau), \quad t = 2, \ldots, T$.

3.1.5 Chance-Constrained Program (SCCP)

Chance-constrained problems are difficult and generally it is necessary to assume convexity and smoothness to deal with them using mathematical programming methods. A "problem with single probabilistic constraints" (PSPC) is a problem of the form,

$$\begin{aligned}
\text{(PSPC)} \quad \min_{x \in X} \quad & \mathbb{E}\{c^T x\} \\
\text{s.t.} \quad & P(\xi | T_i(\xi) x \leq h_i(\xi)) \geq \alpha_i, \; i = 1, \ldots, m. \\
& \alpha \in [0, 1].
\end{aligned}$$

If $T_i(\xi) = T_i$ is constant, then the constraints,

$$P(\{\xi | T_i(\xi) x \leq h_i(\xi)\}) \geq \alpha_i, \; i = \{1, \ldots, m.\},$$

are equivalent to $T_i x \geq F_i^{-1}(\alpha_i)$. Here the single chance constraint turns into a linear constraint. Due to its simplicity, this type of Change-constrained program is one of the most utilized in the industry.

3.1.6 Recourse Problem (RP)

The recourse model paradigm consists to make one decision now and minimize the expected costs (or utilities) of the consequences of that decision. Recourse models can be extended to include more than one stage, known as a multistage problem, in which we make one decision now, wait for some uncertainty to be resolved (realized), and then make another decision based on what's happened. The objective is to minimize the expected costs of all decisions taken.

The simplest and most common form used is the Two-stage Recourse Problem. In this thesis, we also utilize this common form.

3.2 Stochastic Bilevel Programming (SBLP)

Because of the existence of random variables in the bilevel program, it is not possible to calculate exactly the vectors x, y that optimize f on the expectation of $f(x, y(\xi))$.

The general form of the stochastic bilevel program (GSBP) includes a lower-level variational inequality (VI(x,ξ)) problem :

$$
\begin{aligned}
\text{(GSBP)} \quad \min_{x} \quad & \mathbb{E}_{\xi}\{f(x, y(\xi))\} \\
\text{s.t.} \quad & x \in X, \\
& \xi \in \Xi, \\
& y(\xi) \in \mathrm{VI}(x,\xi), \\
& \mathrm{VI}(x,\xi) = \{y \mid T(x, y(\xi))^T \ (y - y') \leq 0, y' \in Z(x,\xi)\},
\end{aligned}
$$

where the feasible set $Z(x, \xi)$ is assumed convex, and corresponds to the feasibility set of the lower-level, and $T(x, y)$ is a continuous utility function. We also need $\xi \in \Xi$ defined in a probability space $(\Xi, \mathcal{F}, P)$.

The objective function of the stochastic bilevel model is a multiple integral over a vector of continuous random variables, then $\mathbb{E}_{\xi}[f(x, y(\xi))] = \int_{\Xi} f(x, y(\xi)) \ dF(\xi)$. This integral could be difficult to evaluate or its expression be unavailable. It is widely used to work with a discretization of the random distributions in order to express the expectation as a sum.

3.2.1 Important Properties of SBLP

To continue, let's consider the following assumptions in Wynter (2001) for a discrete stochastic bilevel problem with a non-linear lower level :

1. X is nonempty and closed.
2. The lower-level constraint set is of the form $Z(x) = \{y \mid g_i(x, y) \leq 0, i = 1, \cdots, K\}$ where $g_i(x, y)$ is continuous and convex in $y, \forall x \in X$, and for each $x \in X$, there is a y such $g_i(x, y) \leq 0$.
3. There is a $(x, y) \in \{(x, y) \in \text{ gr VI} \mid x \in X\}$, such as $f(x, y)$ is bounded from above.

Some properties of interest are :

— **Existence of optimal solutions to the discrete SBLP :** Under the previous hypotheses and T continuous, there is at least one solution to the discrete SBP (Patriksson and Wynter, 1999; Wynter, 2001).

— **Convexity of Two-stage Bilevel Programming (TBLP) where the objective depends only on the objective value of the lower-level** : Generally, either BLPs or SBLPs are non-convex. But there is a special case, the discrete formulation of the SBLP in which the upper level objective function depends only in the value of the solution of the lower-level (Shimizu et al., 1997),

$$
\begin{aligned}
\text{(SBP-OV)} \quad \min_{x} \quad & \sum_{j \in \Xi} p_j F_j(x, v_j(x, \xi^j)) \\
\text{s.t.} \quad & x \in X, \\
& v_j(x, \xi^j) := \inf \{f_j(x, y_j, \xi^j)\}, \\
& y_j \in Z_j(x) = \{y \mid g_i(x, y, \xi^j) \leq 0,\ i = 1, \cdots, K.\}.
\end{aligned}
$$

In this particular case with the former assumptions,

a. Assumptions 1) to 3) and Inf-compactness (f is lower semicontinuous, proper, and has bounded level sets on P),

b. $f_j(x, y)$, are convex,

c. $v_j(x, \xi^j)$, are convex on j,

d. $g_i(x, y)$, $i = \{1, \cdots, K.\}$, are convex,

e. $F_j(x, v)$, are convex and increasing in $v_j(x, \xi^j)$,

then the convexity of SBP-OV is obtained. The reader can look into Shimizu et al. (1997) and Patriksson and Wynter (1999) for more details on this interesting property.

— **Differentiability of the upper level objective** : This is a useful property for some resolution methods. Let's suppose for this property,

1. f is in the class C^2.
2. T is in C^1 and strongly monotone in y, $\forall x \in X$.
3. Uniqueness of the lower-level solution.
4. Let $I(x,y) = \{g_i(x,y) = 0 : i = 1,\ldots,K.\}$, and $\nabla_y g_i(x,y)$, $\forall i \in \{1,\ldots,K\}$, linearly independent.
5. Every $g_i(x,y)$ is in the class C^2.

Under the former assumptions $\mathbb{E}_\xi[f(x,y(\xi)]$ is locally Lipschitz continuous and directionally differentiable (Patriksson and Wynter, 1999).

3.2.2 Applications of SBLP

We can have different applications for SBLP. We present some of the most common applications found in literature.

1. **Optimal Pricing problem :** According to Wynter (2001), either in transportation or in telecommunication problems that are modeled as bilevel or MPECs, uncertainty can be incorporated to add realism. For this, we can set the demand as stochastic, $d(x) := d(x(\xi))$, with $\xi \in (\Xi, \mathcal{F}, P)$ a probability space. The same can be integrated for the capacity on the upper "usage limits" of resources or in the user's cost function, where infrastructure parameters can vary according to a known distribution.
2. **Stackelberg-Nash-Cournot equilibrium :** According to Sherali et al. (1983) and Wynter (2001)
3. **Structural optimization (Kim and Wen (1988); Evgrafov et al. (2003); Christiansen et al. (1997, 1998); Stavroulakis and Günzel (1998))** the (lowerlevel) state variables : displacements, stress and contact forces, $y \in Y$. The matrix $K(x)$ represents the stiffness of the material and is symmetric and positive semi-definite.

   Another similar application can be seen in Kim and Wen (1988), where a bilevel reliability constraint model is developed. In that model, uncertainty is taken into account for modeling loads as a random process. The objective function is constrained by the probabilities of various limit states within the allowable limits. Evgrafov and Patriksson (2003) and Stavroulakis and Günzel (1998) developed other complex applications in the same field.
4. **Multivariable control :** Li and Marlin (2006)

5. **Interdiction of Material Smuggling :** Pan et al. (2003) added to the model when the pair origin-destination (s,t) is unknown but governed by a probability mass function $p^\xi := P(\ (s,t) = (s^\xi, t^\xi), \xi \in \Xi)$. The authors argue that the models follow a nested "min-max" structure because the leader and follower use the same objective function.
6. **Telecommunication :** Werner (2004)

3.2.3 Algorithms for the Resolution of SBLP

Wynter (2001) presents an algorithm (for the continuous case) that solves the problem generating subgradients of f (based on Outrata and Zowe (1995))[2]. Some other methods, which are not detailed here but that are found in literature, are :

— A penalty method making use of a merit function reformulation of the VIP (Wynter (2001)).

— A perturbation method, which uses the KKT conditions of the VIP (Wynter (2001)).

— Methods using random sampling.

— Stochastic quasi-gradient algorithms.

— Stochastic decomposition (mostly for discrete cases).

— Approximation strategies and heuristics.

3.2.4 SBLP models with Recourse

Let's define a SBLPR with recourse as follow :

$$
\text{(SBLPR)} \qquad \begin{aligned} \min_{x_1, y_1} \quad & F(x_1, y_1) + Q(x, y) \\ \text{s.t.} \quad & f_F(x_1, y_1) \leq 0, \\ & y_1 \in \operatorname*{argmin}_{y_1'} g_1(x_1, y_1'), \\ & \qquad\qquad \text{s.t.} \quad h_{g_1}(x_1, y_1') \leq 0, \end{aligned}
$$

2. We note that this algorithm utilizes the local Lipchitz continuity and directional differentiability of the implicit function f, and at least this requires the assumptions presented above.

where, $Q(x_2, y_2) = \mathbb{E}_{\xi\in\Xi}[\Phi(x_2, y_2, \xi)]$, $\xi : \Xi \to \mathbb{R}$, $\forall\xi \in \Xi$ and :

$$\begin{aligned}
\Phi(x_2, y_2, \xi) = \min_{x_2,y_2} {}_{\xi} \quad & G(x_2, y_2, \xi), \\
\text{s.t.} \quad & f_G(x_1, y_1, x_2, y_2, \xi) \le 0, \\
& y_2 \in \operatorname*{argmin}_{y_2'} g_2(x_2, y_2', \xi), \\
& \qquad \text{s.t.} \quad h_{g_2}(x_2, y_2', \xi) \le 0
\end{aligned}$$

Based on this definition, this problem can be transformed into a standard Two-stage Stochastic problem, considering the following conditions : First, we need that both objective and constraints functions, in upper and lower level, are convex, type C^1 in $(x_1, y_1, x_2, y_2)^*$, at the optimal solution, by adopting a suitable constraint qualification. Second, the KKT conditions need to be established, taking μ and δ_ξ (shown in the right margin) as the KKT multipliers, and assuming that $\xi \in \Xi$ is finite, or at least that we can sample ξ_s, for $s \in S$, with S finite, large enough and non-biased.

We note that the Second Stage constraint in Φ does not depend on variables x_1,y_1, but they should remain tied by the objective. Therefore, this problem can be formerly re-written as a Single-stage Stochastic Bilevel problem as :

$$\begin{aligned}
\text{(RSSBP)} \qquad \min_{x_1,y_1} \quad & F(x_1, y_1) + \mathbb{E}_{\xi\in\Xi}[\min_{x_2,y_2} {}_{\xi} \; G(x_2, y_2, \xi)] \\
\text{s.t.} \quad & f_F(x_1, y_1) \le 0, \\
& f_G(x_1, y_1, x_2, y_2, \xi) \le 0, \\
& y_1 \in \operatorname*{argmin}_{y_1'} g_1(x_1, y_1'), \\
& y_2 \in \operatorname*{argmin}_{y_2'} g_2(x_2, y_2', \xi), \quad \forall\xi \in \Xi \\
& h_{g_1}(x_1, y_1') \le 0, \qquad (\mu \ge 0) \\
& h_{g_2}(x_2, y_2', \xi) \le 0, \qquad (\delta_\xi \ge 0) \quad \forall\xi \in \Xi.
\end{aligned}$$

3.3 The Value of Stochastic Solutions

While several decision problems involve uncertainty, utilisation of stochastic programs are rarely used in practice because of their complexity. A common practice is to make approximations or expectations of the random variables involved. After that, both deterministic solutions for different scenarios and sensitivity analysis are utilized to gain knowledge over uncertainty. When the computational cost is too high, this practice could be adequate. Moreover, a common criterion to determine the importance of a stochastic solution is calculating the expected value of perfect information Expected Value of Perfect Information (EVPI), proposed by Raiffa and Schlaifer (1961). This number represents the value of having complete information over uncertainty and it is calculated using "WS" and "RP". WS is commonly known as the "wait and see" value and denotes the value of using the optimal solution for "each scenario" (3.9) ; RP denotes the "here and now" solution, which is the optimal value solution of the "recursion problem". Some of these approximations and related bounds have being revisited and some new ones have being proposed by Maggioni et al. (2012) to consider cases such as Multi-stages Stochastic Problems.

The most common approximation in literature to solve stochastic problems is the "Deterministic Approximation", also known as the "Expected Value Problem", or the Expected Value (EV), as proposed by Birge (1982).

$$\mathrm{EV} = \max_{(x_1,y_1)} \phi((x_1,y_1),\mathbb{E}(\xi)). \tag{3.5}$$

Taking $\mathbb{E}(\xi) = \bar{\xi}$, yield a First Stage solution, $(\bar{x}_1,\bar{y}_1)(\bar{\xi})$. The expected result of using EV solution is then called Expected Solution of the Expected Value (EEV),

$$\mathrm{EEV} = \mathbb{E}_{\xi}[\phi((\bar{x}_1,\bar{y}_1),\xi)]. \tag{3.6}$$

The most standard approximations are the following : First, the Value of the Stochastic Solution (VSS), which will be defined in (3.7), also proposed by Birge (1982), and that illustrates the expected gain from solving a stochastic programming problem rather than using a deterministic approximation and in which stochastic variables are replaced by their expected values.

$$\mathrm{VSS} = \mathrm{RP} \ - \ \mathrm{EEV} \tag{3.7}$$

Large VSS relative values are associated with systems and models, in which uncertainty plays a very important role. The second approximation is the "Value of perfect Information", EVPI (presented in (3.8)), which indicates the difference from solving the stochastic programming problem in comparison to the solution of "knowing" the future for any outcome.

$$\mathrm{EVPI} = \mathrm{WS} \ - \ \mathrm{RP} \tag{3.8}$$

The perfect information behavior is taken from the "Wait-&-See" solutions,

$$\mathrm{WS} = \mathbb{E}(\xi)[\max_{(x_1,y_1)} \phi((x_1, y_1), \xi)]. \tag{3.9}$$

It has been established by Madansky (1960) that EEV $\geq$ RP $\geq$ WS $\geq$ EV, for a minimisation problem.

We refer the reader to Birge (1982) and Escudero et al. (2007); Maggioni et al. (2012) for further explanations and definitions on approximations.

3.4 Some Applications of Stochastic Programming and Bilevel Programming

Stochastic Programming and Bilevel Programming have been developed in multiple fields, in which we can notice the following advances :

1. **Structural mechanics :** Patriksson and Wynter (1997a) propose a model for addressing a structural optimization problem. Christiansen et al. (2001) then formulate a topology optimization model. Wynter (2009) also cover this problem.
2. **Traffic models :** Patriksson (2008) introduces stochastic data fluctuations in a bilevel context.
3. **Network Pricing :** Mirza Alizadeh (2013) continue the Labbé et al. (1998) framework in a two-stage stochastic model, which tries to counter the net-revenue loss of neglecting randomness, and in which each stage became a bilevel structure. In the second-stage, demand and cost are stochastic. Both stages are connected using constraints of leader's prices. In the second part of Mirza Alizadeh (2013), the authors study three variations :
   (a) Tardiness and reliability terms in the commuters' disutility.
   (b) Chance constraints at the leader level as concern for the network administrator.
   (c) Congestion associated with random capacities along the arcs of the transportation network.
4. **Energy biddings :** Fampa et al. (2008) formulate a model to address the strategic-bidding process in the energy market as a Stackelberg Game. To do this, an energy provider maximizes its profit expectation at the first level, and minimizes its operational costs at the inferior level. The bidding process is based on stochastic scenarios. A similar problem was alternatively studied in Hobbs et al. (2000) using also game theoretical approaches but taking a Nash Equilibrium.
5. **Retail Market :** Carrión et al. (2009) model a retailer that optimizes its medium-term revenue with a given risk level in a context in which prices, demand and competitors prices are uncertain.

6. **Energy market modeling :** Kalashnikov et al. (2010) formulate the natural gas cash-out problem as multi-stage bilevel stochastic problem.
7. **Duopoly price fixation :** Cooper et al. (2012) optimizes the performance of price strategies as dynamic prices and demand estimation uncertainties.
8. **Telecommunication :** As in Werner (2005) and Audestad et al. (2006). In the specific case examined by Audestad et al. (2006) several agents (owner, clients and lease operators) are analyzed, in which the objective is to maximize the combined revenue for the owner and lease operators, under either deterministic or stochastic environments.
9. **Multi-product pricing :** Like in Kosuch et al. (2012). A leader "prices" multiple products with the intention of maximizing its revenue. Clients shop for lowest price products, which are under capacity constraints. The capacity is stochastic (it is modeled as a chance constraint).

It can be easily seen that Bilevel Programming, combined with stochastic programming, is a trending paradigm for modeling problems in a context of high competition and commoditization. Uncertainty plays a bigger role in the context of economic stagnation and Eco-responsibility and sustainability issues. In the near future, it would be possible to see new models in the field of parallel computing and data-mining

3.5 Stochastic Programming Models for HRM

In this section we present two different Stochastic Programming HRM models that have been proposed in the literature [3]. These models will be important for the devolpement of the stochastic bilevel models in this thesis. The first model, is the model of Goldman et al. (2002). This model can be considered as a general framework in stochastis programming HRM. In this model, demand can take on a limited number of realizations, which are considered as scenarios. The model is derived from EMSR in the airline industry. The list of parameters and variables of the stochastic model considered for optimizing the room distribution are :

Φ : Demand probability function.

E : Set of Scenarios.

L : Length of stay (LOS), in days.

J : Planning Horizon (in days).

Q : Set of QoS.

T_q : Price for QoS "q", $q \in Q$.

3. The models were reframed to follow the ongoing notation.

C_j : Capacity for day "j".

d^e_{jqk} : Demand on day "j", LOS of "k" days, with QoS "q", $q \in Q$, in scenario "e", $e \in E$.

D_{jqk} : Demand in the discrete set of values : $\{d^1_{jqk} < d^2_{jqk} < \ldots < d^{|E|}_{jqk}\}$

Variable in the model :

x^e_{jqk} : Integer variable, booking limit (number of room) for arrival day "j", QoS "q" and LOS "k", in scenario "e". It represents the demand D_{jqk} in the interval $[d^{e-1}_{jqk}, d^e_{jqk}]$.

The model is,

[HSRM1]

$$\max_x \sum_{e=1}^{|E|} \sum_{j=1}^{|J|} \sum_{q=1}^{|Q|} \sum_{k=1}^{|L|} k\, T_q \Phi(D_{jqk} \geq d^e_{jqk})\, x^e_{jqk}$$

$$\text{s.t.}$$

$$\sum_{e=1}^{|E|} \sum_{l \leq j} \sum_{q=1}^{|Q|} \sum_{(l+k)>j} x^e_{lqk} \leq C_j, \qquad \forall j \in J$$

$$x_{jqk,1} \leq d_{jqk,1} \qquad \forall j, q, k.$$

$$x^e_{jqk} \leq d^e_{jqk} - d^{e-1}_{jqk}, \qquad \forall j, q, k.\ \forall e \in E$$

$$x^e_{jqk} \geq 0 \qquad \forall j, q, k.$$

$$x^e_{jqk} \text{ Integer}$$

The second model, commonly reference in literature is the model in Lai and Ng (2005). This models considers an stochastic network revenue management formulation. The model's robust optimization framework integrates goal programming with scenario realizations. The models takes two parameters to weight the trade-off between optimality and robustness, w and λ.

List of parameters :

p_e : Probability of scenario e.

$T^e_{i,j}$: Price in scenario "e", $e \in E$.

C_i : Capacity on day "i".

λ : Risk trade-off factor. between expected revenue and deviation, for the decision-maker.

w^e_{ij} : Penalty for excess/lack of assignments in day "i" with check-out in day "j", in scenario "e", $e \in E$.

D^e_{ij} : Demand on day "i" with check-out in day "j", in scenario "e", $e \in E$.

In this model, a penalization is introduced because the assignment could not match demand, for each scenario, which means $x_{i,j} \neq D^e_{ij},\ (i,j),\ 0 \leq i < j \leq J$:

Variable in the model :

$x_{i,j}$: Booking limit for arrival day "i" and check-out in day "j".

The reduced model is stated as,

[HSRM2]

$$\max_x \sum_{e=1}^{|E|} p_e \sum_{j=1}^{|J|-1} \sum_{j=i+1}^{|J|} \left\{T^e_{i,j}\, x_{i,j} - w^e_{i,j}|D^e_{ij} - x_{i,j}|\right\} - \lambda \sum_{e=1}^{|E|} p_e \left| \sum_{j=1}^{|J|-1} \sum_{j=i+1}^{|J|} T^e_{i,j}\, x_{i,j} - \sum_{e=1}^{|E|} p_e \sum_{j=1}^{|J|-1} \sum_{j=i+1}^{|J|} T^e_{i,j}\, x_{i,j} \right|$$

s.t.

$$\sum_{i=0}^{k-1} \sum_{j=k+1}^{|J|} x_{i,j} + \sum_{j=k+1}^{|J|} x_{k,j} - \sum_{i=0}^{k-1} x_{i,k} \leq C_k, \qquad \forall k \in J$$

$$\sum_{j=1}^{|J|} x_{0,j} \leq C_0$$

$$x_{i,j} \leq \max_{e \in E}\{D^e_{ij}\}, \qquad \forall (i,j), 0 \leq i < j \leq J.$$

$$x_{i,j} \geq 0, \qquad \forall (i,j), 0 \leq i < j \leq J.$$

In the next Chapter, we will present the bilevel stochastic programming model developed in this thesis.

CHAPTER 4

BILEVEL STOCHASTIC PROGRAMMING MODELS FOR HOTEL REVENUE MANAGEMENT

In this chapter, we propose SBLP for the hotel industry, which are ordered by their level of complexity. Their main differences are in terms of the number of stages considered and the inclusion of penalizations or overbooking. In fact, the first three models only have one stage whilst the last two models have two-stages. Moreover, penalization for revenue variation is only included in MODEL 2 and overbooking is only considered in MODEL 3 through the inclusion of a probabilistic constraint. Finally, Model 4 and Model 5 are two-stage models, whose only difference is the restriction that connect both stages, either an absolute or a proportional constraint.

4.1 SBLP models for the Hotel Industry

Decision making under uncertainty with two or more agents is a new field of research. In the literature, we can find two majors points of view combining bilevel programming, game theory, and stochastic programming. It is possible to find a recent application to network communication in Werner (2004).

For simplicity, we propose to formulate a SBLP as,

$$\begin{aligned} \text{(SBLP)} \quad \min_{x,y} \quad & F(x,y,\xi) \\ \text{s.t.} \quad & g(x,y,\xi) \leq 0 \\ & y \in \operatorname*{argmin}_{y'} f(x,y',\xi) \\ & \qquad \text{s.t.} \quad h(x,y',\xi) \leq 0, \end{aligned}$$

where ξ is a random vector varying over a set $\Xi \subset \mathbb{R}^k$, in the probability space $(\Xi, \mathcal{F}, P)$, with a family $\mathcal{F}$ of measurable events, and whose probability distribution P on $\mathcal{F}$ is given. This model should be more precisely defined to be considered as a contribution for HRM. Several possibilities are open, especially from the point of view of the leader :

1. We can choose that both agents, the leader and the followers, face uncertainty from the environment, which comes from the competition : the price and the QoS, or by changes in hotel capacity.

2. The leader could face uncertainty from the follower's response. We need to limit the scope to assume that the leader will have the information from the response of the followers (the solution of the lower level). But we could also assume that only a part of this information is available by introducing new variables (random variables and upper level constraints) to model this noise and adding them to the real information. The leader will have an estimation of the response [1].
3. Simple probabilistic constraints in the lower level such as variation in demand, arrival date or LOS.

An initial use of bilevel stochastic programs can be found in Patriksson and Wynter (1997b); Christiansen et al. (1997). Those models were developed taking into account the variability of the inputs and their unfeasibility. In our case, we introduce a model with expectations in the leader revenue as,

$$
\begin{aligned}
\min_{x,y} \quad & \mathbb{E}_{\xi}\{F(x,y,\xi)\} \\
\text{s.t.} \quad & g(x,y,\xi) \leq 0 \\
& y \in \operatorname*{argmin}_{y'} f(x,y',\xi) \\
& \qquad \text{s.t.} \quad h(x,y',\xi) \leq 0.
\end{aligned}
$$

However, solving bilevel problems is always challenging, and for some parameters these problems could also become unfeasible. The common difficulty of bilevel problems is non convexity that also can occur in stochastic models. In such cases, the use of "pertinent" recourse functions could help to obtain feasible models.

4.2 SBLP models with Expectations in the Objective

In the first model, the objective is to maximize the expectation over several scenarios. Each scenario follows the determistic HRM framework presented in Chapter 2.

For this model we have

— **First level :** Maximize the expected revenue over all the leader hotels.

— **Second level objective :** Minimize the perceived cost for all the stays in the planning horizon over all the hotels.

— **Model constraints (in lower level) :**

1. To ensure that the maximal capacity of each hotel is not exceeded.

1. An alternative is to include the random vector in the lower level problem. Then the constraint linking the results will be in the lower level.

2. Demand should be satisfied by day and by length of stay.
3. The *assignments* for every group should be non-negative.

The parameters are the following :

E : Set of scenarios, $e \in E$.
A : Set of leader's hotels.
B : Set of competitor's hotels.
p_e : Probability of scenario $e \in E$.
K^a : Maximum capacity for leader's hotels $a \in A$.
K^b : Maximum capacity for competitor's hotels $b \in B$.
$d_{e,j,g,l}$: Demand for day $j \in W, l \in S, g \in G$, and scenario $e \in E$

Decision variables in the model are the following :

$T^i_{e,q}$: Room price $i \in A \cup B$, for the quality $q \in C$ in scenario $e \in E$.
$\pi^i_{e,g,q}$: Perceived price for inventory $i \in \{A \cup B\}, g \in G, q \in C$ and scenario $e \in E$.
$x^i_{e,j,g,l,q}$: Demand for day $j \in W, l \in S, g \in G, q \in C$, and scenario $e \in E$.

We introduce the notation, for the perceived price for group $g \in G$ is shown in (4.1) :

$$\pi^a_{g,q} = \left\{T^i_q + \alpha_g \lambda_i - \beta_g q\right\}, \forall q \in C, \forall q \in C. \tag{4.1}$$

and,

$$\pi^b_{e,g,q} = \left\{T^i_{eq} + \alpha_g \lambda_i - \beta_g q\right\}, \forall q \in C, \forall q \in C, \forall e \in E.$$

The initial Hotel Stochastic Bilevel Program, named Model 1, for many groups, hotels

and quality services, is the following :
MODEL 1 :

$$\max_{T^a} \sum_{e=1}^{|E|} p_e \sum_{j=1}^{W} \sum_{a,g,l,q} l\, T_q^a\, x_{e,j,g,l,q}^a$$

$$g \in G, \forall q \in C, \forall l \in S$$

s.t

$$L(T) = \min_{x,y} \sum_{j=1}^{W} \sum_{a,b} \left\{ \sum_{g,l,q} l\, \pi_{g,q}^a\, x_{e,j,g,l,q}^a + \sum_{g,l,q} l\, \pi_{e,g,q}^b\, y_{e,j,g,l,q}^b \right\}$$

s.t

$$\sum_{q\in C} \sum_{g\in G} \left\{ \sum_{l\in S} x_{e,j,l,q,g}^a + \sum_{k>1} \sum_{l>k} x_{e,(j-l+1),l,q,g}^a \right\} \leq K_e^a, \quad \forall a \in A, \forall j \in W, \forall e \in E$$

$$\sum_{q\in C} \sum_{g\in G} \left\{ \sum_{l\in S} y_{e,j,l,q,g}^b + \sum_{k>1} \sum_{l>k} y_{e,(j-l+1),l,q,g}^b \right\} \leq K_e^b, \quad \forall b \in B, \forall j \in W, \forall e \in E$$

$$\sum_{a\in A} \sum_{q\in C} x_{e,j,g,l,q}^a + \sum_{b\in B} \sum_{q\in C} y_{e,j,g,l,q}^b = d_{e,j,g,l} \qquad \forall e \in E, \forall j \in W, \forall\, g \in G, \forall\, l \in S$$

$$x_{e,j,g,l,q}^a,\ y_{e,j,g,l,q}^b \geq 0$$

$$\forall e \in E, \forall a \in A, \forall b \in B, \forall j \in W, \forall\, g \in G, \forall\, q \in C, \forall\, l \in S$$

It is important to notice that this first model is only a tariffication model and thus we can only obtain the assignments for each scenario but not the booking limits. One possibility would be to introduce a penalization in the objective in order to reduce revenue variations in all the scenarios and thus assignments could be more homogeneous. This is still not a booking limit but it gives the possibility of homogenizing the assignments. Reducing booking limits variability is important for hotels that want to forecast their needs both in services and stocks.

Therefore, in this second model, the objective is to maximize the expectations by subtracting the penalty for the deviation from the weighted mean in absolute value. In other words, we want to penalize big deviations from the average revenue. This second model is close to models utilized in optimal control and finance, in which the purpose is to penalize the notion of risk aversion for decision makers.

The parameters and variables for our second model are equivalent to those presented in our first model, except for the inclusion of the parameter κ that corresponds to the risk factor.

This risk factor allows to the manager to set the amount of risk that it is willing to take.

κ : Weight for the penalty.

Our second stochastic bilevel model, named MODEL 2, for many groups, hotels and quality services, is the following :
MODEL 2 :

$$\max_{T^a} \sum_{e=1}^{|E|} p_e \sum_{j=1}^{W} \sum_{a,g,l,q} l\, T_q^a\, x_{e,j,g,l,q}^a$$

$$-\kappa \sum_{e=1}^{|E|} p_e \left| \sum_{j=1}^{W} \sum_{a,g,l,q} l\, T_q^a\, x_{e,j,g,l,q}^a - \sum_{e=1}^{|E|} p_e \sum_{j=1}^{W} \sum_{a,g,l,q} l\, T_q^a\, x_{e,j,g,l,q}^a \right|$$

$$g \in G, \forall q \in C, \forall l \in S$$

$$\min_{x,y} \sum_{e=1}^{|E|} \sum_{j=1}^{W} \sum_{a,b} \left\{ \sum_{g,l,q} l\, \pi_{e,g,q}^a\, x_{e,j,g,l,q}^a + \sum_{g,l,q} l\, \pi_{e,g,q}^b\, y_{e,j,g,l,q}^b \right\}$$

$$\forall\, g \in G, \forall\, q \in C, \forall\, l \in S$$

s.t

$$\sum_{q\in C} \sum_{g\in G} \left\{ \sum_{l\in S} x_{e,j,l,q,g}^a + \sum_{k>1} \sum_{l>k} x_{e,[(j-l+1) \bmod W],l,q,g}^a \right\} \leq K^a, \quad \forall a \in A, \forall j \in W, \forall e \in E$$

$$\sum_{q\in C} \sum_{g\in G} \left\{ \sum_{l\in S} y_{e,j,l,q,g}^b + \sum_{k>1} \sum_{l>k} y_{e,[(j-l+1) \bmod W],l,q,g}^b \right\} \leq K^b, \quad \forall b \in B, \forall j \in W, \forall e \in E$$

$$\sum_{a\in A} \sum_{q\in C} x_{e,j,g,l,q}^a + \sum_{b\in B} \sum_{q\in C} y_{e,j,g,l,q}^b = d_{e,j,g,l} \qquad \forall e \in E, \forall j \in W, \forall\, g \in G, \forall\, l \in S$$

$$x_{e,j,g,l,q}^a,\ y_{e,j,g,l,q}^b \geq 0$$

$$\forall e \in E, \forall a \in A, \forall b \in B, \forall j \in W, \forall\, g \in G, \forall\, q \in C, \forall\, l \in S$$

Although the introduction of penalization in this second model, we acknowledge that it could not result in homogeneous assignments and that at the same time we are subject to managers' risk aversion. If the manager is not risk averse, the impact of the penalization should be reduced (setting $\kappa \approx 0$) and we will arrive to the same situation presented in MODEL 1. As already mentioned, it is not enough to have just the price "information" T^a, because the industry would like to have a value for the Prices/"booking limits" to compare whether the current season corresponds to a good or a bad scenario before it finishes. Therefore, other models must be explored in this thesis. For instance, we can introduce a second stage and attempt to obtain new Prices/"booking limits" for the remaining allocations in the middle of the season. This possibility will be introduced in the last two models of this chapter. In the meanwhile, we will present a third model that aims to consider over-booking through the use of probabilistic constraints.

4.3 SBLP models with Probabilistic Constraints

In this section we present a third stochastic bilevel model that was developed not only to consider over-booking through the introduction of probabilitic constraints, but also to allow us to incorporate changes in capacity that are not totally controlled by managers.

For example, instead of considering that demand follows a normal distribution, we could consider that hotel capacity varies following the normal distribution. The manager could impose a hard restriction to the maximum number of rooms overbooked in order to limit the risk that is willing to take. Therefore, we have two capacity restrictions for the leader, the first one is a strict constraint that represents the maximum willingness for overbooking (4.2), given by parameter Δ, and the second one corresponds to the probabilistic constraint per se (4.3). We are going to suppose that $\xi_e, \forall e \in E$, follows a Normal distribution, i.i.d $\xi_e \sim \mathcal{N}(0, \sigma^2)$. With the linear constraints (4.2) and (4.3) we seek to model the case in which the hotel available capacity can vary because of wrong management decisions, unexpected damages caused by fire or leaks, overstay guests or clients that do no-show, etc. We want these constraints to be hold with some risk parameters chosen by the manager (as a constant), or as another random variable $i.i.d\ M_e \sim U(0,1)), \forall e \in E$.

Two additional parameters are included in this model compared to the first two models. These parameters are the following :

$\Delta \geq 0$: Max overbooking room units.

σ^2 : Normal Distribution Variance.

Our third stochastic bilevel model, named MODEL 3, is the following :
MODEL 3 :

$$\max_{T^a} \sum_{e=1}^{|E|} p_e \sum_{j=1}^{W} \sum_{a,g,l,q} l\, T_q^a\, x_{e,j,g,l,q}^a,$$

$$g \in G, \forall q \in C, \forall l \in S$$

s.t

$$\min_{x,y} \sum_{j=1}^{W} \sum_{a,b} \left\{ \sum_{g,l,q} l\, \pi_{g,q}^a\, x_{e,j,g,l,q}^a + \sum_{g,l,q} l\, \pi_{e,g,q}^b\, y_{e,j,g,l,q}^b \right\}$$

$$\forall\, g \in G, \forall\, q \in C, \forall\, l \in S$$

s.t

$$\sum_{q \in C} \sum_{g \in G} \left\{ \sum_{l \in S} x_{e,j,l,q,g}^a + \sum_{k>1} \sum_{l>k} x_{e,(j-l+1),l,q,g}^a \right\} \leq K^a + \Delta, \quad \forall a \in A, \forall j \in W, \forall e \in E \tag{4.2}$$

$$P_{\xi_e} \left(\sum_{q \in C} \sum_{g \in G} \left\{ \sum_{l \in S} x_{e,j,l,q,g}^a + \sum_{k>1} \sum_{l>k} x_{e,(j-l+1),l,q,g}^a \right\} \leq K^a + \xi_e' \right) \leq M_{j,e}^a, \quad \forall a \in A, \forall j \in W, \forall e \in E \tag{4.3}$$

$$\sum_{q \in C} \sum_{g \in G} \left\{ \sum_{l \in S} y_{e,j,l,q,g}^b + \sum_{k>1} \sum_{l>k} y_{e,(j-l+1),l,q,g}^b \right\} \leq K^b, \quad \forall b \in B, \forall j \in W, \forall e \in E$$

$$\sum_{a \in A} \sum_{q \in C} x_{e,j,g,l,q}^a + \sum_{b \in B} \sum_{q \in C} y_{e,j,g,l,q}^b = d_{e,j,g,l} \qquad \forall e \in E, \forall j \in W, \forall\, g \in G, \forall\, l \in S$$

$$x_{e,j,g,l,q}^a,\ y_{e,j,g,l,q}^b \geq 0$$

$$\forall e \in E, \forall a \in A, \forall b \in B, \forall j \in W, \forall\, g \in G, \forall\, q \in C, \forall\, l \in S$$

In this case $\xi_e\,'$ is a realization of $\xi_e \sim N(0, \sigma^2)$ but restricted to $-K_a \leq \xi_e' \leq \Delta$ and $M_{j,e}^a, \forall e \in E$ is a realization of $M_e^a \sim U(0,1)$. Every constraint can be converted to a linear constraint, as follow,

$$\Phi_{\xi_e}^{(0,\sigma^2)} \left(\sum_{q \in C} \sum_{g \in G} \left\{ \sum_{l \in S} x_{e,j,l,q,g}^a + \sum_{k>1} \sum_{l>k} x_{e,(j-l+1),l,q,g}^a \right\} \right) \geq 1 - M_{j,e}^a, \quad \forall a \in A, \forall j \in W, \forall e \in E \tag{4.4}$$

where $\Phi_{\xi_e}^{(0,\sigma^2)}$ is the normal probability density function.

Because we have individual probabilistic constraints for each scenario, they maintain their simplicity,

$$\sum_{q\in C}\sum_{g\in G}\left\{\sum_{l\in S} x^a_{e,j,l,q,g} + \sum_{k>1}\sum_{l>k} x^a_{e,(j-l+1),l,q,g}\right\} \geq Q^{(0,\sigma^2)}(1 - M^a_{j,e}), \quad \forall a \in A, \forall j \in W, \forall e \in E \tag{4.5}$$

where $Q^{(0,\sigma^2)}(p)$ is the Gaussian p-quantile.

Like in the other models, the introduction of individual probabilistic constraints does not allow us to obtain booking limits because it solves each scenario independently.

We propose to develop a two-stage stochastic bilivel model in which booking limits are considered in the first stage, and demand stochasticity is allowed in the second stage (through the scenarios). We can incorporate overbooking in the second stage without the complexity of introducing probabilistic constraints considering stochastic capacity in every scenario. Therefore, MODEL 3 is not necessary to be solved. This approach is presented in the following section.

4.4 SHBP with Recourse

In this section, we present two stochastic bilevel models with recourse functions that are relevant for the hotel industry because they allow hotel managers to reconsider decisions taken in the first stage. These models are finally the ones that integrate most of the previous concepts presented in the previous models and thus these are the ones that will be solved in the next chapter.

SBLP models with Recourse were presented previously in 3.2.4. However, in practice, this model is not useful for the industry because the two stages were only “tied” in the objective and thus they are independent. Because of that, we need to introduce another constraint that is pertinent for the hotel industry and that recourse the second stage. This constraint is price variation between these two stages.

The model, including this new constraint, is then formulated as the Stochastic Hotel

Bilevel Program (SHBP) model,

SHBP :

$$\max_{T_a} \sum_{j=1}^{W} \sum_{a,g,l,q} l\, T'_{a,q}\, x'_{a,j,g,l,q} + Q(x,y)$$

$$g \in G, \forall q \in C, \forall l \in S$$

s.t

$$(t_{a,q}, t'_{a,q}) \in C_\delta, \quad \forall a \in A, \forall q \in Q$$

$$T'_{a,q} \leq T'_{a,q+1}, \forall a \in A, \forall q \in \{1 \leq, \ldots, q, \ldots, \leq |C| - 1.\} \tag{4.6}$$

$$L(T) = \min_{x,y} \sum_{j=1}^{W} \sum_{a,b} \left\{ \sum_{g,l,q} l\, \pi'_{a,g,q}\, x'_{a,j,g,l,q} + \sum_{g,l,q} l\, \pi_{b,g,q,1}\, y'_{b,j,g,l,q} \right\} \tag{4.7}$$

$$\forall\, g \in G, \forall\, q \in C, \forall\, l \in S$$

s.t

$$\sum_{q \in C} \sum_{g \in G} \left\{ \sum_{l \in S} x'_{a,j,l,q,g} + \sum_{k>1} \sum_{l>k} x'_{a,(j-l+1),l,q,g} \right\} \leq K'_a, \quad \forall a \in A, \forall j \in W$$

$$\sum_{q \in C} \sum_{g \in G} \left\{ \sum_{l \in S} y'_{b,j,l,q,g} + \sum_{k>1} \sum_{l>k} {y'}^{b}_{(j-l+1),l,q,g} \right\} \leq K'_b, \quad \forall b \in B, \forall j \in W$$

$$\sum_{a \in A} \sum_{q \in C} x'_{a,j,g,l,q} + \sum_{b \in B} \sum_{q \in C} y'_{b,j,g,l,q} = d'_{j,g,l} \qquad \forall j \in W, \forall\, g \in G, \forall\, l \in S$$

$$x'_{a,j,g,l,q},\ y'_{b,j,g,l,q} \geq 0$$

$$\forall a \in A, \forall b \in B, \forall j \in W, \forall\, g \in G, \forall\, q \in C, \forall\, l \in S$$

$$Q(x,y) = \max_{T_a} \sum_{e=1}^{|E|} p_e \sum_{j=1}^{W} \sum_{a,g,l,q} l\, T_{a,q}\, x^a_{e,j,g,l,q}$$

$$g \in G, \forall q \in C, \forall l \in S$$

$$T_{a,q} \leq T_{a,q+1}, \forall a \in A, \forall q \in \{1 \leq, \ldots, q, \ldots, \leq |C| - 1.\} \tag{4.8}$$

s.t

$$\min_{x,y} \sum_{j=1}^{W} \sum_{a,b} \left\{ \sum_{g,l,q} l\, \pi_{a,g,q}\, x^a_{e,j,g,l,q} + \sum_{g,l,q} l\, \pi_{b,g,q,e}\, y^b_{e,j,g,l,q} \right\} \tag{4.9}$$

$$\forall\, g \in G, \forall\, q \in C, \forall\, l \in S$$

s.t

$$\sum_{q \in C} \sum_{g \in G} \left\{ \sum_{l \in S} x^a_{e,j,l,q,g} + \sum_{k>1} \sum_{l>k} x^a_{e,(j-l+1),l,q,g} \right\} \leq K^a_e, \quad \forall a \in A, \forall j \in W, \forall e \in E$$

$$\sum_{q \in C} \sum_{g \in G} \left\{ \sum_{l \in S} y^b_{e,j,l,q,g} + \sum_{k>1} \sum_{l>k} y^b_{e,(j-l+1),l,q,g} \right\} \leq K^b_e, \quad \forall b \in B, \forall j \in W, \forall e \in E$$

$$\sum_{a \in A} \sum_{q \in C} x^a_{e,j,g,l,q} + \sum_{b \in B} \sum_{q \in C} y^b_{e,j,g,l,q} = d_{e,j,g,l} \qquad \forall e \in E, \forall j \in W, \forall\, g \in G, \forall\, l \in S$$

$$x^a_{e,j,g,l,q},\; y^b_{e,j,g,l,q} \geq 0$$

$$\forall e \in E, \forall a \in A, \forall b \in B, \forall j \in W, \forall\, g \in G, \forall\, q \in C, \forall\, l \in S$$

For C_δ, we can consider two interesting cases for the thesis. By setting the range, the manager aims to avoid radical changes in prices. Indirectly it reduces revenue variance. The first one is (4.10),

$$C_{\delta_{\text{ABS}}} = \{(t, t') \,/\, |t - t'| \leq \delta_{\text{ABS}}\}, \tag{4.10}$$

The particular model associated with this constraint is “MODEL 4” :
MODEL 4 :

$$\max_{T_a} \sum_{j=1}^{W} \sum_{a,g,l,q} l\, T'_{a,q}\, x'_{a,j,g,l,q} + Q(x,y)$$

$$g \in G, \forall q \in C, \forall l \in S$$

s.t

$$C_{\delta_{\mathrm{ABS}}} = \{(t,t') \,/\, |t-t'| \le \delta_{\mathrm{ABS}}\}, \tag{4.11}$$

$$T'_{a,q} \le T'_{a,q+1}, \forall a \in A, \forall q \in \{1 \le, \ldots, q, \ldots, \le |C|-1.\} \tag{4.12}$$

$$L(T) = \min_{x,y} \sum_{j=1}^{W} \sum_{a,b} \left\{ \sum_{g,l,q} l\, \pi'_{a,g,q}\, x'_{a,j,g,l,q} + \sum_{g,l,q} l\, \pi_{b,g,q,1}\, y'_{b,j,g,l,q} \right\} \tag{4.13}$$

$$\forall\, g \in G, \forall\, q \in C, \forall\, l \in S$$

s.t

$$\sum_{q\in C} \sum_{g \in G} \left\{ \sum_{l \in S} x'_{a,j,l,q,g} + \sum_{k>1} \sum_{l>k} x'_{a,(j-l+1),l,q,g} \right\} \le K'_a, \quad \forall a \in A, \forall j \in W$$

$$\sum_{q\in C} \sum_{g \in G} \left\{ \sum_{l \in S} y'_{b,j,l,q,g} + \sum_{k>1} \sum_{l>k} {y'}^{b}_{(j-l+1),l,q,g} \right\} \le K'_b, \quad \forall b \in B, \forall j \in W$$

$$\sum_{a \in A} \sum_{q \in C} x'_{a,j,g,l,q} + \sum_{b \in B} \sum_{q \in C} y'_{b,j,g,l,q} = d'_{j,g,l} \qquad \forall j \in W, \forall\, g \in G, \forall\, l \in S$$

$$x'_{a,j,g,l,q},\, y'_{b,j,g,l,q} \ge 0$$

$$\forall a \in A, \forall b \in B, \forall j \in W, \forall\, g \in G, \forall\, q \in C, \forall\, l \in S,$$

This corresponds to an absolute restriction between the First Stage t' and the second stage tariffs, for each category.

The second case corresponds to (4.14),

$$C_{\delta_{\mathrm{PROP}}} = \{(t,t') \,/\, |t-t'| \le \delta_{\mathrm{PROP}}\ t\}, \tag{4.14}$$

The particular model associated with this constraint is "MODEL 5" :
MODEL 5 :

$$\max_{T_a} \sum_{j=1}^{W} \sum_{a,g,l,q} l\, T'_{a,q}\, x'_{a,j,g,l,q} + Q(x,y)$$

$$g \in G, \forall q \in C, \forall l \in S$$

s.t

$$C_{\delta_{\text{PROP}}} = \{(t,t') \,/\, |t-t'| \leq \delta_{\text{PROP}}\ t\}, \tag{4.15}$$

$$T'_{a,q} \leq T'_{a,q+1}, \forall a \in A, \forall q \in \{1 \leq, \ldots, q, \ldots, \leq |C|-1.\} \tag{4.16}$$

$$L(T) = \min_{x,y} \sum_{j=1}^{W} \sum_{a,b} \left\{ \sum_{g,l,q} l\, \pi'_{a,g,q}\, x'_{a,j,g,l,q} + \sum_{g,l,q} l\, \pi_{b,g,q,1}\, y'_{b,j,g,l,q} \right\} \tag{4.17}$$

$$\forall\, g \in G, \forall\, q \in C, \forall\, l \in S$$

s.t

$$\sum_{q\in C} \sum_{g \in G} \left\{ \sum_{l \in S} x'_{a,j,l,q,g} + \sum_{k>1} \sum_{l>k} x'_{a,(j-l+1),l,q,g} \right\} \leq K'_a, \quad \forall a \in A, \forall j \in W$$

$$\sum_{q\in C} \sum_{g \in G} \left\{ \sum_{l \in S} y'_{b,j,l,q,g} + \sum_{k>1} \sum_{l>k} {y'}^{b}_{(j-l+1),l,q,g} \right\} \leq K'_b, \quad \forall b \in B, \forall j \in W$$

$$\sum_{a\in A} \sum_{q \in C} x'_{a,j,g,l,q} + \sum_{b\in B} \sum_{q \in C} y'_{b,j,g,l,q} = d'_{j,g,l} \qquad \forall j \in W, \forall\, g \in G, \forall\, l \in S$$

$$x'_{a,j,g,l,q},\ y'_{b,j,g,l,q} \geq 0$$

$$\forall a \in A, \forall b \in B, \forall j \in W, \forall\, g \in G, \forall\, q \in C, \forall\, l \in S,$$

In which Second stage tariffs must not increase/decrease more than a fraction of the actual price for each category.

Both stages are connected if the constraint is active for a value lower than $\delta' \leq \infty$. If δ is set to infinity, then both stages are disconnected. The converse case is also possible, $\delta = 0$, and both stages should have the same tariffs. This case is very similar (for the discrete or sampled case) as having one single stage with one "fixed scenario", assuming realizations/samples for $\xi \in \Xi$.

Linking these stages by considering prices is advantageous because each one of the stages and the scenarios can be transformed and easily solved. If capacity were used as a recourse variable, KKT conditions could not be taken in each scenario so easily and decomposition could not be possible.

4.5 Some properties for SHBP

An upper bound on the leader's revenue was derived in Labbé et al. (1998) and revisited by Roch et al. (2005) for MaxToll. We can rewrite the same theorem for the deterministic case, taking the new bound as : $B(T) = L_\infty - L(0)$, where $L(T)$ is the second level optimal value for a tariff vector T for SHBP as defined in (4.7). $L(\infty)$ and L_0 are the values for $T = \infty$ and $T = 0$ respectively. Because SHBP never has negative increasing cycles ($L(T)$ is bounded) and the constraints set is not empty, then the theorem also applies when considering the perceived values in the lower level $\Pi(T)$ as function of T. Therefore, for a single stage problem (and the deterministic case), $B(T)$ is an upper bound on the leader's revenue. Considering now $\delta = \infty$, then identical bounds can be found for every scenario w, independently, in the second stage. So the SHBP revenue will be bounded by (4.18) :

$$L(\infty) - L(0) + \max_{T_a} \sum_{e=1}^{|E|} p_e[L_e(\infty) - L_0] \tag{4.18}$$

In practice, this bound is not very tight because the individual revenue scenario diminishes for any $T^*_\delta \neq T^*_{w,\delta=\infty}$, where $T^*_{w,\delta=\infty}$ is the optimal value for the scenario w and $\delta' \leq \delta \leq \infty$. So the optimal revenue decreases with δ, having a minimum for $\delta = 0$.

Mirza Alizadeh (2013) found interesting properties for the bilevel stochastic case in a network. Among special cases, it was proved the continuity for the stochastic MaxToll considering the "absolute value" restriction for connecting both stages and taking δ as an independent variable. Primarily, we need to remember that in the deterministic model, for a given price vector $T \in R_0^+$, any optimal solution is found in an extremal point of the polyhedral formed by the lower level constraints. This extremal point remains optimal as long as T does not exceed a threshold vector value T_{thr}, which carries a new set of assignments. For $T - \Delta \leq T_{thr}$, the leader's objective is the sum of linear functions with constant slopes given for their respective assignments. For $T \geq T_{thr}$ there is a change in the extreme point associated (because it will not remain optimal for the lower level). So, there are several jump discontinuities, from the right. Obviously, the optima is found in one of these local maxima.

The main properties that can be found are :

a. Continuity : The objective function in SHBP is C_1 for $\delta \in \mathbb{R}$.

b. Piecewise linearity : The objective function in SHBP is piecewise linear on $\delta \in \mathbb{R}$.

The complexity of Model 4 makes it very difficult to prove these properties, but the conjecture can be state as :

Conjecture 4.5.1 *Considering the absolute constraints for non degenerated cases*[2]*, the function $R(\delta)$, is continuous and piecewise linear, for $T \geq 0$.*

Continuity Conjecture : Having capacity constraints in the problem, it is not possible to rely in the demonstration found in Mirza Alizadeh (2013), but similar arguments follow for a non-degenerated case in which capacity constraints are non-active. We note that in the case of a degenerated solution, a "little" change in price could introduce a change in assignments, so any change in price needs to be as minimal as possible to prevent any assignment change for the theorem to hold.

Piecewise Linear Conjecture : Having fixed the assignments, and with non-active capacity constraints, the stochastic bilevel problem is reduced to a linear program (LP) with only the vector T (Max LP on $(T_{\mathrm{F}}, T_{\mathrm{S}})$, where F represents the First Stage and S represents the Second Stage). This problem becomes thus a special case of a "Parametric Right-hand side LP", with a single parameterized constraint. Then, it is a concave piecewise-linear function of δ. And by generalization, a concave polyhedral function.

As noted in Mirza Alizadeh (2013) for the problem with Proportional Constraints Case (PROP) constraints (Model 5), it does not follow the conjecture, not even for the continuous part.

2. When multiple solutions are available.

CHAPTER 5

ALGORITHMS FOR THE SHBP MODELS

In this chapter, we examine the most pertinent forms of solving the SHBP model proposed in the previous chapter. We present exact and heuristic methods.

5.1 Exact Method

For simplicity, it is easier to use the decomposition of each one of the scenarios in each stage and rebuild the problem into a unique large combinatorial problem. In fact, all scenarios are linked through the price and each scenario is treated as a deterministic problem. We can thus rebuild the problem by adding a constraint that makes all prices equal for each scenario. Finally, we use Cplex to solve this large combinatorial problem. The advantage of this method is explained by the efficiency of Branch&Cut in Cplex and by the experience obtained in solving the deterministic case Montecinos (2007).

The deterministic hotel bilevel model HRM shown in (2.4) can be transformed into a single level model by replacing the lower level objective by its KKT conditions. We also notice that the first level objective of this model is non-linear but easily convertible into linear by algebraic transformations. Therefore, we obtain,

$$
\begin{aligned}
\max_{x^a, x^b, \eta, \nu} \quad & \sum_{a \in A} \left\{ \sum_{g \in G} \eta_g d_g - \sum_{\substack{g \in G \\ q \in C}} (\pi^b_{g,q} x^b_{g,q} + \alpha_g \lambda x^a_{g,q} - \beta_g q x^a_{g,q}) + \sum_{b \in B} \nu_b K^b \right\} \\
\text{s.t.} \quad & \\
& T_{a,q} \leq T_{a,q+1}, \forall a \in A, \forall q \in \{1 \leq, \dots, q, \dots, |C| - 1.\} \qquad (5.1) \\
& \sum_{q \in C} A^a x^a_{g,q} + \sum_{q \in C} A^b x^b_{g,q} = d_g, \qquad g \in G, q \in C \\
& \sum_{\substack{g \in G \\ q \in C}} G^a x^a_{g,q} \leq K^a, \qquad a \in A \\
& \sum_{\substack{g \in G \\ q \in C}} G^b x^b_{g,q} \leq K^b, \qquad b \in B \\
& \eta_g A^a \leq \pi^a_{g,q}, \qquad g \in G, q \in C, \qquad a \in A
\end{aligned}
$$

$$
\begin{aligned}
&\eta_g A^b + \nu_b G^b \leq \pi^b_{g,q}, \qquad g \in G, q \in C, \qquad b \in B \\
&-\nu_b(K^b - \sum_{\substack{g \in G \\ q \in C}} G^b x^b_{g,q}) = 0, \qquad b \in B && (5.2)\\
&x^a_{g,q}(\pi^a_{g,q} - \eta_g A^a) = 0, \qquad g \in G, q \in C, \qquad a \in A && (5.3)\\
&x^b_{g,q}(\pi^b_{g,q} - \eta_g A^b - \nu_b G^b) = 0, \qquad g \in G, q \in C, \qquad b \in B && (5.4)\\
&x^a_{g,q}, x^b_{g,q} \geq 0, \qquad g \in G, a \in A, b \in B \\
&\nu_b \leq 0 \qquad b \in B \\
&\eta_g \quad \text{free}, \forall g \in G.
\end{aligned}
$$

We note that the transformed model has complementarity constraints. For solving this problem there are some effective methods. In this development, we use both the “big-M” method. The transformed model can be solved as MIP through the use of Cplex.

We introduce a matrix notation for the “big-M” constant used in the transformation.

$N_{x^a}, N_{x^b} \in \mathbb{R}^{(\|S\|\,\|W\|)}$	:	Matrices associated to the constraints (5.2).
$M_{\nu_b} \in \mathbb{R}^{(\|W\|)}$	:	Matrix associated to the constraints (5.3).
$N_{\xi_a}, N_{\nu_b} \in \mathbb{R}^{(\|W\|)}$	:	Matrices associated to the constraints (5.4).

We introduce thus some binary variables vector for the “big-M” method.

$w^b \in \{0,1\}^{1\,(\|C\|\,\|W\|)}$	:	Binary variables column vector associated to the constraints (5.2).
$z^a_{g,q} \in \{0,1\}^{1\,(\|S\|\,\|C\|\,\|W\|)}$	:	Binary variables column vector associated to the constraints (5.3).
$z^b_{g,q} \in \{0,1\}^{1\,(\|S\|\,\|C\|\,\|W\|)}$	:	Binary variables column vector associated to the constraints (5.4).

The "big-M" formulation, is the following :

$$
\begin{aligned}
\max_{T^a, x^a, x^b, \eta, \nu} \quad & \sum_{g \in G} \eta_g d_g - \sum_{\substack{g \in G \\ q \in C}} (\pi^b_{g,q} x^b_{g,q} + \alpha_g \lambda x^a_{g,q} - \beta_g q x^a_{g,q}) + \sum_{b \in B} \nu_b K^b \\
\text{s.t.} \quad & \\
& T_{a,q} \leq T_{a,q+1}, \forall a \in A, \forall q \in \{1 \leq, \ldots, q, \ldots, |C| - 1.\} \\
& A^a x^a_{g,q} + A^b x^b_{g,q} = d_g, \qquad g \in G \\
& \sum_{\substack{g \in G \\ q \in C}} G^a x^a_{g,q} \leq K^a \\
& \sum_{\substack{g \in G \\ q \in C}} G^b x^b_{g,q} \leq K^b \\
& \eta_g A^a \leq \pi^a_{g,q}, \qquad g \in G \\
& \eta_g A^b + \nu_b G^b \leq \pi^b_{g,q}, \qquad g \in G \\
& (K^b - \sum_{\substack{g \in G \\ q \in C}} G^b x^b_{g,q})_i \leq [M_{\nu_b}]_{ii} w^b_i \quad , \forall i \in W \\
& -\nu_i \leq [N_{\nu_b}]_{ii}(1 - w^b_i) \quad , \forall i \in W \\
& (\pi^a_{g,q} - \eta_g A^a)_{g,q} \leq [M_{x^a}]_{g,q} z^a_{g,q} \quad , g \in G, q \in C \\
& x^a_{g,q} \leq [N_{x^a}]_{g,q}(1 - z^a_{g,q}) \quad , g \in G, q \in C \\
& (\pi^b_{g,q} - \eta_g A^b - \nu_a G^b)_{g,q} \leq [M_{x^b}]_{g,q} z^b_{g,q} \quad , g \in G, q \in C \\
& x^b_{g,q} \leq [N_{x^b}]_{g,q}(1 - z^b_{g,q}) \quad , \forall g \in G, q \in C \\
& x^a_{g,q}, x^b_{g,q} \geq 0, \qquad g \in G, q \in C \\
& \nu_b \leq 0 \\
& \eta_g \quad \text{free}, \forall g \in G \\
& z^a_{g,q}, z^b_{g,q} \in \{0,1\}^{1\,(|S|\,|W|)}, \forall g \in G, q \in C \\
& w^a, w^b \in \{0,1\}^{1\,(|W|)}
\end{aligned}
$$

The deterministic model has a great number of binary variables. In Montecinos (2007), the deterministic models were solved first using Cplex. The MIP transformation suggested in Labbé et al. (1998) was utilized because it is able to escape from the local-maximum, keeping the optimization global, by using state of the art MIP solvers. However, we note that the MIP transformation can be very weak to face symmetry, and the linear relaxation is terrible, because of the several big-M constants that multiply auxiliary binary variables.

The lower level problem is frequently degenerated as also happens in many network flow problems. This condition can be treated is several ways, such as adding perturbations to the dual right hand side constraint (RHS). We can notice that in the SHBP (4.4) problem, stochasticity reduces the inconvenient of degeneracy, at least in theory, but it increases the chances of finding several local-maximum in the resolution.

In order to solve large instance models, some heuristics were developed to test and compare their efficiency against exact methods in SHBP.

5.2 A Naive Managerial Approach Heuristics

The market driven naive managerial approach tries to average the perceived prices π (from (4.1)) for each group (α, β) in every day of the rolling horizon for each QoS without considering the LOS. These values correspond to three price list outputs, the "lowest market price", the "average market price", and the "maximum market price". The Service-Quality constraint for each inventory (2.12) is applied to ensure that to higher-quality corresponds higher-price list. If the constraint is not satisfied, a reduction to the price list must be applied from the highest to the lowest QoS. The same price list is replicated in all the inventories, as used in many hotel brands settled in a narrow zone. The manager will decide among these three outputs based on its experience, and thus this approach is very simple. However, it is important to notice that risk is not being measured in these three outputs and thus it is only inherent to manager's aversion.

The manager could believe that at least one of these outputs will assure him/her the maximal revenue in the future. However, because these outputs are not considering stochasticity, this managerial approach is not very "safe", as it will be seen in the numerical results presented in the following chapter. Finally, this approach assumes that group behavior is known by the Leader and managers do not analyze the perceived prices in two-stages. The Naive Market-Price Heuristic is shown in Algorithm 1.

Due to the great difficulty presented in SHBP problems, more complex heuristics are needed to solve them. The developed heuristics are not approximations of the optimal solution because they have the option of either finding the optimal solution or contributing with a good enough solution to re-evaluate the exact solution.

In the next two sections, we present the two heuristics developed for solving our SHBP model.

Algorithm 1 Naive Market-Price Heuristics

INPUT : A problem **SHBP** with competitors Prices T_B.
OUTPUT : An integer solution for **SHBP**.

Set S = Stages in **SHBP**
for $s \in S$ **do**
  Set E = Scenarios in s
  for $e \in E$ **do**
    Set $\Delta_B^{s,e}$ = Distance and Quality Price for Competitors Inventories $(\alpha_g \lambda_b - \beta_g q)_B$
    Set $\Delta_A^{s,e}$ = Distance and Quality Price for Leader Inventories $(\alpha_g \lambda_a - \beta_g q)_A$
    Set $\Delta^{s,e} = \left[\overline{\Delta_B^{s,e}} - \overline{\Delta_A^{s,e}}\right]_{a,q}, (a,q) \in A \times C$
  end for
end for
Generation of the outpus.
Set $T^+ = \min_{i,q}(T_B^{s,e} + \Delta^{s,e}, \forall e \in E, \forall s \in S)$
Set $T^- = \max_{i,q}(T_B^{s,e} + \Delta^{s,e}, \forall e \in E, \forall s \in S)$
Set $\overline{T} = \text{mean}\,(T_B^{s,e} + \Delta^{s,e}, \forall e \in E, \forall s \in S)$
Apply Service Quality Constraints($[T^+, T^-, \overline{T}]$)

5.3 Multipath-based Heuristic (HBP)

Several algorithms have been proposed in the literature to solve bilevel programming problems. For example, sensitivity analysis based algorithms (solving the lower-level for a set of prices and after that re-optimizing the first-level in order to modify prices in the ascent direction) have been tried to take advantage of alternating or switching paths in transportation problems but with the inconvenient of falling in local-maxima. In order to avoid this problem, we developed a multipath-based heuristic [1], whose main idea is to “match” competitors’ prices beginning in any given price vector solution. If by matching one competitor’ price (a tariff by quality of services and inventory) it is possible to obtain a higher revenue, then this new tariff is considered as part of the solution. Otherwise, it is dismissed and another tariff is examined. This is done iteratively to each one of the elements of vector T controlled by the leader, T_A hereinafter.

The heuristics works as follow : First, we begin matching competitors’ prices for the product having the highest QoS for every inventory. Leader’s price is set an ϵ-below competitors’ prices, $\epsilon,\ 0 < \epsilon \ll 1$, in order to attract a large quantity of competitors’ customers. It is important to notice that the “match-below” is done in terms of average perceived prices π (4.1)

1. A multipath is a set of feasible paths in a network O–D. A multipath-based heuristic construct a path per each origin–destination pair.

for each user group instead of matching price list. The reason is that the use of the average perceived price allows us to reduce the large combinatorial problem that would be caused if we only use the price list of each competitor. Second, if the revenue of the leader does not increase after matching its price to the average perceived price, we do not use that price and we continue with the next QoS. Each trial is considered as a step in the heuristic.

To be able to produce a "proper" price reduction, it is important to consider some restrictions on vector T_A (Quality Services Restrictions and non-negativity). It must be noticed that we could find a local-maximum and thus something must be done to get out of those points. Such as accepting some "backtracking steps" or forcing a decrease that is much lower than the value of competitors' prices.

There is not a natural order to perform price reduction one by one on the components of the vector T_A. Moreover, there is not a manner to know if we have fallen in the same local-maximum already explored. Furthermore, a decrease from a local maximum may invariably also lead to a downward path from which there is no improvement.

The form to avoid problems caused by these reductions is to give a tree structure to the problem. The interest behind generating this tree is to make the fewest moves as possible, when exploring the tree nodes one by one, to obtain a very fast and good quality solution. Nevertheless, our intention is to sample the best possible T_A space to find the most profitable sub-space in terms of revenue. In order to create the tree, we need to define a neighborhood. In fact, the neighborhood structure is very important for the algorithm, as we will explain later. In this tree, a node is defined as the basis of simplex in the lower level, in which the perceived prices $\pi_A = \pi(T_A)$ of non-assigned capacities have been reduced as much as possible without affecting the actual solution. In order to do that, their reduced costs [2] are utilized. A second step is to allow that several columns can be used to re-optimize the first level.

Neighbors of this node are all the other basis of simplex to which is possible to arrive by reducing any component of the vector π_A in at least ρ. As we can see, this definition of the oriented network (a tree of sons and siblings) is very dense. However, it is possible to construct an improved neighborhood considering degenerated basis-simplex as nodes connected by small feasible moves in every component of $\pi_A(T_A)$. Feasible moves are those that consider all the constraints on T_A at the same time, among all the scenarios. In fact, any move is not restricted to a small one ρ.

The most technical part of the algorithm relies in the way to generate a succession of visiting nodes that improves the revenue. The pursuit can begin from the highest price vector that allows getting a revenue for the leader. This is the highest vector price that an assignment could have favorable for the leader. We can name this point as an "initial vector price".

2. The smallest of all is selected.

The next step is to generate moves from one basis to another simplex basis. As previously explained, a form of obtaining a first move from the initial vector price is by considering the most strict low reduction on perceived prices π_A (from (4.1)) to modify the vector T_A.

Exploring the tree as defined here can be done by tracking the nodes already visited, to avoid evaluating them again, which generates an exploration tree. As we previously said, computing assignments for a price vector generates a tree-like structure for the lower level. In practice, we can take wider "moves" in T_A and after that to do again the tree projection to any of the nodes.

Just making moves arbitrarily in T_A, without considering the neighborhood, does not take advantages of multiple lower level solutions that allow switches in the inventories for different users. These switches allow us to explore interesting zones for revenue increase but also to get out from the local-maximum areas. We propose to make an exploration of this tree by considering a best-first search approach and by using a priority queue to maintain the best solution in front of us. To improve the searching speed, we allow jumps by choosing a variable ρ steps-size for a reduction over a single T_A, and then projecting back into the neighborhood-tree, leaving several "sons" nodes unexplored. This is especially important at the beginning.

When a jump and its projection conduct us to an already explored node [3], we are in a dead end of the exploration in its respective tree-branch. It is possible that two nodes are linked to similar value solutions or whose vectors T_A resemble closely (using any metrics like L^2 norm). To avoid this, and also to reduce the computation, we consider that nodes are "identical" using a simpler L^1 metric on the associated revenue. It is important to notice that exploring too close nodes early in the tree avoids following parallel paths later. The heuristics implemented can be seen in Algorithm (2).

We define :

— **Best-T :** Vector Price found by the heuristic.

— **Best-Solution :** The best solution to **SHBP**.

— **Dual Variables :** The Second Level Dual Variables : $\nu_b, \forall b \in B, \eta_g, \forall g \in G$.

— **PQueue :** A descending priority queue that keeps all the nodes already explored.

— **The vector** T_A **:** $[T_{a,q}]\ \forall a \in A, \forall q \in C$ Vector of Leader's Prices.

— **The vector** $\Pi_A(T_A)$ **:** $[\pi_{a,g,q}]\ \forall a \in A, \forall g \in G, \forall q \in C\}$ Vector of Perceived Leader's Prices.

— **Second-Level-SHBP :** The lower level problem of **SHBP**.

3. Or to close to a explored node

Algorithm 2 Multipath

INPUT : A problem **SHBP** with Leader vector Prices T_A.
OUTPUT : The solution Best-T, the Second Level Dual Variables, and the flows.

Step 1.
Set $\rho = \rho_0 > 0$, variable step
Set $\rho' > 0$, step constant
Let DualVariables $= \varnothing$, set of Second Level Dual Variables Values
Let Best-T $= \varnothing$, set of T_A
Let Best-Solution $= \varnothing$, set of solutions
Solve every $\mathrm{Second} - \mathrm{Level} - \mathbf{SHBP}(T_A = \infty)$ in both stages
Let Revenue$(T_A) = 0$
Let $R_\pi = \mathrm{ReducedCost}(\mathbf{SHBP}(T_A))$.
Let $\mathrm{Rmin}_{a,q} = \min\{R_{\pi_{a,g,q,e}}, \forall g \in G, \forall e \in E\},\ \forall a \in A, \forall q \in C$.
Update, $T_A = \max_{T'_A}\{T'_A / T'_A \leq (T_A - \mathrm{Rmin}),\ \text{S.T. Constraints for } T'_A.\}$.
Let PQueue := Descending Priority Queue
Add $(\mathrm{Revenue}(T_A), T_A)$ to PQueue

Step 2.
while PQueue $\neq \varnothing$ and not StopCondition(Time, Iterations) **do**
  Remove T_A from PQueue
  Add T_A to Best-T
  Solve every $\mathrm{Second} - \mathrm{Level} - \mathbf{SHBP}(T_A)$ in both stages
  Add Dual **SHBP** to DualVariables
  Let $R_\pi = \mathrm{ReducedCost}(\mathbf{SHBP}(T_A))$
  Let $\mathrm{Rmin}_{a,q} = \min\{R_{\pi a,g,q,e}, \forall g \in G, \forall e \in E\},\ \forall a \in A, \forall q \in C$.
  Solve $\mathrm{Revenue}(T_A) = \mathrm{Upper} - \mathrm{Level} - \mathbf{SHBP}(T_A)$
  for all $(a, q) \in A \times C$ **do**
    Let $T''_A = \max_{T'_A}\{T'_A / T'_A \leq (T_A - \mathrm{Rmin}) - \hat{e}_{a,q} * \rho,\ \text{S.T. Constraints for } T'_A.\}$.
    if $(\mathrm{Revenue}(T_A), T''_A)$ not in PQueue **then**
      Add $(\mathrm{Revenue}(T_A), T''_A)$ to PQueue
    end if
  end for
  if Revenue$(T_A) >$ BestRev **then**
    BestRev = Revenue(T_A)
    $\rho = \min(\rho + \rho', \rho_{max})$
  else
    $\rho = \max(\rho - \rho', 0)$
  end if
  Update Best-Solution set
end while

A multi-assignment is any feasible assignment for the lower level (an optimal allocation that respects capacity restrictions and demand (every component of the demand)). Doubtless, our heuristic could be seen as a "multi-assignment" seeker. It is intuitive to assume that the leader will have high revenue values and thus this multi-assignment is quite expensive for the followers. An upper bound to the leader's revenue under a multi-assignment can be obtained solving an inverse optimization problem, in which leader's prices are re-computed for a given fixed path. Indeed, our model is subject to capacity constraints and to a large number of "groups" multiplied by the "Length of Stay" and Dates "j" (similar to multi-commodities and O-D pairs), but the multi-assignment interactions make possible several short-paths options when products are split among inventories, *e.g.,* many possible "paths" (crossing inventories) composed by just two arcs, and this for every scenario. In summary, the number of possible paths for our model is too huge and thus inverse optimization is less productive.

Our HBP heuristic took some steps from Didi-Biha et al. (2006), Brotcorne et al. (2011) and Brotcorne et al. (2012). For example, Didi-Biha et al. (2006) suggest evaluating multi-assignments iteratively by sorting from the highest upper potential bound to the lowest potential bound and stopping arbitrarily when a sufficient good bound was found or when computation was too much costly. Lately, this idea was taken in the algorithm device of Brotcorne et al. (2011), improved in Brotcorne et al. (2012), where an inverse optimization process with fixed paths was solved. In fact, these authors took the dual of the former problem and used the Dantzig-Wolfe decomposition. The dual was an easier problem with only one complicated constraint. The sub-problem was solved in the column-generation fashion. The improvement consisted on using Tabu-Search to generate paths, avoiding falling in previous false steps. Both algorithms adapted well to the connected network (without capacity), in which the number of pairs O-D is small and the number of commodities can be very large. Therefore, the number of feasible paths was easy to enumerate. The Tabu-Search algorithm elaborated by them improved the enumeration and made it more efficient.

The distinction of our heuristic to the work of Brotcorne et al. (2011) and Brotcorne et al. (2012) is that instead of using the Tabu-search to avoid the same feasible path, we used a best-first search approach and the price reductions.

It is important to highlight that although we could do a local search as a final step in this multipath-based heuristic, we only use its results as an initial solution for another heuristic, named "Restricted SHBP" heuristic, which is presented in the following subsection. In that heuristic we will do a local search for the problem.

5.4 Restricted SHBP

In this heuristic, we focus our attention on a reduced number of binary variables and decrease the variable domain in order to do a local search in the context of the MIP defined in the exact method. We rename this MIP as a “MIP restricted”.

In order to do that, we take all the solutions coming from the previous heuristic and we analyze the range of the prices and dual variables. By reducing and setting the range of dual variables, we are doing stabilization, as is done in column generation. Big-M constants that defined the polyhedral space of the linear relaxation can also be set. This is possible by searching the smallest value of M in a way that the best solutions are feasible.

The restricted MIP has tighter bounds for prices and dual variables. Also some binary variables can be fixed through the inspection of big-M constants associated to each variable. The only computing cost comes from solving several linear problems, one for each constant. Keeping the structure of a MIP assures that we can get-out from any local-maximum. An optimal solution for the MIP restricted problem is a lower bound to the MIP original problem (and a good starting solution). The dual variables corresponding to Demands Constraints vary widely. It is also known from column-generation practitioners that stabilization for dual variables is key in order to avoid a high rotation of columns in a master problem, as stated by Du Merle et al. (1999). There are several techniques to do so, mainly limiting the range of moves for the dual variables and keeping some variables (columns) until they are exhausted.

Most complex problems will finish with hundreds of columns with negative but small reduced-cost. At this point, the dual variables do not need more stabilization because the impact in the objective is negligible for new moves. The problem remains purely combinatorial to prove the optimality. In summary, just limiting the dual range proves to be helpful in several problems.

In this heuristic, the best solution obtained with HBP is used by Cplex as an initial solution. The MIP restricted heuristic is presented in Algorithm 3.

Algorithm 3 MIP-Restricted

(INPUT. : Second Level Dual Variables, Best-T (from step 2 in Algorithm (2))
(OUTPUT. : SHBP-MIP-Restricted solution)

BigM = vector of Big-M constants
Create ConvexEnvelope(DualVariables, Best-T)
Minimize Big-M(ConvexEnvelope(DualVariables, Best-T))
Construct a **SHBP**-MIP-Restricted(BigM)
Fix **SHBP**-MIP-Restricted BinaryVariables
Input Best-Solution to **SHBP**-MIP-Restricted
Solve **SHBP**-MIP-Restricted

5.5 Algorithm MIP-H+

The MIP problem can be solved using Cplex. An improved MIP problem is proposed using a heuristic at every node of the B&B tree. The heuristics is based on the price vector T_A, previously defined. The heuristic consists in relaxing SHBP by replacing all the second lower level problems, in every scenario at every stage, for new linear inequalities. Each new restriction consists in the second lower level objective equal to the optimal value of the second level calculated for T_A considered as a constant. The resulting relaxed problem is a simple linear problem that recalculates the best assignment for the constant vector T_A that maximizes revenue. The new solution can be corrected if it does not satisfy each one of the restrictions of the lower level problem in every scenario at every stage. This new algorithm was called "MIP-H+". 'MIP-H+" helps to find new feasible solutions and sometimes it improves the lower bound. This improvement reduces the integral gap when convergence is too slow or reaches a time limit. The lower level of all the scenarios can be solved and this information is used to improve the solution. The new solution is added to Cplex's solutions pool if the solution improves the lower bound. We note that Cplex has control on the nodes where the algorithm is applied. In most cases, Cplex could have found integral values for $\approx 70\%$ of the integers variables (all stages and scenarios included) but the price vector T_A is still "floating" in the B&B tree until all the binary variables are set. This procedure just helps to find the integrality for all binaries without fixing the previous binary variables found for that node. Exploring the "non-fixed binaries" as a sub-MIP is already done by Cplex in its "proving"[4]. So, this version seems to be the best for our model because the new solution could have a totally different multi-assignment compared to the one that is explored in the branch, and

4. Cplex has an heuristic that tries to fix binary variables by checking logical implications and lifting binary coefficients.

in theory, it should find many solutions at the beginning of the B&B. In Brotcorne et al. (2011), an improved MIP problem also was proposed using inverse optimization applied at every node of the B&B tree. This MIP was called "MIP+". Making reference to this idea, we have call our present algorithm "MIP-H+".

5.6 Numerical Issues : parameter settings for Cplex and choice of an initial solution

Cplex could take a considerable amount of time, first, to obtain an integer solution and the integrality gap information, and finally to obtain the optimal solution and prove optimality, if possible [5]. To cope with this numerical problem, a fine tuning with Cplex was done, and then the main heuristic was developed to apply on more difficult problems [6].

Several runs were made with different instances of the problem to characterize the best parameters to be given to Cplex. These are shown in the Appendix A. These parameters balance the obtainment of good dual bounds (and use of memory) with the obtainment of feasible solutions. It is important to mention that the obtainment of a feasible solution can be very time consuming. The successive tests, without incorporating other external cuts to the model, show the interest to force Cplex in a pure aggressive branch-and-bound (B&B) behavior. All the heuristics for B&B behave very badly when problems grow to thousands of binary auxiliary variables. With a machine using two threads, the test found that several general cuts in Cplex library fit the best to the problems. The most successful of these cuts, in terms of the number of times that these cuts were applied to the problem, were : Mixed integer rounding cuts applied, Flow cuts applied, Disjunctive cuts applied, Implied bound cuts applied, Gomory fractional cuts applied, Cover Cuts. The rest of all possible cuts available in Cplex were set to Off to gain in speed. The tuning improved considerably the computing time for most of the instances solved in this thesis, but it did not help to prove optimality [7]. The experiment was done without using the tools provided by Cplex because they way they work are unknown and we decide not to rely on them, considering the difficulty in solving bilevel models. For example, in one of the trial after tuning Cplex, it took 106 seconds to get optimality (a first solution was find after 70 seconds), the performed manual tuning found the best solution in just 44 seconds, and the parameters totally differed from the solver tuning. Without any tuning, Cplex by default took ≈ 18907 seconds. This last time is an absolutely prohibited attempt for trying more complex problems and thus a tuning to Cplex must be

5. For a reasonable amount of time or cost. In this case one hour was considered reasonable

6. Almost any commercial solver will find different upper bounds that can change the integrality gap calculation merely relying on other heuristics after the dual value calculation.

7. In a reasonable amount of time or cost. In this case one hour was considered as reasonable

performed. Moreover, we realized that a better performance was possible if an initial solution was provided to the solver.

Starting solutions

In our model, it was often easy to compute a first solution for any instance, as the trivial solution, but that is not good enough for Cplex.

A second alternative is to generate several random solutions, which consists in taking a random vector T, to then solving the lower level problem and finally re-optimizing the first level (leader revenue). However, this procedure was extremely inefficient. Because of that, this method was disregarded as a way to obtain an initial solution.

A third alternative is to use the "Feasibility Pump" implemented in Cplex to get an initial solution, as described in Fischetti et al. (2005); Bertacco et al. (2007); Achterberg and Berthold (2007); Bonami et al. (2009); Boland et al. (2011); Fischetti and Salvagnin (2009); De Santis et al. (2010); Boland et al. (2012). However, we did not get any good result for large instances, even after our attempts to re-implement the heuristic independently. Because of that, this heuristic was finally disregarded.

Therefore, we decided to use our own heuristic HBP to provide Cplex this initial solution. One of the main contributions of our heuristic was to provide to Cplex a fairly good feasible solution and not just a lower bound. Practioners have reported that using this technique it is possible to find the optimal in many cases because every new solution sets a new lower bound to the maximization problem. Better bounds dismiss many branches in the exploring tree. We notice that the solution time (comparing the situation without a feasible solution) was reduced in a half for most common problems (for the previous second trial it was a 43% of reduction).

However, proving that the initial solution is the optimal solution (or an integrality gap equal to zero) is impossible for large instances. The stochastic bilevel model considers a large amount of binary variables multiplying each one by a different big M constant. The numerical stability is under stretch and thus the integrality precision was increased from defaults to more strict values (from 10^{-6} to 10^{-8}, big-M values rely in the range $[1, 10^5]$). In fact, the transformation from the original model [8] to the MIP increases the chances that small deviations in "list prices" will false the solution or make it appears "almost true solutions".

This is what happens when the rounded auxiliary binary variables are poor. This situation is particularly difficult with the complementary constraints ("list prices" are embedded in these constraints). Small deviations in the assignment caused by rounding binary variables can also happen, but they do not have a serious impact on the resolution.

8. The model itself is prone to multiple solutions, so to have multiple lower level degenerated basis.

It is important to point out that numerical errors could invalid the improved heuristic solution when tested against the lower level model and when the first level is re-optimized.

Therefore, because there is no easy way to prove that any solution is optimal, we will rely on the B&B solver process to do so. From a practical point of view, the resolution of large instances is useful for economical decision making or analysis even with a relative high gap.

The success of our approach relies on a reasonable good heuristic that works well in almost any instance tested to find a solution with an integrality gap lesser than 30%. However, if Cplex does not accept the solution in its MIP heuristics, the user needs to force Cplex to acquire this solution through several attempts, until Cplex makes it feasible. However, most of the time Cplex downgrades the value of the solution initially provided. In other words, Cplex first checks the feasibility of this initial solution, but this solution has reasonable sources for inexactitude due to the transformation process. So, Cplex will mostly find the solution infeasible and will try to correct it. Appropriated parameters were thus set to accomplish this important task.

5.7 A brief comparison of HBP and MIP-H+

With the advent of parallel machines with multiple threads capabilities, it becomes more difficult to compare the resolution time or the improvement of each heuristic in the same problem.

Both heuristics were implemented with Cplex. Comparing both heuristics is difficult because the implementation is different in nature : MIP-H+ was implemented using Python 2.6 and the Cplex API[9]. Python is a dynamic type language. Python is not comparable in speed to language C or C++. The main reason is that it is not compiled ; it does not use primitives, just objects, and handle them indistinctly. Python could be at least tenth hundred times slower in the test machine. The comparison shown in Figure 5.1 was taken using a single thread and limiting the solve to 5000 linear relaxations for HBP, beyond this scope the performance decays because of memory use. In order to make a fair comparison, MIP-H+ implementation also uses some notation in Python but using a different API. At the same time, MIP-H+ is able to solve 2 times more linear programs than HBP, but it also has a lost in performance for memory issues.

Another important point is that MIP-H+ does not need any parameterization, but HBP step size needs a tuning for some instances : The step size should always remain in the "middle" of all reduced-cost sizes to be effective, but it needs to grow faster at the beginning to get in the "interesting zone" and to get as much samples as possible there, for the second

9. Application programming interface. Software library that facilitates the interaction among software.

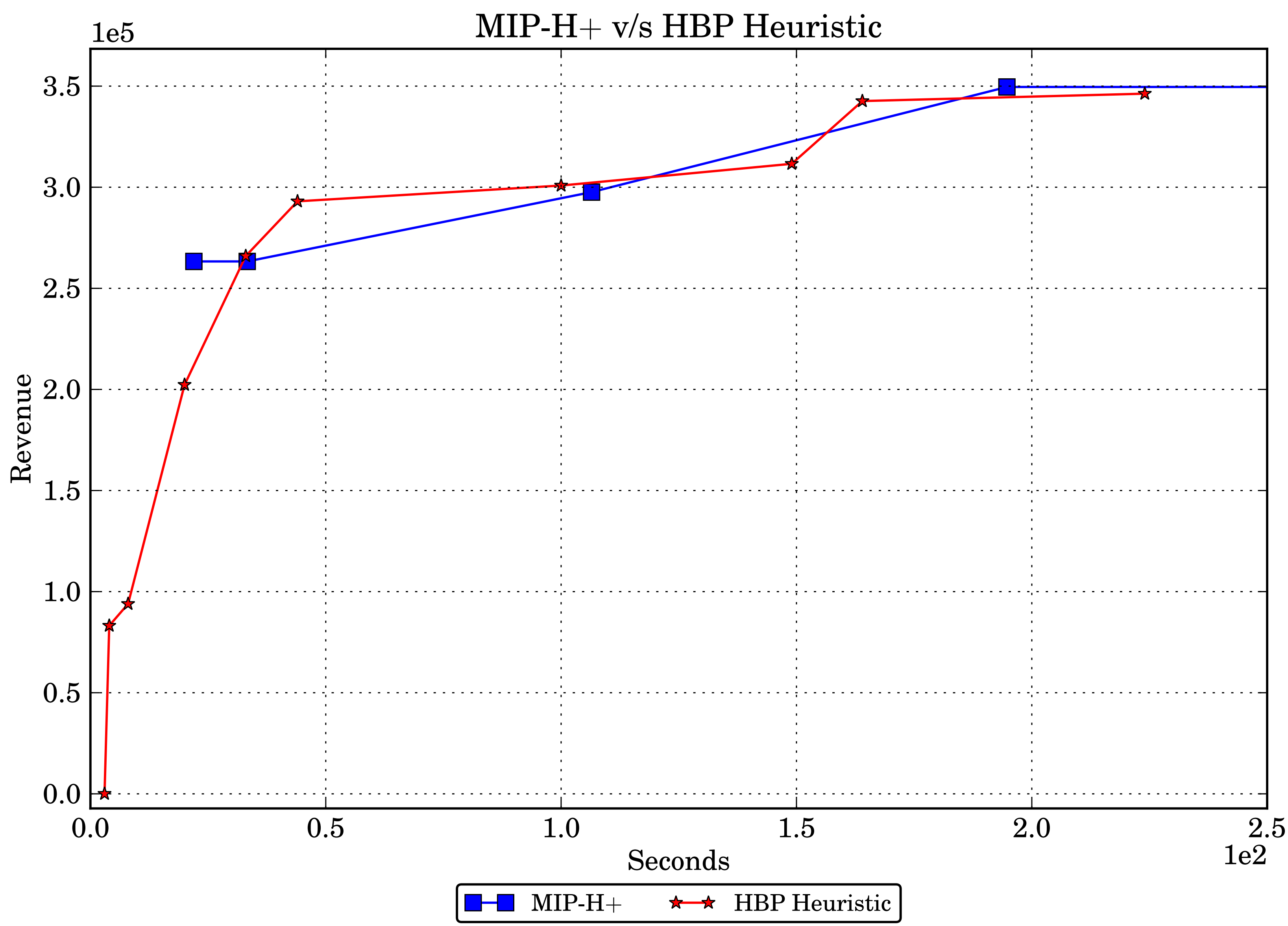


Figure 5.1 MIP-H+ v/s Heuristic HBP : Performance in single thread

part. The algorithm MIP-H+ was always used with the original model, using standard values for Big-M constants (10^5) to avoid any interference with the upper bounds that could end up the B&B earlier.

In the graphic 5.1, it is easy to appreciate that the performance of HBP is very good at the beginning, both lines cross several times. We could infer that both algorithms found identical local maxima and both stopped finding better solutions almost at the same point, based on some milestones and the asymptotic behavior.

In the graphic 5.2, it can be seen that Cplex starts from the same solution found in the previous example. It is also possible to see in the extreme left, the solution supplied to Cplex. Immediately after, Cplex gets the solution, which is accepted or modified (downgrade or upgrade) because of numerical problems. In almost all the cases, Cplex accepts the solution.

We notice that MIP-H+ gets in stagnation very fast and stays like that for a while. In fact, Cplex was capable to find a similar good solution. MIP-H+ at the end of the test was capable to find a small gap, mostly because of the oportunistic behavior of the heuristic. An example is shown as “Number 23” in the following chapter. The graphic also illustrates that Cplex could not find any solution for the comparison, and even with a first solution the results were very deceiving.

Another relevant aspect is the mathematical precision : there is a huge number of auxiliary binary variables that are tied to assignment variables, and thus a small imprecision confuses Cplex to accept solutions, and at the same time, it could create a unstable equilibrium (or false equilibrium). On the other hand, under small variations of vector T, assignments can change dramatically. Because of these technicalities, it was not possible to make an interaction between these different approaches (MIP and multipath) in another heuristic that tied both.

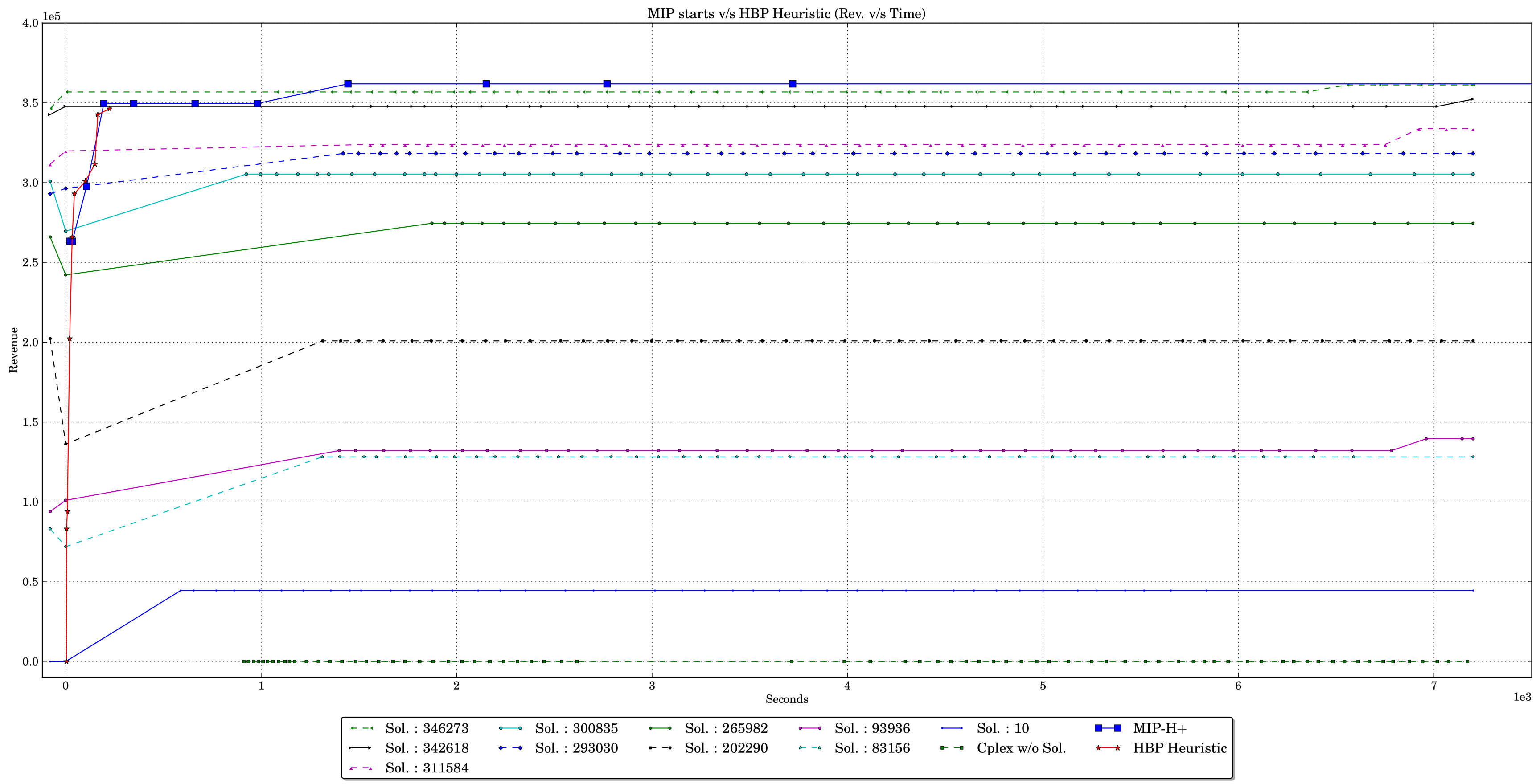


Figure 5.2 Performance MIP Starts v/s Heuristic HBP

CHAPTER 6

NUMERICAL EXPERIMENTATION AND VALIDATION

In this chapter, we present and analyze the solution of several numerical examples, ordered by its level of complexity, from small to large instances. Complexity is related to the number of variables involved in the resolution of the problem, such as number of inventories, number of groups, length of stay, number of scenarios, rolling horizon days, and price range. In all these examples, the leader attempts to optimize its profit, considering two different periods of time (two rolling horizons). These periods of time could be the first week in the month, compared to the first week in the next month ; or a season compared to the next season, etc.

Before presenting and analyzing these examples, we need to present our data generation process. After that, one example corresponding to a small instance is introduced and analyzed, from both a numerical and an economic perspective. Although the small instance is solved until optimality is reached, we also present the results obtained for the naive and HBP heuristics to compare their results with the optimal solution. Moreover, due to the enormous quantity of data that can be obtained for each example, even for small instances, we only present its main results, which are specifically revenue (using the most common stochastic approximations) and price list (first stage and second stage), for the small instance, all this for Model 4 and Model 5. Finally, we perform a sensitivity analysis in terms of price range for Model 4 and Model 5.

In the second part of this chapter, large instances are presented (beginning in "Example 23" and finishing in "Example 51"). We only present the revenue obtained in each example to focus the analysis on the difficulty behind solving them. However, three examples are further analyzed in terms of their economic results. In those cases, we explain the main effects of capacity, group composition, and list prices on revenue.

In the third part, stochasticity in prices and/or demand is introduced to large instances and its effect on revenue and users' distribution are briefly analyzed. Finally, in the last section of this chapter, the impact of changes in capacity on revenue are presented and analyzed.

6.1 Experimental Set-Up and Data Generation

In order to test the SHBP model, we created data that represented a relevant context for the hotel industry. Because of that, we developed a data generation process that allowed us to reduce the possibilities of falling into infeasibility, atypical solutions and thus into unrealistic market situations.

However, it is important to notice that in order to generate this realistic market situation, a trade-off in terms of parameters dependency was required. For instance, the number of inventories and hotel capacity are related to the number of total clients (considering all the groups). Moreover, quality of service is related to Distance Price and Quality Price. Therefore, although in theory we propose the existence of independent variables, our parameters are related in order to generate feasible and realistic situations. In terms of price and demand, these variables were generated in a range that allowed us to obtain a 90% of feasible instances.

Our data generation process was as follows : First of all, we generated random values uniformly distributed for demand, competitors' price, inventory proximity, inventory capacity, distance price α and quality price β for each user's group, scenario probability, and ABS (Model 4) and PROP (Model 5) parameters [1], as presented in Table 6.1.

Table 6.1 Parameters for the Set-Up

Item	**Uniform Range**
Demand	$U[20,80]+U[-10,10]$
Price	$U[50,500]+U[-40,40]$
Inventory Distance	$U[0,50]$
Inventory Capacity	$U[0,500]$
Distance Price, α	$U[0,90]$
Quality Price, β	$U[0,300]$
Scenario Probability	$U[0.0,1.0]$
ABS parameter	$U[2,50]$
PROP parameter	$U[0,20]$ in %

The other parameters such as LOS, rolling horizon, number of user groups, number of scenarios, number of inventories and number of quality services were also randomly generated using a uniform distribution but in a reduced range trying to replicate a more realistic business environment and a pertinent instance to be examined. For instance, in terms of inventories, we assumed that the leader cannot surpass 50% of the market share. We also assume that competitors' inventories are larger or equal in size and at least as numerous as the leader's inventories in all the instances.

Once all these values were determined, they became fixed and the next step was to introduce uncertainty either on price, on demand, or on price and demand simultaneously. Our setting process to generate stochasticity was developed as follow. First, two random

1. In the following tables and descriptions, we will use generically the denomination "Model 4", for "Absolute Constraints Case" case ; and "Model 5", for "Proportional Constraints Case" case to make reference to the respective problems.

uniform vectors for distance were generated, one corresponding to the proximity of leader's inventories to the interest points and the other corresponding to the proximity of competitors' inventories to the same interest points. We argue that inventories located closer to downtown (where most of the interest points are placed) must have a higher average price than those located in the periphery. In order to respect this characteristic, random vector prices were generated for each inventory, following a uniform distribution, to which a uniform noise was added to introduce an stochastic perturbation for everyday in the rolling horizon. We assumed that prices are identical for each day in the rolling horizon within each inventory. Because the number of days considered in the rolling horizon is generally not so large, it is reasonable to argue that list price does not follow any trend rather a fluctuation.

After average prices were calculated for each one of the vectors, they were ordered from high to low to assign the highest price to the shortest distance inventories and the lowest price to the farthest distance inventories. Price vectors were also ordered from high to low with respect to QoS in order to assign the highest price to the highest quality. It is important to mention that, in some specific cases, competitors' prices did not follow the "Quality service Constraints", which in practice is explained by market conditions or strategic decisions, but the spirit of this principle is generally maintained in our resolution.

Regarding to demand, it was also randomly generated following a uniform distribution adding a uniform noise for each specific group, each day, and length of stay [2].

Once stochasticity on price and demand was introduced, we conducted a feasibility analysis to dismiss atypical solutions. In order to do that, we fixed the price of the leader inventories to infinite as a manner to make abstraction of it and we analyzed users' group distributions on competitors' inventories but also inventory capacity. If for example the groups became concentrated in only one inventory and one QoS, the instance was discharged and both competitors' price and demand noise must be set-up again until the solution was feasible. If after some attempts the solution was still atypical, we regenerated certain initial parameters, such as α and β. After getting experience with this analytical process, we can assure that in approximately 90% of the cases our solutions are feasible for the values presented in Table 6.1. The remaining 10% of the cases were unfeasible and thus they were dismissed.

All the set-up parameters were maintained for all the examples presented in this chapter. The problems were solved by using the transformation to a MIP problem for all the scenarios (and stages) in one single problem, because the standard decomposition (Benders' decomposition or L-Shape Method) under bilevel programming became not suitable. The

2. In general terms, stochasticity can exist in both demand and price, individually or simultaneously. However, in the small instance and in the first large examples (from example 23 to 51) only price was stochastic. For the instances utilized in the sensitivity analysis, demand and price were considered as stochastic, individually and simultaneously.

MIP obtained by this mean was solved using Cplex.

6.2 A Small Instance

The purpose of solving and presenting a small instance is to show that when the problem has a reduced size, we can be confident that the result obtained through our HBP heuristic is close to the optimal.

In this instance, the letter **A** refers to leaders' inventories, and **B** designates competitors' inventories. This example is composed of four inventories, which is reasonable due that hotels' chains regularly manage more than one hotel in a geographical zone. We define four quality service categories for each leader's inventory in order to cover from economic rooms to more luxury ones. The leader has one additional QoS category (Q4), which is superior to all the categories offered by the competitor. In that sense, this QoS is considered as the most luxurious of our problem.

In terms of capacity, the leader has one big inventory (hotel), which is similar in size to one of the hotels of its competition, but most of the leader's hotels are medium size. Competitors' hotels are larger and closer to attraction points (which are mainly located in downtown), and its farthest hotels are similarly located to the closest leader's hotels.

Four Group's categories represent users that ranges from the less sensitive to proximity and QoS, and thus mostly sensitive to price, to those sensitive to proximity and QoS. The rolling horizon is equivalent to a weekend.

Problem Definition

In terms of price, this example assumes a certain volatility that is common in this industry and that varies from one inventory to another but also in terms of QoS as shown in Table 6.3. We see that for the inventory B2 for quality of service Q2, the list price of competitors for the second stage in each scenario is lower compared to the first stage. However, in the inventory B4 for the quality of service Q3, the price list of competitors is higher in each scenario.

Specifically regarding the scenarios, this example considers four price scenarios, in which the probability is 1 for the first stage, and respectively $(0.47, 0.29, 0.19, 0.05)$ for each scenario in the second stage, with a discount factor of 1.0 (discount rate $= 0\%$) between both stages and with maximal price difference among stages of $\delta_{\text{ABS}} = 17$ or $\delta_{\text{PROP}} = 0.05$.

The model is described and summarised using two tables, Table 6.2 and Table 6.3. In the first table, inventories characteristics are presented in (a), Users Groups related definitions are presented in (b), and Stage definitions are shown in (c). In the second table, demand is shown in (a) and competitor's list price in (b).

Similarly to the rest of the examples presented in this chapter, "Service-Quality Order" constraints (2.12), were introduced for every scenario.

Table 6.2 Small Instance Definitions A

Inventory	Service Qualities	Distance	Capacity
A1 (Leader)	4	37	301
A2	4	48	176
A3	4	72	188
A4	4	77	154
B1 (competitors)	3	13	347
B2	3	27	365
B3	3	30	303
B4	3	47	266

(a) Inventories

Groups	Distance Value (α)	Quality Value (β)
G1	3.20	4
G2	4.32	49
G3	4.97	101
G4	8.75	165

(b) Group Definitions

Parameter	Value
Price Absolute Difference :	17.0
Price Proportional Difference :	0.05
Discount Factor :	1.0
Rolling Horizon :	2 days

(c) Stage Definition

Table 6.3 Small Instance Definitions B

Probability by Stage and Scenario

First Stage	Second Stage Scenario 1	Second Stage Scenario 2	Second Stage Scenario 3	Second Stage Scenario 4
1.0	0.47	0.29	0.19	0.05

Demand by Stage and Scenario

Day	Groups	Lenght of Stay	First Stage	Second Stage Scenario 1	Second Stage Scenario 2	Second Stage Scenario 3	Second Stage Scenario 4
1	G1	1			201		
1	G2	1			193		
1	G3	1			212		
1	G4	1			208		
2	G1	1			186		
2	G2	1			201		
2	G3	1			229		
2	G4	1			208		

(a) Scenario Probability & Demand Definition

Inventory	Services Qualities	First Stage	Second Stage Scenario 1 [day 1, day 2]	Second Stage Scenario 2 [day 1, day 2]	Second Stage Scenario 3 [day 1, day 2]	Second Stage Scenario 4 [day 1, day 2]
B1 (competitors)	1	317	266.40, 247.81	254.49, 313.36	280.42,293.30	275.65, 265.68
B1	2	366	335.34, 375.54	371.67, 309.34	364.63,343.43	313.00, 337.10
B1	3	383	411.64, 354.38	356.37, 366.22	369.51,362.66	414.14, 363.11
B2	1	212	229.02, 233.76	210.90, 233.88	259.27,234.29	265.21, 269.49
B2	2	410	282.46, 309.03	262.73, 245.5	271.08,262.83	235.57, 265.86
B2	3	414	498.63, 501.23	430.88, 500.32	463.32,497.41	484.46, 460.00
B3	1	102	94.24, 132.10	128.47, 77.14	82.46,128.00	138.43, 139.44
B3	2	299	275.80, 345.41	300.46, 342.05	283.28,285.39	288.96, 327.67
B3	3	432	326.81, 357.55	326.97, 303.41	327.72,343.50	353.60, 355.55
B4	1	148	120.62, 117.11	120.39, 61.99	135.48, 78.90	86.98, 94.74
B4	2	219	221.05, 225.53	182.33, 233.54	178.16,230.07	218.21, 177.40
B4	3	239	442.46, 381.19	417.89, 411.28	405.64,422.84	426.98, 388.91

(b) List Price Definitions

6.2.1 Numerical Results and Sensitivity Analysis

The optimal solutions, for our Model 4, are presented in Table 6.5 and Table 6.6 and for our Model 5 in Table 6.7 and Table 6.8. The solutions provided by the naive heuristic and our HBP heuristic for both Model 4 and Model 5 are presented in Table 6.4

In this example, Model 4 shows changes that are notorious for the basic analysis : the Expected Value (EV) and the Expected Solution of the Expected Value (EEV) are both 6% different compared to the SHBP solution. The Value of the Stochastic Solution (VSS) is equivalent to 6%. The Expected Value of Perfect Information (EVPI) is small (approx. to 1%) but it is relevant from an industrial point of view because 1% can represent thousands of dollars in terms of revenue.

We can argue that the differences found are consistent with the importance of the stochastic solution. If we take a look at prices, we can see that "Scenario 1" is dominant in Recourse Problem (RP), EEV and WS solutions. Moreover, if we examine data coming from "Scenario 4", they are not favorable for a price increase, which is also reflected in all the solutions.

Regarding Model 5, it presents similar results compared to the previous case, mainly because in both situations prices are close. In effect, Model 5 shows that EV is also 6% different. The EEV is also 6% different and the VSS is lower compared to Model 4, just 6.4% than SHBP solution. Finally, the Expected Value of Perfect Information (EVPI) is almost 1% as well. Moreover, in Model 5, the maximal proportional difference is equivalent to a difference in price of $9 [3], which is quite different from $17 in Model 4, presented in Table 6.2, despite similar solutions in terms of price.

We also notice that the price obtained for the second stage for SHBP is the same compared to the prices obtained for the Second stage, "Scenario 1", for the WS type solution. That means that "Scenario 1" is influent in the results obtained for SHBP in the Second stage in both Model 4 and Model 5 because of its weight. In addition, we can see that the First stage can be also influent in the results. A good example of this is the price equal to zero obtained for the leader in inventory 4 for the WS type solution, for both Model 4 and Model 5. In this example, the leader faced low prices coming from competitors' inventories B3 and B4 in the First stage and, at the same time, the leader assumed that there would be very low prices for those inventories in the second stage for "Scenario 4". Therefore, the leader, in a deterministic behaviour, reduced its prices to attract clients [4]. Fortunately for the leader, "Scenario 4" has a low weight in the final solution for this particular example.

Another aspect to highlight is that, compared to the prices obtained for the SHBP type

3. It corresponds to 5% of $432
4. Although this seams to be counter intuitive form an economic perspective, it happens because of the inherent characteristic of bilevel programming.

Table 6.4 Revenue Small Instance, Heuristic Results

Solution Heuristic		Revenue	Diff. to Optimal in %	
Naive Revenue Min		372155	−34.8%	
Naive Revenue Max		404651	−29.1%	
Naive Revenue Mean		402577	−29.4%	
Case	Model 4	Model 5		
HBP Best Revenue	529165	492600	−7.2%	− 12.3%

solution, prices differ across all the other solutions, but this variation is not higher than 15% for every single tariff. Finally, it is important to point out that the EV and EEV solutions are similar to price results for Model 4 and Model 5. This behavior is natural because of its own definition, in which just the second stage changes. The same situation happens between WS and SHBP price solutions presented in Table 6.5 and Table 6.7. Therefore, we can argue in both cases, Model 4 and Model 5, that WS is a good approximation for SHBP.

We conducted a sensitivity analysis of the parameter δ (the parameter that ties both stages) on revenue in order to observe the behavior of the stochastic value, the approximations and the measures. The results are shown in Figure 6.1 and Figure 6.2. In terms of the graph shape of each solution type for Model 4 and Model 5, we can see from those figures clear differences among them. In addition, the shapes of each solution type within each Model are also different. If we look at the shape of the EV for Model 4, we can see an increase until it reaches an asymptote upper value, which occurs in straight lines, without jumps. Moreover, for the EEV (also for Model 4) we can also see an increase, but with jumps or steep slopes. For SHBP (RP) and WS, we cannot assume that we are looking at a straight line. The reason seems to be the piecewise linearity conjecture. However, we can argue that since inventories' prices all increase or decrease in a proportional fashion (in one or both stages) as a function of δ_{ABS}, under small changes of δ, it is clear that the inventory assignment will not be modified under the new reduced/increased prices, and this could be expected for each scenario. Whenever the lower level minimal cost assignments are unique, the SHBP became a linear program in the tariffs T. Then, the value function is piecewise linear, between different assignments.

We can suppose that the discontinuity points are coincident with the points where assignments (or prices) change in a "bang-bang effect"[5]. As we have noted, Model 5 is the most erratic. Looking the curves, there is a local minimum in Model 4-EVPI function and also a

5. Having a sudden, quick and violent effect.

Table 6.5 Small Instance, Model 4, Price Values EV and EEV

Model 4, Solution Aprox.	Revenue	Diff. to Optimal in %
Expected Value (EV)	602714	5.7%
Expected Solution of the Expected Value (EEV)	534793	−6.2%
Wait and See (WS)	576595	1.1%
SHBP (RP)	570364	
HBP Heuris.	529165	−7.2%
Value of the Stochastic Solution (VSS)	35572	6.2%
Expected Value of Perfect Information (EVPI)	6231	1.1%

(a) Model 4, Stochastic Problem Solution

Sol. Aprox.	Inv.	Serv. Qual.	First Stage	Second Stage
EV	1	1	150.00	133.00
	1	2	191.16	174.16
	1	3	240.16	248.00
	1	4	338.00	349.00
	2	1	114.80	97.80
	2	2	143.64	126.64
	2	3	192.64	193.33
	2	4	283.33	294.33
	3	1	38.00	21.00
	3	2	42.00	25.00
	3	3	88.96	74.05
	3	4	164.05	175.05
	4	1	22.00	5.00
	4	2	26.00	9.00
	4	3	67.36	50.36
	4	4	139.20	150.20

Sol. Aprox.	Inv.	Serv. Qual.	First Stage	Second Stage Scen. 1	Scen. 2	Scen. 3	Scen. 4
EEV	1	1	150.00	149.11	152.39	167.00	133.00
	1	2	191.16	190.27	193.55	208.16	174.16
	1	3	240.16	239.27	242.55	257.16	243.83
	1	4	338.00	336.10	321.22	324.51	344.83
	2	1	114.80	113.91	117.19	131.80	97.80
	2	2	143.64	142.75	146.03	160.64	126.64
	2	3	192.64	191.75	195.03	209.64	199.33
	2	4	283.33	292.75	266.55	269.84	300.33
	3	1	38.00	40.62	40.39	55.00	21.00
	3	2	42.00	44.62	44.39	59.00	25.00
	3	3	88.96	91.58	91.35	105.96	80.05
	3	4	164.05	176.98	147.27	154.96	181.05
	4	1	22.00	24.62	24.39	39.00	5.00
	4	2	26.00	28.62	28.39	43.00	9.00
	4	3	67.36	69.98	69.75	84.36	55.20
	4	4	139.20	152.13	122.42	133.36	156.20

(b) Model 4, Price Values

Table 6.6 Small Instance, Model 4, Price Values WS and and SHBP (RP)

Sol. Type	Inv.	Serv. Qual.	First Stage Scen. 1	Scen. 2	Scen. 3	Scen. 4	Second Stage Scen. 1	Scen. 2	Scen. 3	Scen. 4
WS	1	1	166.11	169.39	176.64	133.03	149.11	152.39	167.48	117.04
	1	2	207.27	210.55	217.80	174.19	190.27	193.55	208.64	157.19
	1	3	256.27	259.55	266.80	230.34	239.27	242.55	257.64	213.34
	1	4	326.38	328.37	334.66	331.34	309.38	311.37	317.66	314.34
	2	1	130.91	134.19	141.44	97.83	113.91	117.19	132.28	81.84
	2	2	159.75	163.03	170.28	126.67	142.75	146.03	161.12	109.67
	2	3	208.75	212.03	219.28	175.67	191.75	195.03	210.12	158.67
	2	4	271.71	273.70	279.99	276.67	254.71	256.70	262.99	259.67
	3	1	54.11	57.39	64.64	21.03	37.11	40.39	55.48	5.04
	3	2	58.11	61.39	68.64	25.03	41.11	44.39	59.48	9.04
	3	3	105.07	108.35	115.60	71.99	88.07	91.35	106.44	54.99
	3	4	154.07	157.35	164.60	157.39	137.07	140.35	155.44	140.39
	4	1	38.11	41.39	48.64	5.03	21.11	24.39	39.48	0.00
	4	2	42.11	45.39	52.64	9.03	25.11	28.39	43.48	0.01
	4	3	83.47	86.75	94.00	50.39	66.47	69.75	84.84	33.39
	4	4	132.47	135.75	143.00	132.54	115.47	118.75	133.84	115.54

Sol. Type	Inv.	Serv. Qual.	First Stage	Second Stage
SHBP (RP)	1	1	166.11	149.11
	1	2	207.27	190.27
	1	3	256.27	239.27
	1	4	326.38	309.38
	2	1	130.91	113.91
	2	2	159.75	142.75
	2	3	208.75	191.75
	2	4	271.71	254.71
	3	1	54.11	37.11
	3	2	58.11	41.11
	3	3	105.07	88.07
	3	4	154.07	137.07
	4	1	38.11	21.11
	4	2	42.11	25.11
	4	3	83.47	66.47
	4	4	132.47	115.47

(a) Model 4, Price Values

Table 6.7 Small Instance, Model 5, Price Values, EV and EEV

Model 5, Solution Aprox.	Revenue	Diff. to Optimal in %
EV	596796	6.3%
EEV	525387	−6.5%
WS	569409	1.4%
SHBP (RP)	561670	
HBP Heuris.	492600	−12.3%
VSS	36283	6.46%
EVPI	7738	1.37%

(a) Model 5, Stochastic Problem Solution

Sol. Aprox.	Inv.	Serv. Qual.	First Stage	Second Stage			
EV	1	1	133.00	133.00			
	1	2	174.16	174.16			
	1	3	237.00	237.00			
	1	4	338.00	338.00			
	2	1	97.80	97.80			
	2	2	126.64	126.64			
	2	3	182.33	182.33			
	2	4	283.33	283.33			
	3	1	21.00	21.00			
	3	2	25.00	25.00			
	3	3	71.96	71.96			
	3	4	164.05	164.05			
	4	1	5.00	5.00			
	4	2	9.00	9.00			
	4	3	50.36	50.36			
	4	4	139.20	139.20			
Sol. Aprox.	Inv.	Serv. Qual.	First Stage	Second Stage Scen. 1	Scen. 2	Scen. 3	Scen. 4
EEV	1	1	145.70	148.50	148.50	148.50	123.84
	1	2	186.86	189.66	189.66	189.66	160.14
	1	3	237.00	238.66	238.66	238.66	216.29
	1	4	338.00	309.38	311.37	317.66	317.29
	2	1	110.50	113.30	113.30	113.30	93.92
	2	2	139.34	142.14	142.14	142.14	118.44
	2	3	188.34	191.14	191.14	191.14	161.62
	2	4	283.33	254.71	256.70	262.99	262.62
	3	1	33.70	36.50	36.50	36.50	28.64
	3	2	37.70	40.50	40.50	40.50	32.04
	3	3	84.66	87.46	87.46	87.46	71.96
	3	4	164.05	139.44	139.44	143.71	143.34
	4	1	17.70	20.50	20.50	20.50	15.04
	4	2	21.70	24.50	24.50	24.50	18.44
	4	3	63.06	65.86	65.86	65.86	53.60
	4	4	139.20	118.32	118.32	118.86	118.49

(b) Model 5, Price Values

Table 6.8 Small Instance, Model 5, Price Values WS and and SHBP (RP)

Sol. Type	Inv.	Serv. Qual.	First Stage Scen. 1	Scen. 2	Scen. 3	Scen. 4	Second Stage Scen. 1	Scen. 2	Scen. 3	Scen. 4
WS	1	1	156.32	159.78	175.66	118.92	149.11	152.39	167.48	117.04
	1	2	197.48	200.94	216.82	160.08	190.27	193.55	208.64	157.19
	1	3	246.48	249.94	265.82	230.97	239.27	242.55	257.64	219.43
	1	4	318.23	321.69	331.50	331.98	309.38	311.37	317.66	318.11
	2	1	121.12	124.58	140.46	83.72	115.07	118.35	133.44	81.84
	2	2	149.96	153.42	169.30	112.56	142.75	146.03	161.12	109.67
	2	3	198.96	202.42	218.30	176.31	191.75	195.03	210.12	167.49
	2	4	263.56	267.02	276.83	277.31	254.71	256.70	262.99	263.44
	3	1	44.32	47.78	63.66	6.92	42.11	45.39	60.48	6.58
	3	2	48.32	51.78	67.66	10.92	45.91	49.19	64.28	10.38
	3	3	95.28	98.74	114.62	57.88	90.52	93.80	108.89	54.99
	3	4	144.28	147.74	163.62	158.03	137.07	140.35	155.44	150.12
	4	1	28.32	31.78	47.66	0.00	26.91	30.19	45.28	0.00
	4	2	32.32	35.78	51.66	0.01	30.71	33.99	49.08	0.01
	4	3	73.68	77.14	93.02	36.28	70.00	73.28	88.37	34.47
	4	4	122.68	126.14	142.02	133.18	116.55	119.83	134.92	126.52
Sol. Type	**Inv.**	**Serv. Qual.**	**First Stage**				**Second Stage**			
SHBP (RP)	1	1	156.32				149.11			
	1	2	197.48				190.27			
	1	3	246.48				239.27			
	1	4	318.23				309.38			
	2	1	121.12				115.07			
	2	2	149.96				142.75			
	2	3	198.96				191.75			
	2	4	263.56				254.71			
	3	1	44.32				42.11			
	3	2	48.32				45.91			
	3	3	95.28				90.52			
	3	4	144.28				137.07			
	4	1	28.32				26.91			
	4	2	32.32				30.71			
	4	3	73.68				70.00			
	4	4	122.68				116.55			

(a) Model 5, Price Values

minimum point for Model 5-EVPI function.

Finally, in terms of the results obtained through the use of heuristics, we notice that the best naive heuristic result has a difference of 29% with respect to the optimal solution SHBP, and our heuristic HBP performs better in both models, with only a 7.2% difference compared to the optimal solution in the case of our Model 4, and a 12.3% difference in the case of our Model 5.

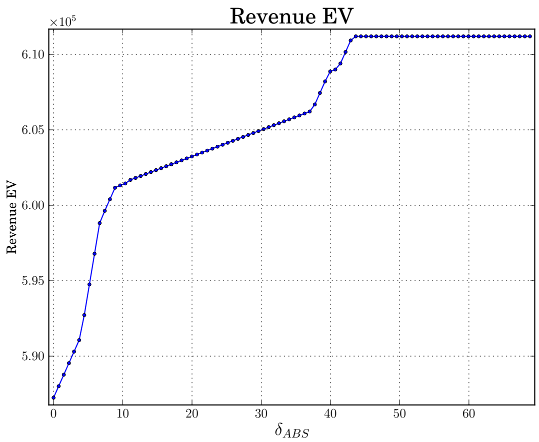
It is important to mention that in order to present these results, an extra complex automatic tuning was done (using the Cplex Tuning Tool) and further extra manual tuning on the solver parameters was also performed, without using other extra heuristics from Cplex. This was necessary to reduce calculation time related to the vast number of graph dots required to present these results. This example took on average 1464 seconds to be solved in a single CPU machine Intel I7 for all problems and both models.

6.2.2 Economic Analysis of the Small Instance

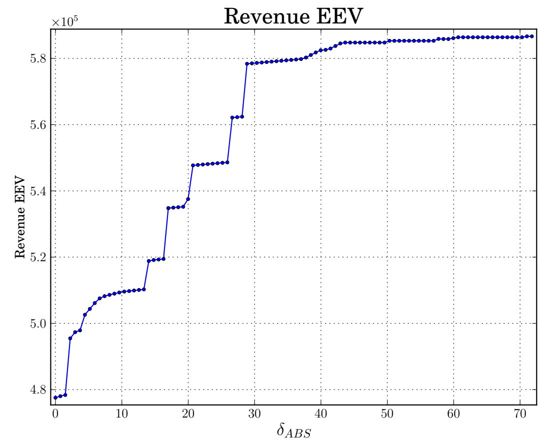
From data presented in the previous pages, we can notice that there is an excess of capacity, because the sum of all rooms available from the leader and its competition is higher than the total demand. Moreover, we can also point out that although the leader has four qualities of services instead of three, and thus it can provide the highest quality, the competitor has a competitive advantage in terms of location, because 3 out of 4 of its hotels are closer to the attraction points. Because of that, clients are tempted to choose competitor's inventories instead of leader's inventories for rooms with the same quality service.

Therefore, the leader must set a price that compensates clients for being far from the attraction points. By looking at Table 6.7, and considering the four scenarios, the leader could expect that competitor's list prices are going to decrease for quality of services Q1 and Q2 and increase for quality of service Q3 in the second stage. The leader could be thus tempted to also increase its list prices for quality services Q3 and Q4 (in Q4 even there is no competition), but the leader finally decided to reduce its prices in the second stage for all its quality services as shown in Table 6.8. As a result, the leader was able to attract all the demand related to quality of service Q3 and Q4 from users' groups G3 and G4, and some users from group G2, to whom it offered a lower price for the same QoS. In terms of revenue, almost 90% was contributed by users G3 and G4.

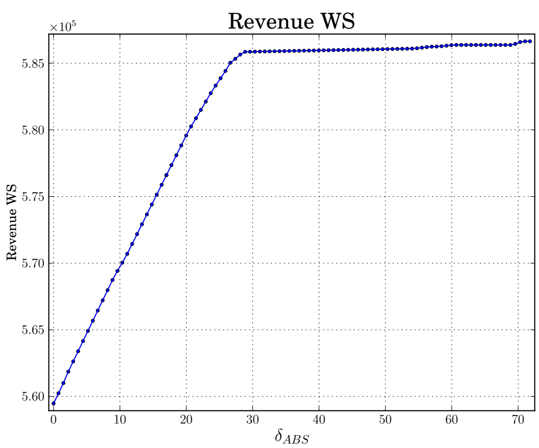
Moreover, in terms of the rationale of the leader to set its list price in the first stage, we can see that the leader's list price for its inventory A2 (closely located to inventory B4) is lower for quality of services Q1, Q2 and Q3 than the price settled by the competitor for the same quality service. However, the leader can set a higher price in its best located inventory (A1) compared to its inventory A2 and to competitor's inventory B4. In order to compete



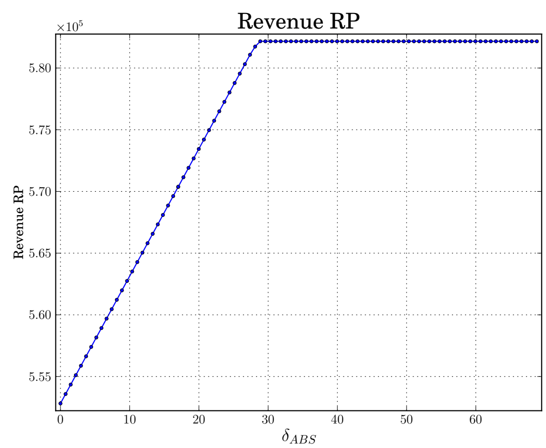
(a) Deterministic Approximation (EV)



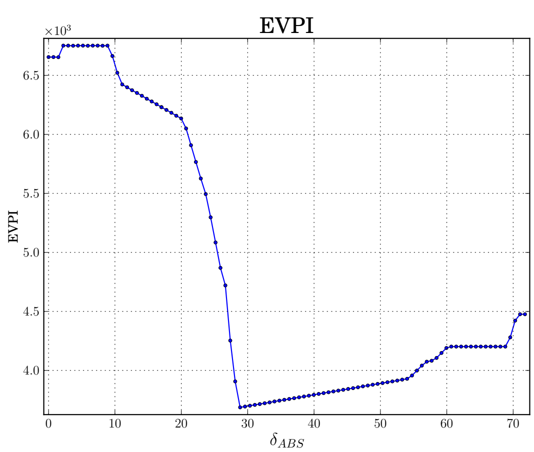
(b) Expected Result for Expected Value



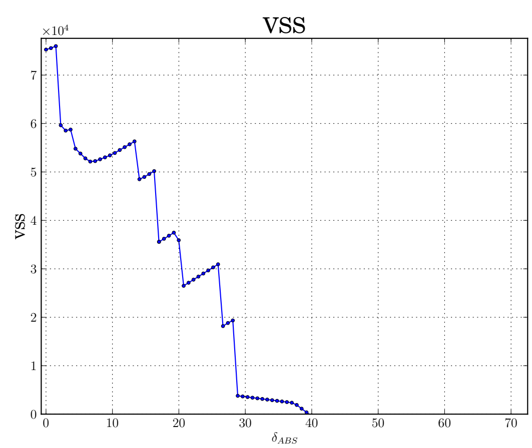
(c) Wait-&-See Value



(d) SHBP Value (RP)

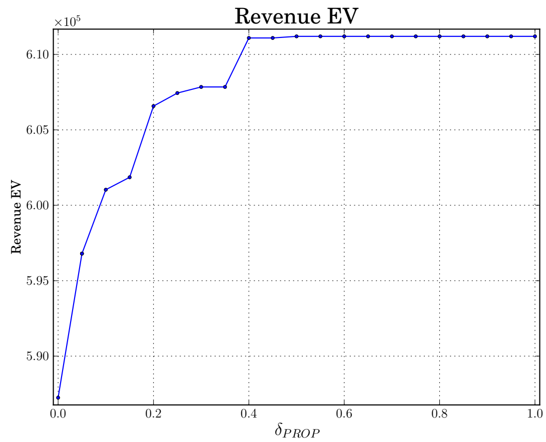


(e) Expected Value of Perfect Information

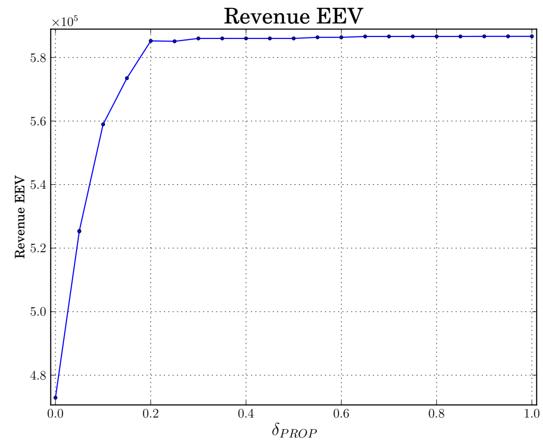


(f) Value of Stochastic Solution

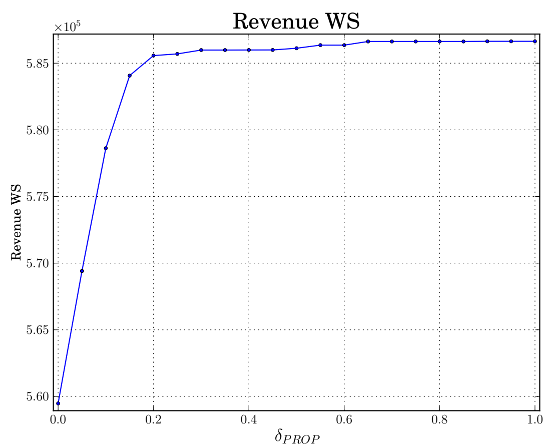
Figure 6.1 Small Instance : Revenue Sensitivity to δ_{ABS} under Model 4



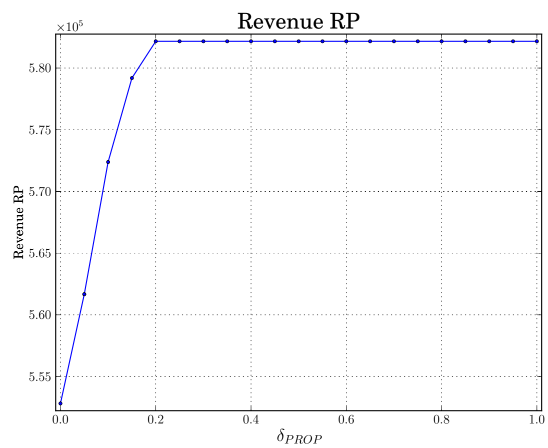
(a) Deterministic Approximation (EV)



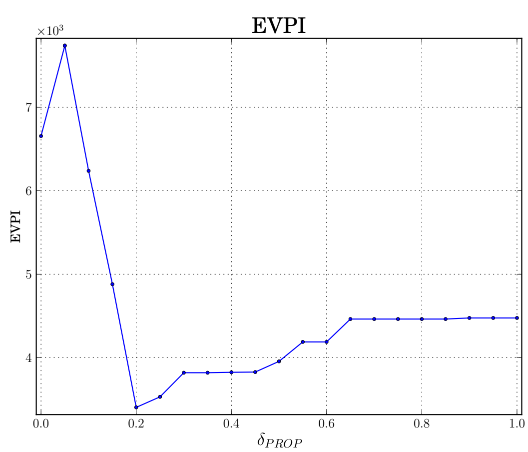
(b) Expected Result for Expected Value



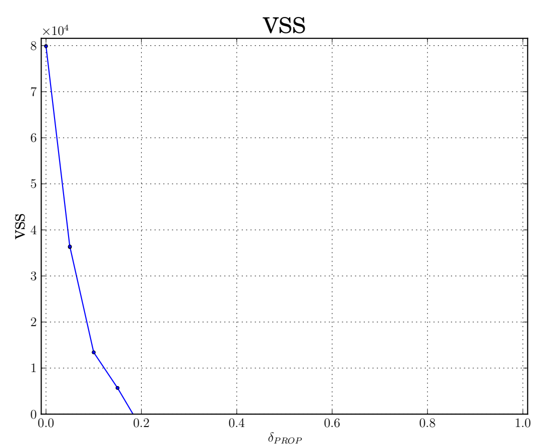
(c) Wait-&-See Value



(d) SHBP Value



(e) Expected Value of Perfect Information



(f) Value of Stochastic Solution

Figure 6.2 Small Instance : Revenue Sensitivity to δ_{PROP} under Model 5 with the best located inventory of the competition, the leader seeks to attract groups that highly value QoS and that are not too sensitive to proximity. For instance, let's examine the leader list price for its inventory A1–quality of service Q3. In fact, that price (256.27) is much lower compared to the price of inventory B3 (432) for the same quality of service, because those inventories (B3) are better located than those of the leader. The reason is that there are two groups (G3 and G4) that valorize a good QoS and thus the leader considers that those clients could be highly interested on staying in farther hotels if the price is attractive for them compared to rooms with equal quality of service but that are closer to the attraction points.

We can also see that the leader sets low prices for the inventories located far away to the ones of the competition (inventories A3 and A4) in the first stage, in all its quality of service, as a strategy to attract groups that highly valuate quality and that can make the effort of being far from the attraction points.

6.3 Large Instances

Twenty nine large instances of the problem were tested for models 4 and 5, trying to be solved to optimality. These instances only consider stochastic pricing.

A description of the notation used in the "Test Instances Description" can be seen in Table 6.9. The description for the notation used in the "Test Results Tables", can be seen in Table 6.10. The "Test Instances" description can be seen in two tables : Part I in Table 6.11 and Part II in Table 6.12.

Table 6.9 Notation used in the "Test Instances Description Tables"

Notation	Description
NA	Number of Inventories for the leader [4 to 5]
QA	Number of Service Qualities [4 to 6]
DA	Distance of the Inventories to the Downtown
CA	Capacity of the Inventories
NB	Number of Inventories for the Competitors
QB	Number of Service Qualities
DB	Distance of the Inventories to the Downtown
CB	Capacity of the Inventories
GR	Number of Clients Groups
α	Price Value of Distance
β	Price Value for Quality of Service
LOS	Lenght of Stay [2 to 3]
RH	Rolling Horizon
ABS	ABS case (Model 4)
PROP	PROP case (Model 5)
DF	Discount Factor
PrF	Probability for Scenarios for First Stage
PrS	Probability for Scenarios in the Second Stage

Table 6.10 Notation used in the "Test Results Tables"

Notation	Description
Example	Example Name
ABS	Results for the Problem with ABS (Model 4)
PROP	Results for the Problem with PROP (Model 5)
Threads	Max. numbers of threads used in the resolution
Total Time	Max Time allowed for Optimization
	The Heuristic HBP using single thread
HBP	Always stop after 5000 iterations ($\approx$ 2800 sec.)
MIP	The MIP problem
MIP Restr	The MIP Restricted with incumbent from HBP
MIP-H+	The Heuristic applied over MIP
MIP Begin	Initial Integral Solution used as Incumbent
Initial Gap	The gap corresponding to Initial Integral Solution used as Incumbent
Best Revenue	Final Solution after solve
Gap	Final Gap corresponding to the Best Solution after solve
Called	Number of calls to MIP-H+
Usefull	Number of Incumbent found with MIP-H+
NaN	Not a Number

Table 6.11 Description of Large Test Instances, Part I

Name	NA	QA	DA	CA	NB	QB	DB	CB	GR
Large23	4	4	37, 56, 72, 77	188, 154, 375, 304	5	5	13, 27, 30, 47, 48	365, 303, 266, 301, 176	10
Large24	4	5	37, 56, 72, 77	188, 154, 375, 304	5	6	13, 27, 30, 47, 48	365, 303, 266, 301, 176	10
Large25	4	4	49, 56, 59, 77	304, 357, 428, 74	6	5	13, 30, 37, 47, 48, 72	266, 301, 176, 188, 154, 375	8
Large26	4	5	37, 56, 72, 77	188, 154, 375, 304	5	7	13, 27, 30, 47, 48	365, 303, 266, 301, 176	12
Large27	4	5	34, 36, 63, 68	454, 342, 442, 352	5	7	19, 38, 69, 69, 76	472, 279, 348, 482, 108	10
Large28	4	6	40, 50, 66, 74	80, 465, 354, 287	6	5	18, 31, 32, 56, 66, 74	76, 225, 216, 205, 461, 386	10
Large29	4	5	33, 39, 57, 75	349, 304, 253, 194	5	4	20, 69, 70, 76, 79	483, 302, 378, 85, 250	8
Large30	4	5	11, 25, 36, 49	291, 177, 115, 331	5	4	14, 24, 32, 43, 46	112, 285, 350, 252, 488	8
Large31	4	4	11, 24, 29, 72	346, 120, 102, 309	4	4	19, 48, 66, 67	405, 482, 302, 305	9
Large32	5	6	20, 34, 50, 59, 66	194, 366, 273, 355, 261	5	6	20, 33, 63, 64, 70	355, 201, 341, 395, 321	10
Large33	4	4	52, 61, 70, 73	191, 350, 275, 443	5	6	10, 22, 24, 25, 68	296, 83, 494, 351, 385	10
Large34	4	5	13, 19, 23, 32	112, 402, 313, 274	5	6	22, 30, 34, 47, 51	88, 108, 306, 340, 317	12
Large35	4	5	12, 14, 46, 73	321, 310, 239, 214	5	6	13, 22, 43, 58, 62	200, 306, 462, 279, 86	11
Large36	4	4	14, 51, 57, 61	188, 411, 114, 204	4	4	10, 40, 56, 79	357, 129, 367, 291	7
Large37	4	4	39, 46, 68, 70	346, 270, 252, 57	4	4	28, 54, 63, 78	341, 268, 464, 494	13
Large38	4	4	32, 36, 64, 71	86, 486, 97, 372	4	4	42, 45, 57, 58	389, 471, 69, 492	13
Large39	4	5	22, 46, 62, 64	68, 65, 429, 168	4	5	10, 26, 58, 69	61, 349, 237, 251	10
Large40	4	5	17, 51, 56, 71	193, 159, 371, 196	5	5	16, 42, 43, 44, 67	484, 243, 193, 411, 430	10
Large41	5	5	13, 44, 48, 53, 76	126, 105, 401, 115, 219	5	5	27, 34, 69, 73, 80	58, 121, 126, 323, 471	10
Large43	5	5	31, 32, 53, 63, 67	421, 53, 443, 240, 126	5	5	28, 35, 65, 70, 78	156, 292, 480, 436, 335	8
Large44	5	5	26, 27, 34, 47, 57	217, 148, 171, 352, 184	5	5	11, 13, 39, 47, 53	64, 448, 362, 120, 416	8
Large45	5	5	25, 27, 53, 59, 77	356, 318, 168, 383, 106	5	5	43, 53, 54, 69, 72	427, 488, 110, 329, 459	8
Large46	5	5	15, 24, 47, 71, 77	319, 124, 83, 261, 400	5	5	14, 24, 36, 79, 79	479, 408, 113, 108, 211	8
Large47	5	5	23, 25, 33, 42, 48	227, 88, 196, 96, 221	5	5	20, 32, 41, 51, 67	111, 272, 372, 240, 354	8
Large48	5	5	10, 12, 24, 63, 75	68, 342, 177, 177, 388	5	5	33, 46, 55, 58, 61	488, 267, 458, 493, 81	8
Large49	5	5	11, 26, 29, 35, 47	200, 147, 213, 495, 55	5	5	12, 12, 21, 39, 67	255, 263, 264, 441, 155	8
Large50	5	5	12, 30, 30, 38, 62	83, 456, 221, 224, 76	5	5	13, 23, 64, 74, 75	469, 445, 500, 381, 97	8
Large51	5	5	15, 23, 39, 65, 77	456, 55, 376, 179, 468	5	5	29, 31, 37, 42, 65	460, 101, 295, 423, 421	8

Table 6.12: Description of Large Test Instances, Part II (cont'd)

Name	**Distance Value (α)**	**Quality Value (β)**	**LOS**	**RH**	**ABS**	**PROP**	**DF**	**PrF**	**PrS**
Large23	9.41, 12.83, 0.20, 0.90, 2.48, 5.44, 7.49, 2.70, 2.94, 11.24	19.33, 22.25, 25, 29, 31, 70, 73.33, 82.50, 86, 87.33	2	5	17	0.05	1.19	1	0.47, 0.29, 0.19, 0.05
Large24	1.53, 5.70, 1.70, 1.44, 0.86, 7.35, 5.51, 5.09, 11.24, 2.25	33.75, 48, 56.60, 57.33, 66, 73.33, 77, 80.33, 87.33, 88.00	2	5	17	0.05	1.19	1	0.47, 0.29, 0.19, 0.05
Large25	3.40, 4, 2.57, 3.12, 1.40, 2.42, 4.29, 3.34	25, 35, 50.25, 50.50, 65, 66, 92, 93.33	2	5	17	0.05	1.19	1	0.40, 0.25, 0.16, 0.05, 0.14
Large26	9.19, 4.25, 2.80, 1.53, 9.29, 3.50, 6.83, 6.29, 10.47, 0.56, 1.50, 2.25	14.50, 18.67, 21, 22, 22.25, 41, 47.17, 49.40, 55.25, 56, 66, 68.67	3	6	17	0.05	1.19	1	0.47, 0.29, 0.19, 0.05
Large27	1.76, 0.54, 2.37, 2, 0.45, 3.23, 1.82, 6, 2.42, 1.52	43.33, 51.33, 51.40, 54.50, 56, 57.75, 61, 62.50, 71, 96.00	3	6	20	0.19	1	1	0.15, 0.66, 0.19
Large28	3.40, 4.72, 2.88, 1.59, 3.16, 3.74, 9.11, 6.50, 0.28, 9.43	32.33, 36, 44.67, 49.25, 50.67, 56.67, 70, 75, 88, 93.00	3	6	13	0.08	1	1	0.19, 0.40, 0.02, 0.40
Large29	13.29, 6.10, 3.86, 6.33, 30.30, 4.86, 4.55, 2.62	58.50, 62.67, 93, 101.40, 108, 115.67, 119, 128.00	3	6	7	0.10	1	1	0.35, 0.08, 0.57
Large30	0.50, 1.38, 6.47, 1.86, 0.83, 0.88, 0.84, 9.41	82.40, 82.67, 89.30, 91.33, 121.33, 123.40, 124.33, 128.00	3	6	5	0.18	1	1	0.17, 0.15, 0.67

Table 6.12 Description of Large Test Instances, Part II (cont'd)

Name	**Distance Value** (α)	**Quality Value** (β)	**LOS**	**RH**	**ABS**	**PROP**	**DF**	**PrF**	**PrS**
Large31	1.26, 0.72, 2.42, 1.88, 0.53, 2.62, 1.43, 5.32, 1.62	67.43, 69.33, 69.40, 83.10, 83.33, 97.33, 98.20, 111.33, 112.00	3	6	7	0.25	1	1	0.43, 0.34, 0.24
Large32	1.27, 1.22, 0.47, 0.74, 1.94, 0.76, 0.08, 2.34, 2.57, 1.32	16.67, 17.33, 50.33, 52.75, 69.50, 69.67, 71.20, 79.67, 88.25, 115.33	2	3	19	0.23	1	1	0.36, 0.44, 0.20
Large33	3.38, 0.36, 8.79, 2.40, 1.93, 10.43, 2, 2.48, 5.50, 1.54	44.33, 60.67, 64, 66, 68, 69, 70.60, 81.75, 82.50, 85.33	2	7	6	0.13	1	1	0.29, 0.44, 0.27
Large34	5.59, 5.75, 6.71, 5.94, 3.48, 4.48, 1.33, 3.71, 4.95, 3.83, 5.58, 0.33	22.67, 28.67, 33.25, 34, 42.50, 43.40, 44.33, 47, 52.67, 58.75, 61.75, 73.67	2	6	7	0.11	1	1	0.20, 0.58, 0.22
Large35	0.11, 1.65, 6.32, 3, 2.14, 0.86, 3.51, 5.73, 3.53, 1.58, 3.50	31.25, 35.50, 38, 45.67, 62, 68.67, 72.20, 73.67, 74.67, 80.33, 99.50	2	6	11	0.20	1	1	0.16, 0.47, 0.30, 0.07
Large36	0.82, 2.71, 4.39, 0.72, 0.36, 3.07, 1.67	47.50, 50, 93.67, 95.60, 97.67, 111.90, 113.33	2	6	19	0.19	1	1	0.22, 0.12, 0.28, 0.17, 0.22
Large37	9.50, 2.09, 4.48, 4.46, 3.97, 6.60, 4.52, 4.75, 2.42, 3.42, 1.26, 0.29, 1.39	57, 59.67, 69.77, 92, 98.90, 111.20, 114, 124.09, 128.90, 133.33, 151.11, 154.25, 168.93	2	6	16	0.14	1	1	0.20, 0.37, 0.43

Table 6.12 Description of Large Test Instances, Part II (cont'd)

Name	Distance Value (α)	Quality Value (β)	LOS	RH	ABS	PROP	DF	PrF	PrS
Large38	9.06, 13.77, 1.27, 11.54, 12.54, 9.40, 1.19, 10.44, 7.69, 0.67, 10.24, 5.82, 6.38	41.40, 44, 59.58, 82.70, 87.19, 88, 90.33, 93.33, 101, 114.40, 123.33, 123.89, 135.24	2	6	7	0.18	1	1	0.40, 0.10, 0.50
Large39	2.44, 0.86, 0.19, 0.17, 2.63, 2.16, 0.01, 2.43, 2.43, 0.99	24.67, 59.75, 68.50, 69.33, 75.33, 77.50, 79.75, 86.33, 88, 91.00	2	6	24	0.21	1	1	0.30, 0.42, 0.28
Large40	25.50, 2.61, 3.68, 0.68, 5.38, 2.33, 0.38, 2.24, 5.63, 3.89	43, 50.67, 52.25, 54, 60.67, 64, 68.25, 74.50, 86.50, 99.00	2	6	25	0.11	1	1	0.26, 0.34, 0.15, 0.25
Large41	2.46, 2.19, 1.29, 1.59, 0.67, 0.98, 1.49, 2.87, 4.22, 1.55	8.33, 26.67, 45.50, 56.75, 59.33, 65, 79.33, 86.25, 93.67, 113.67	3	6	24	0.10	1	1	0.53, 0.31, 0.16
Large43	5.46, 5.67, 6.07, 4.30, 3.15, 1.64, 4.02, 3.93	10.67, 44.67, 57.33, 70.75, 71.67, 90.67, 99.25, 105.00	3	6	22	0.24	1	1	0.21, 0.12, 0.68
Large44	3.15, 5.74, 0.36, 0.64, 4.18, 2.82, 1.71, 6.06	22.33, 67.33, 67.67, 74, 74.25, 80.25, 106.67, 109.00	3	6	10	0.11	1	1	0.27, 0.65, 0.08
Large45	5.50, 5.82, 9.62, 2.89, 4.44, 3.68, 11, 3.69	40.33, 50.33, 62.50, 71, 73.67, 79.25, 85, 86.00	3	6	15	0.25	1	1	0.45, 0.39, 0.16
Large46	2.02, 2.09, 0.18, 0.15, 1.58, 3.52, 0.60, 0.48	37, 53.67, 54.75, 67.25, 71.33, 83.25, 90, 105.67	3	6	23	0.24	1	1	0.27, 0.31, 0.42

Table 6.12 Description of Large Test Instances, Part II (cont'd)

Name	**Distance Value (α)**	**Quality Value (β)**	**LOS**	**RH**	**ABS**	**PROP**	**DF**	**PrF**	**PrS**
Large47	4.26, 3.34, 4.88, 7.62, 1.37, 4.06, 6.34, 4.35	21.33, 41.25, 57, 59.67, 69.33, 82, 85.50, 87.00	3	6	22	0.08	1	1	0.28, 0.37, 0.35
Large48	2.83, 5.83, 5.08, 6.82, 5.23, 4.95, 3.40, 8.50	40, 56, 68.33, 73.75, 87.75, 92, 104.67, 113.67	3	6	17	0.11	1	1	0.34, 0.30, 0.36
Large49	1.07, 0.13, 2.78, 1, 19.89, 1.24, 0.67, 7.22	49.33, 60.75, 65.33, 68.75, 70.67, 81.25, 85, 98.00	3	6	9	0.22	1	1	0.15, 0.29, 0.56
Large50	0.69, 2.31, 2.21, 2.39, 1.18, 0.71, 13.09, 1.35	26.33, 49.33, 75.75, 85.67, 88, 89.50, 91, 110.33	3	6	7	0.17	1	1	0.45, 0.31, 0.24
Large51	10, 7.18, 1.75, 9.88, 0.50, 1.97, 2.07, 1.91	57, 70.50, 78.75, 82.33, 85.25, 91.33, 92.67, 101.67	3	6	24	0.18	1	1	0.20, 0.47, 0.33

The Test Results are shown in Tables 6.13, 6.14, 6.15 and 6.16, for the Parts I to IV respectively. In the Table 6.16 are presented results with a bigger discount factor and longer resolution time to compare the difficulty of the problem.

6.3.1 Numerical Results and Analysis

In this section, we present the revenue of the leader obtained through the different resolution strategies. The main purpose is to show the difficulties solving these large instances and the benefits of using our HBP heuristic.

As presented in the next tables, the problems show themselves as difficult problems, the gap in many cases is high (60% gap or more), and at the moment of stopping the solver, the gap was already in stagnation. Allowing more time did not improve the bounds any more. In many problems with a gap lower than 40%, we could think been close to the optimal solution with the MIP [6]. From the point of view of the restrictions tying both stages, it is difficult to judge that Model 5 is much more complicated to solve than Model 4. We can argue that differently to a network, as reported by (Mirza Alizadeh, 2013), Model 5 restriction does not imply many changes in the assignment (paths for the network case) to explore because the congestion happens inside the inventories, and these are few.

From the point of view of the algorithms, it can be seen that HBP, our proposed heuristic, works well considering the low computing time allowed. In some instances : “Example 32”, “Example 38” and “Example 39” the performance was ranging the 20% of the gap (taken from the MIP-Restricted), and in 23/29, the gap was lower than 50%. In these tests, no special tuning was done for the HBP heuristic, so the results represent how much easy for the heuristic is to adapt to general difficult problems.

The algorithm MIP-H+ was allowed to run for approximately 3600 sec., because of the few times that this algorithm improved the lower bound in each call, and because of the memory management problems described in section 5.7. The improvements are frequent at the beginning, and spread after. In some cases, MIP-H+ identifies much better solutions like in “Model 5-Example 27” compared to a pure MIP and MIP-Restricted. Some runs were carried out by using 4 threads in order to check if parallelisation outperforms previous results. However, no mayor improvement were revealed and therefore no more test were done. This is probably because the dual bound was not closing much faster for any MIP other than MIP-Restricted.

The MIP runs for Model 4 were capable of ameliorating the solution in only 4 examples, and in 1 instance for Model 5. This confirms that the problem is difficult even having a initial solution, and that B&B is week for identifying new solutions and/or closing the dual gap with

6. From a practical point of view, the computational cost exceeds the benefits of looking for a better gap.

Table 6.13 Large Test Instances, Numerical Results Part I

Example	**27**	**28**	**29**	**30**	**31**	**32**
Model 4						
Naive Revenue Min	5.983E+05	8.878E+05	2.532E+06	3.010E+06	1.448E+06	1.503E+06
Naive Revenue Max	3.185E+05	6.319E+05	1.895E+06	2.167E+06	1.204E+06	1.262E+06
Naive Revenue Mean	3.898E+05	6.489E+05	2.348E+06	2.922E+06	1.137E+06	1.646E+06
HBP Best Revenue	3.422E+06	2.577E+06	3.138E+06	3.375E+06	2.303E+06	2.040E+06
HBP Total Time	2846	3069	2511	2443	2413	539
MIP Restr. Begin	3.344E+06	2.467E+06	3.179E+06	3.273E+06	2.366E+06	1.850E+06
MIP Restr. Initial Gap	41.97%	51.00%	47.28%	56.02%	40.87%	54.98%
MIP Restr. Best Revenue	3.344E+06	2.467E+06	3.179E+06	3.273E+06	2.366E+06	2.249E+06
MIP Restr. Gap	37.93%	47.36%	38.59%	51.66%	35.26%	20.49%
MIP Restr. Best Bound	4.612E+06	3.636E+06	4.406E+06	4.964E+06	3.200E+06	2.710E+06
MIP Restr. Total Time	3600	3600	3600	3600	3600	3600
MIP Initial Gap	NaN	NaN	NaN	NaN	NaN	23.37%
MIP Best Revenue	0	0	0	0	0	0
MIP Gap	NaN	NaN	NaN	NaN	NaN	NaN
MIP Best Bound	NaN	NaN	NaN	NaN	NaN	NaN
MIP Total Time	3599	3599	3600	3600	3600	3600
MIP-H+ Called	20	12	28	30	29	53
MIP-H+ Usefull	5	4	6	11	8	7
MIP-H+ Best Revenue	1.934E+06	9.948E+05	2.622E+06	2.212E+06	1.403E+06	1.962E+06
MIP-H+ Gap	146.83%	277.04%	94.24%	124.71%	127.20%	45.85%
MIP-H+ Best Bound	4.774E+06	3.751E+06	5.093E+06	4.971E+06	3.188E+06	2.862E+06
MIP-H+ Total Time	3600	3599	3599	3600	3600	3600
Model 5						
Naive Revenue Min	5.983E+05	8.878E+05	2.532E+06	3.010E+06	1.448E+06	1.503E+06
Naive Revenue Max	3.185E+05	6.319E+05	1.895E+06	2.167E+06	1.204E+06	1.262E+06
Naive Revenue Mean	3.898E+05	6.489E+05	2.348E+06	2.922E+06	1.137E+06	1.646E+06
HBP Best Revenue	1.071E+06	2.328E+06	2.470E+06	3.226E+06	1.634E+06	1.797E+06
HBP Total Time	2528	2995	2257	2409	1950	455
MIP Restr. Begin	9.845E+05	2.260E+06	2.444E+06	2.990E+06	1.623E+06	1.747E+06
MIP Restr. Initial Gap	257.64%	64.96%	55.07%	71.83%	92.86%	62.57%
MIP Restr. Best Revenue	9.845E+05	2.260E+06	2.770E+06	2.990E+06	1.623E+06	2.057E+06
MIP Restr. Gap	244.45%	60.28%	32.18%	61.47%	75.36%	27.82%
MIP Restr. Best Bound	3.391E+06	3.622E+06	3.661E+06	4.829E+06	2.846E+06	2.630E+06
MIP Restr. Total Time	3600	3600	3600	3600	3600	3302
MIP Initial Gap	NaN	NaN	49.88%	NaN	NaN	NaN
MIP Best Revenue	0	0	0	0	0	0
MIP Gap	NaN	NaN	NaN	NaN	NaN	NaN
MIP Best Bound	NaN	NaN	NaN	NaN	NaN	NaN
MIP Total Time	3599	3599	3600	3600	3599	3600
MIP-H+ Called	22	10	36	33	26	NaN
MIP-H+ Usefull	5	6	13	4	13	NaN
MIP-H+ Best Revenue	2.233E+06	1.366E+06	2.432E+06	2.687E+06	1.605E+06	1.69E+06
MIP-H+ Gap	113.59%	175.52%	110.25%	84.81%	101.16%	59.00%
MIP-H+ Best Bound	4.770E+06	3.763E+06	5.113E+06	4.966E+06	3.229E+06	2.842E+03
MIP-H+ Total Time	3572	3534	3600	3600	3600	3600

Table 6.14 Large Test Instances, Numerical Results Part II

Example	**33**	**34**	**35**	**36**	**37**	**38**
Model 4						
Naive Revenue Min	6.797E+05	8.794E+05	1.328E+06	1.723E+06	1.517E+06	1.287E+06
Naive Revenue Max	4.476E+05	5.470E+05	6.268E+05	7.710E+05	8.839E+05	9.351E+05
Naive Revenue Mean	3.868E+05	5.009E+05	7.115E+05	1.488E+06	1.181E+06	1.267E+06
HBP Best Revenue	1.095E+06	3.059E+06	2.761E+06	2.608E+06	2.592E+06	3.586E+06
HBP Total Time	1966	2408	2654	1185	2360	636
MIP Restr. Begin	9.672E+05	3.097E+06	2.723E+06	2.525E+06	2.601E+06	3.606E+06
MIP Restr. Initial Gap	102.95%	39.48%	44.21%	43.78%	43.51%	21.63%
MIP Restr. Best Revenue	1.103E+06	3.097E+06	2.723E+06	2.525E+06	2.601E+06	3.644E+06
MIP Restr. Gap	59.68%	34.01%	39.15%	37.28%	32.95%	14.86%
MIP Restr. Best Bound	1.762E+06	4.150E+06	3.789E+06	3.466E+06	3.459E+06	4.185E+06
MIP Restr. Total Time	3600	3601	3600	3601	3601	3600
MIP Initial Gap	NaN	NaN	NaN	NaN	NaN	22.31%
MIP Best Revenue	0	0	0	0	0	3.502E+06
MIP Gap	NaN	NaN	NaN	NaN	NaN	20.48%
MIP Best Bound	NaN	NaN	NaN	NaN	NaN	4.220E+06
MIP Total Time	3600	3600	3600	3600	3600	3600
MIP-H+ Called	28	32	26	37	39	39
MIP-H+ Usefull	6	11	8	27	5	4
MIP-H+ Best Revenue	1.040E+06	1.705E+06	1.105E+06	1.514E+06	1.596E+06	3.500E+06
MIP-H+ Gap	78.29%	145.19%	243.88%	130.83%	122.39%	20.93%
MIP-H+ Best Bound	1.854E+06	4.181E+06	3.800E+06	3.494E+06	3.548E+06	4.232E+06
MIP-H+ Total Time	3600	3599	3600	3600	3600	3599
Model 5						
Naive Revenue Min	6.797E+05	8.794E+05	1.328E+06	1.723E+06	1.517E+06	1.287E+06
Naive Revenue Max	4.476E+05	5.470E+05	6.268E+05	7.710E+05	8.839E+05	9.351E+05
Naive Revenue Mean	3.868E+05	5.009E+05	7.115E+05	1.488E+06	1.181E+06	1.267E+06
HBP Best Revenue	9.832E+05	1.211E+06	2.350E+06	2.030E+06	2.615E+06	1.541E+06
HBP Total Time	1813	2369	2068	985	2306	623
MIP Restr. Begin	9.526E+05	1.222E+06	2.322E+06	1.968E+06	2.581E+06	2.073E+06
MIP Restr. Initial Gap	106.96%	210.77%	57.50%	82.72%	44.50%	110.08%
MIP Restr. Best Revenue	1.097E+06	1.271E+06	2.322E+06	2.209E+06	2.581E+06	3.731E+06
MIP Restr. Gap	63.31%	168.57%	48.51%	52.91%	33.52%	10.11%
MIP Restr. Best Bound	1.791E+06	3.415E+06	3.449E+06	3.377E+06	3.446E+06	4.108E+06
MIP Restr. Total Time	3600	3600	3600	3600	3106	3600
MIP Initial Gap	NaN	NaN	NaN	64.22%	NaN	23.04%
MIP Best Revenue	0	0	0	2.484E+06	0	3.646E+06
MIP Gap	NaN	NaN	NaN	39.44%	NaN	14.08%
MIP Best Bound	NaN	NaN	NaN	3.464E+06	NaN	4.159E+06
MIP Total Time	3600	3600	3600	3600	3600	3600
MIP-H+ Called	33	40	24	39	42	45
MIP-H+ Usefull	11	23	7	16	4	3
MIP-H+ Best Revenue	1.084E+06	1.999E+06	1.561E+06	1.885E+06	1.967E+06	3.666E+06
MIP-H+ Gap	70.99%	109.37%	144.27%	85.01%	79.75%	15.26%
MIP-H+ Best Bound	1.853E+06	4.185E+06	3.813E+06	3.487E+06	3.535E+06	4.225E+06
MIP-H+ Total Time	3600	3600	3599	3599	3600	3599

Table 6.15 Large Test Instances, Numerical Results Part III

Example	**39**	**40**	**41**	**43**	**44**	**45**
Model 4						
Naive Revenue Min	7.293E+05	1.108E+06	1.443E+06	2.457E+06	1.556E+06	2.953E+06
Naive Revenue Max	6.533E+05	8.161E+05	1.438E+06	1.908E+06	1.481E+03	2.781E+06
Naive Revenue Mean	6.532E+05	9.996E+05	1.437E+06	2.326E+06	1.671E+06	3.085E+06
HBP Best Revenue	2.189E+06	2.228E+06	1.913E+06	3.177E+06	3.596E+06	4.733E+06
HBP Total Time	2389	1952	3204	2945	2833	1041
MIP Restr. Begin	2.135E+06	2.219E+06	1.805E+06	3.111E+06	3.328E+06	4.551E+06
MIP Restr. Initial Gap	29.31%	45.34%	103.01%	53.51%	33.77%	57.14%
MIP Restr. Best Revenue	2.135E+06	2.219E+06	1.805E+06	3.111E+06	3.328E+06	4.836E+06
MIP Restr. Gap	21.50%	35.67%	100.04%	49.69%	28.91%	41.13%
MIP Restr. Best Bound	2.594E+06	3.010E+06	3.612E+06	4.657E+06	4.290E+06	6.825E+06
MIP Restr. Total Time	3600	3600	3600	3600	3600	3600
MIP Initial Gap	NaN	NaN	NaN	NaN	NaN	NaN
MIP Best Revenue	0	0	0	0	0	0
MIP Gap	NaN	NaN	NaN	NaN	NaN	NaN
MIP Best Bound	NaN	NaN	NaN	NaN	NaN	NaN
MIP Total Time	3600	3600	3599	3599	3599	3600
MIP-H+ Called	NaN	32	14	23	27	NaN
MIP-H+ Usefull	NaN	32	14	5	7	NaN
MIP-H+ Best Revenue	6.04E+05	1.261E+06	1.063E+06	2.480E+06	1.684E+06	3.03E+06
MIP-H+ Gap	22.00%	142.91%	239.77%	100.22%	160.29%	43.00%
MIP-H+ Best Bound	2.808E+06	3.064E+06	3.611E+06	4.966E+06	4.383E+06	6.979E+06
MIP-H+ Total Time	3600	3600	3600	3549	3600	3600
Model 5						
Naive Revenue Min	7.293E+05	1.108E+06	1.443E+06	2.457E+06	1.556E+06	2.953E+06
Naive Revenue Max	6.533E+05	8.161E+05	1.438E+06	1.908E+06	1.481E+03	2.781E+06
Naive Revenue Mean	6.532E+05	9.996E+05	1.437E+06	2.326E+06	1.671E+06	3.085E+06
HBP Best Revenue	1.845E+06	2.169E+06	1.814E+06	2.646E+06	2.673E+06	3.689E+06
HBP Total Time	1853	1863	2849	2700	2820	849
MIP Restr. Begin	1.838E+06	2.140E+06	1.876E+06	2.783E+06	2.339E+06	3.604E+06
MIP Restr. Initial Gap	49.79%	50.73%	95.45%	72.90%	88.69%	77.56%
MIP Restr. Best Revenue	1.838E+06	2.140E+06	1.876E+06	2.783E+06	2.339E+06	3.604E+06
MIP Restr. Gap	40.92%	40.68%	92.35%	69.15%	81.37%	67.72%
MIP Restr. Best Bound	2.590E+06	3.010E+06	3.608E+06	4.707E+06	4.242E+06	6.044E+06
MIP Restr. Total Time	3243	3600	3600	3600	3600	3600
MIP Initial Gap	NaN	NaN	NaN	NaN	NaN	NaN
MIP Best Revenue	0	0	0	0	0	0
MIP Gap	NaN	NaN	NaN	NaN	NaN	NaN
MIP Best Bound	NaN	NaN	NaN	NaN	NaN	NaN
MIP Total Time	3600	3600	3599	3599	3599	3600
MIP-H+ Called	NaN	33	12	19	27	36
MIP-H+ Usefull	NaN	10	12	19	7	9
MIP-H+ Best Revenue	8.661E+05	1.606E+06	8.342E+05	1.485E+06	1.992E+06	3.204E+06
MIP-H+ Gap	32.00%	91.11%	333.97%	235.78%	120.28%	117.44%
MIP-H+ Best Bound	2.685E+06	3.069E+06	3.620E+06	4.985E+06	4.388E+06	6.966E+06
MIP-H+ Total Time	3600	3600	3599	3600	3600	3600

Table 6.16 Large Test Instances, Numerical Results Part IV

Example	**46**	**47**	**48**	**49**	**50**	**51**
Model 4						
Naive Revenue Min	1.398E+06	1.787E+06	2.025E+06	2.131E+06	1.069E+06	3.016E+06
Naive Revenue Max	1.322E+06	1.313E+06	2.422E+06	8.710E+05	6.561E+05	2.399E+06
Naive Revenue Mean	1.392E+06	1.643E+06	2.316E+06	1.053E+06	1.010E+06	3.042E+06
HBP Best Revenue	3.378E+06	2.341E+06	3.682E+06	3.246E+06	3.249E+06	6.376E+06
HBP Total Time	2782	2935	2953	2927	2906	1167
MIP Restr. Begin	3.105E+06	2.252E+06	3.534E+06	3.224E+06	3.033E+06	6.184E+06
MIP Restr. Initial Gap	55.31%	53.76%	52.24%	42.16%	41.02%	28.55%
MIP Restr. Best Revenue	3.105E+06	2.252E+06	3.534E+06	3.224E+06	3.033E+06	6.184E+06
MIP Restr. Gap	52.64%	50.64%	48.67%	36.32%	32.00%	25.82%
MIP Restr. Best Bound	4.740E+06	3.393E+06	5.253E+06	4.394E+06	4.004E+06	7.781E+06
MIP Restr. Total Time	2682	3600	3600	3599	3599	3600
MIP Initial Gap	NaN	NaN	NaN	NaN	NaN	NaN
MIP Best Revenue	0	0	0	0	0	0
MIP Gap	NaN	NaN	NaN	NaN	NaN	NaN
MIP Best Bound	NaN	NaN	NaN	NaN	NaN	NaN
MIP Total Time	3599	3599	3600	3599	3600	3600
MIP-H+ Called	20	26	16	NaN	27	29
MIP-H+ Usefull	3	12	8	NaN	27	2
MIP-H+ Best Revenue	1.578E+06	9.834E+05	2.200E+06	2.172E+06	2.681E+05	4.709E+06
MIP-H+ Gap	200.25%	250.87%	142.33%	47.00%	1499.56%	65.62%
MIP-H+ Best Bound	4.739E+06	3.451E+06	5.331E+06	4.612E+06	4.288E+06	7.799E+06
MIP-H+ Total Time	3600	3600	3600	3601	3598	3600
Model 5						
Naive Revenue Min	1.398E+06	1.787E+06	2.025E+06	2.131E+06	1.069E+06	3.016E+06
Naive Revenue Max	1.322E+06	1.313E+06	2.422E+06	8.710E+05	6.561E+05	2.399E+06
Naive Revenue Mean	1.392E+06	1.643E+06	2.316E+06	1.053E+06	1.010E+06	3.042E+06
HBP Best Revenue	2.215E+06	2.576E+06	2.913E+06	1.855E+06	2.048E+06	4.536E+06
HBP Total Time	2645	2743	2855	2721	2863	1006
MIP Restr. Begin	1.844E+06	2.550E+06	2.909E+06	1.667E+06	1.989E+06	4.001E+06
MIP Restr. Initial Gap	156.65%	36.81%	84.50%	167.79%	110.72%	94.64%
MIP Restr. Best Revenue	1.844E+06	2.550E+06	2.909E+06	1.667E+06	3.003E+06	5.279E+06
MIP Restr. Gap	147.90%	34.26%	79.84%	134.77%	30.36%	43.01%
MIP Restr. Best Bound	4.573E+06	3.423E+06	5.232E+06	3.914E+06	3.915E+06	7.550E+06
MIP Restr. Total Time	1694	3600	2761	3599	3600	3600
MIP Initial Gap	NaN	NaN	NaN	NaN	NaN	NaN
MIP Best Revenue	0	0	0	0	0	0
MIP Gap	NaN	NaN	NaN	NaN	NaN	NaN
MIP Best Bound	NaN	NaN	NaN	NaN	NaN	NaN
MIP Total Time	3600	3600	3599	3600	3600	3600
MIP-H+ Called	21	29	18	NaN	29	29
MIP-H+ Usefull	6	10	14	NaN	29	3
MIP-H+ Best Revenue	1.565E+06	1.148E+06	2.438E+06	2.295E+06	1.183E+06	5.378E+06
MIP-H+ Gap	202.53%	199.92%	118.65%	48.00%	261.90%	45.33%
MIP-H+ Best Bound	4.735E+06	3.442E+06	5.331E+06	4.612E+06	4.282E+06	7.816E+06
MIP-H+ Total Time	3600	3600	3600	3599	3598	3600

Table 6.17 Large Test Instances, Numerical Results Part V, Long Time, **DF=1.19**

Example	**23–DF** = 1.19	**24–DF** = 1.19	**25–DF** = 1.19	**26–DF** = 1.19	**41–DF** = 1.18
Naive Revenue Min	5.18E+04	1.57E+05	2.06E+05	6.69E+04	7.43E+05
Naive Revenue Max	4.02E+04	2.46E+05	6.49E+03	4.52E+04	7.30E+05
Naive Revenue Mean	4.02E+04	2.23E+05	4.03E+04	6.77E+04	7.29E+05
Model 4					
Threads	12	4	4	4	32
Total Time	2.88E+04	3.60E+03	3.60E+03	2.88E+04	1.44E+04
HBP Best Revenue	3.46E+05	5.91E+05	7.67E+05	3.93E+05	1.23E+06
MIP Restr. Begin	3.57E+05	6.13E+05	7.18E+05	3.55E+05	1.13E+06
MIP Restr. Initial Gap	43.4%	37.7%	46.2%	72.5%	62%
MIP Restr. Best Revenue	3.90E+05	6.31E+05	7.18E+05	3.55E+05	1.13E+06
MIP Restr. Gap	14.8%	22.2%	32.8%	59.8%	62%
MIP Begin	3.90E+05	6.31E+05	7.18E+05	3.55E+05	1.13E+06
MIP Initial Gap	41.5%	36.7%	46.0%	90.3%	67.6%
MIP Best Revenue	3.90E+05	6.31E+05	8.17E+05	4.00E+05	1.13E+06
MIP Gap	20.0%	30.6%	24.8%	55.9%	67.6%
MIP-H+ Best Revenue	3.21E+05	6.53E+05	8.00E+05	4.07E+05	1.47E+06
MIP-H+ Gap	45.8%	26.2%	28.3%	50.3%	26.0%
MIP-H+ Called	1.74E+03	2.90E+04	3.46E+04	1.35E+04	1.08E+04
MIP-H+ Usefull	2	57	64	19	20
MIP-H+ Total Time	2.88E+04	2.88E+04	2.88E+04	2.88E+04	2.88E+04
MIP-H+ Threads	4	4	4	1	1
Model 5					
Threads	4	4	4	4	32
Total Time	1.44E+04	1.44E+04	1.44E+04	1.44E+04	1.44E+04
HBP Best Revenue	3.64E+05	5.86E+05	6.73E+05	3.80E+05	1.23E+06
MIP Restr. Begin	3.78E+05	5.82E+05	6.41E+05	3.96E+05	1.12E+06
MIP Restr. Initial Gap	38.2%	45.8%	63.4%	57.2%	63.5%
MIP Restr. Best Revenue	3.82E+05	6.55E+05	6.41E+05	3.96E+05	1.12E+06
MIP Restr. Gap	19.8%	21.8%	56.6%	52.2%	63.5%
MIP Begin.	3.82E+05	6.55E+05	6.41E+05	3.96E+05	1.12E+06
MIP Initial Gap	37.3%	31.5%	63.5%	63.8%	69.0%
MIP Best Revenue	3.82E+05	6.55E+05	6.41E+05	3.96E+05	1.12E+06
MIP Gap	26.5%	23.9%	55.1%	56.4%	65.0%
MIP-H+ Best Revenue	3.47E+05	6.22E+05	7.69E+05	4.12E+05	1.25E+06
MIP-H+ Gap	43.3%	33.3%	33.4%	48.9%	48.1%
MIP-H+ Called	3.45E+04	2.89E+04	3.42E+04	1.23E+04	5.80E+03
MIP-H+ Usefull	56	32	95	69	28
MIP-H+ Total Time	2.88E+04	2.88E+04	2.88E+04	2.88E+04	2.89E+04
MIP-H+ Threads	4	4	4	1	1

the prices floating in a big range of bounds. MIP-Restricted has a reduced range of bounds for every variable. On the other hand, MIP-Restricted was able to improve in just 11 cases for Model 4, and in 12 cases for Model 5. MIP-Restricted was not able of finding a lower gap in 1 instance for Model 4, and 1 instance for Model 5. This is probably because of the size of the problems involved. In fact, it could be necessary a larger "sampling" for those instances using the heuristic HBP, but it could be also possible that the best solution could have been cut from the solution space. In the overall process, HBP in cascade with MIP-Restricted is a good tool to solve the general problem, or simply the only available tool to solve the problem under reasonable computing time. MIP-H+ was able to find the best solution in 7 cases, which is few in comparison to HBP and MIP-Restricted together.

6.3.2 Economic Analysis of some Large Instances

We identified three pertinent instances ("Example 51", "Example 27" and "Example 33") selected because they represent some extreme cases. Let's take a look to the example having the highest revenue for Model 4, the "Example 51" int Table 6.16 on page 104.. In this case, the leader and the competitor have the same number of inventory types and the same number of quality of services. However, the leader has 2 hotels better located than competition and only 1 hotel located in a greater distance than the competition. Eight users' groups are considered in this example, which valuate differently distance but more homogenously the quality of service (except for few groups).

In this case, we obtained that this higher revenue comes from the fact that the leader attracted more than 85% of the demand (that is attracted by distance and quality of service), and thus it was able to fulfill its capacity in a similar percentage. More specifically, the leader attracted clients to the highest quality of services and thus it was able to charge high prices to them. In terms of users' groups, the leader attracted the whole group G4 to its highest QoS located in the nearest hotel to the attraction points.

Another pertinent example to examine is "Example 27". In that case, leader capacity is similar to the capacity of "Example 51", and in terms of demand, the leader attracted 65% of the total demand. However, its revenue is almost 50% lower in this example. The main reason is explained by the location of the competitor. Indeed, the competitor has the best located hotel and it has two QoS that the leader does not have. Therefore, the leader was not able to attract the most profitable clients and it had to fulfill its inventories with clients that were willing to pay a lower price.

It is important to mention that in our "Example 27" the initial leader's list price was, in general, cheaper than competitor's list price, and due to the disadvantage of the leader with respect to the location of its inventories, the leader was not able to increase its prices in the second stage. That had a negative impact in its total revenue as well.

Finally, we decided to analyze “Example 33”, which presents the lowest revenue (in terms of restricted value). In this case, competition has a strong competitive advantage with respect to location compared to the leader. In fact, there are 4 competitor’s inventories located close to the interest points, and the closest leader inventory can only compete in proximity with the two farthest inventories of the competition. Moreover, the competition has two additional categories of quality of service in its inventories, which means that the best located inventories and the highest QoS inventories belong to the competition. Therefore, it is not surprising 60% of the total demand chose the competition, and on the other hand, the leader had to set low prices from the beginning (compared to competitor’s inventories) in order to finally attract the 40% remaining. We could think this example resembles the case in which a hotel chain was settled several years ago in a location that was attractive at that time, but with the development of new areas within the city, these hotels become less attractive for customers. Therefore, these hotels finally became economic hotels mainly attractive for low-budget customers.

6.3.3 Stochasticity in Prices and/or Demand for Large Instances

In order to compare the effect of different sources of uncertainty on revenue, gap and client groups distribution, several tests were conducted for examples in which Demand, Price or both were stochastic. The instances were noted using the notation **P** for Stochastic Price, **D** for Stochastic Demand, and **DP** for Stochastic Price & Demand. Similarly to our previous test, all the algorithms were used to determine if any of them was more useful, using both Model 4 and Model 5. We noted that the gap for these instances is in general high, and no algorithm was really dominant over the others. The HBP algorithm nevertheless is able to find a solution in all the cases. The instance used several “threads” (4 threads in total) to look for significant improvements with the parallelizing mode in the solver. The results were putting aside to facilitate the comparison. For some of the instances, the solution was not found using Cplex, because the solver did not accept a first solution or bound. In those cases, the solver runs for all periods without finding a solution by itself. We noticed that all the problems tested were feasible under the stochastic parameters in use. The notation is the same used in Table 6.9. The tables showing the definitions of these instances are Table 6.18 and Table 6.19.

The notation used in the tables showing numerical results is presented in 6.20. The Test Results are shown in Table 6.22, 6.23, 6.24 and 6.25[7]. Instances **604**, **607**, **609**, **610**, **614**

7. To avoid any confusion with other tables, these tables received the notation “600”, and the number of the instances are the tens and units, followed by the respective letter **P**, **D**, **DP**.

Table 6.18 Parameter defintions used in Tables Numerical Results : Stochasticity in Demand **D**, Prices **P** and Both **DP**, Part I

Ex. Name	NA	QA	DA	CA	NB	QB	DB	CB	GR
602	6	6	12, 40, 58, 60, 78, 78	426, 129, 484, 155, 182, 205	6	6	17, 41, 42, 52, 64, 64	376, 375, 360, 406, 291, 155	12
604	6	6	23, 30, 35, 43, 49, 65	122, 150, 250, 87, 62, 121	6	6	23, 34, 36, 39, 45, 78	185, 105, 186, 92, 185, 497	12
607	6	6	28, 39, 69, 69, 75, 79	106, 83, 230, 189, 194, 469	6	6	30, 42, 57, 63, 66, 76	137, 433, 113, 231, 228, 289	12
609	6	6	15, 19, 57, 62, 66, 67	134, 281, 78, 51, 121, 294	6	6	12, 21, 28, 47, 62, 79	146, 319, 448, 401, 474, 250	12
610	5	5	17, 19, 46, 72, 79	344, 412, 442, 106, 281	6	5	22, 24, 40, 42, 49, 71	414, 474, 233, 192, 65, 440	9
611	5	5	17, 18, 31, 42, 66	58, 290, 180, 370, 186	6	5	11, 12, 58, 62, 66, 73	276, 170, 413, 276, 325, 166	9
614	5	5	22, 35, 41, 60, 74	132, 481, 168, 383, 492	6	5	13, 21, 26, 28, 39, 64	184, 148, 463, 280, 429, 476	9

Table 6.19 Parameter defintions used in Tables Numerical Results : Stochasticity in Demand **D**, Prices **P** and Both **DP**, Part II

Ex. Name	Distance Value (α)	Quality Value (β)	LOS	RH	ABS	PROP	DF	PrF	PrS
602	4.33, 4, 3.61, 4.74, 0.56, 2.04, 5.87, 2.79, 1.72, 8.83, 16.09, 3.26	34.50, 37.67, 42.40, 43.67, 46.25, 51.67, 64.75, 70.50, 84.67, 86, 87.25, 98.67	3	7	17	0.19	0.76	1	0.24, 0.40, 0.06, 0.29
604	2.05, 7.67, 6.55, 10, 2.92, 11.60, 14.44, 2.68, 2.02, 1.08, 14.60, 7.23	35.67, 37.25, 44.33, 46.40, 53, 58, 67, 67.33, 71.60, 76.25, 78.75, 87.33	3	7	23	0.08	1.00	1	0.27, 0.44, 0.28, 0.01
607	0.26, 4.78, 9.05, 2.44, 7.79, 3.17, 2.59, 1, 10.37, 2, 2.05, 0.85	45, 65.75, 68.75, 70.40, 71.20, 79.80, 83, 84.67, 87.50, 88, 97, 98.33	3	7	5	0.21	1.00	1	0.01, 0.09, 0.63, 0.27
609	0.41, 4.72, 0.79, 3.59, 2.26, 3.34, 4.37, 1.91, 2.96, 0.64, 1.96, 1.39	15.67, 29.20, 33.25, 37.67, 40, 62.33, 62.50, 66.60, 74.40, 82.50, 89.75, 93.67	3	7	5	0.07	1.00	1	0.18, 0.39, 0.08, 0.35
610	2.72, 0.26, 7.90, 6.96, 4.20, 7.60, 5.64, 12.78, 11.89	25, 45.67, 49, 50, 57.25, 70.75, 79, 97.25, 126.33	3	7	11	0.11	1.00	1	0.52, 0.34, 0.07, 0.07
611	11.07, 2.76, 2.25, 2.90, 9.36, 4.27, 1.30, 0.67, 2.11	33, 48.33, 56.33, 74.50, 82, 92.33, 93, 98.33, 100.00	3	7	11	0.19	1.03	1	0.44, 0.29, 0.23, 0.05
614	3.16, 4.03, 5.11, 16.29, 3, 2.92, 5.38, 1.49, 5.00	25.67, 37.75, 43.67, 65.50, 72.75, 73, 81.67, 87.67, 124.67	3	7	8	0.21	1.00	1	0.39, 0.29, 0.19, 0.13

Table 6.20 Notation used in “Sensitivity Examples”

Notation	Description
Naive Revenue Min	Result for Naive Heuristics Min
Naive Revenue Max	Result for Naive Heuristics Max
Naive Revenue Mean	Result for Naive Heuristics Mean
HBP Best Revenue	Best Integer for Heuristics HBP
HBP Total Time	Total Running Time for Heuristics HBP
MIP Restr. Begin	MIP Restristed Problem Best Initial Integer Solution
MIP Restr. Initial Gap	MIP Restristed Problem Initial Gap Solution
MIP Restr. Best Value	MIP Restristed Problem Best Final Integer Solution
MIP Restr. Gap	MIP Restristed Problem Final Gap Solution
MIP Restr. Best Bound	MIP Restristed Problem Final Bound Solution
MIP Restr. Total Time	Total Running Time for MIP Restricted
MIP Begin	MIP Problem Best Initial Integer Solution
MIP Initial Gap	MIP Problem Initial Gap Solution
MIP Best Value	MIP Problem Best Final Integer Solution
MIP Gap	MIP Problem Final Gap Solution
MIP Best Bound	MIP Problem Bound Gap Solution
MIP Total Time	Total Running Time for MIP Problem
MIP-H+ Called	MIP-H+ Problem Best Initial Integer Solution
MIP-H+ Usefull	MIP-H+ Problem Initial Gap Solution
MIP-H+ Best Value	MIP-H+ Problem Best Final Integer Solution
MIP-H+ Gap	MIP-H+ Problem Final Gap Solution
MIP-H+ Best Bound	MIP-H+ Problem Final Bound Solution
MIP-H+ Total Time	Total Running Time for Heuristics MIP-H+

did not have any solution found by MIP-Original. The gap was not uniform along several instances, which makes quite difficult to compare the quality of the results.

These differences, also noted before for the other instances, are usually related to the way Cplex calculates the Best Bound. We can note that in most cases the stochasticity in demand does not reduce but in some cases improves the results for the leader. Uncertainty in Demand and Price, usually jeopardize the revenue.

The HBP heuristic behave quite well considering the reduced time used, and it was capable of obtaining results in several cases in which Cplex was unable to do it. Cplex improved the GAP in [5%, 20%] from HBP results. This can be seen in Table 6.21.

Table 6.21 Summary of Algorithms Finding the Best Solution

Algorithm Type	Found Best Integer Sol. in	
HBP	22	cases
Restricted Problem	15	...
Original Problem	8	...
MIP-H+	3	...
Total	48	

Economical Analysis

We will analyze in detail "Example 610" and "Example 611" from an economic perspective :

— **Analysis of "Example 610" :** In this example, the leader has a competitive advantage in terms of location, with 2 hotels better located than the competition. Both leader and competitors offer the same five qualities of services and there are 9 users' groups, from which four of them (groups 4, 6, 8, and 9) are sensitive to proximity. Sensitivity analysis was conducted on three cases : stochasticity in price, in demand, and in price-demand jointly. Results for Model 4 shows that the highest revenue is achieved when only demand is stochastic, and the lowest revenue is achieved when both price and demand are stochastic. The difference in revenue is mostly explained by the fact of losing around 3800 clients from group 6, who valuates proximity and quality, impacting negatively on revenue by approximately $700,000. Other lost revenue come from groups 3 and 4, with a negative impact of approximately $160,000 each one. It is important to mention that clients from groups 8 and 9 did not move to the competition but only internally to inventory 2, because this is still better located that the best located hotel of the competition. Therefore, there was a minimum impact on revenue from this new distribution. Another important aspect to point out is the fact that although

Table 6.22 Sensitivity Examples to Stochasticity in Demand **D**, Prices **P** and Both **DP**, Numerical Results Part I

Example	**604–P**	**604–D**	**604–DP**	**607–P**	**607–D**	**607–DP**
Model 4						
Naive Revenue Min	1.436E+06	1.315E+06	1.493E+06	2.187E+06	2.067E+06	2.176E+06
Naive Revenue Max	9.799E+05	1.315E+06	9.990E+05	2.071E+06	2.068E+06	2.065E+06
Naive Revenue Mean	1.227E+06	1.315E+06	1.254E+06	2.003E+06	2.067E+06	2.007E+06
HBP Best Revenue	1.657E+06	1.876E+06	1.710E+06	2.875E+06	3.446E+06	2.915E+06
HBP Total Time	5044	5033	4845	4338	4833	4773
MIP Restr. Begin	1.635E+06	1.765E+06	1.647E+06	2.849E+06	3.178E+06	2.907E+06
MIP Restr. Initial Gap	114.50%	109.48%	118.88%	104.89%	79.57%	101.44%
MIP Restr. Best Revenue	1.635E+06	1.765E+06	1.647E+06	2.849E+06	3.178E+06	2.907E+06
MIP Restr. Gap	114.50%	109.48%	118.88%	104.89%	79.57%	101.44%
MIP Restr. Best Bound	3.506E+06	3.697E+06	3.604E+06	5.837E+06	5.707E+06	5.857E+06
MIP Restr. Total Time	3600	3600	3600	3600	3600	3600
MIP Best Revenue	0	0	0	0	0	0
MIP Gap	NaN	NaN	NaN	NaN	NaN	NaN
MIP Best Bound	NaN	NaN	NaN	NaN	NaN	NaN
MIP Total Time	3601	3600	3600	3602	3603	3603
MIP-H+ Called	4	5	4	2	2	NaN
MIP-H+ Usefull	3	3	3	2	1	NaN
MIP-H+ Best Revenue	1.570E+06	1.307E+06	1.292E+06	1.960E+06	2.354E+06	NaN
MIP-H+ Gap	136.90%	191.97%	192.54%	206.85%	153.64%	NaN
MIP-H+ Best Bound	3.719E+06	3.816E+06	3.780E+06	6.013E+06	5.970E+06	NaN
MIP-H+ Total Time	3597	3598	3598	3603	3598	3598
Model 5						
Naive Revenue Min	1.436E+06	1.315E+06	1.493E+06	2.187E+06	2.067E+06	2.176E+06
Naive Revenue Max	9.799E+05	1.315E+06	9.990E+05	2.071E+06	2.068E+06	2.065E+06
Naive Revenue Mean	1.227E+06	1.315E+06	1.254E+06	2.003E+06	2.067E+06	2.007E+06
HBP Best Revenue	1.625E+06	1.793E+06	1.725E+06	2.695E+06	2.730E+06	2.794E+06
HBP Total Time	4014	3841	4366	3659	4043	3869
MIP Restr. Begin	1.516E+06	1.678E+06	1.603E+06	2.778E+06	2.911E+06	2.832E+06
MIP Restr. Initial Gap	131.44%	120.48%	124.98%	110.02%	97.16%	106.76%
MIP Restr. Best Revenue	1.516E+06	1.678E+06	1.603E+06	2.778E+06	2.911E+06	2.832E+06
MIP Restr. Gap	131.44%	120.48%	124.98%	106.57%	97.16%	103.46%
MIP Restr. Best Bound	3.508E+06	3.699E+06	3.606E+06	5.738E+06	5.740E+06	5.763E+06
MIP Restr. Total Time	3600	3600	3600	3600	3600	3600
MIP Best Revenue	0	0	0	0	0	0
MIP Gap	NaN	NaN	NaN	NaN	NaN	NaN
MIP Best Bound	NaN	NaN	NaN	NaN	NaN	NaN
MIP Total Time	3601	3599	3601	3598	3603	3598
MIP-H+ Called	5	4	3	2	1	NaN
MIP-H+ Usefull	3	3	2	2	1	NaN
MIP-H+ Best Revenue	1.590E+06	1.318E+06	1.227E+06	2.345E+06	2.689E+06	NaN
MIP-H+ Gap	133.51%	190.21%	208.77%	156.12%	130.72%	NaN
MIP-H+ Best Bound	3.713E+06	3.824E+06	3.790E+06	6.006E+06	6.205E+06	NaN
MIP-H+ Total Time	3598	3598	3598	3598	3598	3598

Table 6.23 Sensitivity Examples to Stochasticity in Demand **D**, Prices **P** and Both **DP**, Numerical Results Part II

Example	**609–P**	**609–D**	**609–DP**	**610–P**	**610–D**	**610–DP**
Model 4						
Naive Revenue Min	1.399E+06	1.699E+06	1.409E+06	2.389E+06	2.945E+06	2.393E+06
Naive Revenue Max	1.341E+06	1.700E+06	1.339E+06	3.491E+06	2.945E+06	3.478E+06
Naive Revenue Mean	1.484E+06	1.700E+06	1.505E+06	2.947E+06	2.945E+06	2.931E+06
HBP Best Revenue	1.915E+06	2.170E+06	1.929E+06	5.066E+06	5.342E+06	5.055E+06
HBP Total Time	4979	4201	4467	3407	3366	3317
MIP Restr. Begin	1.850E+06	2.213E+06	1.852E+06	5.034E+06	5.155E+06	4.558E+06
MIP Restr. Initial Gap	109.89%	83.44%	110.74%	40.13%	37.67%	54.65%
MIP Restr. Best Revenue	1.850E+06	2.213E+06	1.852E+06	5.034E+06	5.744E+06	4.558E+06
MIP Restr. Gap	109.89%	83.44%	110.74%	37.04%	20.11%	51.47%
MIP Restr. Best Bound	3.883E+06	4.060E+06	3.902E+06	6.898E+06	6.899E+06	6.904E+06
MIP Restr. Total Time	3600	3600	3600	3600	3600	3600
MIP Best Revenue	0	0	0	0	0	0
MIP Gap	NaN	NaN	NaN	NaN	NaN	NaN
MIP Best Bound	NaN	NaN	NaN	NaN	NaN	NaN
MIP Total Time	3605	3604	3604	3599	3599	3599
MIP-H+ Called	3	2	4	10	13	13
MIP-H+ Usefull	3	2	4	5	11	5
MIP-H+ Best Revenue	1.020E+06	1.024E+06	1.019E+06	2.535E+06	3.107E+06	2.679E+06
MIP-H+ Gap	287.83%	296.86%	288.74%	175.67%	126.05%	160.30%
MIP-H+ Best Bound	3.955E+06	4.065E+06	3.962E+06	6.989E+06	7.022E+06	6.975E+06
MIP-H+ Total Time	3597	3598	3597	3599	3584	3578
Model 5						
Naive Revenue Min	1.399E+06	1.699E+06	1.409E+06	2.389E+06	2.945E+06	2.393E+06
Naive Revenue Max	1.341E+06	1.700E+06	1.339E+06	3.491E+06	2.945E+06	3.478E+06
Naive Revenue Mean	1.484E+06	1.700E+06	1.505E+06	2.947E+06	2.945E+06	2.931E+06
HBP Best Revenue	2.034E+06	2.198E+06	2.027E+06	4.239E+06	3.444E+06	4.104E+06
HBP Total Time	3810	4386	4118	2926	2861	2773
MIP Restr. Begin	1.943E+06	2.111E+06	1.953E+06	3.983E+06	3.813E+06	4.399E+06
MIP Restr. Initial Gap	100.27%	93.06%	100.19%	77.17%	86.13%	60.31%
MIP Restr. Best Revenue	1.943E+06	2.111E+06	1.953E+06	3.983E+06	3.813E+06	4.399E+06
MIP Restr. Gap	100.27%	93.06%	100.19%	72.55%	81.90%	57.23%
MIP Restr. Best Bound	3.890E+06	4.076E+06	3.910E+06	6.873E+06	6.936E+06	6.916E+06
MIP Restr. Total Time	3600	3600	3600	3600	3600	3600
MIP Best Revenue	0	0	0	0	0	0
MIP Gap	NaN	NaN	NaN	NaN	NaN	NaN
MIP Best Bound	NaN	NaN	NaN	NaN	NaN	NaN
MIP Total Time	3605	3604	3605	3599	3599	3599
MIP-H+ Called	4	2	3	15	12	19
MIP-H+ Usefull	3	2	3	5	3	7
MIP-H+ Best Revenue	1.484E+06	1.022E+06	1.017E+06	2.979E+06	3.529E+06	3.019E+06
MIP-H+ Gap	167.74%	301.84%	295.51%	134.16%	98.66%	130.52%
MIP-H+ Best Bound	3.972E+06	4.107E+06	4.024E+06	6.975E+06	7.011E+06	6.961E+06
MIP-H+ Total Time	3598	3598	3598	3599	3599	3599

Table 6.24 Sensitivity Examples to Stochasticity in Demand **D**, Prices **P** and Both **DP**, Numerical Results Part III

Example	**611–P**	**611–D**	**611–DP**	**614–P**	**614–D**	**614–DP**
Model 4						
Naive Revenue Min	9.104E+05	9.817E+05	9.161E+05	3.001E+06	2.541E+06	3.031E+06
Naive Revenue Max	7.985E+05	9.819E+05	8.056E+05	2.521E+06	2.542E+06	2.537E+06
Naive Revenue Mean	7.985E+05	9.818E+05	8.055E+05	2.521E+06	2.542E+06	2.541E+06
HBP Best Revenue	1.606E+06	1.754E+06	1.648E+06	3.981E+06	4.226E+06	3.839E+06
HBP Total Time	3301	3399	3225	3087	3170	3190
MIP Restr. Begin	1.559E+06	1.676E+06	1.599E+06	3.991E+06	4.169E+06	3.849E+06
MIP Restr. Initial Gap	47.07%	41.30%	33.72%	41.15%	39.00%	47.52%
MIP Restr. Best Revenue	1.559E+06	1.822E+06	1.599E+06	3.991E+06	4.169E+06	3.849E+06
MIP Restr. Gap	44.42%	27.04%	30.59%	39.04%	39.00%	44.71%
MIP Restr. Best Bound	2.251E+06	2.315E+06	2.088E+06	5.549E+06	5.796E+06	5.570E+06
MIP Restr. Total Time	4.313E+04	4.315E+04	4.313E+04	3600	3600	3600
MIP Best Revenue	1.559E+06	1.822E+06	1.599E+06	0	0	0
MIP Gap	64.91%	43.78%	60.65%	NaN	NaN	NaN
MIP Best Bound	2.571E+06	2.620E+06	2.569E+06	NaN	NaN	NaN
MIP Total Time	4.320E+04	4.320E+04	4.320E+04	3599	3600	3599
MIP-H+ Called	43	39	48	5	7	6
MIP-H+ Usefull	14	13	11	4	7	5
MIP-H+ Best Revenue	1.001E+06	1.346E+06	1.566E+06	1.294E+06	1.182E+06	1.309E+06
MIP-H+ Gap	150.41%	89.14%	60.18%	335.20%	383.54%	331.29%
MIP-H+ Best Bound	2.506E+06	2.546E+06	2.509E+06	5.631E+06	5.714E+06	5.647E+06
MIP-H+ Total Time	4.319E+04	4.319E+04	4.319E+04	3599	3598	3599
Model 5						
Naive Revenue Min	9.104E+05	9.817E+05	9.161E+05	3.001E+06	2.541E+06	3.031E+06
Naive Revenue Max	7.985E+05	9.819E+05	8.056E+05	2.521E+06	2.542E+06	2.537E+06
Naive Revenue Mean	7.985E+05	9.818E+05	8.055E+05	2.521E+06	2.542E+06	2.541E+06
HBP Best Revenue	1.105E+06	1.277E+06	1.112E+06	2.957E+06	3.130E+06	2.958E+06
HBP Total Time	2907	2935	2812	2809	2871	2947
MIP Restr. Begin	1.087E+06	1.269E+06	1.099E+06	3.222E+06	3.443E+06	3.255E+06
MIP Restr. Initial Gap	91.60%	71.39%	86.39%	74.62%	68.42%	74.71%
MIP Restr. Best Revenue	1.264E+06	1.808E+06	1.220E+06	3.222E+06	3.443E+06	3.255E+06
MIP Restr. Gap	60.87%	16.84%	63.24%	71.61%	68.42%	71.38%
MIP Restr. Best Bound	2.034E+06	2.113E+06	1.992E+06	5.530E+06	5.799E+06	5.578E+06
MIP Restr. Total Time	4.310E+04	4.310E+04	4.314E+04	3600	3600	3600
MIP Best Revenue	1.264E+06	1.808E+06	1.220E+06	0	0	0
MIP Gap	102.45%	44.26%	110.90%	NaN	NaN	NaN
MIP Best Bound	2.559E+06	2.609E+06	2.573E+06	NaN	NaN	NaN
MIP Total Time	4.320E+04	4.32E+04	4.320E+04	3599	3600	3599
MIP-H+ Called	53	45	52	5	5	6
MIP-H+ Usefull	6	10	9	5	5	6
MIP-H+ Best Revenue	1.404E+06	1.719E+06	1.392E+06	1.051E+06	1.221E+06	3.896E+05
MIP-H+ Gap	78.50%	48.56%	80.00%	440.47%	375.51%	1361.45%
MIP-H+ Best Bound	2.506E+06	2.553E+06	2.506E+06	5.682E+06	5.805E+06	5.694E+06
MIP-H+ Total Time	4.319E+04	4.319E+04	4.319E+04	3599	3599	3600

Table 6.25 Sensitivity Examples to Stochasticity in Demand **D**, Prices **P** and Both **DP**, Numerical Results Part IV

Example	**602–P**	**602–D**	**602–DP**	**605–P**	**605–D**	**605–DP**
Model 4						
DF	DF = 0.75			DF = 0.94		
Naive Revenue Min	7.032E+05	5.762E+05	6.989E+05	7.519E+05	6.739E+05	7.701E+05
Naive Revenue Max	2.465E+05	5.765E+05	2.611E+05	5.162E+05	6.741E+05	5.224E+05
Naive Revenue Mean	5.565E+05	5.763E+05	5.644E+05	6.603E+05	6.740E+05	6.689E+05
HBP Best Revenue	1.606E+06	1.684E+06	1.620E+06	9.452E+05	9.703E+05	9.404E+05
HBP Total Time	2817	2705	2788	4339	5137	5004
MIP Restr. Begin	1.518E+06	1.637E+06	1.617E+06	9.308E+05	8.781E+05	9.140E+05
MIP Restr. Initial Gap	42.84%	34.76%	34.39%	95.17%	115.40%	103.04%
MIP Restr. Best Revenue	1.682E+06	1.852E+06	1.713E+06	9.308E+05	8.781E+05	9.140E+05
MIP Restr. Gap	22.40%	13.23%	21.05%	95.17%	115.40%	103.04%
MIP Restr. Best Bound	2.059E+06	2.097E+06	2.074E+06	1.817E+06	1.891E+06	1.856E+06
MIP Restr. Total Time	4.318E+04	4.317E+04	4.318E+04	4.300E+04	4.280E+04	4.293E+04
MIP Best Revenue	1.682E+06	1.852E+06	1.713E+06	9.308E+05	8.781E+05	9.140E+05
MIP Initial Gap	31.91%	22.61%	29.84%	110.23%	128.38%	116.92%
MIP Best Revenue	1.685E+06	1.852E+06	1.727E+06	9.308E+05	8.781E+05	9.835E+05
MIP Gap	27.49%	18.15%	24.40%	110.23%	123.81%	98.16%
MIP Best Bound	2.148E+06	2.189E+06	2.148E+06	1.957E+06	1.965E+06	1.949E+06
MIP Total Time	4.320E+04	4.320E+04	4.320E+04	4.319E+04	4.319E+04	4.319E+04
MIP-H+ Called	62	153	62	15	30	21
MIP-H+ Usefull	11	5	7	8	3	7
MIP-H+ Best Revenue	7.600E+05	1.639E+06	9.284E+05	5.872E+05	6.155E+05	7.089E+05
MIP-H+ Gap	178.73%	31.11%	128.37%	226.38%	212.38%	171.25%
MIP-H+ Best Bound	2.118E+06	2.149E+06	2.120E+06	1.917E+06	1.923E+06	1.923E+06
MIP-H+ Total Time	4.320E+04	4.320E+04	4.320E+04	2.878E+04	2.877E+04	2.878E+04
Model 5						
Naive Revenue Min	7.032E+05	5.762E+05	6.989E+05	7.519E+05	6.739E+05	7.701E+05
Naive Revenue Max	2.465E+05	5.765E+05	2.611E+05	5.162E+05	6.741E+05	5.224E+05
Naive Revenue Mean	5.565E+05	5.763E+05	5.644E+05	6.603E+05	6.740E+05	6.689E+05
HBP Best Revenue	9.019E+05	1.050E+06	9.160E+05	9.052E+05	1.014E+06	9.271E+05
HBP Total Time	2628	2673	2588	3741	4431	3712
MIP Restr. Begin	9.037E+05	1.070E+06	9.168E+05	8.650E+05	9.183E+05	8.830E+05
MIP Restr. Initial Gap	129.50%	103.63%	122.98%	110.24%	106.08%	110.33%
MIP Restr. Best Revenue	1.239E+06	1.699E+06	1.297E+06	8.650E+05	9.183E+05	8.830E+05
MIP Restr. Gap	62.39%	19.55%	51.89%	110.24%	106.08%	110.33%
MIP Restr. Best Bound	2.012E+06	2.031E+06	1.970E+06	1.819E+06	1.892E+06	1.857E+06
MIP Restr. Total Time	4.316E+04	4.317E+04	4.317E+04	4.301E+04	4.279E+04	4.296E+04
MIP Best Revenue	1.239E+06	1.699E+06	1.297E+06	8.650E+05	9.183E+05	8.830E+05
MIP Restr. Initial Gap	79.12%	33.66%	71.47%	126.24%	118.38%	124.53%
MIP Best Revenue	1.253E+06	1.877E+06	1.403E+06	1.017E+06	1.164E+06	8.830E+05
MIP Gap	68.42%	16.73%	50.50%	90.13%	69.23%	124.53%
MIP Best Bound	2.110E+06	2.191E+06	2.111E+06	1.934E+06	1.969E+06	1.983E+06
MIP Total Time	4.320E+04	4.320E+04	4.320E+04	4.319E+04	4.319E+04	4.319E+04
MIP-H+ Called	213	321	243	33	39	40
MIP-H+ Usefull	7	15	8	4	4	10
MIP-H+ Best Revenue	1.011E+06	1.602E+06	1.091E+06	8.026E+05	6.068E+05	7.845E+05
MIP-H+ Gap	107.93%	33.55%	92.24%			
	136.91%	216.60%	143.62%			
MIP-H+ Best Bound	2.102E+06	2.139E+06	2.097E+06	1.901E+06	1.921E+06	1.911E+06
MIP-H+ Total Time	4.320E+04	4.320E+04	4.320E+04	4.317E+04	4.317E+04	4.318E+04

stochasticity on price had a negative impact on revenue compared to stochasticity on demand only, the impact is less important than in the previous cases. This lower revenue is not explained by losing too many clients from group 6 as before rather because users from group 6 moved from quality of service Q5 to quality of service Q4 after the leader set its prices for the second stage. Therefore, the leader lost \$118 per client, which corresponds to a reduction of 26%. In summary, we can conclude from this example that stochasticity on price and on demand have an impact (in this case negative) on revenue, and the effect is higher when both elements are uncertain for the leader. Indeed, the leader is acting defensively trying to protect its best payers' clients, which has the cost of losing users groups or having a new distribution of groups within its inventories. However, Model 5 shows us the contrary effect.

The lowest revenue appears when the demand is stochastic and the highest revenue is obtained when price and demand are stochastic. Excluding the fact that the gaps are different compared to the Model 4, this result is explained by the fact that in Model 5 the leader has a higher maneuver to set higher prices, and thus following its defensive strategy, it prefers to set higher prices to its best payer's clients.

— **Analysis of "Example 611" :**

In this case, the competitor has a competitive advantage in terms of location. It has two inventories well located but its third hotel is very far compared to the best located hotels of the leader. In terms of capacity, the leader has a small hotel as its best located inventory but its second best located hotel has a similar capacity to the competition. Quality of service is equal to five in both cases. In this example, the highest effect of stochasticity (for Model 4) on revenue is also when price and demand are stochastic. Moreover, there is almost no difference in revenue when stochasticity is either on price or on price and demand jointly. However, the effect is less important than compared to "Example 610". In "Example 611" we only obtain a total negative impact of approximately $\$200,000$, which is mainly explained by losing approximately 4700 clients, mostly from group G3, which is a group that was not present in the most expensive rooms, and thus its impact on revenue is less dramatic. It is important to mention that in this example the revenue obtained in Model 5 is not so different from the one of Model 4 as in the previous example. The main reason is that the revenue of the leader is mainly composed by least expensive rooms and thus the higher room for manoeuvre to set prices is not made on high prices as happened in the previous example.

6.3.4 Sensitivity with respect to changes in Capacity for Large Instances

In order to test sensitivity under capacity considerations, we show a dramatic possible change, such ass local disaster. In this disaster, we can suppose a 20% loss of the total capacity for the leader and its competitors. To make sense about these changes, we use a reference that we call "regular" as the base for conducting three inventory restrictions : "20% increment", "20% decrement", and "No capacity restriction" (or infinite capacity). The last restriction is used to show how much would be the maximum revenue for the fixed capacity that the leader could perceive. The "20% decrement", is the highest reduction without making the problem unfeasible (In some cases this reduction is approximately 20%). The details of capacity reductions are presented in Table 6.27 under notation **CA** and **CB**. The test was run for five different examples, with three scenarios for every test, created in the same way than before, considering uniform random variables. Every test is done considering Model 4 and Model 5. The notation used in the descriptions are the same as Table 6.9. In order to avoid repetition, the definition of the common parameters for this sensitivity analysis is presented separately in Table 6.26. The rest of the specific definitions are presented in Table 6.27 and Table 6.28. The notations used for the column titles are the same than the ones used before.

We note that in three cases, ***Naive Revenue Min***, ***Naive Revenue Max***, ***Naive Revenue Mean*** can have a zero value. This happens because the leader's inventories are in average more distant compared to the competitor's inventories. Then, groups with high disutility penalize these "too far" inventories. The Heuristic "Naive Market Prices" would set a negative price to overcome this difference in distance but the "Non-negative constraint on Prices" corrects this and provides them a zero value.

In some instances, the Heuristic MIP-H+ was able to find an incumbent and update its lower-bound, but the solution was not updated, so this solution was not useful for our revenue and group composition analysis. In those cases, another solution was used among the best from Heuristic HBP or MIP-Restricted or MIP-Original (Model 4, instances 1001–DP, 1002–P, 1003–P, 1003–D, 1003–DP, 1004–DP, 1005–P, 1005–D, 1005–DP).

Table 6.26 Common Parameters for Capacity Sensitivity Experiment

Commons Parameters	**NA**	**QA**	**NB**	**QB**	**GR**	**LOS**	**RH**	**PrF**
Values	4	4	6	4	5	3	7	1

Table 6.27 Individual Parameters for Capacity Sensitivity Experiment : Part I

Ex. Name	DA	CA	DB	CB	Distance Value (α)	Quality Value (β)	ABS	PROP	DF	PrS
Ex. 1										
Decr_20 P	11, 64, 73, 79	40×4	10, 15, 18, 33, 49, 64	40×6	1.03, 11.09, 36.88, 0.46, 0.45	25.67, 87, 112, 120.67, 132.33	6	0.09	1	0.25, 0.33, 0.42
Decr_20 D	...	...	...	50×6	...	...	...	...	...	...
Decr_20 DP	...	...	...	...	...	...	...	...	...	...
Regular P	...	50×4	...	...	...	...	...	...	...	...
Regular D	...	...	...	...	...	...	...	...	...	...
Regular DP	...	...	...	...	...	...	...	...	...	...
Incr_20 P	...	60×4	...	60×6	...	...	...	...	...	...
Incr_20 D	...	...	...	...	...	...	...	...	...	...
Incr_20 DP	...	...	...	...	...	...	...	...	...	...
Infinit P	...	∞×4	...	∞×6	...	...	...	...	...	...
Infinit D	...	...	...	...	...	...	...	...	...	...
Infinit DP	...	...	...	...	...	...	...	...	...	...
Ex. 2										
Decr_20 P	22, 48, 64, 75	40×4	10, 17, 44, 67, 68, 74	40×6	4.80, 12.29, 1.74, 1.76, 4.50	42, 54.67, 78.33, 98.67, 108.33	22	0.21	1	0.21, 0.21, 0.58
Decr_20 D	...	...	...	50×6	...	...	...	...	...	...
Decr_20 DP	...	...	...	...	...	...	...	...	...	...
Regular P	...	50×4	...	...	...	...	...	...	...	...
Regular D	...	...	...	...	...	...	...	...	...	...
Regular DP	...	...	...	...	...	...	...	...	...	...
Incr_20 P	...	60×4	...	60×6	...	...	...	...	...	...
Incr_20 D	...	...	...	...	...	...	...	...	...	...
Incr_20 DP	...	...	...	...	...	...	...	...	...	...
Infinit P	...	∞×4	...	∞×6	...	...	...	...	...	...
Infinit D	...	...	...	...	...	...	...	...	...	...
Infinit DP	...	...	...	...	...	...	...	...	...	...

Note. The notation "m $\times n$" means that the value m is repeated n "times".

Table 6.28 Individual Parameters for Capacity Sensitivity Experiment : Part II

Ex. Name	DA	CA	DB	CB	Distance Value (α)	Quality Value (β)	ABS	PROP	DF	PrS
Ex. 3										
Decr_20 P	19, 29, 53, 66	40×4	19, 37, 59, 66, 71, 75	40×6	2.68, 5, 21.44, 1.38, 0.11	64.67, 67.33, 69.67, 92, 95.33	21	0.06	1	0.32, 0.45, 0.23
Decr_20 D	...	...	...	50×6	...	...	...	...	...	...
Decr_20 DP	...	...	...	...	...	...	...	...	...	...
Regular P	...	50×4	...	...	...	...	...	...	...	...
Regular D	...	...	...	...	...	...	...	...	...	...
Regular DP	...	...	...	...	...	...	...	...	...	...
Incr_20 P	...	60×4	...	60×6	...	...	...	...	...	...
Incr_20 D	...	...	...	...	...	...	...	...	...	...
Incr_20 DP	...	...	...	...	...	...	...	...	...	...
Infinit P	...	∞×4	...	∞×6	...	...	...	...	...	...
Infinit D	...	...	...	...	...	...	...	...	...	...
Infinit DP	...	...	...	...	...	...	...	...	...	...
Ex. 5										
Decr_20 P	11, 21, 51, 59	40×4	10, 54, 62, 65, 74, 75	40×6	1.58, 21.40, 2.75, 1.73, 11.67	43, 46.33, 55.67, 86.33, 100.00	21	0.13	1	0.01, 0.11, 0.88
Decr_20 D	...	...	...	50×6	...	...	...	...	...	...
Decr_20 DP	...	...	...	...	...	...	...	...	...	...
Regular P	...	50×4	...	...	...	...	...	...	...	...
Regular D	...	...	...	...	...	...	...	...	...	...
Regular DP	...	...	...	...	...	...	...	...	...	...
Incr_20 P	...	60×4	...	60×6	...	...	...	...	...	...
Incr_20 D	...	...	...	...	...	...	...	...	...	...
Incr_20 DP	...	...	...	...	...	...	...	...	...	...
Infinit P	...	∞×4	...	∞×6	...	...	...	...	...	...
Infinit D	...	...	...	...	...	...	...	...	...	...
Infinit DP	...	...	...	...	...	...	...	...	...	...

Note. The notation “m $\times n$” means that the value m is repeated n “times”.

Sensitivity Results

Results are presented as follows. First, we present the results related to a capacity decrease of 20%. Those results are shown in Tables 6.29 and 6.30. After that, we present the results for the regular cases, which do not behave differently to those cases presented in the "600" examples. We decided to present these results to support the previous tendencies but also to use them in our economic analysis. The results are shown in Tables 6.31 and 6.32.

Our third scenario corresponds to an increase capacity of 20%. Those results are presented in Tables 6.33 and 6.34. Our last scenario corresponds to an infinite case, which is equivalent to a scenario with no capacity constraints. The results are shown in Tables 6.35 and 6.36. In order to support our economic analysis, we also present hotel composition and revenue contribution per group (in percentages). These tables are presented for the three stochasticity options, stochasticity on price, on demand, or on price and demand simultaneously. We only present the results for Model 4 because the behaviour of the results for Model 5 is very similar. Those results are available for consultation in the appendix.

Results regarding hotel composition and revenue contribution per group are shown in Table 6.37 and 6.38 for Stochastic Demand **D**, in Table 6.39 and 6.40 for Stochastic Price **P**, and Table 6.41 and 6.42 for Stochastic Price & Demand **DP**.

In general, we could expect to have an equal repartition of groups (and revenue) when capacity goes to infinity because groups are also created uniformly and the number of leader inventories are almost the same quantity as groups. However, we will see in the economic analysis that in some cases that expectation does not occur.

Table 6.29 Examples **Decrease 20% Capacity**, Numerical Results Part I

Example	901–P	901–D	901–DP	902–P	902–D	902–DP
Model 4						
Naive Revenue Min	1.978E+05	1.821E+05	1.992E+05	2.505E+05	1.020E+05	2.563E+05
Naive Revenue Max	1.876E+05	1.821E+05	1.956E+05	1.847E+05	1.020E+05	1.071E+05
Naive Revenue Mean	1.953E+05	1.821E+05	2E+05	1.719E+05	1.020E+05	1.151E+05
HBP Best Revenue	4.499E+05	4.946E+05	4.868E+05	4.736E+05	4.997E+05	4.660E+05
HBP Total Time	1818	1629	1514	2116	1860	2048
MIP Restr. Begin	4.247E+05	4.973E+05	4.529E+05	4.611E+05	4.709E+05	4.670E+05
MIP Restr. Initial Gap	52.06%	33.11%	45.64%	45.19%	41.17%	41.17%
MIP Restr. Best Revenue	4.495E+05	5.438E+05	5.067E+05	4.611E+05	4.932E+05	4.670E+05
MIP Restr. Gap	36.29%	12.46%	19.62%	40.69%	29.95%	37.15%
MIP Restr. Best Bound	6.127E+05	6.116E+05	6.061E+05	6.487E+05	6.409E+05	6.404E+05
MIP Restr. Total Time	3600	3600	3600	3600	3600	3600
MIP Best Revenue	0	0	0	0	0	0
MIP Gap	NaN	NaN	NaN	NaN	NaN	NaN
MIP Best Bound	NaN	NaN	NaN	NaN	NaN	NaN
MIP Total Time	3600	3600	3600	3600	3600	3600
MIP-H+ Called	111	201	192	82	202	95
MIP-H+ Usefull	6	3	3	11	9	7
MIP-H+ Best Revenue	2.271E+05	3.938E+05	2.392E+05	3.426E+05	3.599E+05	2.922E+05
MIP-H+ Gap	172.74%	59.50%	161.46%	91.14%	76.80%	118.03%
MIP-H+ Best Bound	6.194E+05	6.282E+05	6.254E+05	6.548E+05	6.363E+05	6.370E+05
MIP-H+ Total Time	3599	3605	3616	3600	3600	3600
Model 5						
Naive Revenue Min	1.978E+05	1.822E+05	1.992E+05	2.505E+05	1.020E+05	2.563E+05
Naive Revenue Max	1.873E+05	1.822E+05	1.954E+05	1.847E+05	1.020E+05	1.071E+05
Naive Revenue Mean	1.952E+05	1.822E+05	1.989E+05	1.719E+05	1.020E+05	1.151E+05
HBP Best Revenue	2.552E+05	3.206E+05	4.838E+05	2.731E+05	2.975E+05	3.026E+05
HBP Total Time	1892	1740	1853	1892	1722	1565
MIP Restr. Begin	2.270E+05	3.279E+05	4.660E+05	2.639E+05	2.715E+05	2.641E+05
MIP Restr. Initial Gap	184.70%	102.15%	41.75%	131.38%	140.03%	133.83%
MIP Restr. Best Revenue	2.700E+05	5.370E+05	4.994E+05	3.898E+05	4.068E+05	3.515E+05
MIP Restr. Gap	127.28%	15.77%	24.05%	48.05%	53.74%	62.31%
MIP Restr. Best Bound	6.137E+05	6.217E+05	6.195E+05	5.770E+05	6.255E+05	5.705E+05
MIP Restr. Total Time	3600	3600	3600	2390	2356	954
MIP Best Revenue	0	0	0	0	0	0
MIP Gap	NaN	NaN	NaN	NaN	NaN	NaN
MIP Best Bound	NaN	NaN	NaN	NaN	NaN	NaN
MIP Total Time	3600	3600	3600	3600	3600	3600
MIP-H+ Called	75	179	210	128	178	171
MIP-H+ Usefull	3	2	18	17	10	5
MIP-H+ Best Revenue	2.350E+05	5.201E+05	2.318E+05	3.687E+05	3.135E+05	3.266E+05
MIP-H+ Gap	163.74%	20.79%	170.54%	76.61%	104.18%	95.56%
MIP-H+ Best Bound	6.198E+05	6.282E+05	6.271E+05	6.512E+05	6.401E+05	6.386E+05
MIP-H+ Total Time	3605	3618	3601	3600	3600	3600

Table 6.30 Examples **Decrease 20% Capacity**, Numerical Results Part II

Example	903–P	903–D	903–DP	905–P	905–D	905–DP
Model 4						
Naive Revenue Min	3.672E+05	2.055E+05	3.573E+05	2.955E+05	2.488E+05	2.321E+05
Naive Revenue Max	2.289E+05	2.055E+05	2.051E+05	3.306E+05	2.488E+05	2.648E+05
Naive Revenue Mean	2.415E+05	2.055E+05	2.127E+05	3.109E+05	2.488E+05	2.491E+05
HBP Best Revenue	4.672E+05	6.433E+05	6.324E+05	4.765E+05	4.498E+05	4.305E+05
HBP Total Time	2046	2026	1916	1546	1300	1363
MIP Restr. Begin	4.672E+05	6.083E+05	5.876E+05	5.970E+05	5.607E+05	4.279E+05
MIP Restr. Initial Gap	68.21%	36.82%	39.87%	68.56%	63.37%	108.52%
MIP Restr. Best Revenue	4.808E+05	6.605E+05	5.876E+05	7.195E+05	5.776E+05	5.637E+05
MIP Restr. Gap	60.33%	23.17%	36.83%	34.32%	44.42%	45.50%
MIP Restr. Best Bound	7.708E+05	8.136E+05	8.039E+05	9.665E+05	8.342E+05	8.202E+05
MIP Restr. Total Time	3600	3601	3600	3600	3600	3600
MIP Best Revenue	0	0	0	0	0	0
MIP Gap	NaN	NaN	NaN	NaN	NaN	NaN
MIP Best Bound	NaN	NaN	NaN	NaN	NaN	NaN
MIP Total Time	3600	3600	3600	3600	3600	3600
MIP-H+ Called	153	278	223	35	63	75
MIP-H+ Usefull	14	84	38	10	41	3
MIP-H+ Best Revenue	2.090E+05	3.486E+05	3.425E+05	3.197E+05	2.301E+05	3.108E+05
MIP-H+ Gap	292.95%	134.10%	135.06%	231.83%	329.88%	215.59%
MIP-H+ Best Bound	8.211E+05	8.161E+05	8.051E+05	1.061E+06	9.892E+05	9.807E+05
MIP-H+ Total Time	3600	3600	3600	3567	3600	3591
Model 5						
Naive Revenue Min	3.672E+05	2.055E+05	3.573E+05	2.955E+05	2.488E+05	2.321E+05
Naive Revenue Max	2.289E+05	2.055E+05	2.051E+05	3.306E+05	2.488E+05	2.648E+05
Naive Revenue Mean	2.415E+05	2.055E+05	2.127E+05	3.109E+05	2.488E+05	2.491E+05
HBP Best Revenue	4.383E+05	4.709E+05	4.943E+05	4.760E+05	4.523E+05	4.193E+05
HBP Total Time	1805	1819	1527	1531	1470	1601
MIP Restr. Begin	4.333E+05	4.436E+05	4.742E+05	6.131E+05	5.804E+05	4.387E+05
MIP Restr. Initial Gap	81.86%	84.48%	71.19%	57.61%	57.83%	103.73%
MIP Restr. Best Revenue	4.333E+05	6.692E+05	4.742E+05	7.385E+05	6.638E+05	5.559E+05
MIP Restr. Gap	78.15%	19.02%	67.15%	23.54%	25.42%	47.55%
MIP Restr. Best Bound	7.720E+05	7.965E+05	7.927E+05	9.123E+05	8.326E+05	8.203E+05
MIP Restr. Total Time	3600	3600	3600	3601	3273	3600
MIP Best Revenue	0	0	0	0	0	0
MIP Gap	NaN	NaN	NaN	NaN	NaN	NaN
MIP Best Bound	NaN	NaN	NaN	NaN	NaN	NaN
MIP Total Time	3600	3600	3600	3600	3600	3600
MIP-H+ Called	234	295	324	45	43	45
MIP-H+ Usefull	13	72	42	10	17	5
MIP-H+ Best Revenue	2.680E+05	3.392E+05	3.475E+05	3.364E+05	2.603E+05	3.514E+05
MIP-H+ Gap	206.47%	139.73%	131.83%	213.79%	281.41%	178.54%
MIP-H+ Best Bound	8.213E+05	8.131E+05	8.056E+05	1.055E+06	9.929E+05	9.787E+05
MIP-H+ Total Time	3600	3600	3600	3600	3593	3592

Table 6.31 Examples **Regular Capacity**, Numerical Results Part I

Example	**701–P**	**701–D**	**701–DP**	**702–P**	**702–D**	**702–DP**
Model 4						
Naive Revenue Min	2.088E+05	1.922E+05	2.171E+05	2.799E+05	1.062E+05	2.943E+05
Naive Revenue Max	1.956E+05	1.922E+05	2.019E+05	8.623E+04	1.063E+05	1.071E+05
Naive Revenue Mean	2.027E+05	1.922E+05	2.111E+05	9.716E+04	1.063E+05	1.199E+05
HBP Best Revenue	5.196E+05	5.608E+05	5.264E+05	5.059E+05	5.361E+05	5.102E+05
HBP Total Time	1412	1586	1415	1851	1952	1624
MIP Restr. Begin	4.646E+05	5.750E+05	4.766E+05	4.882E+05	5.237E+05	4.867E+05
MIP Restr. Initial Gap	49.15%	26.03%	51.75%	45.51%	38.99%	48.53%
MIP Restr. Best Revenue	5.372E+05	5.924E+05	5.599E+05	5.360E+05	5.913E+05	4.867E+05
MIP Restr. Gap	18.19%	13.24%	19.74%	26.59%	18.99%	40.58%
MIP Restr. Best Bound	6.350E+05	6.708E+05	6.704E+05	6.785E+05	7.036E+05	6.841E+05
MIP Restr. Total Time	3600	3600	3600	3602	3600	3590
MIP Best Revenue	0	0	0	0	5.913E+05	0
MIP Gap	NaN	NaN	NaN	NaN	20.43%	NaN
MIP Best Bound	NaN	NaN	NaN	NaN	7.121E+05	NaN
MIP Total Time	3600	3600	3600	3600	3600	3600
MIP-H+ Called	151	303	196	40	104	64
MIP-H+ Usefull	56	3	7	4	4	5
MIP-H+ Best Revenue	2.671E+05	3.395E+05	2.731E+05	2.854E+05	3.173E+05	2.980E+05
MIP-H+ Gap	147.28%	103.18%	151.60%	138.36%	120.33%	134.60%
MIP-H+ Best Bound	6.604E+05	6.899E+05	6.872E+05	6.803E+05	6.991E+05	6.992E+05
MIP-H+ Total Time	3653	3647	3635	3600	3630	3600
Model 5						
Naive Revenue Min	2.088E+05	1.922E+05	2.171E+05	2.799E+05	1.062E+05	2.943E+05
Naive Revenue Max	1.954E+05	1.922E+05	2.017E+05	8.623E+04	1.063E+05	1.071E+05
Naive Revenue Mean	2.027E+05	1.922E+05	2.103E+05	9.716E+04	1.063E+05	1.199E+05
HBP Best Revenue	5.315E+05	3.229E+05	5.344E+05	2.999E+05	3.153E+05	3.232E+05
HBP Total Time	1750	1550	1505	1876	1879	1921
MIP Restr. Begin	4.809E+05	2.992E+05	5.007E+05	2.681E+05	2.875E+05	2.979E+05
MIP Restr. Initial Gap	44.29%	142.50%	44.64%	148.53%	145.29%	121.32%
MIP Restr. Best Revenue	5.591E+05	5.928E+05	5.658E+05	3.805E+05	5.327E+05	3.767E+05
MIP Restr. Gap	15.73%	15.72%	20.18%	63.46%	25.82%	63.02%
MIP Restr. Best Bound	6.471E+05	6.860E+05	6.800E+05	6.219E+05	6.703E+05	6.141E+05
MIP Restr. Total Time	3600	3600	3600	3600	3600	3375
MIP Best Revenue	3.253E+05	0	0	0	5.455E+05	0
MIP Gap	104.55%	NaN	NaN	NaN	28.91%	NaN
MIP Best Bound	6.653E+05	NaN	NaN	NaN	7.031E+05	NaN
MIP Total Time	3600	3600	3600	3600	3599	3600
MIP-H+ Called	161	181	156	65	252	85
MIP-H+ Usefull	4	5	4	6	6	11
MIP-H+ Best Revenue	2.702E+05	5.537E+05	2.993E+05	3.300E+05	4.267E+05	3.505E+05
MIP-H+ Gap	145.98%	25.08%	131.00%	106.39%	64.45%	98.17%
MIP-H+ Best Bound	6.645E+05	6.925E+05	6.913E+05	6.810E+05	7.016E+05	6.946E+05
MIP-H+ Total Time	3696	3613	3680	3600	3628	3600

Table 6.32 Examples **Regular Capacity**, Numerical Results Part II

Example	703–P	703–D	703–DP	705–P	705–D	705–DP
Model 4						
Naive Revenue Min	3.854E+05	2.465E+05	3.893E+05	2.181E+05	2.491E+05	2.324E+05
Naive Revenue Max	2.247E+05	2.465E+05	2.395E+05	2.488E+05	2.491E+05	2.650E+05
Naive Revenue Mean	2.415E+05	2.465E+05	2.573E+05	2.342E+05	2.491E+05	2.495E+05
HBP Best Revenue	6.701E+05	7.155E+05	7.161E+05	6.561E+05	4.991E+05	4.666E+05
HBP Total Time	1467	1730	1598	1475	1442	1612
MIP Restr. Begin	6.103E+05	6.826E+05	6.940E+05	6.454E+05	4.932E+05	4.618E+05
MIP Restr. Initial Gap	48.01%	40.83%	34.87%	38.10%	99.42%	107.96%
MIP Restr. Best Revenue	6.103E+05	7.221E+05	7.194E+05	6.636E+05	5.068E+05	4.720E+05
MIP Restr. Gap	43.54%	28.00%	25.72%	25.79%	74.44%	85.83%
MIP Restr. Best Bound	8.760E+05	9.243E+05	9.044E+05	8.347E+05	8.841E+05	8.771E+05
MIP Restr. Total Time	3600	3600	3600	3600	3600	3600
MIP Best Revenue	0	0	0	0	0	0
MIP Gap	NaN	NaN	NaN	NaN	NaN	NaN
MIP Best Bound	NaN	NaN	NaN	NaN	NaN	NaN
MIP Total Time	3600	3600	3600	3600	3600	3600
MIP-H+ Called	379	342	362	51	52	89
MIP-H+ Usefull	12	7	11	6	31	31
MIP-H+ Best Revenue	6.703E+05	6.266E+05	5.697E+05	5.057E+05	5.721E+05	4.172E+05
MIP-H+ Gap	33.12%	48.38%	62.41%	91.95%	81.85%	145.02%
MIP-H+ Best Bound	8.923E+05	9.297E+05	9.252E+05	9.706E+05	1.040E+06	1.022E+06
MIP-H+ Total Time	3600	3600	3600	3595	3600	3600
Model 5						
Naive Revenue Min	3.854E+05	2.465E+05	3.893E+05	2.181E+05	2.491E+05	2.324E+05
Naive Revenue Max	2.247E+05	2.465E+05	2.395E+05	2.488E+05	2.491E+05	2.650E+05
Naive Revenue Mean	2.415E+05	2.465E+05	2.573E+05	2.342E+05	2.491E+05	2.495E+05
HBP Best Revenue	6.427E+05	6.021E+05	6.791E+05	5.682E+05	5.127E+05	4.832E+05
HBP Total Time	1770	1542	1732	1659	1711	1836
MIP Restr. Begin	6.359E+05	5.662E+05	6.140E+05	NaN	5.156E+05	4.565E+05
MIP Restr. Initial Gap	36.49%	59.88%	50.65%	NaN	87.95%	109.28%
MIP Restr. Best Revenue	6.517E+05	6.639E+05	7.093E+05	0	7.572E+05	4.854E+05
MIP Restr. Gap	29.64%	31.75%	26.62%	NaN	17.54%	79.13%
MIP Restr. Best Bound	8.449E+05	8.747E+05	8.981E+05	NaN	8.901E+05	8.695E+05
MIP Restr. Total Time	3601	3600	3600	3600	3600	3600
MIP Best Revenue	0	0	0	0	0	0
MIP Gap	NaN	NaN	NaN	NaN	NaN	NaN
MIP Best Bound	NaN	NaN	NaN	NaN	NaN	NaN
MIP Total Time	3600	3600	3600	3600	3600	3600
MIP-H+ Called	262	347	349	48	151	88
MIP-H+ Usefull	15	54	7	11	3	15
MIP-H+ Best Revenue	5.751E+05	5.337E+05	5.604E+05	5.323E+05	5.735E+05	4.975E+05
MIP-H+ Gap	54.22%	74.84%	64.97%	82.49%	79.44%	105.01%
MIP-H+ Best Bound	8.869E+05	9.332E+05	9.244E+05	9.714E+05	1.029E+06	1.020E+06
MIP-H+ Total Time	3600	3600	3600	3600	3596	3600

Table 6.33 Examples **Increase Capacity in 20%**, Numerical Results Part I

Example	1001–P	1001–D	1001–DP	1002–P	1002–D	1002–DP
Model 4						
Naive Revenue Min	2.171E+05	2.044E+05	2.263E+05	3.136E+05	4.307E+04	3.232E+05
Naive Revenue Max	2.097E+05	2.044E+05	2.096E+05	2.215E+04	4.309E+04	2.511E+04
Naive Revenue Mean	2.126E+05	2.044E+05	2.171E+05	4.853E+04	4.308E+04	6.882E+04
HBP Best Revenue	5.481E+05	5.831E+05	5.700E+05	5.523E+05	5.731E+05	5.463E+05
HBP Total Time	1674	1255	1645	1689	1552	1534
MIP Restr. Begin	5.151E+05	5.917E+05	5.561E+05	5.161E+05	5.351E+05	5.280E+05
MIP Restr. Initial Gap	45.89%	30.27%	39.03%	43.33%	42.90%	43.36%
MIP Restr. Best Revenue	5.824E+05	6.265E+05	5.997E+05	5.552E+05	5.907E+05	5.562E+05
MIP Restr. Gap	15.84%	12.77%	17.32%	20.60%	16.61%	24.17%
MIP Restr. Best Bound	6.747E+05	7.065E+05	7.035E+05	6.695E+05	6.888E+05	6.907E+05
MIP Restr. Total Time	3600	3600	3600	3600	3600	850
MIP Best Revenue	0	0	0	4.086E+05	4.484E+05	0
MIP Gap	NaN	NaN	NaN	69.38%	62.64%	NaN
MIP Best Bound	NaN	NaN	NaN	6.921E+05	7.293E+05	NaN
MIP Total Time	3600	3600	3600	3600	3600	3600
MIP-H+ Called	138	233	202	58	152	119
MIP-H+ Usefull	102	22	139	3	1	2
MIP-H+ Best Revenue	2.664E+05	4.029E+05	2.878E+05	5.096E+05	5.418E+05	3.402E+05
MIP-H+ Gap	155.44%	80.10%	150.96%	34.59%	31.11%	112.53%
MIP-H+ Best Bound	6.805E+05	7.256E+05	7.222E+05	6.859E+05	7.103E+05	7.231E+05
MIP-H+ Total Time	3611	3612	3621	3599	3628	3600
Model 5						
Naive Revenue Min	2.171E+05	2.044E+05	2.263E+05	3.136E+05	4.307E+04	3.232E+05
Naive Revenue Max	2.095E+05	2.044E+05	2.094E+05	2.215E+04	4.309E+04	2.511E+04
Naive Revenue Mean	2.126E+05	2.044E+05	2.171E+05	4.853E+04	4.308E+04	6.882E+04
HBP Best Revenue	5.670E+05	6.011E+05	5.784E+05	2.900E+05	2.844E+05	3.107E+05
HBP Total Time	1543	1342	1574	1505	1658	1557
MIP Restr. Begin	5.407E+05	6.021E+05	5.827E+05	2.688E+05	2.613E+05	2.814E+05
MIP Restr. Initial Gap	39.17%	28.16%	32.78%	149.57%	169.87%	151.29%
MIP Restr. Best Revenue	5.962E+05	6.514E+05	6.109E+05	4.553E+05	6.253E+05	3.471E+05
MIP Restr. Gap	16.60%	10.19%	16.99%	37.67%	5.05%	88.92%
MIP Restr. Best Bound	6.951E+05	7.178E+05	7.147E+05	6.268E+05	6.569E+05	6.557E+05
MIP Restr. Total Time	3600	3600	3600	3602	3600	1431
MIP Best Revenue	0	0	0	5.777E+05	6.364E+05	0
MIP Gap	NaN	NaN	NaN	18.86%	12.89%	NaN
MIP Best Bound	NaN	NaN	NaN	6.866E+05	7.184E+05	NaN
MIP Total Time	3600	3600	3600	3600	3600	3600
MIP-H+ Called	119	318	92	61	301	142
MIP-H+ Usefull	87	4	7	4	1	9
MIP-H+ Best Revenue	2.687E+05	5.448E+05	2.861E+05	5.358E+05	5.928E+05	4.063E+05
MIP-H+ Gap	148.23%	24.38%	154.91%	28.86%	18.99%	74.92%
MIP-H+ Best Bound	6.669E+05	6.776E+05	7.292E+05	6.904E+05	7.053E+05	7.107E+05
MIP-H+ Total Time	3616	3620	3604	3602	3655	3594

Table 6.34 Examples **Increase Capacity in 20%**, Numerical Results Part II

Example	**1003–P**	**1003–D**	**1003–DP**	**1005–P**	**1005–D**	**1005–DP**
Model 4						
Naive Revenue Min	3.815E+05	2.493E+05	3.826E+05	1.484E+05	1.719E+05	1.618E+05
Naive Revenue Max	2.193E+05	2.493E+05	2.404E+05	1.666E+05	1.719E+05	1.829E+05
Naive Revenue Mean	2.418E+05	2.493E+05	2.656E+05	1.569E+05	1.719E+05	1.722E+05
HBP Best Revenue	6.819E+05	6.712E+05	7.138E+05	5.286E+05	4.545E+05	4.344E+05
HBP Total Time	1708	1732	1603	1442	1486	1529
MIP Restr. Begin	6.752E+05	6.343E+05	6.205E+05	5.239E+05	5.370E+05	4.038E+05
MIP Restr. Initial Gap	37.65%	54.57%	57.05%	55.03%	56.92%	121.37%
MIP Restr. Best Revenue	6.909E+05	6.856E+05	7.318E+05	5.466E+05	6.081E+05	5.384E+05
MIP Restr. Gap	30.08%	36.20%	28.37%	35.04%	23.31%	49.59%
MIP Restr. Best Bound	8.988E+05	9.338E+05	9.394E+05	7.381E+05	7.498E+05	8.055E+05
MIP Restr. Total Time	3600	3600	3600	3600	3600	3600
MIP Best Revenue	0	8.301E+05	0	0	7.434E+05	0
MIP Gap	NaN	16.54%	NaN	NaN	37.47%	NaN
MIP Best Bound	NaN	9.674E+05	NaN	NaN	1.022E+06	NaN
MIP Total Time	3600	3589	3600	3600	3597	3600
MIP-H+ Called	234	299	350	189	193	188
MIP-H+ Usefull	2	7	15	26	31	43
MIP-H+ Best Revenue	5.814E+05	6.798E+05	6.805E+05	5.058E+05	6.072E+05	5.527E+05
MIP-H+ Gap	55.53%	40.71%	38.17%	84.80%	59.82%	78.63%
MIP-H+ Best Bound	9.043E+05	9.566E+05	9.402E+05	9.348E+05	9.705E+05	9.872E+05
MIP-H+ Total Time	3600	3603	3600	3595	3600	3596
Model 5						
Naive Revenue Min	3.815E+05	2.493E+05	3.826E+05	1.484E+05	1.719E+05	1.618E+05
Naive Revenue Max	2.193E+05	2.493E+05	2.404E+05	1.666E+05	1.719E+05	1.829E+05
Naive Revenue Mean	2.418E+05	2.493E+05	2.656E+05	1.569E+05	1.719E+05	1.722E+05
HBP Best Revenue	6.762E+05	5.777E+05	5.275E+05	4.173E+05	4.381E+05	4.198E+05
HBP Total Time	1500	1504	1355	1218	1425	1413
MIP Restr. Begin	6.637E+05	5.480E+05	5.040E+05	NaN	4.492E+05	4.224E+05
MIP Restr. Initial Gap	40.05%	75.43%	84.43%	NaN	83.06%	111.36%
MIP Restr. Best Revenue	6.977E+05	6.671E+05	5.621E+05	0	6.125E+05	5.770E+05
MIP Restr. Gap	27.81%	38.58%	58.04%	NaN	15.99%	39.71%
MIP Restr. Best Bound	8.917E+05	9.244E+05	8.884E+05	NaN	7.104E+05	8.061E+05
MIP Restr. Total Time	3600	1729	3600	3600	2779	3600
MIP Best Revenue	0	0	0	0	0	0
MIP Gap	NaN	NaN	NaN	NaN	NaN	NaN
MIP Best Bound	NaN	NaN	NaN	NaN	NaN	NaN
MIP Total Time	3600	3600	3600	3600	3600	3600
MIP-H+ Called	155	351	345	127	274	252
MIP-H+ Usefull	2	69	7	25	24	19
MIP-H+ Best Revenue	5.184E+05	6.464E+05	6.401E+05	5.212E+05	6.211E+05	4.204E+05
MIP-H+ Gap	74.34%	47.49%	47.22%	79.83%	57.19%	133.30%
MIP-H+ Best Bound	9.037E+05	9.533E+05	9.424E+05	9.373E+05	9.764E+05	9.808E+05
MIP-H+ Total Time	3600	3596	3600	3603	3598	3595

Table 6.35 Examples, **Infinite Capacity**, Numerical Results Part I

Example	801–P	801–D	801–DP	802–P	802–D	802–DP
Model 4						
Naive Revenue Min	2.399E+05	2.451E+05	2.478E+05	2.956E+05	1.827E+04	3.014E+05
Naive Revenue Max	2.374E+05	2.451E+05	2.451E+05	0	1.829E+04	0
Naive Revenue Mean	2.374E+05	2.451E+05	2.451E+05	2.321E+04	1.828E+04	2.229E+04
HBP Best Revenue	6.634E+05	6.856E+05	7.054E+05	6.310E+05	7.225E+05	6.787E+05
HBP Total Time	1248	1216	1330	1512	1326	1324
MIP Restr. Begin	6.551E+05	7.044E+05	6.501E+05	6.410E+05	7.328E+05	7.115E+05
MIP Restr. Initial Gap	38.01%	34.39%	44.96%	41.52%	29.27%	31.25%
MIP Restr. Best Revenue	6.921E+05	7.632E+05	7.202E+05	7.027E+05	7.328E+05	7.294E+05
MIP Restr. Gap	3.13%	0.01%	0.95%	11.51%	11.32%	7.67%
MIP Restr. Best Bound	7.138E+05	7.633E+05	7.271E+05	7.836E+05	8.157E+05	7.853E+05
MIP Restr. Total Time	3600	923	3600	3600	3600	3600
MIP Best Revenue	6.921E+05	7.632E+05	7.202E+05	7.027E+05	7.328E+05	7.294E+05
MIP Gap	3.93%	0.01%	5.61%	7.16%	11.10%	9.53%
MIP Best Bound	7.193E+05	7.633E+05	7.606E+05	7.530E+05	8.141E+05	7.989E+05
MIP Total Time	3600	426	3599	3600	3599	3600
MIP-H+ Called	308	417	404	169	311	212
MIP-H+ Usefull	1	3	0	4	1	0
MIP-H+ Best Revenue	6.921E+05	7.633E+05	7.202E+05	6.995E+05	7.328E+05	7.294E+05
MIP-H+ Gap	16.77%	10.41%	16.02%	9.17%	4.93%	7.64%
MIP-H+ Best Bound	8.082E+05	8.427E+05	8.356E+05	7.637E+05	7.689E+05	7.851E+05
MIP-H+ Total Time	3615	3775	3830	3599	3603	3603
Model 5						
Naive Revenue Min	2.399E+05	2.451E+05	2.478E+05	2.956E+05	1.827E+04	3.014E+05
Naive Revenue Max	2.374E+05	2.451E+05	2.451E+05	0	1.829E+04	0
Naive Revenue Mean	2.374E+05	2.451E+05	2.451E+05	2.321E+04	1.828E+04	2.229E+04
HBP Best Revenue	6.653E+05	7.058E+05	6.773E+05	4.283E+05	6.546E+05	4.764E+05
HBP Total Time	1313	1162	1181	1182	1224	1225
MIP Restr. Begin	6.799E+05	7.164E+05	6.964E+05	3.890E+05	5.749E+05	3.770E+05
MIP Restr. Initial Gap	34.84%	34.18%	37.27%	87.51%	41.10%	103.03%
MIP Restr. Best Revenue	7.044E+05	7.875E+05	7.212E+05	6.181E+05	7.233E+05	6.425E+05
MIP Restr. Gap	12.18%	0.01%	6.72%	1.38%	0.01%	1.46%
MIP Restr. Best Bound	7.902E+05	7.876E+05	7.696E+05	6.266E+05	7.234E+05	6.519E+05
MIP Restr. Total Time	3600	1273	3600	3600	164	3600
MIP Best Revenue	7.044E+05	7.875E+05	7.212E+05	7.027E+05	7.519E+05	7.294E+05
MIP Gap	11.38%	0.01%	13.35%	8.17%	2.84%	7.00%
MIP Best Bound	7.846E+05	7.876E+05	8.174E+05	7.601E+05	7.732E+05	7.804E+05
MIP Total Time	3600	327	3600	3600	3600	3600
MIP-H+ Called	257	478	356	168	338	190
MIP-H+ Usefull	2	4	2	5	2	1
MIP-H+ Best Revenue	7.033E+05	7.875E+05	7.211E+05	7.027E+05	7.519E+05	7.294E+05
MIP-H+ Gap	16.54%	7.81%	17.06%	9.41%	3.09%	7.84%
MIP-H+ Best Bound	8.196E+05	8.491E+05	8.441E+05	7.688E+05	7.752E+05	7.866E+05
MIP-H+ Total Time	3952	3731	3651	3600	3601	3600

Table 6.36 Examples, **Infinite Capacity**, Numerical Results Part II

Example	803–P	803–D	803–DP	805–P	805–D	805–DP
Model 4						
Naive Revenue Min	3.180E+05	2.381E+05	3.180E+05	0	0	0
Naive Revenue Max	2.538E+05	2.381E+05	2.538E+05	0	0	0
Naive Revenue Mean	2.359E+05	2.381E+05	2.359E+05	0	0	0
HBP Best Revenue	6.955E+05	7.936E+05	7.172E+05	8.233E+05	9.147E+05	8.519E+05
HBP Total Time	1470	1509	1397	1199	1047	1223
MIP Restr. Begin	6.946E+05	7.459E+05	7.757E+05	8.055E+05	9.230E+05	8.452E+05
MIP Restr. Initial Gap	39.44%	36.77%	29.91%	30.25%	17.30%	28.33%
MIP Restr. Best Revenue	7.402E+05	8.846E+05	7.781E+05	8.756E+05	9.400E+05	9.062E+05
MIP Restr. Gap	9.92%	1.32%	10.17%	6.78%	6.23%	1.83%
MIP Restr. Best Bound	8.136E+05	8.963E+05	8.572E+05	9.350E+05	9.985E+05	9.228E+05
MIP Restr. Total Time	3600	3600	3600	3600	3600	3600
MIP Best Revenue	7.413E+05	8.846E+05	7.749E+05	8.761E+05	9.400E+05	9.069E+05
MIP Gap	13.91%	3.33%	9.95%	4.02%	4.79%	3.23%
MIP Best Bound	8.444E+05	9.141E+05	8.521E+05	9.113E+05	9.851E+05	9.361E+05
MIP Total Time	3600	3600	3599	3600	3600	3600
MIP-H+ Called	85	288	74	173	368	262
MIP-H+ Usefull	2	1	1	4	2	3
MIP-H+ Best Revenue	7.452E+05	8.385E+05	7.574E+05	8.708E+05	9.438E+05	8.945E+05
MIP-H+ Gap	17.75%	9.31%	20.29%	5.78%	6.20%	6.36%
MIP-H+ Best Bound	8.774E+05	9.167E+05	9.110E+05	9.211E+05	1.002E+06	9.514E+05
MIP-H+ Total Time	3599	3602	3600	3605	3601	3602
Model 5						
Naive Revenue Min	3.180E+05	2.381E+05	3.180E+05	0	0	0
Naive Revenue Max	2.538E+05	2.381E+05	2.538E+05	0	0	0
Naive Revenue Mean	2.359E+05	2.381E+05	2.359E+05	0	0	0
HBP Best Revenue	6.856E+05	7.455E+05	7.162E+05	7.254E+05	8.718E+05	7.416E+05
HBP Total Time	1355	1158	1323	1354	1105	1197
MIP Restr. Begin	7.019E+05	7.218E+05	7.333E+05	6.540E+05	7.872E+05	6.703E+05
MIP Restr. Initial Gap	38.72%	41.52%	37.96%	43.47%	35.00%	44.77%
MIP Restr. Best Revenue	7.262E+05	8.733E+05	7.571E+05	8.615E+05	9.700E+05	8.847E+05
MIP Restr. Gap	14.45%	3.52%	12.95%	0.00%	0.69%	0.79%
MIP Restr. Best Bound	8.311E+05	9.041E+05	8.551E+05	8.615E+05	9.767E+05	8.916E+05
MIP Restr. Total Time	3600	3600	3600	2693	3600	3600
MIP Best Revenue	7.262E+05	8.733E+05	7.571E+05	8.967E+05	9.675E+05	9.284E+05
MIP Gap	13.35%	3.04%	14.01%	5.39%	2.90%	3.21%
MIP Best Bound	8.231E+05	8.999E+05	8.632E+05	9.450E+05	9.956E+05	9.582E+05
MIP Total Time	3599	3600	3599	3600	3600	3600
MIP-H+ Called	61	189	72	243	391	320
MIP-H+ Usefull	1	0	1	4	1	6
MIP-H+ Best Revenue	7.427E+05	8.704E+05	7.580E+05	8.812E+05	9.667E+05	9.227E+05
MIP-H+ Gap	18.98%	4.12%	19.33%	6.76%	4.00%	4.81%
MIP-H+ Best Bound	8.836E+05	9.062E+05	9.046E+05	9.408E+05	1.005E+06	9.671E+05
MIP-H+ Total Time	3599	3600	3600	3601	3600	3600

Table 6.37 Summary Examples : **Stochastic Demand** in **Model 4**, Group % in Hotel

Example 1	**D Revenue**	**Group 1**	**Group 2**	**Group 3**	**Group 4**	**Group 5**
Decr_20	5.440E+05	29.4%	16.5%	13.4%	10.0%	30.7%
Regular	5.920E+05	27.5%	18.5%	16.4%	10.1%	27.6%
Incr_20	6.270E+05	27.1%	21.1%	19.9%	14.3%	17.7%
Infinit	7.630E+05	18.3%	18.0%	21.2%	19.5%	23.0%

Example 2	D Revenue	Group 1	Group 2	Group 3	Group 4	Group 5
Decr_20	4.930E+05	27.9%	1.9%	23.6%	27.8%	18.6%
Regular	5.910E+05	24.7%	0.0%	22.4%	26.4%	26.6%
Incr_20	5.910E+05	23.9%	0.2%	25.6%	30.1%	20.3%
Infinit	7.330E+05	25.9%	6.1%	20.2%	23.7%	24.1%

Example 3	D Revenue	Group 1	Group 2	Group 3	Group 4	Group 5
Decr_20	6.610E+05	26.9%	19.2%	18.8%	27.6%	7.5%
Regular	7.220E+05	29.6%	16.5%	15.0%	26.8%	12.2%
Incr_20	8.300E+05	19.7%	18.6%	19.7%	19.1%	23.0%
Infinit	8.850E+05	19.2%	18.2%	19.7%	18.6%	24.3%

Example 5	D Revenue	Group 1	Group 2	Group 3	Group 4	Group 5
Decr_20	5.780E+05	0.0%	4.0%	5.1%	49.1%	41.7%
Regular	5.070E+05	0.0%	2.3%	24.2%	40.8%	32.8%
Incr_20	7.430E+05	21.2%	17.8%	20.0%	20.3%	20.7%
Infinit	9.400E+05	16.0%	22.1%	20.3%	20.6%	21.0%

Table 6.38 Summary Examples : **Stochastic Demand** in **Model 4**, Group Revenue contribution

Example 1	**D Revenue**	**Group 1**	**Group 2**	**Group 3**	**Group 4**	**Group 5**
Decr_20	5.440E+05	7.1%	15.9%	27.6%	17.1%	32.3%
Regular	5.920E+05	6.4%	19.7%	28.3%	16.4%	29.1%
Incr_20	6.270E+05	3.9%	24.2%	30.0%	18.8%	23.1%
Infinit	7.630E+05	6.9%	29.2%	33.3%	14.3%	16.4%

Example 2	D Revenue	Group 1	Group 2	Group 3	Group 4	Group 5
Decr_20	4.930E+05	21.3%	0.7%	20.6%	29.4%	27.9%
Regular	5.910E+05	22.1%	0.0%	20.6%	25.4%	31.8%
Incr_20	5.910E+05	13.8%	0.1%	24.9%	30.2%	31.0%
Infinit	7.330E+05	23.5%	11.1%	19.6%	22.5%	23.2%

Example 3	D Revenue	Group 1	Group 2	Group 3	Group 4	Group 5
Decr_20	6.610E+05	23.8%	23.1%	23.9%	23.1%	6.1%
Regular	7.220E+05	24.5%	19.6%	22.2%	20.0%	13.7%
Incr_20	8.300E+05	18.4%	21.7%	25.5%	16.5%	17.9%
Infinit	8.850E+05	22.3%	21.7%	23.8%	14.4%	17.9%

Example 5	D Revenue	Group 1	Group 2	Group 3	Group 4	Group 5
Decr_20	5.780E+05	0.0%	4.3%	2.3%	33.5%	60.0%
Regular	5.070E+05	0.0%	7.1%	24.3%	38.8%	29.8%
Incr_20	7.430E+05	15.5%	27.2%	18.4%	15.4%	23.5%
Infinit	9.400E+05	10.1%	24.3%	21.3%	22.6%	21.6%

Table 6.39 Summary Examples : **Stochastic Price** in **Model 4**, Group % in Hotel

Example 1	**P Revenue**	**Group 1**	**Group 2**	**Group 3**	**Group 4**	**Group 5**
Decr_20	4.500E+05	28.7%	10.0%	21.6%	16.9%	22.8%
Regular	5.370E+05	29.3%	14.2%	25.8%	14.3%	16.4%
Incr_20	5.820E+05	27.3%	17.4%	27.4%	13.0%	14.8%
Infinit	6.920E+05	23.4%	30.2%	30.9%	7.5%	8.0%
Example 2	P Revenue	Group 1	Group 2	Group 3	Group 4	Group 5
Decr_20	4.610E+05	1.7%	0.0%	26.3%	34.9%	37.1%
Regular	5.360E+05	24.3%	0.0%	21.8%	26.4%	27.5%
Incr_20	5.550E+05	18.9%	0.0%	23.1%	28.0%	29.9%
Infinit	7.030E+05	22.0%	12.3%	20.1%	21.3%	24.3%
Example 3	P Revenue	Group 1	Group 2	Group 3	Group 4	Group 5
Decr_20	4.810E+05	13.1%	39.2%	4.0%	30.8%	12.9%
Regular	6.100E+05	23.4%	19.0%	22.1%	21.7%	13.9%
Incr_20	6.910E+05	22.2%	19.4%	23.5%	21.8%	13.1%
Infinit	7.450E+05	22.9%	22.3%	26.1%	16.2%	12.5%
Example 5	P Revenue	Group 1	Group 2	Group 3	Group 4	Group 5
Decr_20	7.200E+05	0.0%	14.3%	13.5%	33.5%	38.8%
Regular	6.640E+05	1.0%	6.0%	32.7%	37.8%	22.5%
Incr_20	5.470E+05	5.2%	3.3%	29.2%	43.5%	18.7%
Infinit	8.760E+05	17.6%	22.3%	19.6%	20.9%	19.6%

Table 6.40 Summary Examples : **Stochastic Price** in **Model 4**, Group Revenue contribution

Example 1	**P Revenue**	**Group 1**	**Group 2**	**Group 3**	**Group 4**	**Group 5**
Decr_20	4.500E+05	3.3%	10.0%	38.3%	21.6%	26.9%
Regular	5.370E+05	4.6%	15.6%	37.4%	18.7%	23.7%
Incr_20	5.820E+05	4.6%	21.3%	37.3%	16.4%	20.4%
Infinit	6.920E+05	2.9%	33.0%	33.7%	14.7%	15.7%
Example 2	P Revenue	Group 1	Group 2	Group 3	Group 4	Group 5
Decr_20	4.610E+05	1.0%	0.0%	21.1%	33.2%	44.8%
Regular	5.360E+05	11.8%	0.0%	22.4%	27.6%	38.2%
Incr_20	5.550E+05	11.4%	0.0%	22.9%	27.7%	38.0%
Infinit	7.030E+05	21.2%	15.3%	19.5%	20.6%	23.4%
Example 3	P Revenue	Group 1	Group 2	Group 3	Group 4	Group 5
Decr_20	4.810E+05	18.6%	39.2%	5.9%	28.3%	7.9%
Regular	6.100E+05	14.0%	22.7%	27.1%	20.2%	15.9%
Incr_20	6.910E+05	21.8%	20.6%	25.4%	17.4%	14.9%
Infinit	7.450E+05	22.1%	21.6%	25.2%	17.0%	14.2%
Example 5	P Revenue	Group 1	Group 2	Group 3	Group 4	Group 5
Decr_20	7.200E+05	0.0%	14.7%	13.2%	21.6%	50.4%
Regular	6.640E+05	1.0%	7.4%	22.5%	26.6%	42.4%
Incr_20	5.470E+05	2.4%	4.2%	23.4%	33.7%	36.4%
Infinit	8.760E+05	16.7%	22.5%	19.8%	21.2%	19.8%

Table 6.41 Summary Examples : **Stochastic Price & Demand** in **Model 4**, Group % in Hotel

Example 1	**DP Revenue**	**Group 1**	**Group 2**	**Group 3**	**Group 4**	**Group 5**
Decr_20	5.070E+05	31.4%	7.6%	23.8%	16.3%	20.9%
Regular	5.600E+05	28.4%	12.7%	22.9%	16.8%	19.3%
Incr_20	6E+05	27.3%	15.6%	24.9%	14.7%	17.5%
Infinit	7.200E+05	19.5%	19.2%	22.6%	17.6%	21.1%

Example 2	DP Revenue	Group 1	Group 2	Group 3	Group 4	Group 5
Decr_20	4.670E+05	16.9%	0.0%	24.3%	33.6%	25.2%
Regular	4.870E+05	27.4%	0.7%	18.9%	27.7%	25.4%
Incr_20	5.560E+05	22.2%	0.0%	22.8%	31.0%	24.0%
Infinit	7.290E+05	23.7%	14.2%	18.4%	21.7%	22.0%

Example 3	DP Revenue	Group 1	Group 2	Group 3	Group 4	Group 5
Decr_20	5.880E+05	23.9%	18.2%	21.9%	27.0%	9.1%
Regular	7.190E+05	25.5%	16.0%	20.4%	24.1%	14.0%
Incr_20	7.320E+05	22.6%	18.5%	22.2%	20.0%	16.7%
Infinit	7.780E+05	20.2%	19.1%	20.6%	19.4%	20.8%

Example 5	DP Revenue	Group 1	Group 2	Group 3	Group 4	Group 5
Decr_20	5.640E+05	0.0%	10.6%	13.8%	40.0%	35.5%
Regular	4.720E+05	0.1%	2.3%	22.0%	41.8%	33.8%
Incr_20	5.380E+05	0.0%	7.2%	17.9%	47.8%	27.1%
Infinit	9.070E+05	17.7%	21.7%	19.9%	20.2%	20.6%

Table 6.42 Summary Examples : **Stochastic Price & Demand** in **Model 4**, Group Revenue contribution

Example 1	**DP Revenue**	**Group 1**	**Group 2**	**Group 3**	**Group 4**	**Group 5**
Decr_20	5.070E+05	4.8%	7.2%	38.5%	21.4%	28.2%
Regular	5.600E+05	3.9%	14.7%	35.5%	20.3%	25.5%
Incr_20	6E+05	6.4%	19.9%	36.1%	18.2%	19.4%
Infinit	7.200E+05	6.8%	30.9%	35.2%	12.5%	14.4%

Example 2	DP Revenue	Group 1	Group 2	Group 3	Group 4	Group 5
Decr_20	4.670E+05	13.3%	0.0%	20.7%	32.5%	33.5%
Regular	4.870E+05	13.7%	0.3%	18.0%	30.8%	37.2%
Incr_20	5.560E+05	15.4%	0.0%	22.6%	28.5%	33.5%
Infinit	7.290E+05	22.1%	16.4%	18.5%	21.1%	21.9%

Example 3	DP Revenue	Group 1	Group 2	Group 3	Group 4	Group 5
Decr_20	5.880E+05	15.5%	23.4%	28.5%	25.1%	7.4%
Regular	7.190E+05	21.0%	18.8%	25.3%	20.5%	14.3%
Incr_20	7.320E+05	21.5%	20.6%	25.0%	16.8%	16.2%
Infinit	7.780E+05	23.0%	22.3%	24.6%	14.5%	15.6%

Example 5	DP Revenue	Group 1	Group 2	Group 3	Group 4	Group 5
Decr_20	5.640E+05	0.0%	6.6%	19.3%	32.1%	42.0%
Regular	4.720E+05	0.0%	7.4%	23.8%	39.7%	29.2%
Incr_20	5.380E+05	0.0%	5.3%	14.7%	37.8%	42.2%
Infinit	9.070E+05	17.5%	22.2%	19.4%	21.3%	19.6%

We could suppose for the instance that a larger number of inventories would have not shown congestion inside them, because at least one inventory would have been "taken" by one single group in the infinity, and of course this situation is softened by the number of scenarios. A considerable quantity of scenarios would have also greatly softened the results. This situation however was not tested in this thesis. Having a lower number of inventories is good because the bang-bang behavior is kept, which facilitates to perform general analysis. It is important to notice that MIP-H+ was less effective than in the previous runs. This is probably caused by more complicated problems.

Sensitivity Analysis to changes in Capacity :

- **Example 1 :**

  Leader has one good located hotel and thus it gives preference to groups 2 and 3, who are the ones sensitive to proximity to that inventory, and more preference to groups 1, 4 and 5 to the furthest inventories because they are not sensitive to proximity but sensitive to quality. Specifically, groups 4 and 5 are more sensitive to quality than group 1, and thus higher prices can be charged to those clients. The opposite case happens with group 1, who is only willing to pay low prices. Because of that, when the leader faces a capacity reduction it gives priority to group 5 with its farther hotels and then to group 3 with its closest hotel. In this example, higher capacity generates higher revenue for the leader. When capacity reaches the infinite, group distribution tends to be equal (in a range of $[18\%, 23\%]$), but in terms of revenue it does not happen because group 1 has low prices and thus its contribution to the total revenue is marginal. In terms of differences related to stochasticity on price, demand, or both, we notice the same behaviour of the leader when it faces capacity constraint. In other words, the leader prefers groups 3 and 5, but with an increase on group 3 in terms of revenue contribution, when price is stochastic. The reason is that the leader sought to attract more clients from group 3 and keep clients from group 5 by offering lower prices, which finally happened.

- **Example 2 :**

  In this example, the leader is bad located. Therefore, it cannot compete in distance to attract group 2, which is the one that values more proximity in comparison to the other groups. This explains the fact that even with infinite capacity, group 2 is marginal in terms of group participation. However, in terms of contribution to revenue, this group takes more relevance in the case of infinite capacity, because the leader gets some clients from this group by offering an attractive price that does not jeopardize the price charged to the other categories. When demand is stochastic and the leader faces a capacity reduction, the leader prefers clients from groups 4 and 5, who are the ones that provide more value to quality of service, and particularly to group 4 that is not sensitive to proximity. However, when price is stochastic, the leader is interested on moving group 5 clients to its best located inventory (from inventory 2 to inventory 1) in order to be able to charge a higher price. However, the leader had to disregard clients from group 1 in order to have capacity for pursuing that strategy. Finally, when price and demand are stochastic, the leader behaves similarly as when only demand is stochastic, thus preferences are given to groups 4 and 5.

— **Example 3 :**

In this example, the best located inventory of the leader is in front of the best located inventory of the competition. The second best located inventory of the leader is better located than its competition. There is only one group, G3, which values proximity, and one group, G5, which is not sensitive at all to proximity, but it is the most sensitive to quality. Therefore, the leader is more interested on attracting clients from groups 1 and 3 when it faces a regular capacity and demand is stochastic, and less interested in group 5. However, once there is a capacity reduction and demand is stochastic, the leader becomes more interested in groups 2 and 4, and almost not interested at all in group 5. The leader increased its revenue in group 2 and decreased its revenue from group 3 when price were stochastic, because it decided to aggressively increase the price of that group (approximately $250) but that strategy finally moved clients to the competition. Finally, when price and demand were stochastic, the leader came back to the initial situation, *i.e.,* it was interested in all groups except group 5. It is important to mention that given these results (especially having an aggressive price increase of $250), we think that this solution is suboptimal.

— **Example 5 :**

In this example, the best located inventory of the leader is close to the best located inventory of its competitor, but this one has the location advantage. However, its second and third best located inventories are better located than the competition. There are two groups sensitive to proximity, groups 2 and 5, being the last one also sensitive to quality. In the scenario of stochastic demand and regular capacity, the leader is more interested on attracting groups 3, 4 and 5 because group 2 preferred the competition (it has the best located inventory) and group 1 is disregarded due to its lower sensitivity to proximity. Once the leader faces a capacity reduction, it lost group 3 but it was able to move several clients of group 5 from inventory 3 to inventory 1, with a price increase of approximately $450. That had a positive impact on revenue, which explains the fact that although having a capacity reduction, the leader was able to increase its revenue. However, when price is stochastic, groups 2 and 3 also contributed to the revenue of the leader, although it was not as much as group 5. Indeed, the leader was able to increase the price of group 5 in approximately \$50 without losing demand. Finally, when price and demand are stochastic, group 3 increased its participation in the revenue of the leader, although groups 4 and 5 were still the highest contributors. Regarding the increase in group 3, it happened because the leader was able to move this group from farther inventories to inventories 2 and 3, which allowed setting a higher price to that group.

As a general remark is important to mention that leader controls its "quality of service" (QoS) but not distance, and thus competitors play an important role. Because of that, the leader tends to do more aggressive changes to its prices when it is trying to "move" clients from one inventory to another. If there are more groups with cross-preferences, the leader has to do more aggressive price changes.

CHAPTER 7

CONCLUSION

In this chapter we synthesize the main components of this thesis (model, heuristics, and results analysis), point out our main contributions, highlight the main limitations, and suggest future research in this area.

7.1 Synthesis

Our research consists on developing and solving a stochastic bilevel RM model for the hotel industry and on analyzing its results. In order to identify the main areas for possible academic and industrial contribution, we reviewed several studies conducted over the last 40 years in HRM. We also examined the development of the hotel industry and its current trends.

One of the first elements that we noticed was that the hotel industry did not have RM models that considered simultaneously the length of the stay, the price of the competitors, the inventory capacity, and particularly the effect of uncertainty on revenue. Therefore, we developed a model that considered all these elements at the same time, and in which uncertainty was introduced for competitors' prices and demand. Regarding demand, it was modelled for several users' groups that were jointly looking for minimizing their internal disutility function. This minimization considered users' preferences for three criteria, which were price list, proximity to attraction points, and QoS value.

The model was developed in a first attempt under a deterministic frame and then extended to its stochastic frame. We examined the pertinence of linking our model to other related models based on the same paradigms and their properties. Moreover, the model was built up as a Two-stage Recourse Problem, in which both stages were tied through Price using two different restrictions : An Absolute Restriction (Model 4)and a Proportional Restriction (Model 5). This connection was set for every tariff in the leader's inventories. After this, we reviewed some of its properties, already studied in the literature, and we proposed some conjectures regarding its continuity for the Model 4 : Piecewise continuity and non-convex continuous. Because of all these elements contained in our model, we can argue that our first contribution in this field is the introduction and development of a complex stochastic bilevel model for HRM.

In terms of resolution, we developed some strategies that included exact methods, heuristics and exact methods assisted by new heuristics. First, we began by using an exact method, the MIP Original, which was not able to provide a numerical solution to our problem. Therefore, we had to define a better heuristic that could help MIP Original to solve the problem. Our first attempt was through the development of a Min-Max-Mean naive heuristic, which provided a very bad solution but it was useful to compare the result with those that could have been provided by an hotel manager through the analysis of competitors' prices and users' characteristics.

Our second attempt consisted on developing a more complex heuristic, named HBP, which solved our problem with good results. This heuristic consists in simultaneously descending and copying prices in an orderly manner and re-optimizing the first level of the problem. Price exploration was made over a neighborhood in which price vectors were marked and kept to avoid being re-explored afterwards. A second alternative for exploring prices was examined through the development of a MIP restricted problem that takes into account the price vectors obtained in the HBP heuristic as well as other dual variables, in order to reduce the domain and size of the big constants utilized. By doing this, we were able to help Cplex to focus on the most promising domain values for the solution. This strategy allows us not only to complement the HBP heuristic, but also to calculate a gap and thus to be able to compare our results with those obtained through MIP Original.

Finally, our last strategy consisted in developing MIP-H+ ; a heuristic where assignments were recalculated to each node of the B&B in order to obtain the best result. This result was entered into Cplex with the aim of improving the lower bound. We noticed that in most of the cases, combining HBP with MIP Restricted was the best heuristic for solving our problem and thus we can argue that our second main contribution is the development of our HBP heuristic but also its complementary with the MIP restricted problem.

In order to test our model and its heuristics, we began with the resolution of a small instance. We calculated the classic approximations for the result and also analyzed the quality of the heuristics based on these numerical results. After that, we solved large instances, in which the focus was on analyzing revenue composition and distribution per group of users, with the purpose of getting a better understanding of our model. Once these examples were analyzed, we solved other large instances to which we added stochasticity in price and demand with the intention of examining its impact not only on revenue but also on users' group distribution per inventory, and the implications of that distribution on revenue. Finally, we solved more complex problems by assuming different capacity scenarios, which also included stochasticity in price, demand, or both. The objective of this sensitivity analysis was to understand the impact of hotel capacity on revenue but also to observe the strategic distribution of groups to those inventories. We noticed that most of the results were coherent with the economic behavior introduced to the model.

In summary, the main contributions of this thesis are to provide an elaborated model for HRM, to solve small and large instances in a reasonable computing time, to obtain good mathematical results through the use of our HBP heuristic (although we cannot assure it is the optimal solution), and to provide very useful results for decision making in the hotel industry.

7.2 Limitations

There are some research limitations in this thesis that must be acknowledged. First of all, there are some limitations related to the model and its resolution, which are inherent to MIP transformations. Indeed, these transformations have serious problems not also for closing the best-bound (upper bound) and closing the gap in many of the models but also in making more detailed sensitivity analysis on groups assignments. Moreover, there is not certitude that we have found the optimal solution for large instances. However, the results obtained in this thesis are good enough to provide a complete analysis for the industry.

A second limitation is related to stochasticity and specifically to the analysis that can be done about the impact that it has on revenue. Because our model is a two-stage problem and the number of scenarios is reduced, as well as the rolling horizon and the Length-of-stay, we are mainly examining the impact of stochasticity on revenue in terms of volatility but not in terms of trends.

A third limitation is related to the data generation process, which is by itself another problem to model. In fact, instances must be carefully designed to avoid falling into atypical or infeasible cases when we are introducing stochasticity in price and demand.

Finally, our model does not take into account Price-Demand elasticity of users' groups. However, this limitation is reduced when we are using the model for conducting short-term analysis.

7.3 Future Research

Through the development of this thesis and the analysis of the results, we noticed that there are several possibilities for extending our model for future research. For example, it could be interesting to develop a numerical approach that takes bigger advantages of the simple structure, solving a disjunctive problem for every assignment, instead of a MIP transformation that is cumbersome for closing the gap.

Another approach could be the use of a Progressive Hedging (PH) metaheuristic, which is a well-suited technique for solving stochastic programs and that has been successfully applied by Crainic et al. (2011) for network design. We could use our HBP heuristic to solve scenarios and the consensual problem among decision variables with a PH metaheuristic.

As widely treated in the literature by Hilge and Sen (1991); Kall and Wallace (2003); Birge and Louveaux (1997); Kall and Mayer (2005), the Two-stage SLP with Recourse and discrete distributions shows a particular structure named "L-Shaped". However, SHBP has two stages that are Bilevel Problems. It is thus possible to decompose the problem by using a generalization of Benders' algorithm (Non-convex Generalized Benders' decomposition for Stochastic Programs as presented in Li et al. (2011)) using global optimization tools such as BARON developed by Tawarmalani and Sahinidis (2005); Sahinidis (2013).

We could also adapt our HBP heuristic not only to consider the first level reoptimization but also to perform iterations by solving other MIP restricted problems (with some binary variables fixed) and letting the remaining variables to flip in a reduced number of cases to look for a better solution. The HBP heuristic could be easily parallelized to take advantage of different threads and test diverse exploration leaves simultaneously, with a common tree.

It could be also interesting to develop a new approach for generating scenarios, which would allow us to keep the size of the instances reduced, and at the same time, to enhance the understanding of stochasticity effects on revenue. Moreover, even though our model behave very well in economic terms, it could be modified to perform more complex sensitivities, particularly for the parameters α and β. For instance, we could consider a two-dimensional criterion for measuring distance, by proposing more than one interest point zone, or by including two forms for standardizing QoS in order to normalize it with respect to price. Regarding QoS, the model could be adapted to allow more freedom for the leader in terms of setting its prices, especially when the leader faces complex situations. This could be done by eliminating the restrictions related to ordering QoS.

In terms of users groups, we could modify the model to consider identical users' blocks instead of individual users. Moreover, we could introduce users' booking arrival information to re-optimize prices and booking limits locally, if these were sub-optimal. We argue that it could be interesting to re-obtain the booking limits, because it is going to help to determine the impact of protection levels on revenue, particularly when it is performed for low-price inventories.

Another sensitivity analysis could be done in terms of inventory location. We already noticed that location played an important competitive advantage role and thus it could be interesting to examine the effect of adding or eliminating inventories in well located areas on revenue distribution per user group. It could be done either by closing or by opening new competitors' inventories or by allowing the leader to opening a competitive inventory.

The idea of trying a relaxed model for SHBP, allowing different rates for weekend days

(Thursday to Monday) as certain hotels do seems interesting. Intuitively, this problem should be easier to solve compared to our SHBP problem with a unique tariff. This new model is thus considered as a relaxation of the SHBP problem and its revenue should be higher. This relaxation corresponds to separate the price for each day allowing that each day has an independent tariff. Algorithmically, it seems easier to fix $T_{i,q,j}, \forall i \in A, \forall q \in Q, \forall j \in W$ than $T_{i,q}, \forall i \in A, \forall q \in Q$ because there are less binary auxiliary variables involved for each tariff. As a conjecture, we could consider that the number of local maxima will also increase for the deterministic equivalent. However, we must notice that in case of finding feasible solutions those do not define necessarily a vector of feasible auxiliary binary variables to the original SHBP. Further examinations to this relaxation could be possible by modifying our HBP heuristic in order to consider a different tariff per day.

Finally, another interesting aspect for further examination would be to compare our actual model with a new restricted model in order to obtain results that will be independent of group size and length of stay, but still valid in terms of capacity restrictions. To do this, we could perform an independent minimization for each group instead of considering the sum of the disutility of all our users.

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ANNEXE A

Cplex Common General Parameters

Common parameters used for tests :

CPX_PARAM_SCRIND	1
CPX_PARAM_WORKDIR	"/tmp"
CPX_PARAM_MIPDISPLAY	2
CPX_PARAM_NODEFILEIND	3
CPX_PARAM_TRELIM	1.53600000000000e+05
CPX_PARAM_BNDSTRENIND	0
CPX_PARAM_FLOWCOVERS	1
CPX_PARAM_IMPLBD	2
CPX_PARAM_PROBE	3
CPX_PARAM_GUBCOVERS	-1
CPX_PARAM_FRACCUTS	2
CPX_PARAM_MIRCUTS	1
CPX_PARAM_DISJCUTS	3
CPX_PARAM_MIPEMPHASIS	3
CPX_PARAM_SUBMIPNODELIM	250
CPX_PARAM_REPAIRTRIES	10
CPX_PARAM_MIPSEARCH	1
CPX_PARAM_VARSEL	3
CPX_PARAM_BRDIR	1
CPX_PARAM_PRESLVND	2

ANNEXE B

Summary Model 5, Sensitivity Analysis with respect to capacity

For the Model 5, the result are show in Table B.1 for stochastic demand, Table B.3 for stochastic price, and Table B.5 for stochastic price & demand.

Table B.1 Summary Examples : **Stochastic Demand** in **Model 5**, Group % in Hotel

Example 1	**D Revenue**	**Group 1**	**Group 2**	**Group 3**	**Group 4**	**Group 5**
Decr_20	5.370E+05	30.4%	17.5%	10.1%	9.0%	33.0%
Regular	5.930E+05	23.2%	20.0%	15.4%	12.9%	28.5%
Incr_20	6.510E+05	22.2%	22.5%	21.2%	16.1%	18.0%
Infinit	7.880E+05	18.3%	18.0%	21.2%	19.5%	23.0%

Example 2	D Revenue	Group 1	Group 2	Group 3	Group 4	Group 5
Decr_20	4.070E+05	27.0%	2.9%	15.6%	38.7%	15.8%
Regular	5.450E+05	25.1%	1.7%	29.1%	34.2%	9.9%
Incr_20	6.360E+05	20.9%	0.0%	23.7%	27.8%	27.6%
Infinit	7.520E+05	25.9%	6.1%	20.2%	23.7%	24.1%

Example 3	D Revenue	Group 1	Group 2	Group 3	Group 4	Group 5
Decr_20	6.690E+05	28.6%	17.3%	18.7%	28.9%	6.5%
Regular	6.640E+05	18.6%	18.1%	23.7%	32.2%	7.5%
Incr_20	6.670E+05	37.5%	16.0%	17.0%	25.5%	4.0%
Infinit	8.730E+05	19.2%	18.2%	19.7%	18.6%	24.3%

Example 5	D Revenue	Group 1	Group 2	Group 3	Group 4	Group 5
Decr_20	6.640E+05	0.0%	2.9%	29.1%	37.5%	30.4%
Regular	7.570E+05	0.0%	2.7%	33.4%	36.0%	27.9%
Incr_20	6.120E+05	0.0%	1.1%	35.1%	40.4%	23.4%
Infinit	9.700E+05	20.3%	21.0%	19.2%	19.5%	19.9%

Table B.2 Summary Examples : Change in Group Composition **Stochastic Demand** in **Model 5**, Group Revenue contribution

Example 1	**D Revenue**	**Group 1**	**Group 2**	**Group 3**	**Group 4**	**Group 5**
Decr_20	5.370E+05	4.8%	17.5%	24.5%	18.2%	35.1%
Regular	5.930E+05	5.2%	18.5%	27.8%	19.0%	29.5%
Incr_20	6.510E+05	4.6%	24.3%	29.7%	19.6%	21.9%
Infinit	7.880E+05	6.7%	29.3%	33.4%	14.2%	16.3%

Example 2	D Revenue	Group 1	Group 2	Group 3	Group 4	Group 5
Decr_20	4.070E+05	10.9%	0.7%	20.8%	38.2%	29.5%
Regular	5.450E+05	14.1%	0.5%	27.7%	33.8%	23.8%
Incr_20	6.360E+05	11.0%	0.0%	23.6%	28.4%	37.0%
Infinit	7.520E+05	21.4%	10.8%	20.3%	23.4%	24.0%

Example 3	D Revenue	Group 1	Group 2	Group 3	Group 4	Group 5
Decr_20	6.690E+05	23.7%	22.2%	25.9%	24.2%	3.9%
Regular	6.640E+05	19.0%	22.1%	28.7%	25.8%	4.4%
Incr_20	6.670E+05	31.8%	21.3%	24.9%	19.7%	2.4%
Infinit	8.730E+05	22.3%	21.7%	23.9%	14.3%	17.8%

Example 5	D Revenue	Group 1	Group 2	Group 3	Group 4	Group 5
Decr_20	6.640E+05	0.0%	2.9%	26.4%	31.3%	39.4%
Regular	7.570E+05	0.0%	3.4%	19.2%	29.2%	48.3%
Incr_20	6.120E+05	0.0%	1.2%	20.2%	36.5%	42.1%
Infinit	9.700E+05	13.3%	24.7%	19.2%	21.0%	21.9%

Table B.3 Summary Examples : Change in Group Composition **Stochastic Price** in **Model 5**, Group % in Hotel

Example 1	**P Revenue**	**Group 1**	**Group 2**	**Group 3**	**Group 4**	**Group 5**
Decr_20	2.700E+05	33.5%	0.0%	1.0%	25.9%	39.6%
Regular	5.590E+05	28.2%	13.6%	24.5%	17.1%	16.7%
Incr_20	5.960E+05	27.6%	17.5%	27.6%	11.9%	15.4%
Infinit	7.040E+05	25.0%	29.6%	30.2%	7.3%	7.8%
Example 2	P Revenue	Group 1	Group 2	Group 3	Group 4	Group 5
Decr_20	3.900E+05	27.7%	1.6%	27.6%	17.9%	25.3%
Regular	3.800E+05	27.3%	0.0%	24.4%	18.3%	29.9%
Incr_20	5.780E+05	11.7%	0.0%	28.4%	30.0%	29.9%
Infinit	7.030E+05	22.0%	12.3%	20.1%	21.3%	24.3%
Example 3	P Revenue	Group 1	Group 2	Group 3	Group 4	Group 5
Decr_20	4.330E+05	19.9%	37.9%	3.7%	26.0%	12.6%
Regular	6.520E+05	29.9%	25.4%	8.7%	25.2%	10.7%
Incr_20	6.980E+05	22.4%	20.3%	23.7%	20.4%	13.1%
Infinit	7.430E+05	21.6%	21.1%	24.6%	18.3%	14.4%
Example 5	P Revenue	Group 1	Group 2	Group 3	Group 4	Group 5
Decr_20	7.390E+05	0.0%	13.0%	28.0%	23.7%	35.2%
Regular	4.590E+05	0.4%	0.9%	9.0%	36.8%	53.0%
Incr_20	4.820E+05	5.7%	3.6%	37.4%	32.8%	20.5%
Infinit	8.970E+05	16.8%	22.5%	19.8%	21.1%	19.8%

Table B.4 Summary Examples : Change in Group Composition **Stochastic Price** in **Model 5**, Group Revenue contribution

Example 1	**P Revenue**	**Group 1**	**Group 2**	**Group 3**	**Group 4**	**Group 5**
Decr_20	2.700E+05	7.0%	0.0%	2.2%	41.2%	49.7%
Regular	5.590E+05	3.9%	15.4%	36.3%	20.9%	23.4%
Incr_20	5.960E+05	4.4%	21.1%	36.9%	16.7%	20.9%
Infinit	7.040E+05	2.9%	32.3%	33.0%	15.4%	16.5%
Example 2	P Revenue	Group 1	Group 2	Group 3	Group 4	Group 5
Decr_20	3.900E+05	11.7%	0.6%	24.1%	25.1%	38.6%
Regular	3.800E+05	10.9%	0.0%	23.4%	26.2%	39.5%
Incr_20	5.780E+05	8.2%	0.0%	25.0%	27.7%	39.1%
Infinit	7.030E+05	21.2%	15.3%	19.5%	20.6%	23.4%
Example 3	P Revenue	Group 1	Group 2	Group 3	Group 4	Group 5
Decr_20	4.330E+05	17.4%	43.0%	4.1%	27.2%	8.3%
Regular	6.520E+05	21.9%	24.0%	22.1%	19.7%	12.4%
Incr_20	6.980E+05	20.6%	20.9%	26.1%	17.8%	14.6%
Infinit	7.430E+05	22.6%	22.1%	25.8%	15.4%	14.1%
Example 5	P Revenue	Group 1	Group 2	Group 3	Group 4	Group 5
Decr_20	7.390E+05	0.0%	14.4%	18.7%	17.7%	49.2%
Regular	4.590E+05	0.0%	2.1%	5.7%	26.9%	65.4%
Incr_20	4.820E+05	2.3%	2.9%	40.6%	29.2%	25.0%
Infinit	8.970E+05	19.4%	23.3%	17.8%	19.0%	20.5%

Table B.5 Summary Examples : Change in Group Composition **Stochastic Price & Demand** in **Model 5**, % Group in Hotel

Example 1	**DP Revenue**	**Group 1**	**Group 2**	**Group 3**	**Group 4**	**Group 5**
Decr_20	4.990E+05	30.5%	7.3%	25.1%	15.8%	21.4%
Regular	5.660E+05	27.8%	12.2%	23.0%	17.7%	19.2%
Incr_20	6.110E+05	27.8%	15.8%	25.2%	14.8%	16.4%
Infinit	7.210E+05	25.9%	27.3%	32.3%	7.0%	7.5%

Example 2	DP Revenue	Group 1	Group 2	Group 3	Group 4	Group 5
Decr_20	3.510E+05	27.1%	2.1%	21.4%	22.8%	26.5%
Regular	3.770E+05	31.9%	2.2%	19.9%	21.2%	24.8%
Incr_20	3.470E+05	32.8%	0.0%	20.7%	21.1%	25.3%
Infinit	7.290E+05	23.7%	14.2%	18.4%	21.7%	22.0%

Example 3	DP Revenue	Group 1	Group 2	Group 3	Group 4	Group 5
Decr_20	4.740E+05	9.6%	19.6%	31.0%	25.0%	14.8%
Regular	7.090E+05	27.1%	23.4%	9.4%	23.6%	16.5%
Incr_20	6.400E+05	20.0%	14.0%	17.3%	29.1%	19.5%
Infinit	7.580E+05	24.2%	22.8%	24.7%	14.8%	13.4%

Example 5	DP Revenue	Group 1	Group 2	Group 3	Group 4	Group 5
Decr_20	5.560E+05	0.0%	9.9%	30.3%	26.5%	33.2%
Regular	4.850E+05	0.5%	2.5%	17.2%	43.9%	35.9%
Incr_20	5.770E+05	6.8%	2.3%	28.0%	40.5%	22.3%
Infinit	9.280E+05	16.9%	21.9%	20.0%	20.4%	20.8%

Table B.6 Summary Examples : Change in Group Composition **Stochastic Price & Demand** in **Model 5**, Group Revenue contribution

Example 1	**DP Revenue**	**Group 1**	**Group 2**	**Group 3**	**Group 4**	**Group 5**
Decr_20	4.990E+05	3.3%	6.8%	39.3%	24.7%	25.8%
Regular	5.660E+05	5.0%	14.5%	35.9%	21.7%	22.9%
Incr_20	6.110E+05	6.2%	19.7%	35.7%	18.5%	19.8%
Infinit	7.210E+05	3.0%	30.8%	35.1%	15.0%	16.1%

Example 2	DP Revenue	Group 1	Group 2	Group 3	Group 4	Group 5
Decr_20	3.510E+05	10.0%	0.5%	24.5%	29.1%	35.9%
Regular	3.770E+05	13.3%	0.5%	23.4%	27.0%	35.8%
Incr_20	3.470E+05	11.6%	0.0%	23.0%	27.6%	37.7%
Infinit	7.290E+05	22.1%	16.4%	18.5%	21.1%	21.9%

Example 3	DP Revenue	Group 1	Group 2	Group 3	Group 4	Group 5
Decr_20	4.740E+05	7.1%	22.6%	38.4%	24.0%	7.9%
Regular	7.090E+05	21.0%	23.9%	21.7%	18.8%	14.5%
Incr_20	6.400E+05	14.4%	21.2%	26.8%	21.4%	16.3%
Infinit	7.580E+05	22.9%	22.2%	24.5%	16.0%	14.5%

Example 5	DP Revenue	Group 1	Group 2	Group 3	Group 4	Group 5
Decr_20	5.560E+05	0.0%	6.6%	26.5%	24.6%	42.3%
Regular	4.850E+05	0.0%	7.4%	20.8%	40.7%	31.1%
Incr_20	5.770E+05	2.6%	2.1%	23.2%	34.0%	38.2%
Infinit	9.280E+05	17.3%	22.9%	18.8%	20.6%	20.3%