Developing an Active Learning Environment for the Learning of Stereometry

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Abstract

This paper reports on the design of a dynamic environment for the learning of stereometry (DALEST) and the teaching of spatial geometry and visual thinking. The development of the software was in the framework of DALEST project which aimed at developing a dynamic three-dimensional geometry microworld that enables students to construct, observe and manipulate geometrical figures in space, and to focus on modelling geometric situations. The environment will also, support teachers in helping their students to construct a suitable understanding of stereometry. During the developmental process of software applications the key elements of spatial ability and visualization were carefully taken into consideration with emphasis on enhancing dynamic visualization as an act of construction of transformations between external media and student's mind.

Keywords: stereometry, spatial sense and reasoning, visualization, computer applications, software design

Introduction and Theoretical Background

Spatial visualization is an important skill that deserves instructional attention. Space is a fundamental category of thought, one that plays a deep role in many aspects of human cognition. Because space is a fundamental feature of our environment, spatial knowledge and spatial perception play crucial roles in even the most ordinary human problem solving (National Science Foundation, 1997). Strong evidence supports the claim that "measures of mathematical ability tend to be strongly correlated with spatial ability". The Principles and Standards of the National Council of Teachers of Mathematics (2000) recommend that 2D and 3D spatial visualization and reasoning are core skills that all students should develop. There is every reason to believe that time spent helping students to develop their spatial visualization skills will have additional benefits to their mathematical growth (Chavez, Reys & Jones, 2005).

A possible way to improve students' 2D and 3D spatial visualization and reasoning abilities is to provide students with possibilities to explore the properties of 3D objects in appropriately developed dynamic and interactive computer applications. Computer-based learning environments continue to be a seductive notion in mathematics education and the promise is that through using particular software in carefully-designed ways, it is possible for learners simultaneously to use and come to understand important aspects of mathematics, something that in other circumstances can be particularly elusive (Jones, 2001). This holds true especially in the field of space and 3D geometry where students are required to inspect, encode, transform, and construct information from visual displays. Since children's initial representations of space are based on action an important feature of such applications is that they should be action-based, i.e. the students should be able to slide and rotate 3D objects creating dynamic images, combine 3D objects in more complex compositions, measure the basic 3D objects and study their properties and relationships and represent and solve problems using geometric models (Boytchev, Chehlarova & Sendova, 2006). When provided with carefully designed sequences of educational activities within such computer applications the students are expected to develop an appreciation of geometry as a means of describing the physical world.

In recent years, much disquiet has been expressed about the limitations of traditional approaches to the teaching and learning of geometry and specifically of spatial abilities that constrain the visualisation and advancement of teaching and learning in geometry. Most current curriculum materials and computer-based systems address the investigation of 3D geometry by requiring students to operate within two dimensional representations of 3D geometry which is not a natural way of investigating human 3D spatial ability. To overcome the limitations of these representations the *DALEST* project aimed at developing a dynamic three-dimensional geometry microworld that enables students to construct, observe and manipulate geometrical figures in space, and to focus on modelling geometric situations, and supports teachers in helping their students to construct a suitable understanding of stereometry. In this paper we focus on presenting applications' design and capabilities and possible teaching potential with emphasis on dynamic visualization. In the theoretical background below, we delineate fundamental theoretical spatial ability considerations upon which the design of the software applications were based and in the following sections provide a description of the capabilities of the software for the teaching of spatial geometry.

The aim of developing the *Dalest* software was to enhance the abilities and processes in students that are closely associated with the idea of visual imagery as a mental scheme depicting spatial information (Presmeg, 1986). It is generally accepted that learning 3D geometry is strongly associated with spatial and visual ability. From this perspective, spatial abilities are important cognitive factors in learning geometry and incorporating spatial visualization and manipulation into learning activity could improve geometric learning. As a result, mathematics curricula in a range of countries (see, as an example, the National Council of Teachers of Mathematics, 2000) emphasize the importance of spatial abilities in mathematics education and recommend that 2D and 3D spatial visualization and reasoning are core skills that all students should develop.

A 3D dynamic geometry software should provide to the learner a variety and richness of spatial images. The richer and more diverse the store of spatial representations, the easier is to use images in solving problems. Dynamic visualization can be a very powerful tool to gain a greater understanding of many mathematical concepts or it can be a resource to solve mathematical problems (Harel & Sowder, 1998; Presmeg, 1986). Taking into consideration the gap-bridging power of computers which has been extensively reported on in the research literature, we claim that dynamic representations of mathematical processes may enable "the mind to manipulate them in a far more fruitful way than could ever be achieved starting from static text and pictures in a book". Dynamic spatial abilities are those that are required to reason about moving stimuli. The advent of computer testing has led to renewed interest in the ability to reason about motion and the ability to integrate spatial information over time (Hegarty & Waller, 2005). Dynamic visualization is an act of construction of transformations between external media and individual's mind. Individuals who possess this ability can reason about the essential properties of moving, shrinking, and rotating figures, which appear on the screen or, in their mind, and thus they can solve the mathematical problems (Boz, 2005). There are research evidences too, which show that some students spontaneously develop this ability to move figures in their heads, to stretch and shrink them, to rotate them, to see them interact, and hence solve problems by using this kind of technique (Harel & Sowder, 1998).

Dalest Software Design

In the framework of DALEST project several software applications have been developed aiming at enhancing middle school students' 3D geometry understanding and spatial visualization skills by utilizing dynamic visualization images. In this study we present the general characteristics of the stereometry learning environment and describe the application called Cubix Editor.

General Characteristics of Stereometry Learning Environment: The distinguished feature implemented in the software is the ability to construct geometrical objects in successive steps that correspond to their mathematical properties. For example, a cuboid is constructed is three successive movements, defining in each movement the length of one of the bone edges of the solid (see figure 1). The manual construction of solids gives to the user and intuitive conception of the 3D nature of the solid. The geometrical objects created on the screen can be manipulated, moved and reshaped interactively by means of the mouse. The realisation of the software allows students to see a geometric solid represented in several possible ways on the screen and to transform it, the intention here being to help students to acquire and develop abilities of visualization in the context of 3D geometry. As Gutiérrez (1996) argues, when a person handles a real 3-dimensional solid and rotates it, the rotations made with the hands are so fast and unconscious that the person can hardly reflect on such actions. The design of the software facilitates the informed manipulation of directions of rotation.

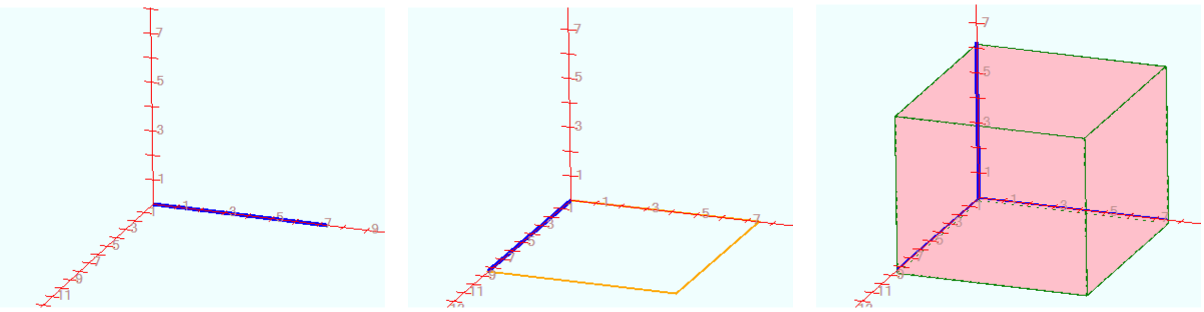


Figure 1: The construction of a cuboid in three successive steps

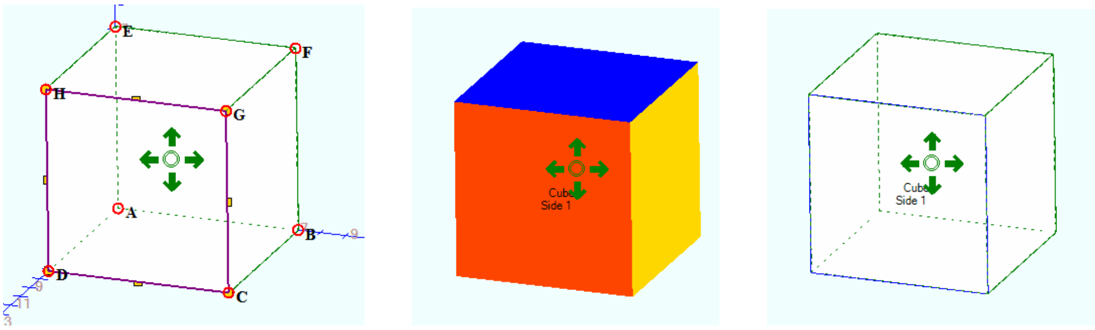
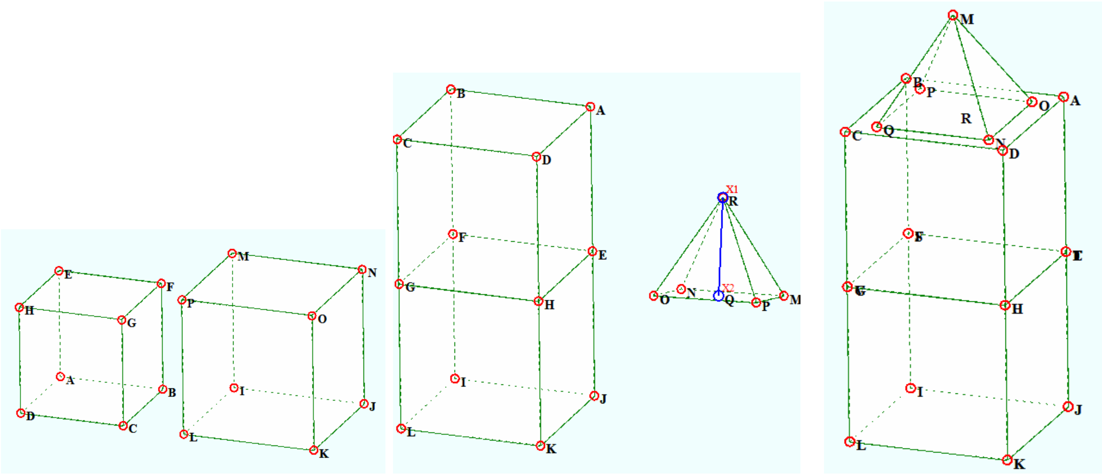


Figure 2: Different representations of a cube

The design of the software focuses on the processes of observation and exploration in the following ways: Observation allows students to see and understand the third dimension by changing the spatial system of reference (axes), choosing perspectives and displaying visual feedback on objects. Students can rotate a geometric object in reference to the three axes and thus gain a holistic view of the object. The software is designed such that the drawing style of any object can be in a ‘solid colour’ view or in a ‘transparent line’ view, as illustrated in Figure 2, and students can select, label and colour the edges and faces of the objects. Exploration allows students to explore and discover the geometrical properties of a figure. This is the main procedure adopted in most of the teaching scenarios that are available to accompany the software. For example, users can take advantage of the grouping wizard of the software to explore how different solids can be grouped to form another solid. As illustrated in Figure 3a the user can group the two cubes to construct the cuboid shown in Figure 3b. In the grouping procedure, the user can select the faces of the cubes that will coincide. In the case that the faces do not have the same dimensions the user has the following options: (a) to resize the first object, (b) to resize the second object, and (c) to group the objects as is, as illustrated in Figure 3c (the two solids in Figure 3b were grouped as is).



(a) (b) (c)

Figure 3: Grouping of cubes and cuboids

The dragging capability of the software enables students to rotate, move and resize 3D objects in much the same way as the commonly available 2D dynamic geometry software environments. The design approach focused on enabling rotation to be executed in all directions through the provision of an on-screen rotation cursor that could also be used to determine the speed of the rotation. In addition, the design was made such that students are able to resize, proportionally, all the dimensions of the object or resize it only in one dimension, according to the requirements of the problem with the use of hot points. For example, students can resize a cone, as illustrated in Figure 4, by shrinking or enlarging the radius BC (by dragging the hot point B), or by shrinking or enlarging the height AC (by dragging the hot point A). The dynamic characteristic of the measurement facility allows learners dynamically to explore the properties within and amongst figures; for example, students can obtain the measure of the volume of the cone and then double its height and see how its volume is altered.

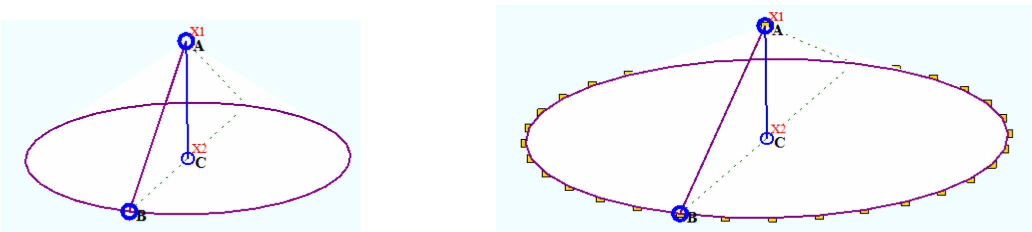


Figure 4: Resizing the dimensions of a cone

Cubix Editor: An editor which can be used by students to create 3D structures built of unit-sized cubes. The Cubix Editor allows the students to save their structures and reload them, thus making a library of various figures (see Figure 5). The user can also recolor the cubes and get measures for the volume and surface area of the solids constructed. A very useful characteristic of the application is the rotation of the whole platform of the construction, procedure which creates dynamic images and gives to the user the capability to develop dynamic visualizations of the front, side and top view of the object. The dragging capability of the application is a valuable tool that can be used to develop didactical situations enhancing visual abilities, such as “Perceptual constancy” and “Mental rotation” by producing dynamic mental images and visualizing a solid in movement, and investigating mathematical properties. As illustrated in Figure 5, student’s visual abilities can be enhanced when are challenged to change the place of one of the unit cubes in Figure 5(a) so that the new construction could take the position of Figure 5(b) in the space.

(a) (b)

Figure 5: Two compositions of unit-size cubes as part of a logical 3D problem

Conclusion

The design of the DALEST applications is in the process of development. As presented earlier, the main purpose of these applications is to enhance students' understanding 3-D geometrical reasoning and visualization and spatial thinking. Thus, during the developmental process the key elements of visualization as defined by several researchers are carefully taken into consideration. We expect that the final library of DALEST applications will constitute a powerful tool in the teaching of stereometry and spatial geometry. We also expect that applications will enhance students' dynamic visualization ability and enable them to gain a greater understanding of 3D mathematical and spatial concepts.

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Main theme: **Designing and using Dynamic Mathematics environments**