Mathematical Modelling Using Technology in Elementary School

N. Mousoulides*, M. Pittalis*, C. Christou*, P. Boytchev **, B. Sriraman*** & D.Pitta*

* Dep. of Education, University of Cyprus, ** Dep. of Informatics, Sofia University, ***Dep. of Mathematics, Montana University
n.mousoulides@ucy.ac.cy, m.pittalis@ucy.ac.cy, edchrist@ucy.ac.cy, pavel@elica.net, sriramanb@mso.umt.edu & dpitta@ucy.ac.cy

Abstract

The purpose of the present study was twofold; first to examine students' mathematization processes as they worked on a mathematical modelling problem and second to investigate students' interactions with *Potters Wheel*, which is a computer software for generating solids by revolution. One group of three students worked for two 40 minute sessions on "Soft Drink Bottle" modelling activity. The activity required students to construct models for a soft drink bottle. The results of the study provide evidence that Potters Wheel assisted students in developing the necessary mathematical constructs and processes to actively engage and solve the problem through meaningful mathematical modelling. Students' mathematical development included creating models for defining appropriate bottles, finding and relating variant and invariant measures such as volume and surface. Finally, implications for further research are discussed.

Keywords: mathematical modelling, software application, visualization

Introduction and Theoretical Framework

During the last three decades, researchers stressed the importance and discussed the role of mathematical modelling and applications in mathematics teaching and learning (Pollak, 1970; Blum & Niss, 1991; Lesh & Doerr, 2003). Henry Pollak (1970) documented that traditional mathematics teaching should move from "Here is a problem; solve it" or "Here is a theorem; prove it", to "Here is a situation; think about it." He also pointed out that there is a strong need to allow students to explore a problematic situation, to set hypotheses and to find out the appropriate tools or theorems they need to use in solving the real world based situation. In recent years, interest in mathematical modelling has increased among mathematics educators, while the National Council of Teachers of Mathematics (NCTM, 2000) recommended that the inclusion of real world based problems in the curriculum can capture students' interest and students will gain both an appreciation of the power of mathematics and some essential mathematical skills. Given the potential value of technology for enhancing learning, students can undertake realistic modelling problems and thereby develop their ideas about and their understanding of related mathematical concepts (Mousoulides et al., 2006).

Mathematical modelling is a non linear process that iteratively involves movement from real world elements to elements from the mathematics world. The first step of mathematical modelling is *understanding* the real world situation by specifying what is needed and by simplifying the problem. The second step is *manipulation* of the problem by identifying variables and relationships in the problem and by mathematizing the problem in building a model. The third step is *prediction* of the behaviour of the real problem by using the constructed model in interpreting the solution to the problem. The fourth step is *verification* of the solution, by evaluating solution, validating and communicating results (Lesh & Doerr, 2003). In the context of modelling activities, the mathematical ideas are embedded within meaningful real world contexts and are elicited by the students as they work in thought-provoking, multifaceted complex problems (Lesh & Doerr, 2003; Lesh & Sriraman, 2006).

An increasing number of researchers have begun focusing their research efforts on mathematical modelling at the school level. Recent research in mathematical modelling indicated that student work with modeling activities assisted students to build on their existing understandings, and to develop important mathematical ideas and processes that students normally would not meet in the traditional school curriculum (English, 2006). English (2006) reported that in her research students' mathematical ideas had improved after they worked in a sequence of modeling activities. More specifically, she reported that modeling activities provided opportunities for elementary school students to explore quantitative relationships, analyze change, and identify, describe, and compare varying rates of change. Quite important parameter is students' use of their informal knowledge in solving a real world problem and the interplay between students' informal and mathematical knowledge. In a number of research paper it is documented that students' informal knowledge helped them relate to and identify the important problem information (e.g., understanding and interpreting the conditions for the solution of a problem) (Mousoulides et al., 2006).

An important parameter that can interfere in students' work in modelling activities is the presence of technological tools. In other words, the availability of technological tools, such as computer software or graphic calculators, might change the way students solve a problem. What is expected is that the use of appropriate tools can enhance students' work and therefore result in better models and solutions. In Blomhøj's (1993) research, a group of 14 year old students were engaged in modeling activities with a specially designed spreadsheet. He reported that students did not find the spreadsheet was a barrier when they were setting up a model. Instead, they often expressed a given relation between variables in the model more easily in spreadsheet notation than in words. In line with previous findings, Christou and his colleagues (2005) reported that the use of a dynamic geometry software in modelling a real world problem assisted students in solving the problem; the use of the software enhanced students in finding alternative solutions, in mathematizing the problem and in verifying their results and documenting their solutions. As a concluding point, the inclusion of appropriate software in modelling activities can provide a pathway in understanding how students approach a real world mathematical task and how their conceptual understanding develops.

The Present Study

Purpose of the Study

The purpose of the present study was to examine students' mathematization processes as they worked on a mathematical modelling problem and to investigate students' interactions with Potters Wheel, a software application, designed for generating solid by revolution.

Participants and Modelling Activity

One group of three 12 year students participated in one modelling activity. All three students had no prior experience in solving problems in a mathematical modelling context. Prior working with the

A famous soft drink company is running a competition!

Your group decided to participate in this competition for designing the best bottle for MarNiko® soft drink. If you want, you can use Potters Wheel application for designing different bottles.

After designing the bottle of your choice, write a letter, explaining and documenting your results, to the president of the competition. Try to convince her that your bottle is the best they could have!

Figure 1: The "Soft Drink Bottle" activity

modelling activity, students had a 15 minute session in getting familiar with the software. The students spent around eighty minutes in completing the modelling activity.

For the purposes of this study, student work on one modelling activity will be presented, namely the "Soft Drink Bottle" activity (see Figure 1). The purpose of the activity was to provide opportunities for students to generate by revolution 3D objects by moving and rotating a simple 2D object and to explore the properties of the constructed 3D objects.

Potter's Wheel Software

The main idea in the *Potter's Wheel* application is that the students have a simple 2D transformable segment passing through two to seven given points. These points can be freely moved according to student's choice. There is an axis on the screen and the object is positioned on the left of the axis. Then the student can use the application to rotate the 2D figure around the axis to produce a 3D rotational image (Figure 2). Finally, as these points are dragged, the volume and surface of the rotational solids can be calculated along with the display of the "algorithm" (Figure 3).

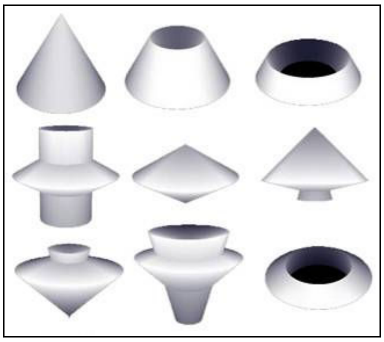


Figure 2: Rotational objects modeled in Potter's Wheel

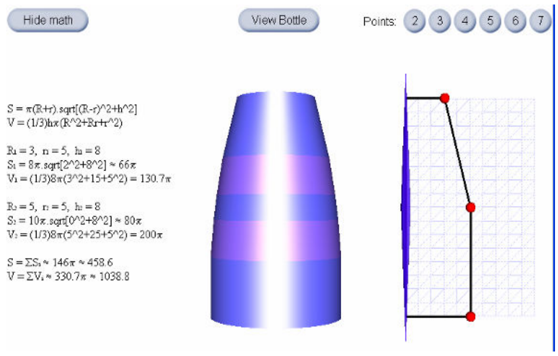


Figure 3: The environment of Potter's Wheel

Data Sources and Analysis

The data for this study were collected through (a) videotape of students' work during the modelling activity, (b) students' files on the computer and students' worksheets detailing the processes used in developing models and solutions, and (c) researchers' field notes. Data were analysed using interpretative techniques (Miles & Huberman, 1994), for evidence of students' interactions with the software and for evidence of students' mathematical developments towards the mathematical concepts appeared in the modelling activity.

Results

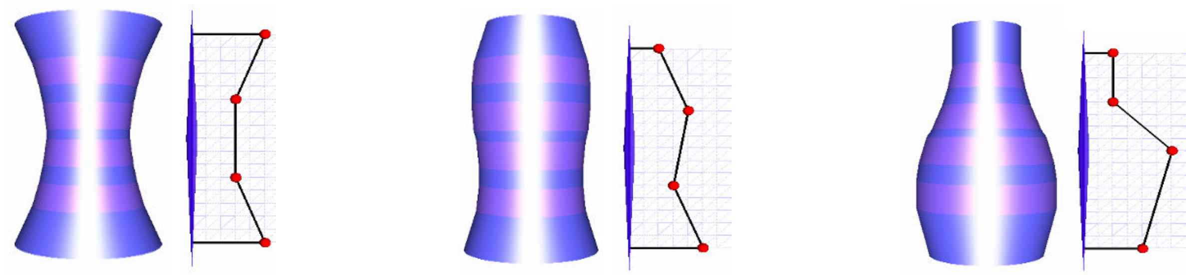
The results of the study focus on the following two dimensions: First, consideration is given to students' interactions with the software, with emphasis on how these interactions assisted students' explorations. Second, the results describe the mathematization processes displayed by students as they worked in the modelling activity. The results are presented in the basis of the two phases that arose from students' work. Students first drew their attempts in designing bottles with no attention to the mathematics part and in the second phase they made modifications in their designs, taking into consideration the surface and volume of the constructed bottle.

Focusing on non mathematical factors

Since in software's initial screen the segment is defined by two points, students' first attempts limited in constructing cylinders and frustum cones. Students commenced that most soft drink cans have cylindrical shape but these shapes are not nice and attractive. During their work, they did not pay any attention to the surface and volume measures. Students' next step was to select the three point segment. After a few explorations, they decided that it would be better to select a four point segment for designing more attractive bottles. The first bottle designed was a cylindrical one. One of the students commented that this bottle is the one used by most of the aluminium cans and it is too common for winning the competition. The same student suggested moving the two middle points closer to the axis and the bottom point to the opposite direction. The new bottle was more attractive. According to students' words, its "rocket" like shape was nice, but not convenient: "How can you hold it?" a girl commented. Students' next experimentations resulted in a number of "sine and curve shaped" bottles. Students commenced that these bottles are practical and attractive, since someone can both hold them easily, directly drink from them and there is enough surface on them for the company's logo. Quite interestingly, students performed a huge number of experimentations. These experimentations were not random. Students moved one point at a time, observing the result in the constructed bottle. After a few experimentations, the software helped them to visualize the expected result from each point movement in the bottle construction!

Beginning mathematization

After reaching a number of possible bottles, a discussion followed. During this discussion, the researchers prompted students to firstly decide the volume of the bottle and then modify the design to fit the volume of the initial cylinder which was 400π cubic units. They decided to work with the three designs they reached earlier, which are illustrated in their final format in Figure 4. They modified their designs to fit the volume, writing in their worksheets bottle's properties. Their initial attempts focused on moving points and observing the changes in bottle's properties. Quite interestingly, at a latter stage, they were hypothesizing what would happen in bottle's properties and more specifically in bottle's surface and volume. When they reached their final three bottles, they wrote in their worksheets the surface and volume measures for each bottle.



(a) (b) (c)

Figure 4: The constructed bottles and the 2D shapes

Since each student was supporting a different bottle, even after writing down the properties' measures, a discussion with researchers took place. During this discussion, researchers encouraged students to think about the different possible factors that the competition board would take into account and the importance of each factor. They all agreed that all bottles were attractive and practical. One of the students commented that the cost is the most important factor for all companies. This comment started a new round of discussion between students to find out the cheapest bottle in terms of aluminium or glass needed. The same student suggested the cost, pointed out that they should consider the surface of each bottle, since the volume was equal. Based on this suggestion, students agreed that the bottle in Figure 4c was the optimal one and the bottle that would cost the most was the one in Figure 4a.

Discussion

An important conclusion of the study is that the participating students were able to work successfully with mathematical modelling activities when presented as meaningful, real world based problems. The learning environment based both on the software as well as on the content of the problem helped students to realize and to get familiar with the activity and thus enhanced their explorations and mathematical understandings (English, 2006). At the same time, the software broadened students' explorations and visualization skills through the process of constructing visual images to analyze the problem, taking into account their informal and visual conception. It was also clear that students identified the structural elements of the problem in developing their final model. The three factors taken into consideration (attractiveness, practical and cost) were employed by students in such a way that they could easily transfer and modify to create successful models for a structurally similar problem (Lesh & Doerr, 2003). An important aspect of students' work is the interaction that took place naturally within the group and the application. The application clearly assisted students to reach models and solutions that they could not probably do with out the use of the software. The application engaged students in analyzing, planning and revising their actions to improve their solutions.

Finally, computer based learning environments for mathematical modeling, at the school level, are a seductive notion in mathematics education. However, further research towards the investigation of their role is needed, to promote both students' conceptual understandings and mathematical developments.

Acknowledgements

This project was co funded by the European Union (grant 224269-CP-1-2005-1-CY-MINERVA-M). The views expressed are those of the authors and do not necessarily reflect the views of the EU or any of its sub-agencies. The authors acknowledge the contribution to the project of all members of the project consortium.

References

Blomhøj, M. (1993). Modelling of dynamical systems at O-level. In J. de Lange, C. Keitel, I. Huntley, & M. Niss (Eds.), *Innovation in mathematics education by modelling and applications* (pp. 257-268). Chichester: Ellis Horwood.

Blum W., & Niss M. (1991). Applied mathematical problem solving, modeling, applications, and links to other subjects: State, trends & issues. *Educational Studies in Mathematics*, *22*, 37-68.

Christou, C., Mousoulides, N., Pittalis, M., Pitta-Pantazi, D. (2005). Problem solving and posing in a dynamic geometry environment. *The Mathematics Enthusiast, A special issue on International Perspectives in Mathematics Teaching using Computer software.*

English, L.D. (2006). Mathematical modeling in the primary school: Children's construction of a consumer guide, *Educational Studies in Mathematics, 63*(3), 303-323.

Lesh, R.A., & Doerr, H.M. (2003). *Beyond Constructivism: A Models and Modeling Perspective on Mathematics Problem Solving, Learning and Teaching.* Hillsdale, NJ: Lawrence Erlbaum.

Lesh, R.A., & Sriraman, B. (2005). Mathematics Education as a design science, *Zentralblatt für Didaktik der Mathematik*, *37*(6), 490-505.

Miles, M. and Huberman, A. (1994). *Qualitative Data Analysis* (2nd Ed.). London: Sage Publications.

Mousoulides, N., Pittalis, M., & Christou, C. (2006). Improving Mathematical Knowledge through Modeling in Elementary School. *Proc. 30th Conf. of the PME* (Vol. 4, pp. 201-208). Prague.

National Council of Teachers of Mathematics: 2000, '*Principles and standards for school mathematics*'. Reston, VA: Author.

Pollak, H. O. (1970). Applications of mathematics. In E. Begle (Ed.) *The sixty-ninth yearbook of the National Society for the Study of Education* (pp. 311-334). Chicago: University of Chicago.

Main Theme: Mathematics Modelling with Technology