IEEE Access
Multidisciplinary : Rapid Review : Open Access Journal

Received September 11, 2018, accepted October 23, 2018, date of publication October 26, 2018, date of current version November 30, 2018.

Digital Object Identifier 10.1109/ACCESS.2018.2878376

Bayesian Belief Network Model Quantification Using Distribution-Based Node Probability and Experienced Data Updates for Software Reliability Assessment

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This work was supported in part by the Nuclear Research & Development Program of the National Research Foundation of Korea Grant, funded by the Korean Government, Ministry of Science, ICT & Future Planning, under Grant 2016M2A8A4946683, Grant 2016M2B2A9909815, and Grant 2015M2A8A4021648.

ABSTRACT Since digital instrumentation and control systems are expected to play an essential role in safety systems in nuclear power plants (NPPs), the need to incorporate software failures into NPP probabilistic risk assessment has arisen. Based on a Bayesian belief network (BBN) model developed to estimate the number of software faults considering the software development lifecycle, we performed a pilot study of software reliability quantification using the BBN model by aggregating different experts' opinions. In this paper, we suggest the distribution-based node probability table (D-NPT) development method which can efficiently represent diverse expert elicitation in the form of statistical distributions and provides mathematical quantification scheme. Besides, the handbook data on U.S. software development and V&V and testing results for two nuclear safety software were used for a Bayesian update of the D-NPTs in order to reduce the BBN parameter uncertainty due to experts' different background or levels of experience. To analyze the effect of diverse expert opinions on the BBN parameter uncertainties, the sensitivity studies were conducted by eliminating the significantly different NPT estimates among expert opinions. The proposed approach demonstrates a framework that can effectively and systematically integrate different kinds of available source information to quantify BBN NPTs for NPP software reliability assessment.

INDEX TERMS Bayesian belief network, nuclear power plant, probabilistic risk assessment, software reliability.

I. INTRODUCTION

The instrumentation and control (I&C) systems in nuclear power plants (NPPs) have recently been replaced with digital-based systems, and the reason for this transition lies in the critical functional advantages that digital systems offer over conventional analog systems. However, the integration of the risk model of a digital I&C system into NPP probabilistic risk assessment (PRA) model presents unique challenges since digital I&C systems have different failure modes and causes, such as software failure, compared to analog systems. Since software failure can significantly affect the risk of digital protection systems in NPPs [1], [2], software reliability must be quantified to guarantee the safety of digitalized NPPs.

Regarding the quantification schemes of safety graded software reliability, a previous study [3] has investigated a spectrum of related methods and identified potential ones that may serve to quantify software on-demand failure probabilities of NPP digital systems, such that the system models can be integrated into an NPP PRA model. Among the various methods investigated in the previous study, a Bayesian belief network (BBN) method was selected as one of the appropriate candidates for the software reliability quantification of the digital protection systems. The BBN method is known as a probabilistic graphical model depicting a set of random variables and their conditional independence via a directed acyclic graph, in which nodes represent random variables with the acyclic graphs not forming any loops [4]. Since the BBN method uses conditional probability tables to represent interdependency among different events, it can potentially combine qualitative information, such as quality in carrying out software life cycle activities, with quantitative information, such as software test and operational data.

In order to estimate the failure probability of the NPP safety graded software and incorporate it into an NPP PRA model, a Bayesian belief network (BBN) model was developed in Kang *et al.* [5] which estimates the number of defects in software programs considering the software development life cycle (SDLC) characteristics. In the model, SDLC characteristics such as the quality of software development and verification and validation (V&V) activities, and software-self characteristics such as program size and complexity, are represented using a hierarchical structure. In order to quantify the various node parameters modeled in the BBN structure, node probability tables (NPTs) are used to contain probability information based on the belief relationships between the parent and child nodes in the model. The root nodes are allocated with a prior distribution, and the others are allocated with their conditional probability distributions where the NPT represents all possible combinations of the parent states for each child node. Since the developed BBN model focuses on safety-related software, NPTs should represent the variability among the class of safety-related software, and their values need to be estimated using the operational data of safety software. Generally, NPTs are derived through various methods such as direct expert judgment, estimation from datasets, and representation using equations to specify the relationship between nodes [6].

Due to a lack of sufficient data from the collection of a safety-related software, previous BBN approaches that have been performed in nuclear safety field including European projects SERENE [7], IMPRESS [8], OECD Halden Reactor project [9], [10], as well as Fenton and Neil [11], Littlewood and Wright [12], and other literature [13]–[17], used expert elicitation to cover quantitative aspects of the BBN model, specifically to quantify the NPTs for the prior probability and the conditional probability of BBN nodes given the state of its parent nodes. In most studies on BBN modeling for software reliability quantification, the NPTs in the BBN model were constructed based on the probability tables composed of point estimate or a single value entered by experts. However, a potential limitation of using expert opinions to estimate the quantities of NPTs specified in the BBN model with point estimates comes from the diversity of the experts' opinions. Since multiple experts may provide widely diverse opinions, those experts' diverse elicitations should be treated in an integrated manner when estimating NPTs for a specific software development process. This variety in the experts' opinions can be better caught in a probabilistic manner where the uncertainty associated with specified NPTs can be represented by using the distribution of the experts' opinions.

Therefore, in this paper, an integrated NPT quantification method is proposed to develop the distribution-based NPT (D-NPT) by aggregating diverse expert opinions and other sources of evidence for the quantification of NPTs in the BBN model. The D-NPTs were used to represent the uncertainty associated with the NPTs for the BBN nodes in this study instead of using discretized point estimates, initially given by each expert, to account for the variability among NPP safety-related software based on the different experts' answers on the BBN model parameter estimates. The variety of the answers from the experts were aggregated and used to derive the probability distributions which gives the best-fit for the overall data points given by experts for each node in the BBN model. The elicitations were conducted by distributing background material and questionnaires to experts, collecting and analyzing the provided answers, resolving the provided comments. The questionnaire and the answers the experts provided are included in the authors' previous report [18].

Since experts have different levels of knowledge on nuclear safety-related software or there is a variation in the quality or error sources of software used in different organizations, a large diversity of opinions can be observed for some nodes in the BBN model. Therefore, available literature data and software development data, such as handbook data on U.S. software development and V&V as well as the testing results for two trial nuclear safety software, were used to Bayesian update the D-NPTs in the BBN model to reduce the BBN parameter uncertainties. Also, sensitivity studies were undertaken to analyze the effect of the elimination of expert estimates that are significantly different from the other estimates on the BBN parameter uncertainties.

II. APPLICATION OF DISTRIBUTED NODE PROBABILITIES IN BBN MODEL QUANTIFICATION

Figs. 1 and 2 depict the high-level BBN structure of the Design phase during software development process and a detailed structure of the "Development quality in Design phase" node, respectively, which are developed in authors' previous research [5]. In Fig. 1, the dashed grey rectangle nodes denote the NPTs of "Defect density" and "Defect detection probability" nodes that are estimated by expert elicitation. The dashed grey circle nodes denote the root nodes of the model. The nodes "Development quality" and "V&V quality" in the model are not directly observable in most software development life cycles, which make data collection challenging. Therefore, in this study, an expert opinion elicitation was performed to estimate a prior distribution for those quality nodes. Similarly, the "Size and Complexity" node was initially estimated in the elicitation, and then a constant value can be used when applied to a specific nuclear safety software. As shown in Fig. 2, various indicators (attribute nodes) were identified and developed to provide indirect indications for these quality nodes. The white rectangle nodes in Fig. 2 are the attribute nodes whose NPTs were estimated

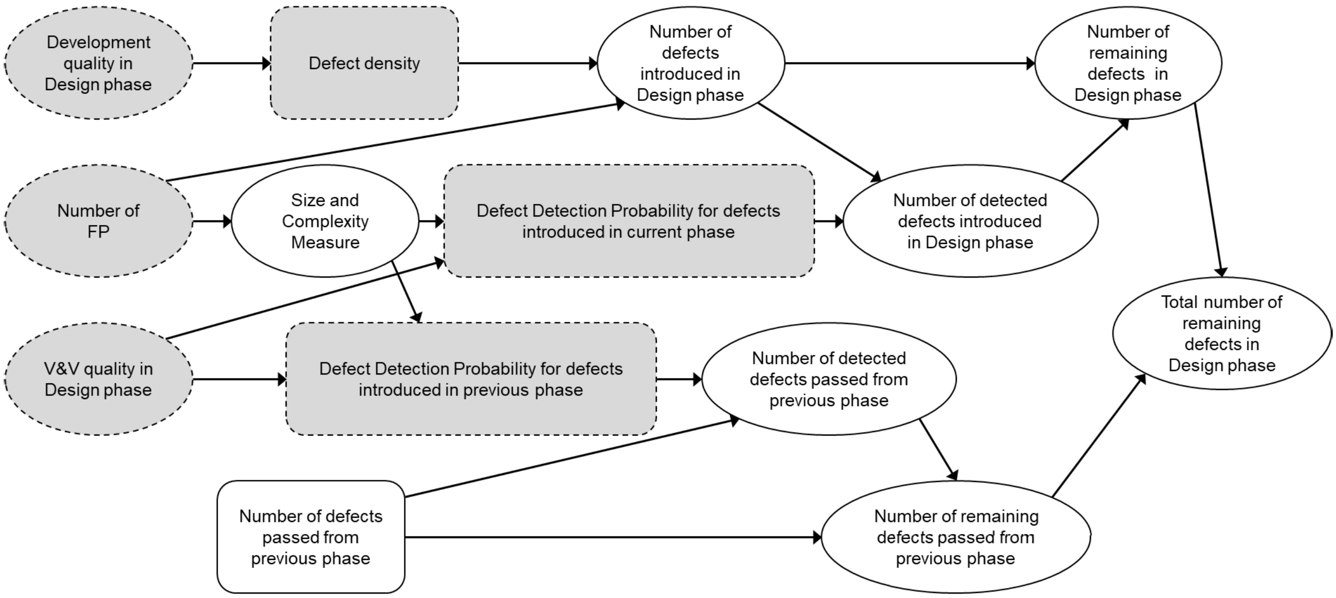


FIGURE 1. BBN model for software design phase – the relationship of expert elicitation to the BBN nodes.

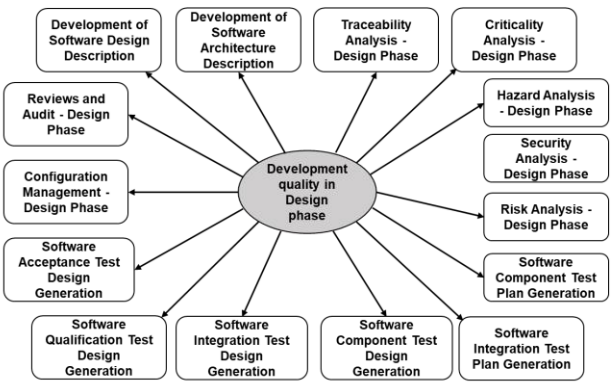


FIGURE 2. BBN model for software design phase – expert elicitation on the attributes nodes for the development quality in design phase.

by expert elicitation which can be replaced with observed evidence from a specific nuclear safety software. The expert elicitation conducted as a part of BBN model construction covers the quantitative aspects of the model as follows: 1) the NPTs of the prior distributions of parent nodes (e.g., the prior distributions for the development and V&V quality nodes, shown in Fig. 1); 2) the conditional distribution of child nodes given the state of parent nodes (e.g., probability distribution of the quality of the attribute nodes given the quality of the nodes "Development quality" or "V&V quality", shown in Fig. 2).

The BBN parameters were answered as either point estimates or three percentile estimates (5th, 50th, and 95th percentile) by the experts. The nodes elicited by the 5th, 50th, and 95th percentile estimates include: 1) the node "number of defects introduced"; 2) the node "probability of defect removal introduced in current phase"; and 3) the node "probability of defect removal passed from the previous phase." As shown in Fig. 1, the expert elicitation on those nodes are the conditional probabilities given either development or V&V quality in that SDLC phase. The nodes elicited by the point estimate include: 1) the node "Development quality"; 2) the node "V&V quality; 3) the node "number of function points (FPs)," and 4) the attribute nodes.

A potential limitation of using expert opinions to estimate the quantities of NPTs in the BBN model comes from the disparity in the experts' opinions. That is, multiple experts may provide widely diverse answers, which should be treated in an integrated manner when estimating the NPTs in order to account for BBN parameter uncertainty representing the variability among the population of safety-related software. This variety of the experts' opinions must be modeled in a probabilistic manner; therefore, in this study, the variance associated with specified NPTs was represented as the tables of probabilistic distributions (D-NPT) based on experts' answers on BBN nodes instead of discretized estimates to account for BBN parameter uncertainty.

In this study, the distributions which gave the best-fit for the empirical distribution generated by combining different experts' opinions were used to define each value of the NPTs in each BBN node, thus, represent the uncertainty associated with the NPTs for the nodes subject to expert elicitation. The BBN parameters including the number of defects inserted or removed in each SDLC phase are estimated. Fig. 3 shows a flowchart of the D-NPT development processes and BBN model quantification using D-NPT. In this process, continuous univariate distributions (Gamma, Logistic, Lognormal, Normal, Weibull, Beta, and Pareto distributions) were employed to fit the empirical distribution obtained from the data provided by the expert opinion by using the distribution-fitting techniques provided by ALLFITDIST toolkit [19].

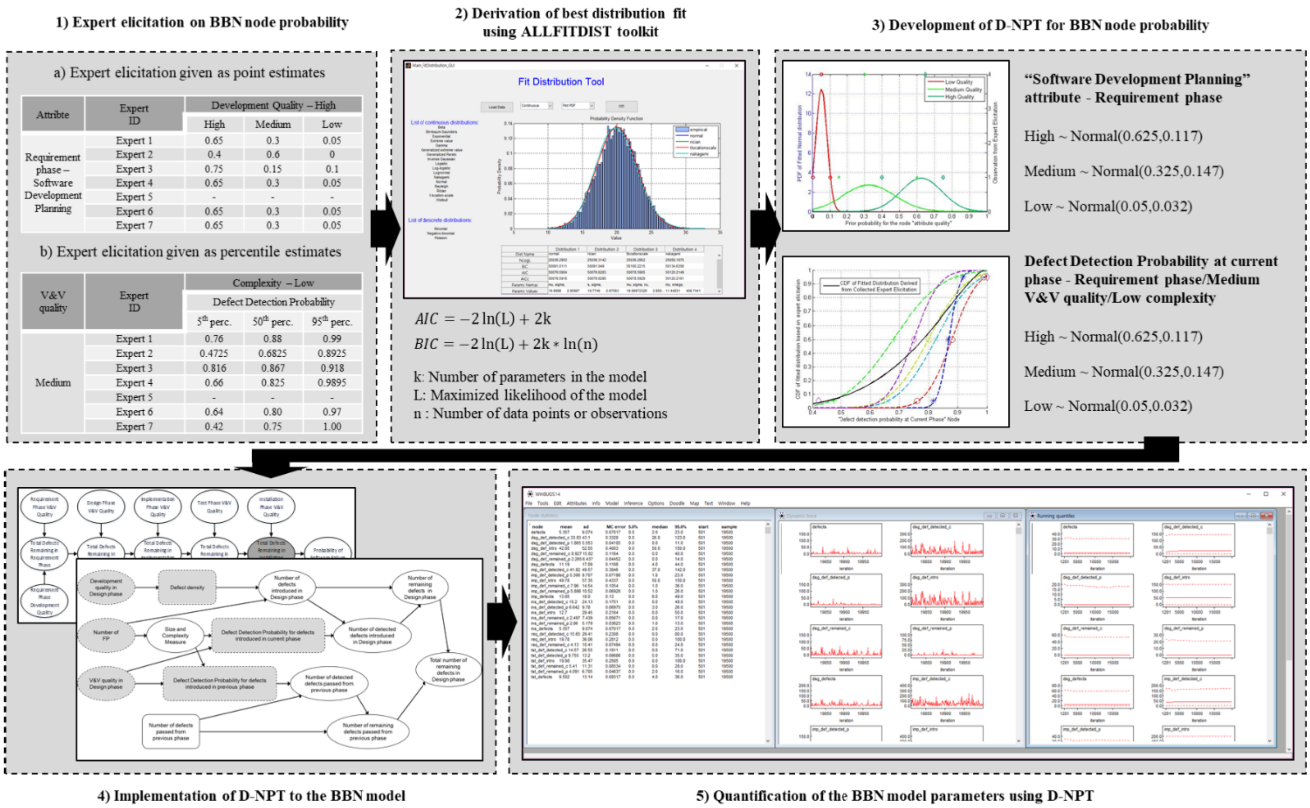


FIGURE 3. An overall scheme of D-NPT development and its implementation to BBN model.

TABLE 1. An example of expert elicitation: The prior distribution for complexity states of safety-related software.

Complexity state	Probability						
	Expert 1	Expert 2	Expert 3	Expert 4	Expert 5	Expert 6	Expert 7
Low	0.3	0.1	0.1	0.1	0.5	0.3	0.6
Medium	0.6	0.5	0.8	0.6	0.3	0.4	0.3
High	0.1	0.4	0.1	0.3	0.2	0.3	0.1

When deriving the D-NPTs of the nodes whose estimates are given as the point values by the experts, the best distribution fit was derived based on the point estimates given by the experts. For the nodes whose estimates were answered as three percentile values (5th, 50th, and 95th percentile), the best-fit distribution was derived by generating an empirical distribution by sampling from experts' elicitations. When finding the probability distribution which best represents the data, Akaike information criterion (AIC) and Bayesian information criterion (BIC) measures were used [20]. Since low values for estimated AIC and BIC imply a low expected information loss, the distribution model with the lower value of AIC and BIC was used for D-NPT development by fitting the distribution to the data points provided by the experts [21].

A. D-NPT OF THE BBN NODES ELICITED BY POINT ESTIMATE

As a part of expert elicitation, BBN parameters such as the prior distribution for quality nodes, the number of FPs, and the conditional probability distribution for the attributes, were elicited as point estimates. For example, regarding the number of FPs used in calculating the defect density and the defect detection probability, point estimates were given by the experts for the probabilities of its three complexity states (Low for $FP \leq 100$, Medium for $100 \leq FP \leq 1000$, and High for $1000 \leq FP \leq 1500$), as shown in Table 1. In this study, these different point estimates given by the experts were assumed to account for the variabilities among the safety-related software population; therefore, the probability distributions were used for the NPTs in the BBN model instead of using point values. In this process, the best distribution fit for prior distribution was analyzed based on the point estimates given as expert opinions over each defined state. In case of the prior distribution for the "Number of FPs" node, Beta distribution was used to represent expert opinion diversity associated with the number of FPs since it showed the lowest AIC and BIC values for most of the data sets. Note that other BBN nodes elicited by the experts as point estimates can

TABLE 2. Description of the D-NPTs estimated based on expert point estimates.

Subject	Parameters to be estimated	Format of expert elicitation	Expert opinion aggregation
Development quality	Prior distribution over High, Medium, and Low quality	Discretized distribution (per phase)	Beta distribution for the 7 or fewer estimates, one per expert
V&V quality	Prior distribution over High, Medium, and Low quality	Discretized distribution (per phase)	Beta distribution for the 7 or fewer estimates, one per expert
Number of function points	Prior distribution over High, Medium, and Low complexity state	Discretized distribution	Beta distribution for the 7 or fewer estimates, one per expert
NPT of attribute nodes	Conditional distribution of attribute nodes given High, Medium, and Low development or V&V quality	Discretized distribution (per condition, per phase	Normal distribution for the 7 or fewer estimates, one per expert

also be represented using probability distributions in the same manner. Table 2 shows the estimated D-NPTs of the BBN nodes elicited by experts as a point estimate.

B. D-NPT OF THE BBN NODES ELICITED BY PERCENTILE ESTIMATE

While experts were asked to provide the point estimates for the NPTs of some nodes, the discretized probability estimates (i.e., 5th, 50th, and 95th percentile estimates) were asked to experts for other BBN nodes such as defect density (the number of defects per FP) and defect detection probability. For example, one expert gave low defect detection probability with high variance, e.g., 0.47, 0.68, and 0.89, while other expert estimated high detection probability with low variance, e.g., 0.82, 0.87, and 0.92, for the 5th, 50th, and 95th percentile estimates respectively, for the detection probability of the defects introduced in current phase at given Medium V&V quality and Low complexity in the Requirements phase as shown in Table 3. In this study, these discrepancies in expert opinion were accounted for the BBN parameter uncertainty representing a population of generic safety-related software, and three requested percentiles were aggregated and further converted to a probability distribution.

TABLE 3. An example of expert elicitation: Conditional NPT for "detection probability for defects introduced in current phase" node in requirement phase.

V&V Quality	Experts	Complexity = Low		
		5th percentile	Median (50th percentile)	95th percentile
Medium	Expert 1	0.76	0.88	0.99
	Expert 2	0.47	0.68	0.89
	Expert 3	0.82	0.86	0.92
	Expert 4	0.67	0.83	0.99
	Expert 5	0.64	0.80	0.96
	Expert 6	0.3	0.75	1.00

For the D-NPTs of "Defect density" and "Defect detection probability" nodes, a Normal distribution was used to fit the 5th, 50th, and 95th percentile estimate given by each expert and a Monte Carlo simulation was then used to generate an empirical distribution by sampling and aggregating the Normal distributions estimated from each expert's elicitation. In the process, the samples from the Normal distributions with values lower than 0 were truncated in case of the defect density defined in $[0, \infty]$. Similarly, for the defect detection probability, the samples whose values fall outside the [0, 1] interval were also truncated.

As a result of deriving the probability distributions defined for BBN NPTs, Gamma and Beta distributions showed the lowest AIC and BIC values for most of the data sets in case of the defect density and the defect detection probability, respectively, and were used to represent the BBN parameter uncertainty in the model. For example, Fig. 4 shows the cumulative distribution function of the fitted distribution based on expert percentile estimations on the defect detection probability at current phase at given Medium V&V quality and Low complexity in the Requirements phase. The colored dotted lines indicate the fitted normal distribution from each expert elicitation, and the marker indicates the expert percentile estimations on the defect detection probability at current phase. The black line indicates the fitted Beta

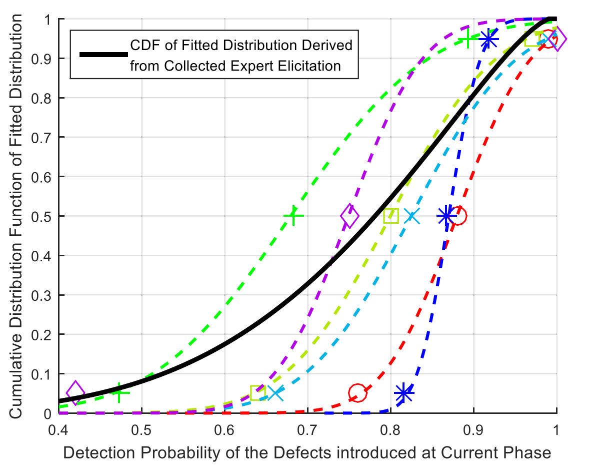


FIGURE 4. Treatment of expert opinion diversity for "Detection probability for defects introduced in current phase" node in requirement phase.

TABLE 4. Description of the D-NPTs estimated based on expert percentile estimates.

Subject	Parameters to be estimated	Format of expert opinions	Expert opinion aggregation
NPT of defect density	Number of defects per function point given High, Medium, and Low development quality and complexity	5%, 50%, and 95% (per phase, per condition)	Gamma distribution for the 7 or fewer estimates, one per expert
NPT of detection probability for current phase	Defect detection probability given High, Medium, and Low V&V quality and complexity	5%, 50%, and 95% (per phase, per condition)	Beta distribution for the 7 or fewer estimates, one per expert
NPT of detection probability for previous phase	Defect detection probability given High, Medium, and Low V&V quality and complexity	5%, 50%, and 95% (per phase, per condition)	Beta distribution for the 7 or fewer estimates, one per expert

distribution representing the probability distribution considering all expert elicitation on defect detection probability. Table 4 shows the summary of the estimated D-NPTs for the BBN nodes elicited by percentile estimates.

III. BAYESIAN UPDATE OF THE DISTRIBUTED NODE PROBABILITIES IN BBN MODEL USING REFERENCE DATA

Considering that experts have different levels of knowledge and experience with nuclear safety-related software, this may lead to significant diversity in the BBN NPT elicitation, and a large variance in the node probability values can be observed for some BBN nodes. One of the critical features of the developed BBN model is that when the evidence for a BBN node is observed from the literature or the field experience data, the NPTs can be updated from the available evidence data based on the Bayesian update method considering the conjugate prior family of probability distributions [22]. Although Bayes' theorem, as shown in Eq. 1, is mathematically simple, practical difficulties of its implementation lie in deriving the normalizing constant, the denominator in Eq. 1 [23]. Here, the product of the prior $P(\vartheta)$ and likelihood function $P(x|\vartheta)$ must be integrated over the domain of the parameters ϑ being estimated.

$$P(\vartheta|x) = \frac{P(\vartheta)\, P(x|\vartheta)}{\int P(\vartheta)\, P(x|\vartheta)\, d\vartheta} \tag{1}$$

In Bayes' theorem, if the posterior distributions $P(\vartheta|x)$ are in the same family as the prior probability distribution $P(\vartheta)$, the prior and posterior are then called conjugate distributions, and the prior is called the conjugate prior for the likelihood function. For example, consider a random variable which consists of the number of successes in n Bernoulli trials with unknown probability of success q in [0, 1]. This random variable will follow the Binomial distribution, with a probability mass function that can be expressed as a function of q, having the form for the parameters a and b, as in Eq. 2.

$$p(x) = \binom{n}{x} q^{x} (1-q)^{x} \propto q^{a} (1-q)^{b} \tag{2}$$

Considering the Beta distribution, which is a conjugate prior for the Bernoulli likelihood, the prior distribution can be derived as follows:

$$p(q) = \frac{q^{\alpha-1}(1-q)^{\beta-1}}{B(\alpha,\beta)} \tag{3}$$

where α and β are chosen to reflect any existing belief or information and q is the parameters of the underlying model. In this context, α and β are called prior hyperparameters (parameters of the prior). Considering that we sample a random variable q, and get s successes and f failures, the posterior distribution, $P(q = x|s, f)$, and its hyperparameters can be derived as follows:

$$P(s,f|q=x) = \binom{n}{s} x^{s} (1-x)^{f} \tag{4}$$

$$P(x) = \frac{x^{\alpha-1}(1-x)^{\beta-1}}{B(\alpha,\beta)} \tag{5}$$

$$P(q=x|s,f) = \frac{P(x)\, P(s,f|x)}{\int P(x)\, P(s,f|x)dx} = \frac{x^{s+\alpha-1}(1-x)^{f+\beta-1}}{B(s+\alpha, f+\beta)} \tag{6}$$

where the posterior is formulated as a Beta distribution with parameters $(s+\alpha, f+\beta)$. Then this posterior distribution can be used as the prior for more samples, with the hyperparameters adding further information as it is introduced.

In this study, handbook data on U.S. software developments and V&V experience as well as the testing result from the development experience of two application systems on the software defect potentials and defect removal efficiency were used as the evidence for the "Defect density" and "Defect detection probability for defects introduced in current phase" nodes in the BBN model and the D-NPTs were updated accordingly.

A. BAYESIAN UPDATE OF THE BBN D-NPTS USING EVIDENCE FROM HANDBOOK DATA

The handbook data on the U.S. software development and V&V experience for the software defect potentials and the defect removal efficiency [24] were utilized to Bayesian update the D-NPTs estimated from the expert elicitation. As previously discussed, a Gamma distribution estimated from the expert elicitation was used to represent the D-NPTs for "Defect density" node throughout each SDLC phase.

In this analysis, it was assumed that the mean defect density was estimated in the expert elicitation process, and the software defect potentials in the handbook data were treated as observations, specifically Poisson likelihood mean. For simplicity, the number of inserted defects was estimated from the mean values of the Poisson process instead of sampling from the Poisson distribution.

Table 5 shows important quality metrics such as defect potentials and defect removal efficiency of various software applications from the handbook data. In this study, it was assumed that the "Capability maturity model (CMM) Level 5 joined with Six Sigma" represents High development or V&V quality, the "CMM Level 4" represents Medium development or V&V quality, and the "Spiral" represents Low development or V&V quality. Since the information was not available regarding how many cases were analyzed to derive the defect potentials and their removal efficiency in the reference, the number of observations was assumed to be one for simplicity.

TABLE 5. Software types and defect characteristic [24].

Software type	Defect potentials per function point	Defect removal efficiency
CMM 5 + Six-Sigma	4.80	98.00%
CMM 4	6.00	93.00%
Spiral	6.50	85.00%

The defect potential in Table 5 refers to the sum of possible errors in the software from five separate sources: errors in requirements, errors in design, errors in source code, errors in user documentation, and errors associated with bad fixes or secondary errors introduced while fixing a primary error. Table 6 lists the overall distribution of software errors among the various categories of origin points across many industry segments from the handbook data. For the Bayesian update of the D-NPTs of the defect density nodes in the BBN model, defect density evidence in each phase of the SDLC was derived as the product of the defect potential and defect origin percentage. Since the defect origin in the installation phase was not reported, the software defect origin for the Installation-and-Checkout phase was not included in the Bayesian update of the D-NPT of defect density node in the BBN model. The code bugs and bad fix bugs were assumed to be defects from the Implementation and Test phases, respectively. The document bugs were not considered in this study. Based on the handbook data evidence, a Poisson distribution was then used to represent the likelihood for the defect density node (conjugate prior) derived from expert elicitation represented as a Gamma distribution.

TABLE 6. Software defect origin allocations [24].

Software type	Defect origin percentage
Requirement bugs	10%
Design bugs	25%
Code bugs	40%
Document bugs	15%
Bad fix bugs	10%
Total	100%

Regarding the D-NPT for "Defect detection probability for defects introduced in current phase" node, the defect removal efficiency reported in the handbook data shown in Table 4 was used as the evidence for the Bayesian update. Here, the defect removal efficiency refers to the percentage of defects removed before the delivery of the software to its users. In this study, the defect removal efficiency reported in the reference was assumed to be the defect detection probability at each SDLC phase. In the analysis, the software defect removal efficiency in the reference data was treated as an observation, specifically the probability (p) of Bernoulli likelihood because the expert elicitation on the defect detection probability corresponds to the distribution of p. Therefore, Bernoulli distribution was used to represent the likelihood for the "Defect detection probability for defects introduced in current phase" node (conjugate prior), which was represented with a Beta distribution derived from expert elicitation. In this study, various software types reported in the handbook data were assumed to have Medium complexity, and the posterior hyperparameters of the Beta distribution for the High and Low complexity cases were estimated by the ratio of the mean of the Beta distribution derived from expert elicitation for High and Low complexity to that of Medium complexity, as in Eqs. 7-9.

$$(Updated\ Mean)_{i,j} = (Updated\ Mean)_{M,j} * \gamma_{i,j} \tag{7}$$

$$(Updated\ Variance)_{i,j} = (Variance)_{i,j} * \frac{(Updated\ Mean)_{i,j}}{(Mean)_{i,j}} \tag{8}$$

$$\gamma_{i,j} = \frac{(Mean)_{i,j}}{(Mean)_{M,j}} \tag{9}$$

where i represents the degree of complexity (H: High, M: Medium, and L: Low) and j represents the degree of V&V quality (H: High, M: Medium, and L: Low). Here, ($Mean$) and ($Variance$) denote the mean and variance of the prior Beta distribution derived from the expert elicitation, and the ($UpdatedMean$) and ($UpdatedVariance$) denote the mean and variance of the posterior Beta distribution updated from the handbook data. By Bayesian updating, the D-NPT for "Defect detection probability for defects introduced in current phase" node with the handbook data, the posterior hyperparameters of the Beta distribution for each V&V quality and complexity at each SDLC phase were estimated.

B. BAYESIAN UPDATE OF THE BBN D-NPTS USING EVIDENCE FROM ANOMALY REPORT DATA OF NPP SAFETY GRADED SOFTWARE

In addition to the handbook data on U.S. software development experience, limited V&V and testing results available for the development of two application systems, namely (1) the Loop Operating Control System (LOCS) of the Advanced Test Reactor at Idaho National Laboratory [25] and (2) the prototype Integrated Digital Protection System–Reactor Protection System (IDiPS-RPS) developed by KAERI [17], were used for the Bayesian update of the D-NPT of "Defect density" node derived from expert opinions to further reduce BBN parameter uncertainty.

Based on the defect estimates data reported in software development anomaly reports of both systems, the defect density D-NPT in the BBN model was Bayesian updated considering the conjugate prior family of distributions. The defect estimates in the anomaly reports for both applications were assumed to be the number of defects detected in each SDLC phase. Tables 7 and 8 show the defect estimates reported in the IDiPS-RPS and LOCS anomaly reports, respectively. The IDiPS-RPS composed of the software in four main processors in each channel: bistable processor (BP), the coincidence processor (CP), the automatic test and interface processor (ATIP), and the cabinet operator module (COM). In case of the IDiPS-RPS, the number of defects detected at the Test phase was assumed to be the sum of defect estimates reported in the Integration phase and Validation phase. The number of defects in the Installation-and-Checkout phase was not considered since IDiPS-RPS had not yet been installed; thus the "Defect density" node was not updated with IDiPS-RPS data. In the case of defect estimates for LOCS, the number of anomaly reports was assumed to be the number of defects detected at each SDLC phase.

TABLE 7. Defect estimates from the IDIPS-RPS anomaly report [17].

Software type	Defect potentials per function point	Defect removal efficiency
	Requirement	6
CMM 5 + Six-Sigma	Design	16
	Implementation	3
CMM 4	Integration	4
Spiral	Validation (System testing)	4

TABLE 8. LOCS defect estimates based on anomaly report [25].

Phase	Defect estimates
Requirements	1
Design	2
Implementation	2
Test	2
Installation-and-Checkout	2
Total	9

Since the defect estimates in the anomaly reports of IDiPS-RPS and LOCS were assumed to be the number of defects detected in each SDLC phase, the number of defects denoted as x_j in Eq. 10 introduced in j SDLC phase can be as:

$$y_i = x_j * \left(P_{j,H} * V_{j,H} + P_{j,M} * V_{j,M} + P_{j,L} * V_{j,L}\right) \quad (10)$$

where y_i is the defect estimates reported in both systems' anomaly reports, $P_{j,i}$ is the defect detection probability at *i* V&V quality in *j* SDLC phase, and $V_{j,i}$ is the posterior distribution for *i* V&V quality in *j* SDLC phase. From the derived number of defects at each SDLC phase (x_j), the number of defects for each development quality was derived using the posterior distribution of development quality for IDiPS-RPS and LOCS as shown in Eqs. 11-15.

$$x_j = N_{j,H} * D_{j,H} + N_{j,M} * D_{j,M} + N_{j,L} * D_{j,L} \quad (11)$$
$$N_{j,H} = \beta_{j,H,M} * N_{j,M} \quad (12)$$
$$N_{j,L} = \beta_{j,L,M} * N_{j,M} \quad (13)$$
$$\beta_{j,H,M} = \frac{(Updated\ Mean)_{j,H}}{(Updated\ Mean)_{j,M}} \quad (14)$$
$$\beta_{j,L,M} = \frac{(Updated\ Mean)_{j,L}}{(Updated\ Mean)_{j,M}} \quad (15)$$

where x_j is the number of defects in each SDLC phase derived from Eq. 10, $N_{j,i}$ is the mean number of defects at *i* development quality in *j* SDLC phase, and $D_{j,i}$ is the posterior distribution for *i* development quality in *j* SDLC phase. Here, $\beta_{j,H,M}$ is the ratio of the updated mean of High development quality to that of Medium, and $\beta_{j,L,M}$ is the ratio of the updated mean of Low development quality to that of Medium. The defect density, or the number of defects per FP, for each SDLC phase was derived by dividing the number of defects by the number of FPs of the two trial systems: 56 and 41 for IDiPS-RPS and LOCS, respectively.

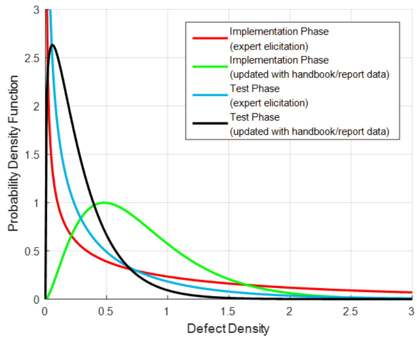
C. BAYESIAN UPDATE RESULT OF THE BBN D-NPTS USING REFERENCE DATA

In this study, both the handbook data and two application systems' anomaly report data were treated as observations used for the Bayesian update of the D-NPTs of "Defect density" node to reduce parameter uncertainty from the diverse expert inputs. As a result of the Bayesian update using reference data, the uncertainty regarding defect density D-NPTs can be decreased considerably over all phases. Particularly, defect density in the SDLC phases where a greater diversity in expert estimations was observed showed a more significant effect from the Bayesian update using both handbook and anomaly data from the two trial systems. Further, the mean of defect density decreased in all phases. As an example, for the Test and Implementation phases, the mean decreased by 27.48% and 48.32%, and the standard deviation decreased by 53.11% and 77.32%, respectively. Fig. 5-(a) shows the updated results for defect density at given High development quality in the Implementation and Test phases.

For the "Defect detection probability for defects introduced in current phase" node, handbook data on the defect removal efficiency of various software applications were used in the Bayesian update to reduce parameter uncertainty. Subsequently, the uncertainty regarding defect detection probability node also decreased over all phases while the mean of defect detection probability was slightly increased. For example, for the Test and Implementation phases, the mean increased by 0.54% and 3.05%, and the standard deviation decreased by 2.52% and 10.30%, respectively.

TABLE 9. BBN model parameters for all medium development and V&V quality.

Phase	Defects introduced in the current phase		Detection probability for defects passed from the previous phase		Detection probability for defects introduced in the current phase		Detected defects passed from the previous phase		Detected defects introduced in the current phase		Defect density		Defects remaining	
	Mean	SD	Mean	SD	Mean	SD	Mean	SD	Mean	SD	Mean	SD	Mean	SD
Requirement	19.71	35.9	-	-	0.79	0.16	-	-	15.64	29.28	0.39	0.72	4.07	10.15
Design	42.61	52.56	0.46	0.26	0.79	0.17	1.86	5.40	33.82	43.23	0.85	1.05	11.00	17.05
Implementation	49.45	56.96	0.48	0.25	0.84	0.15	5.25	9.66	41.49	49.03	0.99	1.14	13.71	17.99
Test	19.88	35.25	0.70	0.16	0.73	0.14	9.61	13.16	14.54	26.42	0.40	0.70	9.45	13.08
Installation/ Checkout	12.63	29.35	0.70	0.19	0.80	0.14	6.64	9.77	10.12	23.88	0.25	0.59	5.32	9.12



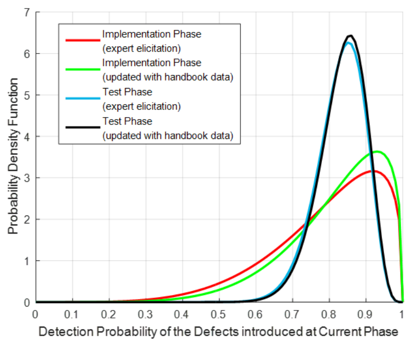


FIGURE 5. Bayesian-updated D-NPTs – (a) "Defect density" D-NPT at given high development quality, and (b) "Detection probability for defects introduced in current phase" D-NPT at given medium complexity and High V&V quality in implementation and test phases.

Fig. 5-(b) shows the updated result of defect detection probability at current phase at given Medium complexity and High V&V quality in the Implementation and Test phases using handbook data on U.S. software development experience.

IV. SENSITIVITY ANALYSIS FOR EXPERT ELICITATION DIVERSITY ON THE BBN PARAMETERS

As previously discussed, different estimations can be given by the experts for BBN parameters due to their different backgrounds and levels of experience in managing and developing safety-related software. This diversity can cause large deviations in the application results as presented in the authors' previous study because all diverse estimations given by experts were considered in quantifying BBN model parameters, such defect density, and defect detection probability. Therefore, demonstrations were undertaken to observe the effects of the elimination of expert estimations that are significantly different from the estimations given by other experts. In this study, sensitivity studies were performed for the BBN parameters, such as the "Attributes," "Defect density," and "Defect detection probability" nodes. The sensitivity analysis results were then compared with the results of the base case where it was assumed that all development quality and V&V quality have Medium quality and the number of FPs is 50. Table 9 shows the evaluation results when the development and V&V quality in all phases have Medium quality for the number of FPs of 50. The model was evaluated using WinBUGS [26], which uses Markov chain Monte Carlo to solve the Bayesian inference problem posed in the model.

A. SENSITIVITY ANALYSIS FOR ATTRIBUTE

The elicited expert opinions on the conditional probabilities of an attribute for a given development quality or V&V quality were estimated by point estimates. Two significantly different estimations were selected by identifying the estimations that were the furthest from the average of the estimations and then excluded from the simulations. In result, slight changes in the mean of both the number of defects introduced and detected were caused by the exclusion of the two most different estimations, as shown in Table 10. The number of defects introduced in the current phase changed by less than 1.83%. The probability of detecting the defects introduced in the current phase and those passed from the previous phase changed by less than 0.70% and 0.64%, respectively. This is because the experts' answers for the

TABLE 10. The number of defects introduced in the current phase and defect detection probabilities following the exclusion of the two most significantly different opinions on the attributes.

Phase	Number of defects introduced in the current phase				Detection probability for defects introduced in the current phase				Detection probability for defects passed from the previous phase			
	Mean	SD	△Mean	△SD	Mean	SD	△Mean	△SD	Mean	SD	△Mean	△SD
Requirement	19.35	34.93	-1.83%	-2.70%	0.80	0.16	0.70%	1.56%	-	-	-	-
Design	42.36	52.30	-0.59%	-0.49%	0.79	0.17	0.38%	-2.18%	0.46	0.26	-0.20%	0.50%
Implementation	50.09	57.44	0.28%	0.84%	0.84	0.15	0.11%	-2.13%	0.48	0.25	-0.31%	0.68%
Test	20.10	35.50	1.11%	0.71%	0.73	0.14	-0.03%	0.43%	0.70	0.16	0.64%	-1.13%
Installation/ Checkout	12.81	29.97	1.43%	2.11%	0.80	0.14	0.46%	-2.00%	0.70	0.19	-0.21%	2.21%

attribute conditional probabilities were relatively consistent, and thus the diverse opinions in attributes did not have much effect on the result.

B. SENSITIVITY ANALYSIS FOR DEFECT DENSITY

The node probability for defect density was given as 5th, 50th, and 95th percentile estimates with significant diversity observed in the expert elicitation. As a sensitivity study, distributions of the smaller variance were selected as the experts' opinions, whose 50th percentile estimation is the furthest from the average of 50th percentile estimation from all experts. Monte Carlo simulation was then used to generate an empirical distribution by sampling from the estimated Normal distributions while excluding the two most different estimations for each specified development quality. Consequently, defect density, having more diverse estimations than other BBN parameters, showed a larger effect from excluding the significantly different estimations. Especially, considerable differences were observed in the Test and Installation-and-Checkout phases. As shown in Table 11, the mean decreased by 21.33% and 59.18%, and the standard deviation decreased by 12.26% and 35.78% in the Test and Installation-and-Checkout phases, respectively.

TABLE 11. LOCS defect estimates based on anomaly report [25].

Phase	Number of defects introduced in each phase			
	Mean	SD	△Mean	△SD
Requirements	20.54	35.27	4.21%	-1.75%
Design	43.45	53.14	1.97%	1.10%
Implementation	51.43	58.09	2.96%	1.98%
Test	15.64	30.93	-21.33%	-12.26%
Installation-and-Checkout	5.155	18.85	-59.18%	-35.78%

C. SENSITIVITY ANALYSIS FOR DETECTION PROBABILITY OF DEFECTS INTRODUCED IN CURRENT PHASE

The experts' opinions for the defect detection probability were given as the estimated 5th, 50th, and 95th percentiles. The same approach of selecting the most significantly different estimations of defect density was used for the sensitivity study on the probability of detecting the defects in the current phase. Monte Carlo simulation was used to generate an empirical distribution by sampling from the estimated Normal distributions while excluding the two most significantly different estimations for each specified V&V quality and complexity. The estimations for the detection probability of the defects introduced in the current phase showed a relatively high consistency. As shown in Table 12, the detection probabilities were changed by −1.13%, 1.85%, −1.42%, 1.33%, and 1.29%, for the Requirements, Design, Implementation, Test, and Installation-and-Checkout phases, respectively.

TABLE 12. Defect detection probability for the defects introduced in the current phase excluding the two most significantly different opinions.

Phase	Detection probability for defects introduced in the current phase			
	Mean	SD	△Mean	△SD
Requirements	0.78	0.17	-1.13%	7.25%
Design	0.80	0.14	1.85%	-15.65%
Implementation	0.83	0.11	-1.42%	-26.53%
Test	0.74	0.07	1.33%	-53.17%
Installation-and-Checkout	0.81	0.08	1.29%	-42.53%

D. SENSITIVITY ANALYSIS FOR DETECTION PROBABILITY FOR DEFECTS PASSED FROM PREVIOUS PHASE

In case of the detection probability of defects passed from the previous phases, significant differences between the expert opinions were observed when compared to those for the defects introduced in the current phase. Similar to the previous sensitivity analysis, Monte Carlo simulation was used to generate an empirical distribution by sampling from the estimated Normal distributions while excluding the two most significantly different estimations for each specified V&V quality and complexity. In result, a larger change was observed by excluding the two most significantly different estimations compared to the case of detection probability of defects introduced in current phase. As depicted in Table 13, the probability of defect detection for defects passed from the previous phase change by 6.17%, 9.35%, −2.40%, and 7.97% for the Design, Implementation, Test, and Installation-and-Checkout phases, respectively.

TABLE 13. Defect detection probability for the defects introduced in the current phase excluding the two most significantly different opinions.

Phase	Detection probability for defects introduced in the current phase			
	Mean	SD	△Mean	△SD
Design	0.49	0.22	6.17%	-15.08%
Implementation	0.52	0.18	9.35%	-29.00%
Test	0.68	0.06	-2.40%	-59.49%
Installation-and-Checkout	0.76	0.07	7.97%	-62.94%

E. SENSITIVITY ANALYSIS FOR IDIPS-RPS APPLICATION

A sensitivity analysis for the IDiPS-RPS application was performed by simultaneously excluding the most significantly different estimations for "Attributes," "Defect density," and "Defect detection probability" nodes mentioned in the previous sections. The results of the analysis are shown in Tables 14 to 17. The defects introduced in each phase changed by 5.28%, 0.53%, 3.58%, −22.24%, and −60.17% for the Requirements, Design, Implementation, Test, and Installation-and-Checkout phases, respectively, as shown in Table 14. Tables 15 and 16 show the defect detection probability for defects introduced in each phase and passed from the previous phase, respectively. While the mean of defect detection probability in both cases changed by less than 10%, the standard deviation was considerably reduced after eliminating the two most divergent estimations. Table 17 shows the number of defects remaining in each phase. The estimated final number of defects in the IDiPS-RPS system decreased from 9.89 to 7.53, and its standard deviation also decreased from 13.19 to 7.92. However, it is notable that due to the proprietary nature of the example software and its development process, the number of defects remaining reported in this study is based on limited information for demonstration purpose only.

TABLE 14. Defects introduced in the current phase of the IDIPS-RPS application excluding two most significantly different opinions.

Phase	Number of defects introduced in each phase			
	Mean	SD	△Mean	△SD
Requirements	22.93	39.53	5.28%	-1.22%
Design	47.56	58.36	0.53%	-0.38%
Implementation	57.30	65.27	3.58%	2.48%
Test	17.45	34.54	-22.24%	-12.84%
Installation-and-Checkout	5.65	21.10	-60.71%	-36.60%

TABLE 15. Detection probability for defects introduced in the current phase of the IDIPS-RPS application excluding two most significantly different opinions.

Phase	Detection probability for defects introduced in the current phase			
	Mean	SD	△Mean	△SD
Requirements	0.61	0.21	1.43%	-12.87%
Design	0.53	0.17	-12.48%	-30.03%
Implementation	0.57	0.18	-5.33%	-27.70%
Test	0.58	0.06	-8.54%	-60.19%
Installation-and-Checkout	0.81	0.08	0.66%	-40.73%

TABLE 16. Detection probability for defects passed from the previous phase of the IDIPS-RPS application excluding two most significantly different opinions.

Phase	Detection probability for defects passed from the previous phase			
	Mean	SD	△Mean	△SD
Requirements	0.24	0.11	9.20%	-17.53%
Design	0.28	0.11	-3.96%	-32.49%
Implementation	0.58	0.06	-8.04%	-64.04%
Test	0.76	0.07	7.99%	-64.25%
Installation-and-Checkout	0.81	0.08	1.29%	-42.53%

TABLE 17. Defects remaining in each phase of the IDIPS-RPS application excluding two most significantly different opinions.

Phase	Number of defects remained in each phase			
	Mean	SD	△Mean	△SD
Requirements	8.96	18.39	3.31%	-6.70%
Design	29.22	33.99	6.56%	-1.16%
Implementation	45.49	40.54	10.01%	-2.67%
Test	26.53	23.36	12.65%	-7.01%
Installation-and-Checkout	7.53	7.92	-24.29%	-40.54%

V. CONCLUSION

Since there are limited operating experience and data of NPP software-related protection systems, one of the challenges of the BBN model development is to systematically integrate the available data required for constructing the BBN nodes and quantifying the NPTs to assess the BBN model. In this study, a framework was proposed which effectively integrates expert opinions and other sources of evidence for NPT development in order to quantify the number of software defects and software reliability with less uncertainty. To adequately accommodate the variability of experts' opinion on the quality of NPP safety-related software when evaluating the BBN model, distribution-based node probabilities were applied in the NPT modeling in the BBN model. Especially, the variety of the node probability values estimated from the experts were aggregated and treated in an integrated manner by deriving the best-fit probability distributions over the experts' estimates on each BBN node.

In addition, in order to reduce the uncertainty caused by the diverse experts' opinions, other sources of evidence such as software development data from literature across many industries and the data for two NPP safety software development projects were used to Bayesian update the D-NPTs related to the defect density and defect detection probability. This study also investigated an outlier treatment to conduct a sensitivity study on the diverse node probability estimates provided by different experts. The outlier-eliminated analysis provided a smaller standard deviation for the number of defects remaining in each phase excluding the most significantly different estimations, which results in less uncertainty of software reliability.

An essential characteristic of the proposed framework includes establishing and quantifying the causal relationships between the software development characteristics, estimating the number of defects remaining, and the software failure probability using expert opinion; probabilistically aggregating multiple expert inputs; and utilizing literature data and available development data to Bayesian update expert inputs to reduce uncertainties introduced from expert opinion. Future research is recommended to reduce the uncertainty regarding the NPTs and to update the BBN model result for software failure probability quantification with less uncertainty. By increasing the number of experts in the elicitation, uncertainty associated with the expert opinions on the BBN parameters is expected to decrease, and so the standard deviation for the number of defects remaining in each phase is expected to decrease considerably. In addition, as more evidence and observations from nuclear safety software operation experiences become available in the future, critical parameters in the NPTs can be Bayesian updated to reduce BBN NPT uncertainties further.

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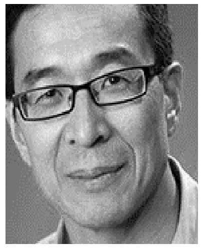

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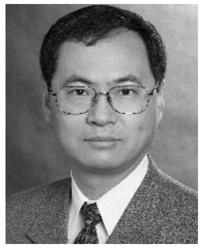

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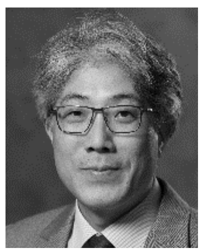

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